

ACI 423.10R-16

Guide to Estimating Prestress Loss

Reported by Joint ACI-ASCE Committee 423



American Concrete Institute
Always advancing



Documents

Document Sharing Hub



Guide to Estimating Prestress Loss

Copyright by the American Concrete Institute, Farmington Hills, MI. All rights reserved. This material may not be reproduced or copied, in whole or part, in any printed, mechanical, electronic, film, or other distribution and storage media, without the written consent of ACI.

The technical committees responsible for ACI committee reports and standards strive to avoid ambiguities, omissions, and errors in these documents. In spite of these efforts, the users of ACI documents occasionally find information or requirements that may be subject to more than one interpretation or may be incomplete or incorrect. Users who have suggestions for the improvement of ACI documents are requested to contact ACI via the errata website at <http://concrete.org/Publications/DocumentErrata.aspx>. Proper use of this document includes periodically checking for errata for the most up-to-date revisions.

ACI committee documents are intended for the use of individuals who are competent to evaluate the significance and limitations of its content and recommendations and who will accept responsibility for the application of the material it contains. Individuals who use this publication in any way assume all risk and accept total responsibility for the application and use of this information.

All information in this publication is provided “as is” without warranty of any kind, either express or implied, including but not limited to, the implied warranties of merchantability, fitness for a particular purpose or non-infringement.

ACI and its members disclaim liability for damages of any kind, including any special, indirect, incidental, or consequential damages, including without limitation, lost revenues or lost profits, which may result from the use of this publication.

It is the responsibility of the user of this document to establish health and safety practices appropriate to the specific circumstances involved with its use. ACI does not make any representations with regard to health and safety issues and the use of this document. The user must determine the applicability of all regulatory limitations before applying the document and must comply with all applicable laws and regulations, including but not limited to, United States Occupational Safety and Health Administration (OSHA) health and safety standards.

Participation by governmental representatives in the work of the American Concrete Institute and in the development of Institute standards does not constitute governmental endorsement of ACI or the standards that it develops.

Order information: ACI documents are available in print, by download, on CD-ROM, through electronic subscription, or reprint and may be obtained by contacting ACI.

Most ACI standards and committee reports are gathered together in the annually revised ACI Manual of Concrete Practice (MCP).

American Concrete Institute
38800 Country Club Drive
Farmington Hills, MI 48331
Phone: +1.248.848.3700
Fax: +1.248.848.3701

www.concrete.org

Guide to Estimating Prestress Losses

Reported by Joint ACI-ASCE Committee 423

Carin L. Roberts-Wollmann*, Chair

Amy M. Reineke Trygestad, Secretary

Theresa M. Ahlborn*
Robert W. Barnes†
Florian G. Barth
Asit N. Baxi†
Roger J. Becker
Kenneth B. Bondy†
Charles W. Dolan†
James P. Donnelly
Pierre Esselinck†

Martin J. Fradua
William L. Gamble
Harry A. Gleich†
Shawn P. Gross
Pawan R. Gupta
William M. Hale
H. R. Trey Hamilton III†
Carol Hayek
Mohammad Iqbal

Donald P. Kline
Larry B. Krauser
Jason J. Krohn†
Mark E. Moore‡
Theodore L. Neff
Sami H. Rizkalla
James Rogers
Bruce W. Russell
David H. Sanders

Thomas C. Schaeffer
Morris Schupack‡
Richard W. Stone
Miroslav F. Vejvoda†
Jeffery S. Volz
H. Carl Walker
Zuming Xia†
Paul Zia*

*Co-chairs of subcommittee responsible for preparation of report.

†Subcommittee members.

‡Deceased

Consulting Members

Robert N. Bruce Jr.
Ned H. Burns
Chunsheng “Steve” Cai

Steven R. Close
Henry J. Cronin Jr.
Ward N. Marianos Jr.

Hani Melhem
Antoine E. Naaman
Thomas E. Nehil

Andrea J. Schokker

This guide is intended for estimation of prestress losses in concrete structures. Methods presented include lump sum, simplified approaches addressing individual source of loss, and additional estimation methods. They address losses in pretensioned and post-tensioned members, including bonded, unbonded, and external tendons. Note that these estimation methods have not been evaluated for relative merits. A discussion of the variability of prestress losses caused by the variability in concrete properties is also presented. Several example problems are included.

Keywords: creep; friction; post-tensioning; prestress loss; prestressed concrete; relaxation; shrinkage.

ACI Committee Reports, Guides, and Commentaries are intended for guidance in planning, designing, executing, and inspecting construction. This document is intended for the use of individuals who are competent to evaluate the significance and limitations of its content and recommendations and who will accept responsibility for the application of the material it contains. The American Concrete Institute disclaims any and all responsibility for the stated principles. The Institute shall not be liable for any loss or damage arising therefrom.

Reference to this document shall not be made in contract documents. If items found in this document are desired by the Architect/Engineer to be a part of the contract documents, they shall be restated in mandatory language for incorporation by the Architect/Engineer.

CONTENTS

CHAPTER 1—INTRODUCTION, p. 2

- 1.1—Introduction, p. 2
- 1.2—Scope, p. 2
- 1.3—Historical development, p. 3
- 1.4—Guide organization and use, p. 3

CHAPTER 2—NOTATION AND DEFINITIONS, p. 4

- 2.1—Notation, p. 4
- 2.2—Definitions, p. 7

CHAPTER 3—LUMP-SUM METHOD, p. 7

- 3.1—Scope, p. 7
- 3.2—Historical code requirements, p. 7
- 3.3—Industry practice, p. 8
- 3.4—Measured losses, p. 8

ACI 423.10R-16 was adopted and published August 2016.

Copyright © 2016, American Concrete Institute

All rights reserved including rights of reproduction and use in any form or by any means, including the making of copies by any photo process, or by electronic or mechanical device, printed, written, or oral, or recording for sound or visual reproduction or for use in any knowledge or retrieval system or device, unless permission in writing is obtained from the copyright proprietor.

CHAPTER 4—INITIAL LOSSES, p. 12

- 4.1—Scope, p. 12
- 4.2—Pretensioning losses before transfer, p. 13
- 4.3—Elastic shortening losses in pretensioned members, p. 16
- 4.4—Post-tensioning losses during tensioning and transfer, p. 18
- 4.5—Elastic shortening loss in post-tensioned members, p. 21
- 4.6—Elastic gain under superimposed loads, p. 22

CHAPTER 5—LONG-TERM LOSSES: SIMPLIFIED METHOD, p. 22

- 5.1—Scope, p. 22
- 5.2—Creep of concrete (Δf_{pCR}), p. 23
- 5.3—Concrete shrinkage (Δf_{pSH}), p. 23
- 5.5—AASHTO LRFD approximate estimate of time-dependent losses, p. 25

CHAPTER 6—LONG-TERM LOSSES: DETAILED METHODS, p. 25

- 6.1—Scope, p. 25
- 6.2—Creep and shrinkage models, p. 25
- 6.3—Age-adjusted effective modulus approaches, p. 26
- 6.4—Incremental time-step method, p. 30
- 6.5—Computer programs, p. 31
- 6.6—Effects of deck temperature during casting of composite deck or topping, p. 31

CHAPTER 7—VARIABILITY OF LOSS CALCULATIONS, p. 32

- 7.1—Objective, p. 32
- 7.2—Scope, p. 32
- 7.3—Contributions to prestress loss, p. 32
- 7.4—Modulus of elasticity, p. 33
- 7.5—Creep, p. 35
- 7.6—Variational analysis, p. 35
- 7.7—Shrinkage case study, p. 36
- 7.8—Self-consolidating concrete, p. 36
- 7.9—Conclusions, p. 37

CHAPTER 8—EXAMPLES, p. 38

- 8.1—Pretensioned double-tee beam, p. 38
- 8.2—Post-tensioned slab with unbonded tendons, p. 48
- 8.3—Post-tensioned beam with bonded tendons, p. 52
- 8.4—Example with heat of hydration during casting, p. 57

CHAPTER 9—REFERENCES, p. 61

- Authored documents, p. 61

CHAPTER 1—INTRODUCTION**1.1—Introduction**

Estimating prestress loss at any given time during the life of a prestressed concrete member is a complex issue. In pretensioned and post-tensioned members, applying prestressing force causes shortening of the concrete member that, in turn, causes a loss of tendon stress. Over time, concrete creep,

concrete shrinkage, and steel relaxation result in additional reductions of tendon stress. In post-tensioned members, losses occur during the stressing operation due to friction between the tendon and sheathing or duct, which is caused by the intended and unintended tendon curvature. There are also losses due to seating of the wedges or nuts as the jacking force is transferred into the anchorage device. These and other sources of prestress loss are examined by the licensed design professional to get an estimate of the total prestress loss and resulting effective prestressing force.

Losses have inherent variability due to variations of material properties and environmental and curing conditions. Some losses may affect others. Time-dependent concrete properties are particularly difficult to estimate accurately, so losses due to creep and shrinkage are expected to be variable. Friction between the tendon and sheathing or duct, movement of wedges within the anchorage device, and modulus of elasticity of concrete are also variables. The variability within each component and the interdependence among the components make it understandable that studies comparing measured prestress losses to predictions have shown that accurate and consistent calculation of prestress loss is difficult to achieve.

The best effort to calculate prestress loss is only an estimate and, therefore, the licensed design professional should consider the consequences of actual losses being higher or lower than the estimated value. Estimation of prestress loss is an important factor for evaluating the serviceability of all types of prestressed members and the calculation of flexural strength of members with unbonded tendons. The estimation of prestress loss, however, is not a significant factor in determination of flexural strength of bonded prestressed members. When computing the shear strength of prestressed members with little or no transverse reinforcement, a conservative estimate of the effective prestressing force is warranted.

1.2—Scope

ACI 318-11 requires that the design of prestressed concrete members allow for prestress loss; however, the required level of detail for calculating losses is unspecified. The friction loss provisions for post-tensioned construction that first appeared in ACI 318-63 were removed from ACI 318-11. Although ACI 318-11 Commentary indicates that the lump sum method is obsolete, the licensed design professional's requirement to choose a method to compute losses remains. This guide is intended to aid the designer in this choice by providing an overview of the various methods available.

Many participants in the design and construction process need information on prestress losses. The licensed design professional, precasters, and post-tensioners all need an understanding of, and method to estimate, aspects of losses. To which entity is responsible for calculation of each type of loss has to be clearly defined in the contract documents.

Total losses, Δf_{pT} , are losses due to friction and seating Δf_{pFS} , elastic shortening Δf_{pES} , creep of concrete Δf_{pCR} , shrinkage of concrete Δf_{pSH} , and relaxation of tendons Δf_{pRE} . This can be expressed as Eq. (1.2).

$$\Delta f_{pT} = \Delta f_{pFS} + \Delta f_{pES} + \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_{pRE} \quad (1.2)$$

This guide presents background information and methods to calculate each type of loss.

Following the introduction and a list of notation and definitions, [Chapter 3](#) includes a historical account of the lump sum method, currently recommended values for preliminary design, and a summary of losses that have been measured in field and laboratory studies.

[Chapter 4](#) discusses the different types of initial losses and addresses the differences between pretensioned and post-tensioned members.

[Chapter 5](#) presents a simplified approach to estimate long-term losses due to creep, shrinkage, and relaxation for pretensioned and post-tensioned concrete members.

Detailed approaches to estimate long-term losses are presented in [Chapter 6](#), which also addresses changes in prestressing force caused by differential shrinkage and hydration of the concrete deck in composite members. The approaches can be used for pretensioned or post-tensioned members.

[Chapter 7](#) discusses the variability of prestress loss calculations caused by concrete material properties, including compressive strength at transfer, modulus of elasticity, and creep and shrinkage.

[Chapter 8](#) presents example problems and compares solutions from different methods.

