

State-of-the-Art Report on Partially Prestressed Concrete

Reported by Joint ACI-ASCE Committee 423

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Partially prestressed concrete construction uses prestressed, or a combination of prestressed and nonprestressed, reinforcement. Partially prestressed concrete falls between the limiting cases of conventionally reinforced concrete and fully prestressed concrete, which allows no flexural tension under service loads. When flexural tensile stresses and cracking are allowed under service loads, the prestressed members have historically been called

partially prestressed. This report is presented as an overview of the current state of the art for partial prestressing of concrete structures. Research findings and design applications are presented. Specific topics discussed include the history of partial prestressing, behavior of partially prestressed concrete members under static loads, time-dependent effects, fatigue, and the effects of cyclic loadings.

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CHAPTER 1—INTRODUCTION

1.1—Historical perspective

Application of prestressing to concrete members imparts a compressive force of an appropriate magnitude at a suitable location to counteract the service-load effects and modifies the structural behavior of the members. Although the concept of prestressed concrete was introduced almost concurrently in the U.S. and in Germany before the turn of the 20th century (Lin and Burns 1981), its principle was not fully established until Freyssinet published his classical study

(Freyssinet 1933). Freyssinet recognized that as the load on a prestressed member is increased, flexural cracks would appear in the tensile zones at a certain load level, which he referred to as the transformation load. Even though the cracks would close as the load was reduced and the structure would recover its original appearance, Freyssinet advocated avoiding cracks under service load so that the concrete would behave as a homogeneous material.

A different design approach, however, was proposed by von Emperger (1939) and Abeles (1940). They suggested using a small amount of tensioned high-strength steel to control deflection and crack width while permitting higher working stresses in the main reinforcement of reinforced concrete. Most of the early work in support of this design concept was done by Abeles (1945) in England. Based on his studies, Abeles determined that eliminating the tensile stress and possible cracking in the concrete is unnecessary in many designs. Abeles also realized that prestress can be applied to counteract only part of the service load so that tensile stress, or even hairline cracks, occur in the concrete under full service load. Abeles did specify that under dead load only, no flexural tension stress should be allowed at any member face where large flexural tensile stresses occurred under maximum load, so as to ensure closure of any cracks that may have occurred at maximum load. Additional bonded and well-distributed nonprestressed reinforcement could be used to help control cracking and provide the required strength. Abeles termed this design approach as “partially prestressed concrete.” Therefore, the design approach advocated by Freyssinet was then termed as “fully prestressed concrete.” In actual practice, nearly all prestressed concrete components designed today would be “partially prestressed” as viewed by Freyssinet and Abeles.

Interest in partial prestressing continued in Great Britain in the 1950s and early 1960s. Many structures were designed by Abeles based on the principle of partial prestressing, and examinations of most of these structures around 1970 revealed no evidence of distress or structural deterioration, as discussed in the technical report on *Partial Prestressing* published by the Concrete Society (1983). Partially prestressed concrete design was recognized in the *First Report on Prestressed Concrete* published by the Institution of Structural Engineers (1951). Provisions for partial prestressing were also included in the *British Standard Code of Practice for Prestressed Concrete* (CP 115) in 1959. In that code, a permissible tensile stress in concrete as high as 750 psi (5.2 MPa) was accepted when the maximum working load was exceptionally high in comparison with the load normally carried by the structure. Presently, the British Code (BS 8110) as well as the *Model Code for Concrete Structures* (1978), published by CEB-FIP, defines three classes of prestressed concrete structures:

Class 1—Structures in which no tensile stress is permitted in the concrete under full service load;

Class 2—Structures in which a limited tensile stress is permitted in the concrete under full service load, and no visible cracking; and

Class 3—Structures in which cracks of limited width (0.2 mm [0.008 in.]) are permitted under full service load. Calculations for Class 3 structures would be based on the hypothetical tensile stress in the concrete assuming an uncracked section. The allowable values of the hypothetical tensile stress vary with the amount, type, and distribution of the prestressed and nonprestressed reinforcement.

Elsewhere in Europe, interest in partial prestressing also developed in the 1950s and 1960s. In the mid-1950s, many prestressed concrete structures in Denmark, especially bridges, were designed using the partial prestressing concept. Their performance was reported as satisfactory after 25 years of service (Rostam and Pedersen 1980). In 1958, the first partially prestressed concrete bridge in Switzerland (Weinland Bridge) was completed near Zurich. Provisions for partial prestressing were introduced in SIA Standard 162, issued by the Swiss Society of Engineers and Architects (1968), and since 1960, more than 3000 bridges have been designed according to this concept with highly satisfactory results (Birkenmaier 1984). Unlike the British Code and CEP-FIP Model Code, the limit of partial prestressing in the Swiss Code was not defined by the hypothetical tensile stress. Instead, it was defined by the tensile stress in the prestressed and nonprestressed reinforcement, and calculated using the cracked section. Under full service load, the allowable stress in the nonprestressed reinforcement was 22,000 psi (150 MPa), and in railroad bridges, the stress increase in the prestressed reinforcement was not to exceed 1/20 of the tensile strength. This value was taken as 1/10 of the tensile strength in other structures. It was required, however, that the concrete be in compression when the structure supported only permanent load.

In the U.S., the design of prestressed concrete in the early 1950s was largely based on the *Criteria for Prestressed Concrete Bridges* (1954) published by the Bureau of Public Roads, which did not permit tensile stress and cracking in concrete under service loads. The ACI-ASCE Joint Committee 323 report (1958), however, recognized that “complete freedom from cracking may or may not be necessary at any particular load stage.” For bridge members, tensile stress was not allowed in concrete subjected to full service load. For building members not exposed to weather or corrosive atmosphere, a flexural tension stress limit of $6\sqrt{f'_c}$ psi* was specified with the provision that the limit may be exceeded if “it is shown by tests that the structure will behave properly under service load conditions and meet any necessary requirements for cracking load or temporary overload.” Thus, partial prestressing was permitted in that first definitive design guide for prestressed concrete, and designers were quick to embrace the idea. When the balanced load design concept was published by Lin (1963), it provided a convenient design tool and encouraged the practical application of partial prestressing.

In 1971, the first edition of the *PCI Design Handbook* was published. Design procedures allowing tension stresses are

illustrated in that guide. The second edition (1978) mentioned the term “partial prestressing,” and by the third edition (1985), design examples of members with combined prestressed and nonprestressed reinforcement were included. Presently, ACI 318 permits a tensile stress limit of $12\sqrt{f'_c}$ psi with requirements for minimum cover and a deflection check. Section 18.4.3 of ACI 318 permits the limit to be exceeded on the basis of analysis or test results. Bridge design guidelines or recommendations, however, did not follow the development until the publication of the *Final Draft LRFD Specifications for Highway Bridges Design and Commentary* (1993), even though most bridge engineers had been allowing tension in their designs for many years.

