

GUIDE FOR THE

ASSESSMENT OF PARAMETRIC ROLL RESONANCE IN THE DESIGN OF CONTAINER CARRIERS SEPTEMBER 2004

Guide to Color Coding Used in Online Version of the Rules

The following summarizes the colors corresponding to Rule Changes, Corrigenda items and editorial changes in the Rules files which are available for download.

Rule Changes:

NOTICE NO. 1 – June 2008 (effective 1 June 2008)

Corrigenda:

CORRIGENDA/EDITORIALS – 17 December 2004

Editorials:

Editorial Changes



GUIDE FOR THE

ASSESSMENT OF PARAMETRIC ROLL RESONANCE IN THE DESIGN OF CONTAINER CARRIERS

SEPTEMBER 2004 (Updated June 2008 – see next page)

**American Bureau of Shipping
Incorporated by Act of Legislature of
the State of New York 1862**

**Copyright © 2004
American Bureau of Shipping
ABS Plaza
16855 Northchase Drive
Houston, TX 77060 USA**

Updates

June 2008 consolidation includes:

- September version plus Notice No. 1, Corrigenda/Editorials

Foreword

The main purpose of this Guide is to supplement the Rules and the other design and analysis criteria that ABS issues for the classification of container carriers in relation to parametric roll resonance phenomenon.

The Guide contains a brief description of the physical phenomenon of parametric roll resonance, which may cause an excessive roll of a containership in longitudinal (head and following) waves. The Guide also contains a description of criteria used to determine if a particular vessel is vulnerable to parametric roll (susceptibility criteria) and how large these roll motions might be (severity criteria). Recommendations are given for further actions if a ship is found to be endangered by the possibility of parametric roll, including numerical simulations and a model test. Means of mitigation of consequences of the parametric roll are briefly considered.

If criteria and requirements included in this Guide are satisfied, ABS may assign an optional class notation as recognition of safety performance in relation to parametric roll resonance.

ABS welcomes comments and suggestions for improvement of this Guide. *Comments or suggestions can be sent electronically to rdd@eagle.org.*

This Page Intentionally Left Blank

11/11/2008 11:11:11 AM

GUIDE FOR THE ASSESSMENT OF PARAMETRIC ROLL RESONANCE IN THE DESIGN OF CONTAINER CARRIERS

CONTENTS

SECTION 1	Introduction	1
1	Parametric Roll Resonance in Longitudinal Waves.....	1
1.1	General	1
1.2	Stability in Longitudinal Waves	1
1.3	Roll Motions in Calm Water.....	2
1.4	Physics of Parametric Roll Resonance	3
1.5	Influence of Roll Damping	5
1.6	Amplitude of Parametric Roll.....	6
1.7	Influence of Ahead Speed and Wave Direction.....	7
1.8	Definitions	8
1.9	Nomenclature.....	8
FIGURE 1	Profile of Waterline in Wave Trough (Solid) vs. Calm Water (Dotted).....	1
FIGURE 2	Profile of Waterline in Wave Crest (Solid) vs. Calm Water (Dotted).....	2
FIGURE 3	Undamped Small Roll Motions in Calm Water.....	2
FIGURE 4	Parametric Roll Resonance	3
FIGURE 5	Development of Parametric Roll Resonance; Case 1: Ship Encounters Roll Disturbance when Stability is Increasing.....	4
FIGURE 6	Development of Parametric Roll Resonance; Case 2: Ship Encounters Roll Disturbance when Stability is Decreasing	5
FIGURE 7	Successively Decreasing Roll Amplitudes due to Roll Damping in Calm Water.....	5
FIGURE 8	Change of Instantaneous GM Value with Increasing Heel Angle.....	7
FIGURE 9	Development of Parametric Roll	7
FIGURE 10	Coordinate System for Hydrostatic Calculations	9
FIGURE 11	Definition of the Draft i -th Station with j -th Position of the Wave Crest	10
FIGURE 12	Definition of the Offsets at i -th Station with j -th Position of the Wave Crest	10

SECTION 2	Parametric Roll Criteria	11
1	General	11
2	Susceptibility Criteria	13
2.1	Design Wave	13
2.2	Stability in Longitudinal Waves	13
2.3	Ahead Speed	16
2.4	Application of Susceptibility Criteria	16
3	Severity Criterion for Parametric Roll Resonance in Head Seas	17
TABLE 1	Wave Heights	13
FIGURE 1	Diagram Showing Selection of Wave Length and Ahead Speed	12
FIGURE 2	Change of Stability in Longitudinal Wave	15
FIGURE 3	GM as a Function of Wave Crest Position	15
FIGURE 4	Restoring Moment as a Function of Wave Position and Heel Angle	19
FIGURE 5	Restoring Term as a Function of Time and Heel Angle	19
SECTION 3	Numerical Simulations	21
SECTION 4	Mitigation of Parametric Roll Resonance	23
1	Operational Guidance	23
2	Anti-Rolling Devices	23
FIGURE 1	Example of Polar Diagram and Color Scale	24
SECTION 5	Optional Class Notation	25
TABLE 1	Optional Class Notations	25
APPENDIX 1	Sample Calculations	27
TABLE 1	Particulars of a Sample Container Carrier	27
TABLE 2	Conditions for Sample Calculations	28
TABLE 3	Calculation of GM Value for Different Positions of Wave Crest along Ship Hull (Simplified Method – 2/2.2)	28
TABLE 4	Sample Results for Susceptibility Criteria	29
TABLE 5	GZ Curves for Different Positions of Wave Crest	31
TABLE 6	Sample Results for Forward Speed Calculations	32
TABLE 7	Sample Input Data for Integration of Roll Equation	33
TABLE 8	Amplitude of Parametric Roll in Degrees	34

