

Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete

Reported by ACI Committee 209



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Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete

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Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete

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This guide is intended for the prediction of shrinkage and creep in compression in hardened concrete. It may be assumed that predictions apply to concrete under tension and shear. It outlines the problems and limitations in developing prediction equations for shrinkage and compressive creep of hardened concrete. It also presents and compares the prediction capabilities of four different numerical methods. The models presented are valid for hardened concrete moist cured for at least 1 day and loaded after curing or later. The models are intended for concretes with mean compressive cylindrical strengths at 28 days within a range of at least 20 to 70 MPa (3000 to 10,000 psi). This document is addressed to designers who wish to predict shrinkage and creep in concrete without testing. For structures that are sensitive to shrinkage and creep, the accuracy of an individual model's predictions can be improved and their applicable range expanded if the model is calibrated with test data of the actual concrete to be used in the project.

Keywords: creep; drying shrinkage; prediction models; statistical indicators.

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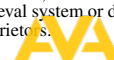
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CHAPTER 1—INTRODUCTION AND SCOPE**1.1—Background**

To predict the strength and serviceability of reinforced and prestressed concrete structures, the structural engineer requires an appropriate description of the mechanical properties of the materials, including the prediction of the time-dependant strains of the hardened concrete. The prediction of shrinkage and creep is important to assess the risk of concrete cracking, and deflections due to stripping-reshoring. As discussed in ACI 209.1R, however, the mechanical properties of concrete are significantly affected by the temperature and availability of water during curing, the environmental humidity and temperature after curing, and the composition of the concrete, including the mechanical properties of the aggregates.

Among the time-dependant properties of concrete that are of interest to the structural engineer are the shrinkage due to cement hydration (self-desiccation), loss of moisture to the environment, and the creep under sustained loads. Drying before loading significantly reduces creep, and is a major complication in the prediction of creep, stress relaxation, and strain recovery after unloading. While there is a lot of data on shrinkage and compressive creep, not much data are available for creep recovery, and very limited data are available for relaxation and tensile creep.

Creep under variable stresses and the stress responses under constant or variable imposed strains are commonly determined adopting the principle of superposition. The limitations of this assumption are discussed in Section 1.3.

Further, the experimental results of Gamble and Parrott (1978) indicate that both drying and basic creep are only partially, not fully, recoverable. In general, provided that water migration does not occur as in sealed concrete or the interior of large concrete elements, superposition can be used to calculate both recovery and relaxation.

The use of the compressive creep to the tensile creep in calculation of beam's time-dependant deflections has been

successfully applied in the work by Branson (1977), Bažant and Ho (1984), and Carreira and Chu (1986).

The variability of shrinkage and creep test measurements prevents models from closely matching experimental data. The within-batch coefficient of variation for laboratory-measured shrinkage on a single mixture of concrete was approximately 8% (Bažant et al. 1987). Hence, it would be unrealistic to expect results from prediction models to be within plus or minus 20% of the test data for shrinkage. Even larger differences occur for creep predictions. For structures where shrinkage and creep are deemed critical, material testing should be undertaken and long-term behavior extrapolated from the resulting data. For a discussion of testing for shrinkage and creep, refer to Acker (1993), Acker et al. (1998), and Carreira and Burg (2000).

1.2—Scope

This document was developed to address the issues related to the prediction of creep under compression and shrinkage-induced strains in hardened concrete. It may be assumed, however, that predictions apply to concrete under tension and shear. It outlines the problems and limitations in developing prediction equations, presents and compares the prediction capabilities of the ACI 209R-92 (ACI Committee 209 1992), Bažant-Baweja B3 (Bažant and Baweja 1995, 2000), CEB MC90-99 (Muller and Hillsdorf 1990; CEB 1991, 1993, 1999), and GL2000 (Gardner and Lockman 2001) models, and gives an extensive list of references. The models presented are valid for hardened concrete moist cured for at least 1 day and loaded at the end of 1 day of curing or later. The models apply to concretes with mean compressive cylindrical strengths at 28 days within a range of at least 20 to 70 MPa (3000 to 10,000 psi). The prediction models were calibrated with typical composition concretes, but not with concretes containing silica fume, fly ash contents larger than 30%, or natural pozzolans. Models should be calibrated by testing such concretes. This document does not provide information on the evaluation of the effects of creep and shrinkage on the structural performance of concrete structures.

1.3—Basic assumptions for development of prediction models

Various testing conditions have been established to standardize the measurements of shrinkage and creep. The following simplifying assumptions are normally adopted in the development of prediction models.

1.3.1 Shrinkage and creep are additive—Two nominally identical sets of specimens are made and subjected to the same curing and environment conditions. One set is not loaded and is used to determine shrinkage, while the other is generally loaded from 20 to 40% of the concrete compressive strength. Load-induced strains are determined by subtracting the measured shrinkage strains on the nonloaded specimens from the strains measured on the loaded specimens. Therefore, it is assumed that the shrinkage and creep are independent of each other.

Tests carried out on sealed specimens, with no moisture movement from or to the specimens, are used to determine autogenous shrinkage and basic creep.

1.3.2 Linear aging model for creep—Experimental research indicates that creep may be considered approximately proportional to stress (L'Hermite et al. 1958; Keeton 1965), provided that the applied stress is less than 40% of the concrete compressive strength.

The strain responses to stress increments applied at different times may be added using the superposition principle (McHenry 1943) for increasing and decreasing stresses, provided strain reversals are excluded (for example, as in relaxation) and temperature and moisture content are kept constant (Le Camus 1947; Hanson 1953; Davies 1957; Ross 1958; Neville and Dilger 1970; Neville 1973; Bažant 1975; Gamble and Parrot 1978; RILEM Technical Committee TC-69 1988). Major deviations from the principle of superposition are caused by the neglect of the random scatter of the creep properties, by hygrothermal effects, including water diffusion and time evolution of the distributions of pore moisture content and temperature, and by material damage, including distributed cracking and fracture, and also frictional microslips. A comprehensive summary of the debate on the applicability of the principle of superposition when dealing with the evaluation of creep structural effects can be found in the references (Bažant 1975, 1999, 2000; CEB 1984; RILEM Technical Committee TC-107 1995; Al Manaseer et al. 1999; Jirasek and Bažant 2002; Gardner and Tsuruta 2004; Bažant 2007).

1.3.3 Separation of creep into basic creep and drying creep—Basic creep is measured on specimens that are sealed to prevent the ingress or egress of moisture from or to its environment. It is considered a material constitutive property and independent of the specimen size and shape. Drying creep is the strain remaining after subtracting shrinkage, elastic, and basic creep strains from the total measured strain on nominally identical specimens in a drying environment. The measured average creep of a cross section at drying is strongly size-dependant. Any effects of thermal strains have to be removed in all cases or are avoided by testing at constant temperature.

In sealed concrete specimens, there is no moisture movement into or out of the specimens. Low-water-cement-ratio concretes self-desiccate, however, leading to autogenous shrinkage. Normal-strength concretes do not change volume at relative humidity in the range 95 to 99%, whereas samples stored in water swell (L'Hermite et al. 1958).

