

A probabilistic framework for quantification of aftershock ground-motion hazard in California: Methodology and parametric study

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SUMMARY

This paper presents a proposed method of aftershock probabilistic seismic hazard analysis (APSHA) similar to conventional ‘mainshock’ PSHA in that it estimates the likelihoods of ground motion intensity (in terms of peak ground accelerations, spectral accelerations or other ground motion intensity measures) due to aftershocks following a mainshock occurrence. This proposed methodology differs from the conventional mainshock PSHA in that mainshock occurrence rates remain constant for a conventional (homogeneous Poisson) earthquake occurrence model, whereas aftershock occurrence rates decrease with increased elapsed time from the initial occurrence of the mainshock. In addition, the aftershock ground motion hazard at a site depends on the magnitude and location of the causative mainshock, and the location of aftershocks is limited to an aftershock zone, which is also dependent on the location and magnitude of the initial mainshock. APSHA is useful for post-earthquake safety evaluation where there is a need to quantify the rates of occurrence of ground motions caused by aftershocks following the initial rupture. This knowledge will permit, for example, more informed decisions to be made for building tagging and entry of damaged buildings for rescue, repair or normal occupancy. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In conventional probabilistic seismic hazard analysis (PSHA), the annual rate of exceedance of a given ground motion intensity measure (typically peak ground acceleration (PGA) or first-mode spectral acceleration) is evaluated for a given structural period and damping ratio. The mainshock

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occurrence rates are assumed to be time-invariant constant values, and a homogeneous Poisson earthquake occurrence model is generally used. Conventional mainshock PSHA, typically based on a truncated exponential magnitude model or a characteristic magnitude model or both, is an established process that is well documented in the literature (e.g., [1]). Mainshock PSHA is performed first by considering the geometry of the neighboring faults with respect to the site of concern as well as the annual rates of earthquakes on these faults. The basic formulation of PSHA requires the development of an earthquake model to address the randomness of the times of occurrences as well as the randomness of the earthquake magnitudes. The randomness of the distance from the site to the possible rupture planes and the distribution of possible ground motions, given an earthquake of a particular magnitude and distance, are also addressed in standard mainshock PSHA procedures.

In the post-mainshock situation, however, aftershock occurrence rates are not time-invariant. Aftershock rates are at their maximum immediately following the occurrence of the mainshock, and they decrease with increasing elapsed time from the mainshock. Aftershock rates are dependent on the magnitude of the causative mainshock. The subsequent aftershock ground motion hazard at a site is also dependent on the location and magnitude of the initial mainshock, which defines the aftershock zone in which aftershocks are expected to occur. In this paper, we propose a method for calculating the post-mainshock ground motion hazard at a site, which we refer to as aftershock probabilistic seismic hazard analysis (APSHA), where we take the above factors into consideration. This method enables us to estimate the likelihood of severe ground motions at a site due to aftershocks.

There are several examples of studies where the results of APSHA would have proved useful.

Gallagher *et al.* [2] presented an ATC technical brief titled ‘Earthquake Aftershocks—Entering Damaged Buildings’, which provided guidelines on the safety evaluation of earthquake-damaged buildings as well as conditions for permitting entry into such buildings that may be structurally unsound. These guidelines are purely based on the aftershock earthquake occurrence rates obtained for California in [3, 4] as a function of magnitude and time. However, the hazard at the site of concern cannot be adequately described without the explicit consideration of the location of the aftershock hazard zone relative to the site and the possible ground motion intensities at the site. Both these factors are considered in the proposed method.

FEMA, in conjunction with the SAC steel project, has published a report titled ‘Recommended Post-earthquake Evaluation and Repair Criteria for Welded Steel Moment-frame Buildings’. FEMA [5] provides guidelines on post-earthquake occupancy criteria for damaged structures based on their ability to resist collapse after the mainshock. In the report, a ‘presumed post-earthquake hazard curve’ for a mainshock-damaged building is presented based on the annual rate of exceedance of the first-mode spectral acceleration in an aftershock, assuming simply that the probable maximum intensity value for aftershock-induced ground shaking at the building site is the same as that experienced in the original damaging mainshock and that the variability in this intensity is normally distributed with a coefficient of variation of 50%. A more precise methodology to quantify the aftershock ground motion hazard is required.

The proposed method of APSHA is similar to the conventional mainshock PSHA in terms of the treatment of uncertainty in magnitude, source-to-site distance and ground motion intensity. The detailed description of conventional PSHA will not be discussed in this paper. The reader is presumed to be familiar with conventional PSHA as described, for example, in [1]. We will explicitly incorporate the unique characteristics of aftershock ground motion hazard mentioned earlier in the formulation of APSHA.

The time-varying nature of aftershock ground motion hazard has also been studied recently in [6, 7]. Their focus has been on generating real-time aftershock PGA maps, in which the parameters of the modified Omori law are updated periodically via Bayesian analysis to reflect the on-going aftershock sequence as the parameters and the aftershock zone may be sequence dependent. Availability of such a real-time system would facilitate the use of aftershock ground motion hazard in making engineering decisions. Such aftershock maps are being developed under the OpenSHA program in the U.S. Geological Survey (USGS). (See [8].)