FIGURE 1	Lines of Sample Container Carrier	27
FIGURE 2	Calculation of GM Value for Different Positions of Wave Crest along Ship Hull (Simplified Method – 2/2.2)	29
FIGURE 3	GZ Curves for Different Positions of Wave Crest	30
FIGURE 4	Solution of the Roll Equation for V_1 and $\mu = 0.1$	34

APPENDIX 2 Sample Polar Diagrams 35

FIGURE 1	Sample Polar Diagram	35
FIGURE 2	Sample Polar Diagram – Full Load, Sea State 9	36
FIGURE 3	Sample Polar Diagram – Full Load, Sea State 8	37
FIGURE 4	Sample Polar Diagram – Full Load, Sea State 7	37
FIGURE 5	Sample Polar Diagram – Partial Load, Sea State 9	38
FIGURE 6	Sample Polar Diagram – Partial Load, Sea State 8	38
FIGURE 7	Sample Polar Diagram – Partial Load, Sea State 7	39

APPENDIX 3 Criteria for Parametric Roll of Large Containerships in Longitudinal Seas 41

This Page Intentionally Left Blank

Copyright 2008 by American Bureau of Shipping

SECTION 1 Introduction

1 Parametric Roll Resonance in Longitudinal Waves

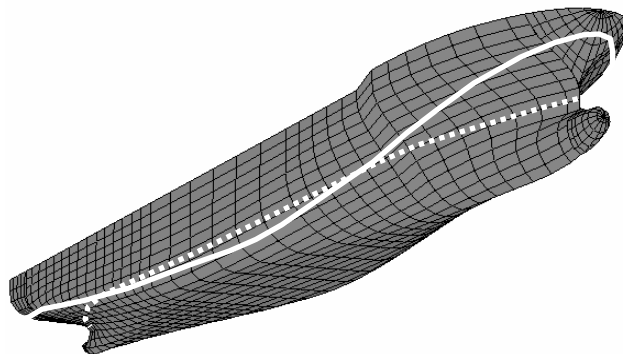
1.1 General

Parametric roll resonance in longitudinal (head and following) seas is observed as a significant amplification of roll motions, which may become dangerous to the ship, its cargo and crew. This phenomenon is related to the periodic change of stability as the ship moves in longitudinal waves at a speed when the ship's wave encounter frequency is approximately twice the rolling natural frequency and the damping of the ship to dissipate the parametric roll energy is insufficient to avoid the onset of a resonant condition.

1.2 Stability in Longitudinal Waves

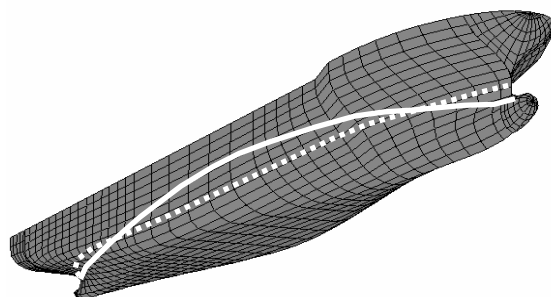
If a ship is located in a wave trough, the average waterplane width is significantly greater than in calm water. The flared parts of the bow and stern are more deeply immersed than in calm water and the wall-sided midship is less deep. This makes the mean, instantaneous waterplane wider than in calm water with the result that the metacentric height (GM) is increased over the calm water value. (See Section 1, Figure 1)

FIGURE 1
Profile of Waterline in Wave Trough (Solid) vs. Calm Water (Dotted)



In contrast to the above, when the wave crest is located amidships, the waterplane at the immersed portions of the bow and stern are narrower than in calm water. Consequently, the average waterplane is narrower and the GM is correspondingly decreased in comparison to calm water (see Section 1, Figure 2). As a result, the roll restoring moment of the ship changes as a function of the wave's longitudinal position along the ship.

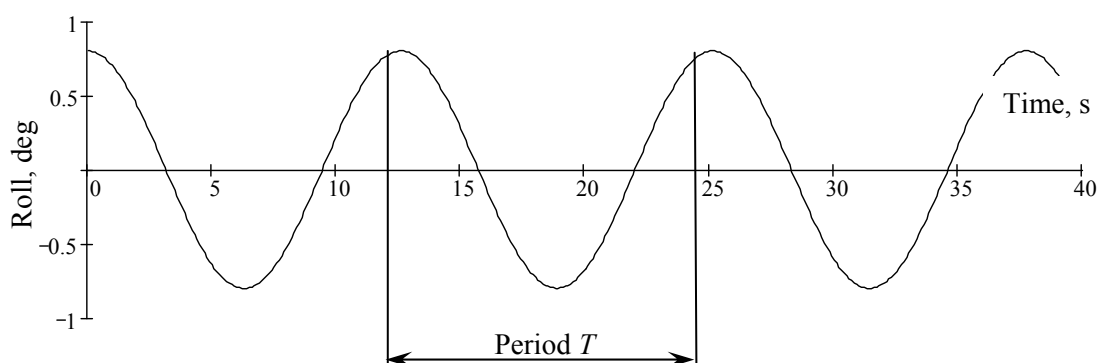
FIGURE 2
Profile of Waterline in Wave Crest (Solid) vs. Calm Water (Dotted)



1.3 Roll Motions in Calm Water

When a ship is in calm water, any disturbance in transversal (as from a wind gust) will lead to roll motions. When the roll equilibrium is disturbed, the hydrostatic restoring moment acts to oppose the instantaneous roll angle and tends to return the ship back to the upright position. Because of inertia, the ship does not stop at the instant when the equilibrium angle is reached but continues to roll at a progressively slower velocity until a maximum roll angle is reached. At this point, the excess roll restoring moment causes the ship to begin to right itself. Once upright, inertia causes the ship to continue to roll. As before, the restoring moment works against further motion and it stops at some roll angle. The restoring moment then again pushes the ship back to the equilibrium, and again, because of inertia, the ship cannot stop at the equilibrium point and the motion cycle is repeated. The period of such roll oscillations in calm water is known as the “natural roll period” and is related to ship stability and mass distribution. The corresponding roll frequency is called the “natural frequency”. A sample of such a free roll oscillation is shown in Section 1, Figure 3.