1.3—Historical development

The concept of prestressing concrete dates back to the late 1800s ([Naaman 2012](#)). The performance of early prestressed concrete structures was adversely affected by time-dependent strains in the concrete—for example, creep and shrinkage, which were nearly as large as the initial steel strain due to prestressing. Before 1940, the initial steel strain induced by prestressing was limited by the low yield strength of steel. French engineer Eugene Freyssinet recognized the significance of prestress losses and the need for steels with high yield strength for prestressed applications. By 1945, higher strength steel became available, making it possible to produce the initial prestressing strain large enough so that the time-dependent strains developed in the concrete would not overcome the initial prestressing strain. As a result, the remaining prestressing force in the steel would be sufficiently large to be effective.

Prestress losses were first addressed by ACI 318 in 1963. Although the provisions catalogued the different causes of prestress loss, they only provided specific instruction on determining friction losses. These code provisions were based on an earlier committee publication that provided similar, slightly more detailed guidance on prestress loss ([ACI-ASCE Committee 323 1958](#)).

In the 1970s, the Precast/Prestressed Concrete Institute ([PCI Committee on Prestress Losses 1975](#)) and [Zia et al. \(1979\)](#) provided more detailed methods to estimate prestress losses. Since the 1970s, others have developed methods to estimate prestress losses ([Tadros et al. 2003](#); [Seguirant and Anderson 1985](#); [Youakim et al. 2007](#); [Garber et al. 2013](#)).

[Gilbert and Ranzi \(2011\)](#) and [Branson \(1977\)](#) provide general approaches to the calculation of a variety of time-dependent effects in concrete structures, including prestress losses. Computer programs have been developed to perform the tedious calculations required for stepwise analyses of prestress loss. However, due to the inherent uncertainties associated with material properties, construction practices, and in-service conditions, even the most refined calculations result in prestress loss predictions that differ from measured values.

1.3.1 Currently available guidance on estimating prestress losses—For pretensioned building products, the *PCI Design Handbook* ([PCI 2010](#)) presents a method to estimate prestress losses based on the method developed by Zia et al. (1979). This method is widely used for building structures and is referenced in the R18.6.1 commentary of [ACI 318-11](#), and presented in this guide in Chapter 5. For bridge beams, the “AASHTO LRFD Bridge Design Specification” ([AASHTO 2012](#)) presents two methods. One is an approximate method and the other a refined method based on several parameters to estimate prestress losses. The refined method could be applied to building products as well. These methods are presented in Chapters 5 and 6.

1.4—Guide organization and use

This guide presents a variety of approaches for estimating prestress losses in pretensioned and post-tensioned members. This section identifies relevant sections of interest in the guide, depending on member type (pretensioned or post-tensioned) and level of effort (lump sum, simplified, or detailed). The lump sum method is only recommended for preliminary designs. The simplified method is appropriate for most typical designs. Detailed methods are most often used for more complex structures, which may have staged construction and prestressing operations.

1.4.1 Pretensioned members—Losses for pretensioned members are classified as initial or long-term. One group of initial loss occurs during stressing and before transfer of prestress due to friction, seating losses, and temperature effects. It is the precaster’s responsibility to understand the magnitude of these losses and account for them to provide the specified strand stress before transfer. Information on these types of losses is found in:

- (a) Anchorage seating—[4.2.1](#)
- (b) Form and abutment deformations—[4.2.2](#)
- (c) Thermal effects—[4.2.4](#)
- (d) Steel relaxation—[4.2.5](#)

Another initial loss is elastic shortening of the member that occurs at the time of transfer. As the prestress force is transferred to concrete, the member shortens. The steel and concrete are fully bonded, so the steel shortens with the concrete. This shortening causes a loss in stress in the prestressing steel, known as the elastic shortening loss, which should be accounted for by the designer. Long-term losses occur due to concrete creep and shrinkage and prestressing steel relaxation. Other changes of tendon force can occur due to temperature effects and external loads on the member at the time of casting and in service.

1.4.1.1 Pretensioned members/lump sum method—The lump sum method presented in [Chapter 3](#) is often used for preliminary design. The values presented in sources referenced in Chapter 3 typically include all losses, both initial and long-term.

1.4.1.2 Pretensioned members/simplified method—The simplified method is a commonly employed approach to estimate prestress losses in typical pretensioned members. The designer needs to calculate four components of loss and add them together for the total prestress loss. Components and applicable sections are:

- (a) Elastic shortening—[4.3.2](#)
- (b) Creep—[5.2](#)
- (c) Shrinkage—[5.3](#)
- (d) Relaxation—[5.4](#)

1.4.1.3 Pretensioned members/detailed method—This guide provides information on more detailed methods of prestress loss estimation. Two alternate methods for a more detailed calculation of elastic shortening losses in pretensioned members are:

- (a) Transformed section method—[4.3.1](#)
- (b) Iterative gross section method with iteration—[4.3.3](#)

More detailed approaches to calculate long-term losses are presented in [Chapter 6](#). These methods are used with a variety of creep and shrinkage models, as opposed to the simplified method, which uses a single model. Detailed methods also allow the designer to consider the influence of a cast-in-place composite deck if needed, whereas the simplified method only accounts for the weight of the deck, but not other factors such as differential shrinkage and internal stress redistributions between the beam and the deck, if acting compositely. The detailed methods are:

- (a) AASHTO LRFD refined method (AASHTO 2012)—[6.3.2](#)
- (b) General age-adjusted effective modulus method (Menn 1990)—[6.3.3](#)
- (c) Incremental time-step method ([Nilson 1987](#))—[6.4](#)

Chapter 6 ([6.6](#)) also provides information on the approximation of changes due to thermal effects of deck casting.

1.4.2 Post-tensioned members—Several approaches can be used to approximate prestress losses in post-tensioned members. Initial losses encompass all prestress loss during the stressing operation, including friction due to wobble and curvature, seating losses, and elastic shortening losses. The estimation of long-term losses for bonded post-tensioned members is essentially the same as for pretensioned members. Calculation of long-term losses in unbonded post-tensioned members is different, because losses are related to the overall change in tendon length, rather than the change in strain at a specific section.

1.4.2.1 Post-tensioned members/lump sum method—The lump sum method, presented in [Chapter 3](#), is typically used only for preliminary designs. Before adopting a value for use in preliminary design, the licensed design professional should determine if the presented value includes friction and seating losses.

1.4.2.2 Post-tensioned members/simplified method—The simplified method can be used to estimate prestress losses in

typical post-tensioned members. The designer needs to calculate five components of loss and add them together for the total prestress loss. Components and applicable sections are:

- (a) Friction and seating loss—[4.4](#)
- (b) Elastic shortening loss—[4.5](#)
- (c) Creep loss—[5.2.1](#) (bonded)
- (d) Creep loss—[5.2.2](#) (unbonded)
- (e) Shrinkage loss—[5.3](#)
- (f) Relaxation loss—[5.4](#)

Note that elastic shortening losses only occur in post-tensioned members with multiple tendons when the tendons are stressed sequentially. Tendons stressed first will incur losses as the concrete shortens due to the stressing of subsequent tendons.

1.4.2.3 Post-tensioned members/detailed methods—As with pretensioned members, long-term prestress loss in post-tensioned members are estimated using more detailed methods presented in Chapter 6, with a detailed description in 1.4.1.3. Initial losses are calculated per [4.3.3](#) and [4.4](#).

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

a	= constant
ab	= eccentricity from the centroid of the beam (gross section) to the centroid of the deck (also centroid of the deck reinforcing steel), in. (mm)
A_c	= area of concrete, in. ² (mm ²)
A_{comp}	= transformed area of the composite section, in. ² (mm ²)
A_d	= area of composite concrete deck, in. ² (mm ²)
A_g	= area of gross concrete section at the cross section considered, in. ² (mm ²)
A_{ps}	= area of prestressing steel, in. ² (mm ²)
A_{sd}	= area of deck steel, in. ² (mm ²)
A_{tr}	= transformed cross-sectional area, in. ² (mm ²)
$b(y)$	= width of cross-section at depth y relative to centroid of section, in. (mm)
C	= factor in calculation of prestress loss due to relaxation according to the <i>PCI Design Handbook</i> (PCI 2010) method
C_c	= creep coefficient
d	= friction loss over length L , psi (MPa)
e	= base of Napierian logarithms
e_b	= basic elongation, in. (mm)
e_c	= eccentricity of centroid of tendons with respect to the centroid of the gross cross section at the center of the beam, in. (mm)
e_d	= distance between centroid of deck and centroid of composite beam, in. (mm)
e_e	= eccentricity of centroid of tendons with respect to the centroid of the gross cross section at the ends of the beam, in. (mm)
e_p	= eccentricity of centroid of tendons with respect to the centroid of the gross concrete at the cross section considered, in. (mm)
e_{pc}	= eccentricity of centroid of tendons with respect to the centroid of the composite cross section at the ends of the beam, in. (mm)

e_{tr}	= eccentricity of centroid of tendons with respect to the centroid of the f_{bpt} transformed concrete at the cross section considered, in. (mm)	I_c	= moment of inertia of the composite cross section, in. ⁴ (mm ⁴)
$E(t)$	= modulus of elasticity at any time t , psi (MPa)	I_d	= moment of inertia of the deck, in. ⁴ (mm ⁴)
E_c	= modulus of elasticity of concrete, psi (MPa)	I_g	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. ⁴ (mm ⁴)
E'_c	= effective modulus of elasticity of concrete, psi (MPa)	I_{tr}	= moment of inertia of transformed concrete section about centroidal axis, including reinforcement, in. ⁴ (mm ⁴)
E''_c	= age-adjusted effective modulus of elasticity of concrete, psi (MPa)	J	= factor in calculation of prestress loss due to relaxation according to the <i>PCI Design Handbook</i> (PCI 2010) method
E_{cd}	= modulus of elasticity of the composite deck, psi (MPa)	k	= wobble friction coefficient per unit length of tendon, per ft (per m)
E_{ci}	= modulus of elasticity of concrete at time of application of prestress, psi (MPa)	k_f	= factor for the effect of concrete strength
$E_c(t_i)$	= modulus of elasticity of concrete at time t_i , psi (MPa)	k_{hc}	= humidity factor for creep
E_p	= modulus of elasticity of the prestressing steel, psi (MPa)	k_s	= factor for the effect of volume-to-surface ratio
f_{anchor}	= strand stress at the anchorage device after seating, psi (MPa)	k_{nd}	= time development factor
f_c	= concrete compressive stress, psi (MPa)	K_{cir}	= modification factor in <i>PCI Design Handbook</i> (PCI 2010) method in calculation of concrete stress due to prestressing force immediately after the prestress has been applied to the concrete
f'_c	= specified compressive strength of concrete, psi (MPa)	K_{cr}	= coefficient in the <i>PCI Design Handbook</i> (PCI 2010) method to account for loss due to creep
f_{cd}	= concrete stress at center of gravity of prestressing force due to all superimposed permanent loads that are applied to the member after it has been prestressed, psi (MPa)	K_{df}	= transformed section coefficient
f'_{ci}	= specified compressive strength of concrete at transfer of prestress, psi (MPa)	K_{es}	= factor in calculation of elastic shortening losses in <i>Zia et al. (1979)</i> method
f_{ci}	= concrete compressive stress immediately after transfer at fiber under investigation, psi (MPa)	K_{id}	= section modification factor from <i>AASHTO (2012)</i> prestress loss method
f_{cir}	= net compressive concrete stress at center of gravity of prestressing force immediately after the prestress has been applied to the concrete, psi (MPa)	K_{re}	= factor in calculation of prestress loss due to relaxation in <i>PCI Design Handbook</i> (PCI 2010) method
f_{cpa}	= average compressive concrete stress at the center of gravity of the tendons immediately after the prestress has been applied to the concrete, psi (MPa)	K_{sh}	= factor in calculation of prestress losses due to shrinkage in <i>PCI Design Handbook</i> (PCI 2010) method
f_{cps}	= concrete stress at center of gravity of prestressing force due to all prestress and applied loads, psi (MPa)	L	= strand length from anchorage to anchorage, ft or in. (m or mm)
f_{dead}	= tendon stress at nonstressing end, psi (MPa)	L_{beam}	= length of beam, ft or in. (m or mm)
f_{jack}	= jacking stress, psi (MPa)	L_{free}	= length of strand outside of beam, ft or in. (m or mm)
f_L	= stress in prestressing steel at a distance L from jacking end, psi (MPa)	M	= bending moment experienced by cross section immediately after transfer (usually due to self-weight), in.-lb (N-mm)
$f_{L/2}$	= stress in prestressing steel at a distance $L/2$ from the jacking end, psi (MPa)	M_b^o	= initial creep-producing moment in the girder, in.-lb (N-mm)
f_{max}	= maximum stress in the prestressing steel along the tendon length, psi (MPa)	M_{deck}	= moment in beam due to the weight of the deck, in.-lb (N-mm)
f_{pbt}	= stress in prestressing steel immediately before transfer, psi (MPa)	M_g	= bending moment due to dead weight of prestressed member and any other permanent loads in place at the time of prestressing, in.-lb (N-mm)
f_{pi}	= prestressing steel stress immediately following transfer, psi (MPa)	M_{sd}	= moment due to all superimposed permanent loads applied after prestressing, in.-lb (N-mm)
f_{po}	= prestressing steel stress after jacking and seating, psi (MPa)	n_p	= modular ratio; modulus of prestressing steel divided by modulus of concrete
$f_{ps}(t)$	= stress in prestressing steel at time t , psi (MPa)	N	= number of sequentially stressed tendons
f_{pt}	= stress in prestressing steel immediately after transfer, psi (MPa)	N_b^o	= initial creep-producing force in the girder, lb (N)
f_{pu}	= specified tensile strength of prestressing steel, psi (MPa)	P	= applied tension force, lb (N)
f_{py}	= specified yield strength of prestressing steel, psi (MPa)	P_{avg}	= average force in the tendon, lb (N)
f_x	= stress in prestressing steel at a distance x from the jacking end, psi (MPa)	P_i	= initial prestress force after anchorage losses, lb (N)

