

CMC: Calculation of Small Color Differences for Acceptability

Developed by AATCC Committee RA36 in 1989; reaffirmed 1990; editorially revised and reaffirmed 1991, 2005; revised and reaffirmed 1992; revised 1998; editorially revised 2006. Partly related to ISO 105-J03.

1. Purpose and Scope

1.1 The CMC ($\ell:c$) formula is a modification of the corresponding CIELAB color-difference formula. It has color-difference symbol ΔE_{cmc} . Even though use of the CIE 1976 $L^*a^*b^*$ (CIELAB) equation provides industry with a 'standard' means of calculating and communicating color differences of trials from their standards (see 8.1), it is recognized that CIELAB color-difference values (ΔE^*) correlate poorly with visual assessments. The lack of correlation is caused by the non-uniformity of CIELAB color space (see Fig. 1). Significantly improved correlation between visual assessments and instrumentally measured color differences is given by the CMC ($\ell:c$) color-difference formula (see 7.1 to 7.3). The improvement in correlation between visual and instrumental color-difference reports yielded by the CMC ($\ell:c$) equation will usually permit the adoption of a single-number tolerance for judging the ac-

ceptability of a color match in most situations, regardless of both the color of the standard and the direction of the color difference of any trial from it. Another equation, CIE94 has been considered, but it has not shown any significant improvement over CMC (see 8.2).

1.2 The CMC ($\ell:c$) formula retains, in modified form, the partitioning of overall color difference into differences in lightness, chroma, and hue components (see Fig. 2) which is present in the CIELAB color-difference formula. Using the ellipsoidal semi-axes (ℓS_L , $c S_C$, and S_H) makes the CMC ($\ell:c$) equation suitable for a wide range of uses.

2. Principle

2.1 The CMC ($\ell:c$) modification to CIELAB provides a unit of measurement for the acceptance volume about a standard color. This volume takes the shape of an ellipsoid whose semi-axes are ℓS_L , $c S_C$, and S_H in the directions of lightness, chroma, and hue differences, respectively, in CIELAB color space. The CMC ($\ell:c$) formula varies the ratio of the lengths of these three semi-axes systematically throughout CIELAB color space according to equality of lengths with visually equal color differences, regardless of both the color of the standard and direc-

tion of difference of any trial from it. Around any given standard color, the ratio $S_L:S_C:S_H$ is fixed regardless of the end use of the trials.

3. Terminology

3.1 **CIE 1976 $L^*a^*b^*$ equation**, n.—a commonly used equation which transforms CIE tristimulus values into a three-dimensional opposed color space. Generally abbreviated as CIELAB (see 8.1).

3.2 **CMC unit**, n.—in color difference evaluation, a measure of acceptability expressed in terms of the boundary for the CMC acceptability ellipsoid of $\Delta E_{cmc} = 1.0$.

3.2.1 CMC is an acronym for the Colour Measurement Committee of The Society of Dyers and Colourists which was largely responsible for the development of the CMC ($\ell:c$) equation.

3.3 **commercial factor (cf)**, n.—in color difference evaluation, a tolerance (specified in terms of ΔE_{cmc} units) which adjusts all axes of the unit CMC volume equally to create a volume of acceptance for commercial use.

3.4 ΔE_{cmc} , n.—in color difference evaluation, a single number defining the total color difference in CMC units of a trial from a standard.

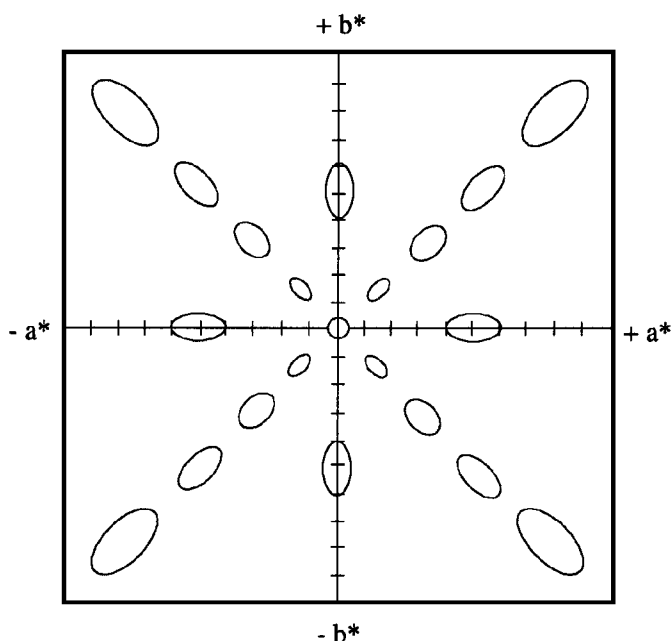


Fig. 1—CMC($\ell:1$) Unit ellipsoid cross sections in CIELAB a^*b^* diagram.