FIGURE 3
Undamped Small Roll Motions in Calm Water

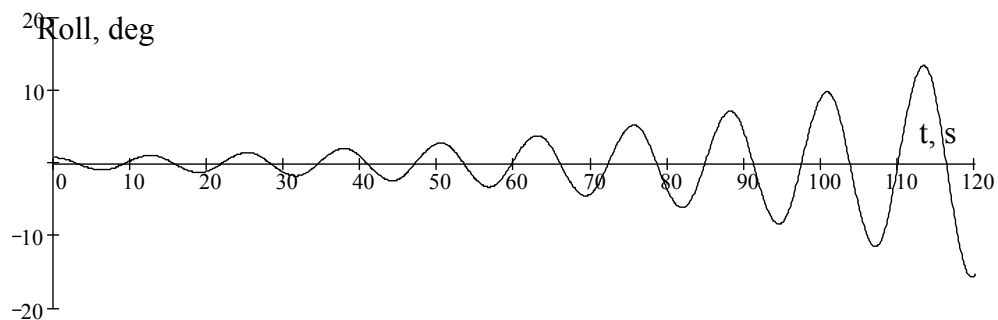


If a ship sailed on a course exactly perpendicular to the crests of head or following seas, there would be no wave-induced heeling moment. However, the ship may experience a very small roll disturbance from some external or internal cause (in reality, roll disturbances can always exist, e.g., wind). Normally, when the roll equilibrium is disturbed in the absence of a wave excitation moment, the ship rolls with its natural roll frequency and the motion time history is similar to that shown in Section 1, Figure 3

1.4 Physics of Parametric Roll Resonance

As described earlier, when a ship is sailing in longitudinal (head or following) or nearly longitudinal seas, its stability increases in the wave trough and decreases on the wave crest. If this oscillatory change in stability occurs at approximately twice the natural roll period, roll motions may increase to a significant, possibly unacceptable, angle as a result of parametric roll resonance. A typical sample time history is shown in Section 1, Figure 4.

FIGURE 4
Parametric Roll Resonance



The most rapid increase of parametric roll motion could be observed when the ship experiences an external roll disturbance at the time when the wave crest is moving away from amidships, *i.e.*, the condition of improving or increasing stability, in combination with an encounter frequency approximately twice that of the natural roll frequency. In this situation, the restoring moment tends to accelerate the ship back to equilibrium with a larger-than-calm-water moment because the ship is entering the wave trough where stability is improved. As a result, at the end of the first quarter of the period T , the roll angle is slightly larger than it would be in calm water. See Section 1, Figure 5.

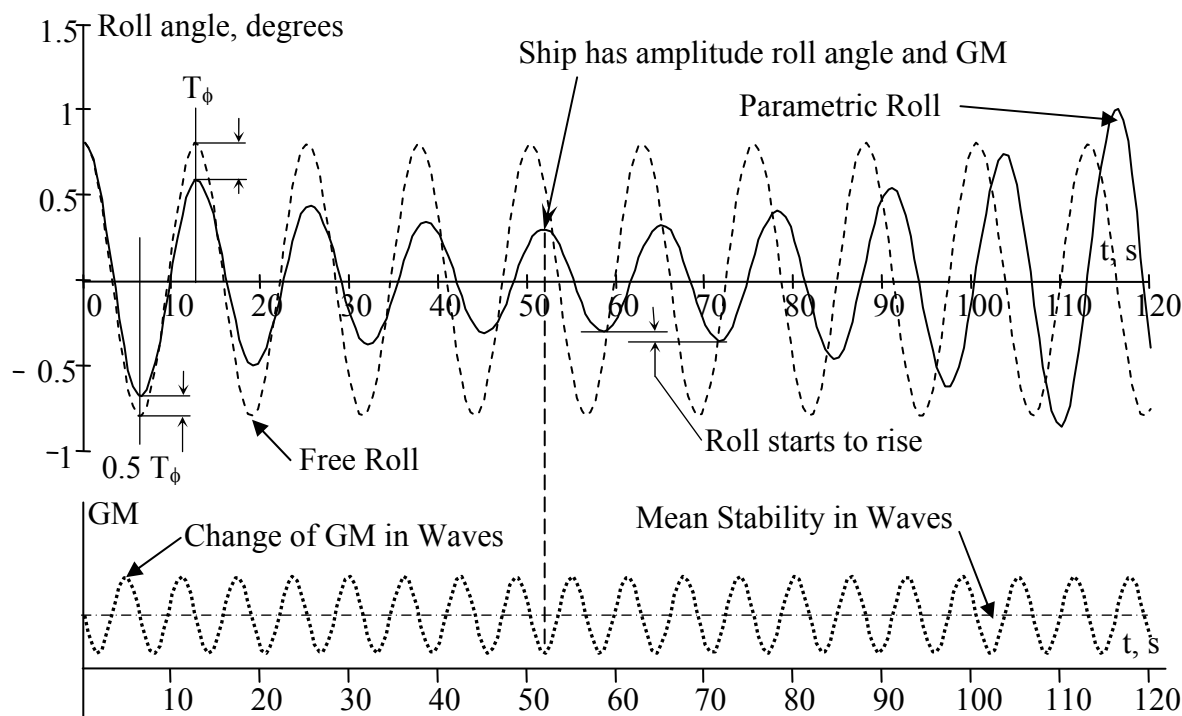
At the end of the first quarter period of roll oscillation, the ship reaches a zero-degree roll angle, which is the upright equilibrium attitude, but the roll motion does not stop there because of the roll inertia.