Thus, APSHA provides us with the aftershock hazard curve that can be compared with the mainshock hazard curve obtained from conventional mainshock PSHA. The aftershock hazard at a site allows accurate quantification of the likelihood of severe ground motions, which enables building owners and stakeholders to make better-informed building-tagging and evacuation decisions. In order to gain insight and familiarity with the nature and level of aftershock ground motion hazard, in the example to follow, we shall study the effects of elapsed time, rupture length, site location, duration and mainshock magnitude on aftershock ground motion hazard. We shall also study the sensitivity of aftershock hazard to changes in the period of the structure.

2. METHODOLOGY

It has been generally observed that several earthquakes occur as a cluster within a limited period of time and confined to a limited interval in space. For the purpose of this study, we assume that we can classify all earthquakes as foreshocks, mainshocks and aftershocks. The mainshock is generally defined to be the earthquake in a sequence, which has the largest magnitude. The foreshocks are usually very limited in number. Our focus in this paper is on the aftershock sequence. For the purpose of this study, we are interested in the aftershocks in the days and perhaps months following the occurrence of a mainshock, and the resulting increased ground motion hazard at the site.

Immediately following the occurrence of a mainshock, the rate of occurrences of aftershocks is at its maximum, and then decreases with increasing elapsed time from the occurrence of the mainshock. This decay is of a power-law form, which is generally referred to as the modified Omori law in which the aftershock rate per day is expressed as a function of $(t+c)^p$, where t is the elapsed time from the occurrence of the mainshock and c and p are constant values for a particular aftershock sequence. See [9, 10]. Because of the time-varying rate of aftershocks immediately following a mainshock, a nonhomogeneous Poisson process is used to model aftershock occurrences.

The modified Omori law has been used by a number of researchers to model aftershock rates immediately after the occurrence of a mainshock. For example, Reasenberg and Jones [3, 4] fitted 62 California aftershock sequences with mainshock magnitudes m_m greater than 5.0 to the modified Omori law using the method of maximum likelihood. The mean aftershock rates are described by the modified Omori law (elapsed-time dependence) and by the Gutenberg-Richter relationship (aftershock magnitude probability distribution). They refer to this model as the ‘generic California’ aftershock model. Here, $\gamma(t, m; m_m)$, the mean daily rate of aftershocks with moment magnitude m or larger at time t following a mainshock of moment magnitude m_m , can be calculated using Equation (1).

$$\gamma(t, m; m_m) = \frac{10^{a+b(m_m-m)}}{(t+c)^p} \quad (1)$$

The generic California aftershock sequence has parameters $a = -1.67$, $b = 0.91$, $p = 1.08$ and $c = 0.05$. Reasenberg and Jones [3,4] used all aftershocks with moment magnitude $m \geq m_m - 3$ to estimate a and p , and $m \geq 2$ to estimate b . Note that Reasenberg and Jones [3,4] studied only aftershock occurrence rates without explicit consideration of the resulting ground motion intensities.

The b -value describes the relative likelihood of large and small mainshocks or aftershocks. For example, USGS [11] obtained a b -value of 0.91 for mainshocks in the San Francisco Bay Region. The generic aftershock sequence for California studied by Reasenberg and Jones also has a b -value of 0.91. Thus, because b -values are usually quite similar in value, the relative likelihood of large and small aftershocks is approximately the same as the relative likelihood of large and small mainshocks. In the example to follow, we assume that the b -values for mainshocks and aftershocks are both equal to 0.91 for a specific site in California. In general, the b -values for mainshocks and aftershocks may be different and should be taken into consideration for ground-motion hazard analysis.

If we define $a_0 = a + bm_m$, $\alpha = a_0 \ln 10$ and $\beta = b \ln 10$, Equation (1) simplifies to the following equation:

$$\gamma(t, m; m_m) = \frac{e^{\alpha - \beta m}}{(t + c)^p} \quad (2)$$

$|\gamma_m(t, m; m_m)|$, defined in the following equation, is the absolute value of the partial derivative of $\gamma(t, m; m_m)$ in Equation (2) with respect to m , and it is the mean daily rate density of aftershocks of magnitude m at time t following a mainshock of magnitude m_m :

$$|\gamma_m(t, m; m_m)| = \left| \frac{\partial \gamma(t, m; m_m)}{\partial m} \right| = \frac{e^{\alpha}}{(t + c)^p} \beta e^{-\beta m} = \mu(t; m_m) f_M(m) \quad (3)$$

In this representation of the generic California model, $\mu(t; m_m) = e^{\alpha} / (t + c)^p$ is the mean daily rate of aftershocks at time t following the mainshock of magnitude m_m , and $f_M(m) = \beta e^{-\beta m}$ is the exponential probability density function of aftershock magnitudes. Here, $m \geq 2$ because Reasenberg and Jones used aftershocks with moment magnitude of 2.0 or more to estimate the value of β in the modified Omori law of the generic California model.

