Theoretical naval architecture

Samuel James Pope Thearle

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Putnam's Advanced Science Series.

THEORETICAL NAVAL ARCHITECTURE:

A TREATISE

ON THE

CALCULATIONS INVOLVED IN NAVAL DESIGN.

BY

SAMUEL J. P. THEARLE,

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VOL. I.—TEXT.



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50 ft., $MG = 177{\cdot}41$ ft.,
3 ft., and $LS = 77$ ft.
forward a distance of
0 ft.

$\frac{77}{180}$ ft. $= 5\frac{3}{17}$ in.

$\frac{73}{180}$ ft. $= 4\frac{13}{17}$ in.

$4\frac{13}{17} = 10$ in.

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12 0 aft.

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$-4\frac{13}{17} = 11 \quad 7\frac{4}{17}$ aft.

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linary displacement sheet
trim one inch, the method
the investigation just gone

$= \frac{1}{1\,2}$ foot.

$$\frac{MG \times D}{P \times d}$$

$$\frac{\times D}{WL}$$

ent sheet (Table I.)

$= 61{\cdot}1$ foot-tons.

alter the trim of this ship
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PREFACE.

THE favourable reception accorded to the Treatise on *Practical Shipbuilding* and *Laying Off*, which I had the honour of contributing to this Series three years ago, has encouraged me to venture upon the present work. The literature of Naval Architecture in the English language is not at all commensurate with the importance and magnitude of the shipbuilding industries of Great Britain, and that literature which exists is practically beyond the reach of most of those who desire and need acquaintance with it. Without particularising the special defects in this respect of existing books on the *Theory of Naval Architecture*, it may be said that either they are too large and costly, or written too obscurely to place them within the pecuniary or mental grasp of the majority of students, draughtsmen, and workmen.

In preparing the present work, it has been my object to adapt it for the student who possesses simply a knowledge of elementary mathematics, and at the same time to provide for the requirements of those whose studies have been much more advanced. The former will find all the information necessary to enable him to perform the usual calculations of the drawing office, expressed in formulæ, or by rules easily applied; while the latter will be enabled, by following out the investigations by which these formulæ and rules have

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been obtained, to gather a tolerably clear idea of the principles upon which they are based.

In offering this work to Naval Architects, it is hardly necessary for me to say that I have been indebted to those who have written before me for a large proportion of the ideas which it contains. It would, indeed, be impossible to write usefully upon a science which has been brought to its present state by the contributions of so many minds without being under such obligations.

An effort has been made to simplify the problems involved in calculating the stability of ships by separating the calculations relating to the form of the vessel from those relating to her weight, and the position of her centre of gravity. A few alterations have been made in the usual nomenclature of the subject with a view to greater exactitude of expression. Some of these alterations were suggested by a recent writer in *Naval Science*. To what extent success, or the contrary, has attended my attempts to secure those very desirable objects—simplicity and accuracy, it will be for the reader to decide. At all events, it is hoped that a work based upon such a plan as the author himself would have desired when a student will be of service to the young naval architect by simplifying his studies, and to the draughtsman and shipbuilder by helping them in their daily duties.

S. J. P. THEARLE.

LONDON, *February 1877*.

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THEORETICAL NAVAL ARCHITECTURE.

INTRODUCTION.

THE work of the Naval Architect is to design a floating structure, stable, seaworthy, and strong; capable of being propelled with facility either by the wind, by steam, or some other motive power, and adapted for the particular service in which it is to be employed, whether as a fighting ship or for carrying merchandise or passengers. In every case certain qualities, viz., stability, sea-worthiness, strength, speed, stowage, and accommodation, have to be fulfilled in a varying ratio according to the purpose for which the ship is intended. In a ship of war certain other qualities have to be obtained, such as power of carrying and fighting guns, and sometimes invulnerability; these too being provided in different ratios of importance according to the duties the ship has specially to perform. It is the art of securing these qualities, and apportioning them in the several degrees necessary to secure the maximum efficiency of the ship in her peculiar vocation, that constitutes the functions of the Naval Architect, and renders his duties among the most difficult and onerous that fall into the hands of man to perform; requiring, as they do, a knowledge of some of the most complex and abstruse physical laws, and a wise discrimination in applying experimental data.

The development of the science of Naval Construction has been very slow until within the last century, notwithstanding that Naval Architecture has a history dating back almost as far as that of land architecture. Indeed, until the application of iron to shipbuilding, no considerable improvement

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could be observed in the mode of constructing ships during several centuries. The advances that have been made during the past twenty years, especially in regard to war ships, are far greater than can be traced during at least one hundred and fifty years previously. But, while the growth in the mechanical or building branches of the art have been most important, resulting, as they have, from the rapid development of new resources, and the bringing of powerful forces of nature more under our control, it is to the enlargement of our knowledge in the scientific—or, as it is termed, the theoretical department—that we should attribute a large share of the credit for the unparalleled rapidity of our advances; as by it we have been guided safely, in the wide departures from the old lines of experience and routine, into modes of construction which have been rendered necessary by the altered circumstances of the present day.

It is to the French that we are indebted for the first definite and consistent application of scientific truths to Naval Construction; and, until within recent years, the principal text-books on naval science were written by Frenchmen. The superiority over our own ships in point of sailing and sea-going qualities displayed by ships of war which we had captured from the French, during the wars of the last century, caused our own naval architects to frequently build upon the French models; and this continued until, in the year 1811, we began, in earnest, the scientific study of Naval Architecture. From that time, until the present day, England has not been without highly trained men, into whose hands she could confide the responsible task of providing her with trustworthy and suitable ships for the several departments of her warlike and mercantile navies.

It has already been stated that a ship should fulfil a number of conditions, some of these being more prominent than others; in each case prominence being given to those qualities which are essential for the service the ship is especially intended to perform. We purpose in the following pages to discuss these several qualities, and show how the possession of each is to be obtained. This discussion will necessarily involve statements and explanations of scientific truths and

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mathematical formulæ, also illustrations of calculations which have to be made in the application of those truths and formulæ.

In considering the *Theoretical Principles of Naval Architecture*, we propose to treat the several branches of the inquiry in the following order:—

1. Calculations relating to the forms and dimensions of ships.
2. Calculations relating to the weights and centres of gravity of ships.
3. Calculations relating to the strengths of ships.
4. Calculations relating to the propulsion of ships by sails.
5. Calculations relating to the propulsion of ships by steam engines.
6. Calculations relating to the steering of ships.

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PART I.

CALCULATIONS RELATING TO THE FORMS AND DIMENSIONS OF SHIPS.

CHAPTER I.

CALCULATION OF AREAS AND VOLUMES.

Buoyancy — Stability — Hydrostatical Principles — Displacement — Areas of Plane Surfaces—Trapezoidal Rule—Simpson's 1st Rule: Proofs—Fractional Intervals—Simpson's 2nd Rule—One-twelfth Rule—Worked Examples—Centre of Gravity of a Plane Area—Centres of Gravity of various Figures—Principle of Moments—Centre of Gravity of a Plane Area bounded by a Curve—Volumes of Solids—Volume of a Solid bounded by a Plane and a Curved Surface — Calculation of Displacement — Centres of Gravity of Solids—Centre of Gravity of a Volume bounded by a Curved Surface and a Plane—Curve of Areas of Midship Section—Of Sectional Areas—Of Tons per inch of Immersion—Of Displacement — Explanation of a Displacement Sheet — Geometrical Method — Woolley's Rule — Proof — Centre of Buoyancy by Woolley's Rule—Geometrical Application of Woolley's Rule—Angular Measurement of Areas—Volumes and Moments of Wedge-shaped Solids — Moments of Inertia — Examples—Co-efficients of Fineness—Table of Co-efficients of Fineness.

1. The essential qualities sought for in a ship are that she shall be able to float and carry a cargo of some kind with safety; and that, too, at a certain speed when under the influence of a propelling force. In order to do this, it is requisite that she shall possess *buoyancy*, *stability*, and *strength;* also that she shall be of such a form as to move through the water without an undue expenditure of force. We purpose, considering in this and the next division of the subject, the two first of these qualities; and leave the discussion of *strength* and *speed* for succeeding chapters.

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2. Buoyancy is that quality whereby a ship, or any other floating body, is enabled to support a certain weight: in the case of a ship, it is necessary that that weight should be carried without her sinking too deeply in the water, or floating too lightly on it.

