

AATCC Evaluation Procedure for Small Color Differences

1. Purpose and Scope

1.1 Ellipsoidal color difference equations are used to calculate a volume of acceptable color difference, based on the color of the Reference and the specified tolerance. *CIEDE2000* and *CMC (l:c)* are both ellipsoidal color difference equations, and they may both be used for color evaluation of textile and other materials in many applications. The current data (see references 6.1 through 6.7) indicate that the *CIEDE2000* metric performs slightly better in terms of agreement with visual evaluations. It is therefore being recommended as the primary color difference method to use for instrumental evaluations.

1.2 The *CIEDE2000* formula is denoted as ΔE_{00} . The *CMC (l:c)* formula is generally shown as ΔE_{cmc} . Both *CIEDE2000* and *CMC (l:c)* are extensions of the *CIE 1976 L*a*b** color-difference formula with corrections for variation in color-difference perception dependent on lightness, chroma, and hue. *CIEDE2000* additionally includes corrections for chroma-hue interaction.

1.3 The *CMC (l:c)* modification to *CIELAB* provides a unit of measurement for the acceptance volume about a reference color. This volume takes the shape of an ellipsoid whose base semi-axes are defined by S_L , S_C , and S_H in the directions of lightness, chroma, and hue differences, respectively, in *CIELAB* color space. The actual size and shape of the ellipsoid is further controlled by modifying parametric factors l and c , see section 3.3. The *CMC (l:c)* formula varies the ratio of the lengths of these three semi-axes systematically throughout *CIELAB* color space according to the equality of lengths with perceptually equal color differences, regardless of both the color of the Reference and the direction of difference of any trial specimen from it. Around any given reference color, the ratio $S_L:S_C:S_H$ is fixed regardless of the industrial application. However, it should be noted that color difference evaluations may vary if the reference and trial specimen are switched. Note: The difference in the resulting total color difference computed in the two directions (reference to specimen vs. specimen to reference) is not significant for color differences of the magnitude that is relevant for textile applications (under 3.0 units).

The *CIEDE2000* modification to *CIELAB* also provides an ellipsoidal acceptance volume about a reference color. It is analogous to *CMC (l:c)* in the use of

three semi-axes for the lightness, chroma, and hue dimensions. *CIEDE2000* was developed to improve on shortcomings identified in the performance of various color difference equations including *CMC (l:c)*. *CIEDE2000* differs from *CMC (l:c)* in an important application, however. *CIEDE2000* determines the arithmetic mean between the Reference and Specimen and uses this as the point in *CIELAB* color space to determine the shape of the ellipsoidal space. This resolves the associated color differences encountered when the reference and the specimen are reversed. This also results in the boundary for the acceptance volume to be somewhat imprecise for a group of specimens compared to a single reference, since the calculations are based on the relation of each specimen to the reference. Because of this difference in computing the acceptance boundary, *CIEDE2000* may not be suited for some applications, especially when visual plotting is utilized.

1.4 Both *CIEDE2000* and *CMC (l:c)* support the use of component color difference metrics, that is DL , DC , and DH values which are weighted for visual acceptance by the *CMC (l:c)* or *CIEDE2000* internal math, respectively. These weighted values of DL , DC , and DH are correlated more strongly to visual color evaluation than legacy metrics such as DL^* , Da^* , Db^* , DC^* , DH^* . For *CMC (l:c)*, these component color difference terms are DL_{cmc} , DC_{cmc} , DH_{cmc} . For *CIEDE2000*, these terms are DL_{00} , DC_{00} , and DH_{00} .

1.5 In applications which require rating within or about a critical tolerance, multiple levels consisting of ellipsoidal volumes representing various levels of color difference may be generated in both ΔE_{00} and ΔE_{cmc} . This will result in a set of concentric volumes/tolerances which provide a uniform grading system when correlated to and associated with a predefined set of terms. An example of this is the association of Gray Scale Grades with word descriptions as shown in Table III of AATCC EP9.

2. Principle

2.1 These equations offer significantly improved correlation between visual assessments and instrumentally measured color differences compared to *CIELAB DE*ab*. Industrial users of these equations are able to adopt a single-number tolerance for judging the acceptability of

a color match in most situations, regardless of both the color of the reference and the direction of the color difference of any specimen from it.

3. Terminology

3.1 **CIE 1976 L*a*b* equation, n.**—a commonly used equation which transforms CIE tristimulus values into a three-dimensional opponent color space. Generally abbreviated as *CIELAB* (see Reference section 6.1).

