

DESIGN HANDBOOK

In Accordance with the Strength Design Method of ACI 318-89

VOLUME 2—Columns

Reported by ACI Committee 340

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FOREWORD

The three volumes of the ACI Design Handbook are intended for use by persons having a general familiarity with the strength design method and with "Building Code Requirements for Reinforced Concrete (ACI 318-89)."

This volume, Volume 2, presents Design Aids for use in the engineering design and analysis of reinforced concrete columns.

The Design Aids are tables and graphs intended to save the designer the effort of repeatedly performing certain routine calculations:

- COLUMNS 1-5 simplify calculations involving slenderness effects.
- COLUMNS 6-8 provide information useful in considering load-moment strength of column cross sections subject to uniaxial bending and axial load.
- COLUMNS 9-12 provide means for applying to columns subjected to biaxial bending and axial load the load-moment strength data for columns subject to uniaxial bending and axial load.
- REINFORCEMENT 1-7 give information on bar dimensions, minimum allowable bar spacing, bar development lengths and splice lengths, maximum allowable spiral pitch, etc.

In addition to the Design Aids (buff pages), this volume includes a Commentary (gray pages) explaining how each Design Aid was calculated, and Design Examples (white pages) illustrating the use of the Design Aids. A note at the bottom of each Design Aid tells which Design Examples illustrate use of the table or graph.

This handbook identifies column sections by letter and number designations such as:

R5-60.75

in which

- R = rectangular tied column with bars on four faces
- 5 = specified compressive strength of concrete f'_c of 5 ksi
- 60 = specified yield strength of longitudinal steel f_y of 60 ksi
- .75 = γ , the ratio of distance between centroids of opposite bars and thickness of gross column cross section in the direction of bending, of 0.75.

Table 1 shows the symbols used to identify the types of column cross sections and lists the categories of column sections for which load-moment interaction diagrams and other Design Aids are provided.

The load-moment strength interaction diagrams (COLUMNS 7) and tables (COLUMNS 8) given in this volume use (1) an axial stress $\phi P_n/A_g$ rather than axial load, (2) an index of moment $\phi M_n/A_g h$ rather than moment itself, (3) the ratio of eccentricity to column size e/h , and (4) the ratio ρ_g of total area of longitudinal reinforcement to gross area of column cross section.

Thus the load-moment strength interaction diagrams and tables are not related to specific column sizes, loads, moments, or numbers and sizes of bars. The designer can determine the optimum design for a particular situation instead of merely selecting the best of several possible combinations of column size and quantity and size of bars.

This volume is intended to be a tool for use by structural engineers already familiar with column design rather than a textbook. Before using the Design Aids, the designer will probably find it desirable to peruse the Design Examples.

The general procedure used in the Design Examples for design or analysis of column sections subjected to axial load and uniaxial bending is:

1. If column size is not known, select a trial column size, using the axial load-moment strength interaction data (COLUMNS 7).

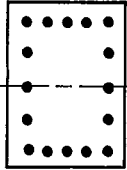
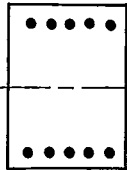
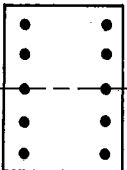
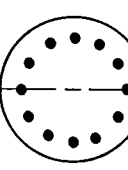
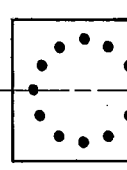
2. Consider column slenderness effects, using COLUMNS 1-5.

3. Determine reinforcement ratio ρ_g using the value of γ (which may be determined or estimated from COLUMNS 6) plus known load and moment with the appropriate load-moment strength interaction data. (COLUMNS 7 presents these load-moment strength interaction data in graphical form; COLUMNS 8 presents critical load-moment strength interaction coordinates to greater accuracy in tabular form.) Using the value of reinforcement ratio ρ_g thus determined, compute the required cross section area A_{st} of longitudinal reinforcement.

4. For the required area of steel, select the optimum combination of bar quantity and size, on the basis of such economic considerations as cost of various sizes of bars, required size of ties, ease of fabricating and placing reinforcement, and labor rates, REINFORCEMENT 1-7 aid in determining the possible choices of bars, splice type, spiral pitch, etc., from which the most cost-efficient design may be selected.

For rectangular columns subjected to *biaxial* bending and axial load, Design Examples 11-16 illustrate how to apply one or the other of two approximation methods. These are the "reciprocal load" method and the "load contour" method, which are described in the Commentary on COLUMNS 9-12. Both methods reduce the biaxial moments to an approximately equivalent uniaxial bending moment. This moment can be used with the load-moment strength interaction

TABLE I—IDENTIFICATION OF COLUMN TYPES FOR WHICH DESIGN AIDS ARE PROVIDED IN THIS HANDBOOK

Letter designation	R	E	L	C	S		
Description	<i>Rectangular tied column with bars on four faces</i>	<i>Rectangular tied column with bars on end faces</i>	<i>Rectangular tied column with bars on lateral faces</i>	<i>Circular spiral column*</i>	<i>Square spiral column*</i>		
Symbol							
	f_c' , ksi	f_y , ksi					
Column types for which	3	40	R3-40	E3-40	L3-40		
	4	40	R4-40	E4-40	L4-40		
	3	60	R3-60	E3-60	L3-60		
Design Aids are provided	4	60	R4-60	E4-60	L4-60	C4-60	S4-60
	5	60	R5-60	E5-60	L5-60	C5-60	S5-60
	6	60	R6-60	E6-60	L6-60	C6-60	S6-60
	8	60	R8-60	E8-60	L8-60	C8-60	

*Interaction diagrams for spirally reinforced columns may be used for tied columns of the same section shape, f_c' , and f_y by appropriate compensation for the difference in strength reduction factor (see ACI 318-89 Section 9.3.2 and Commentary on COLUMNS 7 and 8).

diagrams (COLUMNS 7) for uniaxial bending.

This edition of this volume differs from the last (1985) edition in these respects:

—REINFORCEMENT 5.2 is a new Design Aid calculated in accordance with the revised requirements in ACI 318-89 Chapter 12 for development length and splice length of bars in tension.

—To REINFORCEMENT 7.3 a note has been added explaining that the use of spacers is no longer required to hold spirals in place in spirally reinforced columns, but that the Commentary on the Code Requirements recommends that if spacers are used, the minimums given in this Design Aid be used for guidance.

—References to sections of the Building Code Requirements (ACI 318) and the Commentary on the Code Requirements (ACI 318R) have been updated to ACI 318-89 and ACI 318R-89, respectively.

—References to publications of the American Society for Testing and Materials (ASTM) and the Concrete Reinforcing Steel Institute (CRSI) have been updated to current editions.

—Minor editorial corrections have been made.

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—Noel J. Everard of the University of Texas at Arlington and Edward Cohen of Ammann and Whitney, who in the early 1960s developed equations for graphs and tables for use in design of reinforced concrete columns in accordance with ACI 318-63—equations which are the basis of the interaction diagrams in this volume; and

—Professor Richard W. Furlong of the University of Texas, who developed the load-moment interaction diagrams and tables in the form in which they appear in this volume.

HANDBOOK USERS' COMMENTS

ACI Committee 340, which has been charged with the continuing responsibility to keep current the ACI Design Handbook volumes, welcomes users' comments aimed at improving future editions. Such comments should be addressed to Committee 340 in care of the American Concrete Institute, P.O. Box 19150, Detroit, Michigan 48219, U.S.A.

NOTATION

A_b = area of an individual bar, sq in.	h_{core} = core diameter of a spiral column = outside column dimension minus cover, in.
A_c = area of core of spirally reinforced column measured to outside diameter of spiral, sq in.	h_e = effective thickness of a column for slenderness considerations, in.
A_g = gross area of column cross section, sq in.	I = moment of inertia of section resisting externally applied factored loads, in. ⁴
A_{sp} = area of bar or wire from which spiral is formed, sq in.	I_c = moment of inertia of gross cross section of column, in. ⁴
A_{st} = total area of longitudinal reinforcement in a column cross section, sq in.	I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. ⁴
b = overall cross section dimension of a rectangular column, in.	I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section, in. ⁴
C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram (For members braced against side-sway and without transverse loads between supports, $C_m = 0.6 + 0.4 (M_{1b}/M_{2b})$ but not less than 0.4. For all other cases, C_m shall be taken as 1.0.)	k = effective length factor for compression members
d_b = nominal diameter of bar, in.	K = a constant relating to EI and having the same units as EI
d_{sp} = nominal diameter of bar or wire from which spiral is formed, in.	l = span length of beam or slab, as defined in ACI 318-89, Section 8.7, in.
D = dead loads, or related internal moments and forces	l_c = height of column, center-to-center of floors or roof, in.
e = eccentricity of axial load measured from the centroid of the gross section, in.	l_d = development length, in.
e_x = eccentricity e along x-axis, in.	l_{db} = basic development length, in.
e_y = eccentricity e along y-axis, in.	l_{dc} = development length for bars in compression, in.
E_c = modulus of elasticity of concrete, psi. (For normal weight concrete, E_c may be taken as $57,000 \sqrt{f'_c}$ psi.)	l_u = unsupported length of compression member, in.
E_s = modulus of elasticity of reinforcement, psi (= 29,000,000 psi for steel)	L = magnified factored moment to be used for design of column
EI = flexural stiffness of cross section for frame analysis	M_d = service dead load moment, kip-ft
= flexural stiffness of compression members specified	M_ℓ = service live load moment, kip-ft
f'_c = specified compressive strength of concrete, psi or ksi	M_n = nominal moment strength at section
f_y = specified yield strength of reinforcement, psi or ksi	M_w = service wind load moment, kip-ft
h = diameter of a round column or side of a rectangular column, in.	M_{nx} = nominal moment strength about x-axis
	M_{ny} = nominal moment strength about y-axis
	M_{nox} = equivalent uniaxial moment strength about x-axis
	M_{noy} = equivalent uniaxial moment strength about y-axis
	M_u = factored moment at section

M_{ux} = factored moment about x-axis
 M_{uy} = factored moment about y-axis
 M_{1b} = value of smaller factored end moment on compression member due to the loads that result in no appreciable sideways, calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature, kip-ft
 M_{2b} = value of larger factored end moment on compression member due to the loads that result in no appreciable sidesway, calculated by conventional elastic frame analysis, always positive, kip-ft
 n = number of longitudinal bars
 P_b = nominal axial load strength at balanced strain conditions
 P_c = critical axial load, kips
 P_d = service axial dead load, kips
 P_l = service axial live load, kips
 P_n = nominal axial load strength at given eccentricity
 P_{ni} = approximation of nominal axial load strength at eccentricities e_x and e_y , kips
 P_{nx}^* = nominal axial load strength for eccentricity e_y along y-axis only, x-axis being axis of bending
 P_{ny}^* = nominal axial load strength for eccentricity e_x along x-axis only, y-axis being axis of bending
 P_o = nominal axial load strength at zero eccentricity, kips
 P_u = factored axial load at given eccentricity ϕP_n , kips
 P_{ux} = factored axial load for eccentricity e_y along y-axis only $\leq \phi P_{nx}$
 P_{uy} = factored axial load for eccentricity e_x along x-axis only $\leq \phi P_{ny}$
 r = radius of gyration of cross section of a compression member, in.
 s = center-to-center spacing of bars, in.
 S = pitch of spiral, center-to-center of bar
 S_b = clear spacing between bars, in.
 w_c = weight of concrete, lb per cu ft

β (beta) = biaxial bending design constant = constant portion of uniaxial factored moment strengths M_{nox} and M_{noy} which may be permitted to act simultaneously on the column cross section
 β_d = ratio of maximum factored axial dead load to maximum total factored axial load, where the load is due to gravity effects only in the calculation of P_c in Eq (10-7) of ACI 318-89, or ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in that story in the calculation of P_c in Eq (10-8) of ACI 318-89
 β_1 = a coefficient relating the depth of the equivalent rectangular stress block to the depth from the compression face to the neutral axis
= 0.85 for $f'_c \leq 4.0$ ksi
= $0.85 - 0.05 (f'_c - 4.0)$ for $f'_c > 4.0$ but not less than 0.65 (f'_c in ksi)
 γ (gamma) = ratio of distance between centroid of outer rows of bars and thickness of cross section, in the direction of bending
 δ_b (delta) = moment magnification factor for columns braced against sidesway
 δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads
 ξ (xi) = dimensionless constant used in computing I_g and I_{se}
 ρ_g (rho) = A_{st}/A_g = ratio of total reinforcement area to cross-sectional area of column
 ϕ (phi) = strength reduction factor as defined in Section 9.3 of ACI 318-89
 ψ (psi) = ratio of sum of stiffness $\Sigma(I/l_c)$ of compression members to $\Sigma(I/l)$ of flexural members in a plane at one end of a compression member

*Definitions for P_{nx} and P_{ny} differ from those in ACI 318R-89 in Sections 10.3.5 and 10.3.6.