The concept of partial prestressing was developed half a century ago. Over the years, partial prestressing has been accepted by engineers to the extent that it is now the normal way to design prestressed concrete structures. Bennett’s work (1984) provides a valuable historical summary of the development of partially prestressed concrete.

1.2—Definition

Despite a long history of recognition of the concept of partial prestressing, both in the U.S. and abroad, there has been a lack of a uniform and explicit definition of the term, “partial prestressing.” For example, Lin and Burns (1981) state: “When a member is designed so that under the working load there are no tensile stresses in it, then the concrete is said to be fully prestressed. If some tensile stresses will be produced in the member under working load, then it is termed partially prestressed.” On the other hand, Naaman (1982a) states: “Partial prestressing generally implies a combination of prestressed and nonprestressed reinforcement, both contributing to the resistance of the member. The aim is to allow tension and cracking under full service loads while ensuring adequate strength.” According to Nilson (1987), “Early designers of prestressed concrete focused on the complete elimination of tensile stresses in members at normal service load. This is defined as full prestressing. As experience has been gained with prestressed concrete construction, it has become evident that a solution intermediate between full prestressed concrete and ordinary reinforced concrete offers many advantages. Such an intermediate solution, in which a controlled amount of concrete tension is permitted at full service, is termed partial prestressing.”

A unified definition of the term “partial prestressing” should be based on the behavior of the prestressed member under a prescribed loading. Therefore, this report defines partial prestressing as: “An approach in design and construction in which prestressed reinforcement or a combination of prestressed and non-prestressed reinforcement is used such that tension and cracking in concrete due to flexure are allowed under service dead and live loads, while serviceability and strength requirements are satisfied.”

For the purposes of this report, fully prestressed concrete is defined as concrete with prestressed reinforcement and no flexural tension allowed in the concrete under service loads. Conventionally reinforced concrete is defined as concrete with no prestressed reinforcement, and generally, there is

* In this report, when formulas or stress values are taken directly from U.S. codes and recommendations, they are left in U.S. customary units.

flexural tension in concrete under service loads. Partially prestressed concrete falls between these two limiting cases. Serviceability requirements include criteria for crack widths, deformation, long-term effects (such as creep and shrinkage), and fatigue.

By the previous definition, virtually all prestressed concrete that uses unbonded tendons is “partially prestressed,” as codes require that a certain amount of bonded reinforcement be provided to meet strength requirements. Most pretensioned members used in routine applications such as building decks and frames, and bridges spanning to approximately 100 ft (30 m) will allow flexural tension under full service load. The addition of nonprestressed reinforcement is used only in special situations, such as unusually long spans or high service loads, or where camber and deflection control is particularly important.

1.3—Design philosophy of partial prestressing

The basic design philosophy for partial prestressing is not different from that of conventionally reinforced concrete or fully prestressed concrete. The primary objective is to provide adequate strength and ductility under factored load and to achieve satisfactory serviceability under full service load.

By permitting flexural tension and cracking in concrete, the designer has more latitude in deciding the amount of prestressing required to achieve the most desirable structural performance under a particular loading condition. Therefore, partial prestressing can be viewed as a means of providing adequate control of deformation and cracking of a prestressed member. If the amount of prestressed reinforcement used to provide such control is insufficient to develop the required strength, then additional nonprestressed reinforcement is used.

In the production of precast, pretensioned concrete members, serviceability can be improved by placing additional strands, as this is more economical than placing reinforcing bars. When this technique is used, the level of initial prestress in some or all of the strands is lowered. This is also a useful technique to keep transfer stresses below the maximum values prescribed by codes. At least for purposes of shear design, the ACI Building Code treats any member with effective prestress force not less than 40% of the tensile strength of the flexural reinforcement as prestressed concrete.

1.4—Advantages and disadvantages of partial prestressing

In the design of most building elements, the specified live load often exceeds the normally applied load. This is to account for exceptional loading such as those due to impact, extreme temperature and volume changes, or a peak live load substantially higher than the normal live loads. By using partial prestressing, and by allowing higher flexural tension for loading conditions rarely imposed, a more economical design is achieved with smaller sections and less reinforcement.

Where uniformity of camber among different members of a structure is important, partial prestressing will enable the designer to exercise more control of camber differentials. In

multispan bridges, camber control is important in improving riding comfort as a vehicle passes from one span to the next. The relatively large mild steel bars used in partially prestressed members result in a transformed section that can be significantly stiffer than a comparable section that relies solely on prestressing strand, thus reducing both camber and deflection.

Nonprestressed reinforcement used in partially prestressed members will enhance the strength and also control crack formation and crack width. Under ultimate load, a partially prestressed member usually demonstrates greater ductility than a fully prestressed member. Therefore, it will be able to absorb more energy under extreme dynamic loading such as an earthquake or explosion.

Because mild steel does not lose strength as rapidly as prestressing strands at elevated temperature, it is sometimes added to prestressed members to improve their fire-resistance rating. See Chapter 9 of the *PCI Design Handbook* (1992) and *Design for Fire Resistance of Precast Prestressed Concrete* (1989) for more information.

Partial prestressing is not without some disadvantages. Under repeated loading, the fatigue life of a partially prestressed member can be a concern. In addition, durability is a potential problem for partially prestressed members because they can be cracked under full service load. Recent studies (Harajli and Naaman 1985a; Naaman 1989; and Naaman and Founas 1991), however, have shown that fatigue strength depends on the range of stress variation of the strand (refer to [Chapter 4](#)) and that durability is related more to cover and spacing of reinforcement than to crack width, so these concerns can be addressed with proper design and detailing of the reinforcement (Beeby 1978 and 1979).

1.5—Partial prestressing and reinforcement indexes

Several indexes have been proposed to describe the extent of prestressing in a structural member. These indexes are useful in comparing relative performances of members made with the same materials, but caution should be exercised in using them to determine absolute values of such things as deformation and crack width. Two of the most common indexes are the degree of prestress λ , and the partial prestressing ratio (*PPR*). These indexes are defined as

$$\lambda = \frac{M_{dec}}{M_D + M_L} \quad (1-1)$$

where

M_{dec} = decompression moment (the moment that produces zero concrete stress at the extreme fiber of a section, nearest to the centroid of the prestressing force, when added to the action of the effective prestress alone);

M_D = dead-load moment; and

M_L = live-load moment

and

$$PPR = \frac{M_{np}}{M_n} \quad (1-2)$$

where

M_{np} = nominal moment capacity provided by prestressed reinforcement; and

M_n = total nominal moment capacity.

In the previous expressions, all moments are computed at critical sections. This report will generally use the PPR to describe the extent of prestressing in flexural members. The tests, studies, and examples described in this report usually concern members with $PPR < 1$, and the members are pre-tensioned unless otherwise noted.

