

Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures

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This report reviews the methods for predicting creep, shrinkage and temperature effects in concrete structures. It presents the designer with a unified and digested approach to the problem of volume changes in concrete. The individual chapters have been written in such a way that they can be used almost independently from the rest of the report.

The report is generally consistent with ACI 318 and includes material indicated in the Code, but not specifically defined therein.

Keywords: beams (supports); buckling; camber; composite construction (concrete to concrete); compressive strength; concretes; concrete slabs; cracking (frao turing); creep properties; curing; deflection; flat concrete plates; flexural strength; girders; lightweight-aggregate concretes; modulus of elasticity; moments of inertia; precast concrete; prestressed concrete: prestress loss; reinforced concrete: shoring; shrinkage; strains; stress relaxation; structural design; temperature; thermal expansion; two-way slabs: volume change; warpage.

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The 1992 revisions became effective Mar. 1, 1992. The revisions consisted of minor editorial changes and typographical corrections.

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CHAPTER 1-GENERAL

1.1-Scope

This report presents a unified approach to predicting the effect of moisture changes, sustained loading, and temperature on reinforced and prestressed concrete structures. Material response, factors affecting the structural response, and the response of structures in which the time change of stress is either negligible or significant are discussed.

Simplified methods are used to predict the material response and to analyze the structural response under service conditions. While these methods yield reasonably good results, a close correlation between the predicted deflections, cambers, prestress losses, etc., and the measurements from field structures should not be expected. The degree of correlation can be improved if the prediction of the material response is based on test data for the actual materials used, under environmental and loading conditions similar to those expected in the field structures.

These direct solution methods predict the response behavior at an arbitrary time step with a computational effort corresponding to that of an elastic solution. They have been reasonably well substantiated for laboratory conditions and are intended for structures designed using the ACI 318 Code. They are not intended for the analysis of creep recovery due to unloading, and they apply primarily to an isothermal and relatively uniform environment.

Special structures, such as nuclear reactor vessels and containments, bridges or shells of record spans, or large ocean structures, may require further considerations which are not within the scope of this report. For structures in which considerable extrapolation of the state-of-the-art in design and construction techniques is achieved, long-term tests on models may be essential to provide a sound basis for analyzing serviceability response. Reference 109 describes models and modeling techniques of concrete structures. For mass-produced concrete members, actual size tests and service inspection data will result in more accurate predictions. In every case, using test data to supplement the procedures in this report will result in an improved prediction of service performance.

1.2-Nature of the problem

Simplified methods for analyzing service performance are justified because the prediction and control of time-dependent deformations and their effects on concrete structures are exceedingly complex when compared with the methods for analysis and design of strength performance. Methods for predicting service performance involve a relatively large number of significant factors that are difficult to accurately evaluate. Factors such as the nonhomogeneous nature of concrete properties caused by the stages of construction, the histories of water content, temperature and loading on the structure and their effect on the material response are difficult to quantify even for structures that have been in service for years.

The problem is essentially a statistical one because most of the contributing factors and actual results are inherently random variables with coefficients of variations of the order of 15 to 20 percent at best. However, as in the case of strength analysis and design, the methods for predicting serviceability are primarily deterministic in nature. In some cases, and in spite of the simplifying assumptions, lengthy procedures are required to account for the most pertinent factors.

According to a survey by ACI Committee 209, most designers would be willing to check the deformations of their structures if a satisfactory correlation between computed results and the behavior of actual structures could be shown. Such correlations have been established for laboratory structures, but not for actual structures. Since concrete characteristics are strongly dependent on environmental conditions, load history, etc., a poorer correlation is normally found between laboratory and field service performances than between laboratory and field strength performances.

With the above limitations in mind, systematic design procedures are presented which lend themselves to a computer solution by providing continuous time functions for predicting the initial and time-dependent average response (including ultimate values in time) of structural members of different weight concretes.

The procedures in this report for predicting time-dependent material response and structural service performance represent a simplified approach for design purposes. They are not definitive or based on statistical results by any means. Probabilistic methods are needed to accurately estimate the variability of all factors involved.

1.3-Definitions of terms

The following terms are defined for general use in this report. It should be noted that separability of creep and shrinkage is considered to be strictly a matter of definition and convenience. The time-dependent deformations of concrete, either under load or in an unloaded specimen, should be considered as two aspects of a single complex physical phenomenon.⁸⁸

1.3.1 Shrinkage

Shrinkage, after hardening of concrete, is the decrease

with time of concrete volume. The decrease is due to changes in the moisture content of the concrete and physico-chemical changes, which occur without stress attributable to actions external to the concrete. The converse of shrinkage is swelling which denotes volumetric increase due to moisture gain in the hardened concrete. Shrinkage is conveniently expressed as a dimensionless strain (in./in. or m/m) under steady conditions of relative humidity and temperature.

The above definition includes drying shrinkage, autogenous shrinkage, and carbonation shrinkage.

- a) Drying shrinkage is due to moisture loss in the concrete
- b) Autogenous shrinkage is caused by the hydration of cement
- c) Carbonation shrinkage results as the various cement hydration products are carbonated in the presence of CO₂

Recommended values in Chapter 2 for shrinkage strain (ϵ_{sh})_t are consistent with the above definitions.

1.3.2 Creep

The time-dependent increase of strain in hardened concrete subjected to sustained stress is defined as creep. It is obtained by subtracting from the total measured strain in a loaded specimen, the sum of the initial instantaneous (usually considered elastic) strain due to the sustained stress, the shrinkage, and the eventual thermal strain in an identical load-free specimen which is subjected to the same history of relative humidity and temperature conditions. Creep is conveniently designated at a constant stress under conditions of steady relative humidity and temperature, assuming the strain at loading (nominal elastic strain) as the instantaneous strain at any time.

The above definition treats the initial instantaneous strain, the creep strain, and the shrinkage as additive, even though they affect each other. An instantaneous change in stress is most likely to produce both elastic and inelastic instantaneous changes in strain, as well as short-time creep strains (10 to 100 minutes of duration) which are conventionally included in the so-called instantaneous strain. Much controversy about the best form of "practical creep equations" stems from the fact that no clear separation exists between the instantaneous strain (elastic and inelastic strains) and the creep strain. Also, the creep definition lumps together the basic creep and the drying creep.

- a) Basic creep occurs under conditions of no moisture movement to or from the environment
- b) Drying creep is the additional creep caused by drying

In considering the effects of creep, the use of either a unit strain, δ_c (creep per unit stress), or creep coefficient, ν_c (ratio of creep strain to initial strain), yields the same

results, since the concrete initial modulus of elasticity, E_{ci} , must be included, that is:

$$v_i = \delta_i E_{ci} \quad (1-1)$$

This is seen from the relations:

$$\begin{aligned} \text{Creep strain} &= \sigma \delta_i \\ &= \epsilon_i v_i, \text{ and} \\ E_{ci} &= \sigma / \epsilon_i \end{aligned}$$

where, σ is the applied constant stress and ϵ_i is the instantaneous strain.

The choice of either of δ_i or v_i is a matter of convenience depending on whether it is desired to apply the creep factor to stress or strain. The use of v_i is usually more convenient for calculation of deflections and prestressing losses.

1.3.3-Relaxation

Relaxation is the gradual reduction of stress with time under sustained strain. A sustained strain produces an initial stress at time of application and a deferred negative (deductive) stress increasing with time at a decreasing rate.⁸⁹

1.3.4-Modulus of elasticity

The static modulus of elasticity (secant modulus) is the linearized instantaneous (1 to 5 minutes) stress-strain relationship. It is determined as the slope of the secant drawn from the origin to a point corresponding to 0.45 f_c' on the stress-strain curve, or as in ASTM C 469.

1.3.5-Contraction and expansion

Concrete contraction or expansion is the algebraic sum of volume changes occurring as the result of thermal variations caused by heat of hydration of cement and by ambient temperature change. The net volume change is a function of the constituents in the concrete.

CHAPTER 2-MATERIAL RESPONSE

2.1-Introduction

The procedures used to predict the effects of time-dependent concrete volume changes in Chapters 3, 4, and 5 depend on the prediction of the material response parameters; i.e., strength, elastic modulus, creep, shrinkage and coefficient of thermal expansion.