DESIGN EXAMPLES

COLUMNS EXAMPLE 1—Selection of reinforcement for a rectangular tied column with bars on four faces (slenderness ratio found to be *below* critical value)

For a rectangular tied column with bars equally distributed along four faces, select the reinforcement.

Given: Loading

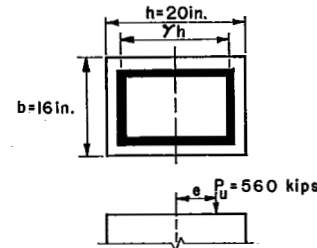
Factored axial load $P_u = 560$ kips
Factored moment $M_u = 3920$ kip-in.

Materials

Compressive strength of concrete $f'_c = 4$ ksi
Yield strength of reinforcement $f_y = 60$ ksi
Nominal maximum size of aggregate is 1 in.

Design conditions

Unsupported length of columns $l_u = 10$ ft
Column is braced against sidesway



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Step 1—Determine column section size.	Given: $h = 20$ in. $b = 16$ in.	
10.11.4.1	Step 2—Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected.		
10.11.5.1	If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b (as in Columns Example 2, Step 2).		
10.11.3	A) Compute M_{1b}/M_{2b} , and read critical value of kl_u/h	In this case, it is given that $M_{1b} = 3920$ kip-in, but M_{2b} is not known. However, for rectangular columns, for all values of M_{1b}/M_{2b} , slenderness may be neglected where $kl_u/h < 6.6$	COLUMNS 1
10.11.4.1	B) Compute kl_u/h and compare with critical value; determine whether slenderness effects must be considered	For columns braced against sidesway: $k = 1.0$ Given: $l_u = 10 \text{ ft} = 120 \text{ in.}$ $\frac{kl_u}{h} = \frac{(1.0)(120)}{20} = 6.0$ < 6.6 $\therefore$ Slenderness effects may be neglected	
10.11.4.1	Step 3—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	Given: $P_u = 560$ kips $M_u = 3920$ kip-in. $h = 20$ in. $b = 16$ in. $\therefore A_g = b \times h = 20 \times 16 = 320 \text{ in.}^2$	
9.3.2.2(b) 10.2.7	A) Compute $\frac{P_u}{A_g}$ B) Compute $\frac{M_u}{A_g h}$	$\frac{P_u}{A_g} = \frac{560}{320} = 1.75 \text{ ksi}$ $\frac{M_u}{A_g h} = \frac{3920}{320 \times 20} = 0.61 \text{ ksi}$	

EXAMPLE 1

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	C) Estimate $\gamma \approx \frac{h-5}{h}$ D) Determine appropriate interaction diagram(s) E) Read ρ_R for $P_u/A_R (\leq \phi P_n/A_R)$ and $M_u/A_R h (\leq \phi M_n/A_R h)$ F) Compute required A_{st} from $A_{st} = \rho_R A_R$	$\gamma \approx \frac{20-5}{20} = 0.75$ For a rectangular tied column with bars along four faces, $f_c' = 4$ ksi, $f_y = 60$ ksi, and an estimated γ of 0.75, use R4-60.75. For $P_u/A_R = 1.75$ from Step 3A and $M_u/A_R h = 0.61$ from Step 3B: $\rho_R = 0.040$ Required $A_{st} = 0.040 \times 320 \text{ in.}^2 = 12.8 \text{ in.}^2$				COLUMNS 7.4.3 (R4-60.75)
10.9.2	Step 4—Select optimum reinforcement. A) Assume trial bar quantities	4 (2/face)	8 (3/face)	12 (4/face)	16 (5/face)	REINFORCE- MENT 1.2
10.9.1	B) Determine smallest bar size to provide A_{st} , list resulting A_{st} , in. ² , compute resulting $\rho_R = A_{st}/A_R$, and check that $0.01 \leq \rho_R \leq 0.08$	#18 16.00 0.050 OK	#14 18.00 0.056 OK	#10 15.24 0.048 OK	#9 16.00 0.050 OK	
7.10.5.1	C) List tie size	#4	#4	#3	#3	COLUMNS 6.1
	D) Refine γ and, if necessary, repeat from Step 3E	0.69	0.72	0.75	0.76	
		(γ for 8, 12, and 16 bars is close enough to estimate of 0.75 used in Step 3E, but if 4 bars appears to be optimum choice, ρ_R should be interpolated from interaction diagrams for $\gamma = 0.60$ and $\gamma = 0.75$)				
3.3.3(c) 7.6.3 7.6.4 7.7.1 7.10.5.1 12.14.2.1	E) Check whether reinforcement can be accommodated along smaller face with —Bearing splices —Normal lap splices —Tangential lap splices	OK Not permitted Not permitted	OK Not permitted Not permitted	OK NO NO	NO NO NO	REINFORCE- MENT 2 REINFORCE- MENT 2 REINFORCE- MENT 2
7.10.5.2	F) Determine tie spacing as least of —16 longitudinal bar diameters, in. —48 tie bar diameters, in. —Least dimension of column, in.	36 24 16	27 24 16	20 18 16		REINFORCE- MENT 1.1
7.8 7.9 7.10	G) Consider special design details	Omitted in this example				
	H) Select most cost-efficient reinforcement			(Probable) first choice		

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
		(If 4 bars appears to be a choice, repeat Step 3 with L4-60 diagram(s), redetermine required A_{st} , and check adequacy of choice)	Commentary on Columns 7
	Solution	Use 12 #10 bars with bearing splices and #3 ties spaced not more than 16 in. apart. (Choice is based on minimum steel requirement, use of #3 ties instead of #4, and ease of handling #10 bars instead of larger bars.)	

E
X
A
M
P
L
E
1

COLUMNS EXAMPLE 2—Selection of reinforcement for a rectangular tied column with bars on four faces (slenderness ratio found to be *above* critical value)

For a 22 × 20 in. rectangular tied column with bars equally distributed along four faces, select the reinforcement

Given: Loading

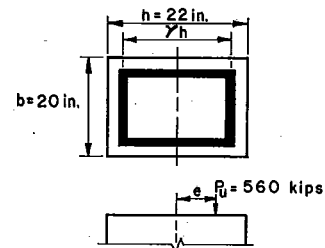
Service dead load $P_d = 160$ kips
 Total factored axial load $P_u = 560$ kips
 Factored end moment at top of column $M_u = M_{2b} = +3920$ kip-in.
 Dead load moment unfactored at top of column $M_d = 1120$ kip-in.
 Factored end moment at bottom of column $M_{1b} = +2940$ kip-in.
 No transverse loading on member

Materials

Compressive strength of concrete $f'_c = 4$ ksi
 Yield strength of reinforcement $f_y = 60$ ksi
 Nominal maximum size of aggregate = 1 in.

Design conditions

Unsupported length of column $l_u = 27.5$ ft
 Column braced against sidesway



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Step 1—Determine column section size.	Given: $h = 22$ in. $b = 20$ in.	
10.11.4.1	Step 2—Check whether slenderness ratio is less than critical value. If so, slenderness effects may be neglected.		
10.11.5.1	If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b .		
10.11.4.1	A) Compute M_{1b}/M_{2b} and read critical value of kl_u/h .	$M_{1b}/M_{2b} = \frac{2940}{3920} = 0.75$	
10.11.3			
10.11.4.1		Critical $\frac{kl_u}{h} = 7.5$	COLUMNS 1
10.11.2.1	B) Determine k	For columns braced against sidesway: $k = 1.0$	
	C) Compute kl_u/h and compare with critical value.	Given: $l_u = 27.5$ ft $= 27.5 \times 12 = 330$ in. $\frac{kl_u}{h} = \frac{1.0 \times 330}{22.0} = 15.0 > 7.5$	
10.11.5.1	D) Determine moment magnification factor δ_b —Determine appropriate moment magnifier graph	$\therefore$ Slenderness effects must be considered	
9.2	—Compute $\beta_d = 1.4 P_d/P_u$	For rectangular tied column with $f'_c = 4$ ksi: use COLUMNS 5.2 $\beta_d = \frac{1.4 \times 160}{560} = 0.4$	
10.0	—Compute $\frac{P_u (1 + \beta_d)}{A_g}$	$\frac{P_u (1 + \beta_d)}{A_g} = \frac{560 \times 1.4}{22 \times 20} = 1.78$ ksi	

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION			HANDBOOK REFERENCE
10.11.5.3	—Since there is no transverse load, compute C_m from $C_m = 0.6 + 0.4 (M_{1b}/M_{2b})$ —Estimate $\gamma \approx \frac{h-5}{h}$ —Determine appropriate interaction diagram	$C_m = 0.6 + 0.4 \left(\frac{2940}{3920} \right) = 0.9$ $\gamma = \frac{22-5}{22} = 0.77 \approx 0.75$ For rectangular tied column with steel along four faces, $f'_c = 4$ ksi, $f_y = 60$ ksi, and $\gamma \approx 0.75$, use R4-60.75			COLUMNS 7.4.3 (R4-60.75)
9.3.2.2(b) 10.2 10.3	—Assume a series of trial values of ρ_g —Read h_e/h	0.02 1.00	0.03 1.10	0.04 1.20	
8.5.1 9.3.2.2 10.11.5.1 10.11.5.2	—Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} + \frac{h_e}{h}$ —Read δ_b/C_m from moment magnifier graph determined above For values of $\frac{P_u(1+\beta_d)}{A_g}$ and $\frac{kl_u}{h_e}$ determined above —Compute $\delta_b = C_m \times \frac{\delta_b}{C_m}$ from C_m and $\frac{\delta_b}{C_m}$ determined above E) Compute $\delta_b M_u$, kip-in.	$\frac{15.0}{1.00} = 15.0$ 1.91	$\frac{15.0}{1.10} = 13.6$ 1.64	$\frac{15.0}{1.20} = 12.5$ 1.49	COLUMNS 5.2
9.3.2.2(b) 10.2.7	Step 3—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s), and compute required cross-section area A_{st} of longitudinal reinforcement. A) Compute $\frac{P_u}{A_g}$	$A_g = 22 \times 20 = 440 \text{ in.}^2$ $\frac{P_u}{A_g} = \frac{560}{440} = 1.27 \text{ ksi}$			
10.11.5.1	B) Compute $\frac{\delta_b M_u}{A_g h}$, ksi	For assumed $\rho_g = 0.02$ 6740 440×22.0 = 0.696	For assumed $\rho_g = 0.03$ 5800 440×22.0 = 0.599	For assumed $\rho_g = 0.04$ 5250 440×22.0 0.542	

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	C) Read ρ_g for $\frac{P_u}{A_g} \left(\leq \frac{\phi P_n}{A_g} \right)$ and $\frac{\delta_b M_u}{A_g h} \left(\leq \frac{\phi M_n}{A_g h} \right)$ on interaction diagram for estimated γ of 0.75, and compare with assumed value of ρ_g Read h_e/h for assumed ρ_g Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} \div \frac{h_e}{h}$ Read δ_b/C_m Compute $\delta_b = C_m \times \frac{\delta_b}{C_m}$ Compute $\delta_b M_u$, kip-in. Compute $\frac{\delta_b M_u}{A_g h}$, ksi	For assumed $\rho_g = 0.02$	For assumed $\rho_g = 0.03$	For assumed $\rho_g = 0.04$	COLUMNS 7.4.3 (R4-60.75)	
		0.042 $\neq 0.02$	0.032 ≈ 0.03	0.027 $\neq 0.04$		
		9.3.2.2(b) 10.2 10.3				COLUMNS 7.4.3 (R4-60.75)
8.5.1 9.3.2.2 10.11.5.1 10.11.5.2		8.5.1 9.3.2.2 10.11.5.1 10.11.5.2				COLUMNS 5.2
9.3.2.2(b) 10.2 10.3	Read ρ_g for $\frac{P_u}{A_g} \left(\leq \frac{\phi P_n}{A_g} \right)$ and $\frac{\delta_b M_u}{A_g h} \left(\leq \frac{\phi M_n}{A_g h} \right)$ D) Compute required A_{st} $A_{st} = \rho_g A_g$	9.3.2.2(b) 10.2 10.3				COLUMNS 7.4.3 (R4-60.75)
		10.9.1				
		10.9.2 7.10.5.1				
10.9.1	Step 4—Select optimum reinforcement.	10.9.1				REINFORCE- MENT 1.2
10.9.2 7.10.5.1	A) Assume trial bar quantities	4 (2/face)	8 (3/face)	12 (4/face)	16 (5/face)	
	B) Determine smallest bar size to provide A_{st} , List resulting A_{st} , in. ² , Compute resulting $\rho_g = A_{st}/A_g$, And check that $0.01 \leq \rho_g \leq 0.08$	#18 16.0 0.036 OK #4 0.72	#14 18.0 0.041 OK #4 0.74	#10 15.2 0.035 OK #3 0.77	#9 16.0 0.036 OK #3 0.78	
	C) List tie size					COLUMNS 6.1
	D) Refine γ and compare with estimated γ	γ is close enough to estimate of 0.75 used in Steps 2D and 3C.				