1.3.4 Differential shrinkage and creep or shrinkage and creep gradients are neglected—The shrinkage strains determined according to ASTM C157/C157M are measured along the longitudinal axis of prismatic specimens; however, the majority of reported creep and shrinkage data are based on surface measurements of cylindrical specimens (ASTM C512). Unless finite element analysis (Bažant et al. 1975) or equivalent linear gradients (Carreira and Walser 1980) are used, it is generally assumed that shrinkage and creep strains in a specimen occur uniformly through the specimen cross section. Kristek et al. (2006) concluded that for box girder bridges, the classical creep analysis that assumes the shrinkage and creep properties to be uniform throughout the cross section is inadequate. As concrete ages, differences in strain gradients reduce (Carreira and Walser 1980; Aguilar 2005).

1.3.5 Stresses induced during curing phase are negligible—Most test programs consider the measurement of strains from the start of drying. It is assumed that the restrained stresses due to swelling and autogenous shrinkage are negligible because of the large creep strains and stress relaxation of the concrete at early ages. For restrained swelling, this assumption leads to an overestimation of the tensile stresses and, therefore, it may be an appropriate basis for design when predicting deflections or prestress losses. For predicting the effects of restrained autogenous shrinkage or relaxation, however, the opposite occurs. Limited testing information exists for tensile creep.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

a, b	=	constants used to describe the strength gain development of the concrete, ACI 209R-92 and GL2000 models
a	=	aggregate content of concrete, kg/m ³ or lb/yd ³ , B3 model
$C_o(t, t_o)$	=	compliance function for basic creep at concrete age t when loading starts at age t_o , B3 model
$C_d(t, t_o, t_c)$	=	compliance function for drying creep at concrete age t when loading and drying starts at ages t_o and t_c , respectively, B3 model
c	=	cement content of concrete, kg/m ³ or lb/yd ³ , ACI 209R-92 and B3 models
$d = 4V/S$	=	average thickness of a member, mm or in., ACI 209R-92 model
E	=	modulus of elasticity, MPa or psi
E_{cm}	=	mean modulus of elasticity of concrete, MPa or psi
E_{cm28}	=	mean modulus of elasticity of concrete at 28 days, MPa or psi
E_{cmt}	=	mean modulus of elasticity of concrete at age t , MPa or psi
E_{cmto}	=	mean modulus of elasticity of concrete when loading starts at age t_o , MPa or psi
$e = 2V/S$	=	effective cross section thickness of member or notional size of member according to B3 or CEB MC90 and CEB MC90-99 models, respectively, in mm or in.; defined as the cross-section divided by the semi-perimeter of the member in contact with the atmosphere, which coincides with the actual thickness in the case of a slab
f_{cm}	=	concrete mean compressive cylinder strength, MPa or psi
f_{cm28}	=	concrete mean compressive cylinder strength at 28 days, MPa or psi
f_{cmt}	=	concrete mean compressive cylinder strength at age t , MPa or psi
f_{cmto}	=	concrete mean compressive cylinder strength when drying starts at age t_o , MPa or psi
f_{cmto}	=	concrete mean compressive cylinder strength when loading starts at age t_o , MPa or psi

f'_c	=	concrete specified cylinder strength at 28 days, MPa or psi	β_{sc}	=	correction coefficient that depends on type of cement, CEB MC90 model
$H(t)$	=	spatial average of pore relative humidity at concrete age t , B3 model	$\beta_{s,T(t-t_c)}$	=	correction coefficient to account for effect of temperature on time development of shrinkage, CEB MC90 model
h	=	relative humidity expressed as a decimal	$\epsilon_{cas}(t)$	=	autogenous shrinkage strain at concrete age t , mm/mm or in./in., CEB MC90-99
$J(t, t_o)$	=	compliance at concrete age t when loading starts at age t_o , 1/MPa or 1/psi	$\epsilon_{cds}(t, t_c)$	=	drying shrinkage strain at concrete age t since the start of drying at age t_c , mm/mm or in./in., CEB MC90-99 model
$J(t_o, t_o)$	=	elastic compliance at concrete age t_o when loading starts at age t_o , 1/MPa or 1/psi	ϵ_{cso}	=	notional shrinkage coefficient, mm/mm or in./in., CEB MC90 model
$k_h, \beta_{RH}(h)$ or $\beta(h)$	=	correction term for effect of humidity on shrinkage according to B3, CEB MC90 and CEB MC90-99, or GL2000 models, respectively	$\epsilon_{caso}(f_{cm28})$	=	notional autogenous shrinkage coefficient, mm/mm or in./in., CEB MC90-99 model
k_s	=	cross-section shape factor, B3 model	$\epsilon_{cdso}(f_{cm28})$	=	notional drying shrinkage coefficient, mm/mm or in./in., CEB MC90-99 model
q_1	=	inverse of asymptotic elastic modulus, 1/MPa or 1/psi, B3 model	$\epsilon_{sh}(t, t_c)$	=	shrinkage strain at concrete age t since the start of drying at age t_c , mm/mm or in./in.
$S(t-t_c)$, $\beta_s(t-t_c)$ or $\beta(t-t_c)$	=	correction term for effect of time on shrinkage according to B3, CEB MC90, or GL2000 models, respectively	ϵ_{shu} or $\epsilon_{sh\infty}$	=	notional ultimate shrinkage strain, mm/mm or in./in., ACI 209R-92 and GL2000 models and B3 model, respectively
s	=	slump, mm or in., ACI 209R-92 model. Also, strength development parameter, CEB MC90, CEB MC90-99, and GL2000 models	$\phi(t, t_o)$	=	creep coefficient (dimensionless)
T	=	temperature, °C, °F, or °K	$\phi_{28}(t, t_o)$	=	28-day creep coefficient (dimensionless), CEB MC90, CEB MC90-99, and GL2000 models
t	=	age of concrete, days	ϕ_o	=	notional creep coefficient (dimensionless), CEB MC90 and CEB MC90-99 models
$t-t_c$	=	duration of drying, days	$\phi_{RH}(h)$	=	correction term for effect of relative humidity on notional creep coefficient, CEB MC90 and CEB MC90-99 models
t_c	=	age of concrete when drying starts at end of moist curing, days	$\Phi(t_c)$	=	correction term for effect of drying before loading when drying starts at age t_c , GL2000 model
t_o	=	age of concrete at loading, days	ϕ_u	=	ultimate (in time) creep coefficient, ACI 209R-92 model
V/S	=	volume-surface ratio, mm or in.	γ_c	=	unit weight of concrete, kg/m ³ or lb/ft ³
w	=	water content of concrete, kg/m ³ or lb/ft ³ , B3 model	γ_{sh} and γ_c	=	shrinkage and creep correction factor, respectively; also used as product of all applicable corrections factors, ACI 209R-92 model
α	=	air content expressed as percentage, ACI 209R-92 model	τ_{sh}	=	shrinkage half-time, days, ACI 209R-92 and B3 models
α_1 or k	=	shrinkage constant as function of cement type, according to B3 or GL2000 models, respectively	ψ	=	ratio of fine aggregate to total aggregate by weight expressed as percentage, ACI 209R-92 model
α_2	=	shrinkage constant related to curing conditions, B3 model			
$\alpha_{as}, \alpha_{ds1}$ and α_{ds2}	=	correction coefficients for effect of cement type on autogenous and drying shrinkage, CEB MC90-99 model			
$\beta_{as}(t)$	=	function describing time development of autogenous shrinkage, CEB MC90-99 model			
$\beta_c(t-t_o)$	=	correction term for effect of time on creep coefficient according to CEB MC90 and CEB MC90-99 models			
$\beta_{ds}(t-t_c)$	=	function describing time development of drying shrinkage, CEB MC90-99 model			
β_e	=	factor relating strength development to cement type, GL2000			
$\beta_{RH,T}$	=	correction coefficient to account for effect of temperature on notional shrinkage, CEB MC90 model			

2.2—Definitions

autogenous shrinkage—the shrinkage occurring in the absence of moisture exchange (as in a sealed concrete specimen) due to the hydration reactions taking place in the cement matrix. Less commonly, it is termed basic shrinkage or chemical shrinkage.

basic creep—the time-dependent increase in strain under sustained constant load of a concrete specimen in which moisture losses or gains are prevented (sealed specimen).

compliance $J(t, t_o)$ —the total load induced strain (elastic strain plus creep strain) at age t per unit stress caused by a unit uniaxial sustained load applied since loading age t_o .

creep coefficient—the ratio of the creep strain to the initial strain or, identically, the ratio of the creep compliance to the compliance obtained at early ages, such as after 2 minutes.