P_j	= maximum prestress force during jacking operation, lb (N)	Δf_{pCR1}	= prestress loss due to creep of girder concrete between transfer and deck placement, psi (MPa); (AASHTO 2012)
P_o	= prestress force before release, lb (N)	Δf_{pEG}	= increase in prestress (elastic gain) due to addition of superimposed permanent loads, psi (MPa)
P_{shd}	= force to fully restrain the shrinkage of the composite deck, lb (N)	Δf_{pES}	= prestress loss due to elastic shortening, psi (MPa)
RH	= average ambient relative humidity in percent	Δf_{pFS}	= prestress loss due to friction and seating, psi (MPa)
s	= slope of stress in prestress versus distance line, psi/ft (MPa/m)	Δf_{pLT}	= long-term prestress loss, psi (MPa)
S_b	= section modulus with respect to the bottom fiber of the beam, in. ³ (mm ³)	Δf_{pR1}	= prestress loss due to relaxation of prestressing strands between time of transfer and deck placement, psi (MPa)
t	= time under consideration from time of release, days	Δf_{pR2}	= prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time, psi (MPa)
t_c	= age of concrete, days	Δf_{pRE}	= prestress loss due to relaxation, psi (MPa)
t_d	= time since end of cure to time of deck placement, days	Δf_{pSD}	= prestress loss due to shrinkage of girder concrete between time of deck placement and final time, psi (MPa)
t_f	= final time under consideration from time of release, days	Δf_{pSH}	= prestress loss due to shrinkage, psi (MPa)
t_i	= 1-day steam cured	Δf_{pSR}	= prestress loss due to shrinkage of girder concrete between transfer and deck placement, psi (MPa)
t_o	= time of initial loading from time of release, days	Δf_{pSS}	= prestress gain due to shrinkage of deck in composite section, psi (MPa)
T_0	= prestressing force at stressing end, lb (N)	Δf_{pT}	= total prestress loss, psi (MPa)
T_x	= prestressing force at point x, lb (N)	ΔL_{jack}	= elongation within the jack, in. (mm)
$T(y)$	= temperature of cross section at distance y from centroid, °F (°C)	ΔM_b	= change in moment in the beam, in.-lb (N-mm)
V/S	= ratio of volume to surface area of concrete element, in. ³ /in. ² (mm ³ /mm ²)	ΔM_d	= change in moment in the deck, in.-lb (N-mm)
w_c	= unit weight of normalweight concrete or equilibrium density of lightweight concrete, lb/ft ³ (kg/m ³)	ΔN_b	= change in force in the beam, lb (N)
w_t	= weight of topping, lb/ft ² (N/m ²)	ΔN_d	= change in force in the deck, lb (N)
x	= length of tendon from stressing end to point x, ft or in. (m or mm)	ΔN_{ps}	= change in prestress force, lb (N)
x_s	= length influenced by anchor set, ft or in. (m or mm)	ΔN_{relax}	= change in prestress force due to relaxation (no associated strain), lb (N)
y	= distance from centroid of cross section to location under consideration, in. (mm)	ΔN_{sd}	= change in the force in the deck reinforcement, lb (N)
y_{bott}	= the distance from the centroid of the composite section to the bottom of the section, in. (mm)	ΔP	= change in force, lb (N)
y_{tr}	= distance from centroid of transformed section to concrete fiber under investigation, in. (mm)	ΔP_c	= change in force in concrete, lb (N)
α	= total angular change from jacking end to point x, radians	ΔP_{ps}	= change in force in prestressing steel, lb (N)
α_c	= coefficient of thermal expansion of concrete, /°F (/°C)	Δs	= anchor set, in. (mm)
α_{ps}	= coefficient of thermal expansion of prestress, /°F (/°C)	ΔT_1	= temperature rise, °F (°C)
β	= constant such that $\beta = (28 - \alpha)/28$	ΔT_2	= temperature change, °F (°C)
γ_h	= correction factor for ambient relative humidity	$\Delta \epsilon_b$	= change in strain in the beam at the gross section centroid
γ_{st}	= correction factor for specified concrete compressive strength at transfer	$\Delta \epsilon_c$	= change in strain in concrete at the center of gravity of the prestressing force
Δ	= tendon elongation, in. (mm)	$\Delta \epsilon_{cr}$	= change in strain in concrete due to creep
Δf_{cdf}	= change in concrete stress at the center of gravity of the prestressing force due to the differential shrinkage force, psi (MPa)	$\Delta \epsilon_d$	= change in strain at the centroid of the deck
Δf_{cgp}	= change in concrete stress at the center of gravity of the prestressing force due to application of superimposed load, psi (MPa)	$\Delta \epsilon_p$	= change in strain in prestressing steel
Δf_{pA}	= change in stress due to anchor set, psi (MPa)	$\Delta \epsilon_{pA}$	= change in strain in prestressed reinforcement due to anchor set
Δf_{pCD}	= prestress loss due to creep of girder concrete between time of deck placement and final time, psi (MPa)	$\Delta \epsilon_{free}$	= change in strain in free length of prestressing steel
Δf_{pCR}	= prestress loss due to creep, psi (MPa)	$\Delta \epsilon_{shb}$	= change in strain in the beam due to shrinkage
		$\Delta \epsilon_{shb}(t_d)$	= shrinkage strain in the beam concrete at the time the deck is placed
		$\Delta \epsilon_{shb}(t_f)$	= shrinkage strain in the beam concrete at the final time considered
		$\Delta \epsilon_{sd}$	= change in strain in the deck steel
		$\Delta \kappa$	= change in curvature, /in. (/mm)
		$\Delta \sigma(t_i)$	= change in stress at time t_i , psi (MPa)

$\epsilon(t)$ = concrete strain at time t
 $\epsilon(t_o)$ = initial concrete strain
 $\epsilon_{shb}(t)$ = shrinkage strain in the prestressed beam at time t following the end of cure
 $\epsilon_{shb}(t_d)$ = shrinkage strain of the girder at the time the deck is placed
 $\epsilon_{shb}(t_f)$ = total shrinkage strain of the beam at the time of analysis
 $\epsilon_{shd}(t)$ = shrinkage strain in the composite deck at time t following the end of cure
 $\epsilon_{shd}(t_f)$ = shrinkage of the deck concrete at final time under consideration
 μ = curvature friction coefficient
 $\sigma(t_o)$ = stress at time t_o , psi (MPa)
 $\phi(t, t_i)$ = creep coefficient at time t for loads applied at time t_i
 $\phi(t, t_o)$ = creep coefficient at time t for loads applied at time t_o
 $\phi(t_f, t_d)$ = creep coefficient at time t_f for loads applied at time t_d
 $\phi(t_f, t_o)$ = creep coefficient at time t_f for loads applied at time t_o
 $\phi_d(t_f, t_d)$ = creep coefficient of the deck concrete at time t_f for loads applied at time t_d
 χ = aging coefficient

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, “ACI Concrete Terminology,” <https://www.concrete.org/store/productdetail.aspx?ItemID=CT16>. Definitions provided herein complement this source.

anchorage—in post-tensioning, a device used to anchor the tendon to the concrete member; in pretensioning, a device used to maintain the elongation of a tendon during the time interval between stressing and release.

modulus of elasticity—the ratio of normal stress to corresponding strain for tensile or compressive stress below the proportional limit of the material.

post-tensioning—method of prestressing reinforced concrete in which tendons are tensioned after concrete has attained a specified minimum strength or specified minimum age.

CHAPTER 3—LUMP-SUM METHOD

3.1—Scope

In some cases, use of a simple lump-sum value for prestress loss may be adequate instead of detailed calculations. This chapter summarizes lump-sum values from past codes and literature, and discusses their use in design. In addition, data from the measurement of prestress losses are presented and summarized.

3.2—Historical code requirements

The Federal Highway Administration (FHWA), known as the **Bureau of Public Roads (1954)** at the time, published design criteria for prestressed concrete bridges in which losses due to creep, relaxation, shrinkage, and elastic deformation were specified as

$$\begin{aligned}
 \text{pretensioned concrete} &= 6000 + 16f_{cps} + 0.04f_{pi} \text{ (psi)} \\
 \text{pretensioned concrete} &= 41.4 + 16f_{cps} + 0.04f_{pi} \text{ (MPa)}
 \end{aligned}
 \quad (3.2a)$$

$$\begin{aligned}
 \text{post-tensioned concrete} &= 3000 + 11f_{cps} + 0.04f_{pi} \text{ (psi)} \\
 \text{post-tensioned concrete} &= 20.7 + 11f_{cps} + 0.04f_{pi} \text{ (MPa)}
 \end{aligned}
 \quad (3.2b)$$

where f_{cps} is the concrete stress at the center of gravity of the prestressing force due to all prestress and applied loads, psi (MPa); and f_{pi} is prestressing steel stress immediately following transfer, psi (MPa).

In formulating the criteria, it was assumed that the efficiency of the anchorage was 100 percent with no seating loss and applicable to normalweight concrete only. The amount of losses was to be determined by tests for lightweight concrete. In Eq. (3.2a) and (3.2b), the first term represents loss due to shrinkage, the second represents combined losses due to elastic shortening and creep, and the third represents loss due to steel relaxation. For post-tensioned concrete, elastic shortening was ignored. In the early 1950s, only lower strengths of concrete and steel were available for prestressed concrete applications. For typical designs, the value of f_{cps} was approximately 1300 psi (8.96 MPa) and the value of f_{pi} was a maximum of 200,000 psi (1380 MPa). Using these typical values for f_{cps} and f_{pi} , the criteria stated (Eq. (3.2a) and Eq. (3.2b)) for losses equates to 34,800 psi (240 MPa) for pretensioned concrete and 25,300 psi (174 MPa) for post-tensioned concrete.