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		4-#18	8-#14	12-#10	16-#9	
3.3.3(c) 7.6.3 7.6.4 7.7.1 7.10.5.1 12.14.2.1 15.8.2	E) Check whether reinforcement can be accommodated along smaller face with —Bearing splices —Normal lap splices —Tangential lap splices	OK Not permitted Not permitted	OK Not permitted Not permitted	OK OK OK	OK OK NO	REINFORCE- MENT 2 REINFORCE- MENT 2 REINFORCE- MENT 2
7.10.5.2	F) Determine tie spacing as least of —16 longitudinal bar diameters, in. —48 tie bar diameters, in. —Least dimension of column, in.	36 24 20	27 24 20	20 18 20	18 18 20	REINFORCE- MENT 1.1 REINFORCE- MENT 1.1
7.8 7.9 7.10	G) Consider special design details	Omitted in this example				
	H) Select most cost-efficient reinforcement			Probable first choice		
	Solution	Use 12 #10 bars with #3 ties spaced not more than 18.0 in. apart. (Choice is based on minimum steel requirement, use of #3 ties instead of #4 ties, ease of handling #10 bars instead of larger bars, and suitability to all three types of splice.)				

COLUMNS EXAMPLE 3—For a specified reinforcement ratio, selection of column section size and reinforcement for a rectangular tied column having bars on end faces only

For minimum longitudinal reinforcement ($\rho_g \approx 0.01$) and column section dimension $h = 16$ in., select column dimension b and reinforcement for a rectangular tied column with bars on end faces only.

Given: Loading

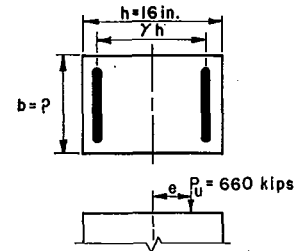
Factored axial load $P_u = 660$ kips
Factored moment $M_u = 2790$ kip-in.

Materials

Compressive strength of concrete $f'_c = 4$ ksi
Yield strength of reinforcement $f_y = 60$ ksi
Nominal maximum size of aggregate = 1 in.

Design conditions

Slenderness effects may be neglected because kl_u/h is known to be below critical value



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION			HANDBOOK REFERENCE
	Step 1—Determine trial column dimension b corresponding to known values of variables on appropriate interaction diagram(s).	Given: $P_u = 660$ kips $M_u = 2790$ kip-in. $\rho_g \approx 0.01$ $f'_c = 4$ ksi $f_y = 60$ ksi $h = 16$ in.			
	A) Assume a series of trial column sizes b , in., and compute $A_g = b \times h$, in.	24 384	26 416	28 448	
	B) Compute $\frac{P_u}{A_g}$, ksi	$\frac{660}{384}$ = 1.72	$\frac{660}{416}$ = 1.59	$\frac{660}{448}$ = 1.47	
	C) Compute $\frac{M_u}{A_g h}$, ksi	$\frac{2790}{384 \times 16}$ = 0.45	$\frac{2790}{416 \times 16}$ = 0.42	$\frac{2790}{448 \times 16}$ = 0.39	
	D) Estimate $\gamma = \frac{h - 5}{h}$	0.69	0.69	0.69	
	E) Determine appropriate interaction diagram(s)	For a rectangular tied column with steel on end faces only, $f'_c = 4$ ksi, $f_y = 60$ ksi, and an estimated γ of 0.69, use E4-60.60 and E4-60.75			
9.3.2.2(b) 10.2 10.3	F) Read ρ_g for P_u/A_g ($\leq \phi P_n/A_g$) and $M_u/A_g h$ ($\leq \phi M_n/A_g h$) for $\gamma = 0.60$ and $\gamma = 0.75$ and interpolate for $\gamma = 0.69$ and select dimension corresponding to ρ_g nearest desired value of $\rho_g = 0.01$	0.022 0.018 0.020	0.017 0.014 0.015	0.013 0.011 0.012	COLUMNS 7.11.2 (E4-60.60) COLUMNS 7.11.3 (E4-60.75)
		∴ Try a 16 × 28-in. column			

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION					HANDBOOK REFERENCE
10.11.4.1	Step 2 —Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b (as in Columns Example 2, Step 2).	Given that kl_u/h is below critical value, slenderness effects may be neglected.					
10.11.5.1	Step 3 —Determine reinforcement ratio ρ_g , and compute required cross-section area A_{st} of longitudinal reinforcement.	For a 16 × 28 in. column: $\rho_g = 0.012$ and $A_g = 448 \text{ in.}^2$ (from Step 1F) $\therefore$ Required $A_{st} = 0.012 \times 448 = 5.4 \text{ in.}^2$					
7.10.5.1	Step 4 —Select minimum ρ_g reinforcement.						REINFORCE- MENT 1.2 COLUMNS 6.1 REINFORCE- MENT 2 REINFORCE- MENT 1.1 REINFORCE- MENT 1.1
	A) Assume trial bar quantities	4 (2/face)	6 (3/face)	8 (4/face)	10 (5/face)	12 (6/face)	
	B) Determine smallest bar size to provide A_{st} , list resulting A_{st} , in. ²	#11 6.24	#9 6.00	#8 6.32	#7 6.00	#7 7.20	
	C) List tie size	#4	#3	#3	#3		
	D) Refine γ for bar and tie size, and using this γ interpolate for accurate ρ_g (Step 1F); if ρ_g differs from that associated with dimension chosen in Step 1F recompute required A_{st} , in. ² , and compare with A_{st} provided; if necessary, revise selection of bars	0.66 0.012 OK	0.70 0.012 OK	0.70 0.012 OK	0.71 0.012 OK		
	E) Check whether reinforcement can be accommodated with						
	—Bearing splices	OK	OK	OK	OK		
	—Normal lap splices	OK	OK	OK	OK		
	—Tangential lap splices	OK	OK	OK	OK		
	F) Determine tie spacing as least of						
7.8 7.9 7.10	—16 longitudinal bar diameters, in.	23	18	16	14		REINFORCE- MENT 1.1 REINFORCE- MENT 1.1
	—48 tie bar diameters, in.	24	18	18	18		
	—Least dimension of column, in.	16	16	16	16		
	G) Consider special design details	Omitted in this example					
	H) Select minimum- ρ_g reinforcement (which is also minimum- A_{st} reinforcement)		First choice				
		$\therefore$ Try 16 × 28 in. column with six #9 bars and #3 ties spaced not more than 16 in. apart. (While A_{st} is same for six #9 and ten #7, it is easier to handle six bars.)					
	Solution	Use a 16 × 28 in. column with three #9 bars on each of two end faces and #3 ties spaced not more than 16.0 in. apart.					

COLUMNS EXAMPLE 4—Selection of reinforcement for a square spiral column with reverse curvature (slenderness ratio found to be below critical value)

For the square spiral column section shown, select the reinforcement

Given: Loading

Factored axial dead load $P_d = 236$ kips

Total factored axial load $P_u = 660$ kips

Factored moment at top of column $M_{1b} = -660$ kip-in.

Factored moment at bottom of column $M_{2b} = M_u = +2640$ kip-in.

$\beta_d = 1.4 P_d/P_u = 0.5$

No transverse loading

Materials

Compressive strength of concrete $f'_c = 4$ ksi

Yield strength of reinforcement $f_y = 60$ ksi

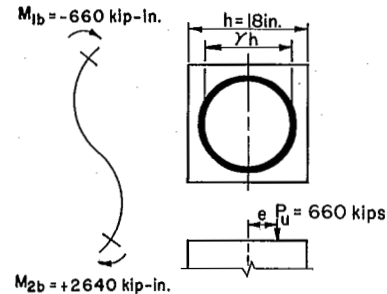
Nominal maximum size of aggregate = 1 in.

Design conditions

Column section size $h = b = 18$ in.

$l_u = 12$ ft 10 in.

Column is braced against sidesway



Note: Examples 4 and 5 are similar except for magnitude and sign of moment M_{1b} .

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Step 1 —Determine column section size.	Given: $h = b = 18$ in.	
10.11.4.1 10.11.5.1	Step 2 —Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b .		
10.11.4.1 10.11.3 10.11.4.1	A) Compute $\frac{M_{1b}}{M_{2b}}$, and read critical value of kl_u/h	$\frac{M_{1b}}{M_{2b}} = \frac{-660}{2640} = -0.25$ Critical value of $\frac{kl_u}{h}$ is 11.1	COLUMNS 1
10.11.2.1	B) Compute kl_u/h and compare with critical value to determine whether slenderness effects must be considered	Given column braced against sidesway $\therefore k = 1.0$ Given: $l_u = 12$ ft 10 in. $= 154$ in. $\frac{kl_u}{h} = \frac{(1.0 \times 154)}{18} = 8.56$ < 11.1 $\therefore$ Slenderness effects may be neglected.	
10.11.4.1			
9.3.2.2(b) 10.2.7	Step 3 —Determine reinforcement ratio using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	Given: $P_u = 660$ kips $M_u = 2640$ kip-in. $h = b = 18$ in. $\therefore A_g = 324$ in.²	