28-day creep coefficient—the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days ($\phi_{28}(t, t_o) = \phi(t, t_o) \cdot E_{cm28}/E_{cmto}$).

creep strain—the time-dependent increase in strain under constant load taking place after the initial strain at loading.

drying creep—the additional creep to the basic creep in a loaded specimen exposed to a drying environment and allowed to dry.

drying shrinkage—shrinkage occurring in a specimen that is allowed to dry.

elastic compliance or the nominal elastic strain per unit stress $J(t_o, t_o)$ —the initial strain at loading age t_o per unit stress applied. It is the inverse of the mean modulus of elasticity of concrete when loading starts at age t_o .

initial strain at loading or nominal elastic strain—the short-term strain at the moment of loading and is frequently considered as a nominal elastic strain as it contains creep that occurs during the time taken to measure the strain.

load-induced strain—the time-dependent strain due to a constant sustained load applied at age t_o .

shrinkage—the strain measured on a load-free concrete specimen.

specific creep—the creep strain per unit stress.

total strain—the total change in length per unit length measured on a concrete specimen under a sustained constant load at uniform temperature.

CHAPTER 3—PREDICTION MODELS

3.1—Data used for evaluation of models

In 1978, Bažant and Panula started collecting shrinkage and creep data from around the world and created a computerized databank, which was extended by Muller and Panula as part of collaboration between the ACI and the CEB established after the ACI-CEB Hubert Rusch workshop on concrete creep (Hillsdorf and Carreira 1980). The databank, now known as the RILEM databank, has been extended and refined under the sponsorship of RILEM TC 107-CSP, Subcommittee 5 (Kuttner 1997; Muller et al. 1999).

Problems encountered in the development of the databank have been discussed by Muller (1993) and others (Al-Manaseer and Lakshmikantham 1999; Gardner 2000). One problem involves which data sets should be included. For example, some investigators do not include the low-modulus sandstone concrete data of Hansen and Mattock (1966), but do include the Elgin gravel concrete data from the same researchers. A further problem is the data of some researchers are not internally consistent. For example, the results from the 150 mm (6 in.) diameter specimens of Hansen and Mattock are not consistent with the results from the 100 and 200 mm (4 and 8 in.) diameter specimens. Finally, it is necessary to define the relative humidity for sealed and immersed concrete specimens.

A major problem for all models is the description of the concrete. Most models are sensitive to the type of cement and the related strength development characteristics of the material. Simple descriptions, such as ASTM C150 Type I,

used in the databank are becoming increasingly difficult to interpret. For example, many cements meet the requirements of Types I, II, and III simultaneously; also, the multiple additions to the clinker allowed in ASTM C595 or in other standards are unknown to the researcher and designer. Nominally identical concretes stored in different environments, such as those tested by Keeton (1965), have different strength development rates. If this information exists, it should be taken into account in model development.

In addition, cement descriptions differ from country to country. The data obtained from European cement concretes may not be directly compared with that of United States cement concretes. Some researchers have suggested that correlation should only be done with recent and relevant data and that different shrinkage and creep curves should be developed for European, Japanese, North American, and South Pacific concretes (McDonald 1990; McDonald and Roper 1993; Sakata 1993; Sakata et al. 2001; Videla et al. 2004; Videla and Aguilar 2005a). While shrinkage and creep may vary with local conditions, research has shown that short-term shrinkage and creep measurements improve the predictions regardless of location (Bažant 1987; Bažant and Baweja 2000; Aguilar 2005). For this reason, the committee recommends short-term testing to determine the shrinkage, creep, and elastic modulus of the concrete to improve the predictions of the long-term deformations of the concrete.

Other issues include:

- The databank does not include sufficient data to validate modeling that includes drying before loading or loading before drying, which are common occurrences in practice;
- Many of the data sets in the databank were measured over relatively short durations, which reduces the usefulness of the data to predict long-term effects; and
- Most of the experiments were performed using small specimens compared with structural elements. It is debatable if the curing environment and consequent mechanical properties of concrete in the interior of large elements are well represented by small specimen experiments (Bažant et al. 1975; Kristek et al. 2006).

Despite these limitations, it is imperative that databanks such as the RILEM databank are maintained and updated as they provide an indispensable source of data in addition to a basis for comparing prediction models.

3.2—Statistical methods for comparing models

Several methods have been used for the evaluation of the accuracy of models to predict experimental data. Just as a single set of data may be described by its mean, mode, median, standard deviation, and maximum and minimum, a model for shrinkage or creep data may have several methods to describe its deviation from the data. The committee could not agree on a single method for comparison of test data with predictions from models for shrinkage and creep. Reducing the comparison between a large number of experimental results and a prediction method to a single number is fraught with uncertainty. Therefore, the committee strongly recommends designers to perform sensitivity analysis of the response of the structure using the models in this report and

to carry out short-term testing to calibrate the models to improve their predictions. The summary of the statistical indicators given in Chapter 4 provides the user with basis for comparison without endorsing any method.

One of the problems with the comparison of shrinkage and creep data with a model's prediction is the increasing divergence and spread of data with time, as shown in the figures of Chapter 4. Thus, when techniques such as linear regression are used, the weighting of the later data is greater than that of the earlier data (Bažant 1987; Bažant et al. 1987). On the contrary, comparison of the percent deviation of the model from the data tends to weight early-age data more than later-age data. The divergence and spread are a measure of the limitation of the model's capabilities and variability in the experimental data.

Commonly used methods for determining the deviation of a model from the data include:

- Comparison of individual prediction curves to individual sets of test data, which requires a case-by-case evaluation;
- Comparison of the test data and calculated values using linear regression;
- Evaluation of the residuals (measured-predicted value) (McDonald 1990; McDonald and Roper 1993; Al-Manaseer and Lakshmikantham 1999). This method does not represent least-square regression and, if there is a trend in the data, it may be biased; and
- Calculation of a coefficient of variation or standard error of regression normalized by the data centroid.

In the committee's opinion, the statistical indicators available are not adequate to uniquely distinguish between models.

3.3—Criteria for prediction models

Over the past 30 years, several models have been proposed for the prediction of drying shrinkage, creep, and total strains under load. These models are compromises between accuracy and convenience. The committee concludes that one of the primary needs is a model or models accessible to engineers with little specialized knowledge of shrinkage and creep. Major issues include, but are not restricted to:

- How simple or complex a model would be appropriate, and what input information should be required;
- What data should be used for evaluation of the model;
- How closely the model should represent physical phenomena/behavior;
- What statistical methods are appropriate for evaluating a model.

