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Guide to Design of Reinforced Two-Way Slab Systems

Reported by Joint ACI-ASCE Committee 421



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Guide to Design of Reinforced Two-Way Slab Systems

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Guide to Design of Reinforced Two-Way Slab Systems

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This guide presents analysis methods, design procedures, slab reinforcement and detailing practices, and strength and serviceability considerations, as well as information for the resistance to lateral forces for slab-column frames. It also covers the design for flexure and shear and torsion, as well as the effect of openings. Both two-way nonprestressed slabs and post-tensioned slabs are included.

Keywords: analysis method; deflection; direct design; flat plates; flat slabs; post-tension; reinforcement; shear; shearhead; slab-column frame; two-way slabs.

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CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Introduction: history of two-way slab system

Two-way flat slab construction in the United States evolved, and was invented and patented, in the early 1900s (Cohen and Heun 1979). Early two-way flat slab construction was built and subjected to load tests in place and scaled models were later tested in laboratories. While the amount of reinforcement in slab construction varied dramatically, flat slab systems were found to be economical for heavy live load occupancy. As the number of flat slab projects increased steadily worldwide, design rules were established and formalized (Sozen and Seiss 1963).

Prior to the 1950s, two-way waffle slabs and two-way flat slabs were designed and constructed with column capitals and some with drop panels. The hollow tile and concrete slab is a type of waffle slab that dates back to at least 1918 (Gamble et al. 1964). Column capitals were used to increase slab shear strength and drop panels to reduce the flexural reinforcement over columns, which allowed for thinner slabs. In the post-1970s era, field labor to construct formwork for column capitals and drop panels became costly; the introduction of reusable forms led to construction of flat plates, which are two-way flat slabs without column capitals or drop panels.

The lift-slab system for multistory construction was popular in the 1960s and 70s, but is no longer commonly

used. The slabs were cast in a stack at ground level, post-tensioned, and then lifted to their final elevations using jacks lifting on steel collars embedded in the slabs.

Draped post-tensioning can be designed to balance part of the gravity loads. Combining unbonded post-tensioned tendons and nonprestressed reinforcement results in reduced slab thickness. In addition, the use of nonprestressed reinforcement supplements prestressed tendons to meet the required nominal strength and control slab cracking.

1.2—Scope

The performance record of various two-way slab systems is well established based on results of extensive tests and practical construction improvements in the twentieth century. The ACI Building Code permits design of slab systems, both nonprestressed and post-tensioned, based directly on fundamental principles of structural mechanics that satisfy equilibrium and compatibility. This guide provides classic solutions based on linearly elastic continuum, as well as prescriptive procedures used in common practice for analysis and design of slab systems. The fundamental principles in this guide are applicable to all planar structural slab systems subjected to gravity loads and, in certain conditions, those combined with lateral forces.

This guide addresses recommended practice in the selection and distribution of flexural reinforcement, and guidelines to transmit loads from slabs to columns by flexure, torsion, and shear. Detailing practices for post-tensioned two-way slabs are found in ACI 423.3R-05. This guide also discusses aspects and parameters where two-way slabs without beams are incorporated in ordinary or intermediate moment frames with ductile detailing and toughness.

While two-way slab systems have more than 100 years of service history, various practical refinements and research programs continue to develop new materials and technologies that support sustainable construction of two-way slabs.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

- A_{cf} = larger gross cross-sectional area of the slab-column strips in the two orthogonal equivalent frames intersecting at a column in a two-way slab, ft² (m²)
- A_{sb} = area of reinforcement through the column core used as integrity reinforcement
- b_1 = dimension of the critical section b_o measured in the direction of the span for which moments are determined, in. (mm)
- b_2 = dimension of the critical section b_o measured in the direction perpendicular to b_1 , in. (mm)
- b_e = effective slab width, in. (mm)
- b_o = perimeter of critical section at $d/2$ from face of support, in. (mm)
- C = cross-sectional constant to define torsional properties of slab and beam
- c_1 = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction

