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The Shear Strength of Reinforced Concrete Members

Reported by ACI-ASCE Committee 426

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The state of the art of the knowledge of shear strength and behavior of reinforced and prestressed beams, deep beams, brackets, reinforced and prestressed concrete slabs, and other members is reviewed. Design criteria are discussed but specific design recommendations are not made.

The report is divided into five chapters. The first two discuss the general concepts of shear in concrete. The third, fourth, and fifth chapters discuss shear with respect to beams, special members, and slabs, respectively.

Keywords: beams (supports); brackets; building codes; columns (supports); concrete slabs; concretes; cracking (fracturing); deep beams; dowels; failure; joints (junctions); lightweight concrete; prestressed concrete; punching shear; reinforced concrete; reinforcing steels; reviews; shear properties; shear strength; shear stress; shear walls; stresses; structural analysis; structural design; web reinforcement.

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THE SHEAR STRENGTH OF REINFORCED CONCRETE MEMBERS

By the Joint ASCE-ACI Task Committee 426 on Shear and Diagonal
Tension of the Committee on Masonry and Reinforced Concrete
of the Structural Division

CHAPTER 1

1.1 Introduction

In the design of concrete structures an adequate margin of safety must be provided against any mode of failure that might occur under the forces that act upon the structure during its lifetime. One general type of failure that must be prevented is the so-called "shear failure," which in reality is a failure under combined shearing force and bending moment, plus, occasionally, axial load, or torsion, or both. Such failures reduce the strength of structural elements below the flexural capacity and considerably reduce the ductility of the elements. Especially for the latter reason, shear failures are generally considered undesirable.

The understanding and knowledge of the shear transfer mechanisms in various types of concrete structural elements has progressed significantly since the previous report issued by this Committee (7). This advance has occurred in three ways: though greater understanding of the fundamental mechanisms of shear transfer, better quantitative evaluation of the shear strength of structures, and the study of new types of structures or loading conditions.

This report reviews recent research results and design proposals in an attempt to establish the current state-of-the-art in our knowledge of shear transfer in reinforced concrete structures. Although specific design recommendations are not presented, it is hoped that this report will help designers, researchers, and specification writers to arrive at simple, universally applicable design concepts. Despite the tremendous number of references on this subject, the question of shear strength is far from being settled. In some instances the explanations of behavior and the design concepts that are presented are somewhat speculative and may change as more information becomes available.

Throughout the report an attempt is made to compare the shear transfer mechanisms in various types of members. The different modes of shear failure are listed in this chapter but will be discussed more fully in subsequent chapters. The strength of concrete under combined loadings is discussed briefly in Chapter 2, followed by a review of the fundamental methods of shear transfer commonly found in various structural members. Use will be made of this information in subsequent chapters.

Considering the importance of the shear problem in beams, Chapter 3 analyzes the nature of the shear strength of beams, the factors affecting strength, and the design of beams to prevent shear failures.

Chapter 4 deals with special types of members such as deep beams, brackets, walls, etc. A future continuation of this report will consider the shear strength of slabs.

Although this report covers the subject extensively, some peripheral material such as the historical background presented in the previous report