The equations recommended in this chapter are simplified expressions representing average laboratory data obtained under steady environmental and loading conditions. They may be used if specific material response parameters are not available for local materials and environmental conditions.

Experimental determination of the response parameters using the standard referenced throughout this report and listed in Section 2.10 is recommended if an accurate prediction of structural service response is desired. No prediction method can yield better results than testing actual materials under environmental and

loading conditions similar to those expected in the field. It is difficult to test for most of the variables involved in one specific structure. Therefore, data from standard test conditions used in connection with the equations recommended in this chapter may be used to obtain a more accurate prediction of the material response in the structure than the one given by the parameters recommended in this chapter.

Occasionally, it is more desirable to use material parameters corresponding to a given probability or to use upper and lower bound parameters based on the expected loading and environmental conditions. This prediction will provide a range of expected variations in the response rather than an average response. However, probabilistic methods are not within the scope of this report.

The importance of considering appropriate water content, temperature, and loading histories in predicting concrete response parameters cannot be overemphasized. The differences between field measurements and the predicted deformations or stresses are mostly due to the lack of correlation between the assumed and the actual histories for water content, temperature, and loading.

2.2-Strength and elastic properties

2.2.1-Concrete compressive strength versus time

A study of concrete strength versus time for the data of References 1-6 indicates an appropriate general equation in the form of Eq. (2-1) for predicting compressive strength at any time.^{6,7}

$$(f_c')_t = \frac{t}{a + \beta t} (f_c')_{28} \quad (2-1)$$

where a in days and β are constants, $(f_c')_{28}$ = 28-day strength and t in days is the age of concrete.

Compressive strength is determined in accordance with ASTM C 39 from 6 x 12 in. (152 x 305 mm) standard cylindrical specimens, made and cured in accordance with ASTM C 192.

Equation (2-1) can be transformed into

$$(f_c')_t = \frac{t}{a/\beta + t} (f_c')_u \quad (2-2)$$

where a/β is age of concrete in days at which one half of the ultimate (in time) compressive strength of concrete, $(f_c')_u$ is reached.⁹²

The ranges of a and β in Eqs. (2-1) and (2-2) for the normal weight, sand lightweight, and all lightweight concretes (using both moist and steam curing, and Types I and III cement) given in References 6 and 7 (some 88 specimens) are: $a = 0.05$ to 9.25, $\beta = 0.67$ to 0.98.

The constants a and β are functions of both the type of cement used and the type of curing employed. The use of normal weight, sand lightweight, or all-lightweight aggregate does not appear to affect these constants significantly. Typical values recommended in References 7 are given in Table 2.2.1. Values for the time-ratio, $(f_c')_t / (f_c')_{28}$ or $(f_c')_t / (f_c')_u$ in Eqs. (2-1) and (2-2) are given also in Table 2.2.1.

"Moist cured conditions" refer to those in ASTM C 132 and C 511. Temperatures other than 73.4 ± 3 F (23 ± 1.7 C) and relative humidities less than 35 percent may result in values different than those predicted when using the constant on Table 2.2.1 for moist curing. The effect of concrete temperature on the compressive and flexural strength development of normal weight concretes made with different types of cement with and without accelerating admixtures at various temperatures between 25 F (-3.9 C) and 120 F (48.9 C) were studied in Reference 90.

Constants in Table 2.2.1 are not applicable to concretes, such as mass concrete, containing Type II or Type V cements or containing blends of portland cement and pozzolanic materials. In those cases, strength gains are slower and may continue over periods well beyond one year age.

"Steam cured" means curing with saturated steam at atmospheric pressure at temperatures below 212 F (100 C).

Experimental data from References 1-6 are compared in Reference 7 and all these data fall within about 20 percent of the average values given by Eqs. (2-1) and (2-2) for constants α and β in Table 2.2.1. The temperature and cycle employed in steam curing may substantially affect the strength-time ratio in the early days following curing.¹⁰⁷

2.2.2 Modulus of rupture, direct tensile strength and modulus of elasticity

Eqs. (2-3), (2-4), and (2-5) are considered satisfactory in most cases for computing average values for modulus of rupture, f_r , direct tensile strength, f_t' , and secant modulus of elasticity at $0.4(f_c')$, E_{ct} respectively of different weight concretes.^{1,4-12}

$$f_r = g_r [w(f_c')_t]^{1/2} \quad (2-3)$$

$$f_t' = g_t [w(f_c')_t]^{1/2} \quad (2-4)$$

$$E_{ct} = g_{ct} [w^3(f_c')_t]^{1/2} \quad (2-5)$$

For the unit weight of concrete, w in pcf and the compressive strength, $(f_c')_t$ in psi

$$g_r = 0.60 \text{ to } 1.00 \text{ (a conservative value of } g_r = 0.60 \text{ may be used, although a value } g_r = 0.60 \text{ to } 0.70 \text{ is more realistic in most cases)}$$

$$g_t = 1/3$$

$$g_{ct} = 33$$

For w in Kg/m^3 and $(f_c')_t$ in MPa

$$g_r = 0.012 \text{ to } 0.021 \text{ (a conservative value of } g_r = 0.012 \text{ may be used, although a value of } g_r = 0.013 \text{ to } 0.014 \text{ is more realistic in most cases)}$$

$$g_t = 0.0069$$

$$g_{ct} = 0.043$$

The modulus of rupture depends on the shape of the tension zone and loading conditions Eq.(2-3) corresponds to a 6×6 in. (150×150 mm) cross section as in ASTM C 78, Where much of the tension zone is remote from the neutral axis as in the case of large box girders or large I-beams, the modulus of rupture approaches the direct tensile strength.

Eq. (2-5) was developed by Pauw¹¹ and is used in Subsection 8.5.1 of Reference 27. The static modulus of elasticity is determined experimentally in accordance with ASTM C 649.

The modulus of elasticity of concrete, as commonly understood is not the truly instantaneous modulus, but a modulus which corresponds to loads of one to five minutes duration.⁸⁶

2.3—Theory for predicting creep and shrinkage of concrete

The principal variables that affect creep and shrinkage are discussed in detail in References 3, 6, 13-16, and are summarized in Table 2.2.2. The design approach presented^{6,7} for predicting creep and shrinkage: refers to "standard conditions" and correction factors for other than Standard conditions. This approach has also been used in References 3, 7, 17, and 83.

Based largely on information from References 3-6, 13, 15, 18-21, the following general procedure is suggested for predicting creep and shrinkage of concrete at any time.⁷

$$v_t = \frac{t^\psi}{d + t^\psi} v_u \quad (2-6)$$

$$(\epsilon_{sh})_t = \frac{t^\alpha}{f + t^\alpha} (\epsilon_{sh})_u \quad (2-7)$$

where d and f (in days), ψ and α are considered constants for a given member shape and size which define the time-ratio part, v_u is the ultimate creep coefficient defined as ratio of creep strain to initial strain, $(\epsilon_{sh})_u$ is the ultimate shrinkage strain, and t is the time after loading in Eq. (2-6) and time from the end of the initial curing in Eq. (2-7).

When ψ and α are equal to 1.0, these equations are the familiar hyperbolic equations of Ross¹⁵ and Lorman²¹ in slightly different form.

The form of these equations is thought to be convenient for design purposes, in which the concept of the ultimate (in time) value is modified by the time-ratio to yield the desired result. The increase in creep after, say, 100 to 200 days is usually more pronounced than shrinkage. In percent of the ultimate value, shrinkage usually increases more rapidly during the first few months. Appropriate powers of t in Eqs. (2-6) and (2-7) were found in References 6 and 7 to be 1.0 for shrinkage (flatter hyperbolic form) and 0.60 for creep (steeper curve for

larger values of t). This can be seen in Fig. (2-3) and (2-4) of Reference 7.

Values of ψ , d , v_u , α , f , and $(\epsilon_{sh})_u$ can be determined by fitting the data obtained from tests performed in accordance to ASTM C 512.