There is no agreement upon which information should be required to calculate the time-dependent properties of concrete; whether the mechanical properties of the concrete specified at the time of design should be sufficient or if the mixture proportions are also required.

At a minimum, the committee believes that shrinkage and creep models should include the following information:

- Description of the concrete either as mixture proportions or mechanical properties such as strength or modulus of elasticity;
- Ambient relative humidity;
- Age at loading;

- Duration of drying;
- Duration of loading; and
- Specimen size.

Models should also:

- Allow for the substitution of test values of concrete strength and modulus of elasticity;
- Allow the extrapolation of measured shrinkage and creep compliance results to get long-term values; and
- Contain mathematical expressions that are not highly sensitive to small changes in input parameters and are easy to use.

As described in ACI 209.1R, it has long been recognized that the stiffness of the aggregate significantly affects the shrinkage and creep of concrete. Some models account for the effect of aggregate type by assuming that the effects of aggregate are related to its density or the concrete elastic modulus. Models that use concrete strength can be adjusted to use a measured modulus of elasticity to account for aggregate properties. Models that do not use the mechanical characteristics of the concrete and rely on mixture proportion information alone may not account for variations in behavior due to aggregate properties.

Until recently, autogenous shrinkage was not considered significant because, in most cases, it did not exceed 150 microstrains. For concretes with water-cement ratios (w/c) less than 0.4, mean compressive strengths greater than 60 MPa (8700 psi), or both, however, autogenous shrinkage may be a major component of the shrinkage strain.

Some models consider that basic creep and drying creep are independent and thus additive, while other models have shrinkage and creep as dependent, and thus use multiplicative factors. The physical phenomenon occurring in the concrete may be neither.

3.4—Identification of strains

Equations (3-1) and (3-2) describe the additive simplification discussed in 1.3.1

$$\text{total strain} = \text{shrinkage strain} + \text{compliance} \times \text{stress} \quad (3-1)$$

$$\text{compliance} = \frac{(\text{elastic strain} + \text{basic creep} + \text{drying creep})}{\text{stress}} \quad (3-2)$$

The total and shrinkage strains are measured in a creep and shrinkage test program from which the compliance is determined. Errors in the measured data result in errors in the compliance. The elastic strain is determined from early-age measurement, but as discussed previously, it is difficult to separate early-age creep from the elastic strain. Thus, the assumed elastic strain is dependent on the time at which the strain measurement is made and, therefore, on the ignored early creep.

Basic creep and drying creep are determined from the compliance by subtracting the elastic strain, which may have implicit errors, from the strains measured on drying and nondrying specimens. Errors in the measured elastic strain used to determine the modulus of elasticity (E_c), in the measured total strain, or in the measured shrinkage

strain, are all reflected in the calculated creep strain, the compliance, and creep coefficient.

For sealed specimens, the equations for compliance and total strain simplify significantly if autogenous shrinkage is ignored as in Eq. (3-3) and (3-4)

$$\text{total strain} = \text{compliance} \times \text{stress} \quad (3-3)$$

$$\text{compliance} = \frac{(\text{elastic strain} + \text{basic creep})}{\text{stress}} \quad (3-4)$$

3.5—Evaluation criteria for creep and shrinkage models

In 1995, RILEM Committee TC 107 published a list of criteria for the evaluation of shrinkage and creep models (RILEM 1995; Bažant 2000). In November 1999, ACI Committee 209, which has a number of members in common with RILEM TC 107, discussed the RILEM guidelines and agreed on the following:

- Drying shrinkage and drying creep should be bounded. That is, they do not increase indefinitely with time;
- Shrinkage and creep equations should be capable of extrapolation in both time and size;
- Shrinkage and creep models should be compared with the data in the databank limited by the conditions of applicability of the model(s). That is, some experimental data, such as those with high water-cement ratios or low-modulus concrete, may not be appropriate to evaluate a model;
- Equations should be easy to use and not highly sensitive to changes in input parameters;
- The shape of the individual shrinkage and creep curves over a broad range of time (minutes to years) should agree with individual test results;
- Creep values should be compared as compliance or specific creep rather than as the creep coefficient. The immediate strain/unit stress and the modulus of elasticity are dependent on the rate of loading; however, for developing the creep equations to determine long-term deformations, this effect should not play a major role;
- Creep expressions should accommodate drying before loading. Results by Abiar reported by Acker (1993) show that predried concrete experiences very little creep. Similarly, the very late-age loaded (2500 to 3000 days) results of Wesche et al. (1978) show reduced creep compared with similar concrete loaded at early ages. The effect of predrying may also be significantly influenced by the size of the specimen;
- Shrinkage and creep expressions should be able to accommodate concretes containing fly ash, slag (Videla and Gaedicke 2004), natural pozzolans (Videla et al. 2004; Videla and Aguilar 2005a), silica fume and chemical admixtures (Videla and Aguilar 2005b);
- The models should allow for the effect of specimen size; and
- The models should allow for changes in relative humidity.

Success in achieving the following guidelines is consequent to the method of calculation; that is, if the principle of super-

position is valid and if the model includes drying before loading, and how they are considered under unloading:

- Recovery of creep strains under complete unloading should not exceed the creep strain from loading, and should asymptotically approach a constant value; and
- Stress relaxation should not exceed the initially applied stress.

Yue and Taerwe (1992, 1993) published two related papers on creep recovery. Yue and Taerwe (1992) commented, “It is well known that the application of the principle of superposition in the service stress range yields an inaccurate prediction of concrete creep when unloading takes place.” In their proposed two-function method, Yue and Taerwe (1993) used a linear creep function to model the time-dependent deformations due to increased stress on concrete, and a separate nonlinear creep recovery function to represent concrete behavior under decreasing stress.

CHAPTER 4—MODEL SELECTION

There are two practical considerations in the models for prediction of shrinkage and creep, namely:

- Mathematical form of their time dependency; and
- Fitting of the parameters and the resulting expressions.

If the mathematical form of the model does not accurately describe the phenomena, extrapolations of shrinkage and creep results will deviate from reality. After the mathematical form has been justified, the fit of the prediction to measured results should be compared for individual data sets.

The models selected for comparison are the ACI 209R-92 (ACI Committee 209 1992), the Bažant-Baweja B3 developed by Bažant and Baweja (1995, 2000), the CEB Model Code 1990-99 (CEB MC90-99) (Muller and Hillsdorf 1990; CEB 1991, 1993, 1999), and the GL2000 developed by Gardner and Lockman (2001). Table 4.1 lists the individual model’s applicable range for different input variables (adapted from Al-Manaseer and Lam 2005). Comparison of models with experimental data is complicated by the lack of agreement on selection of appropriate data and on the methods used to compare the correlation. Descriptions of the ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000 models are given in Appendix A. Kristek et al. (2001) and Sassone and Chiorino (2005) developed design aids for determination of shrinkage, compliance, and relaxation for ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000 models.