	tion of the span for which moments are being determined, in. (mm)	y	= longer overall dimension of rectangular part of cross section, in. (mm)
c_2	= dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction perpendicular to c_1 , in. (mm)	V_c	= nominal shear strength provided by concrete, lb (N)
c_t	= distance from the interior face of the column to slab edge measured parallel to c_1 , but not exceeding c_1 , in. (mm)	V_p	= vertical component of all effective prestress forces crossing the critical section, lb (N)
d	= distance from extreme compression fiber to the centroid of tension reinforcement, in. (mm)	V_{se}	= unfactored shear force, but not less than twice the unfactored dead load shear, lb (N)
E_{cb}	= modulus of elasticity of beam concrete, psi (MPa)	V_{ug}	= factored shear force on the slab critical section for two-way action due to gravity loads, lb (N)
E_c	= modulus of elasticity of slab concrete, psi (MPa)	α_f	= ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam
f'_c	= specified compressive strength of concrete, psi (MPa)	α_{fm}	= average value of α_f for all beams on edges of a panel
f_{pc}	= average compressive stress in the two directions at centroid of concrete cross section after allowing for all prestress losses, psi (MPa)	α_{f1}	= α_f in direction of ℓ_1
f_y	= specified yield stress of reinforcement, psi (MPa)	α_{f2}	= α_f in direction of ℓ_2
g	= distance between adjacent stirrup legs or studs, measured in a parallel direction to a column face	α_s	= constant used to compute V_c in slabs
h	= slab thickness, in. (mm)	β	= ratio of long side-to-short side of the column, concentrated load, or reaction area
I_b	= moment of inertia of gross section of beam about centroidal axis, in. ⁴ (mm ⁴)	β_p	= factor used to compute V_c in prestressed slabs
I_s	= moment of inertia of gross section of slab about centroidal axis defined for calculating α_f and β_t , in. ⁴ (mm ⁴)	β_t	= ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports
K_c	= stiffness of columns based on moment of inertia at any cross section outside the joint	γ_f	= factor used to determine the unbalanced moment transferred by flexure at slab-column connections
K_{ec}	= stiffness of equivalent column	γ_v	= factor used to determine the unbalanced moment transferred by eccentricity of shear at slab-column connections
K_{FP}	= modification factor accounting for reduction in joint confinement at exterior connections	ρ_b	= ratio of A_s to b_d at balanced condition when concrete and reinforcement both reach their respective yield strain
K_t	= torsional stiffness	$\sum A_{sb}$	= total area of reinforcing steel passing through the column core, summed on all four sides of the column of an interior column
ℓ_1	= length of span in direction that moments are being determined, measured center-to-center of supports, in. (mm)	χ	= ℓ_2/ℓ_1
ℓ_2	= length of span in direction perpendicular to ℓ_1 , measured center-to-center of supports, in. (mm)		
ℓ_3	= distance measured from the column centerline to the edge of the slab, in. (mm)		
ℓ_n	= length of clear span measured face-to-face of supports, in. (mm)		
ℓ_t	= span of member under load test, taken as the shorter span for two-way slab systems, in. (mm); span is the smaller of: a) distance between centers of supports; and b) clear distance between supports plus thickness h of member.		
M_o	= total factored static moment, in.-lb (kNm)		
M_{sc}	= portion of slab factored moment balanced by support moment, in.-lb (kNm)		
N_c	= resultant tensile force acting on the portion of the concrete cross section that is subjected to tensile stresses due to the combined effects of service loads and effective prestress, lb (N)		
q_{Du}	= factored dead load per unit area, lb/ft ² (kPa)		
q_{Lu}	= factored live load per unit area, lb/ft ² (kPa)		
q_u	= factored load per unit area, lb/ft ² (kPa)		
x	= shorter overall dimension of rectangular part of cross section, in. (mm)		

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, “ACI Concrete Terminology,” <https://www.concrete.org/store/productdetail.aspx?ItemID=CT13>. Definitions provided herein complement that source.

design story drift—design displacement of one level, or floor, relative to the level above or below.

design story drift ratio—design story drift divided by the story height.

headed shear stud reinforcement—individual headed studs or groups of studs with anchorage provided by a head at each end or a common base rail consisting of a steel plate or shape.

lateral-force-resisting system—portion of the structure composed of members designed to resist forces related to wind or earthquake effects.

podium—a thick slab that supports a light frame building above parking or commercial levels.

shear cap—a projection below the slab used to increase the slab shear strength.