Normal ranges of the constants in Eqs. (2-6) and (2-7) were found to be:^{6,7}

$$\begin{aligned}\psi &= 0.40 \text{ to } 0.80, \\ d &= 6 \text{ to } 30 \text{ days}, \\ v_u &= 1.30 \text{ to } 4.15, \\ \alpha &= 0.90 \text{ to } 1.10, \\ f &= 20 \text{ to } 130 \text{ days}, \\ (\epsilon_{sh})_u &= 415 \times 10^{-6} \text{ to } 1070 \times 10^{-6} \text{ in./in. (m/m)}\end{aligned}$$

These constants are based on the standard conditions in Table 2.2.2 for the normal weight, sand lightweight, and all lightweight concretes, using both moist and steam curing, and Types I and III cement as in References 3-6, 13, 15, 18-20, 23, 24.

Eqs. (2-8), (2-9), and (2-10) represent the average values for these data. These equations were compared with the data (120 creep and 95 shrinkage specimens) in Reference 7. The constants in the equations were determined on the basis of the best fit for all data individually. The average-value curves were then determined by first obtaining the average of the normal weight, sand lightweight, and all lightweight concrete data separately, and then averaging these three curves. The constants v_u and $(\epsilon_{sh})_u$ recommended in References 7 and 96 were approximately the same as the overall numerical averages, that is $v_u = 2.35$ was recommended versus 2.36; $(\epsilon_{sh})_u = 800 \times 10^{-6}$ in./in. (m/m) versus 803×10^{-6} for moist cured concrete, and 730×10^{-6} versus 788×10^{-6} for steam cured concrete.

The creep and shrinkage data, based on 20-year measurements^{7,18} for normal weight concrete with an initial time of 28 days, are roughly comparable with Eqs. (2-8) to (2-10). Some differences are to be found because of the different initial times, stress levels, curing conditions, and other variables.

However, subsequent work⁵⁹ with 479 creep data points and 356 shrinkage data points resulted in the same average for $v_u = 2.35$, but a new average for $(\epsilon_{sh})_u = 780 \times 10^{-6}$ in./in. (m/m), for both moist and steam cured concrete. It was found that no consistent distinction in the ultimate shrinkage strain was apparent for moist and steam cured concrete, even though different time-ratio terms and starting times were used.

The procedure using Eqs. (2-8) to (2-10) has also been independently evaluated and recommended in Reference 60, in which a comprehensive experimental study was made of the various parameters and correction factors for different weight concrete.

No consistent variation was found between the different weight concretes for either creep or shrinkage. It was noted in the development of Eq. (2-8) that more consistent results were found for the creep variable in the

form of the creep coefficient, v_t (ratio of creep strain to initial strain), as compared to creep strain per unit stress, δ_t . This is because the effect of concrete stiffness is included by means of the initial strain.

2.4-Recommended creep and shrinkage equations for standard conditions

Equations (2-8), (2-9), and (2-10) are recommended for predicting a creep coefficient and an unrestrained shrinkage strain at any time, including ultimate values.⁶⁻⁷ They apply to normal weight, sand lightweight, and all lightweight concrete (using both moist and steam curing, and Types I and III cement) under the standard conditions summarized in Table 2.2.2.

Values of v_u and $(\epsilon_{sh})_u$ need to be modified by the correction factors in Sections 2.5 and 2.6 for conditions other than the standard conditions.

Creep coefficient, v_t for a loading age of 7 days, for moist cured concrete and for 1-3 days steam cured concrete, is given by Eq. (2-8).

$$v_t = \frac{t^{0.60}}{10 + t^{0.60}} v_u \quad (2-8)$$

Shrinkage after age 7 days for moist cured concrete:

$$(\epsilon_{sh})_t = \frac{t}{35 + t} (\epsilon_{sh})_u \quad (2-9)$$

Shrinkage after age 1-3 days for steam cured concrete:

$$(\epsilon_{sh})_t = \frac{t}{55 + t} (\epsilon_{sh})_u \quad (2-10)$$

In Eq. (2-8), t is time in days after loading. In Eqs. (2-9) and (2-10), t is the time after shrinkage is considered, that is, after the end of the initial wet curing.

In the absence of specific creep and shrinkage data for local aggregates and conditions, the average values suggested for v_u and $(\epsilon_{sh})_u$ are:

$$v_u = 2.35 \gamma_c \text{ and}$$

$$(\epsilon_{sh})_u = 780 \gamma_{sh} \times 10^{-6} \text{ in./in., (m/m)}$$

where γ_c and γ_{sh} represent the product of the applicable correction factors as defined in Sections 2.5 and 2.6 by Equations (2-12) through (2-30).

These values correspond to reasonably well shaped aggregates graded within limits of ASTM C 33. Aggregates affect creep and shrinkage principally because they influence the total amount of cement-water paste in the concrete.

The time-ratio part, [right-hand side except for v_u and $(\epsilon_{sh})_u$] of Eqs. (2-8), (2-9), and (2-10), appears to be applicable quite generally for design purposes. Values from the standard Eqs. (2-8) to (2-10) of v_t/v_u and

$(\epsilon_{sh})_t/(\epsilon_{sh})_u$ are shown in Table 2.4.1. Note that v is used in Eqs. (4-11), (4-20), and (4-22), hence, $v_t/v_u = v_s/v_u$ for the age of the precast beam concrete at the slab casting.

It has also been shown²⁴ that the time-ratio part of Eqs. (2-8) and (2-10) can be used to extrapolate 28-day creep and shrinkage data determined experimentally in accordance with ASTM C 512, to complete time curves up to ultimate quite well for creep, and reasonably well for shrinkage for a wide variety of data. It should be noticed that the time-ratio in Eqs. (2-8) to (2-10) does not differentiate between basic and drying creep nor between drying autogenous and carbonation shrinkage. Also, it is independent of member shape and size, because d, f, ψ , and α are considered as constant in Eqs. (2-8), (2-9), and (2-10).

The shape and size effect can be totally considered on the time-ratio, without the need for correction factors. That is, in terms of the shrinkage-half-time τ_{sh} , as given by Eq. (2-35) by replacing t by t/τ_{sh} in Eq. (2-9) and by $0.1t/\tau_{sh}$ in Eq. (2-8) as shown in 2.8.1. Also by taking $\psi = \alpha = 1.0$ and $d = f = 26.0 [\exp 0.36(v/s)]$ in Eqs. (2-6) and (2-7) as in Reference 23, where v/s is the volume to surface ratio, in inches. For v/s in mm use $d = f = 26.0 \exp [1.42 \times 10^{-2} (v/s)]$.

References 61, 89, 92, 98 and 101 consider the effect of the shape and size on both the time-ratio (time-dependent development) and on the coefficients affecting the ultimate (in time) value of creep and shrinkage.

ACI Committee 209, Subcommittee I Report⁶ is recommended for a detailed review of the effects of concrete constituents, environment and stress on time-dependent concrete deformations.

2.5-Correction factors for conditions other than the standard concrete composition'

All correction factors, y , are applied to ultimate values. However, since creep and shrinkage for any period in Eqs. (2-8) through (2-10) are linear functions of the ultimate values, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

Correction factors other than those for concrete composition in Eqs. (2-11) through (2-22) may be used in conjunction with the specific creep and shrinkage data from a concrete tested in accordance with ASTM C 512.

2.5.1 Loading age

For loading ages later than 7 days for moist cured concrete and later than 1-3 days for steam cured concrete, use Eqs. (2-11) and (2-12) for the creep correction factors.

$$\text{Creep } \gamma_{\ell a} = 1.25(t_{\ell a})^{-0.118} \text{ for moist cured concrete (2-11)}$$

$$\text{Creep } \gamma_{\ell a} = 1.13(t_{\ell a})^{-0.094} \text{ for steam cured concrete (2-12)}$$

where $t_{\ell a}$ is the loading age in days. Representative values are shown in Table 2.51. Note that in Eqs. (4-11), (4-20), and (4-22), the Creep $\gamma_{\ell a}$ correction factor must be used when computing the ultimate creep coefficient of the present beam corresponding to the age when slab is cast, v_{us} . That is:

$$v_{us} = v_u \times (\text{Creep } \gamma_{\ell a}) \quad (2-13)$$

2.5.2 Differential shrinkage

For shrinkage considered for other than 7 days for moist cured concrete and other than 1-3 days for steam cured concrete, determine the difference in Eqs. (2-9) and (2-10) for any period starting after this time.