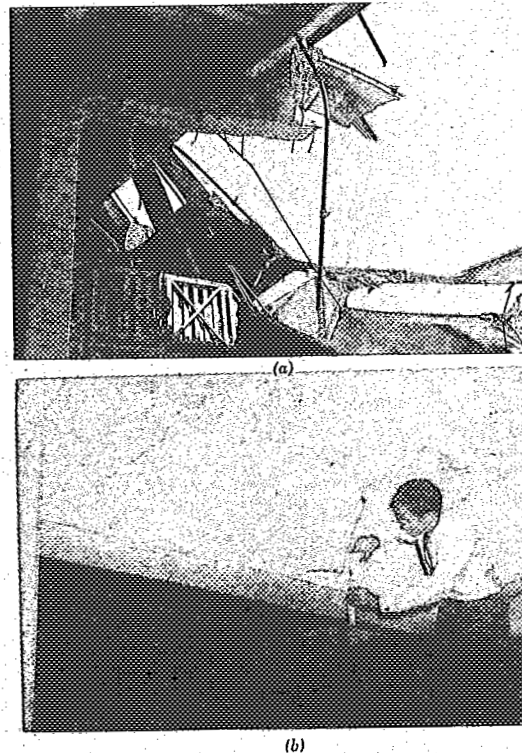


FIG. 1.1.—Shear Failure and Inclined Cracks in Continuous Roof Beams

of the Shear Committee (7) and in other papers (24,88) is not repeated herein. In addition, the interaction of shear and torsional effects is not considered. Design recommendations for this type of loading are given in Ref. 6, 41, and 114, and also in the reports of ACI Committee 438, "Torsion."

1.2 Shear Failures and Shear Distress in Structural Members

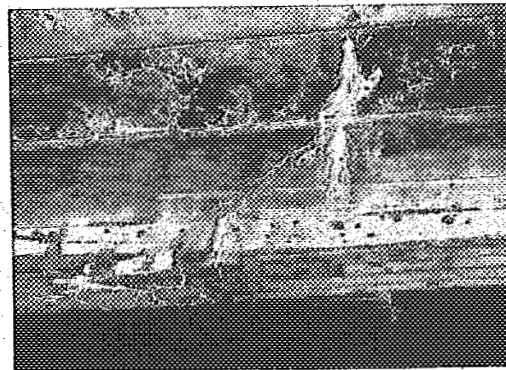
Shear distress and failures have been reported in beams, columns, walls, slabs, brackets, and other members. In general, each type of member exhibits different modes of cracking and failure, although the mechanisms by which shear is transferred within the member may be similar. For a beam subjected to a concentrated load, the major variable affecting the mode of failure is probably the ratio of the distance, a , from the load to the support to the depth of the member, d . As shown in Section 3.1, this ratio can also be expressed in terms of M/Vd . The latter form can be used to describe a wider variety of loading conditions, and therefore is frequently used in design recommendations. The relative magnitude of the flexural stresses, shear stresses, and vertical compression stresses varies, causing changes in behavior, as the M/Vd ratio is varied.

(a) Beams

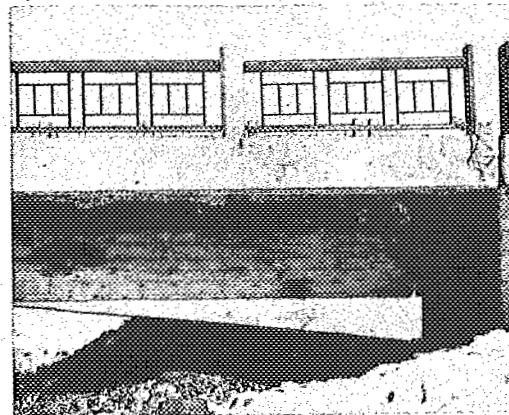
Reinforced concrete and prestressed concrete beams of moderate slenderness (a/d or $M/Vd = 2-6$) develop inclined cracks due to the combination of shear and flexural stresses. As explained in Chapter 3, beams may exhibit a number of different modes of shear failure, the most common of which is the crushing or shearing of the compression flange over the inclined crack which is often accompanied or initiated by splitting along the tension reinforcement.

Fig. 1.1(a) shows a shear failure in a continuous roof beam supporting precast roof panels. This failure, in an Air Materiel Warehouse, led to a revision in the shear provisions in the 1956 ACI Building Code and subsequently to considerable research into shear strength. An inclined crack in a beam and slab roof is shown in Fig. 1.1(b).

Fig. 1.2 shows inclined cracks in a three-span continuous highway bridge. This bridge has not collapsed although the cracks are wide



(a)



(b)

FIG. 1.2.—Inclined Cracks in T-Beam Highway Bridge

and deflections and corrosion of the reinforcement have become a problem.