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION					HANDBOOK REFERENCE
	A) Compute $\frac{P_u}{A_g}$ B) Compute $\frac{M_u}{A_g h}$ C) Estimate $\gamma \approx \frac{h-5}{h}$ D) Determine appropriate interaction diagram(s) E) Read ρ_g for $P_u/A_g (\leq \phi P_n/A_g)$ and $M_u/A_g h (\leq \phi M_n/A_g h)$ F) Compute required A_{st} $A_{st} = \rho_g A_g$	$\frac{P_u}{A_g} = \frac{660}{324} = 2.04 \text{ ksi}$ $\frac{M_u}{A_g h} = \frac{2640}{324 \times 18} = 0.45 \text{ ksi}$ $\gamma \approx \frac{18-5}{18} = 0.72$ For a square spiral column, $f_c' = 4 \text{ ksi}$, $f_y = 60 \text{ ksi}$, and an estimated $\gamma = 0.72$, use S4-60.60 and S4-60.75 For $P_u/A_g = 2.04 \text{ ksi}$, $M_u/A_g h = 0.45 \text{ ksi}$ and, $\gamma = 0.60: \rho_g = 0.035$ $\gamma = 0.75: \rho_g = 0.026$ for $\gamma = 0.72: \rho_g = 0.028$ $A_{st} = 0.028 \times 324 = 9.07 \text{ in.}^2$					COLUMNS 7.26.2 and 7.26.3 (S4-60.60 and S4-60.75)
9.3.2.2(b) 10.2 10.3	Step 4—Select optimum reinforcement. A) Assume trial bar quantities B) Determine smallest bar size to provide A_{st}, list resulting A_{st}, in.² C) Select spiral size and pitch	6 #11	7 #11	8 #10	9 #10	10 #9	REINFORCE- MENT 1.2
3.3.3(c) 7.10.4.2 7.10.4.3 10.0 10.9.3		9.36 #4 2 in.	10.92 #4 2 in.	10.16 #4 2 in.		10.00 #4 2 in.	REINFORCE- MENT 7.2
	D) Refine γ for bar and tie size, and using this γ interpolate for accurate ρ_g (Step 3E), recompute required A_{st} , and compare with A_{st} provided (from B above)	0.70 0.029	0.70 0.029	0.71 0.028		0.72 0.028	COLUMNS 6.1
3.3.3 7.6.3 7.6.4 7.7.1 7.10.4.2 10.9.1 10.9.2	E) Check whether reinforcement can be accommodated in column with —Bearing splices —Normal lap splices —Tangential lap splices	9.40 Not adequate	9.40 OK	9.07 OK		9.07 OK	
recommendation in ACI 318R-89 Section 7.10.4 7.8 7.9 7.10	F) Determine recommended number of spacers, if spacers are used		2	2		2	REINFORCE- MENT 4.1.1 REINFORCE- MENT 4.2.1 REINFORCE- MENT 4.3.1 REINFORCE- MENT 7.3
	G) Consider special design details	Omitted in this example					
	H) Select most cost-efficient reinforcement			(Probable) first choice			
	Solution	Use eight #10 bars with #4 spirals at 2 in. pitch (Choice is based on amount of steel required, ease of assembling eight bars instead of ten, and availability of #4 spirals.) If spacers are used, recommend two spacers.					

COLUMNS EXAMPLE 5—Selection of reinforcement for a square spiral column with single curvature (slenderness ratio found to be *above* critical value)

For the square spiral column section shown, select the reinforcement.

Given: Loading

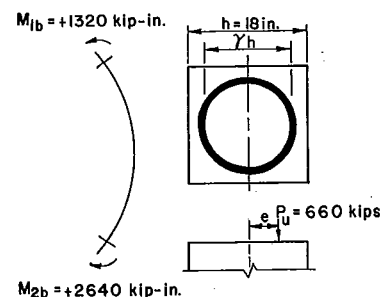
Factored axial load $P_u = 660$ kip
 Factored moment at top of column $M_{1b} = +1320$ kip-in.
 Factored moment at bottom of column $M_{2b} = M_u = +2640$ kip-in.
 $\beta_d = 1.4 P_d / P_u = 0.5$
 No transverse loading

Materials

Compressive strength of concrete $f'_c = 4$ ksi
 Yield strength of reinforcement $f_y = 60$ ksi
 Nominal maximum size of aggregate = 1 in.

Design conditions

Column section size $h = b = 18$ in.
 $l_u = 12$ ft 10 in.
 Column is braced against sidesway



Note: Examples 4 and 5 are similar except for magnitude and sign of moment M_{1b} .

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Step 1—Determine column section size.	Given: $h = b = 18$ in.	
10.11.4.1 10.11.5.1	Step 2—Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b		
10.11.3 10.11.4.1	A) Compute $\frac{M_{1b}}{M_{2b}}$, and read critical value of kl_u/h	$\frac{M_{1b}}{M_{2b}} = \frac{+1320}{+2640} = +0.5$ Critical value of kl_u/h is 8.4	COLUMNS 1
10.11.2.1	B) Compute kl_u/h and compare with critical value to determine whether slenderness effects must be considered.	Given columns braced against sidesway $\therefore k = 1.0$ Given: $l_u = 12$ ft 10 in. $= 154$ in. $\frac{kl_u}{h} = \frac{1.0 \times 154}{18} = 8.56$ > 8.4 $\therefore$ Slenderness effects must be considered	
	C) Compute $\beta_d = \frac{1.4 P_d}{P_u}$	Given $\beta_d = 0.5$	

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION			HANDBOOK REFERENCE
	D) Compute $\frac{P_u(1 + \beta_d)}{A_g}$	Given: $P_u = 660$ kips $\beta_d = 0.5$ $h = b = 18$ in. $\therefore A_g = 324$ in. ² $\frac{P_u(1 + \beta_d)}{A_g} = \frac{660 \times 1.5}{324} = 3.06$ ksi			
	E) Estimate $\gamma = \frac{h - 5}{h}$	$\gamma = \frac{18 - 5}{18} = 0.72$			
	F) Determine appropriate interaction diagram(s)	For a square spiral column, $f_c' = 4$ ksi, $f_y = 60$ ksi; and an estimated $\gamma = 0.72$, use S4-60.60 and S4-60.75.			
	G) Assume ρ_g and find h_e/h for $\gamma = 0.60$ $\gamma = 0.75$ and, interpolating, for $\gamma = 0.72$	0.030	0.035	0.040	
9.3.2.2(b)		1.00	1.00	1.00	COLUMNS
10.2		1.00	1.04	1.09	7.26.2 and
10.3		1.00	1.03	1.07	7.26.3
					(S4-60.60 and S4-60.75)
	H) Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} + \frac{h_e}{h}$	8.56	8.31	8.00	
	I) Determine appropriate moment magnifier graph	For a square spiral column with $f_c' = 4$ ksi use COLUMNS 5.12			
8.5.1	J) Read δ_b/C_m for kl_u/h_e and $P_u(1 + \beta_d)/A_g$	1.33	1.32	1.27	COLUMNS
9.3.2.2(b)					5.12
10.11.5.1	K) Compute C_m for braced column with $M_{3b} = 0$ as $C_m = 0.6 + 0.4 \frac{M_{1b}}{M_{2b}} \geq 0.4$ $= 0.6 + 0.4 (1320/2640)$	0.8	0.8	0.8	
10.11.5.3		1.06	1.06	1.02	
Eq. (10-12)	L) Compute $\delta_b = (\delta_b/C_m) \times C_m$ M) Compute $\delta_b M_u = \delta_b (2640)$ kip-in.	2800	2800	2690	
9.3.2.2(b)	Step 3—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s), and compute required cross section area A_{st} of longitudinal reinforcement.				
10.2.7	A) Compute $\frac{P_u}{A_g} = \frac{660}{324}$ ksi	2.04	2.04	2.04	
	B) Compute $\frac{\delta_b M_u}{A_g h} = \frac{\delta_b M_u}{324 \times 18}$ ksi	0.48	0.48	0.46	
9.3.2.2(b)	C) Read ρ_g for P_u/A_g ($\leq \phi P_n/A_g$) and $M_u/A_g h$ ($\leq \phi M_n/A_g h$) for $\gamma = 0.60$ $\gamma = 0.75$ and interpolating for $\gamma = 0.72$; compare with assumed value of ρ_g	0.041 0.031 0.033	0.041 0.031 0.033	0.036 0.026 0.028	COLUMNS
10.2					7.26.2
10.3					and 7.26.3
					(S4-60.60 and S4-60.75)
	D) Compute required $A_{st} = \rho_g A_g$	$\therefore$ Use $\rho_g = 0.033$ $A_{st} = 0.033 (324) = 10.7$ in. ²			

EXAMPLE 5

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION						HANDBOOK REFERENCE
10.9.2	Step 4—Select optimum reinforcement. A) Assume trial bar quantities	6 #14	7 #11	8 #11	9 #10	10 #10	11 #9	REINFORCE- MENT 1.2
7.10.4.2	B) Determine smallest bar size to provide A_{st}, list resulting A_{st}, in.²	13.50 #4	10.92 #4	12.48 #4	11.43 #4	12.70 #4	11.00 #4	REINFORCE- MENT 1.2
7.10.4.3 Eq. (10-5)	C) Select spiral size and pitch	2 in.	2 in.	2 in.	2 in.	2 in.	2 in.	
	D) Refine γ for bar and spiral size, and using this γ interpolate for accurate ρ_g (Step 3C), recompute required A_{st}, in.², and compare with A_{st} provided (from B above)	0.68 0.036	0.70 0.034	0.70 0.034	0.71 0.034	0.71 0.034	0.72 0.033	COLUMNS 6.1
		11.7 OK	11.0 Not ade- quate	11.0 OK	11.0 OK	11.0 OK	10.7 OK	
3.3.3	E) Check whether reinforcement can be accommodated in column with							
7.6.3								
7.6.4								
7.7.1	—Bearing splices	OK		OK	OK	OK	OK	REINFORCE- MENT 4.1
7.10.4.2								
10.9.1	—Normal lap splices	Not per- mitted		OK	OK	NO	OK	REINFORCE- MENT 4.2
10.9.2								
12.14.2.1	—Tangential lap splices	Not per- mitted		NO	NO	NO	NO	REINFORCE- MENT 4.3
recommendation in ACI 318R-89 Section 7.10.4	F) Determine recommended number of spacers, if spacers are used	2		2	2	2	2	
7.8	G) Consider special design details	Omitted in this example						
7.9					(Prob- able) first choice			
7.10	H) Select most cost-efficient reinforcement							
	Solution	Use nine #10 bars with bearing or normal lap splices and #4 spirals with 2 in. pitch. If spacers are used, recommend two spacers.						

COLUMNS EXAMPLE 6—Design of rectangular tied column section subject to very small design moment*

For a square tied column select column section size and choose reinforcement. Use relatively high proportion of longitudinal steel (i.e., $\rho_g \approx 0.04$) equally distributed on four faces.

Given: Loading

Factored axial load $P_u = 940$ kips

Factored moment at top of column $M_u = \pm 480$ kip-in.

Materials

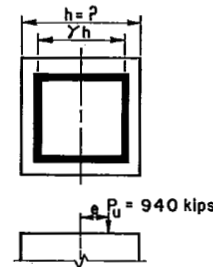
Compressive strength of concrete $f'_c = 5$ ksi

Yield strength of reinforcement $f_y = 60$ ksi

Nominal maximum size of aggregate = 1.0 in.

Design Conditions

Effective column length $kl_u = 90$ in.



*Columns Examples 6 and 7 are similar except that Example 6 considers a square *tied* column with bars on four faces while Example 7 considers a circular *spiral* column.

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION			HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	Step 1 —Determine trial column section size h corresponding to known values of variables on appropriate interaction diagram(s).				COLUMNS 7.5.1, 7.5.2, 7.5.3 (R5-60.45 R5-60.60, R5-60.75)
	A) Assume a series of trial column sizes h , in., and compute A_g , in. ²	12 144	16 256	20 400	
	B) Estimate $\gamma = \frac{h-5}{h}$	0.58	0.69	0.75	
	C) Determine appropriate interaction diagram(s)	R5-60.45 R5-60.60	R5-60.60 R5-60.75	R5-60.75	
	D) Compute $\frac{M_u}{A_g h} = \frac{480}{h^3}$ ksi	0.28	0.12	0.06	
	E) Read $\frac{\phi P_n}{A_g}$ ksi for $\rho_g = 0.04$ and				
	for lower value of γ	3.58	3.63		
	higher value of γ	3.63	3.63		
	and interpolate for estimated value of γ	3.62	3.63	3.63	
	F) Compute $A_g = \phi P_n + \frac{\phi P_n}{A_g}$ $= \frac{940}{\phi P_n / A_g}$ ksi	260	259	259	
	G) Compute $h = \sqrt{A_g}$	16.1	16.1	16.1	
		∴ Try $h = 16$ in.			