That is, the shrinkage strain between 28 days and 1 year, would be equal to the 7 days to 1 year shrinkage minus the 7 days to 28 days shrinkage. In this example for moist cured concrete, the concrete is assumed to have been cured for 7 days. Shrinkage γ_{cp} factor as in 2.5.3 below, is applicable to Eq. (2-9) for concrete moist cured during a period other than 7 days.

2.5.3 Initial moist curing

For shrinkage of concrete moist cured during a period of time other than 7 days, use the Shrinkage γ_{cp} factor in Table 2.5.3. This factor can be used to estimate differential shrinkage in composite beams, for example.

Linear interpolation may be used between the values in Table 2.5.3.

2.5.4 Ambient relative humidity

For ambient relative humidity greater than 40 percent, use Eqs. (2-14) through (2-16) for the creep and shrinkage correction factors.^{7,20,22}

$$\text{Creep } \gamma_{\lambda} = 1.27 - 0.0067\lambda, \text{ for } \lambda > 40 \quad (2-14)$$

$$\text{Shrinkage } \gamma_{\lambda} = 1.40 - 0.0102, \text{ for } 40 \leq \lambda \leq 80 \quad (2-15)$$

$$= 3.00 - 0.030\lambda, \text{ for } 80 > \lambda \leq 100 \quad (2-16)$$

where λ is relative humidity in percent. Representative values are shown in Table 2.5.4.

The average value suggested for $\lambda = 40$ percent is $(\epsilon_{sh})_u = 780 \times 10^{-6}$ in./in. (m/m) in both Eqs. (2-9) and (2-10). From Eq. (2-15) of Table 2.5.4, for $\lambda = 70$ percent, $(\epsilon_{sh})_u = 0.70(780 \times 10^{-6}) = 546 \times 10^{-6}$ in./in. (m/m), for example. For lower than 40 percent ambient relative humidity, values higher than 1.0 shall be used for Creep γ_{λ} and Shrinkage γ_{λ} .

2.5.5 Average thickness of member other than 6 in. (150 mm) or volume-surface ratio other than 1.5 in. (38 mm)

The member size effects on concrete creep and shrinkage is basically two-fold. First, it influences the time-ratio (see Equations 2-6, 2-7, 2-8, 2-9, 2-10, and 2-35). Secondly, it also affects the ultimate creep coefficient, μ_u and the ultimate shrinkage strain, $(\epsilon_{sh})_u$.

Two methods are offered for estimating the effect of

member size on v_u and $(\epsilon_{sh})_u$. The average-thickness method tends to compute correction factor values that are higher, as compared to the volume-surface ratio method,⁵⁹ since Creep $\gamma_h = \text{Creep } \gamma_{vs} = 1.00$ for $h = 6$ in. (150 mm) and $v/s = 1.5$ in. (38 mm), respectively; that is, when $h = 4v/s$.

2.5.5.a Average-thickness method

The method of treating the effect of member size in terms of the average thickness is based on information from References 3, 6, 7, 23 and 61.

For average thickness of member less than 6 in. (150 mm), use the factors given in Table 2.5.5.1. These correspond to the CEB⁶¹ values for small members. For average thickness of members greater than 6 in. (150 mm) and up to about 12 to 15 in. (300 to 380 mm), use Eqs. (2-17) to (2-18) through (2-20).

During the first year after loading:

$$\text{Creep } \gamma_h = 1.14 - 0.023 h, \quad (2-17)$$

For ultimate values:

$$\text{Creep } \gamma_h = 1.10 - 0.017 h, \quad (2-18)$$

During the first year of drying:

$$\text{Shrinkage } \gamma_h = 1.23 - 0.038 h, \quad (2-19)$$

For ultimate values:

$$\text{Shrinkage } \gamma_h = 1.17 - 0.029 h, \quad (2-20)$$

where h is the average thickness in inches of the part of the member under consideration.

During the first year after loading:

$$\text{Creep } \gamma_h = 1.14 - 0.00092 h, \quad (2-17a)$$

For ultimate values:

$$\text{Creep } \gamma_h = 1.10 - 0.00067 h, \quad (2-18a)$$

During the first year after loading:

$$\text{Shrinkage } \gamma_h = 1.23 - 0.00015 h, \quad (2-19a)$$

For ultimate values:

$$\text{Shrinkage } \gamma_h = 1.17 - 0.00114 h, \quad (2-20a)$$

where h is in mm.

Representative values are shown in Table 2.5.5.1.

2.5.5.b Volume-surface ratio method

The volume-surface ratio equations (2-21) and (2-22) were adapted from Reference 23.

$$\text{Creep } \gamma_{vs} = \frac{2}{3}[1 + 1.13 \exp(-0.54 v/s)] \quad (2-21)$$

$$\text{Shrinkage } \gamma_{vs} = 1.2 \exp(-0.12 v/s) \quad (2-22)$$

where v/s is the volume-surface ratio of the member in inches.

$$\text{Creep } \gamma_{vs} = \frac{2}{3}[1 + 1.13 \exp(-0.0213 v/s)] \quad (2-21a)$$

$$\text{Shrinkage } \gamma_{vs} = 1.2 \exp(-0.00472 v/s) \quad (2-22a)$$

where v/s in mm.

Representative values are shown in Table 2.5.5.2.

However, for either method γ_{sh} should not be taken less than 0.2. Also, use $\gamma_{sh}(\epsilon_{sh})_u \geq 100 \times 10^{-6}$ in./in., (m/m) if concrete is under seasonal wetting and drying cycles and $\gamma_{sh}(\epsilon_{sh})_u \geq 150 \times 10^{-6}$ in./in. (m/m) if concrete is under sustained drying conditions.

2.5.6 Temperature other than 70 F (21 C)

Temperature is the second major environmental factor in creep and shrinkage. This effect is usually considered to be less important than relative humidity since in most structures the range of operating temperatures is small,⁶⁸ and high temperatures seldom affect the structures during long periods of time.

The effect of temperature changes on concrete creep⁶⁸ and shrinkage is basically two-fold. First, they directly influence the time ratio rate. Second, they also affect the rate of aging of the concrete, i.e. the change of material properties due to progress of cement hydration. At 122 F (50 C), creep strain is approximately two to three times as great as at 68-75 F (19-24 C). From 122 to 212 F (50 to 100 C) creep strain continues to increase with temperature, reaching four to six times that experienced at room temperatures. Some studies have indicated an apparent creep rate maximum occurs between 122 and 176 F (50 and 80 C).⁸⁶ There is little data establishing creep rates above 212 F (100 C). Additional information on temperature effect on creep may be found in References 68, 84, and 85.

2.6-Correction factors for concrete composition

Equations (2-23) through (2-30) are recommended for use in obtaining correction factors for the effect of slump, percent of fine aggregate, cement and air content. It should be noted that for slump less than 5 in. (130 mm), fine aggregate percent between 40-60 percent, cement content of 470 to 750 lbs. per yd^3 (279 to 445 kg/m^3) and air content less than 8 percent, these factors are approximately equal to 1.0.

These correction factors shall be used only in connection with the average values suggested for $v_u = 2.35$ and $(\epsilon_{sh})_u = 780 \times 10^{-6}$ in./in. (m/m). As recommended in 2.4, these average values for v_u and $(\epsilon_{sh})_u$ should be used only in the absence of specific creep and shrinkage data for local aggregates and conditions determined in accordance with ASTM C 512.

If shrinkage is known for local aggregates and conditions, Eq. (2-31) as discussed in 2.6.5, is recommended.

The principal disadvantage of the concrete composition correction factors is that concrete mix characteristics are unknown at the design stage and have to be estimated. Since these correction factors are normally not excessive and tend to offset each other, in most cases, they may be neglected for design purposes.