(b) Deep Beams

Shearing failures of deep beams, brackets, and similar members differ considerably from those in normal beams, as shown in Figs. 1.3 and 1.4. This is largely due to the much steeper inclined cracks in deep members. This, in turn, changes the relative importance of the various shear transfer mechanisms as compared to normal beams. The shear strength of deep members is discussed in Chapter 4.

Figure 1.3(a) shows inclined cracks in corbels supporting precast beams in an industrial building. Similar failures in laboratory specimens are shown in Fig. 1.3(b) and (c). Fig. 1.4 shows inclined cracks in laboratory tests of deep beams.

(c) Shear Walls

Depending on their height to width ratio, shear walls represent a special case of beams or deep members. Such members may have significant axial compressions and occasionally axial tensions due to gravity loads and overturning moments. The transverse loads are generally applied to such walls by floor slabs attached to the full width of the wall, and frequently are cyclic in nature.

(d) Columns

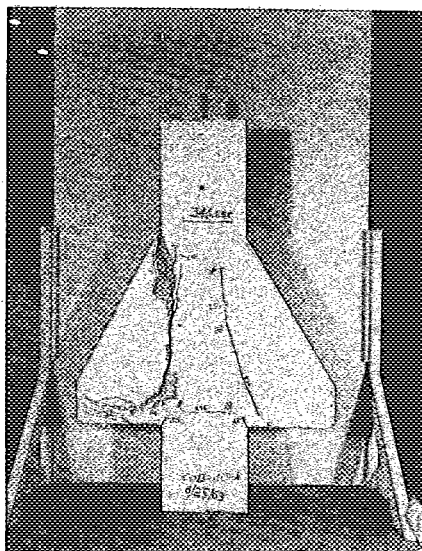
Columns may fail in shear in earthquakes. These failures appear to be of two types: either similar to that in an axially loaded beam involving inclined cracking, as shown in Fig. 1.5(a); or complete destruction of the column core apparently prior to inclined cracking, as shown in Fig. 1.5(b). Shear failures in columns are discussed in Section 4.9.

(e) Beam-Column Joints

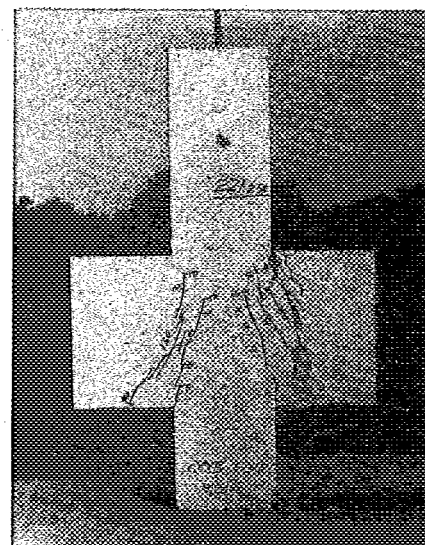
Inclined cracks may develop within the joints in reinforced concrete structures subjected to unbalanced gravity loads or seismic loads. Fig. 1.6 shows such a crack in a parking garage subjected only to gravity loads. This problem is discussed in Section 4.8.



(a)

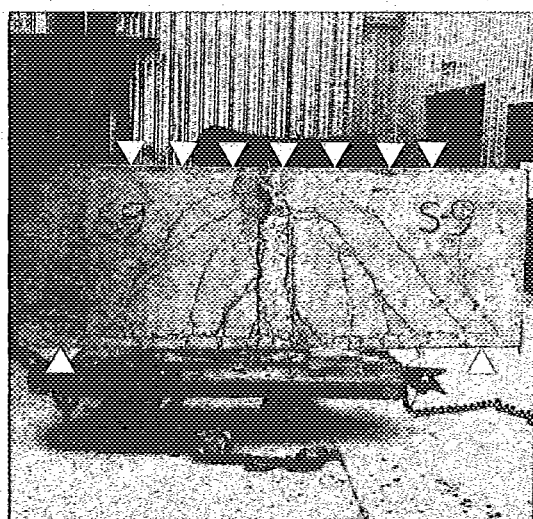


(b)