During the second quarter of the period, the ship encounters a wave crest and its stability is decreased. Meanwhile, the roll motion inertia makes the ship continue to roll. The restoring moment now resists further motion, but with a less-than-calm-water value since ship stability is lessened on the wave crest. As a result, the ship rolls more than it would in calm water with the same roll disturbance, consequently, after the second quarter, the increase in roll angle is even greater than after the first quarter. This is shown in Section 1, Figure 5.

In the third quarter, the ship enters the wave trough and an increased restoring moment pushes it back with an increased force. The situation is analogous to that observed during the first quarter. The observations in the fourth quarter are similar to those in the second quarter, and the roll angle continues to increase, as shown in Section 1, Figure 5.

With no further change in wave amplitude and ship speed, this combination of restoring (with a larger-than-calm-water) and resisting the roll (with less-than-calm-water) can cause the roll angle to progressively increase to a large and possibly dangerous level. This constitutes the parametric roll resonance phenomenon.

FIGURE 5
Development of Parametric Roll Resonance;
Case 1: Ship Encounters Roll Disturbance when Stability is Increasing



If the ship experiences the roll disturbance while approaching a wave crest, i.e., when the stability is decreasing, the evolution of parametric roll development is different.

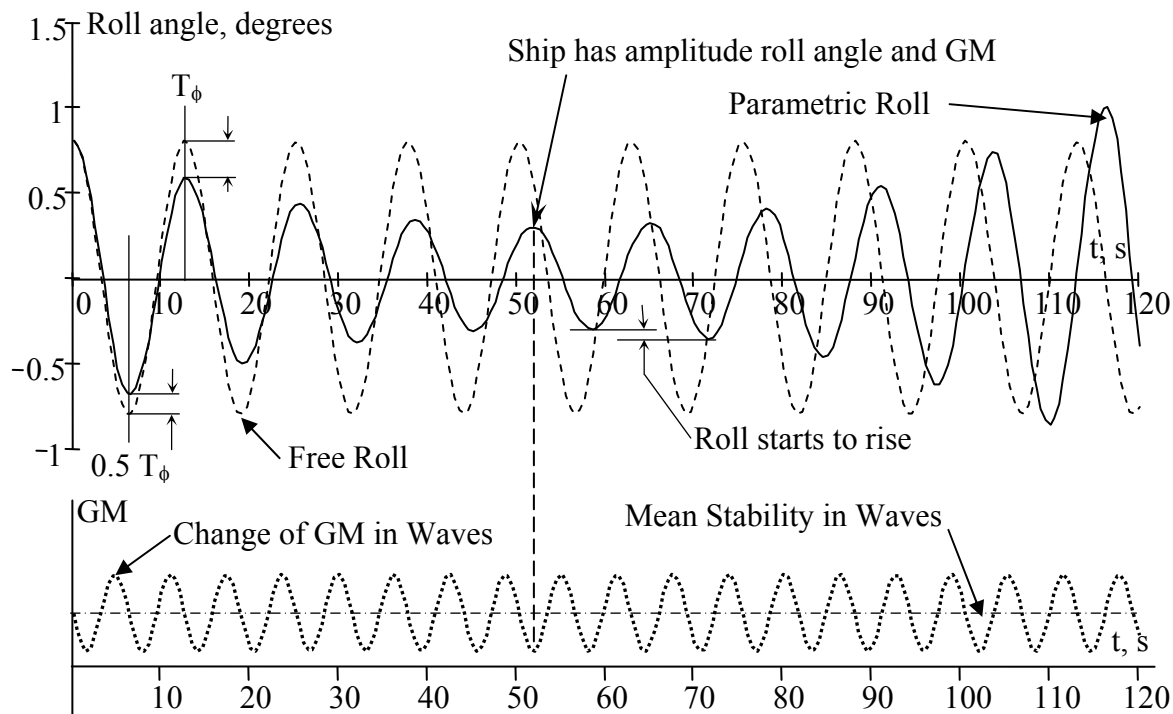
The same factors that were increasing roll in the first case now damp the roll motion. When the ship is just disturbed, it approaches a wave crest with its stability decreased and the “push back” is made with a smaller moment than in calm water. Once the ship reaches equilibrium, its stability starts to improve and it reaches a less-than-in-calm-water angle at the end of the first period. See Section 1, Figure 6.

Such a combination of decreasing and increasing roll restoring moments is capable of significantly decreasing roll. However, this situation does not last long. The changing stability leads to a slight change in the natural period. As a result, the roll in waves lags behind in comparison with the roll in calm water. See Section 1, Figure 6.

As can be seen from Section 1, Figure 6, the shifting phase leads to a situation where the ship reaches a peak value of roll angle and as its GM is just about to start to increase. This situation is similar to the conditions considered in the previous case.

The two considered sample scenarios represent two extreme possibilities with the most and least favorable conditions for the development of parametric roll. The real situation is usually somewhere in between.

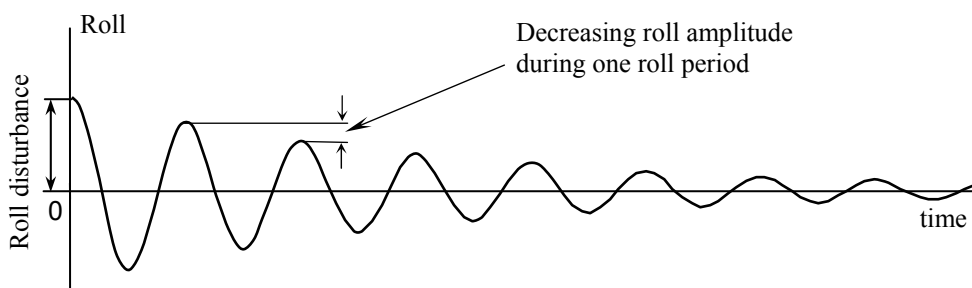
FIGURE 6
Development of Parametric Roll Resonance;
Case 2: Ship Encounters Roll Disturbance when Stability is Decreasing



1.5 Influence of Roll Damping

When a ship rolls in calm water after being disturbed, the roll amplitudes decrease successively due to roll damping. See Section 1, Figure 7. A rolling ship generates waves and eddies, and experiences viscous drag. All of these processes contribute to roll damping.