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slab-on-girder system—a grid of girders in both plan directions with a solid slab spanning between girders; may include beams between girders in both directions.

slab panel—a slab bounded by column, beam, or wall centerlines on all sides.

two-way slab—slab with or without beams that meets a particular aspect ratio.

two-way wide-band system—slab with paneled ceiling with shallow and wide beams spanning between columns in each direction.

waffle slab—two-way concrete joist construction using prefabricated pan (dome) forms.

CHAPTER 3—ANALYSIS METHODS

3.1—General

3.1.1 Fundamental principles of structural mechanics—Analysis of two-way slab can be based directly on fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all strength and serviceability criteria in Chapters 4 and 6 of this guide are satisfied. Based on theory of elasticity, slab design should satisfy force equilibrium and strain compatibility. Slab analysis can be achieved through the use of classic solutions based on linearly elastic continuum theories; numerical solutions based on discrete elements; yield-line analyses; or strip method analyses, including in all cases evaluation of the stress conditions around the supports in relation to shear and torsion, as well as flexure. Regardless of analysis method, deviations in physical dimensions of the slab from common practice should be justified on the basis of knowledge of the expected loads, reliability of the calculated stresses, and deformations of the structure.

Analysis of a two-way slab system should consider the aspect ratio of each slab panel and the relative stiffness of the slab panels, supporting beams (if any), and supporting columns or walls. Analysis and design of two-way slab systems using the Direct Design Method (DDM) and Equivalent Frame Method (EFM) are discussed in 3.2.

3.1.2 Slab stiffness—During the life of a structure, construction loads, ordinary occupancy loads, anticipated overloads, and volume changes can cause cracking of slabs. Excessive cracking exposes concrete to moisture infiltration, which can cause corrosion of reinforcement and deterioration of structural elements. Under sustained gravity loads, cracking can lead to large vertical deflection resulting in damage to nonstructural elements.

Cracking reduces stiffness of the slabs, and increases lateral displacement when lateral forces act on the structure. Cracking of slabs should be considered in stiffness assumptions so drift caused by wind or earthquake is not grossly underestimated. Conservatively, analysis of slab-column connections can use a stiffness reduction factor that provides higher reduction in stiffness to ensure lateral displacement and that the design forces are not underestimated. Refer to 7.2.2 for further discussions.

3.1.3 Total factored static moment—Total factored static moment M_o for a span of a rectilinear interior panel may be

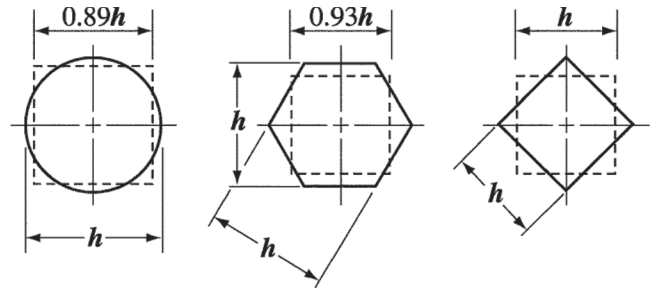


Fig. 3.1.3—Example of equivalent square section for supporting members.

determined in a strip bounded laterally by the centerline of a panel on each side of the centerline of supports. The absolute sum of positive and average negative factored moments in each direction should not be less than

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \quad (3.1.3)$$

In the case of an edge panel, use $0.5\ell_2$, where ℓ_2 is the transverse span.

Equation (3.1.3) follows directly from Nichols (1914) with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate the static moment for two adjacent half panels, which includes a column strip with a half middle strip along each side. The clear span ℓ_n is taken as the distance between columns, capitals, brackets, or walls. The value of ℓ_n used in Eq. (3.1.3) should not be less than $0.65\ell_1$. Circular or regular polygon-shaped supports should be treated as square supports with the same area (Fig. 3.1.3).

3.2—Analysis methods

Both the DDM and EFM are based on analysis of the results of an extensive series of tests (Jirsa et al. 1963) and the well-established performance record of various slab systems. This guide covers these two methods in depth.