3. Stability is that quality, governed both by the form of the ship, and the positions of the weights carried, whereby, when she is inclined out of the upright position, she immediately seeks to recover herself; and, in passing, it may be remarked that this amount of stability should be sufficient to prevent her from heeling, whether under the influence of the wind or waves, to such an extent as to endanger her safety, or produce inconvenience; and, at the same time, should not be so great as to produce abruptness in her movements sufficient to strain or damage the structure or its contents. The influence of the positions of the weights carried upon the stability of a ship will be considered in Part II.; for the present, we propose to confine ourselves to a discussion of the influence of a ship's *form* upon her buoyancy and stability.

4. Hydrostatical Principles.—A body floating in a fluid, in a state of equilibrium, *i.e.*, with no disposition to sink deeper, or rise higher with regard to the surface of the fluid, displaces a volume of that fluid exactly equal to its own weight. For instance, suppose a cubical block of wood, each of whose edges is one foot long, when placed in fresh water to sink to a depth of six inches, so that exactly one half of its volume is immersed, and then to float in a state of repose; we at once conclude that, bulk for bulk, the wood weighs just half as much as the water. In other words, taking the weight of a cubic foot of water at 1000 ounces, the cubic foot of wood weighs 500 ounces; for that is the weight of the half of a cubic foot of water that would fill the space occupied by the portion of the block of wood that is immersed.* Should the block of wood sink until the surface

* As a fact, it should be here observed that a cubical block of wood, which weighs a half that of an equal volume of water, will not float in equilibrium in the manner here supposed, with one of its sides horizontal, but with one of its angular points upwards. This can be verified by experiment.

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of its uppermost face is level with that of the water, we conclude that, bulk for bulk, the wood and water weigh the same.

This may be proved experimentally in the following manner: Fill a bowl with water, and place it in a dish capable of containing water. Then gently place any body, that will float, upon the surface of the water. In sinking to its proper depth of immersion, it will displace a certain quantity of the fluid, which will flow over the edge of the bowl into the dish. When the body is in equilibrium, remove it; and by weighing the body, and then the whole of the water that flowed over the edge into the dish, their weights will be found to be the same. Replace the body, and load it until it sinks deeper into the fluid; weigh the water that again flows over the edge of the bowl, and its weight will be found to be the same as that of the material with which the body was loaded.

Before passing on, it should be remarked that in the case of the cubical block that sank to half it depth in the fluid, the wood of which it was composed was bulk for bulk half as heavy as water; assuming that the water was distilled, then it is said of the wood that its *specific gravity* is $\frac{1}{2}$ or $\cdot 5$; the term *specific gravity* meaning the ratio of the weight of any body to that of an equal volume of distilled water. In the second case, when the wood weighed the same as an equal volume of water, its *specific gravity* was unity. Should the body sink to the bottom of the fluid, its *specific gravity* is determined by placing it in a bowl full of water, and comparing its weight with the weight of the fluid that flows over; the volumes being evidently identical. By this it will be seen that the specific gravity of a floating body is less than unity, while that of a body that sinks is greater than unity. The accompanying table of specific gravities of materials used in the construction of ships is given, not because of its immediate use at this part of our subject, but because of its bearing upon the principles under consideration. The figures are given upon the authority of Professor Rankine.

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TABLE OF SPECIFIC GRAVITIES OF MATERIALS USED IN THE CONSTRUCTION OF SHIPS.

Materials.	Specific Gravity. Pure Water=1.	Materials.	Specific Gravity. Pure Water=1.
Timber.		**Metals.**	
Cedar, ...	·486	Brass, cast, ...	7·8 to 8·4
Cowdie, ...	·579	Copper, cast, ...	8·6
Elm, ...	·544	,, sheet, ...	8·8
Fir, Red Pine, ...	·48 to ·7	,, hammered, ...	8·9
,, Spruce, ...	·48 to ·7	Iron, cast, average, ...	7·11
,, American Yellow Pine, ...	·46	,, wrought, average	7·69
,, Larch, ...	·5 to ·56	Lead, ...	11·4
Greenheart, ...	1·001	Steel, ...	7·8 to 7·9
Lignum Vitæ, ...	·65 to 1·33	Zinc, ...	6·8 to 7·2
Mahogany, Honduras,	·56		
,, Cuba, ...	·85		
Mora, ...	·92		
Oak, European, ...	·69 to ·99		
,, American Red,	·87		
Teak, Indian, ...	·66		
,, African, ...	·98		

It must, however, be clearly understood that it is not essential to the buoyancy of a vessel that she should be constructed, even in part, of materials having a specific gravity less than unity. All that is necessary is that *the volume of water displaced*, when floating at the required depth, should be of the same weight as that of the vessel and her contents. As an illustration, we may cite the common phenomena of a porcelain tea-cup or a glass bottle—the materials of each of which have a higher specific gravity than water—floating so long as no more than a certain weight of water, or any other substance is contained in them. The buoyancy depends upon the form as well as the specific gravity of the floating body, and it is to the former quality that we now wish to direct attention.

5. Displacement.—The weight of any body floating at rest in a fluid being equal to that of a quantity of the fluid, equal in volume to the portion of the body that is immersed, it consequently follows that if we want to know the weight

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of the body, we can determine it by first calculating that volume in terms of some unit of measurement—say a cubic foot—and then multiplying the volume by the weight of a cubic foot of the fluid. Now it happens that in the case of fresh-water, at its ordinary temperature, a cubic foot of it weighs 1000 ounces, or 62½ pounds avoirdupois; in other words, 35·84 cubic feet weigh 1 ton. Salt-water being somewhat denser, 35 cubic feet are found to weigh 1 ton.

It will thus be seen that when the immersed portion of a floating body is of some regular form, admitting of the application of a simple rule or formula in the determination of its volume, the problem of calculating the weight of the body, by the knowledge of this law, is of a very easy character. This was exemplified in the case of the floating cube of wood already referred to. But the bottom of an ordinary ship is not of this regular and well-known form, and, consequently, the problem of determing the *volume of displacement*, or the "*displacement*," as it is usually termed, requires the application of one or other of several rules, which have been investigated by the aid of principles, founded upon the integral calculus, and which are given in such forms as to admit of ready use, without requiring, on the part of the calculator, a knowledge of those principles upon which they are founded. The two principal of these rules are known as "Simpson's* 1st and 2nd Rules," while another is known as "Woolley's Rule." In each case the name is that of the inventor of the formula. These rules we shall presently state.

It must first, however, be understood that displacement calculations are made for two purposes:—*First*, In order to insure, when designing a vessel, that there is a sufficient volume in the portion of the ship when immersed up to a certain height, chosen for the level of the water surface, that the weight of an equivalent volume of water may be as great as that of the total weight which the ship and the cargo she has to carry is expected to have.

Second, In order that when a ship is afloat, we may determine, accurately, the total weight of herself and the cargo on board; it being obviously quite impracticable, if not impossible, to determine that weight in any other way, with certainty.

* These are sometimes known as Stirling's Rules.

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The elementary principles of buoyancy being thus established, and the reasons given for obtaining the volume, and thence the weight of displacement, we will proceed to explain how that volume is determined in the case of a ship. It is, however, necessary to first consider the areas of plane surfaces, such as those obtained by cutting transverse slices off a ship, and then advance to the discussion of the modes of obtaining the volume of the whole body immersed.

6. Areas of Plane Surfaces Bounded by Straight Lines.—The simplest form of plane area that we have to measure is that of a *square* (fig. 1), or *rectangle* (fig. 2), which, it is

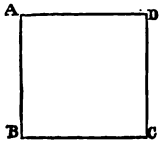


Fig. 1

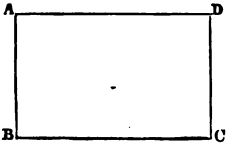


Fig. 2.

hardly necessary to say, is obtained by multiplying together two of its adjacent sides, as AB and AD; these being equal in the case of the square. The next form is that of the *triangle* (fig. 3); and since, in the cases we have to consider, it can always be measured in any way thought desirable, its area is simply found by multiplying one side, as BC, by the perpendicular AD, drawn to it from the opposite angle, and then dividing the product

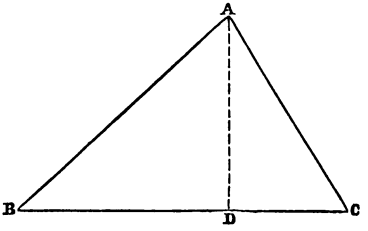


Fig. 3.