FIGURE 7
Successively Decreasing Roll Amplitudes due to
Roll Damping in Calm Water



Roll damping may play a critical role in the development of parametric roll resonance. If the “loss” of energy per cycle caused by damping is more than the energy “gain” caused by the changing stability in longitudinal seas, the roll angles will not increase and the parametric resonance will not develop. Once the energy “gain” per cycle is more than the energy “loss” due to damping, the amplitude of the parametric roll starts to grow.

There is then a roll damping threshold for parametric roll resonance. If the roll damping moment is higher than the threshold, then parametric roll resonance is not possible. If the roll damping moment is below the threshold, then the parametric roll resonance can take place.

During the parametric roll resonance the combination of harder push-backs due to the increased stability on the wave trough and larger achieved roll angles due to the decreased stability on the wave crest, which occur about twice during the roll period, makes the roll angle grow significantly. The only other condition that has to be met is that the energy loss due to roll damping is not large enough to completely consume the increase of energy caused by parametric roll resonance – the roll damping is below the threshold value.

1.6 Amplitude of Parametric Roll

The shape of the GZ curve is one of the most important factors determining the amplitude of parametric roll. As discussed in 1/1.4, the development of parametric roll requires the encounter wave frequency to be approximately twice the roll natural frequency. There is a range of encounter wave frequencies around this value that is capable of causing parametric roll resonance.

It is known that the instantaneous value of GM is a function of roll angle (see Section 1, Figure 8). It is also known that the natural roll period and natural roll frequency depend on GM value. While the GZ curve usually is practically linear in the first 10-12 degrees of heel angle, the GM does not change, so both natural roll period and frequency remain constant for small values of roll angle (up to about 10-12 degrees).

Once the roll angle increases beyond the linear portion of the GZ curve, the instantaneous GM value changes as the GZ curve bends (see Section 1, Figure 8). This causes the natural roll period and natural roll frequency to change as well. Since the wave encounter frequency remains the same, the roll natural frequency may no longer be close to twice the encounter frequency. As a result, parametric resonance conditions no longer exist and roll motions no longer receive additional energy at each cycle. This causes parametric roll to stop increasing and a certain amplitude of roll is established (see Section 1, Figure 9).

FIGURE 8
Change of Instantaneous GM Value
with Increasing Heel Angle

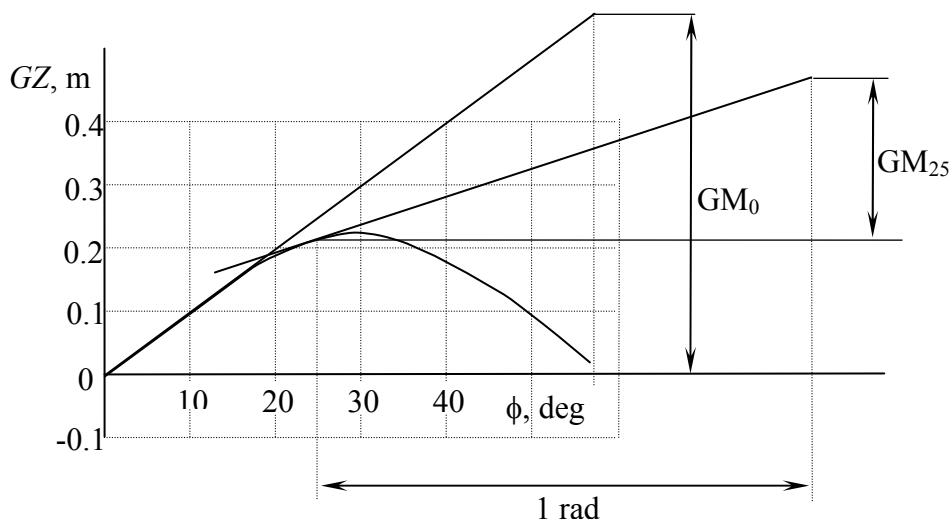
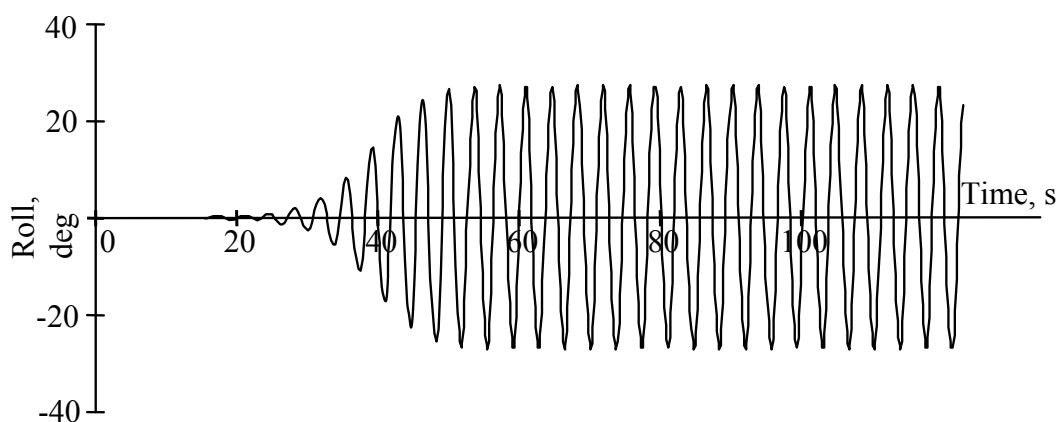


FIGURE 9
Development of Parametric Roll



1.7 Influence of Ahead Speed and Wave Direction

Longitudinal waves (head and following) cause the most change in ship's intact stability and, therefore, create maximum parametric excitation. This Guide considers head and following seas only, as these situations potentially pose the greatest danger of development of parametric roll resonance.