3.2.1 Direct Design Method (DDM)—The DDM consists of a set of rules for distributing moments to slab and beam sections to satisfy strength requirements. The DDM can be used for slab systems with regular column layouts subjected to gravity loads only. Three fundamental steps are involved:

- 1) Determination of M_o
- 2) Distribution of M_o along each span to supports and midspan for negative factored moments and positive factored moment, respectively
- 3) Distribution of the negative and positive factored moments transversely to the column and middle strips and to the beams, if any. (The same distribution of moments to column and middle strips is also used in EFM, which is discussed in 3.2.2.6.)

3.2.1.1 Limitations—The DDM was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple analysis and construction procedures, and

precedents supplied by performance of slab systems. Consequently, based on ACI 318-14, Section 8.10.1.1, the analysis of slab systems using the DDM should conform to the following limitations:

a) Slab system has a minimum of three continuous spans in each direction;

b) Panels are rectangular, with a ratio of longer-to-shorter panel span taken from center-to-center of each panel not greater than 2;

c) Successive span lengths center-to-center of supports in each direction do not differ by more than one-third the longer span;

d) Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between center-lines of successive columns is permitted;

e) All loads are due to gravity only and uniformly distributed over an entire panel and unfactored live load does not exceed two times unfactored dead load;

f) For a panel with beams between supports on all sides, Eq. (3.2.1.1a) should be considered for beams in the two perpendicular directions:

$$0.2 \leq \frac{\alpha_{f1} \ell_2^2}{\alpha_{f2} \ell_1^2} \leq 5.0 \quad (3.2.1.1a)$$

where α_{f1} and α_{f2} are calculated in accordance with Eq. (3.2.1.1b)

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \quad (3.2.1.1b)$$

g) Moment redistribution as permitted by ACI 318-14, Section 6.6.5, is not applicable for slab systems analyzed by the DDM;

h) Variations from these limitations are permitted if demonstrated by analysis from 3.1.1 of this guide.

Rules given for the DDM assume that the slab system at the first interior support is neither fixed against rotation nor discontinuous. The primary reason for limitation a), which requires a minimum of three spans, is that the magnitude of the negative moments at the interior support in a structure with only two continuous spans is higher by approximately 10 percent. Under limitation b), when the ratio of the two spans (long span/short span) of a panel exceeds 2, the slab resists the moment in the shorter span essentially as a one-way slab.

Limitation c) relates to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as discussed in a subsequent section. Limitation d) permits columns to be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit when using the DDM.

Limitation e) is included because the DDM is based on tests (Jirsa et al. 1969) for uniform gravity loads and resulting column reactions determined by statics. Lateral loads such

Table 3.2.1.2—Distribution coefficient of end span total factored static moment, M_o

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

as wind or seismic require a frame analysis. Inverted foundation mats analyzed as two-way slabs involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis should be performed. The limit of applicability of the DDM for ratios of live-to-dead load was reduced in ACI 318-95 from 3 to 2. Because in most slab systems the live-to-dead load ratio is less than 2, it is unnecessary to check the effects of pattern loading. For a slab system supporting a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer need not consider the live load limitation e) because there is no need to consider pattern loading.

The elastic distribution of moments will deviate significantly from those assumed in the DDM unless the stiffness under limitation f) is met. Limitation g) forbids the use of moment redistribution as permitted by ACI 318-14, Section 6.6.5, which is not intended for use where approximate values for bending moments are used. For the DDM, 10 percent modification is allowed as described in 3.2.2.5 of this guide.

Limitation h) permits a designer to use the DDM even when the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure.

3.2.1.2 Distribution of total factored moments—As stated in 3.1.3, the clear span is based on distance between faces of supports. If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it should be treated as a square support having the same area, as illustrated in Fig. 3.1.3.

In an interior span, for M_o , calculated based on Eq. (3.1.3) should be distributed as follows:

a) Negative factored moment: 0.65

b) Positive factored moment: 0.35

The M_o , in an end span, should be distributed to slab midspan and supports as shown in Table 3.2.1.2.

The moment coefficients for an end span are based on the equivalent column stiffness expressions (Jirsa et al. 1963; Corley and Jirsa 1970). The coefficients for

Table 3.2.1.3a—Distribution in percent of interior negative factored moment to column strips

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Note: Linear interpolations can be made between values shown.

an unrestrained edge would be used, for example, if the slab is simply supported on a masonry or concrete wall. Those for a fully restrained edge would apply if the slab is constructed integrally with a concrete wall having a flexural stiffness so large compared to that of the slab that little rotation occurs at the slab-to-wall connection. Note that Table 3.2.1.2 is a major simplification of values that is computed on the basis of relative stiffnesses (ACI 318-77) and that, in some cases, these moments could deviate substantially larger from theoretic values. This is especially true for the exterior negative moment case because there are no limits on column stiffness other than pinned or fixed.