2.6.1 Slump

$$\text{Creep } \gamma_s = 0.82 + 0.067s \quad (2-23)$$

$$\text{Shrinkage } \gamma_s = 0.89 + 0.041s \quad (2-24)$$

where s is the observed slump in inches. For slump in mm use:

$$\text{Creep } \gamma_s = 0.82 + 0.00264s \quad (2-23 \text{ a})$$

$$\text{Shrinkage } \gamma_s = 0.89 + 0.00161s \quad (2-24a)$$

2.6.2 Fine aggregate percentage

$$\text{Creep } \gamma_\psi = 0.88 + 0.0024\psi \quad (2-25)$$

For $\psi \leq 50$ percent

$$\text{Shrinkage } \gamma_\psi = 0.30 + 0.014\psi \quad (2-26)$$

For $\psi > 50$ percent

$$\text{Shrinkage} = 0.90 + 0.002\psi \quad (2-27)$$

where ψ is the ratio of the fine aggregate to total aggregate by weight expressed as percentage.

2.6.3 Cement content

Cement content has a negligible effect on creep coefficient. An increase in cement content causes a reduced creep strain if water content is kept constant; however, data indicate that a proportional increase in modulus of elasticity accompanies an increase in cement content.

If cement content is increased and water-cement ratio is kept constant, slump and creep will increase and Eq. (2-23) applies also.

$$\text{Shrinkage } \gamma_c = 0.75 + 0.00036c \quad (2-28)$$

where c is the cement content in pounds per cubic yard. For cement content in Kg/m^3 , use:

$$\text{Shrinkage } \gamma_c = 0.75 + 0.00061c \quad (2-28a)$$

2.6.4 Air content

$$\text{Creep } \gamma_\alpha = 0.46 + 0.09\alpha, \quad \text{but not less than 1.0} \quad (2-29)$$

$$\text{Shrinkage } \gamma_\alpha = 0.95 + 0.008\alpha \quad (2-30)$$

where α is the air content in percent.

2.6.5 Shrinkage ratio of concretes with equivalent paste quality⁹¹

Shrinkage strain is primarily a function of the shrinkage characteristics of the cement paste and of the aggregate volume concentration. If the shrinkage strain of a given mix has been determined, the ratio of shrinkage strain of two mixes $(\epsilon_{sh})_1/(\epsilon_{sh})_2$, with different content of paste but with equivalent paste quality is given in Eq. (2-31).

$$\frac{(\epsilon_{sh})_{u1}}{(\epsilon_{sh})_{u2}} = \frac{1 - (v_1)^{1/3}}{1 - (v_2)^{1/3}} \quad (2-31)$$

where v_1 and v_2 are the total aggregate solid volumes per unit volume of concrete for each one of the mixes.

2.7-Example

Find the creep coefficient and shrinkage strains at 28, 90, 180, and 365 days after the application of the load, assuming that the following information is known: 7 days moist cured concrete, age of loading $t_{la} = 28$ days, 70 percent ambient relative humidity, shrinkage considered from 7 days, average thickness of member 8 in. (200 mm), 2.5 in. slump (63 mm), 60 percent fine aggregate, 752 lbs. of cement per yd^3 (446 Kg/m^3), and 7 percent air content.⁷ Also, find the differential shrinkage strain, $(\epsilon_{sh})_\delta$ for the period starting at 28 days after the application of the load, $t_{la} = 56$ days.

The applicable correction factors are summarized in Table 2.7.1. Therefore:

$$v_u = (2.35)(0.710) = 1.67$$

$$(\epsilon_{sh})_u = (780 \times 10^{-6})(0.68) = 530 \times 10^{-6}$$

The results from the use of Eqs. (2-8) and (2-9) or Table 2.4.1 are shown in Table 2.7.2.

Notice that if correction factors for the concrete composition are ignored for v_t and $(\epsilon_{sh})_t$, they will be 10 and 4 percent smaller, respectively.

2.8-Other methods for predictions of creep and shrinkage

Other methods for prediction of creep and shrinkage are discussed in Reference 61, 68, 86, 87, 89, 93, 94, 95, 97, and 98. Methods in References 97 and 98 subdivide creep strain into delayed elastic strain and plastic flow (two-component creep model). References 88, 89, 92, 99, 100, 102, and 104 discuss the conceptual differences between the current approaches to the formulation of the creep laws. However, in dealing with any method, it is important to recall what is discussed in Sections 1.2 and 2.1 of this report.

2.8.1 Remark on refined creep formulas needed for special structures^{93,94,95}

The preceding formulation represents a compromise between accuracy and generality of application. More accurate formulas are possible but they are inevitably not as general.

The time curve of creep given by Eq. (2-8) exhibits a decline of slope in log- t scale for long times. This property is correct for structures which are allowed to lose their moisture and have cross sections which are not too massive (6 to 12 in., 150 to 300 mm). Structures which are insulated, or submerged in water, or are so massive they cannot lose much of their moisture during their lifetime, exhibit creep curves whose slope in log- t scale is not decreasing at end, but steadily increasing. For example, if Eq. (2-8) were used for extrapolating short-time creep data for a nuclear reactor containment into long times, the long-term creep values would be seriously underestimated, possibly by as much as 50 percent as shown in Fig. 3 of Ref. 81.

It has been found that creep without moisture exchange (basic creep) for any loading age $t_{\ell a}$ is better described by Equation (2-33).^{86,80,83,95} This is called the double power law.

In Eq. (2-33) ψ_1 is a constant, and strain ϵ is the sum of the instantaneous strain and creep strain caused by unit stress.

$$\epsilon = \frac{1}{E_o} + \frac{\psi_1}{E_o} (t_{\ell a})^{-1/8} t^{1/8} \quad (2-33)$$

where $1/E_o$ is a constant which indicates the lefthand asymptote of the creep curve when plotted in log t -scale (time $t = 0$ is at $-\infty$ in this plot). The asymptotic value $1/E_o$ is beyond the range of validity of Eq. (2-33) and should not be confused with elastic modulus. Suitable values of constants are $\psi_1 = 0.97\nu_u$ and $1/E_o = 0.84/E_{ct}$, being E_{ct} the modulus of concrete which does not undergo drying. With these values, Eq. (2-33) and Eq. (2-8) give the same creep for $t_{\ell a} = 28$ days, $t = 10,000$ days and 100 percent relative humidity ($\gamma_A = 0.6$), all other correction factors being taken as one.

Eq. (2-33) has further the advantage that it describes not only the creep curves with their age dependence, but also the age dependence of the elastic modulus E_{ct} in absence of drying. E_{ct} is given by $\epsilon = 1/E_{ct}$ for $t \approx 0.001$ day, that is:

$$\frac{1}{E_{ct}} = \frac{1}{E_o} + \frac{\psi_1}{E_o} (0.001)^{1/8} (t_{\ell a})^{-1/8} \quad (2-34)$$

Eq. (2-33) also yields the values of the dynamic modulus, which is given by $\epsilon = 1/E_{dyn}$ when $t = 10^{-7}$ days is substituted. Since three constants are necessary to describe the age dependence of elastic modulus (E_o , ψ , and $1/8$), only one additional constant (i.e., $1/8$) is needed to describe creep.

In case of drying, more accurate, but also more complicated, formulas may be obtained⁹⁴ if the effect of cross section size is expressed in terms of the shrinkage half-time, as given in Eq. (2-35) for the age t_d at which concrete drying begins.

$$\tau_{sh} = 600 \left[\frac{\lambda_s d_c^2}{150} \right] \frac{C_1}{(C_1)_a} \quad (2-35)$$

where:

- d_c = characteristic thickness of the cross section, or twice the volume-surface ratio
- $(d_c) = 2 \text{ v/s in mm}$
- C_1 = Drying diffusivity of the concrete (approx. 10 mm/day if measurements are unavailable)
- $(C_1)_a$ = age dependence coefficient
- $= C_7 \lambda_T (0.05 + \sqrt{6.3/t_d})$
- $C_7 = \frac{w}{8} - 12$, if $C_7 < 7$, set $C_7 = 7$
if $C_7 > 21$, set $C_7 = 21$
- λ_s = coefficient depending on the shape of cross section, that is:
- $\lambda_s = 1.00$ for an infinite long slab
- $= 1.15$ for an infinite long cylinder
- $= 1.25$ for an infinite long square prism
- $= 1.30$ for a sphere
- $= 1.55$ for a cube
- λ_T = temperature coefficient
- $= \frac{T}{t_o} \exp \left(\frac{5000}{T_o} - \frac{5000}{T} \right)$
- T = concrete temperature in kelvin
- T_o = reference temperature in kelvin
- w = water content in kg/m^3

By replacing t in Eq. (2-9) t/τ_{sh} , shrinkage is expressed without the need for the correction factor for size in Section 2.5.5.