A ship moving through the waves encounters them with a different frequency than a ship that is not moving. This frequency is called "frequency of encounter" or "encounter frequency". It is smaller for following seas (ship speed is subtracted from wave celerity) and larger for head seas (wave celerity is added to ship ahead speed).

Frequency of encounter is a frequency with which a ship passes through wave crests and troughs. The encounter period (wave period corresponding to the wave frequency of encounter) is the time that passes while a ship encounters two adjacent wave crests or two adjacent wave troughs. It is also a frequency of change of ship's stability.

Parametric roll resonance develops when the frequency of stability change is nearly twice that of natural roll frequency or when the frequency of encounter is nearly twice that of natural roll frequency. The value of natural roll frequency mostly depends on GM value (transversal distribution of weight also may have an influence). Therefore, whether parametric roll resonance may occur in following or head seas depends mostly on current GM value. Wave length also has an influence because it is related to the wave frequency on which the frequency of encounter is dependent.

Despite the physical background of parametric roll resonance in following and head seas being identical, the latter one may experience the influence of heave and pitch motions, which are more pronounced in head seas. At this moment, the Guide does not consider such an influence.

1.8 Definitions

Parametric roll – roll motion in the regime of *parametric resonance*

Parametric roll resonance – dynamic amplification of roll motions caused by periodic change of ship stability in longitudinal (head or following) waves

Stability in longitudinal waves – improving ship stability in the wave trough and decreasing stability on the wave crest, provided that the ship is moving or is located in head or following waves

Stability on the wave crest – stability of a ship measured by conventional characteristics such as metacentric height (GM) and GZ curve, with wave crest amidships. Usually, stability characteristics are decreased in comparison with calm water

Stability in the wave trough - stability of a ship measured by conventional characteristics such as metacentric height (GM) and GZ curve, with wave trough amidships. Usually, stability characteristics are increased in comparison with calm water.

Wave period – time between two consecutive wave crests or wave troughs passing the fixed reference point.

Wave frequency – frequency corresponding to wave period

Wave celerity – velocity of wave propagation

Period of encounter - time between two consecutive wave crests or wave troughs passing the moving reference point

Frequency of encounter – frequency corresponding to Period of encounter

Free roll oscillations – roll motion in calm water that take place if a ship was inclined on angle less than 10 degrees and then was set free to roll.

Natural roll period – period of free roll oscillations or time necessary for a ship to roll from minimum roll angle to maximum roll angle, or from maximum roll angle to minimum roll angle

Natural roll frequency – frequency corresponding to natural roll period

1.9 Nomenclature

All ship-related values are defined in the coordinate system shown in Section 1, Figure 10

B molded breadth

d_m a molded draft amidships

D_m a molded depth amidships

$d(x_i, x_{Cj})$ a molded draft of i -th station with j -th position of the wave crest, see Section 1, Figure 11

$y(x_i, x_{Cj})$	offsets – half-breadth of i -th station at draft d_{Ci} , see Section 1, Figure 12
$BM(x_{Cj})$	metacentric radius for calculated i -th position of the wave crest
GM	metacentric height (GM value) defined as a distance between transverse roll metacenter and center of gravity in calm water
$GM_{\min}$	minimum metacentric height (GM value) in waves
$GM_{\max}$	maximum metacentric height (GM value) in waves
GM_a	magnitude of change of metacentric height (GM value) between wave trough and wave crest and is calculated as the difference between $GM_{\min}$ and $GM_{\max}$
GM_m	mean value of metacentric height (GM value) averaged between wave trough and wave crest: $(GM_{\max} + GM_{\min})/2$
L	length of a ship measured between perpendiculars
V_s	ahead speed, knots
V_{pr}	ahead speed, corresponding to encounter frequency twice of roll natural frequency – “parametric roll speed”, knots
V	ahead speed, m/s
$VCB(x_{Cj})$	vertical position of buoyancy center calculated i -th position of the wave crest
T_W	period of wave
ω_0	roll natural frequency in calm water
ω_m	roll natural frequency corresponding to mean value of metacentric height
ω_a	roll natural frequency corresponding to magnitude of change of metacentric height
ω_W	frequency of wave
λ	length of wave
μ	roll damping expressed as fraction of critical roll damping

FIGURE 10
Coordinate System for Hydrostatic Calculations

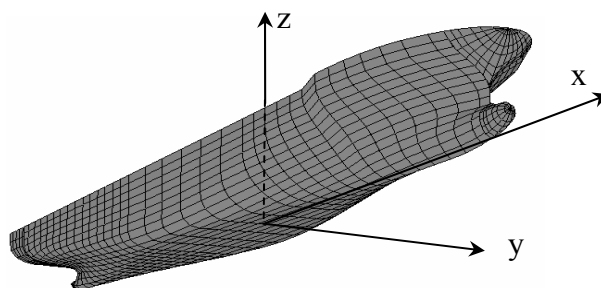


FIGURE 11
Definition of the Draft i -th Station with
 j -th Position of the Wave Crest