EXAMPLE 6

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
10.11.4.1	Step 2 —Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected.		
10.11.5.1	If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b (see Columns Example 5, Step 2).		
10.11.3 10.11.4.1	A) Compute M_{1b}/M_{2b} , and read critical value of kl_u/h	In this case, M_{1b} and M_{2b} are not known, but for rectangular columns, for all values of M_{1b}/M_{2b} , slenderness may be neglected where $kl_u/h < 6.6$	COLUMNS 1
10.11.2.1	B) Compute kl_u/h and compare with critical value; determine whether slenderness effects must be considered.	$kl_u/h = \frac{90}{16} = 5.6 < 6.6$ $\therefore$ Slenderness effects may be neglected	
9.3.2.2(b) 10.2 10.3	Step 3 —Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement. A) Estimate $\gamma \approx \frac{h-5}{h}$ B) Determine appropriate interaction diagrams C) Read ρ_g for $P_u/A_g (\leq \phi P_n/A_g)$ and $M_u/A_g h (\leq \phi M_n/A_g h)$ D) Compute required A_{st} $A_{st} = \rho_g A_g$	Known: $P_u = 940$ kips $\therefore \frac{P_u}{A_g} = \frac{940}{16 \times 16} = 3.67 \text{ ksi}$ $\frac{M_u}{A_g h} = \frac{4.80}{(16)^3} = 0.12$ $\gamma \approx \frac{16-5}{16} \approx 0.69$ For a rectangular column with steel along four faces, $f'_c = 5$ ksi, $f_y = 60$ ksi, and an estimated γ of 0.69, use R5-60.60 and R5-60.75 For $P_u/A_g = 3.67$ ksi, $\frac{M_u}{A_g h} = 0.12$ ksi, and $\gamma = 0.60$; $\rho_g = 0.041$ $\gamma = 0.75$; $\rho_g = 0.041$ $\therefore$ for $\gamma = 0.69$; $\rho_g = 0.041$ Required $A_{st} = 0.041(16)^2$ $= 10.5 \text{ in.}^2$	COLUMNS 7.5.2 and 7.5.3 (R5-60.60 and R5-60.75)

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
	Step 4 —Select optimum reinforcement.					
10.9.2	A) Assume trial bar quantities B) Determine smallest bar size to provide A_{st} , list resulting A_{st} , in. ²	4 #18 16.00 (obviously not an economic choice)	8 #11 12.48	12 #9 12.00	16 #8 12.64	REINFORCE- MENT 1.2 REINFORCE- MENT 1.2
7.10.5.1	C) Select tie size D) Refine γ for bar and tie size, and using this γ interpolate for accurate ρ_g (Step 3C), recompute required A_{st} , in. ² and compare with A_{st} provided (from B above)		#4 0.66 0.046 11.8 OK	#3 0.70 0.045 11.5 OK	#3 0.70 0.045 11.5 OK	COLUMNS 6.1
3.3.3 7.6.3 7.7.1 7.10.5.1 10.9.1 10.9.2	E) Check whether reinforcement can be accommodated in column with —Bearing splices —Normal lap splices —Tangential lap splices		OK OK OK	OK OK NO	OK NO NO	REINFORCE- MENT 3.1 REINFORCE- MENT 3.2 REINFORCE- MENT 3.3
7.10.5.2	F) Determine tie spacing as least of —16 longitudinal bar diameters, in. —48 tie bar diameters, in. —Least dimension of column		23 24 16	18 18 16	16 18 16	REINFORCE- MENT 1.1 REINFORCE- MENT 1.1
	G) Consider special design details	Omitted in this example				
	H) Select most cost-efficient reinforcement		(Probable) second choice	(Probable) first choice		
	Solution	Therefore, use a 16 × 16-in. column with 12 #9 bars with bearing or normal lap splices and #3 ties spaced 16 in. apart.				

COLUMNS EXAMPLE 7—Design of circular spiral column section subject to very small design moment*

For a circular spiral column, select column section diameter h and choose reinforcement.
Use relatively high proportion of longitudinal steel (i.e., $\rho_g \approx 0.04$).

Given: Loading

Factored axial load $P_u = 940$ kips

Factored moment at top of column $M_u = \pm 480$ kip-in.

Materials

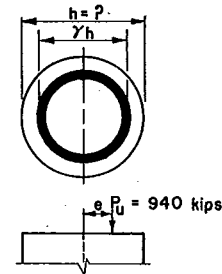
Compressive strength of concrete $f'_c = 5$ ksi

Yield strength of reinforcement $f_y = 60$ ksi

Maximum size of aggregate = 1.00 in.

Design Conditions

Effective column length $kl_u = 90$ in.



*Columns Examples 6 and 7 are similar except that Example 6 considers a square tied column with bars on four faces while Example 7 considers a circular spiral column.

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION			HANDBOOK REFERENCE
	Step 1—Determine trial column section diameter h corresponding to known values of variables on appropriate interaction diagram(s).	Knowns: $M_u = 480$ kip-in. $\rho_g \approx 0.04$			
	A) Assume trial column sizes h, in.	12	16	20	
	B) Compute estimated $\gamma \approx \frac{h-5}{h}$	0.58	0.69	0.75	
	C) Determine appropriate interaction diagram(s)	C5-60.45 C5-60.60	C5-60.60 C5-60.75	C5-60.75	
	D) Compute $M_u / A_g h = \frac{480}{\pi \left(\frac{h}{2}\right)^2 h}$	0.35	0.15	0.08	
9.3.2.2(b) 10.2 10.3	E) Read $\phi P_n / A_g$ ksi for $M_u / A_g h$ ($\leq \phi M_n / A_g h$), $\rho_g = 0.04$, and for $\gamma =$ lower value $\gamma =$ higher value and interpolate for $\gamma =$ value from Step 1B	3.26 3.57 3.54	4.12 4.12 4.12	4.12 4.12 4.12	COLUMNS 7.23.1, 7.23.2, 7.23.3 (C5-60.45, C5-60.60, C5-60.75)
	F) Compute $A_g = \frac{\phi P_n}{\phi P_n / A_g} = \frac{940}{\phi P_n / A_g}$ in.²	266	228	228	
	G) Compute $h = 2 \sqrt{\frac{A_g}{\pi}}$ in.	18.4	17.0	17.0	
		∴ Try 17 in. diameter column			

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
10.11.4.1 10.11.5.1 10.11.3 10.11.4.1	Step 2—Check whether slenderness ratio kl_u/h is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_u by factor δ_b (see Columns Example 5, Step 2) A) Compute M_{1b}/M_{2b}, and read critical value of kl_u/h B) Compute kl_u/h and compare with critical value; determine whether slenderness effects must be considered.	In this case, M_{1b} and M_{2b} are not known, but for circular columns, for all values of M_{1b}/M_{2b} slenderness may be neglected where $kl_u/h < 5.5$. $kl_u/h = \frac{90}{17} = 5.3 < 5.5$ ∴ Slenderness effects may be neglected.	COLUMNS 1
9.3.2.2(b) 10.2 10.3	Step 3—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s), and compute required cross section area A_{st} of longitudinal reinforcement. A) Compute P_u/A_g, ksi B) Compute $M_u/A_g h$, ksi C) Estimate $\gamma \approx \frac{h-5}{h}$ D) Determine appropriate interaction diagram(s) E) Read ρ_g for P_u/A_g ($\leq \phi P_n/A_g$) Required $A_{st} = \rho_g A_g$	$A_g = \pi \left(\frac{17}{2} \right)^2 = 227 \text{ in.}^2$ $\frac{P_u}{A_g} = \frac{940}{227} = 4.14 \text{ ksi}$ $M_u/A_g h = \frac{480}{227 \times 17} = 0.12 \text{ ksi}$ $\gamma = \frac{15-5}{15} = 0.67$ C5-60.60 and C5-60.75 For $\frac{P_u}{A_g} = 4.14 \text{ ksi}$, $\frac{\phi M_n}{A_g h} = 0.12 \text{ ksi}$, and $\gamma = 0.60$: $\rho_g = 0.041$ $\gamma = 0.75$: $\rho_g = 0.041$ ∴ For $\gamma = 0.67$: $\rho_g = 0.041$ $A_{st} = 0.041 \pi \left(\frac{17}{2} \right)^2 = 9.31 \text{ in.}^2$	COLUMNS 7.23.2 COLUMNS 7.23.3

EXAMPLE 7

ACI 318-89 REFERENCE	PROCEDURE	CALCULATION						HANDBOOK REFERENCE
10.9.1	Step 4—Select optimum reinforcement. A) Assume trial bar quantities B) Determine smallest bar size to provide A_{st} ; list resulting A_{st} , in. ²	6 #11 9.36	7 #11 10.92	8 #10 10.16	9 #10 11.43	10 #9 10.00	11 #9 11.00	REINFORCE- MENT 1.2 REINFORCE- MENT 1.2 REINFORCE- MENT 7.1
7.10.4.2 7.10.4.3 Eq. (10-5) 3.3.3 7.6.3 7.6.4 7.7.1 7.10.4.2 10.9.1 10.9.2	C) Select spiral size and pitch, in. D) Check whether reinforcement can be accommodated in column with —Bearing splices —Normal lap splices —Tangential lap splices	#4 3 OK OK OK	 3 OK OK OK	#4 3 OK OK OK	 OK OK OK	#4 3 OK OK NO	 OK OK NO	REINFORCE- MENT 4.1 REINFORCE- MENT 4.2 REINFORCE- MENT 4.3 REINFORCE- MENT 7.3
recommendation in ACI 318R-89 Section 7.10.4 7.8 7.9 7.10	E) Determine recommended number of spacers, if spacers are used F) Consider special design details G) Select most cost-efficient reinforcement	2 First choice	 	2 	 	2 	 	
	Solution	Omitted in this example						
		Use a 17 in. diameter column with six #11 bars and #4 spirals at 3 in. pitch. If spacers are used, recommend two spacers.						

COLUMNS EXAMPLE 8—Determination of moment magnification factors δ_b for each column and δ_s for each level and required reinforcement ratio ρ_g for columns in the first two stories of an unbraced frame

Note: This example illustrates the determination of moment magnification factors δ_b and δ_s for columns in a structure that relies on column shears to resist horizontal force (unbraced frames). The illustration involves the total structure instead of one individual column in order to emphasize the fact that factors δ_s reflect the stability of the entire structure, whereas braced frame factors δ_b reflect the slenderness aspects of individual columns.

The structure contains 12 columns in each level. The four corner columns must resist the same sets of maximum forces, and the four columns A2, A3, C2, and C3 likewise resist identical maximum forces. Columns B1 and B4 are alike, and column B2 resists the same design forces as column B3. Thus, the structure contains four different types of columns. Reinforcement must be assigned for all five levels of the structure, but this example includes only the columns that support level 1 and level 2.

In this example, the required reinforcement ratio ρ_g must

be determined for the more severe of two loading conditions:

- gravity load only
- gravity load plus lateral forces on columns

Initially, the amount of reinforcement ρ_g is not known, and ACI 318-89 Eq. (10-11) can be used for column stiffness EI estimates. If resulting values of ρ_g are very large, ACI 318-89 Eq. (10-10) may produce larger EI values and permit somewhat less required ρ_g . If factors $2K_c$ in COLUMNS 3 are less than the amount of $K_c + K_s$ in COLUMNS 3, column stiffness was undervalued by Eq. (10-11).

With final ρ_g values determined from initial EI according to Eq. (10-11), the example is reworked with initial EI according to Eq. (10-10) in order to demonstrate the procedure with COLUMNS 3 and to illustrate typical reductions in ρ_g when Eq. (10-10) is employed for more heavily reinforced columns.

Given the structure shown on the sketch on the next page

Loading

Loads and moments obtained by performing a first-order elastic frame analysis	Columns A1, A4, C1, C4		Columns A2, A3, C2, C3		Columns B1, B4		Columns B2, B3	
	supporting level		supporting level		supporting level		supporting level	
	2	1	2	1	2	1	2	1
Dead load thrust P_d , kips	112	136	160	193	178	215	251	303
Live load thrust P_l , kips	128	158	198	244	208	257	327	404
x-axis bending moments applied to columns								
Dead load thrust M_d , ft-kips	30	30	34	34	0	0	0	0
Live load thrust M_e , ft-kips	61	61	75	75	59	59	84	84
Wind moment M_w , ft-kips on columns	50	61	50	61	67	82	86	105
y-axis bending moments applied to columns	negligible							

Materials

Compressive strength of concrete $f'_c = 4$ ksi

Yield strength of reinforcement $f_y = 60$ ksi

Design Conditions

Rectangular tied column with reinforcement on four faces

Column cross-sectional dimensions

Columns A1, A2, A3, A4, B1, B4, C1, C2, C3, and C4: $b = 14$ in., $h = 20$ in.