Requirements for evaluating prestress loss first appeared in **ACI 318-63**, in new Chapter 29 on prestressed concrete. ACI 318-63 commentary recommended the following lump-sum losses, excluding friction and seating losses, based on the **ACI-ASCE Committee 323 (1958)** report: pretensioning = 35,000 psi (241 MPa); and post-tensioning = 25,000 psi (172 MPa).

In published discussions, **Abeles (1958)** argued that these recommended lump-sum values should be eliminated or increased to allow designers to assess the losses for specific conditions. If the lump-sum values were to remain, he suggested the following ranges of values: pretensioning = 30,000 to 40,000 psi (207 to 276 MPa); and post-tensioning = 20,000 to 30,000 psi (138 to 207 MPa).

Similarly, the Precast/Prestressed Concrete Institute (PCI) published building code requirements for prestressed concrete (**Structural Engineers Association of Northern California 1959**) specifying the following lump-sum values for estimating prestress losses, excluding friction prestressing steel loss: pretensioning = 25,000 to 35,000 psi (172 to 241 MPa); and post-tensioning = 15,000 to 25,000 psi (103 to 172 MPa).

In **ACI 318-83**, specific lump sum recommendations in the commentary were replaced by the following statement: “Lump-sum values of prestress losses for both pretensioned and post-tensioned members that were indicated before the 1983 Commentary are considered obsolete.” The commentary also provides several references that can be used to compute prestress loss (ACI-ASCE Committee 323 1958; **ACI 435R**; **PCI Committee on Prestress Losses 1975**; **Zia et al. 1979**). Although **ACI 318-11** does not explicitly prohibit use of lump-sum losses, the commentary is clear in warning that previously cited lump-sum losses are considered obsolete.

3.3—Industry practice

The Post-Tensioning Institute's manual (PTI 2006) provides a discussion of long-term losses and a detailed discussion of friction losses, which are addressed elsewhere in this guide, including 4.4.2. While PTI refers to Zia et al. (1979), it is noted that precise determination of losses in post-tensioned members is not critical. The discussion is closed with the statement, "A 100% variance in the estimate for total long-term losses generally results in less than a 10% difference in stress in the prestressing steel at nominal strength." Kelley (2000) indicates that for approximating prestress losses, tendons are typically assumed to be stressed to $0.8f_{pu}$; for ASTM A416/A416M Grade 270 (1860 MPa), this equates to a tendon stress of 216,000 psi (1490 MPa). In addition, it is typically assumed that the average tendon stress along a tendon after seating is $0.7f_{pu}$. Based on an assumed final effective prestress of 175,000 psi (1210 MPa), the total prestress loss is 41,000 psi (282 MPa), or 19 percent of the initial jacking stress. This results in a force of 26,800 lb (119,000 N), often rounded to 27,000 lb (120,100 N), for a single 0.5 in. (12.7 mm) diameter strand.

Before low-relaxation strand gained popularity in the 1970s, use of stress-relieved strand resulted in higher losses due to relaxation of the prestressing strand. At that time, a total loss of 52,000 to 57,000 psi (359 to 393 MPa) was typically assumed, resulting in a final effective prestress force of approximately 24,000 to 25,000 lb (106,800 to 111,200 N) per strand.

The PCI Design Handbook (PCI 2010) indicates that losses have no effect on the ultimate strength of a flexural component unless the tendon is unbonded or has an effective prestress of less than $0.5f_{pu}$. In addition, the handbook indicates that overestimated or underestimated losses may impact service limit states such as camber, deflection, or cracking. PCI (2010) also provides an estimate of total losses for typical concrete components:

(a) Normalweight concrete: 25,000 to 50,000 psi (172 to 345 MPa) (12 percent to 25 percent)

(b) Sand lightweight concrete: 30,000 to 55,000 psi (207 to 379 MPa) (15 percent to 27 percent)

These values are presented as total losses for typical components. Losses in terms of the percentage of an assumed initial jacking force of $0.75f_{pu}$ are also given. For refined estimation of losses, PCI (2010) refers to Zia et al. (1979).

3.4—Measured losses

Measurement of prestressing losses is challenging because it requires that concrete strain be determined from the time when the prestress is transferred until the end of the measurement period. This typically excludes use of bonded electrical resistance foil gauges because of drift due to temperature change and lack of durability when exposed to concrete and the surrounding environment. Consequently, losses are typically measured using vibrating wire strain gauges embedded in concrete or mechanical gauges, which measure relative movement of gauge points adhered to the concrete surface. These movements are used to calculate average concrete strain between the points and to track change in concrete

strain over time. If perfect bond is assumed between prestressing steel and concrete, then change in concrete strain is equal to change in steel strain.

If steel strain has been tracked from the initial value, which is typically the initial stressing, then the measured strain is used to compute the change in steel stress due to change in steel strain. The conversion from strain to stress requires that the elastic modulus of the prestressing steel (E_p) be known. While determining E_p for single wires and bar is relatively straightforward, the nature of the seven-wire strand makes accurate and precise measurement of E_p more difficult. Additionally, strain measurement methods cannot account for time-dependent losses due to relaxation. For low-relaxation prestressing strand, losses due to relaxation are usually minor relative to other time-dependent losses. For other materials more susceptible to relaxation, a force-measuring device is required, which is not typically a practical alternative. The measured change in steel stress is equal to the prestress loss from the time when the initial measurement was taken to when the measurements were terminated. The ultimate value of long-term losses is practically unattainable due to the time over which measurements are taken. Fortunately, most of the long-term losses occur in the first 6 months after prestress transfer. Consequently, relatively short-term measurements of losses, which range from 6 months to 2 years, can provide useful information regarding ultimate long-term losses.

Measurement techniques described previously require that instrumentation be installed during fabrication; readings are then taken before prestress transfer and regularly during the monitoring period. For in-service girders not previously instrumented, measurement of crack opening during loading may be used to estimate the effective prestress (Pessiki et al. 1996). While not as reliable or accurate as vibrating wire or mechanical gauges, it does provide a method for estimating the effective prestress in noninstrumented, in-service girders.

Table 3.4 presents measured losses reported in the literature. Nearly all data are from measurements on precast, pretensioned, I-shaped bridge girders. Spans range from less than 10 ft (3 m) to more than 270 ft (82.3 m). In addition, concrete compressive strengths vary from 4000 psi (27.6 MPa) to more than 15,000 psi (103 MPa) with an average of approximately 9000 psi (62.0 MPa). While little data are available on prestressed concrete elements used specifically in building construction, all or part of the bridge data presented herein is useful as a benchmark against which, for future evaluation, either bridge or building components can be compared.

Measured total losses were gathered from each reference and include instantaneous and long-term losses for the associated measurement period indicated in the table. No adjustments for final total losses have been made. Losses are reported in terms of stress in the prestressing steel and as a percentage of an assumed initial prestress force equal to $0.75f_{pu}$. Overall average prestress losses were approximately 40,700 psi (281 MPa), or 21 percent, with a coefficient of variation (COV) of 40 percent that indicates a wide data scatter. While the data are quite scattered, average total losses

Table 3.4—Measured losses from literature

Reference	Total loss, ksi	Percent loss	f_c' , psi	Age at f_c' test	Span, ft	Age at loss test	Girder type*	Instrumen- tation†	Prestressing steel type‡	Comments§
Greuel et al. (2000)	35.4	17.5	11,800	56 days	115.5	6 to 9 months	ODOT B42-48	c	270LL	
Pessiki et al. (1996)		17.2	8760	28 years	89	28 years	PennDOT 24 x 60 I	c	Unknown	
		18.1	8180	28 years	89	28 years	PennDOT 24 x 60 I	c	Unknown	
Mossiossian and Gamble (1972)	32.5	16.0	Data unavailable							
	35.1	17.3								
Kebraei et al. (1997)	17.9	8.8								
	36.8	18.2								
Shenoy and Frantz (1991)	30.5	15.1	7100	27 years	54	27 years	36W x 27D box	c	250SR	
Stanton et al. (1999)	34	16.8	Data unavailable							
	65.6	32.4								
	55.1	27.2								
	69.3	34.2								
	36.1	17.8								
Seguirant and Anderson (1985)	41.7	20.6								
	35	17.3								
	35.7	17.6								
Gross and Burns (2000) <i>Louetta Bridge</i>	43.1	21.3	13,630	56 days	132.9	761 days	TxDOT U54	v	270LL	2
	37.9	18.7	14,320	56 days	118.2	748 days	TxDOT U54	v	270LL	2
	40.3	19.9	13,290	56 days	119.7	1262 days	TxDOT U54	v	270LL	2
	33.8	16.7	13,410	56 days	132.3	1221 days	TxDOT U54	v	270LL	2
Gross and Burns (2000) <i>San Angelo Eastbound</i>	50.6	25.0	13,700	56 days	127.8	422 days	AASHTO IV	v	270LL	2
	57.2	28.3	13,700	56 days	127.8	422 days	AASHTO IV	v	270LL	2
	51.5	25.4	14,240	56 days	152.2	404 days	AASHTO IV	v	270LL	2
	52.0	25.7	14,830	56 days	152.2	746 days	AASHTO IV	v	270LL	2
	57.4	28.4	13,750	28 days	145.2	316 days	AASHTO IV	v	270LL	2
	58.2	28.7	14,490	56 days	145.2	309 days	AASHTO IV	v	270LL	2
	55.6	27.5	14,550	56 days	144.5	305 days	AASHTO IV	v	270LL	2
Gross and Burns (2000) <i>San Angelo Westbound</i>	34.7	17.1	10,130	28 days	127.8	771 days	AASHTO IV	v	270LL	
	34.4	17.0	10,130	28 days	127.8	771 days	AASHTO IV	v	270LL	
	32.7	16.1	10,130	28 days	127.8	771 days	AASHTO IV	v	270LL	
	30.6	15.1	10,260	28 days	127.8	766 days	AASHTO IV	v	270LL	
Al-Omaishi et al. (2009)	32	15.8	9025	28 days	127	468 days	NU2000	v	270LL	
	35.7	17.6	9025	28 days	127	469 days	NU2000	v	270LL	
	43.5	21.5	10,050	28 days	110	490 days	NE1400BT	v	270LL	
	42.3	20.9	10,050	28 days	110	491 days	NE1400BT	v	270LL	
	25.4	12.5	10,670	28 days	129.2	398 days	U54B	v	270LL	
	42.1	20.8	10,280	28 days	159	385 days	W83G	v	270LL	
	40	19.8	10,280	28 days	159.8	385 days	W83G	v	270LL	
Dwairi et al. (2010)	26	12.8	10,740	100 days	57.4	<1 year	AASHTO III	v	270LL	2,12
	38	18.8	9750	100 days	91.9	<1 year	AASHTO IV	v	270LL	2,12