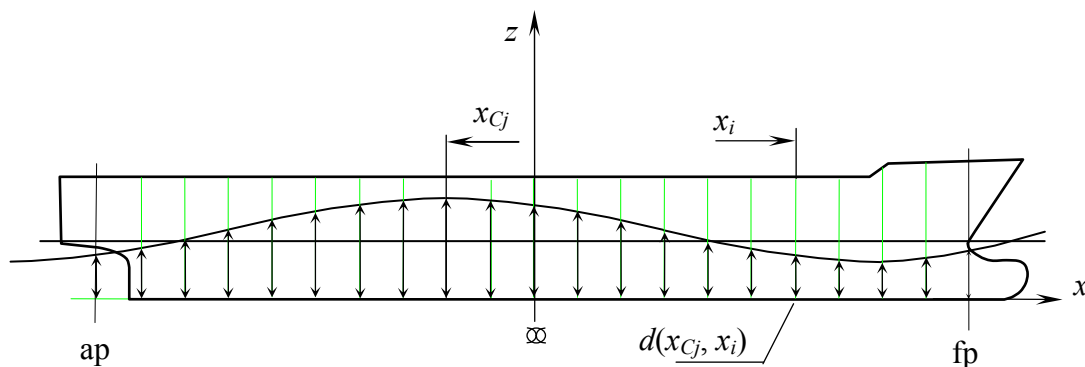
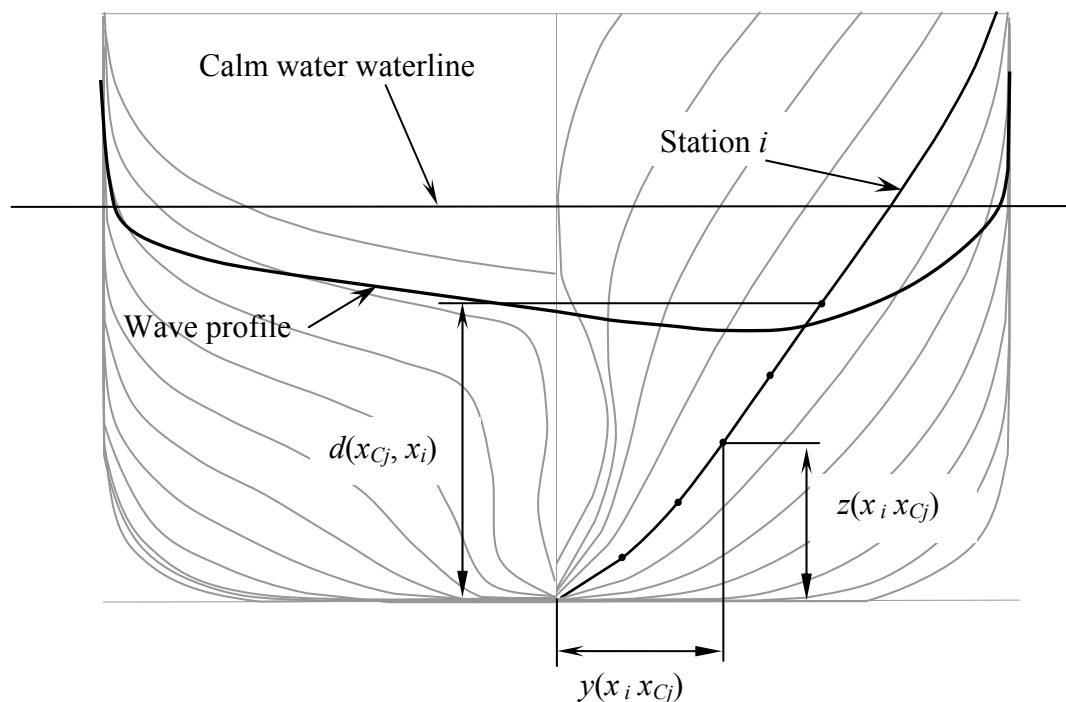


FIGURE 12
Definition of the Offsets at i -th Station with
 j -th Position of the Wave Crest



SECTION 2 Parametric Roll Criteria

1 General

Application of any criteria or performing any calculations related to parametric roll resonance requires reliable data on roll damping. At this moment, the only robust method to obtain roll damping data is a roll decay test. The roll decay test, performed by an ABS-recognized facility (normally, a towing tank – member of ITTC – International Towing Tank Conference and possessing proper certification), is required for an optional class notation (see Section 5). As an alternative, roll damping can be assumed as recommended in 2/2.4.

The general sequence of calculations and checks is described below.

The calculations and checks are done for design wave and ahead speed which will most likely lead to the development of parametric roll. These conditions are: wave length equals ship length and ahead speed results in a frequency of encounter that is about twice the roll natural frequency. If such a speed falls into the operational range of speeds, all further calculations are done for the above conditions.

If such speed does not fall into the operational range, the calculations are done for highest operational speed (design speed) and wave length equal to ship length. Additionally, the calculations are to be done for the wave length leading to encounter frequency about twice that of the roll natural frequency and the design speed. See the diagram in Section 2, Figure 1 for the choice of wave and speed conditions.