Columns B2 and B3: $b = 18$ in., $h = 20$ in.

Unsupported length of column $l_u = 128$ in.

Vertical distance between floors $l_c = 150$ in.

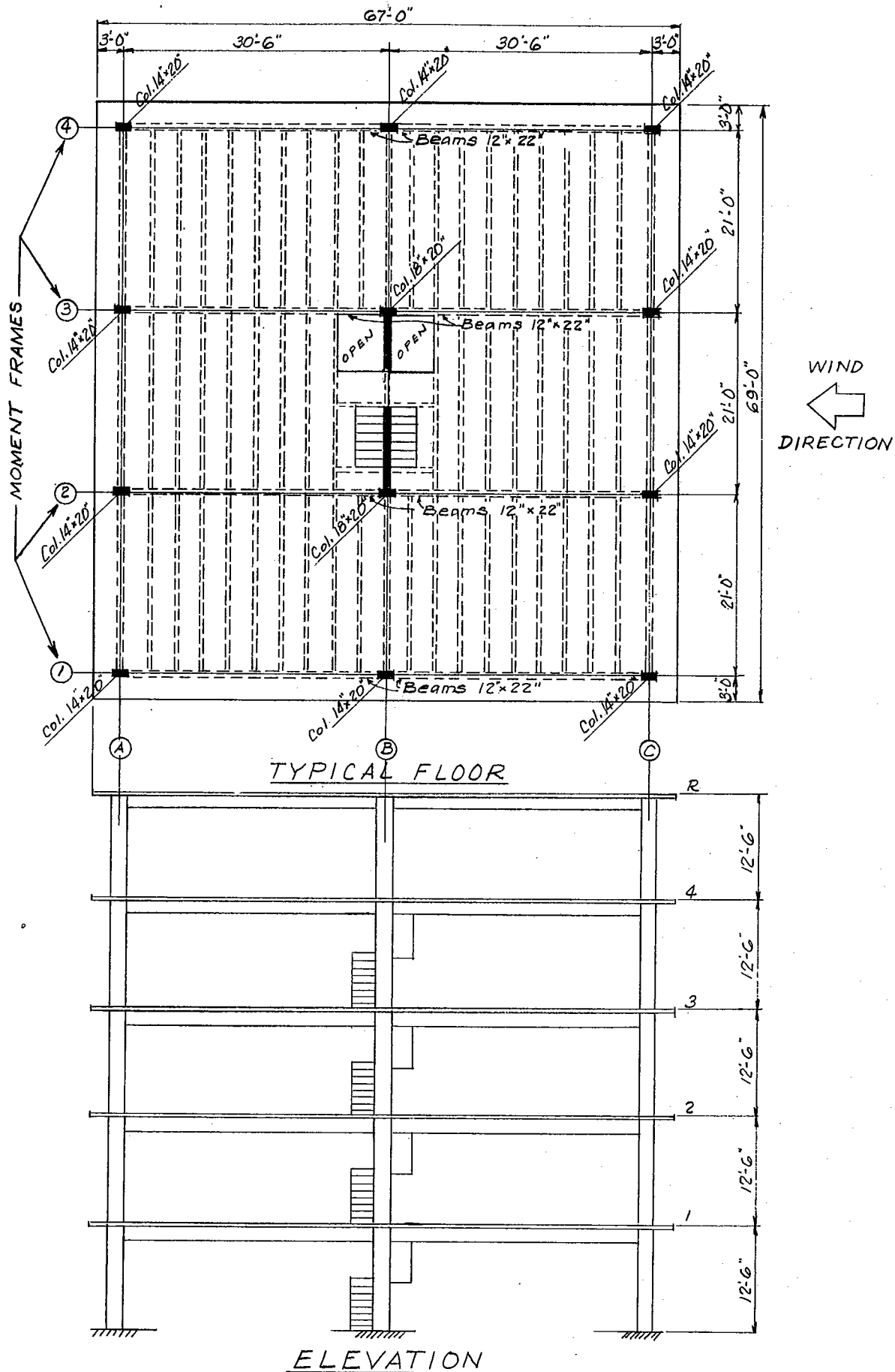
Beam cross sectional dimensions

Beam width $b_w = 12$ in.

Beam thickness $h = 22$ in.

Span length of beam $l = 30.5$ ft

EXAMPLE
8



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	Given b	14 14	14 14	14 14	18 18	
	Given h	20 20	20 20	20 20	20 20	
	Step 1—Calculate $A_g = (b)(h)$	280 280	280 280	280 280	360 360	
10.11.4.1	Step 2—Check whether slenderness ratio $\frac{kl_u}{r}$ is less than critical value of 22. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_u by factors δ_b and δ_s .	$k > 1.0$, but exact value cannot be determined at this point. For unbraced frame, estimate $k > 1.3$ $\frac{kl_u}{r} \geq \frac{(1.3)(128)}{(0.3)(20)} = 27.7 > 22$ $\therefore$ slenderness effects must be considered.				
10.11.22	A) Determine k					
10.11.4.3	Note: If $kl_u/h > 100$, a second-order frame analysis as defined in Section 10.10.1 must be made.					
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	Step 3—For gravity load only, determine moment magnification factor δ_b , using COLUMNS 5. To do so, these values are needed for each column: C_m , $P_u(1 + \beta_d)/A_g$, and kl_u/h .					
10.11.5.3	A) For C_m use 0.6 for exterior columns and 1.0 for interior columns	0.6 0.6	0.6 0.6	1.0 1.0	1.0 1.0	
9.2.1	B) Calculate P_u from $P_u = 1.4P_d + 1.7P_l$, kips	374 459	561 685	603 738	907 1111	
	C) Calculate M_u and β_d $M_u = 1.4M_d + 1.7M_l$, ft-kips $\beta_d = 1.4P_d/P_u$	146 146 0.42 0.41	175 175 0.40 0.39	100 100 0.41 0.41	143 143 0.39 0.38	
10.0	D) Compute $P_u(1 + \beta_d)/A_g$ where A_g is from Step 1	1.90 2.31	2.81 3.40	3.04 3.72	3.50 4.26	
	E) Determine kl_u/h by assuming $kl_u/h_e = l_u/h$ where l_u and h are given $kl_u/h_e = l_u/h$	6.4 6.4	6.4 6.4	6.4 6.4	6.4 6.4	

EXAMPLE 8

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	F) Read δ_b/C_m from the appropriate COLUMNS 5 graph and calculate δ_b . For a rectangular tied column with f'_c = 4 ksi, use COLUMNS 5.2					
Eq. (10-7) Eq. (10-9) Eq. (10-10)	δ_b/C_m from COLUMNS 5.2	1.10 1.13	1.17 1.20	1.18 1.23	1.21 1.27	COLUMNS 5.2
	$\delta_b = C_m(\delta_b/C_m) \geq 1.0$	1.0 1.0	1.0 1.0	1.18 1.23	1.21 1.27	
7.7.1	Step 4 —Determine ρ_g for gravity load, using appropriate COLUMNS 7 graph. A) Select appropriate COLUMNS 7 graph after estimating $\gamma \approx \frac{h - [(2)(2 \frac{1}{2})]}{h}$ $\approx \frac{20 - 5}{20} = 0.75$ $\therefore$ use COLUMNS 7.4.3					
	B) Compute P_u/A_g , ksi where P_u is from Step 3B and A_g from Step 1	1.34 1.64	2.00 2.45	2.15 2.64	2.52 3.09	
	C) Compute $(12 \text{ in./ft})(\delta_b M_u)/(A_g h)$, ksi	0.31 0.31	0.38 0.38	0.25 0.26	0.29 0.30	
9.3.2.2(c) 10.2 10.3	D) Read ρ_g from COLUMNS 7.4.3	0.010 0.010	0.021 0.032	0.010 0.024	0.022 0.039	COLUMNS 7.4.3
Eq. (10-8) 9.3.2.2(b) Eq. (10-9)	Step 5 —Determine moment magnification factor δ_s from $\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{\phi \Sigma P_c}}$ where $P_u = 0.75(1.4P_d + 1.7P_l)$, $\phi = 0.70$, and $P_c = \frac{\pi^2 EI}{(kl_u)^2}$ Note that ΣP_u remains constant when the structure is subjected to lateral force. The change in axial load of individual columns need not be considered for factor δ_s .					
9.2.2	A) Calculate gravity load part of the wind and gravity case $P_u = 0.75(1.4P_d + 1.7P_l)$ and ΣP_u , kips below level 2 and ΣP_u , kips below level 1	281 344 4(281) + 4(344) = 5068 4(344) + 4(514) + 2(553) + 2(833) = 6204	420 514 4(420) + 4(514) + 2(553) + 2(833) = 6204	452 553 4(452) + 4(553) + 2(680) + 2(833) = 6204	680 833 4(680) + 4(833) + 2(553) + 2(833) = 6204	

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
8.5.1 8.5.2 10.11.5.2	B) Calculate P_c and ϕP_c below level 2 and ϕP_c below level 1 Taking $\pi^2 EI = 2K_c \frac{bh^3}{(1 + \beta_d)}$, $\phi P_c = \frac{\phi(2K_c)bh^3}{(1 + \beta_d)(kl_u)^2}$ where $2K_c$ is read from the appropriate COLUMNS 3 graph. For these rectangular columns with reinforcement on four faces, read $2K_c$ from COLUMNS 3.1					COLUMNS 3
8.5.1 8.5.2 10.11.5.2 10.0	Assume no sustained lateral load, $\therefore \beta_d = 0$ To read K from COLUMNS 2 requires ψ_{top} and ψ_{bottom} where $\psi = \frac{\Sigma(EI/l_c)_{column}}{\Sigma(EI/l)_{beam}} = \frac{\Sigma(I/l_c)_{column}}{\Sigma(I/l)_{beam}}$ $\Sigma I_{column}/l_c = \frac{\Sigma bh^3}{12}/l_c, \text{ in.}^3$ $\Sigma I_{beam}/l = \frac{b_w h}{12}/l, \text{ in.}^3$ where b_w, h, and l are given for Columns A and C: $\frac{\Sigma \frac{b_w h}{12}}{l} = \frac{(12)(22)^3}{(30.5)(12 \text{ in./ft}) - 20}$ for Columns B: $\frac{\Sigma \frac{b_w h}{12}}{l} = \frac{(2) \frac{(12)(22)^3}{12}}{(30.5)(12 \text{ in./ft}) - 20}$	1190				COLUMNS 3
		0 0	0 0	0 0	0 0	
Fig. 10.11.2 of ACI 318R-89		146 146	146 146	146 146	188 188	COLUMNS 2
		31 31	31 31			
				62 62	62 62	

EXAMPLE
8

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	ψ_{top} ψ_{bottom}	4.7 4.7 4.7 0.2*	4.7 4.7 4.7 0.2*	2.4 2.4 2.4 0.2*	3.0 3.0 3.0 0.2*	
		*Value of ψ_{bottom} for columns supporting level 1 is taken as 0.2 to reflect the fixed end condition at the base of columns at ground level (for example, framing into basement walls).				
10.11.2	Read k from COLUMNS 2	2.15 1.51	2.15 1.51	1.69 1.35	1.82 1.40	COLUMNS 2
	$\phi P_c = \frac{\phi(2K_c)(bh^3)}{(1 + \beta_d)(kl_u)^2}, \text{ kips}$ where b , h , and l_u are given and $\phi \Sigma P_c$, kips below level 2 and $\phi \Sigma P_c$, kips below level 1	1232 2497	1232 2497	1994 3124	2210 3735	
	C) Calculate					
	$\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{\phi \Sigma P_c}} \text{ below level 2}$ and δ_s below level 1		$\delta_s = \frac{1}{1 - \frac{5068}{18,264}} = 1.38$			
			$\delta_s = \frac{1}{1 - \frac{6204}{33,694}} = 1.23$			
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
9.2.2	Step 6 —Determine ρ_g for gravity load plus lateral force, using COLUMNS 7. To do so, these values are needed: P_u/A_g and $M_c/A_g h$.					
	A) Calculate $0.75P_u/A_g$	1.00 1.23	1.50 1.84	1.61 1.98	1.89 2.32	
	B) Calculate $M_c/A_g h$ where A_g is from Step 1, h is given, and $M_c = 0.75(\delta_b M_u + 1.7\delta_s M_w), \text{ ft-kips}$ $= (12 \text{ in./ft})(0.75)(\delta_b M_u + 1.7\delta_s M_w), \text{ in.-kips}$ $M_c/A_g h$, ksi	2370 2462	2631 2723	2477 2650	3373 3610	
9.3.2.2(c)	C) Read ρ_g from COLUMNS 7.4.3	0.42 0.44	0.47 0.49	0.44 0.47	0.47 0.50	
10.2		0.012 0.016	0.022 0.031	0.021 0.029	0.028 0.038	COLUMNS 7.4.3
10.3						