Characterizing the total amount of flexural reinforcement in a member is also important. This will be done with the reinforcement index ω

$$\omega = \rho \frac{f_y}{f'_c} + \rho_p \frac{f_{ps}}{f'_c} - \rho' \frac{f_y}{f'_c} \quad (1-3)$$

where

$$\rho = \frac{A_s}{bd}$$

$$\rho' = \frac{A'_s}{bd}$$

$$\rho_p = \frac{A_{ps}}{bd_p}$$

and

A_{ps} = area of prestressed reinforcement in tension zone, in.² (mm²);

A_s = area of nonprestressed tension reinforcement, in.² (mm²);

A'_s = area of nonprestressed compression reinforcement, in.² (mm²);

b = width of compression face of member, in. (mm);

d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in. (mm);

d_p = distance from extreme compression fiber to centroid of prestressed reinforcement, in. (mm);

f'_c = specified compressive strength of concrete, psi (MPa);

f_{ps} = stress in prestressed reinforcement at nominal strength, psi (MPa); and

f_y = yield strength of nonprestressed reinforcement, psi (MPa).

1.6—Report objective

The objective of this report is to summarize the state of the art of the current knowledge as well as recent developments in partial prestressing so that engineers who are not experienced in prestressed concrete design will have a better understanding of the concept.

CHAPTER 2—PARTIALLY PRESTRESSED MEMBERS UNDER STATIC LOADING

2.1—Behavior

There are a number of investigations on the behavior of partially prestressed concrete beams under static loading (Abeles 1968; Burns 1964; Cohn and Bartlett 1982; Harajli 1985; Harajli and Naaman 1985a; Shaikh and Branson 1970; Thompson and Park 1980a; and Watcharannuay 1984). The following results were observed for beams having the same ultimate resistance in flexure but reinforced with various combinations of prestressed and nonprestressed reinforcement:

- Partially prestressed beams show larger ultimate deflections, higher ductility, and higher energy absorption than fully prestressed beams;
- Partially prestressed beams tend to crack at lower load levels than fully prestressed beams. Average crack spacing and crack widths are smaller. The stiffness of partially prestressed beams after cracking is larger;
- For a given reinforcement index ω , the moment-curvature relationship is almost independent of the ratio of the tensile reinforcement areas (prestressed versus nonprestressed);
- Changing the effective prestress in the prestressing tendons does not lead to any significant change in the ultimate resistance and curvature of flexural members; and
- A decrease in effective prestress leads to an increase in yield curvature and a decrease in curvature ductility.

2.2—Methods of analysis

Several methods can be followed to analyze partially prestressed concrete members subjected to bending. In terms of assumptions, purpose, and underlying principles, they are identical to those used for reinforced and prestressed concrete (Nilson 1976, Naaman and Siriakson 1979, Siriakson and Naaman 1979, Al-Zaid and Naaman 1986, and Tadros 1982).

2.2.1 Linear elastic analysis—In the elastic range of behavior, the analysis must accommodate either a cracked or an uncracked section subjected to bending, with or without prestress in the steel. The usual assumptions of plane strain distribution across the section, linear stress-strain relations, and perfect bond between steel and concrete remain applicable. Linear elastic analysis under service loads assuming an uncracked section is used for prestressed concrete. In the U.S., the design of reinforced concrete is predominantly based on strength requirement, but a linear elastic analysis under service loads is also necessary to check serviceability limitations such as crack widths, deflections, and fatigue.

Prestressed concrete beams can act as cracked or uncracked sections, depending on the level of loading. In contrast to reinforced concrete, the centroidal axis of the cracked section does not coincide with the neutral axis point of zero stress (Fig. 2.1). Moreover, the point of zero stress does not remain fixed, but moves with a change in applied load. When the effective prestress tends toward zero, the point of zero stress and the centroidal axis tend to coincide. Generalized equations have been developed to determine the zero stress point based on satisfying equilibrium, strain compatibility, and stress-strain relations (Nilson 1976; Naaman and Siriakson 1979; Siriakson and Naaman 1979; Al-Zaid and Naaman 1986).

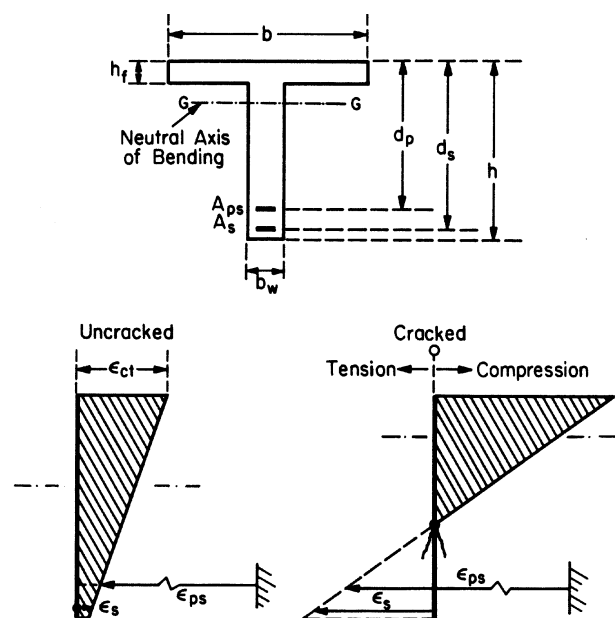


Fig. 2.1—Assumed stress or strain distribution in linear elastic analysis of cracked and uncracked sections (Naaman 1985).

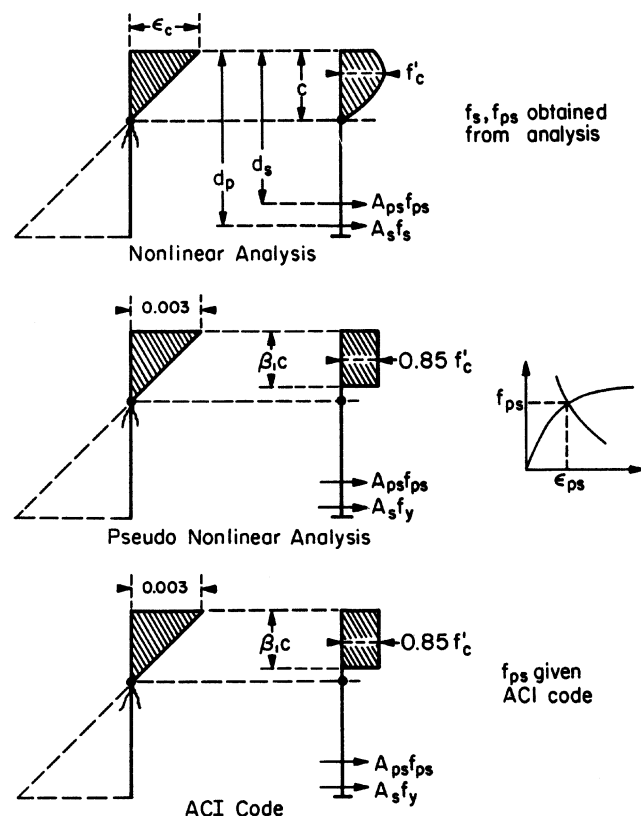


Fig. 2.2—Assumed strain distribution and forces in: (a) nonlinear analysis; (b) approximate nonlinear analysis; and (c) ultimate strength analysis by ACI Code (Naaman 1985).