3.2 **ΔE_{cmc} , n.**—in color difference evaluation, a single number defining the total color difference in *CMC (l:c)* units of a specimen from a reference.

3.3 **semi-axes (S_L , S_C , and S_H), n.**—in *CMC (l:c)* color difference evaluation, basic individual dimensions of the *CMC* tolerance volume, related to lightness, chroma and hue respectively, whose ratios are determined by the position of the reference in *CIELAB* space. The actual semi-axes of the ellipsoid are determined by these values and modified by parametric factors l and c .

3.4 **parametric factors (l and c) n.**—in *CMC (l:c)* color difference equation, weighting factors used to adjust the component difference ratios, lightness and chroma respectively, based on the effect of the surface characteristics of the stimuli, such as texture. The values $l = 2$ and $c = 1$ are usually used for commercial pass/fail tolerance predictions of textile samples, although l may be varied depending on the texture of the material. The value of c is always set to one.

3.5 **ΔE_{00} , n.**—in color difference evaluation, a single number defining the mean total color difference in *CIEDE2000* units of a specimen from a reference and vice versa.

3.6 **parametric factors (k_L , k_C , and k_H), n.**—in *CIEDE2000* color difference equation, weighting factors used to adjust the component difference ratios, lightness, chroma and hue respectively, based on the effect of the surface characteristics of the stimuli, such as texture.

4. Calculations

In all calculations below, the subscript **R** refers to the Reference material and the subscript **S** refers to the Specimen. In some documents this pair is referred to as **S** for Standard and **B** for Batch. In other documents these are also referred to as **S** for Standard and **T** for Trial or **0** and **1**, respectively. Users should clearly define the method being used.

4.1 Calculation of CIELAB Values

4.1.1 Calculate the *CIELAB* L^* , a^* , b^* , C_{ab}^* and h_{ab} values from the CIE tristimulus values X , Y , Z of each specimen using Equations (1), (2), (3) and (4):

$$L^* = 116f(Q_Y) - 16 \quad (1)$$

$$a^* = 500[f(Q_X) - f(Q_Y)] \quad (2)$$

$$b^* = 200[f(Q_Y) - f(Q_Z)] \quad (3)$$

where:

$$Q_X = \left(\frac{X}{X_n}\right); Q_Y = \left(\frac{Y}{Y_n}\right); Q_Z = \left(\frac{Z}{Z_n}\right)$$

and

$$f(Q_i) = Q_i^{1/3} \text{ if } Q_i > \left(\frac{6}{29}\right)^3$$

else

$$f(Q_i) = \left(\frac{841}{108}\right)Q_i + \frac{4}{29} \text{ if } Q_i \leq \left(\frac{6}{29}\right)^3$$

Here, i varies as X , Y and Z .

$$C_{ab}^* = \sqrt{(a^*)^2 + (b^*)^2} \quad (4)$$

$$h_{ab} = \begin{cases} \arctan\left(\frac{b^*}{a^*}\right) & \text{if } a^* > 0 \text{ and } b^* \geq 0 \\ \arctan\left(\frac{b^*}{a^*}\right) + 360^\circ & \text{if } a^* > 0 \text{ and } b^* < 0 \\ \arctan\left(\frac{b^*}{a^*}\right) + 180^\circ & \text{if } a^* < 0 \\ 90^\circ & \text{if } a^* = 0 \text{ and } b^* > 0 \\ 270^\circ & \text{if } a^* = 0 \text{ and } b^* < 0 \end{cases} \quad (5)$$

$$h_{ab} = 0 \quad \text{if } a^* = 0 \text{ and } b^* = 0 \quad (6)$$

4.1.2 For these equations, X_n , Y_n , and Z_n are the CIE tristimulus values for the chosen illuminant and observer. The preferred combination, for textile applications, is *CIE standard illuminant D65* and the *CIE 1964 supplementary standard (10°) observer*. Values for this combination and for some others are given in Table I. Values for combinations not in Table I should be obtained by referring to ASTM E308 (see Reference section 6.11) or, only if the required combination is not therein, CIE 015:2018 (see Reference section 6.10).