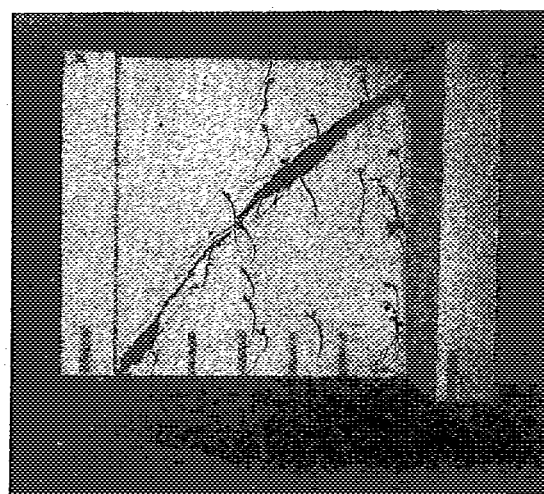


(c)

FIG. 1.3.—Shear Failures in Corbels: (a) Precast Building; (b) Laboratory Test; (c) Laboratory Test (Photos Courtesy of Portland Cement Association)



(a)



(b)

FIG. 1.4.—Shear Failures in Deep Beams

(f) Construction Joints and Joints in Precast Construction

Frequently shearing forces must be transferred across an interface such as a construction joint in a shear wall, the interface between a beam and a composite slab, or at the supports of precast elements. This action will be referred to as interface shear transfer and will be discussed more fully in Sections 2.2.2 and Chapter 4.

(g) Slabs and Footings

So-called punching shear failures occur in slabs and footings in the manner shown in Fig. 1.7. The shear capacity decreases as the moment transferred from the column to the slab, or vice versa, increases. Therefore, failures are more likely for exterior column connections, or connections where moment transfer occurs because of either the loading conditions or the location of perforations adjacent to the column. This type of member will be discussed in a future committee report.

1.3 Design for Shear

The last report of the Shear Committee, published in 1962, presented a basic design procedure for reinforced concrete members subjected to shear forces. The recommendations from this report still are the basis for such design in North America and are contained in the 1971 ACI Building Code (6). Two significant changes have occurred during the past decade:

1. The 1962 report dealt primarily with beams and punching shear failures in slabs. Since then, considerable work has been done on shear in deep beams, brackets, walls, etc., and this has led to design equations for such members. The development of these equations will be discussed in later chapters, particularly in Chapter 4.

2. The basic philosophy of the ACI Building Code with respect to shear failures has changed. Stirrup reinforcement restrains growth of inclined cracking, providing increased ductility and a warning in situations in which the sudden formation of inclined cracking in an unreinforced web may lead directly to distress. Such reinforcement may be of great value where members are subjected to unexpected tensile forces, settlements, or catastrophic loadings. For these reasons the 1971 ACI Building Code requires a minimum amount of shear reinforcing in most members.

During the next decade it is hoped that the design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner.

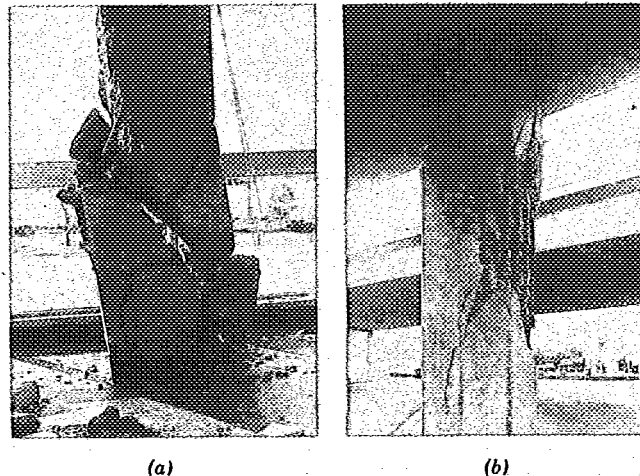
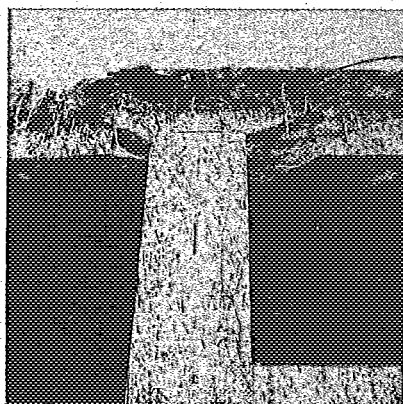