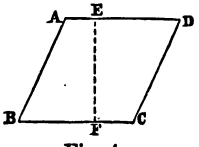


Fig. 4.

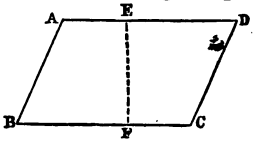


Fig. 5.

by 2. The *rhombus* (fig. 4) and *rhomboid* (fig. 5), can each

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be divided into two equal triangles, and so calculated; or, which is the same thing, the area in each case is found by multiplying either of its sides, say BC, by the perpendicular distance EF, between that side and the side parallel to it. The *trapezoid* (fig. 6), is a four-sided rectilineal, or straight lined, plane figure, only two of whose sides are parallel. Its area is obtained by adding together the lengths of the two parallel sides, AD and BC, and dividing the sum by 2, then multiply this result by the perpendicular distance EF between these sides, and the product is the area. This process is equivalent to obtaining the mean breadth of the figure, and multiplying it by the length; or, in other words, obtaining the area of the equivalent rectangle. The area of the *trapezium* (fig. 7), and that of any other irregular rectilineal plane figure (as in fig. 8), is obtained by dividing

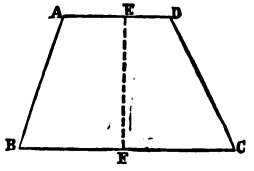


Fig. 6.

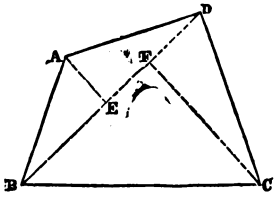


Fig. 7.

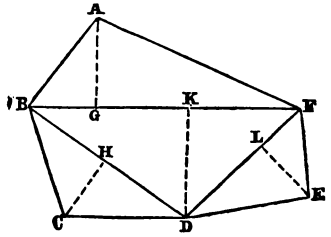


Fig. 8.

the area into triangles, each of which can be calculated in the manner already described, and then summing the results.

7. Areas of Plane Surfaces wholly or partially bounded by Curved Lines.—There is a great number of plane figures, wholly or partially bounded by curved lines, the areas of which can be determined by simple rules; for instance, the circle, segments and sectors of circles, ellipses, parabolas, etc. For our purpose, however, in calculating the areas of such plane figures as are given by the sections of a ship, we do

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not concern ourselves with these simple figures, each of which is measured by a separate rule; but attend rather to a rule, or rules, applicable to all plane surfaces bounded by curved lines, and giving results which, while more or less approximate, can always be depended upon for a close degree of accuracy, if sufficient pains are taken in their application. As we shall see presently, these rules are *true* only for certain figures; nevertheless the assumption of their universal accuracy for all curves in a ship's form is not productive of a greater degree of error than is admitted in measuring with a scale from an ordinary drawing.

It is hardly necessary to state that the two "dimensions," or directions for measurement of plane areas, are termed "*length*" and "*breadth*." We will now dispense with these terms, and use in their stead two others which are necessarily employed in the measurement of curved areas. These are "*ordinate*" and "*abscissa*." In figs. 1, 2, 4, and 5, BC and AB are usually termed the length and breadth; in the nomenclature now to be used, AB will be an "ordinate," and BC an "*abscissa*." In Algebraical language, BC is generally known as x, and AB as y.

In fig. 9, adjoining, ABCD is a trapezoid, the sides AB and CD being parallel. According to the rule already given, the area of this figure is obtained by adding AB and CD together, and then multiplying half of their sum by BC. In Algebraical language, AB would be termed "the ordinate y_1," and CD—"the ordinate y_2"; also BC would be styled "an abscissa x." Consequently, this rule expressed in a formula would appear thus:—

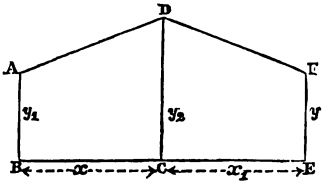


Fig. 9.

$$\text{Area} = \frac{y_1 + y_2}{2} x.$$

Similarly, if another trapezoid be constructed, such as DCEF, having CE in a straight line with BC, and the side DC common to the two figures; then styling EF as y_3, and

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CE as x_1, the area of DCEF is equal to $\frac{y_2+y_3}{2}x_1$. Hence the area of the whole figure ABEF is equal to

$$\frac{y_1+y_2}{2}x+\frac{y_2+y_3}{2}x_1 \quad \text{..... (A)}$$

Let BC = CE, or $x=x_1$, and the area becomes

$$\frac{y_1+2y_2+y_3}{2}x \quad \text{..... (B)}$$

This simple case will serve to familiarise the student with the meaning of the terms *abscissa* and *ordinate;* also of the symbols used to express them, and the manner in which they are employed in expressing the value of an area.

In preparing a plane area for calculation in this manner, the ordinates are always drawn perpendicular to the abscissa, and they are spaced either all equidistant, or else some of the ordinates are a multiple, such as one-half of the distance apart from each other; so that the abscissæ are either all equal, or perhaps a few are some multiple of the other abscissæ.

+ **8. Trapezoidal Rule for Plane Areas.**—It will, no doubt, have already suggested itself to the student, that the area of a plane surface, bounded by a straight line and a curve, can be easily approximated to by dividing it into a sufficient number of trapezoids, as in fig. 1, Plate I. Suppose the area be required of the plane surface *ABECD*, bounded by the straight line *AD* and the curve *BEC*; also, for simplicity sake, by two end ordinates *AB* and *CD*. Divide the base *AD* into a certain number of equal parts at *a,b,c,F,d*, etc., and draw the ordinates *af,bg,ch,FE,dk*, etc., to the curve; these we shall hereafter term *dividing ordinates*. Then by considering each of the figures *BAaf, fabg, gbch*, etc., as a trapezoid, and summing their areas, we obtain a result which is less than the true area of *ABECD*, by the collective areas of the small spaces enclosed between the ticked straight lines *Bf, fg, gh*, etc., and the curve *BEC*. It is evident that by taking a sufficient number of ordinates we may obtain, in this way, an area which will differ from the true one by as

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small an amount as we please. Also, that in the case of a very flat or nearly straight curve, fewer ordinates are required than if the curve is very convex or concave. Again, when a portion of the curve is more concave or convex than the remainder, in order the better to approximate to the area, the ordinates should be closer at that part than elsewhere. Let the lengths of the ordinates *AB*, *ef*, *hg*, etc., in fig. 1, Plate I., be expressed by the symbols y_1, y_2, y_3, etc., and the abscissa or common interval between the ordinates, viz., *Aa* or *ab*, etc., be expressed by x. Then by formula (B), in Art. 7, the total area of the trapezoids becomes

$$\frac{y_1+2y_2+y_3}{2}x+\frac{y_3+2y_4+y_5}{2}x+\frac{y_5+2y_6+y_7}{2}x+\frac{y_7+y_8}{2}x \quad \text{... (A)}$$

adding, we get

$$\left(y_1+2y_2+2y_3+2y_4+2y_5+2y_6+2y_7+y_8\right)\frac{x}{2} \quad \text{.....(B)}$$

or

$$\left(\frac{y_1+y_8}{2}+y_2+y_3+y_4+y_5+y_6+y_7\right)x \quad \text{..... (C)}$$

and by choosing a sufficient number of ordinates, and thereby reducing the size of each trapezoid, we may make this expression represent, as nearly as we please, the area of the figure *ABECD*.

Written out, formula (C) may be stated in the following rule, termed the *trapezoidal rule*:—

Divide the base into a sufficient number of equal parts, and draw and measure ordinates at the points of division. Add together all the dividing ordinates, and one-half of the end-most ordinates; multiply the sum by the common interval: the product will be the required area, nearly.

Should it be considered desirable, owing to the greater convexity or concavity of the curve at some parts, to introduce intermediate ordinates at such places, then the areas of these lesser trapezoids, also of the portion made up of the others, must be calculated separately. For further particulars on the mode of dealing with subdivided areas, the student is referred to Art 12, where the mode of expressing the total area in such a case, by one formula, is explained.