1986). They usually are third-order equations with respect to member depth. Although they can be solved iteratively, charts, tables, and computer programs have been developed for their solution (Tadros 1982, Moustafa 1977). These equations

provide unified treatment for cracked reinforced, prestressed, and partially prestressed sections.

2.2.2 Strength analysis—At ultimate or nominal moment resistance, the assumptions related to the stress and strain distributions in the concrete, such as the compression block in ACI 318, or the stress and strain in the steel (such as yielding of the reinforcing steel) are identical for reinforced, prestressed, and partially prestressed concrete (Fig. 2.2). The corresponding analysis is the same and leads to the nominal moment resistance of the section. Numerous investigations have shown close correlation between the predicted (based on ACI 318) and experimental values of nominal moments. The ACI 318 analysis, however, resulted in conservative predictions of section curvatures at ultimate load, leading to erroneous estimates of deformations and deflections (Wang et al. 1978, Naaman et al. 1986). To improve the prediction of nominal moment and curvature, either a nonlinear or a simplified nonlinear analysis may be followed.

Simplified nonlinear analysis—In the simplified nonlinear analysis procedure (also called pseudo-nonlinear analysis), the actual stress-strain curve of the steel reinforcement is considered while the concrete is represented by the ACI 318 compression block. A solution can be obtained by solving two nonlinear equations with two unknowns, namely the stress and the strain in the prestressing steel at nominal moment resistance (Naaman 1977, Naaman 1983b).

Nonlinear analysis—The best accuracy in determining nominal moments and corresponding curvatures is achieved through a nonlinear analysis procedure (Cohn and Bartlett 1982, Naaman et al. 1986, Harajli and Naaman 1985b, Moustafa 1986). Nonlinear analysis requires as input an accurate analytical representation of the actual stress-strain curves of the component materials (concrete, reinforcing steel, and prestressing steel). Typical examples can be found in two references (Naaman et al. 1986, Moustafa 1986).

2.3—Cracking

Partially prestressed concrete permits cracking under service loads as a design assumption. To satisfy serviceability requirements, the maximum crack width should be equal to, or smaller than, the code-recommended limits on crack width.

The maximum allowable crack widths recommended by ACI Committee 224 (1980) for reinforced concrete members can be used, preferably with a reduction factor for prestressed and partially prestressed concrete members. To select the reduction factor, consideration should be given to the small diameter of the reinforcing elements (bars or strands), the cover, and the exposure conditions.

Only a few formulas are used in the U.S. practice to predict crack widths in concrete flexural members. Because the factors influencing crack widths are the same for reinforced and partially prestressed concrete members, existing formulas for reinforced concrete can be adapted to partially prestressed concrete. Five formulas (ACI 224 1980; Gergely and Lutz 1968; Nawy and Potyondy 1971; Nawy and Huang 1977; Nawy and Chiang 1980; Martino and Nilson 1979; and Meier and Gergely 1981) applicable to partially prestressed beams are summarized in Table 2.1 (Naaman 1985). The vari-

Table 2.1—Crack width prediction equations applicable to partially prestressed beams (Naaman 1985)

Source	Equation* with U.S. system, (in., ksi)	Equation* with SI system, (mm., N/mm ²)
Gergely and Lutz (1968) ACI Code (1971, 1977, and 1983) ACI Committee 224 (1980)	$W_{max} = 7.6 \times 10^{-5} \beta f_s \sqrt[3]{d_c A_b}$ f_s = tensile stress in reinforcing steel d_c = concrete cover to center of closest bar layer A_b = concrete tensile area per bar β = ratio of distances from tension face and steel centroid to neutral axis	Multiply expression by 0.1451 Note: ACI Committee 224 recommends multiplication factor of 1.5 when strands, rather than deformed bars, are used nearest to beam tensile face.
Nawy and Potyondy (1971)	$W_{max} = 1.44 \times 10^{-4} (f_s - 8.3)$	$= 5.31 \times 10^{-4} (f_s - 57.2)$
Nawy and Huang (1977) Nawy and Chiang (1980)	$W_{max} = \alpha \times 10^{-5} \beta \frac{A_t}{\Sigma O} \Delta f_{ps}$ A_t = area of concrete tensile zone ΣO = sum of perimeters of bonded reinforcing elements Δf_{ps} = net stress change in prestressing steel after decompression $\alpha = \begin{cases} 5.85 & \text{if pretensioning} \\ 6.51 & \text{if post-tensioning} \end{cases}$	Multiply expression by 0.1451
Martino and Nilson (1979)	$W_{max} = 14 \times 10^{-5} d'_c f_s + 0.0031$ d'_c = concrete clear cover	$= 2 \times 10^{-5} d'_c f_s + 0.08$
Meier and Gergely (1981)	$\begin{cases} W_{max} = C_1 \epsilon_{ct} d_c & (1) \\ W_{max} = C_2 \epsilon_{ct} d_c \sqrt[3]{A_b} & (2) \end{cases}$ C_1, C_2 = bond coefficients For reinforcing bars: $C_1 = 12$; $C_2 = 8.4$ For strands: $C_1 = 16$; $C_2 = 12$ ϵ_{ct} = nominal concrete tensile strain at tensile face	(1) Same equation (2) Multiply by 220

*In the formulas shown, f_s can be replaced by Δf_{ps} when applied to partially prestressed concrete.

able tensile stress in the reinforcing steel f_s should be replaced by the stress change in the prestressing steel after decompression Δf_{ps} . The ACI 318 formula initially developed by Gergely and Lutz (1968) for reinforced concrete could be used as a first approximation for partially prestressed concrete. Meier and Gergely (1981), however, suggested a modified form (shown in Table 2.1) for the case of prestressed concrete. This alternate formula uses the nominal strain at the tensile face of the concrete (instead of the stress in the steel), and the cover to the center of the steel d_c . Both the stress in the steel and the clear concrete cover are found to be the controlling variables in the regression equation derived by Martino and Nilson (1979). The two prediction equations proposed by Nawy and Huang (1977) and Nawy and Chiang (1980) contain most of the important parameters found in the cracking behavior of concrete members except the concrete cover, which is accounted for indirectly. Moreover, they are based on actual experimental results on prestressed and partially prestressed beams.