Reference	Total loss, ksi	Percent loss	f'_c , psi	Age at f'_c test	Span, ft	Age at loss test	Girder type*	Instrumen- tation†	Prestressing steel type‡	Comments§
Czaderski and Motavalli (2006)		19	8100	38 years	56	38 years	450W I	d	unknown	1,13
		39	8100	38 years	56	38 years	450W I	d	unknown	1,13
		20	8100	38 years	56	38 years	450W I	d	unknown	1,13
Yang and Myers (2006)	41.9	21	10,160	56 days	55	2 years	MoDOT II	v	270LL	2
Roller et al. (1993, 1995)	23	11.4	9660	660 days	69	660 days	AASHTO BT-54	l	270LL	10
Kahn and Lopez (2005)	37.2	18.4	10,229	103 days	40	100 days	AASHTO II	v	270LL	9,14
	29.6	14.6	10,868	144 days	40	100 days	AASHTO II	v	270LL	9,14
Labia et al. (1997)	62	30.6	7864	20 years	70	20 years	1213W x 838D box	c	270LL	15
	62	30.6	8265	20 years	70	20 years	1213W x 838D box	c	270LL	15
Stallings et al. (2003)	44.8	22.1	9950	56 days	112	10 months	AASHTO BT-54	v	270LL	2
Ahlborn et al. (2000)	55.1	27.2	12,600	480 days	132.8	840 days	Mn/DOT 45M	v	270LL	
	53.3	26.3	11,100	480 days	132.8	860 days	Mn/DOT 45M	v	270LL	
Gamble (1970)	31	20.0	7730	667 days	71	1212 days	48D I-beam	d	250SR	
Houdeshell et al. (1972)	33	19.6	6920	420 days	70	782 days	48D I-beam	d	250SR	
	31	18.5	6920	420 days	67.3	783 days	48D I-beam	d	250SR	
Anderson et al. (1972)	57.3	28.3	6405	28 days	9	500 days	6D I-beam	d	16	17
	66.2	32.7	6278	28 days	9	490 days	6D I-beam	d	16	17
	72.6	35.8	6052	28 days	9	480 days	6D I-beam	d	16	17
	76.2	37.6	6170	28 days	9	420 days	6D I-beam	d	16	17
	68.2	33.7	6250	28 days	9	410 days	6D I-beam	d	16	17
	76.7	37.9	6303	28 days	9	400 days	6D I-beam	d	16	17
Gamble (1979)	28.1	17.8	7030	367 days	46	400 days	42D I-beam	d	250SR	
	28.1	17.8	7030	367 days	46	400 days	42D I-beam	d	250SR	
Hamilton and O'Neill (2009)	20.2	10.0	8400	<1 years	15	<1 years	24W x 20D I.T.	c	270LL	1
	45.8	22.6	8400	<1 years	16	<1 years	24W x 20D I.T.	c	270LL	
	17.8	9	8000	4 months	18	4 months	14W x 20D I-beam	v	270LL	
	20.9	10	8000	5 months	18	5 months	14W x 20D I-beam	v	270LL	
	22.7	11	8000	6 months	18	6 months	14W x 20D I-beam	v	270LL	
	26.1	13	8000	3.5 months	18	3.5 months	14W x 20D I-beam	v	270LL	
	24.5	12	8000	3.5 months	18	3.5 months	14W x 20D I-beam	v	270LL	
	34.2	17	8000	3.5 months	18	3.5 months	14W x 20D I-beam	v	270LL	
Barr et al. (2000)	57.6	28	10,000	56 days	133	3 years	W74MG	v	270LL	2,18
	49.7	25	10,000	56 days	133	3 years	W74MG	v	270LL	2,18
	60.6	30	10,000	56 days	133	3 years	W74MG	v	270LL	2,18
	31.9	16	10,000	56 days	76.4	3 years	W74MG	v	270LL	2,18
	32.6	16	10,000	56 days	76.4	3 years	W74MG	v	270LL	2,18

Reference	Total loss, ksi	Percent loss	f'_c , psi	Age at f'_c test	Span, ft	Age at loss test	Girder type*	Instrumen- tation†	Prestressing steel type‡	Comments§
Osborn et al. (2012)	45.3	27.1	7100	42 years	22.5	42 years	AASHTO II	c	250SR	
	55.3	33.0	7100	42 years	22.5	42 years	AASHTO II	c	250SR	
	31.6	18.8	7100	42 years	22.5	42 years	AASHTO II	c	250SR	
	34.0	20.3	7100	42 years	22.5	42 years	AASHTO II	c	250SR	
	40.1	24.0	7100	42 years	22.5	42 years	AASHTO II	c	250SR	
Attanayake and Aktan (2010)	40.6	23.2	8300	48 years	48.5	48 years	915W x 533D box	c	250SR	19
Azizinamini et al. (1996)	36.2	20.7	4330	25 years	55	25 years	Nebraska III	c	250SR	
Boone (2008)	29.8	14.7	13,785	56 days	132	10 months	AASHTO VI	v	270LL	8
	16.1	8.0	13,785	56 days	82	7 months	AASHTO VI	v	270LL	8
Brewer and Myers (2010)	66.5	32.8	8210	243 days	15	196 days	12D T-beam	d	270LL	6
	70.7	34.9	8210	243 days	15	197 days	12D T-beam	d	270LL	6
	64.5	31.9	8210	243 days	15	198 days	12D T-beam	d	270LL	6
	62.9	31.1	8210	243 days	15	199 days	12D T-beam	d	270LL	6
	67.4	33.3	8210	243 days	15	200 days	12D T-beam	d	270LL	6
	57.7	28.5	8210	243 days	15	201 days	12D T-beam	d	270LL	6
Canfield (2005)	37	18.3	15,287	160 days	89	161 days	AASHTO IV	v	270LL	2
	40.3	19.9	15,145	184 days	89	182 days	AASHTO BT-56	v	270LL	2
Furr et al. (1968)	26.3	14	6600	140 days	50	140 days	TxDOT B	d	250SR	7
	25.3	13.5	7291	140 days	50	140 days	TxDOT B	d	250SR	
	30.9	16.5	6360	140 days	50	140 days	TxDOT B	d	250SR	7
	30.9	16.5	7080	140 days	50	140 days	TxDOT B	d	250SR	
	29.9	15.9	5160	28 days	40	660 days	TxDOT B	d	250SR	7
	28.0	14.9	5160	28 days	40	660 days	TxDOT B	d	250SR	7
	40.9	21.8	5160	28 days	56	660 days	TxDOT B	d	250SR	7
	39.1	20.8	5160	28 days	56	660 days	TxDOT B	d	250SR	7
	27.3	14.6	3990	7 days	56	660 days	TxDOT B	d	250SR	
Hale and Russell (2006)	53.4	26.4	15,610	360 days	24	360 days	24D I-beam	d	270LL	
	74	36.5	11,460	360 days	24	360 days	24D I-beam	d	270LL	
	58.4	28.8	11,460	360 days	24	360 days	24D I-beam	d	270LL	
	51.8	25.6	15,610	360 days	24	360 days	24D I-beam	d	270LL	
Halsey and Miller (1996)	29.3	19.5	11,790	25 years	27	25 years	12D inverted-T	c	Unknown	
	35	23.3	11,790	25 years	27	25 years	12D inverted-T	c	Unknown	
Idriss and Solano (2002)	26.5	13.1	10,151	56 days	97	1 years	BT 1600	f	270LL	2,19
Lewis and Karbhari (2006)	8.6	4.2	6050	28 days	264.5	363 days	multiple cell box	v	270LL	1
	9	4.4	5600	28 days	271.7	303 days	multiple cell box	v	270LL	1
Kukay (2008)	48.5	27.7	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	49.1	28.0	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	42.1	24.1	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	54.9	31.4	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	56.9	32.5	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	45.6	26.0	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	30.0	17.2	9300	40 years	34.5	40 years	AASHTO II	c	250SR	
	38.9	22.2	9300	40 years	34.5	40 years	AASHTO II	c	250SR	

Reference	Total loss, ksi	Percent loss	f'_c , psi	Age at f'_c test	Span, ft	Age at loss test	Girder type*	Instrumentation†	Prestressing steel type‡	Comments§
Roller et al. (2011)	28.3	14.0	11,670	90 days	131.2	650 days	BT-78	v	270LL	19
Ruiz et al. (2008)	28.6	14.1	12,520	28 days	18	265 days	6.5W x 12D	v	270LL	11
	22.8	11.3	10,700	28 days	18	258 days	6.5W x 12D	v	270LL	11
	23.8	11.8	13,100	28 days	18	258 days	6.5W x 12D	v	270LL	11
	28.2	13.9	10,340	28 days	18	270 days	6.5W x 12D	v	270LL	5
	28.2	13.9	12,890	28 days	18	255 days	6.5W x 12D	v	270LL	5
	25.1	12.4	11,320	28 days	18	290 days	6.5W x 12D	v	270LL	4
	21.3	10.5	11,420	28 days	18	286 days	6.5W x 12D	v	270LL	4
	24.1	11.9	11,740	28 days	18	286 days	6.5W x 12D	v	270LL	4
	24.6	12.1	11,000	28 days	18	274 days	6.5W x 12D	v	270LL	4
	23.5	11.6	12,030	28 days	18	274 days	6.5W x 12D	v	270LL	4
Saïidi et al. (1998)	33	16.3	5430	28 days	150	300 days	multiple cell box	e	270LL	
Saïidi et al. (1996)	36	17.8	6100	28 days	155	30 months	multiple cell box	e	270LL	
Tabatabai and Dickson (1993)	29	17.5	10,800	34 years	42.3	34 years	AASHTO II	c	240W	1
Wang et al. (2011)	76.7	37.9	6930	28 days	9.8	260 days	200W x 300D	e	270LL	
	70.7	34.9	6930	28 days	9.8	260 days	200W x 300D	e	270LL	
	77.9	38.5	6930	28 days	9.8	260 days	200W x 300D	e	270LL	
	72	35.6	6930	28 days	9.8	260 days	200W x 300D	e	270LL	
Average	41.2	21%	9460							
COV	39%	38%								
Maximum	77.9	39%	15,610							
Minimum	8.6	4%	3990							

*Notes on girder specifications: 45D indicates 45 in. deep; 533W indicates 533 mm wide.

†Notes on instrumentation: c = crack reopening; v = vibrating wire gauge; e = electrical resistance gauge; f = fiber optic gauge; d = demountable mechanical gauge (DEMEC); and l = Carleson strain gauge.

‡Notes on prestressing steel type: 270LL = ASTM A416/A416M Gr270 low-relaxation seven-wire strand; 250SR = ASTM A416/A416M Gr250 stress-relieved seven-wire strand; 240W = Grade 240 prestressing wire.

§Comments: 1. Post-tensioned member; 2. High-performance concrete; 3. Lightweight concrete; 4. Self-consolidating concrete (SCC) with Type I cement; 5. Self-consolidating concrete with Type III cement; 6. High-strength SSC; 7. Lightweight concrete; 8. Self-consolidating high-performance concrete; 9. High-performance lightweight concrete; 10. Steam cured; 11. High-strength concrete; 12. Possible gauge malfunction; 13. Direct measurement of strain in tendons; 14. Creep (CR) and shrinkage (SH), predicted at 40 years; 15. Losses averaged for both girders, possible excessive SH due to temperature; 16. 0.25 in. (6.4 mm) diameter three-strand AMGAL galvanized oceanographic rope ($f'_c = 277,000$ psi [1910 MPa] at failure); 17. Relaxation not included, high shrinkage due to high surface area to cross section area ratio; 18. Specified concrete strength; 19. Losses averaged for four beams.

is within the range of estimated lump-sum losses of 25,000 to 50,000 psi (172 to 345 MPa) given in **PCI (2010)**.

Wide scatter of these measurements is partially due to variation in measurement techniques and the length of time over which measurements were taken. It is also an indication that variations in material properties, design characteristics, and exposure conditions ultimately lead to prestress losses that are naturally variable and almost impossible to predict with either great precision or accuracy.

CHAPTER 4—INITIAL LOSSES

4.1—Scope

Chapter 4 addresses initial losses, sometimes called short-term, instantaneous, or immediate losses, for pretensioned and post-tensioned members. These losses include friction and seating losses during tensioning operations, and elastic shortening losses that occur when

the prestressing force is transferred to concrete. Instantaneous changes in prestress force caused by applied loads are discussed later in this chapter.

Friction and seating losses are discussed for pretensioned and post-tensioned members. Basic friction concepts due to curvature and wobble are covered. The friction and seating losses that occur during the stressing operations in a precast plant are the responsibility of the precaster. Precasters understand the sources and magnitudes of the losses and compensate for them during initial stressing, as described in 4.2. The designer does not typically need to consider these losses for pretensioned members. For post-tensioned members, friction and seating loss calculations are required. The differences between unbonded, bonded, single-strand, and multi-strand tendons are shown in 4.4, along with recommended friction and wobble coefficients specific to the type of duct used. Seating losses for wedge anchorages and bar nuts are also discussed in 4.4.