For other than unrestrained or fully restrained edges, coefficients in Table 3.2.1.2 were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment strength for most slab systems is governed by spacing of minimum reinforcement to control cracking. The final coefficients in the table have been adjusted so that the absolute sum of the positive and average negative moments equal M_o .

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of Column (2) of Table 3.2.1.2 apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of Columns (3) or (4) apply, without or with an edge (spandrel) beam, respectively. Design should be based on the larger of the two interior negative factored moments on either side of a common column or other type of support. If an analysis is made to distribute unbalanced moments, flexural stiffness could be obtained on the basis of the gross concrete section of the members involved.

The gravity load moment transferred by eccentricity within the critical section for shear between slab and edge column should be $0.3M_o$. Proportion edge beams or edges of slab to resist exterior negative factored moments. Torsional stresses caused by the moment assigned to the slab should be investigated.

3.2.1.3 Factored moments in column strips—The rules for assigning moments to the column strips, beams, and middle strips are based on studies (Gamble 1972) of moments in linearly elastic slabs with different beam stiffness tempered by the moment coefficients that have been used successfully. Column strips should be proportioned to resist the portions in percent tabulated in Table 3.2.1.3a for interior negative factored moments, and in Table 3.2.1.3b for exterior negative factored moments.

In Table 3.2.1.3b, linear interpolations can be made between values shown, where β_t is calculated in Eq. (3.2.1.3a) and C is calculated in Eq. (3.2.1.3b).

Table 3.2.1.3b—Distribution in percent of exterior negative factored moment to column strips

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Table 3.2.1.3c—Distribution in percent of positive factored moment to column strips

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Note: Linear interpolations can be made between values shown.

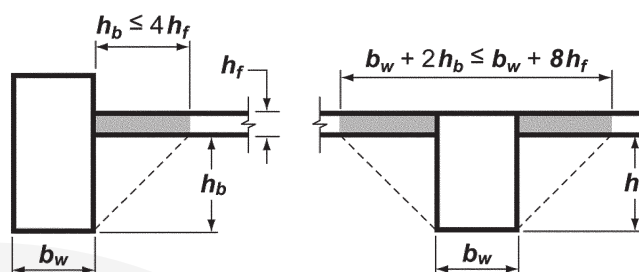


Fig. 3.2.1.3—Example of portion of slab to be included with beam.

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \quad (3.2.1.3a)$$

$$C = \sum \left(1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3} \quad (3.2.1.3b)$$

The constant C for T- or L-sections can be evaluated by dividing the section into separate rectangular parts, as defined in Fig. 3.2.1.3, and summing the values of C for each part.

The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of β_t , the shear modulus has been taken as $E_{cb}/2$. Where walls are used as supports along column lines, they can be regarded as very stiff beams with a $\alpha_1\ell_2/\ell_1$ value greater than 1. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined, β_t may be taken as zero if the wall is of masonry without torsional resistance, and β_t may be taken as 2.5 for a concrete wall with large torsional resistance that is monolithic with the slab.

For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall, ℓ_n in Eq. (3.1.3) may be assumed equal to ℓ_n of the parallel adjacent column-to-column span, and the wall may be considered as a beam having a moment of inertia, I_b , equal to infinity. Where supports consist of columns or walls extending for a distance equal to or greater than $2.4\ell_2$ used to compute

Table 3.2.1.6—Distribution in percent of factored moment in beams

$(\alpha_1 \ell_2 / \ell_1) \geq 1.0$	85
$(\alpha_1 \ell_2 / \ell_1) = 0$	0

Note: Linear interpolations can be made between values shown.

M_o , negative moments should be considered to be uniformly distributed across ℓ_2 .

Column strips should be proportioned to resist the portions in percent of positive factored moments shown in Table 3.2.1.3c.