As pointed out by Siriaksorn and Naaman (1979), large differences can be observed in predicted crack widths depending on the prediction formula used. Harajli and Naaman (1989) compared predicted crack widths with observed crack widths from tests on twelve partially prestressed concrete beams. They considered the three prediction equations recommended by Gergely and Lutz (1968), Nawy and Hua-

ng (1977), and Meier and Gergely (1981). Although none of the three equations gave sufficiently good correlation with experimental data for all conditions, the following observations were made (Fig. 2.3):

- The Gergely and Lutz equation gave a lower prediction in all cases (Fig. 2.3(a));
- The Meier and Gergely equation gave the worst correlation (Fig. 2.3(c)); and
- The Nawy and Huang equation gave a higher prediction in most cases (Fig. 2.3(b)).

Although more experimental data are needed to improve the accuracy of crack-width prediction equations available in U.S. practice, there is sufficient information to judge if the serviceability, with respect to cracking or crack width under short-term loading, is satisfactory for a partially prestressed member. The effects of long-term loading and repetitive loading (fatigue) on the crack widths of partially prestressed members need to be further clarified. A research investigation provided an analytical basis to deal with the problem (Harajli and Naaman 1989); however, the proposed methodology is not amenable to a simple prediction equation that can be easily implemented for design.

2.4—Deflections

Fully prestressed concrete members are assumed to be uncracked and linearly elastic under service loads. Therefore, short-term deflections are determined using general

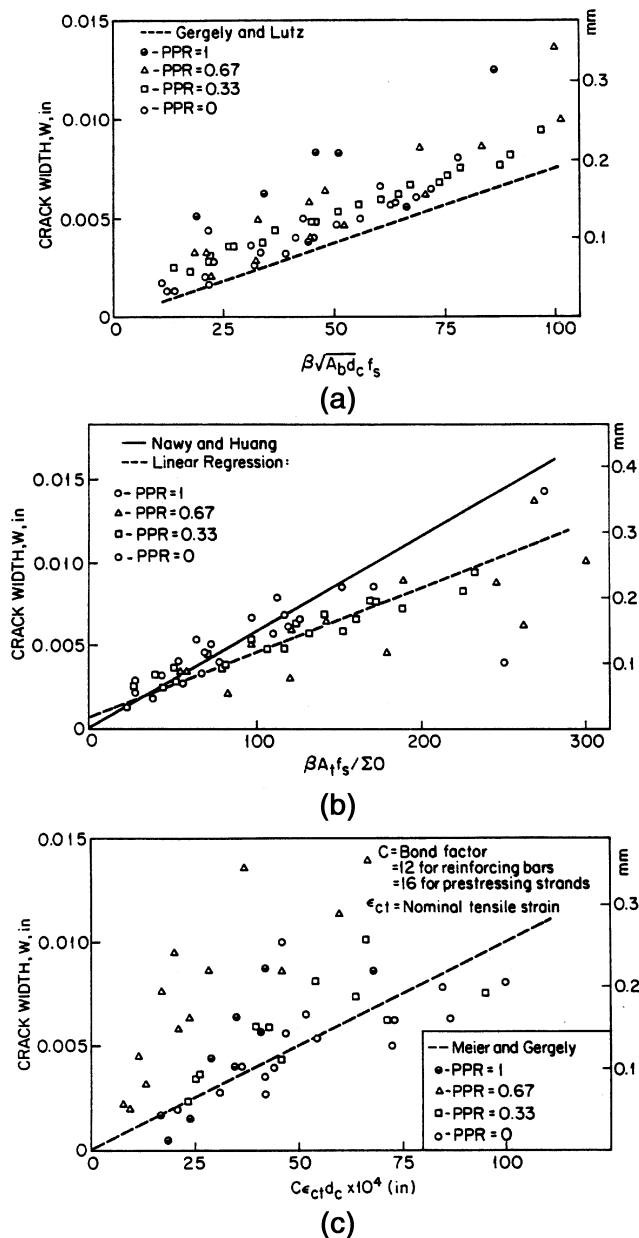


Fig. 2.3—Comparison of observed and theoretically predicted crack widths (Naaman 1985).

principles of mechanics. To compute short-term deflections, customary U.S. practice is to use the gross moment of inertia I_g for pretensioned members, or the net moment of inertia I_n for members with unbonded tendons, and the modulus of elasticity of concrete at time of loading or transfer E_{ci} . Several approaches proposed by various researchers to compute short-term and long-term deflections in prestressed or partially prestressed uncracked members are summarized in Table 2.2 (Branson and Kripanarayanan 1971; Branson 1974; Branson 1977; Naaman 1982a; Naaman 1983a; Branson and Trost 1982a; Branson and Trost 1982b; Martin 1977; Tadros et al. 1975; Tadros et al. 1977; Dilger 1982; and Moustafa 1986). Although no systematic evaluation or comparison of these different approaches has been undertaken, for common cases they lead to results of the same order.

The widely accepted concept of the effective moment of inertia I_{eff} , initially introduced by Branson (1977) for reinforced concrete, has been examined by several researchers and modified accordingly to compute the deflection in cracked prestressed and partially prestressed members. The modified effective moment of inertia is defined (Naaman 1982a) as

$$I_{eff} = I_{cr} + ((M_{cr} - M_{dec}) / (M_a - M_{dec}))^3 (I_g - I_{cr}) \leq I_g \quad (2-1)$$

where

I_g = gross moment of inertia, in.⁴ (mm⁴);
 I_{cr} = moment of inertia of cracked section, in.⁴ (mm⁴);
 M_{cr} = cracking moment, in.-k (mm-N);
 M_{dec} = decompression moment, in.-k (m-N); and
 M_a = applied moment, in.-k (m-N).

Although there is general agreement for the use of the previous expression, substantial divergence of opinion exists as to the computation of I_{cr} and M_{dec} . The computation difference is whether the moment of inertia of the cracked section should be computed with respect to the neutral axis of bending or with respect to the zero-stress point, and whether the decompression moment should lead to decompression at the extreme concrete fiber or whether it should lead to a state of zero curvature in the section. The discussion of Tadros' paper (1982) by several experts in the field is quite informative on these issues. A systematic comparison between the various approaches, combined with results from experimental tests, is given in work by Watcharaumnay (1984), who observed that the use of I_{cr} with respect to the neutral axis of bending is preferable, while the use of M_{dec} as that causing decompression at the extreme concrete fiber, is easier and leads to results similar to those obtained using the zero curvature moment.

2.5—Shear and torsion

2.5.1 General—Nonprestressed and fully prestressed concrete (tensile stress in the concrete under full service load is zero) are the two limiting cases of steel-reinforced concrete systems. Partially prestressed concrete represents a continuous transition between the two limit cases. A unified approach in design to combined actions including partial prestressing would offer designers a sound basis to make the appropriate choice between the two limits (Thurlimann 1971).