The effect of drying on creep may then be expressed by adding two shrinkage-like functions ν_d and ν_p to the double power law for unit stress.⁹⁶ Function ν_d expresses the additional creep during drying and function ν_p , being negative, expresses the decrease of creep by loading after an initial drying. The increase of creep during drying arises about ten times slower than does shrinkage and so function ν_d is similar to shrinkage curve in Eq. (2-9) with t replaced by $0.1 t/\tau_{sh}$ in Eq. (2-8).

This automatically accounts also for the size effect, without the need for any size correction factor. The decrease of creep rate due to drying manifests itself only very late, after the end of moisture loss. This is apparent from the fact that function τ_{sh} is similar to shrinkage curve in Eq. (2-9) with t replaced by $0.01 t/\tau_{sh}$. Both ν_d and ν_p include multiplicative correction factors for relative humidity, which are zero at 100 percent, and function ν_d further includes a factor depending on the time lag from the beginning of drying exposure to the beginning of loading.

2.9-Thermal expansion coefficient of concrete

2.9.1 Factors affecting the expansion coefficient

The main factors affecting the value of the thermal coefficient of a concrete are the type and amount of aggregate and the moisture content. Other factors such as mix proportions, cement type and age influence its magnitude to a lesser extent.

The thermal coefficient of expansion of concrete usually reflects the weighted average of the various constituents. Since the total aggregate content in hardened concrete varies from 65 to 80 percent of its volume, and the elastic modulus of aggregate is generally five times that of the hardened cement component, the rock expansion dominates in determining the expansion of the composite concrete. Hence, for normal weight concrete with a steady water content (degree of saturation), the thermal coefficient of expansion for concrete can be regarded as directly proportional to that of the aggregate, modified to a limited extent by the higher expansion behavior of hardened cement.

Temperature changes affect concrete water content, environment relative humidity and consequently concrete creep and shrinkage as discussed in Section 25.6. If creep and shrinkage response to temperature changes are ignored and if complete histories for concrete water content, temperature and loading are not considered, the actual response to temperature changes may drastically differ from the predicted one.⁷⁹

2.9.2 Prediction of thermal expansion coefficient

The thermal coefficients of expansion determined when using testing methods in ASTM C 531 and CRD 39 correspond to the oven-dry condition and the saturated conditions, respectively. Air-dried concrete has a higher coefficient than the oven-dry or saturated concrete, therefore, experimental values shall be corrected for the expected degree of saturation of the concrete member. Values of e_{mc} in Table 2.9.1 may be used as corrections to the coefficients determined from saturated concrete samples. In the absence of specific data from local materials and environmental conditions, the values given by Eq. (2-32) for the thermal coefficient of expansion e_{th} may be used.⁷⁶ Eq. (2-32) assumes that the thermal coefficient of expansion is linear within a temperature change over the range of 32 to 140 F (0 to 60 C) and applies only to a steady water content in the concrete.

For e_{th} in $10^{-6}/F$:

$$e_{th} = e_{mc} + 1.72 + 0.72 e_a \quad (2-32)$$

For e_{th} in $10^{-6}/C$:

$$e_{th} = e_{mc} + 3.1 + 0.72 e_a \quad (2-32a)$$

where:

e_{mc} = the degree of saturation component as given in Table 2.9.1

1.72 = the hydrated cement past component (3.1)

e_a = the average thermal coefficient of the total aggregate as given in Table 2.9.2

If thermal expansion of the sand differs markedly from that of the coarse aggregate, the weighted average by solid volume of the thermal coefficients of the sand and coarse aggregate shall be used.

A wide variation in the thermal expansion of the aggregate and related concrete can occur within a rock group. As an illustration, Table 2.9.3 summarizes the range of measured values for each rock group in the research data cited in Reference 76.

For ordinary thermal stress calculations, when the type of aggregate and concrete degree of saturation are unknown and an average thermal coefficient is desired, $e_{th} = 5.5 \times 10^{-6}/F$ ($e_{th} = 10.0 \times 10^{-6}/C$) may be sufficient. However, in estimating the range of thermal movements (e.g., highways, bridges, etc.), the use of lower and upper bound values such as $4.7 \times 10^{-6}/F$ and $6.5 \times 10^{-6}/F$ ($8.5 \times 10^{-6}/C$ and $11.7 \times 10^{-6}/C$) would be more appropriate.

2.10-Standards cited in this report

Standards of the American society for Testing and Materials (ASTM) referenced in this report are listed below with their serial designation:

ASTM A 416	“Standard Specification for Uncoated Seven-Wire Stress-Relieved Strand for Prestressed Concrete”
ASTM A 421	“Standard Specification for Uncoated Stress-Relieved Wire for Prestressed Concrete”
ASTM C 33	“Standard Specifications for Concrete Aggregates”
ASTM C 39	“Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens”
ASTM C 78	“Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)”
ACI C 192	“Standard Method of Making And Curing Concrete Test Specimens in the Laboratory”
ASTM C 469	“Standard Method for Static Modulus of Elasticity and Poisson’s Ratio of Concrete in Compression”
ASTM C 511	“Standard Specification for Moist Cabinets and Rooms Used in the Testing Hydraulic Cements and Concretes”
ASTM C 512	“Standard Test Method for Creep of Concrete in Compression”
ASTM C 531	“Standard Method for Preparing, and Testing Specimens from Hardened Lightweight Insulating Concrete for Compressive Strength”

ASTM E 328 "Standard Recommended Practice for Stress-Relaxation Tests for Materials and Structures"

The following standard of the U.S. Army Corps of Engineers (CRD) is referred in Section 2.9 of this report:

CRD C39 "Method of Test for Coefficient of Linear Thermal Expansion of Concrete"

CHAPTER 3-FACTORS AFFECTING THE STRUCTURAL RESPONSE-ASSUMPTIONS AND METHODS OF ANALYSIS

3.1-Introduction

Prediction of the structural response of reinforced concrete structures to time-dependent concrete volume changes is complicated by:

- a) The inherent nonelastic properties of the concrete
- b) The continuous redistribution of stress
- c) The nonhomogeneous nature of concrete properties caused by the stages of construction
- d) The effect of cracking on deflection
- e) The effect of external restraints
- f) The effect of the reinforcement and/or prestressing steel
- g) The interaction between the above factors and their dependence on past histories of loadings, water content and temperature

The complexity of the problem requires some simplifying assumptions and reliance on empirical observations.