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	Step 7—Compare ρ_g from Step 4D and ρ_g from Step 6 and list larger ρ_g , which governs.					
	governing ρ_g	0.012 0.016	0.022 0.032	0.021 0.029	0.028 0.039	
Eq. (10-10) Eq. (10-11)	Step 8—Redetermine δ_b using larger value of EI —that is, using kl_u/h_e instead of kl_u/h in COLUMNS 5.2. To do so, these values are needed: h_e/h , kl_u/h_e , and $P_u(1 + \beta_d)/A_g$					
Eq. (10-10) Eq. (10-11)	A) Read h_e/h from COLUMNS 7.4.3	1.00 1.00	1.02 1.12	1.01 1.09	1.08 1.19	COLUMNS 7.4.3
	B) Calculate $kl_u/h_e = k(l_u/h)/(h_e/h)$ assuming $k = 1.0$	6.4 6.4	6.3 5.7	6.3 5.9	5.9 5.4	
	C) List $P_u(1 + \beta_d)/A_g$ from Step 3D	1.90 2.31	2.81 3.40	3.04 3.72	3.50 4.26	
8.5.1 9.3.2.2(b) 10.11.5.1 10.11.5.2	D) Since $C_m = 1$, $\delta_b = \delta_b/C_m$; for kl_u/h_e and $P_u(1 + \beta_d)/A_g$, read δ_b/C_m from COLUMNS 5.2 and list as δ_b	1.00 1.00	1.00 1.00	1.18 1.19	1.17 1.17	COLUMNS 5.2
	Step 9—Redetermine ρ_g for gravity load, using δ_b from Step 8 and COLUMNS 7. To do so, these values are needed: P_u/A_g and $\delta_b M_u/A_g h$					
	A) List P_u/A_g from Step 4B	1.34 1.64	2.00 2.45	2.15 2.64	2.52 3.09	
	B) Calculate $(\delta_b M_u)/A_g h$. To do so, these values are needed: δ_b , M_u from Step 3C, A_g from Step 1, and h as given.					
	To find δ_b , list kl_u/h_e from Step 8B and $P_u(1 + \beta_d)/A_g$ from Step 3D and using these, read δ_b/C_m from COLUMNS 5.2	6.4 6.4 1.90 2.31 1.09 1.11	6.3 5.7 2.81 3.40 1.12 1.14	6.3 5.9 3.04 3.72 1.11 1.12	5.9 5.4 3.50 4.26 1.12 1.12	
	list C_m from Step 3A calculate $\delta_b = \delta_b/C_m \geq 1.0$ then calculate $(12 \text{ in./ft})(\delta_b M_u)/(A_g h)$, ksi	0.6 0.6 1.00 1.00 0.31 0.31	0.6 0.6 1.00 1.00 0.38 0.38	1.0 1.0 1.18 1.19 0.25 0.26	1.0 1.0 1.17 1.17 0.28 0.28	
9.3.2.2(c) 10.2 10.3	C) Read ρ_g from COLUMNS 7.4.3	0.010 0.010	0.021 0.031	0.010 0.024	0.022 0.038	COLUMNS 7.4.3

Documents

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ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION				HANDBOOK REFERENCE
		Columns A1, A4, C1, C4	Columns A2, A3, C2, C3	Columns B1, B4	Columns B2, B3	
		support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	support- ing level 2 1	
	Step 11 —Redetermine ρ_g for gravity load plus lateral force, using δ_s from Step 10 and COLUMNS 7. To do so, these values are needed: P_u/A_g and $M_c/A_g h$					
9.2.2	A) List $0.75P_u/A_g$ from Step 6A	1.00 1.23	1.50 1.84	1.61 1.98	1.89 2.32	
	B) Calculate $M_c/A_g h$ where $M_c = 0.75(\delta_b M_u + 1.7\delta_s M_w)$, ft-kips = (12 in./ft) (0.75) ($\delta_b M_u$ + $1.7\delta_s M_w$), in-kips	2362 2425	2623 2686	2466 2564	3564 3418	
	$M_c/A_g h$, ksi	0.42 0.43	0.47 0.48	0.44 0.46	0.50 0.47	
9.3.2.2(c) 10.2 10.3	C) Read ρ_g from COLUMNS 7.4.3	0.012 0.014	0.022 0.029	0.020 0.028	0.033 0.036	COLUMNS 7.4.3
	Step 12 —Compare ρ_g from Step 9C and ρ_g from Step 11 and list larger value, which governs Solution: required ρ_g	0.012 0.014	0.022 0.031	0.020 0.028	0.033 0.038	

COLUMNS EXAMPLE 9—Computation of $\pi^2 EI$ for ACI 318-89 Eq. (10-9) [where $EI_{(a)}$ from ACI 318-89 Eq. (10-10) is found to be greater than $EI_{(b)}$ from Eq. (10-11)]

For a rectangular tied column with bars on four faces, compute $\pi^2 EI$ using COLUMNS 3.

Given: Loading

Ratio of maximum factored axial dead load to maximum factored axial total load $\beta_d = 0$

Materials

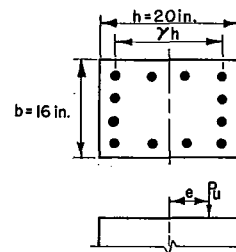
Compressive strength of concrete $f'_c = 4$ ksi

Twelve #10 bars (four on each face) with #3 ties

Design Conditions

Column thickness $h = 20$ in.

Column width $b = 16$ in.



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
10.11.5.1 Eq. (10-10) Eq. (10-11)	Step 1—Compute $\pi^2 EI$ as the greater of $\pi^2 EI_{(a)} = (K_c + K_s) \frac{bh^3}{1 + \beta_d}$ and $\pi^2 EI_{(b)} = 2K_c \frac{bh^3}{1 + \beta_d}$ using appropriate stiffness diagram		
8.5.1 8.5.2 10.11.5.2	A) Determine appropriate stiffness diagram B) Read K_c and $2K_c$ for f'_c C) Read γ for bar size and column size D) Compute ρ_g from $\rho_g = A_{st}/A_g = \frac{nA_b}{h \times b}$	For a rectangular tied column with bars on four faces, use COLUMNS 3.1 For $f'_c = 4$ ksi, $K_c = 595$ ksi and $2K_c = 1190$ ksi For #10 bars and $h = 20$ in., $\gamma = 0.75$ $\rho_g = \frac{12 \times 1.27}{20 \times 16} = 0.048$	COLUMNS 3.1
8.5.1 8.5.2 10.11.5.2	E) Read K_s for γ and ρ_g F) Compare K_c and K_s G) If $K_c < K_s$, compute $\pi^2 EI_{(a)} = (K_c + K_s) \frac{bh^3}{1 + \beta_d}$ If $K_c \geq K_s$, compute $\pi^2 EI_{(b)} = 2K_c \frac{bh^3}{1 + \beta_d}$	For $\gamma = 0.75$ and $\rho_g = 0.048$, $K_s = 1290$ ksi $K_c < K_s$ $\pi^2 EI_{(a)} = (595 + 1290) \frac{16 (20)^3}{1 + 0} = 241 \times 10^6 \text{ kip-in.}$	COLUMNS 3.1
	Solution	$\pi^2 EI = 241 \times 10^6 \text{ kip-in.}$	

COLUMNS EXAMPLE 10—Computation of $\pi^2 EI$ for ACI 318-89 Eq. (10-9) [where $EI_{(b)}$ from ACI 318-89 Eq. (10-11) is found to be greater than $EI_{(a)}$ from Eq. (10-10)]

For a circular spiral column, compute $\pi^2 EI$ using COLUMNS 3 and 4.

Given: Loading

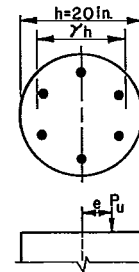
Ratio of maximum factored axial dead load to maximum factored axial total load $\beta_d = 0.5$

Materials

Compressive strength of concrete $f'_c = 6$ ksi
Six #9 bars with #3 spiral

Design conditions

Column diameter $h = 20$ in.



ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
10.11.5 Eq. (10-10) Eq. (10-11) 8.5.1 8.5.2 10.11.5.2	Step 1—Compute $\pi^2 EI$ as the greater of $\pi^2 EI_{(a)} = (K_c + K_s) \frac{h^4}{1 + \beta_d}$ $\pi^2 EI_{(b)} = 2K_c \frac{h^4}{1 + \beta_d}$ using appropriate stiffness EI diagram A) Determine appropriate stiffness diagram B) Read K_c and $2K_c$ for f'_c C) Read γ for bar size and column size D) Compute ρ_g from $\rho_g = A_{st}/A_g$ $= \frac{nA_b}{\pi \left(\frac{h}{2}\right)^2}$ E) Read K_s for γ and ρ_g F) Compare K_c and K_s G) If $K_c < K_s$, compute $\pi^2 EI_{(a)} = (K_c + K_s) \frac{h^4}{1 + \beta_d}$ If $K_c \geq K_s$, compute $\pi^2 EI_{(b)} = 2K_c \frac{h^4}{1 + \beta_d}$	For a circular spiral column, use COLUMNS 3.4 For $f'_c = 6$ ksi, $K_c = 428$ ksi and $2K_c = 856$ ksi For #9 bars and $h = 20$ in., $\gamma = 0.76$ $\rho_g = \frac{6 \times 1.00}{\pi \left(\frac{20}{2}\right)^2} = 0.019$ For $\gamma = 0.76$ and $\rho_g = 0.019$, $K_s = 310$ ksi $K_c > K_s$ $\pi^2 EI_{(b)} = 856 \frac{(20)^4}{1 + 0.5} = 91.3 \times 10^6 \text{ kip-in.}$	COLUMNS 3.4 COLUMNS 3.4
8.5.1 Eq. (10-11)	Alternate Step 1E) If $K_s > K_c$ read $[(E_c I_g)/2.5] \times 10^{-5}$ and compute $\pi^2 EI = \frac{\pi^2 [(E_c I_g)/2.5] \times 10^{-5} \times 10^5}{1 + \beta_d}$	For $h = 20$ in. diameter, $[(E_c I_g)/2.5] \times 10^{-5} = 139$ $\pi^2 EI = \frac{\pi^2 (139) \times 10^5}{1.5} = 91.4 \times 10^6 \text{ kip-in.}$	COLUMNS 4.4
	Solution	$\pi^2 EI = 91.4 \times 10^6 \text{ kip-in.}$	

COLUMNS EXAMPLE 11—Determination of adequacy of square tied column section subject to biaxial bending, using reciprocal load method with $1/P_{ni}$ equation

Determine adequacy of column shown; use reciprocal load method and $\frac{1}{P_{ni}}$ equation.

Given: Loading

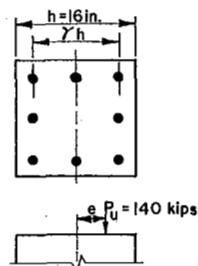
Factored axial load $P_u = 140$ kips
 Factored moment about x-axis $M_{ux} = 1404$ kip-in.
 Factored moment about y-axis $M_{uy} = 636$ kip-in.