Also addressed are changes in prestress force that result from elastic deformations of a prestressed concrete member. The discussion is limited to ranges of member behavior in which the concrete and prestressed and nonprestressed reinforcement exhibit linearly elastic material behavior. For the purposes of estimating initial prestress losses, this is assumed when the member is designed to satisfy the ACI 318 prestressing steel stress limits and concrete stress limits at prestress transfer.

Sections 4.2 and 4.3 discuss losses before and at transfer in pretensioned members, while 4.4 and 4.5 present these losses for post-tensioned members. Section 4.6 discusses subsequent changes to the effective prestress due to externally applied loads.

4.2—Pretensioning losses before transfer

In pretensioned construction, the prestressing takes place in a precasting facility and the producer is usually responsible for providing the correct prestress force immediately before transfer. The licensed design professional has only limited information regarding fabrication procedures.

During stressing operations and concrete curing, strand stress varies. Sources of stress variation are discussed herein, along with strand elongation, which is typically measured to ensure that the strands have the expected level of stress before prestress transfer.

4.2.1 Anchor set—Anchor set is the movement of strands within the anchors as the strands are stressed and anchored. Wedges in the anchorage typically grip the strand and move forward slightly as the jaws seat in the tapered wedge cavity.

Stressing-end anchor set occurs during single-strand tensioning as the final jacking force is transferred from the stressing device to the anchorage. A measurable decrease in strand elongation occurs as the wedges move into the tapered wedge cavity, and the strand stress decreases accordingly. The change in prestress due to anchor set Δf_{pA} is computed

$$\Delta f_{pA} = E_p \Delta \epsilon_{pA} = E_p \frac{\Delta s}{L} \quad (4.2.1)$$

where E_p is modulus of elasticity of prestressed steel, psi (MPa); $\Delta \epsilon_{pA}$ is change in strain in prestressed reinforcement due to anchor set, in./in. (mm/mm); Δs is anchor set, in. (mm); and L is strand length anchorage to anchorage, ft or in. (m or mm).

The decrease in elongation associated with this anchor set typically ranges from 1/8 to 3/8 in. (3 to 10 mm). For a specific anchor set value, the magnitude of the prestress loss due to anchor set decreases with increasing length between anchorages. In some pretensioning operations, the prestressing beds are long enough, which is commonly 300 to 500 ft (90 to 150 m), that the anchorage seating losses are relatively small. Seating losses, however, will be larger in shorter prestressing beds. Additional forces are applied during stressing to compensate for the anticipated seating loss effect.

In the case of multiple-strand tensioning operations, the stressing-end wedges are seated in the anchorage as the



Fig. 4.2.2.1a—Self-stressing form for a double tee.

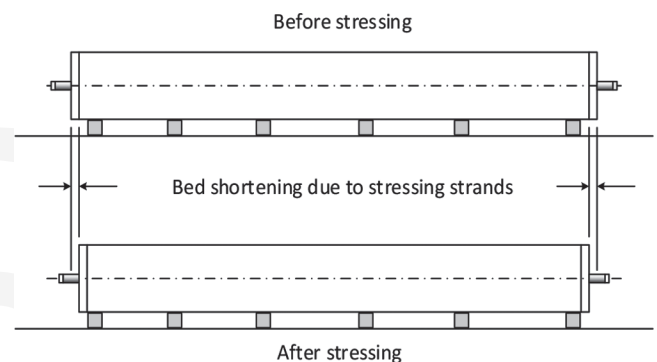


Fig. 4.2.2.1b—Self-stressing form shortening (adapted from the Precast/Prestressed Concrete Institute (PCI 2013)).

force is applied. Therefore, no loss of prestress force occurs relative to the measured value. Likewise, seating loss for fixed-end anchorages and splicing anchorages also occurs during tensioning, and there is no loss in prestress force relative to the measured value.

4.2.2 Form or abutment deformations—Two strand-restraint systems—self-stressing forms and fixed-abutment beds—are considered separately in this section.

4.2.2.1 Self-stressing forms—Self-stressing steel forms have strand anchorages bearing against plates that transfer the jacking force directly to the forms. Therefore, form shortening occurs during the tensioning operation. Figure 4.2.2.1a shows a self-stressing form.

The form shortens incrementally as each strand is stressed. This shortening, which is illustrated in Fig. 4.2.2.1b, might not be uniform for each strand due to flexure of the form, stressing sequence, and friction between the form and its supports.

If strands are stressed sequentially, the stress in each strand will decrease when subsequent strands are stressed. The amount of form shortening that takes place should be measured periodically for different numbers of strands in common configurations. The initial tensioning force may be increased to compensate for the anticipated form shortening effect on the average strand



Fig. 4.2.2.2a—Fixed abutments.

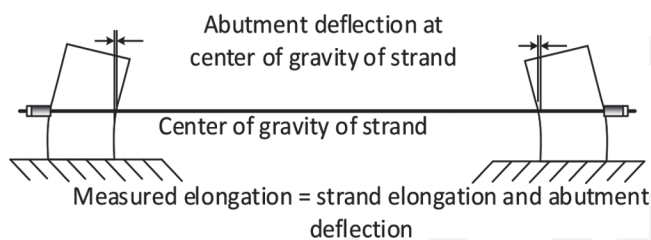


Fig. 4.2.2.2b—Fixed-abutment deformations.

4.2.2.2 Fixed-abutment beds—Abutment movement or movement of anchorages occurs during tensioning of fixed-abutment beds. Figure 4.2.2.2a shows two fixed abutments.

Even large, well-designed abutments will rotate and deflect slightly when loaded, as illustrated in Fig. 4.2.2.2b. Although the movement is small, it should be monitored and evaluated for strand patterns commonly used. The influence of this movement decreases with increasing bed length. As with self-stressing forms, if strands are tensioned sequentially, the tensioning force may need to be increased to compensate for the effects of abutment movement on previously stressed strands.

4.2.3 Elongation calculation and correction—A prestressing strand's length increases in proportion to the applied tension. This increase in length is called elongation. Measured strand elongations are typically compared to expected values to verify the prestress force before transfer. Elongation is commonly discussed using three terms: basic, gross, and net.

The basic elongation e_b is the length the strand is expected to elongate based on the following factors only:

- Applied tension force P , lb (N)
- Strand length from anchorage to anchorage, L , in. (mm)
- Strand cross-sectional area A_{ps} , in.² (mm²)
- Strand modulus of elasticity, E_p , psi (MPa)

ACI 318-11 and AASHTO (2012) design specification stress limits ensure that the prestressing reinforcement remains in the elastic range during tensioning. Therefore,

Eq. (4.2.3) for calculating the basic elongation of straight strands is

$$e_b = \frac{PL}{A_{ps} E_p} \quad (4.2.3)$$

A more complex formulation may be warranted for harped (draped) strands.

In a pretensioning operation, an initial force removes slack from the strands during installation, and the elongation measurements follow after the initial force is applied. The applied tension force for elongation computation, therefore, is determined by subtracting the initial force from the final force measured during tensioning. Strand length is the length between the stressing plates on which the strand chucks bear at each end of the casting bed. Strand cross-sectional area and modulus of elasticity are taken from the strand manufacturer's mill report for each pack of strand.

Strand area and modulus of elasticity may vary for different heat numbers, which identify specific production runs as well as manufacturers. Average values of area and modulus of elasticity for different packs of strand may be used for elongation calculations, provided they vary less than 2.5 percent for all strands in use. Otherwise, the elongation is recomputed using the actual strand properties.

Elongation is typically measured as the displacement of a mark on the strand relative to a reference point on the stressing-end prestressing chuck or anchor. The elongation measured before stressing-end anchorage seating is known as gross elongation. Net elongation is the elongation measured after anchorage seating. Gross and net elongation are used to check strand tension before and after seating loss, respectively.

Because elongation is measured at the stressing-end anchorage, measured elongations may include operational components that do not reflect actual strand elongation, such as strand seating displacements at fixed-end or splicing anchorages, or deformation of the abutments or self-stressing forms during stressing. The characteristics of each pretensioning setup are determined and appropriate compensations made when calculating the expected elongation values.

4.2.4 Thermal effects

4.2.4.1 Stress changes from thermal effects—When strands are anchored between fixed abutments, changes in strand temperature after tensioning change the strand force. Strands tensioned on a cold morning will lose force when warmer concrete raises the strand temperature because the length between anchorages is not affected by the concrete temperature. The opposite effect could occur for strands tensioned on a hot afternoon. The strand stress can decrease approximately 1000 psi (6.89 MPa) for a differential temperature increase of 5°F (2.8°C). Thermal stress changes will not occur with self-stressing forms because the form will expand and contract with temperature changes similar to the strand.

Several researchers have noted that the heat of hydration during curing of a pretensioned beam may affect the force in the prestressing steel (Gross 1963; Barr et al. 2000).

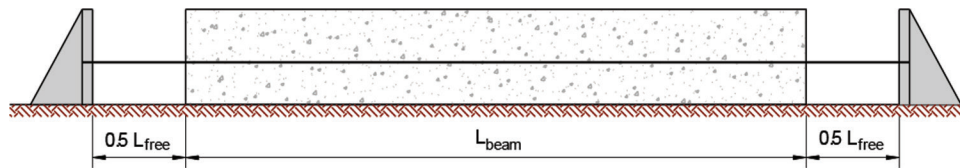


Fig. 4.2.4.2—Beam in casting bed.

Because the strands are anchored in place at the abutments, they have a constant length from the time they are anchored until they are released. If the strands, under a constant strain, are heated by the hydrating concrete, they will experience a loss in stress. The loss is difficult to quantify because it depends on when the concrete and steel begin to bond. After the concrete has reached its peak temperature, the concrete and steel begin to cool. However, because the strand contraction is restrained by the abutments, increased tensile stresses will develop in the strand, and tension may develop in the concrete beam. The gain in tension in the strand outside the beam upon cooling is not the same as the loss of tension during heating, because the steel is bonded to the concrete. However, all changes in stresses that occur due to temperature changes after the concrete and steel are fully bonded are recovered after the strands are cut. After transfer of the prestress, temperature changes will have a small influence on the prestress force because the concrete and steel will expand and contract as a unit. There is a very small difference in the coefficient of thermal expansion between the concrete and steel. The example problem given in 8.4 indicates that these thermal effects are not a significant source of loss.

4.2.4.2 Example of beam with concentric tendon—

Consider the simple case of a concentric tendon in a pretensioned beam as shown in Fig. 4.2.4.2. For this discussion, assume that the beam is steam cured overnight, with the steam shut off at some point to allow the beam to cool before the strands are cut.

Assuming the abutments are completely fixed, if the temperature rises ΔT_1 along the entire tendon length while there is no bond between the strand and the concrete, the tension in the tendon will relax by $\Delta T_1 \alpha_{ps} E_p$, where α_{ps} is the coefficient of thermal expansion of the prestressing steel and E_p is the modulus of elasticity.

To simplify the analysis, assume that when the concrete reaches its peak temperature, which is typically maintained for several hours, perfect bond develops between the concrete and steel, and the concrete develops strength and stiffness, with a modulus of elasticity of E_c . For this discussion, another simplifying assumption is that the strand segments outside the beam and inside the beam are the same temperature.