9. Simpson's First Rule.—The basis of Simpson's rules for the calculation of plane areas is founded upon the assump-

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tion, which is practically true for the curves met with in a ship's hull, viz., that they are members of the group known as *parabolic*. The term *parabolic*, as here used, has a wider meaning than that of the ordinary parabola of the conic sections. In the sense the term is here employed, it refers to all curves, any ordinate of which can be expressed by means of one or more terms, each of which is proportional to some power of the abscissa corresponding to that ordinate. The parabolic curve is said to be of the *second order*, the *third order*, etc., according to the exponent of the highest power of the abscissa.

Simpson's First Rule, however, is founded upon the assumption that the parabola is of the second order,* and therefore that of the conic sections. The result obtained by its application to the curves of ships is found sufficiently correct for all practical purposes. We will now proceed to investigate it by a simple method, based upon a fundamental property of the parabola of the second order, which property it is not necessary to demonstrate in a work of this kind.

10. Simpson's First Rule—Proof First.—A parabola of the second order is a curve, such that *the area of any one of its segments is two-thirds of the product of the base and deflection of that segment.* Another property of the common parabola is, that *the deflection of any arc is proportional to the square of its base.*

In fig. 2, Plate I., let CKD be a portion of the arc of a parabola PKQ upon the base PQ. Join CD, and through the points C and D draw CA and DB, ordinates perpendicular to the base PQ. Bisect AB at E; also bisect AE and EB at F and G, respectively. Through the points F and G draw the ordinates FH and GL perpendicular to PQ. Join CK and KD. Then KO, the length of the ordinate EK intercepted between the chord CD and the curve, is termed the *deflection* of the arc CKD, also MH and NL are the *deflections* of the arcs CHK and KLD, respectively. The property just stated, viz., *that the deflection of any arc is*

* Mr. C. W. Merrifield has shown that Simpson's First Rule is also true for a cubic parabola, or parabola of the third order. See *Transactions of the Institution of Naval Architects*, Vol. VI.

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proportional to the square of its base, will be perhaps better understood when we put it thus:

$$\begin{aligned} OK : HM &:: AB^2 : AE^2 \\ \text{and since } AB : AE &:: 2 : 1 \\ \therefore\ AB^2 : AE^2 &:: 4 : 1 \\ \because\ OK : HM &:: 4 : 1 \end{aligned}$$

$$\text{and thus } HM = \frac{OK}{4}$$

$$\text{Similarly, } NL = \frac{OK}{4} = HM.$$

Suppose it be required to find the area of the figure *ACKDB*, bounded by the base *AB*, the two end ordinates *AC* and *BD*, and the parabolic curve *CKD*. Employing the trapezoidal rule, stated at Art. 8, and using only one dividing ordinate, viz., *EK;* the area we obtain is that of the two trapezoids forming the right-lined figure *ACMKNDB*, which is less than the true area by the sum of the areas of the parabolic segments *CHK* and *KLD*. Again, if we use three dividing ordinates, we obtain the area of the figure *ACHKLDB*, composed of four trapezoids, which is less than the true area by the sum of the four parabolic segments *CH*, *HK*, *KL*, and *LD*. This latter result is more nearly true than the former, inasmuch as it includes the areas of the two triangles *CHK* and *KLD*, which were neglected in the former. It will readily be seen that by bisecting the portions *AF*, *FE*, *EG*, etc., of the base, and so erecting four intermediate ordinates, we approach the true area by the sum of the four triangles in the segments *CH*, *HK*, *KL*, and *LD;* and by increasing the number of the ordinates indefinitely, we shall approach the truth as nearly as we please. Hence the area of the whole figure *ACKDB* is made up of the following:—

Trapezoid ACDB + triangle CKD + triangle CHK + triangle KLD + triangles in segments CH, HK, KL, LD + etc., etc.

Neglecting the trapezoid *ACDB* for the present, we will find the sum of the areas of all the triangles, such as are drawn in the figure, which, *in the limit*, compose the parabolic segment *CKD*.

The area of the triangle *CKD* is equal to

$$\frac{AB \times KO}{2}$$

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Also the areas of the triangles *CHK* and *KLD* are each equal to

$$\frac{AB}{2}\times\frac{HM}{2} \text{ or } \frac{AB}{2}\times\frac{NL}{2}.$$

$$\text{but } HM=NL=\frac{OK}{4}.$$

Therefore the areas of the triangles are each equal to

$$\frac{AB}{2}\times\frac{OK}{8}=\frac{AB\times OK}{16},$$

$$\text{and the sum of the two}=\frac{AB\times OK}{8},$$

$$\text{whence it follows that their sum}=\frac{\text{area CKD}}{4}.$$

Similarly, it may be shown that the sum of the areas of the triangles in the segments *CH*, *HK*, *KL*, and *LD*, is equal to one-fourth the sum of the areas of the triangles *CHK* and *KLD*, or one-sixteenth the area of the triangle *CKD*, and so on.

Hence we have for the area of the parabolic segment

$$\frac{AB\times KO}{2}\left(1+\tfrac{1}{4}+\tfrac{1}{16}+\tfrac{1}{64}+\tfrac{1}{256}+\text{etc}\right).$$

By adding together a sufficient number of terms of this endless series, we can approach the true area of the parabolic segment as nearly as we please.

The limit of the series can be found exactly as follows:—If we take the fraction $\frac{3}{4}$ of any unit, and add to it $\frac{3}{4}$ of the remaining quarter, or $\frac{3}{16}$; then $\frac{3}{4}$ of the remaining sixteenth or $\frac{3}{64}$, and so on, we obtain successively the sums:

$$\tfrac{3}{4},\ \tfrac{15}{16},\ \tfrac{63}{64},\ \tfrac{255}{256},\ \text{etc.},$$

which obviously approach nearer and nearer to unity, and may be made to approach as near to unity as we please by carrying on the series far enough. Therefore the *limit* of the sum of the endless series,

$$\tfrac{3}{4}+\tfrac{3}{16}+\tfrac{3}{64}+\tfrac{3}{256}+\text{etc., is 1;}$$

and dividing by 3, we find that the limit of the sum of the endless series,

$$\tfrac{1}{4}+\tfrac{1}{16}+\tfrac{1}{64}+\tfrac{1}{256}+\text{etc.},$$

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is one-third. Consequently, the exact area of the parabolic segment is

$$\frac{AB \times KO}{2}(1+\tfrac{1}{3}) = \frac{2 \times AB \times KO}{3},$$

as already stated.

Hence the area of the whole figure $ACKDB$ is equal to

$$AB \times \frac{AC+BD}{2} = \text{area of trapezoid,}$$

added to $\tfrac{2}{3}(AB \times KO) = \ldots\ldots$parabolic segment.

But $\tfrac{2}{3}(AB \times KO) = \tfrac{2}{3}AB(EK - FO)$

$$= \tfrac{2}{3}AB\left(EK - \frac{AC+BD}{2}\right).$$

Hence whole area

$$= AB \times \frac{AC+BD}{2} + \tfrac{2}{3}AB\left(EK - \frac{AC+BD}{2}\right)$$

$$= AB\left(\tfrac{2}{3}EK + \frac{AC+BD}{6}\right) \quad \text{(a)}$$

$$= \frac{AB}{6}(AC + 4EK + BD). = \frac{AE}{3}(AC + 4EK + BD)$$

The same result may be demonstrated when the parabolic arc is concave, by taking $A_1E_1B_1$, parallel to AEB, as the base, finding the area A_1CKDB_1; and then subtracting the parabolic segment CKD from the trapezoid, A_1CDB_1.

This is the basis of Simpson's First Rule. In the proof we have given, there is only one dividing ordinate; in order to obtain the rule in its general form, we have to place, side by side on a common base, a number of such elementary parabolic areas, as shown by fig. 2, Plate I., and then determine an expression which will represent the area of the whole parabolic figure. In fig. 1, Plate II., let $PRSTQ$ be a parabolic figure on base PQ, and divided by the equidistant ordinates b, c, d, etc.; a and g being the two end ordinates, and the common interval h. Then the area of the space included between the ordinates:

a and $c = \tfrac{1}{3}h(a+4b+c)$
between c and $e = \tfrac{1}{3}h(c+4d+e)$
between e and $g = \tfrac{1}{3}h(e+4f+g)$.