Materials

Compressive strength of concrete $f'_c = 3$ ksi
 Yield strength of reinforcement $f_y = 40$ ksi
 Eight #9 bars with #3 ties

Design Conditions

Column section size $h = b = 16$ in.
 Slenderness ratio is below critical value so slenderness effects need not be considered
 1-1/2 in. concrete cover over reinforcement



Note: Columns Examples 11, 12, and 13 use different methods to check the adequacy of the same column section subject to the same loading conditions. Columns Example 11 uses the reciprocal load method with the $1/P_{ni}$ equation. Columns Example 12 uses the reciprocal load method and COLUMNS 9. Columns Example 13 uses the load contour method and COLUMNS 11 and 12.

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Step 1—Determine eccentricity ratios about x- and y-axes. $\frac{e}{h} = \frac{M_u}{P_u h}$ $\frac{e_y}{h} = \frac{M_{ux}}{P_u h}$ $\frac{e_x}{h} = \frac{M_{uy}}{P_u h}$	Given: $h = b = 16$ in. $M_{ux} = 1404$ kip-in. $M_{uy} = 636$ kip-in. $P_u = 140$ kips $\frac{e_y}{h} = \frac{1404}{140 \times 16} = 0.627$ $\frac{e_x}{h} = \frac{636}{140 \times 16} = 0.284$	
	Step 2—Find $\frac{P_{ux}}{A_g}$ associated with $\frac{M_{ux}}{A_g}$ and $\frac{P_{uy}}{A_g}$ associated with $\frac{M_{uy}}{A_g}$ A) Compute $\rho_g = \frac{A_{st}}{A_g} = \frac{nA_b}{h \times b}$ B) Find γ C) Determine appropriate interaction diagram(s)	$\rho_g = \frac{8 \times 1.0}{16 \times 16} = 0.031$ For #9 bars with #3 ties in a 16 × 16-in. column: $\gamma = 0.70$ For square tied column, $f'_c = 3$ ksi, $f_y = 40$ ksi, and $\gamma = 0.70$; use R3-40.60 and R3-40.75	COLUMNS 6.1

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	D) Find $P_{ux}/A_g = \phi P_{nx}/A_g$	For $\frac{e_y}{h} = 0.627$, $\rho_g = 0.031$, and $\gamma = 0.60$: $\frac{\phi P_{nx}}{A_g} = 0.62$ ksi	COLUMNS 7.1.2 (R3-40.60)
9.3.2.2(b) 10.2 10.3		$\gamma = 0.75$: $\frac{\phi P_{nx}}{A_g} = 0.71$ ksi	COLUMNS 7.1.3 (R3-40.75)
		$\therefore$ For $\gamma = 0.70$: $\frac{\phi P_{nx}}{A_g} = 0.68$ ksi	
9.3.2.2(b) 10.2 10.3	E) Find $P_{uy}/A_g = \phi P_{ny}/A_g$	For $\frac{e_x}{h} = 0.284$, $\rho_g = 0.031$, and $\gamma = 0.60$: $\frac{\phi P_{ny}}{A_g} = 1.26$ ksi	COLUMNS 7.1.2 (R3-40.60)
		$\gamma = 0.75$: $\frac{\phi P_{ny}}{A_g} = 1.37$ ksi	and COLUMNS 7.1.3 (R3-40.75)
		$\gamma = 0.70$: $\frac{\phi P_{ny}}{A_g} = 1.33$ ksi	
9.3.2.2(b) 10.2 10.3	Step 3—Find nominal axial load strength at zero eccentricity ϕP_o as $\phi P_o/A_g$ using basic limits table(s).	For all values of γ , For $\rho_g = 0.030$: $\frac{\phi P_n}{A_g} = 2.057$ $\rho_g = 0.040$: $\frac{\phi P_n}{A_g} = 2.267$ $\therefore$ For $\rho_g = 0.031$: $\frac{\phi P_n}{A_g} = 2.078$ and $\frac{\phi P_o}{A_g} = \frac{2.078}{0.8} = 2.598$	COLUMNS 8.1 (R3-40)
	Step 4—Compute axial load strength P_{ni} at e_x and e_y using $\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$ and compare with factored axial load P_u .	Dividing both sides of the equation by ϕ/A_g , $\frac{1}{\phi P_{ni}/A_g} = \frac{1}{\phi P_{nx}/A_g} + \frac{1}{\phi P_{ny}/A_g} - \frac{1}{\phi P_o/A_g}$ $\frac{1}{\phi P_{ni}/A_g} = \frac{1}{0.68} + \frac{1}{1.33} - \frac{1}{2.60}$ $= 1.84 \text{ in.}^2/\text{kip}$ $\phi P_{ni} = \frac{A_g}{1.84} = \frac{16 \times 16}{1.84} = 139 \text{ kips}$	Commentary on Columns 9
	Solution	The calculated ϕP_{ni} of 139 kips is within 1% of the given P_u of 140 kips; since this approximate method of analysis is known to be conservative, the section can be considered to be adequate.	

COLUMNS EXAMPLE 12—Determination of adequacy of square tied column section subject to biaxial bending, using reciprocal load method and COLUMNS 9

Determine adequacy of column in Columns Example 11; use reciprocal load method and skew bending chart.

Note: Columns Examples 11, 12, and 13 use different methods to check the adequacy of the same column section subject to the same loading conditions. Columns Example 11 uses the reciprocal load method with the $1/P_{ni}$ equation. Columns Example 12 uses the reciprocal load method and COLUMNS 9. Columns Example 13 uses the load contour method and COLUMNS 11 and 12.

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
	Steps 1–3—Same as for Columns Example 11.	$\phi P_{nx}/A_g = 0.68$ ksi $\phi P_{ny}/A_g = 1.33$ ksi $\phi P_o/A_g = 2.60$ ksi	
9.3.2.2(b) 10.2 10.3	Step 4—Determine axial load strength P_{ni} at e_x and e_y using skew bending chart and compare with factored axial load P_u .	For $\phi P_{nx}/A_g = 0.68$ ksi (from Step 2D), $\phi P_{ny}/A_g = 1.33$ ksi (from Step 2E), and $\phi P_o/A_g = 2.60$ (from Step 3): $\phi P_{ni}/A_g = 0.544$ ksi $\phi P_{ni} = 0.544 (16 \times 16) = 139$ kips $\approx P_u$ of 140 kips	COLUMNS 7.1.2 COLUMNS 7.1.3 COLUMNS 9
	Solution	The calculated ϕP_{ni} of 139 kips is within 1% of the given P_u of 140 kips. Since this approximate method of analysis is known to be conservative, the section can be considered to be adequate.	

COLUMNS EXAMPLE 13—Determination of adequacy of square tied column section subject to biaxial bending, using load contour method and COLUMNS 11 and 12

Determine adequacy of square tied column in Columns Example 11; use load contour method.

Note: Columns Examples 11, 12, and 13 use different methods to check the adequacy of the same column section subject to the same loading conditions. Columns Example 11 uses the reciprocal load method with the $1/P_{ni}$ equation. Columns Example 12 uses the reciprocal load method and COLUMNS 9. Columns Example 13 uses the load contour method and COLUMNS 11 and 12.

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	Step 1—Determine nominal axial load strength at zero eccentricity ϕP_o. A) Find $\frac{\phi P_o}{A_g}$ B) Compute $\phi P_o = \frac{\phi P_o}{A_g} \times A_g$	$\frac{\phi P_o}{A_g} = 2.60 \text{ ksi (from Step 3 of Columns Example 11)}$ $\phi P_o = 2.60 \times 256 = 666 \text{ kips}$	COLUMNS 8.1
	Step 2—Determine biaxial bending design constant β. A) Compute $\rho_g \frac{f_y}{f'_c}$ B) Compute $\frac{P_u}{\phi P_o} = \frac{\phi P_n}{\phi P_o}$ C) Determine appropriate graph for biaxial bending design constant β D) Read β for $\frac{P_u}{\phi P_o}$ and $\rho_g f_y / f'_c$	Knowns: $h = b = 16 \text{ in.}$ $\therefore A_g = 256 \text{ in.}^2$ 8-#9 bars $\therefore A_{st} = 8 \times 1.0 = 8.0 \text{ in.}^2$ and $\rho_g = \frac{8.0}{256} = 0.031$ $f'_c = 3 \text{ ksi}$ $f_y = 40 \text{ ksi}$ $P_u = 140 \text{ kips}$ $\rho_g \frac{f_y}{f'_c} = 0.031 \frac{40}{3} = 0.41$ $\phi P_o = 666 \text{ kips (from Step 1B)}$ $\frac{\phi P_n}{\phi P_o} = \frac{140}{666} = 0.210$ For a rectangular column with steel along four faces, eight bars (three bars on each face), and $f_y = 40 \text{ ksi}$; use COLUMNS 11.2.1 For $\frac{\phi P_n}{\phi P_o} = 0.210$ and $\rho_g f_y / f'_c = 0.41$: $\beta = 0.62$	COLUMNS 11.2.1
	Step 3—Determine M_{nox}, the uniaxial moment capacity about the x-axis associated with P_u. A) Compute $\frac{\phi P_n}{A_g}$	Knowns: $P_u \leq \phi P_n = 140 \text{ kips}$ $h = b = 16 \text{ in.} \therefore A_g = 256 \text{ in.}^2$ and $\rho_g = 0.031$ nine #8 bars equally distributed on four faces and #3 ties $\therefore \gamma = 0.70$ $f'_c = 3 \text{ ksi}$ $f_y = 40 \text{ ksi}$ $\frac{\phi P_n}{A_g} = \frac{140}{256} = 0.547 \text{ ksi}$	COLUMNS 6.1

ACI 318-89 REFER- ENCE	PROCEDURE	CALCULATION	HANDBOOK REFERENCE
9.3.2.2(b) 10.2 10.3	B) Determine appropriate interaction diagram(s) C) Read $\frac{\phi M_{nox}}{A_g h}$ for $\frac{P_u}{A_g} \left(\leq \frac{\phi P_n}{A_g} \right)$ and ρ_g	For rectangular column with steel along four faces, $f'_c = 3$ ksi, $f_y = 40$ ksi, and $\gamma = 0.70$: use R3-40.60 and R3-40.75 For $\frac{\phi P_n}{A_g} = 0.547$ ksi, $\rho_g = 0.031$, and $\gamma = 0.60$: $\frac{\phi M_{nox}}{A_g h} = 0.38$ ksi $\gamma = 0.75$: $\frac{\phi M_{nox}}{A_g h} = 0.43$ ksi $\therefore$ For $\gamma = 0.70$: $\frac{\phi M_{nox}}{A_g h} = 0.41$ ksi $\phi M_{nox} = 0.41 (256 \times 16) = 1680$ kip-in.	COLUMNS 7.1.2 (R3-40.60) COLUMNS 7.1.3 (R3-40.75)
	Step 4—Determine ϕM_{noy} , the uniaxial moment strength about the y-axis.	Because of symmetry, $\phi M_{nox} = \phi M_{noy} = 1680$ kip-in.	
	Step 5 Determine M_{uy} , the factored moment about the y-axis A) Compute $\frac{M_{ux}}{\phi M_{nox}} \leq \frac{\phi M_{nx}}{\phi M_{nox}}$ B) Read $\frac{M_{ny}}{M_{noy}}$ for $\frac{M_{nx}}{M_{nox}}$ and β	Given: $M_{ux} = 1404$ kip-in. $\frac{\phi M_{nx}}{\phi M_{nox}} = \frac{1404}{1680} = 0.84$ For $\frac{\phi M_{nx}}{\phi M_{nox}} = 0.84$ and $\beta = 0.62$ (from Step 2D) $\frac{M_{ny}}{M_{noy}} = 0.36$	COLUMNS 12
	Step 6—Determine moment strength ϕM_{ny} under biaxial bending and compare with factored M_{uy} . $M_{uy} \leq \phi M = \frac{M_{ny}}{M_{noy}} \times \phi M_{noy}$	Given: factored $M_{uy} = 636$ kip-in. $\phi M_{oy} = 1680$ kip-in. (from Step 4) $M_{uy} = \phi M_{ny} = 0.36 \times 1680 = 605$ kip-in. < factored M_{uy} of 636 kip-in.	
	Solution	This analytic method, which is very conservative, implies about 5% less strength than is required for the smaller of the moments when required P_u and required M_{ux} are satisfied.	