Figures 4.1 through 4.8 (Gardner 2004) compare the predicted values for two sets of input information for RILEM data sets extending longer than 500 days, concrete 28-day mean cylinder strengths f_{cm28} between 16 and 82 MPa (2320 and 11,890 psi), water-cement ratios between 0.4 and 0.6, duration of moist curing longer than 1 day (possibly biased against ACI 209R-92 because this model was developed for standard conditions considering 7 days of moist curing and 7 days of age at loading), age of loading greater than the duration of moist curing, and volume-surface ratios V/S greater than 19 mm (0.4 in.). The humidity range for compliance was 20 to 100% and below 80% for

Table 4.1—Parameter ranges of each model

Input variables	Model				
	ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
f_{cm28} , MPa (psi)	—	17 to 70 (2500 to 10,000)	20 to 90 (2900 to 13,000)	15 to 120 (2175 to 17,400)	16 to 82 (2320 to 11,900)
a/c	—	2.5 to 13.5	—	—	—
Cement content, kg/m ³ (lb/yd ³)	279 to 446 (470 to 752)	160 to 720 (270 to 1215)	—	—	—
w/c	—	0.35 to 0.85	—	—	0.40 to 0.60
Relative humidity, %	40 to 100	40 to 100	40 to 100	40 to 100	20 to 100
Type of cement, European (U.S.)	R or RS (I or III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)
t_c (moist cured)	≥ 1 day	≥ 1 day	< 14 days	< 14 days	≥ 1 day
t_c (steam cured)	1 to 3 days	—	—	—	—
t_o	≥ 7 days	$t_o \geq t_c$	> 1 day	> 1 day	$t_o \geq t_c \geq 1$ day

shrinkage. Consequently, swelling was not included even if some specimens were initially moist cured.

Two sets of comparisons are shown in each figure. One set, identified as “ f_{cm} only,” assumes that only the measured 28-day strength f_{cm} is known. The second set, identified as “all data,” uses the f_{cm} calculated as the average of the measured f_{cm} , and that back-calculated from the measured E_{cm} using the elastic modulus formula of the method and mixture proportions if required by the model. Calculated compliance is the calculated specific creep plus calculated elastic compliance for the f_{cm} graphs and the calculated specific creep plus measured elastic compliance for the all data graphs. The reported mixture composition was used for ACI 209R-92 and Bažant-Baweja B3. It was assumed that if mixture data were available, the strength development data and elastic modulus would also be available. Cement type was determined by comparison of measured strength gain data with the GL2000 strength gain equations. The same cement type was used for predictions in all methods. For CEB MC90-99, ASTM C150 Type I was taken as CEB Type N cement, Type III as CEB Type R, and Type II as CEB Type SL.

It should be noted that each model should use an appropriate value of elastic modulus for which the model was calibrated. Therefore, for CEB, the elastic modulus was taken as $E_{cm} = 9500(f_{cm})^{1/3}$ in MPa ($262,250[f_{cm}]^{1/3}$ in psi). For Bažant-Baweja B3, using the shape factor $k_s = 1.00$ in τ_s (the shrinkage time function) improved the results of the statistical analysis, and all concretes were assumed moist cured; that is, $\alpha_2 = 1.20$ for calculations using the Bažant-Baweja B3 model.

To calculate a coefficient of variation (Gardner 2004), the durations after drying or application of load were divided into seven half-log decade intervals: 3 to 9.9 days, 10 to 31 days, 32 to 99 days, 100 to 315 days, 316 to 999 days, 1000 to 3159 days, and greater than 3160 days. That is, each duration is 3.16 times the previous half-log decade; these are similar to the CEB ranges. The root mean square (RMS) (calculated-observed) was calculated for all comparisons in each half-log decade. The coefficient of variation was the average RMS/average experimental value for the same half-log decade.

4.1—ACI 209R-92 model

The model recommended by ACI Committee 209 (1971) was developed by Branson and Christiason (1971), with minor modifications introduced in ACI 209R-82 (ACI Committee 209 1982). ACI Committee 209 incorporated the developed model in ACI 209R-92 (ACI Committee 209 1992). Since then, it has not been revised or updated to the RILEM databank, and it is compared with very recent models. This model, initially developed for the precast-prestressing industry (Branson and Ozell 1961; Branson 1963, 1964, 1968; Branson et al. 1970; Meyers et al. 1970; Branson and Kripanayanan 1971; Branson and Chen 1972), has been used in the design of structures for many years.

Advantages of this model include:

- It is simple to use with minimal background knowledge; and
- It is relatively easy to adjust to match short-term test data simply by modifying ultimate shrinkage or creep to produce the best fit to the data.

Its disadvantages include:

- It is limited in its accuracy, particularly in the method of accommodating member size when its simplest form is used. This disadvantage, however, can be overridden if the methods provided for accommodating the shape and size effect on the time-ratio are applied; and
- It is empirically based, thus it does not model shrinkage or creep phenomena.

At its most basic level, the ACI 209R-92 method only requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Curing method;
- Relative humidity expressed as a decimal;
- Volume-surface ratio or average thickness; and
- Cement type.

This model calculates the creep coefficient rather than the compliance, which may introduce problems due to the assumed value of elastic modulus. Figures 4-1 and 4-2 show

the calculated and measured shrinkages and compliances, respectively. The comparison of shrinkage data in Fig. 4.1 clearly shows that the ACI 209R-92 model overestimates measured shrinkage at low shrinkage values (equivalent to short drying times) and underestimates at high shrinkage values (typical of long drying times). This result indicates the limitation of the model's equation used to predict shrinkage. The ACI 209R-92 compliance comparison is rather insensitive to using all of the available data, including mixture proportions, compared with just using the measured concrete strength.

4.2—Bažant-Baweja B3 model

The Bažant-Baweja B3 model (Bažant and Baweja 1995, 2000) is the culmination of work started in the 1970s (Bažant et al. 1976, 1991; Bažant and Panula 1978, 1984; Jirasek and Bažant 2002), and is based on a mathematical description of over 10 physical phenomena affecting creep and shrinkage (Bažant 2000), including known fundamental asymptotic properties that ought to be satisfied by a creep and shrinkage model (Bažant and Baweja 2000, RILEM Technical Committee TC 107 1995). This model has been found to be useful for those dealing with simple as well as complex structures. The Bažant-Baweja B3 model uses the compliance function. The compliance function reduces the risk of errors due to inaccurate values of the elastic modulus. The model clearly separates basic and drying creep.

The factors considered include:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Aggregate content in concrete;
- Cement content in concrete;
- Cement type;
- Concrete mean compressive strength at 28 days;
- Curing method;
- Relative humidity;
- Shape of specimen;
- Volume-surface ratio; and
- Water content in concrete.

Both Bažant-Baweja B3 shrinkage and creep models may require input data that are not generally available at time of design, such as the specific concrete proportions and concrete mean compressive strength. Default values of the input parameters can be automatically considered if the user lacks information on some of them. The authors suggest when only f_{cm28} is known, the water-cement ratio can be determined using Eq. (4-1), and typical values of cement content and aggregate cement ratio should be assumed

$$\begin{aligned} w/c &= [(f_{cm28}/22.8) + 0.535]^{-1} \quad \text{in SI units} \\ w/c &= [(f_{cm28}/3300) + 0.535]^{-1} \quad \text{in in.-lb units} \end{aligned} \quad (4-1)$$

Equation (4-1) represents the best-fit linear regression equation to the values reported in Tables A1.5.3.4(a) and A6.3.4(a) of ACI 211.1-91 (ACI Committee 211 1991) for non-air-entrained concretes made with Type 1 portland cement; for air-entrained concretes, similar equations can be

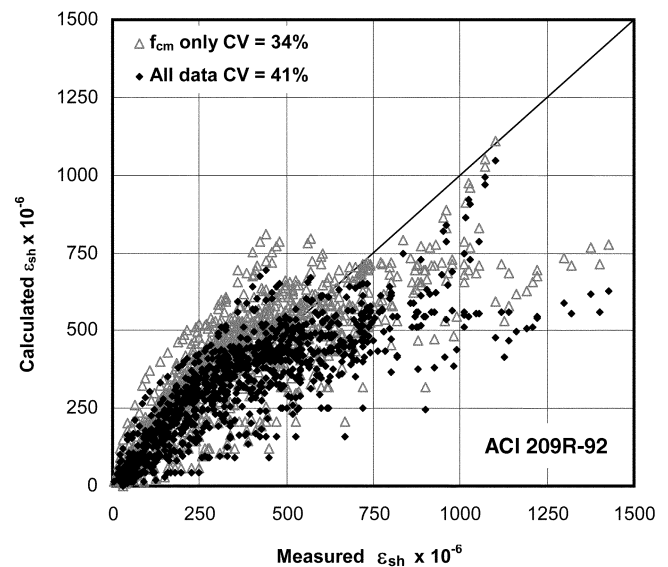


Fig. 4.1—ACI 209R-92 versus RILEM shrinkage databank (Gardner 2004).