The equivalent load concept provides a simple and efficient design of prestressed concrete structures under combined actions (Nilson 1987). For example, this approach allows the designer to calculate the shear component of the prestress anywhere in the beam, simply by drawing the shear diagram due to the equivalent load resulting from a change in the vertical alignment of the tendon (Fig. 2.4). That equivalent load, together with the prestressing forces acting at the ends of the member through the tendon anchorage, may be looked upon as just another system of external forces acting on the member. This procedure can be used for both statically determinate and indeterminate structures, and accounts for the effects of secondary reactions due to

Table 2.2—Deflection prediction equations for prestressed and partially prestressed beams (from Naaman 1985)

Source	Short-term instantaneous deflection	Long-term or additional long-term deflection	Remarks
ACI 435 (1963)	Δ_t is obtained from elastic analysis using F_t , E_{ct} , and I_g .	Long-term deflection obtained by integrating curvatures with due account for creep effects and prestress losses with time.	- Uncracked section; and - No provisions for A_s and A'_s.
ACI Code Section 9.5 (1971, 1977, and 1983)	Δ_t shall be obtained from elastic analysis using I_g for uncracked sections.	Δ_{add} shall be computed, taking into account stresses under sustained load, including effects of creep, shrinkage, and relaxation.	No provisions for partial prestressing (cracking, A_s and A'_s).
Branson et al. (1971, 1974, and 1977)	Δ_t is obtained from elastic analysis using E_{ct} and I_g .	$\Delta_{add} = \left[\eta - 1 + \left(\frac{1 + \eta}{2} \right) k_r C_{cu} \right] (\Delta_t)_{F_t} + k_r C_{CU} (\Delta_t)_G + K_{CA} k_r C_{CU} (\Delta_t)_{SD}$ where C_{CU} = ultimate creep coefficient of concrete; $\eta = F/F_t$; $k_r = 1/(1 + A_s/A_{ps})$; and K_{CA} = age at loading factor for creep	- Uncracked section; k_r is applicable only when $(\Delta_t)_{F_t + G}$ is a camber; and F_t = initial prestressing force immediately after transfer.
Naaman (1982 and 1983)	Δ_t is obtained using I_g and the predicted elastic modulus at time of loading $E_c(t)$.	The long-term deflection is estimated from: $\Delta(t) = \phi_1(t) \frac{l^2}{8} + [\phi_2(t) - \phi_1(t)] \frac{l^2}{48}$ where $\phi_1(t)$ = midspan curvature at time t ; $\phi_2(t)$ = support curvature at time t ; $\phi(t) = M/[E_{ce}(t) \times I]$; and $E_{ce}(t)$ = equivalent modulus	- Uncracked section; - The pressure line is assumed resulting from the sustained loadings; - The profile of the pressure line is assumed parabolic; - Prestress losses must be estimated a priori; - Design chart is provided for the equivalent modulus; and A_s and A'_s are accounted for through I_t and neutral axis of bending.
Bronson and Trost (1982)	For cracked members, the short-term deflection is computed using I_{eff} modified for partial prestressing.	Long-term deflection is not addressed but it is assumed that for a given Δ_t , the earlier method is applicable.	Cracked members.
Martin (1977)	Δ_t is obtained from elastic analysis using E_{ct} and I_g .	$\Delta_{add} = \lambda_1 (\Delta_t)_G + \lambda_2 (\Delta_t)_{F_t}$ $\lambda_1 = k_r \frac{E_{ci}}{E_c} \alpha$ $\alpha = (2 - 1.2 A'_s / A_s) \geq 0.6$ $\lambda_2 = \eta \lambda_1$	k_r = same as Branson; - Uncracked section; - Design values of λ_1 and λ_2 were recommended; and - The method is adopted in <i>PCI Design Handbook</i>.
Tadros et al. (1975 and 1977)	Δ_t is obtained from elastic analysis using $E_c(t)$ and I_g .	The long-term deflection is obtained by integrating the curvatures modified by a creep recovery parameter and a relaxation reduction factor that are time-dependent.	- Uncracked sections; and - For common loading cases, only the curvatures at the support and midspan sections are needed.
Dilger (1982)	Δ_t is obtained from long-term deflection expression at initial loading time. The age adjusted effective modulus and a creep transformed moment of inertia are used.	The long-term deflection is obtained by integrating the curvature along the member. The time-dependent curvature is modified by the effect of an equivalent force acting at the centroid of the prestressing steel due to creep and shrinkage strain. $\phi(t) = \phi_i C_c(t) - \frac{M_c}{I_{tr} E_{ca}(t)}$ I_{tr} = transformed moment of inertia; M_c = moment due to equivalent transformed force; and $E_{ca}(t)$ = age adjusted modulus	- Uncracked sections; and - A relaxation reduction factor is used.
Moustafa (1986)	Δ_t is obtained from nonlinear analysis using actual material properties.	The nonlinear analysis takes both creep and shrinkage into account, using ACI creep and shrinkage functions and a time step method.	A computer program is available from PCI to perform the nonlinear analysis.