Adding these areas together we obtain

$$\text{Total area PRSTQ} = \tfrac{1}{3}h(a+4b+2c+4d+2e+4f+g),$$

which is the rule for seven ordinates; and, similarly, the

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value of the area, in terms of any other odd number of equidistant ordinates and their common intervals, may be expressed by a similar formulæ.

Simpson's First Rule may therefore be thus stated:—

Divide the base into any even number of equal lengths, and through these points draw ordinates to the curve, which ordinates will consequently be odd in number. *Multiply the length of each of the even ordinates by* 4, *and that of each of the odd ordinates by* 2, *except the first and last, or bounding ordinates, which multiply by unity. The sum of these products, multiplied by one-third of the common interval between the ordinates, will give the area required.*

The proof we have given depends upon the integral calculus, although the notation of that branch of mathematics is not employed. This will be seen by the assumption made regarding the limit of the sum of the series $\frac{3}{4} + \frac{3}{16} + \frac{3}{64}$, etc., which we have, for very good reasons, taken as unity, without rigidly demonstrating such to be the case.

The nature of the problem is such as to necessitate the employment of the calculus in an exact demonstration. This we now proceed to give. (See fig. 2, Plate II.)

11. Simpson's First Rule—Second Proof.—To find the area of the plane figure *BCEGH*, bounded by the base *BH*, the parabolic curve *CEG*, and the end ordinates *BC* and *GH*. Draw the three dividing ordinates *OD*, *AE*, and *KF*, so that there are five equidistant ordinates, which we will term α, β, γ, δ, and ε.

Consider the curve *CEG* as a portion of the parabola, whose equation is

$$y = a + bx + cx^2.$$

Let *O* be the origin of co-ordinates, and let

$$\text{BO} = \text{OA} = \text{AK} = \text{KH} = h.$$

Then the area CBA $= \int y dx$, or

$$\text{area} = \int_{-h}^{h} (a + bx + cx^2) dx = 2\left(ah + \frac{ch^3}{3}\right)$$

$$= \tfrac{2}{3}h(3a + ch^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now when $x = o$ $\quad y = a = \beta$ (as seen by the figure)

$x = h$ $\quad y = a + bh + ch^2 = \gamma$

$x = -h$ $\quad y = a - bh + ch^2 = \alpha$

$\therefore \alpha + \gamma = 2(a + ch^2)$

and $4\beta = 4a$

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Hence, adding

$$\alpha+4\beta+\gamma=2(3a+ch^2),\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and the area CBAE by (1) and (2)

$$=\frac{h}{3}(\alpha+4\beta+\gamma).$$

$$\text{Similarly, the area EAHG}=\frac{h}{3}(\gamma+4\delta+\iota),$$

$$\therefore \text{ the whole area CBHG}=\frac{h}{3}(\alpha+4\beta+2\gamma+4\delta+\iota),$$

which proves the Rule.

A very ingenious demonstration, due to Mr. F. K. Barnes, will be found at page 9 of *Shipbuilding, Theoretical and Practical*, which is too lengthy for insertion here.

12. Fractional Intervals.—As already mentioned (Art. 8), when a portion of the bounding curve has very great or sudden convexity or concavity, it is desirable, with a view to greater accuracy, to draw ordinates at one-half, or even one-fourth the common interval apart. In such a case, in order to express the total area in terms of the common interval, and the lengths of the ordinates, the multipliers must be altered in the following manner. In fig. 3, Plate II., the curve *RST* is very concave between *S* and *T*, with regard to the base *PQ*. Let *a*, *b*, *c*, *d*, *e*, be an odd number of equidistant ordinates, *h* being the common interval. Draw the ordinate *f* midway between *c* and *d*, the ordinate *g* midway between *d* and *e*, the ordinate *k* midway between *d* and *g*, and the ordinate *l* midway between *g* and *e*, so that we have nine ordinates. Then the area of the space enclosed between

$$a \text{ and } c=\frac{h}{3}(a+4b+c)$$

$$c \text{ and } d=\frac{h}{6}(c+4f+d)$$

$$d \text{ and } g=\frac{h}{12}(d+4k+g)$$

$$g \text{ and } e=\frac{h}{12}(g+4l+e).$$

Summing these results, we have

$$\text{Total area PRSTQ}=\frac{h}{3}\left(a+4b+c+\frac{c}{2}+2f+\frac{d}{2}+\frac{d}{4}+k+\frac{g}{4}+\frac{g}{4}+l+\frac{e}{4}\right)$$

$$=\frac{h}{3}\left(a+4b+\frac{3c}{2}+2f+\frac{3d}{4}+k+\frac{g}{2}+l+\frac{e}{4}\right)$$

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If a quadrant of a circle (see fig. 4, Plate II.) of a certain radius, say 12 feet, be divided by 9 ordinates in this manner, and the lengths of these ordinates be carefully measured and substituted in this formula, the area so obtained will be 112·93 square feet; whereas the area obtained by taking $\pi = 3{\cdot}1416$ will be 113·1. Thus an error of $113{\cdot}1 - 112{\cdot}93 = {\cdot}17$ or ·16 per cent. is made by applying Simpson's First Rule in such an extreme case, using the number of intermediate ordinates shown in the figure.

13. Simpson's Second Rule for Plane Areas.—This rule is founded upon the assumption that the bounding curve is a parabola of the third order, *i.e.*, one whose equation may be expressed thus,

$$y = a + bx + cx^2 + dx^3.$$

Let *ABCD* (fig. 1, Plate III.), be a plane area, bounded by a base *AD*, two end ordinates *AB* and *CD*, and a parabolic curve of the third order, *BHC*. Divide *AD* into three equal parts, each equal to *h*, at the points *E* and *F*, and draw through these points the ordinates *EG* and *FH*. Let the lengths of the four ordinates be represented by *a*, *b*, *c*, and *d*.

$$\text{Then area ABCD} = \tfrac{3}{8}h(a + 3b + 3c + d).$$

This may be proved by a somewhat similar demonstration to that given of the First Rule at Art. 11. It is not necessary to state the proof here, the more so as the rule does not generally give quite so close an approximation to the true area as the First Rule.

It will be readily seen that, by dividing the base up into six equal lengths, and drawing ordinates through the divisions, we have two elementary figures as above; and if *a*, *b*, *c*, *d*, *e*, *f*, and *g* be the ordinates, and *h* the common interval,

$$\begin{aligned}\text{Then area} &= \tfrac{3}{8}h\,(a + 3b + 3c + d) + \tfrac{3}{8}h\,(d + 3e + 3f + g)\\ &= \tfrac{3}{8}h\,(a + 3b + 3c + 2d + 3e + 3f + g),\end{aligned}$$

from which the Second Rule may be thus stated :—

Divide the base into equal lengths so that their number may be a multiple of 3, and draw ordinates through the dividing points and the extremities of the base, so that their total number, when divided by 3, gives a remainder of 1. Call the 4th, 7th, 10th, etc., ordinates, *dividing ordinates;* and the others, except the endmost, *intermediate ordinates*.

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Add together the endmost ordinates, twice the dividing ordinates, and three times the intermediate ordinates; multiply the sum by three-eighths of the common interval: the product will be the required area, nearly.

14. Area of Subdivision of a Parabolic Figure.—It is sometimes required to calculate separately the area of one of the two subdivisions (see fig. 2, Plate I.) into which an elementary parabolic figure $ACKDB$ is divided by the middle ordinate EK. For example, supposing it is required to calculate separately the area of the subdivision $ACKE$; let AC be called the *near end ordinate*, and BD the *far end ordinate*, then the rule will be as follows:—*To eight times the middle ordinate, add five times the near end ordinate, and subtract the far end ordinate; multiply the remainder by one-twelfth of the common interval: the product will be the area required.*

The truth of this will be readily seen by considering that the area is made up of the trapezoid $ACOE$, and the half segment CKO. For the area of the half segment CKO is equal to

$$\frac{AB}{3}\left(EK-\frac{AC+BD}{2}\right) \text{ (See Art. 10),}$$

and the area of the trapezoid $ACOE$ is equal to

$$\frac{AB}{2}\left(\frac{AC+OE}{2}\right)=\frac{AB}{2}\left(\frac{AC}{2}+\frac{AC+BD}{4}\right)$$

$$=\frac{AB}{2}\left(\frac{2AC+AC+BD}{4}\right)=\frac{AB}{8}\left(3AC+BD\right).$$

Hence area of figure $ACKE$

$$=\frac{AB}{3}\left(EK-\frac{AC+BD}{2}\right)+\frac{AB}{8}\left(3AC+BD\right)$$

$$=\frac{AB}{24}\left(5AC+8EK-BD\right)$$

$$=\frac{AE}{12}\left(5AC+8EK-BD\right)$$

This is sometimes known as the *five-eight rule.*

15. Example of the Application of Simpson's First Rule.—The following example will illustrate the application of Simpson's First Rule for measuring a plane area bounded by a curve.