4.2 Calculation of CIELAB Color Difference Values

Table I—Tristimulus Values for Selected Illuminant-Observer Combinations

Illuminant-Observer Combinations	Tristimulus Values		
	X_n	Y_n	Z_n
D ₆₅ /10°	94.811	100.000	107.304
D ₆₅ /2°	95.047	100.000	108.883

4.2.1 Calculate the *CIELAB* ΔL^* , ΔC_{ab}^* , and ΔH_{ab}^* color difference values as follows:

$$\Delta L^* = L^*_S - L^*_R \quad (7)$$

$$\Delta a^* = a^*_S - a^*_R \quad (8)$$

$$\Delta b^* = b^*_S - b^*_R \quad (9)$$

$$\Delta C_{ab}^* = C_{ab,S}^* - C_{ab,R}^* \quad (10)$$

$$\Delta E_{ab}^* = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2} \quad (11)$$

$$\Delta H_{ab}^* = pq \quad (12)$$

where:

$p = 1$ if $m \geq 0$ or $p = -1$ if $m < 0$ and $q = 1$ if $|m| \leq 180$ or $q = -1$ if $|m| > 180$
 $m = h_{ab,S} - h_{ab,R}$ in which [...] indicates that the positive value is to be used regardless of the sign of the expression between the two lines.

4.2.2 An equivalent method of calculating ΔH_{ab}^* is to use Equation (13):

$$\Delta H_{ab}^* = t [2(C_{ab,S}^* C_{ab,R}^* - a_{S,R}^* a_{R,S}^* - b_{S,R}^* b_{R,S}^*)]^{1/2} \quad (13)$$

where:

$$t = 1 \quad \text{if } a_{S,R}^* b_{R,S}^* \leq a_{R,S}^* b_{S,R}^* \\ \text{or } t = -1 \quad \text{if } a_{S,R}^* b_{R,S}^* > a_{R,S}^* b_{S,R}^*$$

4.3 Calculation of CIEDE2000

4.3.1 Symbols and Definitions

L^*	<i>CIELAB</i> Lightness
a^*, b^*	<i>CIELAB</i> a^*, b^* coordinates
C_{ab}^*	<i>CIELAB</i> chroma
h_{ab}	<i>CIELAB</i> hue angle
L'	<i>CIEDE2000</i> lightness
$\bar{L}'$	arithmetic mean of the <i>CIEDE2000</i> lightnesses of two color stimuli
a', b'	<i>CIEDE2000</i> a', b' coordinates
C'	<i>CIEDE2000</i> chroma

$\bar{C}'$	arithmetic mean of the <i>CIEDE2000</i> chromas of two color stimuli
h'	<i>CIEDE2000</i> hue angle
$\bar{h}'$	arithmetic mean of the <i>CIEDE2000</i> hue angles of two color stimuli
G	switching function used in the modification of a^*
$\Delta L'$	<i>CIEDE2000</i> lightness difference
$\Delta C'$	<i>CIEDE2000</i> chroma difference
$\Delta h'$	<i>CIEDE2000</i> hue-angle difference
$\Delta H'$	<i>CIEDE2000</i> hue difference
ΔE_{00}	<i>CIEDE2000</i> color difference
S_L	lightness weighting function
S_C	chroma weighting function
S_H	hue weighting function
T	T-function for hue weighting
R_T	rotation function
$\Delta \theta$	hue dependence of rotation function
R_C	chroma dependence of rotation function
k_L	lightness parametric factor
k_C	chroma parametric factor
k_H	hue parametric factor

4.3.2 CIEDE2000 Calculations

All angular quantities in this Evaluation Procedure shall be evaluated in degrees. *CIELAB* L^* , a^* , b^* and C_{ab}^* coordinates of the two samples shall be calculated according to ISO 11664-4:2008/CIE S 014-4/E:2008 which are shown above in Equations (1) through (13). Modified *CIELAB* coordinates shall be calculated according to Equations (14) to (20).

$$L' = L^* \quad (14)$$

$$a' = (1 + G)a^* \quad (15)$$

$$b' = b^* \quad (16)$$

$$C' = (a'^2 + b'^2)^{1/2} \quad (17)$$

$$h' = \begin{cases} \arctan\left(\frac{b'}{a'}\right) & \text{if } a' > 0 \text{ and } b' \geq 0 \\ \arctan\left(\frac{b'}{a'}\right) + 360^\circ & \text{if } a' > 0 \text{ and } b' < 0 \\ \arctan\left(\frac{b'}{a'}\right) + 180^\circ & \text{if } a' < 0 \\ 90^\circ & \text{if } a' = 0 \text{ and } b' > 0 \\ 270^\circ & \text{if } a' = 0 \text{ and } b' < 0 \end{cases} \quad (18)$$

$$h' = 0^\circ \quad \text{if } a' = 0 \text{ and } b' = 0 \quad (19)$$