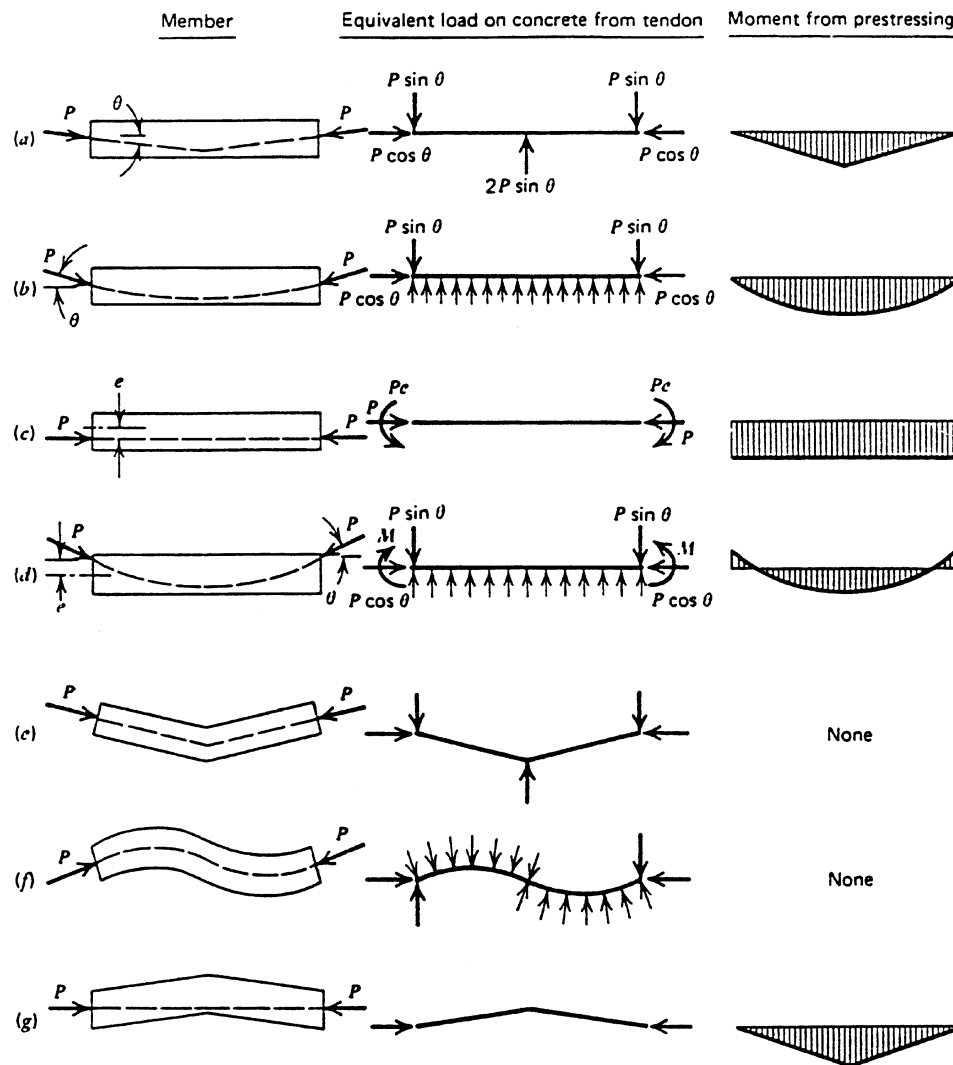


Fig. 2.4—Equivalent loads and moments produced by prestressing tendons (Nilson 1987);
 P = prestressing force.

prestressing, as well. This approach allows the designer to treat a prestressed concrete member as if it was a nonprestressed concrete member. The prestressing steel is treated as mild (passive) reinforcement for ultimate conditions, with a remaining tensile capacity of $(f_{ps} - f_{pe})$, where f_{ps} is the stress in the reinforcement at nominal strength, and f_{pe} is the effective stress in the prestressed reinforcement (after allowance for all losses).

Most codes of practice (ACI 318; AASHTO *Bridge Design Specifications*, Eurocode 2; and CSA *Design of Concrete Structures for Buildings*) use sectional methods for design of conventional beams under bending, shear, and torsion. Truss models provide the basis for these sectional design procedures that often include a term for the concrete contribution (Ramirez and Breen 1991). The concrete contribution supplements the sectional truss model to reflect test results in beams and slabs with little or no shear reinforcement and to ensure economy in the practical design of such members.

In design specifications, the concrete contribution has been taken as either the shear force or torsional moment at

cracking, or as the capacity of an equivalent member without transverse reinforcement. Therefore, detailed expressions have been developed in terms of parameters relevant to the strength of members without transverse reinforcement. These parameters include the influence of axial compression, member geometry, support conditions, axial tension, and prestress.

2.5.2 Shear—The following behavioral changes occur in partially prestressed members at nominal shear levels, as some of the longitudinal prestressing steel in the tension face of the member is replaced by mild reinforcement, but the same total flexural strength is maintained:

- Due to the lower effective prestress, the external load required to produce inclined cracking is reduced. This results in an earlier mobilization of the shear reinforcement; and
- After inclined cracking, there is a reduction in the concrete contribution. The reduction is less significant as the degree of prestressing decreases. This can be explained as follows:

- The addition of mild reinforcement results in an increase in the cross-sectional area of the longitudinal

tension reinforcement and the reinforcement stiffness; and

- The increase in stiffness of the longitudinal tension reinforcement delays the development of the cracking pattern, so that the cracks are narrower and the flexural compression zone is larger than in fully prestressed members of comparable flexural strength.

These behavioral changes are well documented in a series of shear tests carried out by Caflich et al. (1971). In this series of tests, the only variable was the degree of prestressing. The cross sections of the prestressing steel and the reinforcing steel were selected so that all the beams had the same flexural strength. These tests also showed that for the same external load, a higher degree of prestressing delays the onset of diagonal cracking and results in a decrease in the stirrup forces. The decrease in stirrup forces can be explained by the fact that a higher degree of prestressing in the web of the member results in a lower angle of inclination of the diagonal cracks. The lower angle of inclination of the cracks leads to the mobilization of a larger number of stirrups.

In ACI 318, a cursory review of the design approach for shear indicates that partially prestressed members can be designed following the same procedure as for fully prestressed members. In ACI 318, it is assumed that flexure and shear can be handled separately for the worst combination of flexure and shear at a given section. The analysis of a beam under bending and shear using the truss approach clearly indicates that, to resist shear, the member needs both stirrups and longitudinal reinforcement. The additional longitudinal tension force due to shear can be determined from equilibrium conditions of the truss model as $(V \cot \theta)$, where V is the shear force at the section, and θ is the angle of inclination of the inclined struts with respect to the longitudinal axis of the member.

In the shear provisions of ACI 318, no explicit check of the shear-induced force in the longitudinal reinforcement is performed (Ramirez 1994). The difference between the flexural strength requirements for the prestress reinforcement and the ultimate tensile capacity of the reinforcement can be used to satisfy the longitudinal tension requirement. The 1994 AASHTO LRFD Bridge Design Specifications, in the section for shear design, includes a check for longitudinal reinforcement. These recommendations are based on a modified compression field theory (Vecchio and Collins 1986).

2.5.3 Torsion—ACI 318 includes design recommendation for the case of torsion or combined shear and torsion in prestressed concrete members. These provisions model the behavior of a prestressed concrete member before cracking as a thin-walled tube and after cracking using a space-truss model with compression diagonals inclined at an angle θ around all faces of the member. For prestressed members, θ can be taken equal to 37.5 degrees if the effective prestressing force is not less than 40% of the tensile strength of the prestressed reinforcement. For other cases, θ can be taken equal to 45 degrees. This approach is based on the work carried out in the 1960s and 1970s by European investigators led by Thurlimann (1979). This work proposed a method

supported by the theory of plasticity, in which a space truss with variable inclination of compression diagonals provides a lower-bound (static) solution.