3.2-Principal facts and assumptions

3.2.1 Principal facts

- a) Each loading change produces a resulting deformation component continuous for an infinite period of time⁷⁰
- b) Applied loads in homogeneous statically indeterminate structures cause no time-dependent change in stress and all deformations are proportional to creep coefficient ν , as long as the support conditions remain unchanged⁷⁰
- c) The secondary, statically indetermined moments due to prestressing are affected in the same proportion as prestressing force by time-dependent deformations, which is a relatively small effect that is usually neglected
- d) In a great many cases and except when instability is a factor, time-dependent strains due to actual loads do not significantly affect the load capacity of a member. Failure is controlled by very large

strains that develop at collapse, regardless of previous loading history.⁷¹ In these cases, time-dependent strains only affect the structure serviceability. When instability is a factor, creep increment of the eccentricity in beam-columns under sustained load will decrease the member capacity with time

- e) Change in concrete properties with age, such as elastic, creep and shrinkage deformations, must be taken into account

3.2.2 Assumptions

- a) Concrete members including their creep, shrinkage and thermal properties, are considered homogeneous
- b) Creep, shrinkage and elastic strains are mutually additive and independent
- c) For stresses less than about 40 to 50 percent of the concrete strength, creep strains are assumed to be approximately proportional to the sustained stress and obey the principle of superposition of strain histories.^{70,80}

However, tests in References 105 and 106 have shown the nonlinearity of creep strain with stress can start at stresses as low as 30 to 35 percent of the concrete strength. Also, strain superposition is only a first approximation because the individual response histories affect each other as can be seen with recovery curves after unloading
- d) Shrinkage and thermal strains are linearly distributed over the depth of the cross section. This assumption is acceptable for thin and moderate sections, respectively, but may result in error for thick sections
- e) The complex dependence of strain upon the past histories of water content and temperature is neglected for the purpose of analyzing ordinary structures
- f) Restraint by reinforcement and/or prestressing steel is accounted for in the average sense without considering any gradual stress transfer between reinforcement and concrete
- g) The creep time-ratio for various environment humidity conditions and various sizes and shapes of cross section are assumed to have the same shape

Even with these simplifications, the theoretically exact analysis of creep effects according to the assumptions stated,⁶⁶ is still relatively complicated. However, more accurate analysis is not really necessary in most instances, except special structures, such as nuclear reactor vessels, bridges or shells of record spans, or special ocean structures. Therefore, simplified methods of analysis^{66,80} are being used in conjunction with empirical methods to account for the effects of cracking and reinforcement restraint.

3.3-Simplified methods of creep analysis

In choosing the method of analysis, two kinds of cases are distinguished.

3.3.1 Cases in which the gradual time change of stress due to creep and shrinkage is small and has little effect

This usually occurs in long-time deflection and pre-stress loss calculations. In such cases the creep strain is accounted for with sufficient accuracy by an elastic analysis in which the actual concrete modulus at the time of initial loading, is replaced with the so-called effective modulus as given by Eq. (3-1).

$$E_e = E_{ci}/(1 + \nu_t) \quad (3-1)$$

This approach is implied in Chapter 4. To check if the assumption of small stress change is true, the stress computed on the basis of E_{ci} should be compared with the stress computed on the basis of E_e .

3.3.2 Cases in which the gradual time change of stress due to creep and shrinkage is significant

In such cases, the age-adjusted effective modulus method^{67,68,69} is recommended as discussed in Chapter 5.

3.4-Effect of cracking in reinforced and prestressed members

To include the effect of cracking in the determination of an effective moment of inertia for reinforced beams and one-way slabs, Eq. (3-2)^{10,25,26} has been adopted by the ACI Building Code (ACI 318).²⁷

$$I_e = (M_{cr}/M_{max})^3 I_g + [1 - (M_{cr}/M_{max})^3] I_{cr} \quad (3-2)$$

where M_{cr} is the cracking moment, M_{max} denotes the maximum moment at the stage for which deflection is being computed, I_g is the moment of inertia of the gross section neglecting the steel and I_{cr} is the moment of inertia of the cracked transformed section.

Eq. (3-2) applied only when $M_{max} \geq M_{cr}$; otherwise, $I_e = I_g$.

I_e in Eq. (3-2) has limits of I_g and I_{cr} , and thus provides a transition expression between the two cases given in the ACI 318 Code.^{12,27} The moment of inertia I , of the uncracked transformed section might be more accurately used instead of the moment inertia of the gross section I_g in Eq. (3-2), especially for heavily reinforced members and lightweight concrete members (low E_c and hence high modular ratio E_s/E_{ci}).

Eq. (3-2) has also been shown²⁸ to apply in the deflection calculations of cracked prestressed beams.

For numerical analysis, in which the beam is divided into segments or finite elements, it has been shown²⁵ that I , values at individual sections can be determined by modifying Eq. (3-2). The power of 3 is changed to 4 and the moment ratio in both terms is changed to M_{cr}/M , where M is the moment at each section. Such a numerical procedure was used in the development of Eq. (3-2).²⁵

The above cracking moment is given in Eqs. (3-3) and

(3-4).

For reinforced members:

$$M_{cr} = f_r I_g / y_t \quad (3-3)$$

For noncomposite prestressed members:

$$(M_L)_{cr} = Fe + (FI_g)/A_g y_t + (f_r I_g)/y_t - M_D \quad (3-4)$$

The cracking moment for unshored and shored composite prestressed beams is given in Eq. (41) and (42) of Reference 63.

Equation (3-2) refers to an average effective I for the variable cracking along the span, or between the inflection points of continuous beams. For continuous members (at one or both ends), a numerical procedure may be needed although the use of an average of the positive and negative moment region values from Eq. (3-2) as suggested in Section 9.5.2.4 of Reference 27 should yield satisfactory results in most cases. For spans which have both ends continuous, an effective average moment of inertia I_{ea} is obtained by computing an average for the end region values, I_{e1} and I_{e2} and then averaging that result with the positive moment region value obtained for Eq. (3-2) as shown in Eq. (3-5).

$$I_{ea} = [(I_{e1} + I_{e2})/2 + I_{ep}]/2 \quad (3-5)$$

In other cases, a weighted average related to the positive and negative moments may be preferable. For example, the weighted average moment of inertia I_{ew} would be given by Eq. (3-6).⁷²

$$I_{ew} = \left\{ I_{ep} \left(1 - \left[\frac{M_{e1} + M_{e2}}{2M} \right]^\beta \right) + \left[\frac{I_{e1} + I_{e2}}{2} \right] \left[\frac{M_{e1} + M_{e2}}{2M} \right]^\beta \right\} \quad (3-6)$$

where, I_{ep} is the effective moment of inertia for the positive zone of the beam and β is a positive integer that may be equal to unity for simplicity or equal to two, three or larger for a modest increase in accuracy.

For a span with one end continuous, the $(I_{e1} + I_{e2})/2$ in Eqs. (3-5) and (3-6) shall be substituted for I for the negative end zone.

For a flat plate and two way slab interior panels, it has been shown²⁹ that Eq. (3-2) can be used along with an average of the positive and negative moment region values as follows:

Flat plate-both positive and negative values for the long direction column strip.

Two way slabs-both positive and negative values for the short direction middle strip

The center of interior panels normally remains uncracked in common designs of these slabs.

For the effect of repeated load cycles on cracking range, see Reference 63.

3.5-Effective compression steel in flexural members

Compression steel in reinforced flexural members and nontensioned steel in prestressed flexural members tend to offset the movement of the neutral axis caused by creep. The net movement of the neutral axis is the resultant of two movements. A movement towards the tensile reinforcement (increasing the concrete compression zone, which results in a reduction in the moment arm). This movement is caused by the effect of creep plus a reduction in the compression zone due to the progressive cracking in the tensile zone.

The second movement is produced by the increase in steel strains due to the reduction of the internal moment arm (plus the small effect, if any, of repeated live load cycles). As cracking progresses, steel strains increase further and reduce the moment arm.

The reduced creep effect resulting from the movement of the neutral axis and the presence of compression steel in reinforced members A_s' , and the inclusion of nontensioned high strength or mild steel (as specified below) in prestressed members is given by the reduction factor ξ_r in Eqs. (3-7) and (3-9).

The approximate effect of progressive cracking under creep loading and repeated load cycles is also included in the factor ξ_r . Eq. (3-8) refers to the combined creep and shrinkage effect in reinforced members.

For reinforced flexural members, creep effect only:¹⁰

$$\xi_r = 0.85 - 0.45 (A_s'/A_s), \text{ but not less than } 0.40 \quad (3-7)$$

For reinforced flexural members, creep and shrinkage effect:^{26,30}

$$\xi_r = 1 - 0.60 (A_s'/A_s), \text{ but not less than } 0.30 \quad (3-8)$$

For prestressed flexural members:^{28,63}

$$\xi_r = 1/[1 + A_s'/A_s] \quad (3-9)$$

Approximately the same results are obtained in Eqs. (3-7), (3-8), and (3-9) as shown in Table 35.1. It is assumed in Eq. (3-9) that the nontensioned steel and the prestressed steel are on the same side of the section centroid and that the eccentricities of the two steels are approximately the same. See Reference 28 when the eccentricities are substantially different.