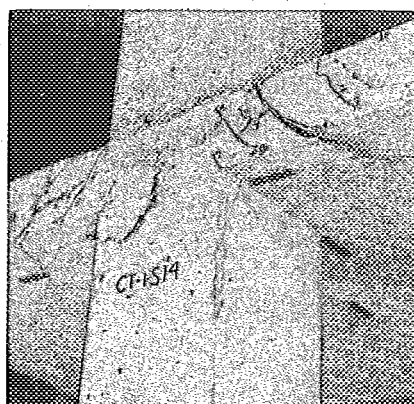
FIG. 1.5.—Shear Failures in Columns in San Fernando Earthquake (Photos Courtesy of National Bureau of Standards)



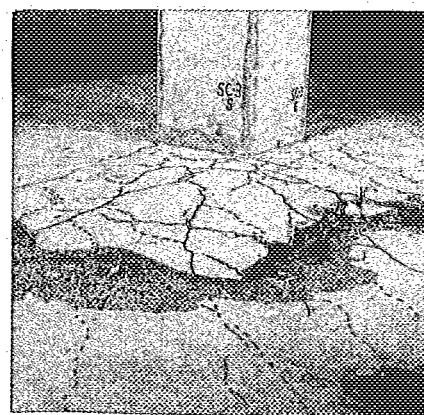
FIG. 1.6.—Inclined Crack in Beam-Column Joint



(a)



(b)



(c)

FIG. 1.7.—Shear Failures in Slabs: (a) Exterior Column in Building; (b) Exterior Column in Laboratory Test; (c) Interior Column in Laboratory Test (Photos Courtesy of Portland Cement Association)

CHAPTER 2

2. Basic Mechanisms of Shear Transfer

2.1 Failure Criteria for Concrete

The various modes of shear failure all involve cracking or crushing of concrete under complex states of stress as, e.g., when diagonal cracking, shear-compression failure, splitting, or web crushing occurs. Several studies in recent years have attempted to use refined concrete failure theories to investigate shear failure mechanisms.

The strength of concrete under combined shear and direct stresses seems to be predicted well by the octahedral stress theory (25). In this regard several investigators have emphasized that the intermediate principal stress is important in failure theories of concrete subjected to complex three-dimensional states of stress (142). In many applications, however, such as the study of relatively slender beams, it is sufficiently accurate to use simpler theories of failure, and these are generally used in discussions of the shear strength of structures.

The simplest cracking criterion is based on the principal tensile stress or principal tensile strain theories of failure. These approaches have been shown to be useful in predicting tensile failure when applied to certain simple states of stress (46) and have been shown to give reasonable