If CIELAB were uniform, these sections would be equal-sized circles.

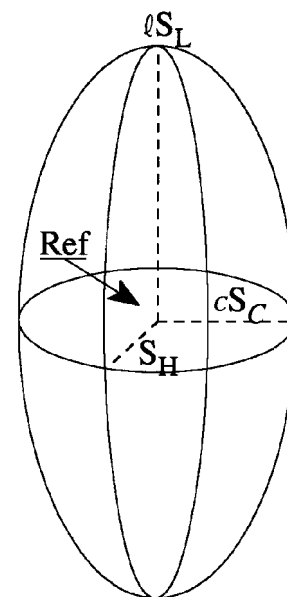


Fig. 2—Unit volume of acceptance.

Distance from standard to surface is one CMC($\ell:c$) unit.

3.5 semi-axes (ℓS_L , cS_C , and S_H), n.—in color difference evaluation, individual dimensions of the CMC volume which are used to calculate a ΔE_{cmc} value.

3.6 volume of acceptability, n.—in color difference evaluation, the volume of the ellipsoid obtained when each semi-axis (ℓS_L , cS_C , and S_H) is multiplied by cf—creating an agreed upon volume describing the limits of commercial acceptability for the color difference about a standard.

4. Procedure of Calculation

4.1 Calculation of CIELAB values.

4.1.1 Calculate the CIELAB L^* , C_{ab}^* , and h_{ab} values from the CIE tristimulus values X , Y , Z of each specimen using equations 1, 2, 3 and 4:

$$L^* = 116 (Y/Y_n)^{1/3} - 16 \quad (1)$$

if $Y/Y_n > 0.008856$
but

$$L^* = 903.3 (Y/Y_n) \quad (2)$$

if $Y/Y_n \leq 0.008856$
 $a^* = 500[f(X/X_n) - f(Y/Y_n)]$
 $b^* = 200[f(Y/Y_n) - f(Z/Z_n)]$

where
 $f(X/X_n) = (X/X_n)^{1/3}$

if $X/X_n > 0.008856$
but
 $f(X/X_n) = 7.787(X/X_n) + 16/116$

if $X/X_n \leq 0.008856$
 $f(Y/Y_n) = (Y/Y_n)^{1/3}$

if $Y/Y_n > 0.008856$
but
 $f(Y/Y_n) = 7.787(Y/Y_n) + 16/116$

if $Y/Y_n \leq 0.008856$
 $f(Z/Z_n) = (Z/Z_n)^{1/3}$

if $Z/Z_n > 0.008856$
but
 $f(Z/Z_n) = 7.787(Z/Z_n) + 16/116$

if $Z/Z_n \leq 0.008856$

$$C_{ab}^* = (a^{*2} + b^{*2})^{1/2} \quad (3)$$

$$h_{ab} = \arctan b^*/a^* \quad (4)$$

expressed on a 0-360° scale with the a^* axis being 0° and the b^* axis at 90°.

4.1.2 For these equations, X_n , Y_n , and Z_n are the CIE tristimulus values for the chosen illuminant and observer. The preferred combination is CIE standard illuminant D₆₅ and the CIE 1964 supplementary standard (10°) observer. Values for this combination and for some others are given in Table I. Values for combinations not in Table I should be obtained by referring to ASTM E 308 (see 8.3) or,

Table I—Tristimulus Values for Four Illuminant-Observer Combinations

Illuminant-Observer Combinations	Tristimulus Value		
	X_n	Y_n	Z_n
D ₆₅ /10°	94.811	100.000	107.304
D ₆₅ /2°	95.047	100.000	108.883
C/10°	97.285	100.000	116.145
C/2°	98.074	100.000	118.232

only if the required combination is not therein, CIE 15.2 (see 8.1).