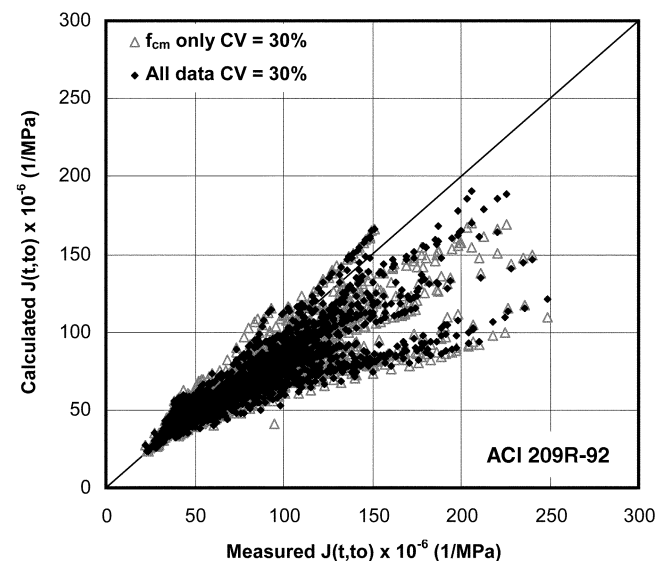


Fig. 4.2—ACI 209R-92 versus RILEM compliance databank (Gardner 2004).

derived by regression analysis of the reported values on ACI 211.1-91. For other cement types and cementitious materials, ACI 211.1-91 suggests that the relationship between water-cement or water-cementitious material ratio and compressive strength of concrete be developed for the materials actually to be used.

Figures 4.3 and 4.4 show the comparison between the calculated and measured shrinkages and compliances, respectively. The shrinkage equation is sensitive to the water content.

The model allows for extrapolation from short-term test data using short-term test data and a test of short-term moisture-content loss.

4.3—CEB MC90-99 model

In 1990, CEB presented a model for the prediction of shrinkage and creep in concrete developed by Muller and

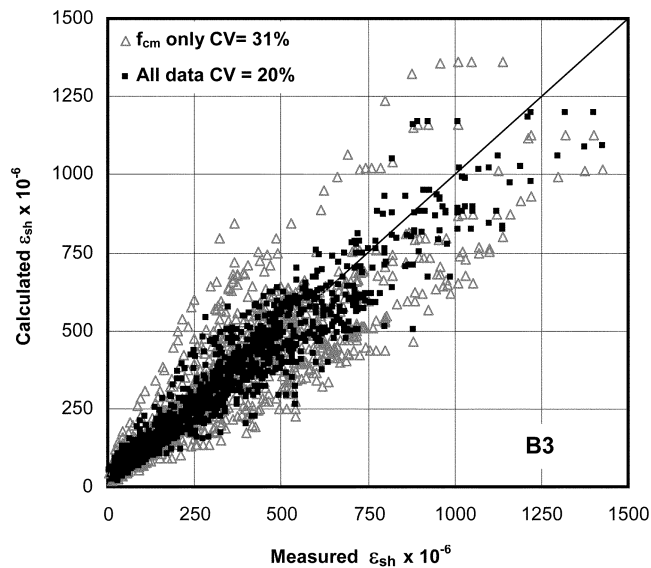


Fig. 4.3—Bažant-Baweja B3 versus RILEM shrinkage databank (Gardner 2004).

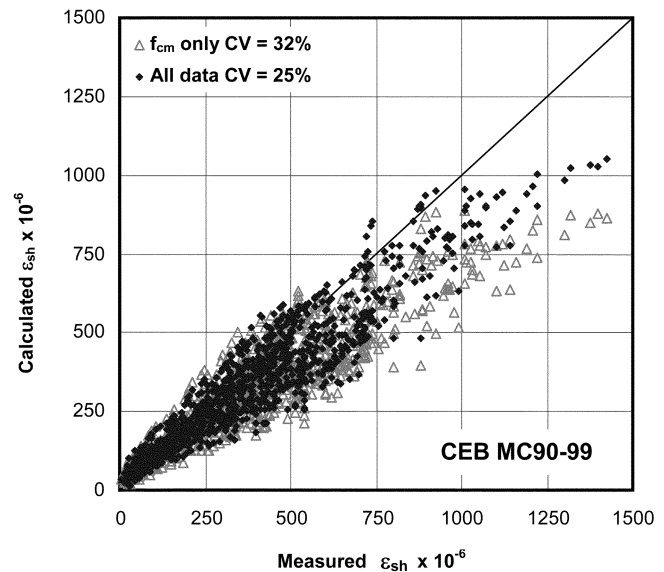


Fig. 4.5—CEB MC90-99 versus RILEM shrinkage databank (Gardner 2004).

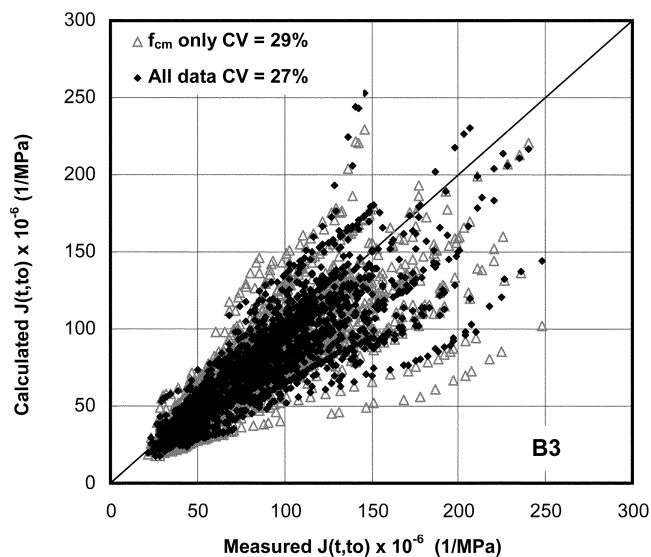


Fig. 4.4—Bažant-Baweja B3 versus RILEM compliance databank (Gardner 2004).