A more useful representation of $|\gamma_m(t, m; m_m)|$ will require us to introduce lower and upper bound aftershock magnitudes of m_l and m_u to the exponential probability density function in Equation (3). m_l represents the minimum aftershock magnitude of engineering interest, typically taken to be 5.0. m_u represents the upper bound magnitude of the fault in consideration. For the case of aftershocks, following [3], m_u is typically considered to be the mainshock magnitude, m_m . Truncating the exponential probability density function at m_l and m_u , and taking $m_u = m_m$, we obtain the following equation:

$$|\gamma_m(t, m; m_m)| = \frac{10^{a+b(m_m-m_l)} - 10^a}{(t + c)^p} \frac{\beta e^{-\beta(m-m_l)}}{1 - e^{-\beta(m_m-m_l)}}, \quad m_l \leq m \leq m_m \quad (4)$$

Similar to Equation (3), we can also represent $|\gamma_m(t, m; m_m)|$ using Equation (5) where $\mu(t; m_m)$ is the mean daily rate density of aftershocks with magnitudes between m_l and m_m at time t following a mainshock of magnitude m_m , and is equal to $(10^{a+b(m_m-m_l)} - 10^a) / (t + c)^p$. The

function, $f_M(m; m_m) = \beta e^{-\beta(m-m_1)} / (1 - e^{-\beta(m_m-m_1)})$, is the truncated exponential probability density function of aftershock magnitudes, with truncations at m_1 and m_m :

$$|\gamma_m(t, m; m_m)| = \mu(t; m_m) f_M(m; m_m), \quad m_1 \leq m \leq m_m \quad (5)$$

The mean number of aftershocks with magnitudes between m_1 and m_m in the time interval $[t, t+T]$ following a mainshock of magnitude m_m (denoted as $\mu^*(t, T; m_m)$) can be found using the following equation:

$$\mu^*(t, T; m_m) = \int_t^{t+T} \mu(\tau; m_m) d\tau = \frac{10^{a+b(m_m-m_1)} - 10^a}{p-1} [(t+c)^{1-p} - (t+T+c)^{1-p}] \quad (6)$$

For seismic hazard analysis, we first define the site ground motion intensity measure as Y . In conventional mainshock PSHA, the rate of mainshocks is a constant value in time. If we denote the mean annual rate of mainshocks with magnitudes between m_1 and m_u by v , we can compute the mean number of mainshocks exceeding site ground motion y in duration T with source-to-site distance R , denoted as $\tilde{v}(y, T)$, using the following equation:

$$\tilde{v}(y, T) = vT \int_R \int_{m_1}^{m_u} P(Y > y|m, r) f_{R|M}(r|m) f_M(m) dm dr \quad (7)$$

T is usually one year in mainshock PSHA such that in Equation (7), the term T does not appear. In Equation (7), $f_M(m)$ is the probability density function of mainshock magnitudes (with upper and lower magnitude bounds at m_1 and m_u). $f_{R|M}(r|m)$ is the conditional probability density function of the closest distance between the site and the plane of rupture given an event of a particular magnitude. Lastly, $P(Y > y|m, r)$ is the conditional probability of exceeding a particular ground motion intensity measure given an event of a particular magnitude at a particular distance from the site. This conditional probability is usually evaluated assuming that the distribution of the ground motion intensity measure given M and R is lognormal, where $E(\ln Y)$ and $\sigma_{\ln(Y)}$ are obtained from a suitable attenuation law.

In Equation (7), we have assumed that only one source contributes to the mainshock hazard at the site. It is straightforward to extend the formulation to include the contributions from multiple sources by summing the contributions to the total hazard from all such sources. We also need to assume a spatial distribution of earthquakes of random magnitudes up to m_u , coupled with a suitable attenuation law for the ground motion intensity measure of interest.

Analogously, in APSHA, we want to calculate the mean number of aftershocks in $[t, t+T]$ exceeding site ground motion y , where there is explicit dependence on both t and T . We represent this mean number as $\tilde{\mu}(y, t, T; m_m)$, which can be calculated using the following equation:

$$\tilde{\mu}(y, t, T; m_m) = \mu^*(t, T; m_m) \int_R \int_{m_1}^{m_m} P(Y > y|m, r) f_{R|M}(r|m) f_M(m; m_m) dm dr \quad (8)$$

Note that the formulation for APSHA is almost identical to that of conventional PSHA, except that $\tilde{v}(y, T)$ is replaced by $\tilde{\mu}(y, t, T; m_m)$, which is a function of t , T and m_m .

Provided Equation (5) holds, Equation (8) is quite general. In practice, the aftershock zone is defined in a complex manner. It has also been observed that aftershocks tend to occur more frequently at the ends of the rupture zones of the initial mainshock. See [12] for details. Such complexity in the definition of the aftershock zone can be considered in APSHA, by adopting an appropriate distribution $f_{R|M}(r|m)$, which can be estimated by considering discrete aftershock