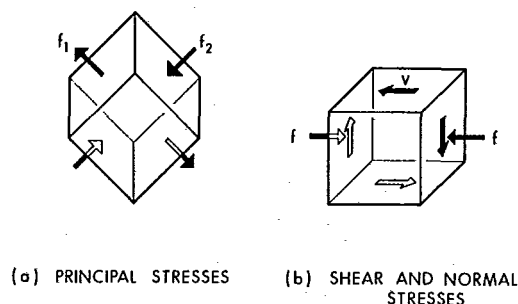


FIG. 2.1.—Biaxial States of Stress

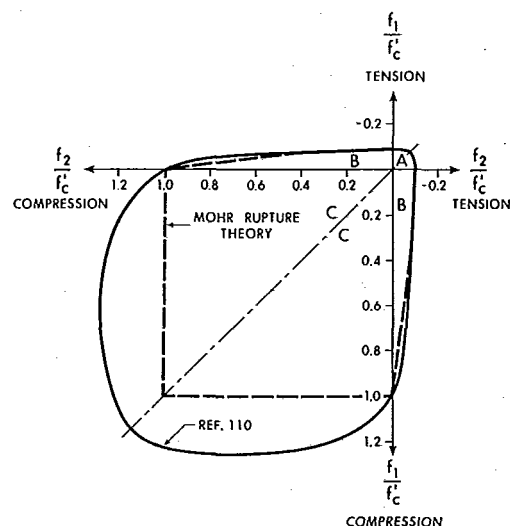


FIG. 2.2.—Biaxial Strength of Concrete

results when applied to the tensile cracking of a beam under combined shear and bending moment (105).

In pure flexure or combined flexure and axial load the limiting compressive strain concept gives reasonable predictions of failure loads. For the computation of plastic hinge relations the value of the limiting strain is sometimes taken to be a function of the M/Vd ratio (44).

Frequently the stresses in a structure can be idealized to a biaxial state of stress with the stress in the third direction equal to zero as

shown in Fig. 2.1(a) and 2.1(b). Kupfer, Hilsdorf, and Rüschi (110) and more recently Liu, et al. (121) have shown that as f_1 and f_2 are varied,

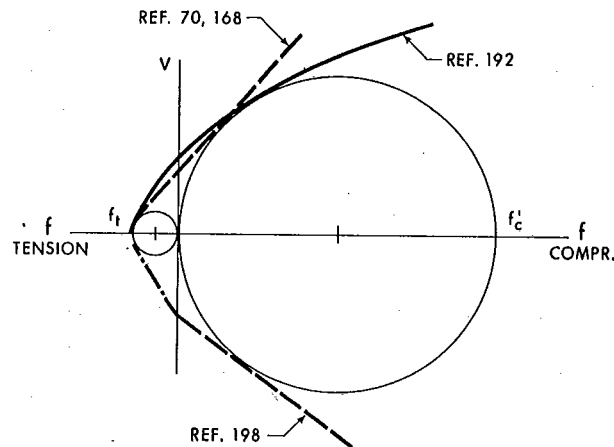


FIG. 2.3.—Possible Mohr Rupture Envelopes for Concrete

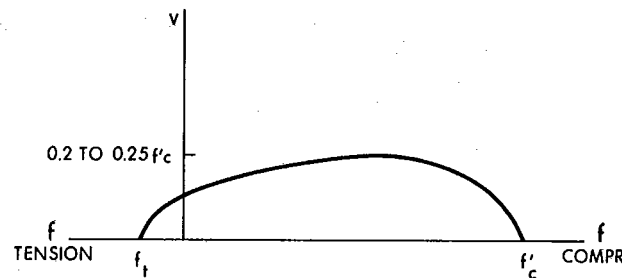


FIG. 2.4.—Combinations of f and v Corresponding to Failure of Element Shown in Fig. 2.1(b)—See Refs. 70, 168, 192

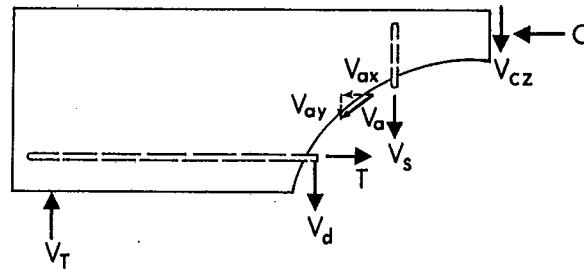


FIG. 2.5.—Forces Acting at Inclined Crack

the element shown in Fig. 2.1(a) will have the strengths shown by the solid lines in Fig. 2.2. This diagram is symmetrical about a 45° axis and can be divided into quadrants A, B, and C as shown. Biaxial tension (Quadrant A) has relatively little effect on the tensile strengths and biaxial compression (Quadrants C) tends to increase the compressive strength over that for uniaxial compression. Combined tension and compression loadings significantly reduce both the tensile and compressive stresses at failure, however, as shown by the quadrants labelled B. The latter case is particularly important when shears are present since they give rise to principal tensile and principal compressive stresses on elements in the web of a beam.