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Let the area of the figure *ACB* (fig. 2, Plate III.) be required, of which the base AB is 40 feet long. Divide AB into a certain even number of, say 8, equal lengths, each of which will therefore measure 5 feet. Erect ordinates through these points; these will necessarily be 9 in number, including the end ordinates; and therefore fitted for the application of Simpson's First Rule. Measure the lengths of these ordinates, which lengths we will suppose to be represented by the dimensions marked against them in the diagram.

Numbers of Ordinates.	Simpson's Multipliers.	Lengths of Ordinates.	Functions of Ordinates.
1	1	·2	·2
2	4	3·5	14·0
3	2	5·2	10·4
4	4	6·6	26·4
5	2	6·8	13·6
6	4	6·7	26·8
7	2	5·8	11·6
8	4	4·7	18·8
9	1	1·6	1·6
			123·4

5 = Common Interval

Divisor by Rule = 3)617·0

205·66 = Area, nearly.

In proceeding to apply the Rule as in the above table, four *columns* are ruled, and as many *rows* as there are ordinates. The first column shows the distinguishing numbers of the ordinates; the second column contains Simpson's Multipliers, arranged in their proper order, and opposite to their respective ordinates; the third column contains the lengths of the respective ordinates, numbered in the first column; and the fourth column contains what is termed "*the functions of the ordinates*," *i.e.*, the products of the ordinates and their respective multipliers.

The sum of the figures in the fourth column is 123·4, and this has to be multiplied by one-third the common interval, which, in this case, is 5 feet. Hence, 123·4 is multiplied by 5 ft. and divided by 3; thus giving 205·6 sq. ft., which is the area required, nearly.

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Some calculators, in order to reduce the figures in the fourth column, use as multipliers the figures $\frac{1}{2}$, 2, 1, 2, . . . 2, $\frac{1}{2}$, instead of 1, 4, 2, 4, . . 4, 1, and consequently the sum so obtained has to be multiplied by 2 after being multiplied by one-third of the common interval.

16. Example of the Application of Simpson's Second Rule.—In this case the number of divisions of the base must be a multiple of 3, and there will, of course, be one more ordinate than division.

Divide the base AB (fig. 3, Plate III.) into, say 6, equal lengths, each being 5 ft., also draw the ordinates and measure them as before.

2d Rule | 1st Rule

Numbers of Ordinates.	Simpson's Multipliers.	Lengths of Ordinates.	Functions of Ordinates.	Multipliers	Products
1	1	2·1	2·1	1	2.1
2	3	3·6	10·8	4	14.4
3	3	5·3	15·9	2	10.6
4	2	5·6	11·2	4	22.4
5	3	5·4	16·2	2	10.8
6	3	5·0	15·0	4	20.0
7	1	2·5	2·5	1	2.5
					82.8

73·7
5 = Common Interval
368·5
3
8)1105·5
138·19 = Area, nearly.

The arrangement of the columns is the same as in the preceding example; the number of ordinates and the multipliers being different according to the Rule. The sum of the figures in the fourth column is 73·7, which is multiplied by three-eighths of the common interval, or $\frac{3}{8} \times 5$ ft., and gives 138·19 ft. which is the area, nearly.

17. Example of the Application of the Trapezoidal Rule.—We will use the same diagram and dimensions as in Art. 16,

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Numbers of Ordinates.	Multipliers.	Ordinates.	Functions of Ordinates.
1	$\frac{1}{2}$	2·1	1·05
2	1	3·6	3·60
3	1	5·3	5·30
4	1	5·6	5·60
5	1	5·4	5·40
6	1	5·0	5·00
7	$\frac{1}{2}$	2·5	1·25
			27·2
			5 = Common Interval
			136·0 = Area, approximately.

The area found by this rule is thus 136 sq. ft., whereas by Simpson's Second Rule it was found to be 138·19 sq. ft. If the area be determined by the First Rule, the result will be 138 sq. ft. Simpson's Rules thus practically agree, and as the First Rule can be depended upon to, at least, the same degree of accuracy as the Second, we may take 138 as the area. Thus it will be seen that in this case the use of the Trapezoidal Rule involves an error of 138 – 136 = 2 sq. ft., or about 1·5 per cent. With a curve of greater convexity or concavity the error would, of course, be still more.

18. Example of the Application of the Rule for Subdivision of a Parabolic Figure.—(See Art. 14.) In fig. 2, Plate I., let AC = 5·3, EK = 7·2, BD = 3·1, and AE = 8.

$$\text{Then area ACKE} = \frac{AE}{12}\left(8\,EK + 5\,AC - BD\right)$$

$$= \frac{8}{12}\left(57{\cdot}6 + 26{\cdot}5 - 3{\cdot}1\right) = \frac{2}{3} \times 87 = 58 \text{ sq. ft.}$$

By treating the figure as a trapezoid—

$$\text{Area} = \frac{8}{2}\left(7{\cdot}2 + 5{\cdot}3\right) = 50 \text{ sq. ft.}$$

The Trapezoidal Rule thus being relatively in error, 58 – 50 = 8 sq. ft., or 14 per cent. in this particular case.

19. Centres of Gravity of Plane Areas.—The centre of gravity of a plane area, termed its *geometrical centre of gravity*, is that point at which, if the surface is uniformly weighted, it can be poised in equilibrium. To illustrate our meaning, we may consider the plane as the surface of a sheet of metal

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having a uniform thickness, then the centre of gravity of a plate of any form is that point where the plate, if supported thereat, will balance or remain poised in a state of equilibrium. In all our considerations regarding the centre of gravity of a plane area the surface is supposed to be heavy, and the weight of every unit of the area to be the same.

20. Centres of Regular Figures.—It will at once be seen that the geometrical centre of gravity* of a circular plane area is its centre. Also, that the geometrical centre of gravity of a square or rectangular area is the point where the two diagonals intersect; for as each diagonal cuts the figure into two similar and equal parts, the centre of gravity must be in each diagonal; and, therefore, at the point common to the two, or where they intersect. The same is true of a rhombus or rhomboid. The centre of gravity of a regular polygon is the centre of the circumscribed circle; for since every diameter of the circle passing through the angular points of the polygon cuts the latter into two similar and equal parts, therefore the centre of gravity of the figure must lie in each diameter, and consequently at the point common to them all, or the centre of the circumscribed circle. The circumscribed and inscribed circles being concentric, the centre of gravity of the latter circle is also that of the regular polygon.

The Centre of Gravity of a Triangle is found in the following manner:—*Bisect two sides and join the points of bisection with the opposite angles, then the point of intersection of the two lines so drawn is the centre of gravity of the triangle.*

Let *ABC* (fig. 1, Plate IV.) be the surface of a triangular plate of uniform weight; bisect *BC* in *E*, and join *AE*; draw *ceb* parallel to *CEB*, cutting *AE* at any point *e*. Then, by similar triangles—

$$\begin{aligned} & ce : CE :: Ae : AE \\ \text{and} \quad & be : BE :: Ae : AE \\ \therefore \quad & ce : CE :: be : BE \\ \text{but} \quad & CE = BE, \quad \therefore \quad ce = be. \end{aligned}$$

Hence *AE* bisects every line in the triangle drawn parallel to *BC*. Therefore each of the strips similar to *ceb*, into which we may suppose the triangle to be divided, will balance on

* Hereafter the geometrical centre of gravity will be understood to be meant when referring to forms without weight.

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AE, and consequently the centre of gravity must be in the line AE.

Bisect AC in F, and join BF; let this cut AE in G. Then, as before, the centre of gravity must be in BF; but it has been shown to be in AE, therefore G, the intersection of AE and BF, is the centre of gravity.