This procedure is representative of the behavior of thin-walled tubes in torsion. For these members, the shear stresses induced by torsion can be determined using only equilibrium relationships. Because the wall of the tube is thin, a constant shear stress can be assumed across its thickness. In the longitudinal direction, equilibrium conditions dictate that the torsion-induced shear stresses be resisted by a constant shear flow around the perimeter of the section. For other sections before cracking, the strength in torsion can be computed from the elastic theory (de Saint-Venant 1956) or from the plastic theory (Nadai 1950). Rather than using these more complex approaches, an approximate procedure is used in ACI 318 based on the concept that most torsion is resisted by the high shear stresses near the outer perimeter of the section. In this approach, the actual cross section before cracking is represented by an equivalent thin-walled tube with a wall thickness t of

$$t = 0.75 \frac{A_{cp}}{P_{cp}} \quad (2-2)$$

where A_{cp} = area enclosed by outside perimeter of concrete cross section, and P_{cp} = outside perimeter of the concrete cross section. While the area enclosed by the shear flow path, A_o , could be calculated from the external dimensions and wall thickness of the equivalent tube, it is reasonable to approximate it as equal to $2A_{cp}/3$. Cracking is assumed to occur when the principal tensile stress reaches $4\sqrt{f'_c}$. For prestressed members, the cracking torque is increased by the prestress. A Mohr's Circle analysis based on average stresses indicates that the torque required to cause a principal tensile stress equal to $4\sqrt{f'_c}$ is the corresponding cracking torque of a nonprestressed beam times

$$\sqrt{1 + \frac{f_{pc}}{4\sqrt{f'_c}}} \quad (2-3)$$

where f_{pc} in psi is the average precompression due to prestress at the centroid of the cross section resisting the externally applied loads or at the junction of web and flange if the centroid lies within the flange.

In ACI 318, the design approach for combined actions does not explicitly consider the change in conditions from one side of the beam to the other. Instead, it considers the side of the beam where shear and torsional effects are additive. After diagonal cracking, the concrete contribution of the shear strength V_c remains constant at the value it has when there is no torsion, and the torsion carried by the concrete is taken as zero. The approach in ACI 318 has been compared with test results by MacGregor and Ghoneim (1995).

In the AASHTO LRFD Specifications, the modified compression field theory proposed for members under shear has

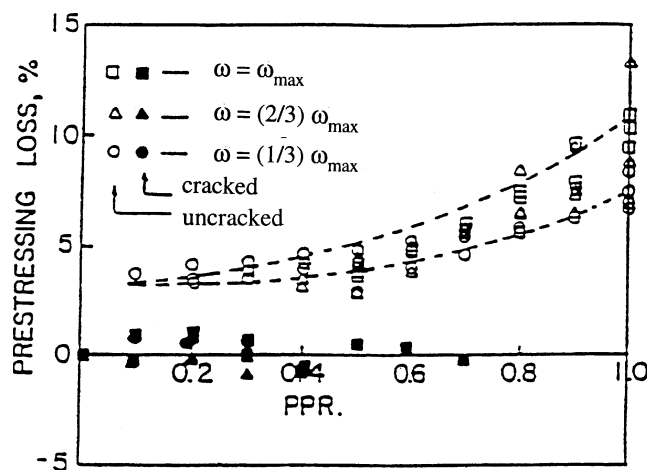


Fig. 3.1—Typical stress change in prestressing steel at end of service life for sections cracked and uncracked under permanent loads (Watcharaumnay and Naaman 1985).

been extended to include the effects of torsion. Similar to the ACI 318 procedure, the AASHTO approach concentrates on the design of the side of the beam where the shear and torsional stresses are additive.

As in the case of shear, torsion leads to an increase in the tensile force on the longitudinal reinforcement. The longitudinal reinforcement requirement for torsion should be superimposed with the longitudinal reinforcement requirement for bending that acts simultaneously with the torsion. In ACI 318, the longitudinal tension due to torsion can be reduced by the compressive force in the flexural compression zone of the member. Furthermore, in prestressed beams, the total longitudinal reinforcement, including tendons at each section, can be used to resist the factored bending moment plus the additional tension induced by torsion at that section.

ACI 318 and AASHTO Specifications recognize that, in many statically indeterminate structures, the magnitude of the torsional moment in a given member will depend on its torsional stiffness. Tests have shown (Hsu 1968) that when a member cracks in torsion, its torsional stiffness immediately after cracking drops to approximately 1/5 of the value before cracking, and at failure can be as low as 1/16 of the value before cracking. This drastic drop in torsional stiffness allows a significant redistribution of torsion in certain indeterminate beam systems. In recognizing the reduction of torsional moment that will take place after cracking in the case of indeterminate members subjected to compatibility-induced torsion, ACI 318R states that a maximum factored torsional moment equal to the cracking torque can be assumed to occur at the critical sections near the faces of supports. This limit has been established to control the width of the torsional cracks at service loads.

CHAPTER 3—TIME-DEPENDENT BEHAVIOR

3.1—Prestress losses

The stress in the tendons of prestressed concrete structures decreases continuously with time. The total reduction in stress during the life span of the structure is termed “total prestress loss.” The total prestress loss consists of several

components, which are generally grouped into instantaneous and time-dependent losses.

Instantaneous losses are due to elastic shortening, anchorage seating, and friction. They can be computed for partially prestressed concrete in a manner similar to that for fully prestressed concrete.

Time-dependent losses are due to shrinkage and creep of concrete and relaxation of prestressing steel. Several methods are available to determine time-dependent losses in prestressed concrete members: lump-sum estimate of total losses, lump-sum estimates of separate losses (such as loss due to shrinkage or creep), and calculation of losses by the time-step method. Because the numerical expressions—described, for instance, in the AASHTO’s *Standard Specifications for Highway Bridges* or the *PCI Design Handbook* (1992)—for the first two methods were developed assuming full prestressing, they are not applicable to partially prestressed members.

The combined presence of nonprestressed reinforcing bars and a lower level of prestress in partially prestressed concrete should lead to smaller time-dependent prestress losses than for fully prestressed concrete. Another significant factor is that a partially prestressed member can be designed as a cracked member under sustained loading. A computerized time-step analysis of the concrete and steel stresses along partially prestressed sections shows that the effect of creep of concrete on the stress redistribution between the concrete and steel tends to counteract the effect of prestress losses over time. This is particularly significant for members that are cracked under permanent loads. The result is illustrated in Fig. 3.1 (Watcharaumnay and Naaman 1985), which was derived from the analysis of 132 beams with various values of the partially prestressed ratio (PPR) and the reinforcement index ω . While time-dependent prestress losses can be 14% for uncracked fully prestressed sections, they remain low for cracked sections up to relatively high values of the PPR. Fig. 3.1 also shows that for uncracked sections, time-dependent prestress losses decrease with a decrease in PPR. Creep redistribution of force to reinforcing steel may reduce the precompression in the concrete.

An investigation under NCHRP Project 12-33 has addressed prestress losses in partially prestressed normal and high-strength concrete beams. This work was described by Naaman and Hamza (1993) and was adopted in the *Final Draft LRFD Specifications for Highway Bridge Design and Commentary* (Transportation Research Board 1993). It led to lump-sum estimates of time-dependent losses for partially prestressed beams that are assumed uncracked under the design sustained loading. These are summarized in Table 3.1 and can be used as a first approximation in design.

3.2—Cracking

Crack widths in reinforced and cracked partially prestressed concrete members subjected to sustained loads are known to increase with time. Bennett and Lee (1985) reported that crack widths increase at a fast rate during the early stages of loading then tend toward a steady-state increase. This is not surprising because deflections and