After the steam is shut off, the tarp is removed, or forms are stripped, the beam and strand are exposed to ambient temperature, and the steel and concrete will cool before prestress transfer by some amount, ΔT_2 . As the concrete and steel cool and try to shorten, the contraction is restrained, so the strand stress becomes more tensile. There will be a change in force, ΔP , which is the same in the free length of strand as it is in the beam. Within the beam, however, a

portion of this force is carried by the concrete, ΔP_c , and the remainder is carried by the prestressing steel, ΔP_{ps} . Strain compatibility should be maintained between the concrete and prestressing steel within the beam, so the following Eq. (4.2.4.2a) and (4.2.4.2b) are written.

$$\Delta \epsilon_p = \Delta \epsilon_c \quad (4.2.4.2a)$$

$$\Delta T_2 \alpha_{ps} - \frac{\Delta P_{ps}}{E_p A_{ps}} = \Delta T_2 \alpha_c - \frac{\Delta P_c}{E_c A_c} \quad (4.2.4.2b)$$

Equation (4.2.4.2b) is written assuming that the force that develops is tensile. The other relationship that can be written is the total length change of the tendon is zero

$$\Delta \epsilon_p L_{beam} + \Delta \epsilon_{free} L_{free} = 0 \quad (4.2.4.2c)$$

where $\Delta \epsilon_{free}$ is the strain in the free tendon length, and L_{free} is the total free tendon length. The strain in the free length is calculated as Eq. (4.2.4.2d)

$$\Delta \epsilon_{free} = \Delta T_2 \alpha_{ps} - \frac{\Delta P}{E_p A_{ps}} \quad (4.2.4.2d)$$

When performing example calculations, assuming that the temperature rise and temperature fall are the same, the strand outside the beam will experience a net increase in tension and the concrete will have a net tensile stress as the composite beam resists the restraint forces. Upon release, the restraint forces are removed, and the prestress force at transfer is lower than the original jacking force due to the loss caused by increased temperatures between tensioning and bond on the fixed length bed. Precasters have reported the effects of the tensile stress in the concrete prior to release as causing occasional vertical cracking in prestressed girders before release of the strands (Gross 1998). Typically, these cracks close at prestress transfer.

Based on the prior analysis, concrete has a tensile stress before transfer, and the strand outside the beam has a higher tensile force than the original jacking force.

With these relationships, a rough idea of strand and concrete stresses are established. Many factors, however, complicate the quantification of these stresses. They include the variable nature of early-age concrete modulus of elasticity and the coefficient of thermal expansion of the young concrete. While the cooling occurs over a period of several hours, the modulus and coefficient of thermal expansion are changing and there will be restraint of thermal movements

by the formwork; both are difficult to quantify. Finally, prior discussion used a concentric tendon. The tendon's eccentricity will also affect result.

In an analysis, Gross (1998) used a similar method to determine effects of temperature during casting. However, rather than trying to quantify the effect of the developing stiffness of the concrete, a factor was calibrated that accounted for the concrete's contribution to the stiffness of the embedded strand.

4.2.5 Steel relaxation—Relaxation is a time-dependent property of the prestressing steel, which results in a loss of stress under a constant strain. Relaxation begins immediately after stressing, so it should be considered in the calculation of losses prior to transfer. The following equation, Eq. (4.2.5), is widely accepted for calculating relaxation losses for low-relaxation strands

$$f_{ps}(t) = f_{po} \left[1 - \frac{\log t}{45} \left(\frac{f_{po}}{f_{py}} - 0.55 \right) \right] \quad (4.2.5)$$

where t is duration of load, in hours; f_{po} is initial strand stress after jacking and seating, psi (MPa); and f_{py} is yield stress of strand, psi (MPa).

If concrete is cast soon after stressing, and the force is released within 24 hours, the relaxation loss will be relatively small (~1 percent) and can be compensated with an initial overstress. Producers might account for this, but designers would generally not consider this component of the relaxation loss.

4.3—Elastic shortening losses in pretensioned members

Any length change of prestressed reinforcement results in a corresponding change in the effective prestress force. During prestress transfer in a pretensioned member, the prestressing strands shorten, as illustrated in Fig. 4.3, as the surrounding concrete is compressed until equilibrium is achieved. The strain decrease in the strand is accompanied by a corresponding decrease in the strand tension. The resulting change in effective prestress is known as the elastic shortening loss.

Reasonably accurate estimation of elastic shortening prestress loss is important for two reasons:

1. Stresses in the concrete and prestressed reinforcement immediately after transfer should satisfy code-prescribed limits. These limits effectively constrain the member size, prestress amount, and concrete strength at transfer.

2. The instantaneous and time-dependent creep responses of prestressed concrete members are strongly dependent on the initial elastic shortening and state of stress at transfer. Thus, an error in the estimation of the elastic shortening loss can result in magnified errors in predicted camber and long-term prestress losses.

Members with larger concrete precompression stresses, such as bridge girders, usually experience larger elastic shortening losses than more lightly prestressed members. Prestress losses attributable to elastic shortening typically range from 4 to 10 percent of the prestress force before

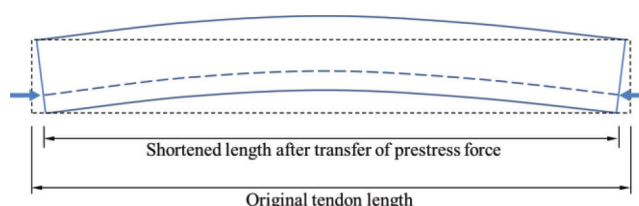


Fig. 4.3—Elastic shortening of prestressed reinforcement during prestress transfer.

transfer in fully pretensioned members—for example, ACI 318 Class U or Class T flexural members. Higher losses are possible in members with lower concrete stiffness, such as those constructed with lightweight concrete. Average elastic shortening losses in post-tensioned members are usually smaller and depend heavily on the sequence of stressing operations.

4.3.1 Transformed-section approach—Computation of stresses in concrete and prestressing steel immediately after transfer is straightforward using the transformed-section approach. Assuming that: a) there is a linearly elastic material behavior; b) there is a perfect bond between concrete and reinforcement; and c) the plane sections remain plane, the following relationships (Eq. (4.3.1a) and (4.3.1b)) result from satisfying axial force and moment equilibrium on a cross section immediately after transfer.

$$f_{ci} = -\frac{f_{pbt} A_{ps}}{A_{tr}} - \frac{f_{pbt} A_{ps} e_{tr} y_{tr}}{I_{tr}} + \frac{M y_{tr}}{I_{tr}} \quad (4.3.1a)$$

and

$$f_{pt} = f_{pbt} - n_p \left(\frac{A_{ps}}{A_{tr}} + \frac{e_{tr}^2 A_{ps}}{I_{tr}} \right) f_{pbt} + n_p \frac{M e_{tr}}{I_{tr}} \quad (4.3.1b)$$

where f_{ci} is concrete compressive stress immediately after transfer at fiber under investigation, psi (MPa); f_{pbt} is stress in prestressing steel immediately before transfer, psi (MPa); f_{pt} is stress in prestressing steel immediately after transfer, psi (MPa); A_{ps} is area of prestressed reinforcement, in.² (mm²); A_{tr} is transformed area of cross section (transforming reinforcement area to concrete area of equivalent stiffness), in.² (mm²) (4.3.2); I_{tr} is second moment of transformed area of cross section (moment of inertia), in.⁴ (mm⁴); e_{tr} is eccentricity of prestress force with respect to centroid of transformed area, in. (mm); y_{tr} is distance from centroid of transformed section to concrete fiber under investigation, in. (mm); $n_p = E_p/E_{ci}$ = modular ratio of prestressing reinforcement with respect to concrete at transfer; E_p is modulus of elasticity of prestressed reinforcement, psi (MPa); E_{ci} is modulus of elasticity of concrete at time of prestress transfer, psi (MPa); and M is bending moment experienced by cross section immediately after transfer (usually due to self-weight), in.-lb (N-mm).

For continuous members, the total prestress moment (primary plus secondary) replaces $f_{pbt}A_{ps}e_{tr}$ in the second term of Eq. (4.3.1a).

Equation (4.3.1a) computes the concrete stress at the level of the prestressing steel immediately after transfer, f_{cir} , by setting y_{tr} equal to e_{tr} . Equation (4.3.1b) can then be simplified to yield Eq. (4.3.1c)

$$f_{pt} = f_{pbt} + n_p f_{cir} \quad (4.3.1c)$$

where f_{cir} is concrete stress at the level of the prestressing steel immediately after transfer, psi (MPa).

Although these equations use the prestress level immediately before transfer, the stresses that result from Eq. (4.3.1a) through (4.3.1c) represent stresses immediately after transfer. Thus, when using the transformed-section approach, the post-transfer concrete stresses are computed directly from the pretransfer prestress force. Accordingly, the prestress loss attributable to elastic shortening, Δf_{pES} , is simply the difference between f_{pt} and f_{pbt} , as shown in Eq. (4.3.1d) and (4.3.1e).

$$\Delta f_{pES} = -n_p \left(\frac{A_{ps}}{A_{tr}} + \frac{e_{tr}^2 A_{ps}}{I_{tr}} \right) f_{pbt} + n_p \frac{M e_{tr}}{I_{tr}} \quad (4.3.1d)$$

or

$$\Delta f_{pES} = n_p f_{cir} \quad (4.3.1e)$$

The first two terms in Eq. (4.3.1d) represent the effect of strand shortening as the concrete is compressed; the third term represents the opposing effect of the self-weight moment on the cross section.

4.3.2 Gross-section approximation—If computation of transformed-section properties is considered overly burdensome, the elastic shortening loss can be estimated with reasonable accuracy using gross-section properties. This is specifically true for lightly prestressed members. For linearly elastic material behavior and perfect bond between reinforcement and concrete, the change in prestress during transfer is computed by multiplying concrete stress at the level of the prestressing steel after transfer (f_{cir}) by the modular ratio (n_p), as indicated in Eq. (4.3.1e).

The concrete stress f_{cir} is computed using gross-section properties, but prior knowledge of the effective prestress force after transfer is required. The *PCI Design Handbook* (PCI 2010) simplifies the method using an approximate version of this technique. A single iteration of the calculation in Eq. (4.3.1e) is performed after an initial assumption of a 10 percent prestress loss due to elastic shortening. This assumption is embodied in the handbook's value of $K_{cir} = 0.9$ for pretensioned members. Equations (4.3.2a) and (4.3.2b) are used to calculate elastic shortening loss

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cir} \quad (4.3.2a)$$

$$f_{cir} = K_{cir} \left(\frac{P_i}{A_g} + \frac{P_i e_p^2}{I_g} \right) - \frac{M_g e_p}{I_g} \quad (4.3.2b)$$

where P_i is initial prestress force (after anchorage seating loss), lb (N); e_p is eccentricity of prestressed reinforcement with respect to the gross section, in. (mm); A_g is area of the gross cross section, in.² (mm²); I_g is second moment of area of gross cross section (moment of inertia), in.⁴ (mm⁴); and M_g is bending moment due to dead weight of prestressed member and any other permanent loads in place at the time of stressing, in.-lb (N-mm).

4.3.3 Iterative gross-section approach—A more exact value of the elastic shortening loss is obtained using gross-section properties when an iterative solution technique is employed. A trial value of elastic shortening loss is assumed and used to compute f_{cir} . After Eq. (4.3.1e) is used to update elastic shortening loss, the designer can evaluate the accuracy of the initial estimate and reiterate if desired. Two or three iterations are usually adequate to get a reasonable level of accuracy. Alternatively, a closed-form solution for directly computing the elastic shortening loss with gross-section properties is employed to condense repeated iterations to a single step.

$$\Delta f_{pES} = \frac{e_p M_g A_g - A_{ps} f_{pbt} (I_g + e_p^2 A_g)}{A_{ps} (I_g + e_p^2 A_g) + \frac{A_g I_g}{n_p}} \quad (4.3.3)$$

Equation (4.3.3) is functionally equivalent to the expression recommended in the **AASHTO LRFD (2011)** commentary.

Analysis with gross-section properties results in an elastic shortening approximation that is close to the transformed-section value if: a) the closed-form solution of Eq. (4.3.3) is employed; or b) an iterative approach is pursued until convergence is achieved. If the iterative approach is used with net section properties, the solution will nearly match the transformed section properties solution. However, accuracy of the single-iteration approximation of the *PCI Design Handbook* (PCI 2010) depends on how closely the assumed elastic shortening loss of 10 percent matches the final calculated value.