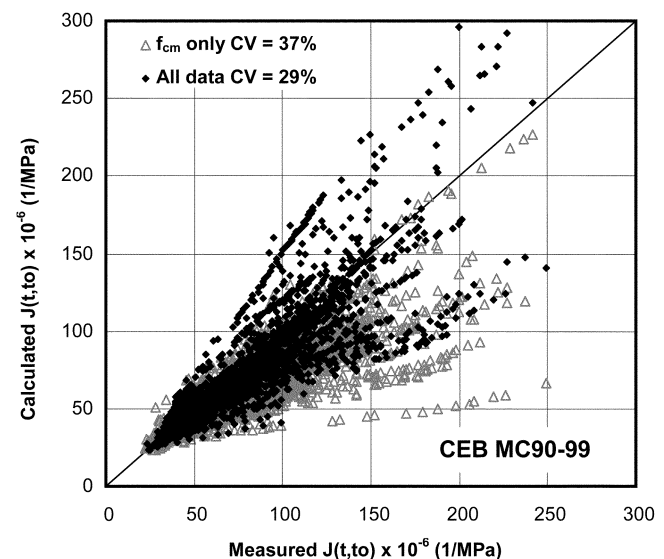


Fig. 4.6—CEB MC90-99 versus RILEM compliance databank (Gardner 2004).

Hilsdorf (1990). The model was revised in 1999 (CEB 1999) to include normal- and high-strength concretes and to separate the total shrinkage into its autogenous and drying shrinkage components, and it is called CEB MC90-99. While the revised models for the drying shrinkage component and for the compliance are closely related to the approach in CEB MC90 (Müller and Hilsdorf 1990, CEB 1993), for autogenous shrinkage, new relations were derived, and some adjustments were included for both normal- and high-strength concrete. For these reasons, the CEB 1990 and the revised CEB 1999 models are described in Appendix A. Some engineers working on creep and shrinkage-sensitive structures have accepted this model as preferable to the ACI 209R-92 model (based on the 1971 Branson and Christiason model). The CEB models do not require any information regarding the duration

of curing or curing condition. The duration of drying might have a direct impact on the shrinkage and creep of concrete, and should not be ignored when predicting the shrinkage and compliance. The correction term used for relative humidity in the creep equation is extremely sensitive to any variation in relative humidity. Figures 4.5 and 4.6 compare the calculated and measured shrinkages and compliances, respectively.

The method requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Concrete mean compressive strength at 28 days;
- Relative humidity expressed as a decimal;
- Volume-surface ratio; and
- Cement type.

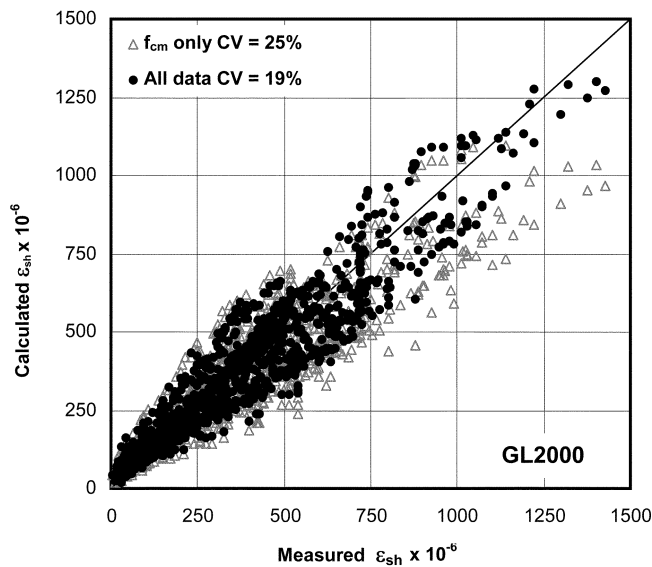


Fig. 4.7—GL2000 versus RILEM shrinkage databank (Gardner 2004).

Using only the data with reported concrete strength, the model generally underestimates the shrinkage of North American concretes, and substantially underestimates the shrinkage of concretes containing basalt aggregates found in Hawaii, Australia, and New Zealand (McDonald 1990; McDonald and Roper 1993; Robertson 2000). The main reason is that primarily European concretes (lower cement content and other types of cement) were considered when optimizing the model. The shrinkage model does not respond well to early-age extrapolation using the simple linear regression method suggested by Bažant (1987); however, the creep model does (Robertson 2000).

4.4—GL2000 model

The GL2000 model was developed by Gardner and Lockman (2001), with minor modifications introduced by Gardner (2004). The model is a modification of the GZ Atlanta 97 model (Gardner 2000) made to conform to the ACI 209 model guidelines given in Section 3.5. Except for the concrete compressive strength, the model only requires input data that are available to engineer at time of design. Figure 4.7 and 4.8 compare the calculated and measured shrinkages and compliances, respectively.

The method requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Relative humidity expressed as a decimal;
- Volume-surface ratio;
- Cement type; and
- Concrete mean compressive strength at 28 days.

4.5—Statistical comparisons

As stated previously, there is no agreement as to which statistical indicator(s) should be used, which data sets should be used, or what input data should be considered. To avoid revising any investigator's results, the statistical comparisons of

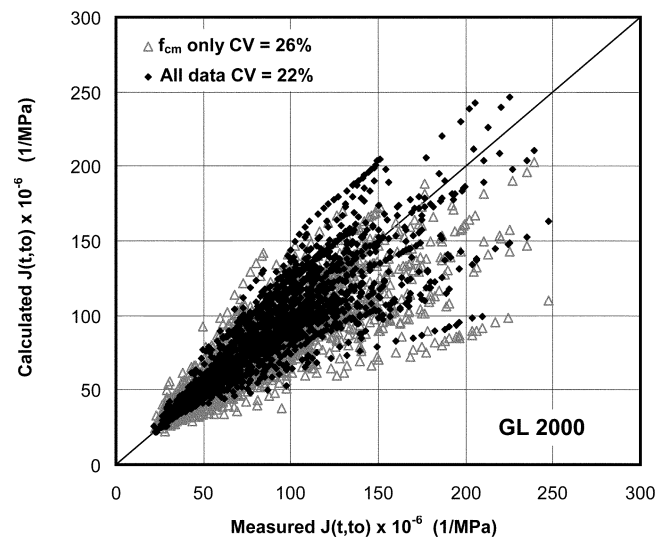


Fig. 4.8—GL2000 versus RILEM compliance databank (Gardner 2004).

Bažant and Baweja (2000), Al-Manaseer and Lam (2005), and Gardner (2004) are summarized in Table 4.2 for shrinkage and in Table 4.3 for compliance. As the statistical indicators represent different quantities and the investigators used different experimental results, comparisons can only be made across a row, but cannot be made between lines in the tables. Descriptions of the statistical indicators are given in Appendix B.

Al-Manaseer and Lam (2005) noted that careful selection and interpretation of concrete data and the statistical methods can influence the conclusions on the performance of model prediction on creep and shrinkage.

Brooks (2005) also reported the accuracy of five prediction models, including ACI 209R-92, Bažant-Baweja B3, CEB MC90, and GL2000 models, in estimating 30-year deformation, concluding that most methods fail to recognize the influence of strength of concrete and type of aggregate on creep coefficient, which ranged from 1.2 to 9.2. Brooks (2005) also reported that shrinkage ranged from 280 to 1460 × 10⁻⁶, and swelling varied from 25 to 35% of shrinkage after 30 years.

4.6—Notes about models

The prediction capabilities of the four shrinkage and compliance models were evaluated by comparing calculated results with the RILEM databank. For shrinkage strain prediction, Bažant-Baweja B3 and GL2000 provide the best results. The CEB MC90-99 underestimates the shrinkage. For compliance, GL2000, CEB MC90-99, and Bažant-Baweja B3 give acceptable predictions. The ACI 209R-92 method underestimates compliance for the most of the RILEM databank. It should be noted that for shrinkage predictions, Bažant-Baweja B3 using Eq. (4-1) instead of experimental values for water, cement, and aggregate masses provides less accurate, but still acceptable, results.