3.2.1.4 Factored moments in middle strips—That portion of the negative and positive factored moments not resisted by column strips should be proportionately assigned to corresponding half middle strips. Each middle strip should be proportioned to resist the sum of the moments assigned to its two half middle strips. A middle strip adjacent to and parallel with a wall-supported edge should be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

3.2.1.5 Modification of factored moments—A modification of 10 percent in negative and positive factored moments, calculated in accordance with 3.2.2.2, is permitted provided the total static moment for a panel in the direction considered is not less than M_o required by Eq. (3.1.3). For example, a 10 percent reduction of negative factored moment should follow with a corresponding increase of positive factored moment. This is intended to recognize a limited amount of inelastic behavior and moment redistribution can occur in slabs that were analyzed with the DDM.

3.2.1.6 Factored moments in beams—Factored moments in beams between supports are proportioned in percent as shown in Table 3.2.1.6.

Loads assigned directly to beams are in addition to the uniform dead load of the slab; uniform superimposed dead loads, such as the ceiling, floor finish, or assumed equivalent partition loads; and uniform live loads. All of these loads are normally included with q_u in Eq. (3.1.3). Linear loads applied directly to beams include partition walls over or along beam centerlines and additional dead load of the projecting beam stem. Concentrated loads include posts above or hangers below the beams or induced force from seismic overturning. For the purpose of assigning directly applied loads, only loads located within the width of the beam stem should be considered as directly applied to the beams. The effective width of a beam as defined in Fig. 3.2.1.3 is solely for strength and relative stiffness calculations. Line loads and concentrated loads located on the slab away from the beam stem require special consideration to determine their apportionment to slab and beams. For slabs with beams between supports, the slab portion of column strips should be proportioned to resist that portion of the column strip moments not resisted by beams.

3.2.1.7 Factored shear in slab systems with beams—Resistance to total shear occurring on a panel based on computation of slab shear strength on the assumption that loads are distributed to supporting beams is permitted. Shear

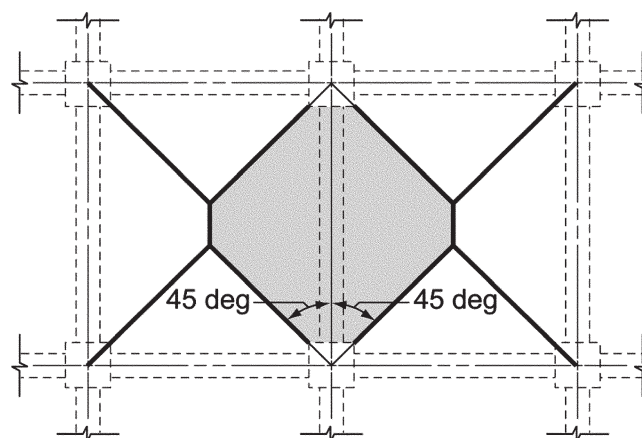


Fig. 3.2.1.7—Tributary area for shear on an interior beam.

strength for beams should satisfy ACI 318-14, Sections 9.5.3 and 22.5.

Beams with $\alpha_1 \ell_2 / \ell_1$ equal to or greater than 1.0 are proportioned to resist shear caused by factored loads on tributary areas that are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides. The tributary area for computing shear on an interior beam is shown shaded in Fig. 3.2.1.7.

If the stiffness for the beam, $\alpha_1 \ell_2 / \ell_1$, is less than 1.0, the shear on the beam can be obtained by linear interpolation, assuming beams resist no load at $\alpha_1 = 0$. In such cases, the beams framing into the column will not account for all of the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column, which should be checked in the same manner as for flat slabs.

In addition to shears calculated from slab tributary areas, beams should be proportioned to resist shear caused by factored loads applied directly on beams. Note that the proportional shear in slab systems with beams described in this section do not apply to calculation of torsional moments on the beams. Torsional moments should be based on the calculated flexural moments acting on the sides of the beam.

3.2.1.8 Factored moments in columns and walls—Columns and walls built integrally with a slab system should resist moments caused by factored loads on the slab system. At an interior support, columns or walls above and below the slab should resist the factored moment specified by Eq. (3.2.1.8) in direct proportion to their stiffnesses unless a general analysis is made.

$$M_u = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q'_{Du}\ell_2'(\ell_n')^2] \quad (3.2.1.8)$$

where q'_{Du} , ℓ_2' , and ℓ_n' refer to the shorter span.