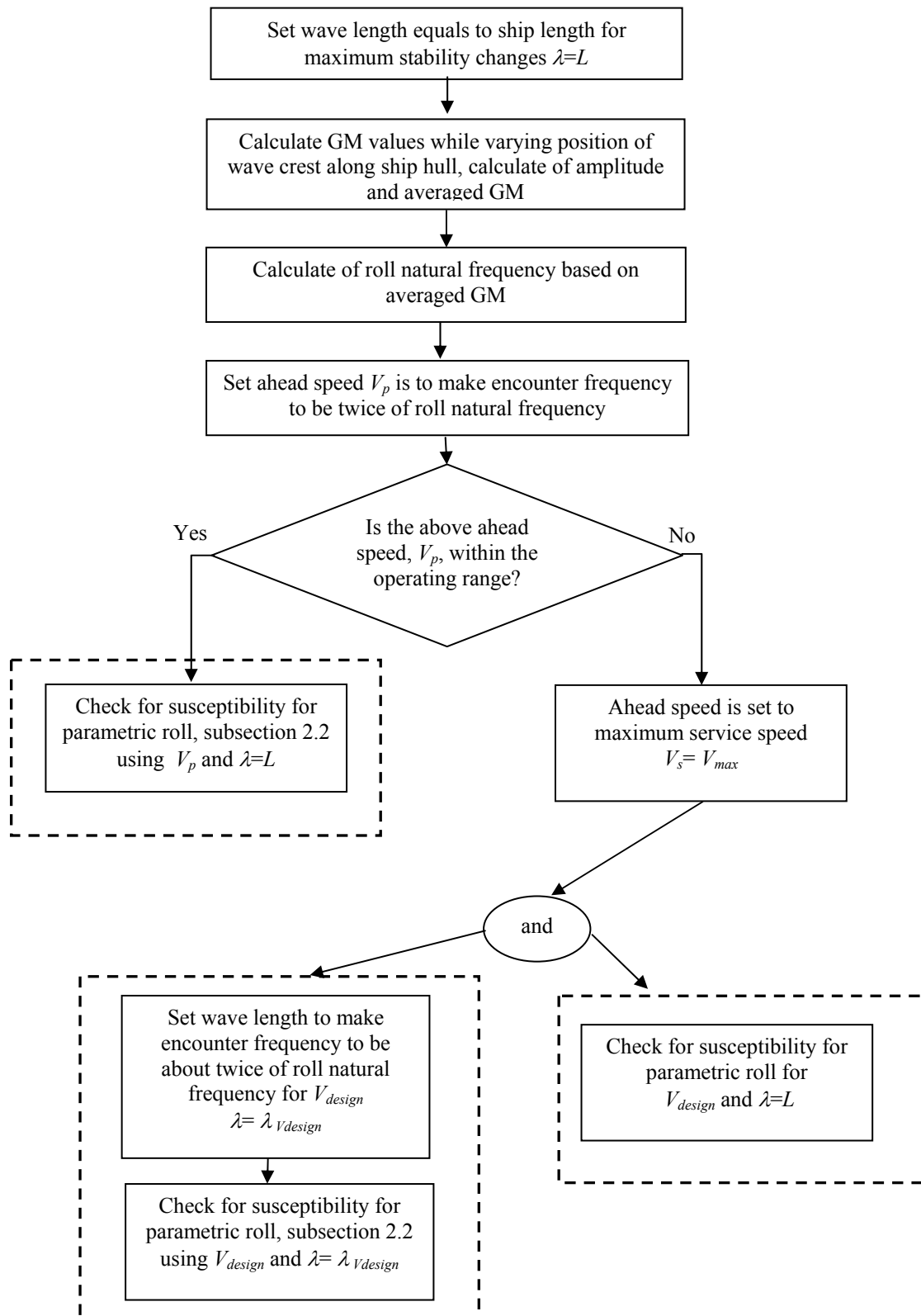
The calculations include the following steps. Stability in waves is to be computed for a number of positions of wave crest along the hull, then maximal and minimal GM values are to be evaluated. Maximal GM value is expected when the wave trough is close to amidships and the minimum GM value is expected when the wave crest is close to (but not necessarily exactly at) amidships.

The difference between maximum and minimum values defines the amplitude of parametric excitation. In order for the susceptibility criteria to be used, the parametric excitation is presented in the form of a value oscillating as sinusoidal function about the mean value, which is obtained by averaging the GM from the above-mentioned calculations. This averaged GM is used to evaluate ahead speed. The procedure is illustrated in Section 2, Figure 1.

If any of these checks indicate susceptibility, the severity of parametric roll is to be checked with a simplified numerical procedure described in Subsection 2/3. A sample of both susceptibility and severity checks can be found in Appendix 1.

Indicated severity is to be considered as a warning of a possible problem with parametric roll, which has to be addressed during further design. Sophisticated numerical simulations and model tests are to be considered. General requirements for such numerical simulations and model tests are described in Section 3. Based on the results of the numerical simulations and model tests, operational guidance is to be developed. One of the possible formats of this guidance is presented in Appendix 2.

FIGURE 1
Diagram Showing Selection of Wave Length and Ahead Speed



2 Susceptibility Criteria

2.1 Design Wave

Stability in longitudinal seas and parametric roll susceptibility criteria is to be calculated for the design wave. The length of the design wave, λ , is equal to the length between perpendiculars, L .

$$\lambda = L$$

The wave height is specified from Section 2, Table 1, which is based on the wave scatter table from IACS Recommendation No. 34, "Standard Wave Data" and represents waves of different lengths with the same probability of encounter.

TABLE 1
Wave Heights

Wave length λ , m	50	100	150	200	250	300	350	400	450
Wave height h_w , m	5.9	11.6	14.2	15.1	15.2	14.6	13.6	12.0	9.9

Linear interpolation may be used for intermediate wave lengths. The wave height need not exceed $2(D_m - d_m)$. Where D_m is a molded depth amidships and d_m is a molded draft amidships

Period of wave T_w corresponding to the accepted wave length should be calculated as:

$$T_w = \sqrt{\frac{2\pi\lambda}{g}} = 0.8\sqrt{\lambda}, \text{ seconds, for } L \text{ in meters}$$

Wave frequency:

$$\omega_w = \frac{2\pi}{T_w}, \text{ rad/s}$$

2.2 Stability in Longitudinal Waves

Stability in longitudinal waves is to be calculated by any appropriate method. The accuracy of the method is to be demonstrated to the satisfaction of ABS.

Alternatively, it is acceptable to use the method described below (assumed coordinate system shown in Section 1, Figure 10).

Note: This method assumes that the draft of the ship in the wave, as measured relative to the mean wave surface, is the same as the still water draft. Therefore, the wave-induced pitch and heave motions are ignored.

It is recommended to use offsets presented for at least 21 stations as well as at least 21 equally spaced wave crest positions.

The draft at each station at x_i for each wave crest position x_{Cj} (see Section 1, Figure 11 for the reference) should be calculated as:

$$d(x_i, x_{Cj}) = d_m + 0.5h_w \cos\left(\frac{2\pi(x_i - x_{Cj})}{\lambda}\right)$$

The half beam $y(x_i, x_{Cj})$ at each station (see Section 1, Figure 12 for the reference) for each wave crest position should be determined by linear interpolation at station drafts $d(x_i, x_{Cj})$, calculated above.