This point G is situated at one-third the length of AE from the point E. To prove this, join EF. Then, because $CE = BE$ and $CF = AF$, therefore EF is parallel to AB and $AB = 2FE$; also by similar triangles—

$$\text{AB} : \text{EF} :: \text{AG} : \text{EG};$$
$$\text{EG} = \tfrac{1}{2}\,\text{AG}, \text{ or } \text{EG} = \tfrac{1}{3}\,\text{AE}.$$

Hence, to find the centre of gravity of a triangle, *bisect any side, join the point of bisection with the opposite angle, and the centre of gravity lies a third of the way up this line.*

It will be useful to observe that the centre of gravity of a triangle coincides with the centre of gravity of three equal weights placed at the angular points of the triangle. For, to find the centre of gravity of three equal weights placed at the points A, B, C, respectively (see fig. 1, Plate IV.), we join CB and bisect it in E; then E is the centre of gravity of the weights at C and B. Suppose these weights collected at E, then join AE, and divide AE in G, so that EG may be to AG as the weight at A is to that of the two at E, that is as 1 : 2; then G is the centre of gravity of the three equal weights. From the construction, G is also the centre of gravity of the triangle ABC, as already shown.

21. Centre of Gravity of a Trapezium.—From this we may proceed to determine the centre of gravity of a trapezium by drawing a diagonal, and thus dividing it into two triangles. The centre of gravity of these are found, and the points joined by a straight line. Next, the other diagonal is drawn, and the centres of gravity of the two resulting triangles are joined by a straight line. As the centre of gravity of the trapezium is in each of these straight lines, it is evidently the point of their intersection.

The ordinary mode of constructing the figure is, however, as follows:—Let $ABCD$ (fig. 2, Plate IV.) be a trapezium; draw the diagonals AC, BD intersecting at E. Let AE be

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greater than EC, and BE greater than ED. Take $AF = EC$, and $BG = ED$. Set off $ES = \frac{1}{3}\ EF$, and $ET = \frac{1}{3}\ EG$; draw SO parallel to BD, and TO parallel to AC; their intersection O is the centre of gravity of the trapezium. It is evident if we join GF, then O is also the centre of gravity of the triangle EGF.

The following is the proof of the accuracy of this construction. It will be observed that it is required to demonstrate that the lines LM and NP joining the centres of gravity of the two sets of triangles, into which the diagonals successively divide the trapezium, are parallel to AC and BD respectively; also that $SO = \frac{1}{3}\ EG$, and $TO = \frac{1}{3}\ EF$.

Since $HL = \frac{1}{3}\ AH$, and $HM = \frac{1}{3}\ HC$, therefore LM is parallel to AC. Similarly, NP is parallel to BD. This gives the first requirement.

Again, since $BG = ED$, and $BH = HD$, $\therefore$ $GH = HE$. Similarly, $FK = KE$. Also, because LM is parallel to AC, and $HM = \frac{1}{3}\ HC$, $\therefore$ $HT = \frac{1}{3}\ HE$, and $TE = \frac{2}{3}\ HE = \frac{1}{3}\ GE$. Similarly, $SE = \frac{1}{3}\ EF$, whence the construction.

The centre of gravity of the trapezium being coincident with that of the triangle EFG, a very simple mode of determining the point by construction is to bisect EF and EG at K then H respectively; then joining FH and GK, their point of intersection is O, the centre of gravity of the triangle, and therefore of the trapezium.

22. The Centre of Gravity of a Trapezoid is easily found in the following manner: Let $ABCD$ (fig. 3, Plate IV.) be a trapezoid, AD and BC being the parallel sides. Bisect AD in E, and BC in F, and join EF. Then divide EF at the point G, so that

[redacted] EG : GF. = 2 BC + AD : 2 AD + BC

G is the centre of gravity of the trapezoid. The truth of this is too obvious to need a proof here.

23. The Centre of Gravity of an Ellipsoid is evidently at its centre, or at the intersection of the major and minor axes.

24. Principle of Moments.—In obtaining the centres of gravities of the plane areas already examined, we have depended merely upon the symmetry of the figures, whereby we were enabled to draw two lines, each of which contained the centre of gravity, the latter being therefore situated at

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the intersection of these lines. We shall presently show how to determine the centre of gravity when there are no symmetrical conditions to assist us. Consequently, we must have recourse to the elements of statical equilibrium as enunciated in the *principles of moments*. The forces we have to consider being those due to gravity, they are therefore parallel, and act in the same direction; a condition which materially simplifies our investigations.

The *moment* of a weight about any point is the product of the weight into the perpendicular distance of its line of action from that point (see fig. 4, Plate IV.). Let the weight W act at the point A, perpendicular to the straight line AB, then the moment of the weight about the point B is $W \times AB$.

If there is more than one weight acting at different points on the straight line, perpendicular to it, then the sum of the products of the weights into the distances of their points of application from B is the total moment acting about that point. For instance, W_1, W_2, and W_3 (fig. 5, Plate IV.) acting at the points A, C, D, respectively, have a total moment about $B = W_1.AB + W_2.CB + W_3.DB$. It will be readily seen that the three weights W_1, W_2, and W_3 may be applied at some point in the line, so that their moment may be equal to the sum of the three moments as they are at present situated. Let x be the distance of that point from B; then

$$x(W_1 + W_2 + W_3) = W_1.AB + W_2.CB + W_3.DB$$

$$x = \frac{W_1.AB + W_2.CB + W_3.DB}{W_1 + W_2 + W_3}$$

from which x may be readily found by substituting the values of W_1, W_2, W_3, and AB, CD, and DB. If P be the position of the point so found, then P is the centre of gravity of the weights; being the point where all the weights collected have the same statical effect as when they were situated as shown in the figure.

In the figure, AB, CB, and DB are the leverages of the several weights; B is the point about which, it is said, *moments are taken*, and $x(W_1 + W_2 + W_3)$ is the *resultant moment*. Moments may be taken about any other point as well as B. For instance, suppose we happen to fix upon C as the point

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about which moments are taken, then in that case x = zero; also

$$W_1.AP = W_2.CP + W_3.DP.$$

When the point, about which moments are taken, is situated between any of the points of application of the weights, then the distances measured on one side of that point are considered positive, and on the other side negative. If the sum of the moments on one side of the point is equal to that of those on the other, then the weights are in equilibrium about that point, and it is therefore their common centre of gravity, as in the above case, when the moments are taken about P. But if the sum of one set of moments is in excess of the other, then the centre of gravity is on that side upon which the greater moments are. For instance, in fig. 6, Plate IV., in which the weights W_1, W_2, W_3, W_4, act at the points A, B, C, and D, respectively, on the straight line AD; then, taking moments about any point P,

$$(W_1.AP + W_2.BP) - (W_3.CP + W_4.DP)$$

is the resultant moment. Let G be the centre of gravity of the weights; then

$$PG(W_1 + W_2 + W_3 + W_4) = (W_1.AP + W_2.BP) - (W_3.CP + W_4.DP)$$

$$PG = \frac{(W_1.AP + W_2.BP) - (W_3.CP + W_4.DP)}{W_1 + W_2 + W_3 + W_4}$$

If $W_1.AP + W_2.BP$ is greater than $W_3.CP + W_4.DP$, the point G will be between P and A; but if less, then the point G will be between P and D. In the figure, we have assumed the latter case.

The value of PG is usually expressed as the *Algebraical sum of the moments* about P, divided by the sum of the weights; the *Algebraical sum* being the collective value of the moments when affected by their proper signs, *i.e.*, *plus* or *minus*.

25. Centre of Gravity of Plane Figure bounded by a Curve.—To apply the preceding results in obtaining the centre of gravity of a plane figure, such as is commonly met with in Naval Architecture, and to which we have been applying Simpson's Rules for finding its area, we proceed as follows:—

First observe that the figures are in almost every case

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symmetrical about a middle line, so that knowing the centre of gravity is somewhere in that line, we have to determine where it is.