Equation (3.2.1.8) refers to two adjoining spans, with one span longer than the other, and with full dead load plus one-half live load applied on the longer span with only dead load applied on the shorter span. Design and detailing of the reinforcement transferring the moment from the slab to edge column is critical to both performance and safety of flat

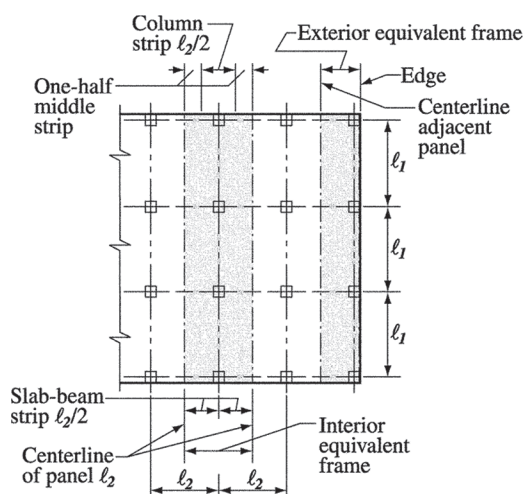


Fig. 3.2.2—Definitions of equivalent frame.

slabs or flat plates without edge beams, or cantilever slabs. Complete details are shown on construction documents, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

3.2.2 Equivalent Frame Method (EFM)—The EFM involves the representation of a three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in plane of frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with 3.2.1.3 (column strips), 3.2.1.4 (middle strips), and 3.2.1.6 (beams). The EFM is based on studies by Corley et al. (1961), Jirsa et al. (1963), and Corley and Jirsa (1970). Unlike the DDM, the EFM can be used for slab systems with irregular column layouts and pattern loading with or without lateral forces.

The EFM was developed when the primary structural analysis tool was the moment-distribution method. Specialized computer programs have been developed for this analysis, and give a way of using ordinary plane-frame structural analysis programs for this task (Schaeffer 1999).

Application of the equivalent frame to a regular structure is illustrated in Fig. 3.2.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) oriented along column or support centerlines, with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

The equivalent frame comprises three parts:

- 1) The horizontal slab strip, including any beams spanning in the direction of the frame
- 2) The columns or other vertical supporting members, extending above and below the slab
- 3) The elements of the structure that provide moment transfer between the horizontal and vertical members.

Where metal column capitals are used, it is permitted to consider their contributions to stiffness and resistance to moment and shear. Change in column and slab length due to direct stress and deflections due to shear can be neglected.

3.2.2.1 Analysis and design considerations—All sections of slabs and supporting members analyzed by EFM should be proportioned for moments and shears obtained:

a) The structure is considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building

b) Each frame consists of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports

c) Columns or supports are assumed to be attached to slab-beam strips by torsional members (3.2.2.4) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column

d) Frames adjacent and parallel to an edge are bounded by that edge and the centerlines of adjacent panels

e) Analysis of each equivalent frame in its entirety is permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed is also permitted

f) Where slab-beams are analyzed separately, determination of the moment at a given support, assuming that the slab-beam is fixed at any support two panels distant therefrom, is permitted, provided the slab continues beyond the points of fixity.

The simplifications permitted under e) and f) were concessions to manual calculation procedures such as moment distribution.

3.2.2.2 Slab-beams—In a slab-beam system, a support is defined as a column, capital, bracket, or wall; a beam is not considered to be a support member for the equivalent frame. Determining the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete is permitted. Consider variation in moment of inertia along the axis of slab-beams. The moment of inertia of slab-beams from the center column to the face of the column, bracket, or capital is assumed equal to the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/\ell_2)^2$, where c_2 and ℓ_2 are measured transverse to the direction of the span for which moments are being determined.

3.2.2.3 Columns—Column stiffness is based on column length from middepth of the slab above to middepth of the slab below. The column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any. Consider variation in moment of inertia along the axis of columns. Moment of inertia of columns from top to bottom of the slab-beam at a joint can be assumed to be infinite.

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness of the slab-beam and torsional member into a composite element, is used. The column is modified to account for the torsional flexibility of the slab-beam.