4.2 Calculation of CIELAB color difference values.

4.2.1 Calculate the CIELAB ΔL^* , ΔC_{ab}^* , and ΔH_{ab}^* color difference values as follows, where subscripts S and R refer to the specimen and reference CIELAB values:

$$\begin{aligned} \Delta L^* &= L^*_S - L^*_R \\ \Delta a^* &= a^*_S - a^*_R \\ \Delta b^* &= b^*_S - b^*_R \\ \Delta C_{ab}^* &= C_{ab,S}^* - C_{ab,R}^* \\ \Delta E_{ab}^* &= [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2} \\ \Delta H_{ab}^* &= pq[\Delta E_{ab}^*]^2 - (\Delta L^*)^2 - (\Delta C_{ab}^*)^2]^{1/2} \end{aligned}$$

where $p = 1$ if $m \geq 0$ or $p = -1$ if $m < 0$ and $q = 1$ if $|m| \leq 180$ or $q = -1$ if $|m| > 180$ where $m = h_{ab,S} - h_{ab,R}$ in which $|\dots|$ indicates that the positive value is to be used regardless of the sign of the expression between the two lines

4.2.2 An equivalent method of calculating ΔH_{ab}^* is to use Eq. 5:

$$\Delta H_{ab}^* = t[2(C_{ab,S}^* C_{ab,R}^* - a^*_S a^*_R - b^*_S b^*_R)]^{1/2} \quad (5)$$

where $t = 1$ if $a^*_S b^*_R \leq a^*_R b^*_S$ or $t = -1$ if $a^*_S b^*_R > a^*_R b^*_S$

4.3 Calculation of ΔE_{cmc} .

4.3.1 Calculate the color difference in CMC ($\ell:c$) units using Eq. 6:

$$\begin{aligned} \Delta E_{cmc} &= \left[\left(\frac{\Delta L^*}{\ell S_L} \right)^2 + \left(\frac{\Delta C_{ab}^*}{c S_C} \right)^2 \right. \\ &\quad \left. + \left(\frac{\Delta H_{ab}^*}{S_H} \right)^2 \right]^{1/2} \quad (6) \end{aligned}$$

where for the standard:

$$\text{for } L^*_S > 16 \quad S_L = \frac{0.040975 L^*}{1 + 0.01765 L^*}$$

$$\text{for } L^*_S \leq 16 \quad S_L = 0.511$$

$$S_C = \frac{0.0638 C_{ab}^*}{1 + 0.0131 C_{ab}^*} + 0.638$$

$$S_H = (FT + 1 - F) S_C$$

where

$$F = \left[\frac{(C_{ab}^*)^4}{(C_{ab}^*)^4 + 1900} \right]^{1/2}$$

and

$$T = 0.36 + abs [0.4 \cos (35 + h_{ab})]$$

unless h_{ab} is between 164° and 345° then

$$T = 0.56 + abs [0.2 \cos (168 + h_{ab})]$$

For the last two equations, 'abs' indicates the absolute—i.e., positive value—of the term inside the square brackets.

Note: If required, the components of CMC ($\ell:c$) color difference (ΔL_{cmc} , ΔC_{cmc} , and ΔH_{cmc}) may be calculated using the terms in the round brackets in the first line of the above equation, namely:

$$\Delta L_{cmc} = \frac{\Delta L^*}{\ell S_L}$$

$$\Delta C_{cmc} = \frac{\Delta C_{ab}^*}{c S_C}$$

$$\Delta H_{cmc} = \frac{\Delta H_{ab}^*}{S_H}$$

4.3.1.1 When $\ell = 2.0$ this equation fixes the ratio of the three terms ($S_L:S_C:S_H$) to correlate with visual assessment of typical textile samples. Other values of ℓ may be required in cases where the surface characteristics are significantly different. For example, other values may be required when measuring very dark trials, but the user should assume an ℓ value of 2.0 until actual results indicate a need for adjustment. In practice, "c" is always set to unity and it may be omitted from the equation.

4.3.2 Concept of unit volume/tolerance. The equation for $\Delta E_{cmc} \leq 1.0$ describes an ellipsoidal volume, with axes in the direction of lightness, chroma and hue centered about a standard. Ellipsoid semi-axis lengths of ℓS_L , $c S_C$, and S_H calculated for a given standard describe a unit volume of acceptance within which all samples are less than 1.0 ΔE_{cmc} unit from the standard.

NOTE: Size and orientation of the diagram shown in Fig. 2 varies considerably depending on location of standard in color space and is used here as a means of conceptualizing the semi-axes.

4.3.2.1 This volume—and the size and ratio of its semi-axes—become the basis for the establishment of an appropriately sized volume of acceptability for a given commercial situation by the application of a commercial factor (cf) to all dimensions.

4.3.2.2 The Volume of Acceptance is defined by $\Delta E_{cmc} \leq cf$.

4.4 Reporting of results. The use of illuminant D₆₅ and 10° observer calculations and ($\ell:c$) ratio of 2:1 are recommended as the standard for calculating ΔE_{cmc} values. If other illuminants, observers or CMC ($\ell:c$) ratios are used they must be specified as part of the value