Eqs. (3-8) and (4-3) are used in ACI 318²⁷ with a time-dependent factor for both creep and shrinkage, $\tau_u = 2.0$. As the ratio, A_s'/A_s , increases, these two sets of factors approach the same value, since shrinkage warping is negligible when the compression reinforcement is high.

The effects of creep plus shrinkage are arbitrarily lumped together in Eq. (3-8).

In Reference 74, Branson notes that Eq. (3-8), as used in ACI 318²⁷ is likely to overestimate the effect of the

compression steel in restraining time-dependent deflections of members with low steel percentage (e.g. slabs) and recommends the alternate Eq. (3-10).

$$\xi_r \tau_u = \tau_u/[1 + 50 \rho'] \quad (3-10)$$

where $\xi_r \tau_u$ is a long time deflection multiplier of the initial deflection and ρ' is the compressive steel ratio A_s'/bd . He further suggests that a factor, $\tau_u = 2.5$ for beams and $\tau_u = 3.0$ for slabs, rather than 2.0, would give improved results.

The calculation of creep deflection as $\xi_r \tau_i$ times the initial deflection a_i , yields the same results as that obtained using the "reduced or sustained modulus of elasticity, E_{ci} , method," provided the initial or short-time modular ratio, n , (at the time of loading) and the transformed section properties are used. This can be seen from the fact that E_{ci} used for computing the initial deflection, is replaced by E_e as given by Eq. (3-1), for computing the initial plus the creep deflection. The factor 1.0 in Eq. (3-1) corresponds to the initial deflection. Except for the calculation of I in the sustained modulus method (when using or not using an increased modular ratio) and I_e/ξ_r in the effective section method, the two methods are the same for computing long-time deflections, exclusive of shrinkage warping.

The reduction factor ξ_r , for creep only (not creep and shrinkage) in Eq. (3-7) is suggested as a means of taking into account the effect of compression steel and the offsetting effects of the neutral axis movement due to creep as shown in Figure 3 of Ref. 10. These offsetting effects appear normally to result in a movement of the neutral axis toward the tensile reinforcement such that:

$$\frac{\epsilon_{cp}}{\epsilon_i} = \xi_r \frac{\phi_{cp}}{\phi_i} = \xi_r \frac{a_{cp}}{a_i} \quad (3-11)$$

in which ξ_r from Eq. 3-7 is less than unity. (See Table 3.5.1). Subscripts cp and i refer to the creep and initial strains, curvatures ϕ , and deflections a , respectively.

The use of the long-time modular ratio, $n_l = n(1 + \nu_l)$, in computing the transformed section properties has also been shown^{31,32} to accomplish these purposes and to provide satisfactory results in deflection calculations.

In all appropriate equations herein, $\nu_p, \nu_w, \tau_p, \tau_u$, are replaced by $\xi_r \nu_p, \xi_r \nu_w, \xi_r \tau_p, \xi_r \tau_u$ respectively, when these effects are to be included.

3.6-Deflections due to warping

3.6.1 Warping due to shrinkage

Deflections due to warping are frequently ignored in design calculation, when the effects of creep and warping are arbitrarily lumped together.²⁷ For thin members, such as canopies and thin slabs, it may be desirable to consider warping effects separately.

For the case in which the reinforcement and eccentricity are constant along the span and the same in the positive and negative moment regions of continuous

beams, shrinkage deflections for uniform beams are computed by Eq. (3-12).

$$a_{sh} = \xi_w \phi_{sh} \ell^2 \quad (3-12)$$

where ξ_w is a deflection coefficient defined in Table 4.2.1 for different boundary conditions, and ϕ_{sh} is the curvature due to shrinkage warping. For more practical cases, some satisfactory compromise can usually be made with regard to variations in steel content and eccentricity, and for nonuniform temperature effects.

3.6.2-Methods of computing shrinkage curvature

Three methods for computing shrinkage curvature were compared in References 10 and 25 with experimental data: the equivalent tensile force method,^{31,33,36,37} Miller's method³⁸ and an empirical method based on Miller's approach extended to include doubly reinforced beams.^{10,25} The agreement between computed and measured results was reasonably good for all three of the methods.

The equivalent tensile force method (a fictitious elastic analysis), as modified in References 10 and 25 using $E_c/2$ and the gross section properties for better results, is given by Eq. (3-13).

$$\phi_{sh} = (T_s e_g) / (E_c I_g / 2) \quad (3-13)$$

where $T_s = (A_s + A_s') \epsilon_{sh} E_s$, and e_g and I_g refer to the gross section.

Miller's method³⁸ assumes that the extreme fiber of the beam furthest from the tension steel (method refers to singly reinforced members only) shrinks the same amount as the free shrinkage of the concrete, ϵ_{sh} . Following this assumption, the curvature of the member is given by Eq. (3-14).

$$\phi_{sh} = \frac{\xi_{sh} - \epsilon_s}{d} = \frac{\epsilon_{sh}}{d} \left[1 - \frac{\epsilon_s}{\epsilon_{sh}} \right] \quad (3-14)$$

where ϵ_s is the steel strain due to shrinkage. Miller suggested empirical values of $(\epsilon_s / \epsilon_{sh}) = 0.1$ for heavily reinforced members and 0.3 for moderately reinforced members.

The empirical method represents a modification of Miller's method. The curvature of a member is given by Eqs. (3-15) and (3-16) which are applicable to both singly and doubly reinforced members. The steel percentage in these equations are expressed in percent ($\rho = 3$ for 3 percent steel, for example).

For $(\rho - \rho') \leq 3.0$ percent:

$$\phi_{sh} = (0.7) \frac{\epsilon_{sh}}{h} (\rho - \rho')^{1/2} \left[\frac{\rho - \rho'}{\rho} \right]^{1/2} \quad (3-15)$$

For $(\rho - \rho') > 3.0$ percent:

$$\phi_{sh} = \epsilon_{sh} / h \quad (3-16)$$

where h is the overall thickness of the section.

For singly reinforced members, $\rho' = 0$, and Eq. (3-15) reduces to Eq. (3-17).

$$\phi_{sh} = (0.7) \frac{\epsilon_{sh}}{h} \rho^{1/3} \quad (3-17)$$

which results in:

$\phi_{sh} = 0.56 (\epsilon_{sh})/h$, when $\rho' = 0.5$ percent	
0.70	1.0
0.88	2.0
1.01	3.0

Eqs. (3-15), (3-16), and (3-17) were adapted from Miller's approach. For example, his method results in the following expression for singly reinforced members:

$$\phi_{sh} = 0.7 \epsilon_{sh} / d \text{ for "moderately" reinforced beams}$$

$$\phi_{sh} = 0.9 \epsilon_{sh} / d \text{ for "heavily" reinforced beams}$$

which approximately correspond to $\rho = 1.0$ and $\rho = 2.0$ in Eq. 3-17.

The use of the more convenient thickness, h , instead of the effective depth, d , in Eqs. (3-15), (3-16), and (3-17) was found to provide closer agreement with the test data.

3.6.3-Warping due to temperature change

Since concrete and steel reinforcement have similar thermal coefficients of expansion (i.e., 4.7 to $6.5 \times 10^{-6}/^\circ\text{F}$ for concrete and $6.5 \times 10^{-6}/^\circ\text{F}$ for steel), the stresses produced by normal temperature range are usually negligible.

When the temperature change is constant along with the span, thermal deflections for uniform beams are given by Eq. (3-18).

$$a_T = \xi_w \phi_{th} \ell^2 \quad (3-18)$$

where ξ_w is the deflection coefficient (Table 4.2.1). The curvature ϕ_{th} due to temperature warping is given by Eq. (3-19).

$$\phi_{th} = (e_{th} t_h) / h \quad (3-19)$$

where e_{th} is the thermal coefficient of expansion and t_h is the difference in temperature across the overall thickness h .

The values of ν_t , ϵ_{sh} , and e_{th} Usually correspond to steady state conditions. A sustained nonuniform change in temperature will influence creep and shrinkage. As a result, significant redistribution of stresses in statically indeterminate structures may occur to such an extent that the thermal effects caused by heating may be completely nullified. A nonuniform temperature reversal may cause a stress reversal.⁷⁹