The example pretensioned double-tee beam in 8.1 provides a basis for comparison between the methods. Transformed-section analysis (8.1.2) results in a computed elastic shortening loss of 15,700 psi (108 MPa), which is approximately 7.8 percent of the pretransfer stress. Application of Eq. (4.3.3) using gross-section properties yields a computed loss of 15,550 psi (108 MPa) (7.7 percent). The single-iteration approximation from the *PCI Design Handbook* (PCI 2010) simplified method using $K_{cir} = 0.9$ is 14,900 psi (103 MPa) (7.4 percent). For this example, the relative error among these three computational techniques is less than 0.5 percent of the initial prestress level, which is small compared to the uncertainty associated with the actual material properties.

In general, the small relative error between transformed section (and iterative with net section) and gross-section

analyses will increase as the ratio of bonded reinforcement to member cross-sectional area increases.

4.3.4 Variation of elastic shortening loss along the member length—The bending moment due to self-weight varies along the member length. Therefore, the magnitude of the elastic shortening loss is not uniform along the pretensioned member. Variation in the eccentricity or quantity of the bonded prestressed reinforcement is often employed to control concrete stresses after transfer. ACI 318 requires that member sections satisfy the prescribed stress limits. It is usually adequate to compute elastic shortening losses at cross sections that are likely to be critical for concrete stress checks immediately after transfer or under service loads. If little variation in the effective prestress force is expected along the member, it is common practice to use the elastic shortening loss computed for the midspan cross section. In members with single-point strand draping and a parabolic moment diagram from gravity loads, the cross section located at 40 percent of the simple span is often selected. End regions are often critical for concrete stress checks immediately after transfer.

4.3.5 Sensitivity to concrete stiffness—Inspection of Eq. (4.3.1d) indicates that the accuracy of computed elastic shortening loss is dependent on accurate estimation of the concrete modulus of elasticity at prestress transfer (E_{ci}). The elastic shortening loss depends directly on concrete material stiffness, a property that is also one of the most uncertain of the parameters involved in computing this loss. As discussed in 7.4, actual measured E_{ci} values can vary greatly from values predicted in accordance with simple, code-prescribed equations; therefore, codes often allow the use of mixture-specific stiffness values if they are known. Knowledge of the stiffness characteristics of commonly used aggregates and concrete mixtures in a region are used to improve the accuracy of the E_{ci} value assumed for design. Even when the relationship between concrete compressive strength and E_{ci} is well established, variability is often experienced because measured concrete compressive strengths usually exceed specified values to reduce the time that the prestressed element remains in the casting bed.

4.4—Post-tensioning losses during tensioning and transfer

4.4.1 Initial losses and tendon types—The initial losses in post-tensioning (PT) tendons are friction, anchor set, and elastic shortening. Although there are many types of PT tendons, only the most commonly used ones are discussed herein. The stressing sequence affects the initial losses of all tendons.

Unbonded single-strand tendons are coated with PT coating and sheathed with extruded high-density polyethylene (HDPE) sheathing. The PT coating lubricates the strand that can move freely within the sheathing, which reduces friction. These tendons are stressed with a single-strand jack and the elongations measured after anchor set and after the stressing jack has been removed from the tendon.

Multistrand tendons typically consist of multiple bare strands in a metal or plastic duct. The strands are stressed

together with a center-hole multistrand jack; however, the elongations are measured before anchor set. Multistrand tendons are normally grouted with cementitious grout after stressing and elongation verification, and are often called bonded tendons.

Bar tendons typically consist of a single bar in a metal or plastic duct. The bar is stressed with a bar jack using a coupler and a pulling bar. The elongation is typically measured after anchor set, or the jacking force is verified with a lift-off test. Bar tendons are grouted after stressing.

4.4.2 Friction losses—During stressing, the prestressing force along a tendon decreases from its maximum value at the stressing end to a minimum at the fixed end. This decrease is due to friction losses between the prestressing strand and its sheathing or duct due to the intended tendon curvature and unintended tendon curvature, which is also called wobble. Once the jacking force reaches its prescribed value, the tendon is released and anchored at the stressing end. Locking the tendon is achieved in most cases by the engagement of serrated wedges, which are activated as the strand retracts and draws the wedges into a conical wedge cavity in the anchor or a wedge plate. Therefore, during stressing, there are two components of initial stress loss: one from friction and the other from anchor set. An example of a final stress diagram of a tendon with one stressing end immediately after anchor set is shown in Fig. 4.4.2a. The slope of the curve, $-s$, between f_{jack} and f_L , represents the average friction intensity. The triangular area between f_{jack} , f_{anchor} , and f_{max} divided by E_p represents the loss of elongation due to the anchor set. The curve between f_{anchor} , f_{max} , and f_L represents the strand stress along the length after anchor set. The area under this curve divided by E_p is the net tendon elongation after anchor set.

The difference in strand stress between the stressing end and the fixed end increases with increasing tendon length, greater friction coefficient, and larger angular change in the tendon profile.

In typical building construction with unbonded single-strand tendons, the slope of the stress curve will be relatively flat as the friction is small. For normal-length tendons, the average tendon stress is assumed to act at all critical sections along the length. For long tendons, over 120 ft (37 m) in length when stressed from one end only, or over 240 ft (73 m) in length when stressed from both sides, additional tendons may be required to provide the total prestressing force required at a given location in the member. For short tendons less than 40 ft (12 m) in length, the effect of the anchor set can be significant and should be considered as discussed in 4.4.3.

The stress distribution for a tendon stressed from both ends is illustrated in Fig. 4.4.2b. It is not necessary to stress the tendon at both ends simultaneously. The stressing at the second end typically takes place after stressing of the first end is completed.

As shown in Fig. 4.4.2b, when the resulting stress distribution is symmetrical when the tendon is stressed from both ends, the average friction is also symmetrical. Figure 4.4.2b also illustrates that the elongation before anchor set obtained from the first-end stressing is proportional to the

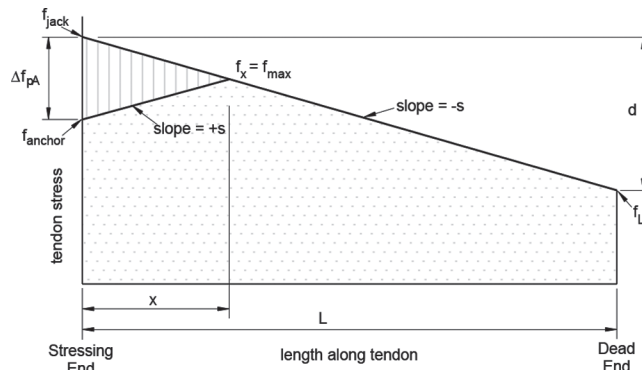


Fig. 4.4.2a—Stress distribution in tendons with one stressing end.

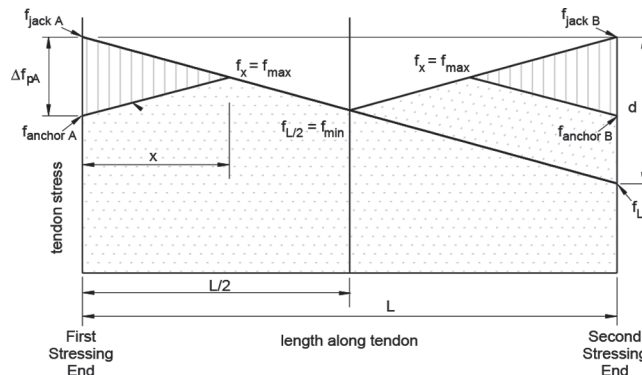


Fig. 4.4.2b—Stress distribution in tendons with two stressing ends.

area under the stress curve, f_{jackA} and f_L , will be much larger than the remaining elongation obtained from the second-end stressing, proportional to the area between $f_{L/2}$, f_{jackB} , and f_L .

Bonded multi-strand tendons are usually used in bridges with deeper members, resulting in a greater angular deviation. They are also used in buildings, in transfer girders, in beams, in slabs, and also in other structures such as water tanks. As the friction increases over the supports where the angular deviation is larger, each tendon is analyzed separately so that the actual force in each span and at each critical section is evaluated. Figure 4.4.2c shows the stress diagram with variable friction intensity. In bonded tendons, no force equalization occurs, as the tendons are typically grouted shortly after stressing and an average force for the tendons is typically not used.

Friction losses are subdivided into intended tendon curvature, represented by a curvature friction coefficient μ , and the unintended tendon curvature, represented by a wobble friction coefficient k . The stiffness of the duct contributes to a lower wobble friction, and a more flexible duct could contribute to increased wobble friction. The typically stiffer metal ducts have a larger maximum support spacing requirement than the more flexible plastic ducts. The effect of friction on the force along the tendon is calculated according to the formula

$$T_x = T_o e^{-(\mu\alpha + kx)} \quad (4.4.2)$$

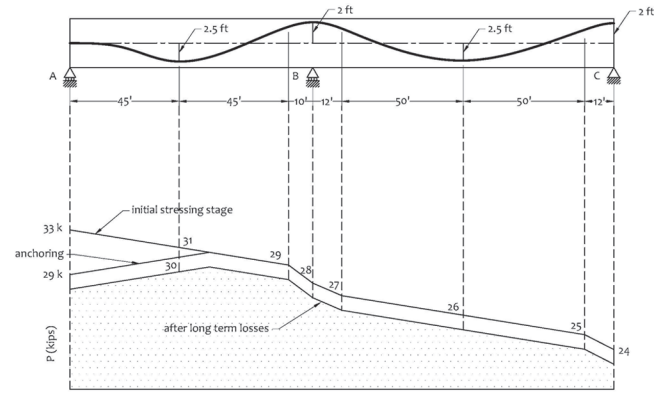


Fig. 4.4.2c—Stress distribution in tendons reflecting variable friction intensity. (Note: 1 ft = 0.305 m; 1 kip = 4.45 kN.)

where T_x is prestressing force at point x , lb (N); T_o is prestressing force at stressing end, lb (N); e is base of Napierian logarithms; μ is curvature friction coefficient (Table 4.4.2); α is total angular change in radians from jacking end to point x , rad; k is wobble friction coefficient per unit length of tendon (Table 4.4.2); and x is length of tendon from stressing end to point x , ft (m).

When different friction intensities exist along the tendon, tendon lengths with constant friction between two points are considered separately. Examples include high point to inflection point and inflection point to low point. Computer software is available to generate prestressing forces at all locations along the tendon path. The effect of the anchor set is discussed in 4.4.3.

Friction coefficients are specific to tendon type and configuration. For unbonded single-strand tendons, the type of PT coating and characteristics of the extruded plastic sheathing determine the friction values. As the PT coating and extruded plastic sheathing materials improve in quality and consistency, predictability of the friction values increases in accuracy. For bonded multi-strand tendons, the type of duct is of great importance. For example, plastic ducts typically have lower friction values than corrugated metal ducts. Recommended friction coefficients are shown in Table 4.4.2 from the Post-Tensioning Institute's guide specifications (PTI M50.3-12 Table 5.1).

Bar tendons are typically straight and relatively short with no friction along the tendon length as long as the bar is not touching the duct.

4.4.3 Anchor set—A typical value for the anchor set is 1/4 in. (6 mm) when stressing equipment with a power seating device is used. When single-strand tendons are stressed with jacks without a power seating device, anchor setup to 3/8 in. (19 mm) is common.

The effect of anchor set on tendon stresses is calculated in accordance with the formulas that follow. The anchor set is computed as the integration of the stress from the jacking point to the anchor set influence distance divided by the modulus of elasticity.

$$\Delta s = \frac{\int (\text{final stress} - \text{initial stress}) dx}{E_p} \quad (4.4.3a)$$