Let $ACBD$ in fig. 1, Plate V., be a plane figure, symmetrical about a middle line AB, then we know the centre of gravity of the area is in AB, also that the centre of gravity of the half area ACB, is in the ordinate passing through that point. In accordance with our previous definition, the area is assumed to be uniformly weighted, such as a sheet of metal of uniform thickness. Divide the straight line AB into an even number of equal parts, and draw the ordinates aa_1, bb_1, cc_1, etc., through the points of division. There is thus (including the end ones), an odd number of ordinates, the value of the end ordinates being in this case zero. Assume each of the ordinates to be a narrow strip, so that they each have a weight, the latter being therefore in proportion to the length of the ordinate. Now, if we merely wished to find the centre of gravity of these strips, it would be the same as finding the centre of gravity of nine weights, acting at the points A, a, b, c, d, etc., the values of the weights being zero, aa_1, bb_1, cc_1, dd_1, etc., respectively. It is evident that if the ordinates be very close together, the sum of their moments, about any point on AB, would approximate very closely to the total moment of the area ACB about the same point; and by increasing the divisions of the line AB, thereby increasing the number of ordinates and reducing the breadths of the strips they represent, we may make the approximation as close as we please. It is here that Simpson's Rules come to our aid, as by assuming the curve ACB to be parabolic, the Rules give us the sum of the areas of all the *very* narrow strips represented by the ordinates when the latter are *very* close together. Hence, if we multiply every ordinate by its distance from the point about which the moments are taken, and consider the results so obtained as the ordinates of a new curve, then the area enclosed by that curve, and the straight line AB, will represent the moment of the area ACB about the point.

It will be observed that as the ordinates are equidistant, the distance of every ordinate from any of the points A, a, b, etc., is a multiple of the common interval; hence, to simplify

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the work, the multiple only is used in applying the Rule, the result being afterwards multiplied by the common interval.

Let the lengths of the ordinates in feet be those given in the figure, the common interval being 4 ft. Taking moments about B, the following is the mode of obtaining the centre of gravity of the area:—

Number of Ordinate.	Simpson's Multiplier.	Length of Ordinate.	Function of Ordinate.	Multiplier for Leverage.	Function of Moment.
1	1	0	0	0	0
2	4	2·5	10·0	1	10·0
3	2	4·3	8·6	2	17·2
4	4	5·1	20·4	3	61·2
5	2	5·5	11·0	4	44·0
6	4	5·4	21·6	5	108·0
7	2	4·8	9·6	6	57·6
8	4	4·0	16·0	7	112·0
9	1	0	0	8	0
			97·2		410·0
			4		4
			3)388·8		3)1640·0
		sq. ft.	129·6 = Area		546·66
					4
					2186·64
		129·6)2186·64			= moment
		16·87 ft.			

Hence the centre of gravity of the area is 16·87 ft. from B, or ·87 from d, on the side of the latter nearest A.

It will be seen that the multipliers for leverage, 0, 1, 2, 3, etc., represent the number of times the common interval of 4 ft. that the ordinates B, gg_1, ff_1, ee_1, etc., are respectively from B, the point about which moments are taken. If the actual leverages were inserted in the fifth column instead of 0, 1, 2, 3, 4, etc., the figures would be 0, 8, 12, 16, etc., and the figures in the fifth column would be 0, 40, 68·8, 244·8, 176, etc.; instead of using these large numbers we afterwards multiply by 4, and so obtain the total moment 2186·64. It may be further remarked that 546·66 represents the area of a plane surface, whose ordinates are multiples of the moments of the ordinates 0, 2·5, 4·3, etc., about the point B, that is, whose ordinates are 0, 2·5, 8·6, 15·3, etc.

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It is usual to avoid multiplying the sums of columns Nos. 4 and 6 by $\frac{1}{3}$ the common interval, as it is a needless process, one product having afterwards to be divided by the other. In the above case the work would be:—

$$\begin{array}{r} 410{\cdot}0 \\ \hline 4 \\ 97{\cdot}2\overline{)1640{\cdot}0} \\ \hline 16{\cdot}87 \text{ ft.} \end{array}$$

Again, in practice, it is customary to still further reduce the figures employed in the calculation by taking moments about an ordinate near the middle of the length of AB. As, for instance, taking moments about dd_1, or No. 5 ordinate.

Number of Ordinate.	Simpson's Multiplier.	Length of Ordinate.	Function of Ordinate.	Multiplier for Leverage.	Function of Moments.
1	1	0	0	4	0
2	4	2·5	10·0	3	30·0
3	2	4·3	8·6	2	17·2
4	4	5·1	20·4	1	20·4
5	2	5·5	11·0	0	67·6
6	4	5·4	21·6	1	21·6
7	2	4·8	9·6	2	19·2
8	4	4·0	16·0	3	48·0
9	1	0	0	4	0
			97·2		88·8
					67·6
					21·2
					4
					97·2)84·8
					·87 ft.

Here we have two sets of moments, one positive and the other negative; their difference, or Algebraical sum, is 21·2, which, multiplied by 4 and divided by 97·2, gives ·87 ft., the distance of the centre of gravity of the area from ordinate No. 5 or dd_1, and on the side of it nearest to A; which is the same result as was obtained before. In this case, the largest multiplier for leverage being 4, the figures in the sixth column are not so large as when the multipliers continually increased to 8. The saving of labour is more apparent in such large areas as are met with in actual practice.

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In the preceding examples, Simpson's First Rule has been applied; the same method is adopted with the Second Rule, the number of the ordinates being, of course, different.

In the case of intermediate ordinates being used, the multipliers are, of course, fractional; as for instance, 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 4, etc., where two sets of half intervals occur next to the ordinate about which moments are taken. Examples of this kind will be given hereafter.

26. Volumes of Solids.—The rules for finding the volumes of regular solids are to be found in any work on Mensuration, so that we shall simply state a few of those most usually required, before proceeding to examine the method of finding the volume of such a solid figure as that of the immersed body of a ship.

27. Volume of a Sphere.—Multiply the cube of the diameter by $\cdot 5236 = \frac{\pi}{6}$; or the cube of the radius by $4 \cdot 1888 = \frac{4\pi}{3}$

28. Volume of Ellipsoid.—Multiply the product of the three axes by $\cdot 5236$, or the product of the three semi-axes by $4 \cdot 1888$.

29. Volume of Pyramid or Cone.—Multiply the area of the base by one-third of the height—the height being measured perpendicular to the base.

30. Volume of a Solid Figure bounded by a Plane and a Curved Surface.—Such a figure is to be found in the case of the immersed body of a ship, the bounding plane being that of the water surface.

The body has first to be divided into a number of segments by equidistant parallel planes, which are perpendicular to the bounding plane. *The area of each plane section is found by either of Simpson's Rules, and the results are treated as the ordinates of a new curve, the common interval being the perpendicular distance between the planes; the area of this curve is the volume of the figure.* It is evident that the ordinates of the new curve being in square measure, the area of the curve will therefore be in cubic measure.

In calculating the volume of the immersed body of a ship by this method, the figure being symmetrical about the longitudinal vertical middle line plane of the ship, only one-half of the body is calculated. It is unimportant whether the dividing planes are drawn parallel to the water plane or not

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pendicular to both the water and middle line planes. In the former case, the bounding curves of the areas are "water lines," and in the latter "square stations" (see *Shipbuilding and Laying Off*, p. 17).* It is usual to calculate the volume by using both sets of dividing planes, in order that one result may check the other, and so prevent inaccuracy. The vertical dividing planes in a large ship are generally spaced from about five to seven times the distance apart that is considered requisite for the horizontal dividing planes.

31. Example of Calculation of Volume of a Ship's Displacement.—As a first example of the method of calculating the volume of a ship's displacement, we will assume that the areas of the sections at the dividing planes are already found; those of the endmost sections are evidently zero.

In (fig. 2, Plate V.) showing the immersed body of a small vessel, $ADBC$ is the water plane, and AB its middle line. This latter is divided into 8 equal lengths, each equal to 4 ft., and sections of the body are made by planes passing through these divisions, perpendicular to AB and the plane $ADBC$. The half areas of these planes in square feet are marked upon the figure; those at the extremities A and B being zero.

Number of Section.	Half Area of Section.	Simpson's Multipliers.	Function of Half Area.
1	0	1	0
2	15	4	60
3	25	2	50
4	30	4	120
5	32	2	64
6	28	4	112
7	23	2	46
8	13	4	52
9	0	1	0
			504
			4
			3)2016
			672 = Half Volume
			2
			1344 = Total Volume in cubic feet.

* Wm. Collins, Sons & Co.'s *Advanced Science Series;* also *Elementary Laying Off*, p. 18.

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