Except for ACI 209R-92, using more information improved the prediction for all other methods. The predictions from the CEB, GL2000, and Bažant-Baweja B3 models were significantly improved by using measured strength development

Table 4.2—Statistical indicators for shrinkage

Investigator	Indicator	Model				
		ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
Bažant and Baweja (2000)	ϖ_{BP}^*	55%	34%	46%	—	—
Al-Manaseer and Lam (2005)	V_{CEB}^*	46%	41%	52%	37%	37%
	F_{CEB}^*	83%	84%	60%	65%	84%
	$M_{CEB}^\dagger$	1.22	1.07	0.75	0.99	1.26
	ϖ_{BP}^*	102%	55%	90%	48%	46%
Gardner (2004), f_{cm} only	ω_G^*	34%	31%	—	32%	25%
Gardner (2004), all data	ω_G^*	41%	20%	—	25%	19%

*Perfect correlation = 0%.

†Perfect correlation = 1.00.

Table 4.3—Statistical indicators for compliance

Investigator	Indicator	Model				
		ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
Bažant and Baweja (2000), basic creep	ϖ_{BP}^*	58%	24%	35%	—	—
Bažant and Baweja (2000), drying creep	ϖ_{BP}^*	45%	23%	32%	—	—
Al-Manaseer and Lam (2005)	V_{CEB}^*	48%	36%	36%	38%	35%
	F_{CEB}^*	32%	35%	31%	32%	34%
	$M_{CEB}^\dagger$	0.86	0.93	0.92	0.89	0.92
	ϖ_{BP}^*	87%	61%	75%	80%	47%
Gardner (2004), f_{cm} only	ω_G^*	30%	29%	—	37%	26%
Gardner (2004), all data	ω_G^*	30%	27%	—	29%	22%

*Perfect correlation = 0%.

†Perfect correlation = 1.00.

and measured elastic modulus of the concrete to modify the concrete strength used in creep and shrinkage equations.

It should be noted that the accuracy of the models is limited by the many variables outlined previously and measurement variability. For design purposes, the accuracy of the prediction of shrinkage calculated using GL2000 and Bažant-Baweja B3 models may be within $\pm 20\%$, and the prediction of compliance $\pm 30\%$. Parametric studies should be made by the designer to ensure that expected production variations in concrete composition, strength, or the environment do not cause significant changes in structural response.

The coefficients of variation for shrinkage measured by Bažant et al. (1987) in a statistically significant investigation were 10% at 7 days and 7% at 1100 days, and can be used as a benchmark for variations between batches. A model that

could predict the shrinkage within 15% would be excellent, and 20% would be adequate. For compliance, the range of expected agreement would be wider because, experimentally, compliance is determined by subtracting two measured quantities of similar magnitude.

There is not an accepted sign convention for stress and strain. In this document, shortening strains and compressive stresses are positive. For all models, it is necessary to estimate the environmental humidity. The Precast/Prestressed Concrete Institute's *PCI Design Handbook* (2005) gives values of the annual average ambient relative humidity throughout the United States and Canada that may be used as a guide. Care should be taken when considering structures, such as swimming pools or structures near water. Although the models are not sensitive to minor changes in input values, the effect of air conditioning in moist climates and exposure to enclosed pool in dry climates can be significant. Therefore, the effects of air conditioning and heating on the local environment around the concrete element should be considered.

Relaxation, the gradual reduction of stress with time under sustained strain, calculated using ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000, agreed with Rostasy et al.'s (1972) experimental results indicating that the principle of superposition can be used to calculate relaxation provided that calculations are done keeping any drying before loading term constant at the initial value (Lockman 2000).

Lockman (2000) did a parametric comparison of models based upon the work of Chiorino and Lacidogna (1998a,b); see also Chiorino (2005). CEB MC90 and ACI 209R-92 underestimate the compliance compared with the GL2000 and Bažant-Baweja B3 models using the same input parameters. Relaxations calculated by Bažant-Baweja B3 are significantly different than those calculated for the three other models. The elastic strains, calculated at 30 seconds after loading, for the Bažant-Baweja B3 model are very different from those calculated by the other three models. The method of calculating the elastic strain is unique to this model, and the initial stresses of relaxation differ radically from other models.

For all ages of loading, especially in a drying environment, Bažant-Baweja B3 predicts more relaxation than the other models. Unlike the other models, Bažant-Baweja B3 uses an asymptotic elastic modulus (fast rate of loading), and not the conventional elastic modulus, which typically includes a significant early-age creep portion. The use of a larger asymptotic elastic modulus explains the comments about relaxation curves obtained from the Bažant-Baweja B3 model. For early ages of loading, the relaxations calculated using CEB MC90-99 and ACI 209R-92 are nearly 100% of the initial stress, with residual stresses close to zero.

For creep recovery, GL2000 and Bažant-Baweja B3 are the only models that predict realistic recoveries by superposition. For partial creep recovery, that is, superposition not assumed, with complete removal of the load, no model provides realistic results. Calculating recovery by superposition is subject to more problems than calculating relaxation by superposition. If recovery is to be calculated by superposition, both basic and drying creep compliance functions have to be

parallel in time to give a constant compliance after unloading. As drying before loading reduces both basic and drying creep, it is not yet possible to determine a formulation that permits calculating recovery by superposition in a drying environment. Experimental evidence (Neville 1960) is inconclusive on whether either drying creep or basic creep is completely recoverable.

High-strength concretes with water-cement ratios less than 0.40 and mean concrete strengths greater than 80 MPa (11,600 psi) experience significant autogenous shrinkage. The magnitude of the autogenous shrinkage also depends on the availability of moisture during early-age curing. Concretes containing silica fume appear to behave differently from conventional concretes. Few data on such concretes are held in the databank and hence, caution should be exercised using equations justified by the databank for such concretes. The models, however, can be used in such circumstances if they are calibrated with test data.

CHAPTER 5—REFERENCES

5.1—Referenced standards and reports

The latest editions of the standards and reports listed below were used when this document was prepared. Because these documents are revised frequently, the reader is advised to review the latest editions for any changes.

American Concrete Institute

- 116R Cement and Concrete Terminology
- 209.1R Report on Factors Affecting Shrinkage and Creep of Hardened Concrete

ASTM International

- C150 Specification for Portland Cement
- C595 Specification for Blended Hydraulic Cements
- C157 Test Method for Length Change of Hardened Hydraulic Cement, Mortar, and Concrete
- C512 Test Method for Creep of Concrete in Compression
- C469 Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression

5.2—Cited references

ACI Committee 209, 1971, "Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures," *Designing for the Effects of Creep, Shrinkage and Temperature*, SP-27, American Concrete Institute, Farmington Hills, MI, pp. 51-93.

ACI Committee 209, 1982, "Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures," *Designing for Creep and Shrinkage in Concrete Structures, A Tribute to Adrian Pauw*, SP-76, American Concrete Institute, Farmington Hills, MI, pp. 193-300.

ACI Committee 209, 1992, "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures (ACI 209R-92)," American Concrete Institute, Farmington Hills, MI, 47 pp.

ACI Committee 211, 1991, "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1-91) (Reapproved 2002)," American Concrete Institute, Farmington Hills, MI, 38 pp.

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