Alternatively, Mohr's Theory of Failure yields acceptable strength predictions for either of the cases shown in Fig. 2.1 except that conservative results are obtained in quadrants C of Fig. 2.2. Parabolic (192) and straight line (70,198) envelopes similar to those shown in Fig. 2.3 have been proposed for the family of Mohr's circles representing failure conditions. Any stress condition that corresponds to a Mohr's circle that is tangent to or intersects this envelope is assumed to represent a failure condition.

For the loading shown in Fig. 2.1(a) this failure theory leads to a strength diagram similar to that shown in dashed lines in Fig. 2.2. The dashed and solid lines differ because the effects of the intermediate principal stress are ignored in the Mohr Theory.

Based on either the Mohr Theory of Failure or the Kupfer, Hilsdorf, and Rüschi diagrams, relationships have been derived for the strength of elements stressed as shown in Fig. 2.1(b) (70,168,192). One such relationship is plotted in Fig. 2.4. This envelope presents all the combinations of shearing and axial stresses on the vertical plane of the element shown in Fig. 2.1(b) which result in failure of the element.

The implication of such a diagram is that the presence of shearing stresses will reduce the compression strength of concrete. Combined shear and compression may occur in columns or in the compression zones of beams.

In recent years a number of more complex failure theories have been proposed (39,150,169). These are particularly important when large triaxial stresses exist as in reactor vessels. A state of triaxial stress also exists in the compression zone under the column in a slab and as a result this region has a high capacity for shear and axial forces.

2.2 Mechanisms of Shear Transfer in Reinforced Concrete

Shear is transmitted from one plane to another in various ways in reinforced concrete members. The behavior, including the failure modes, depends on the method of shear transmission. Our appreciation of some of the principal shear mechanisms is relatively recent and therefore the definite evaluation of the contribution of these shear carrying components is yet only tentative.

The main types of shear transfer are the following: (a) Shear stress in the uncracked concrete; (b) interface shear transfer; (c) dowel action; (d) arch action; and (e) shear reinforcement.

These mechanisms occur to widely varying extents in various types of structural elements. For example, the forces in a reinforced concrete beam with a diagonal crack are shown in Fig. 2.5. The longitudinal forces, T and C , are related to the flexural resistance of the member. The forces along the diagonal tension crack, V_{ax} and V_{ay} , are due to interface shear transfer (also called aggregate interlock). The V_s and V_d forces are the forces in the stirrups and the "dowel shear" carried by the longitudinal steel, respectively and V_{cz} = the shear carried by the compression zone. Thus,

$$V_T = V_s + V_{cz} + V_d + V_{ay} \dots \dots \dots (2.1)$$

Most modern design procedures assign portions of the total shear to the compression zone and to the stirrups. However, it has become evident in recent years that the shear carried by the longitudinal steel (dowel effect) and the interface shear transfer along the crack significantly affect the capacity and mode of failure (60,179). These basic concepts will be described briefly in this chapter with specific cases looked at in more detail in subsequent chapters.

2.2.1 Shear Transfer by Concrete Shear Stress

The simplest method of shear transfer is by shearing stresses. This occurs in uncracked members or in the uncracked portions of structural members. The interaction of shear stresses with tensile and compressive stresses produces principal stresses which may cause inclined cracking or a crushing failure of the concrete. The strength of concrete subjected to principal stresses or combined shear and normal stresses is shown in Figs. 2.3 and 2.4, respectively.

2.2.2 Interface Shear Transfer

There are several instances in which shear must be transferred across a definite plane or surface where slip may occur. Researchers have called this mechanism aggregate interlock, surface roughness shear transfer, shear friction, tangential shear transfer. Since none of these names are sufficiently general, "interface shear transfer" is used in this report to denote the tangential shear force transmitted across a crack or plane (such as a construction joint). Several types of interface shear transfer are described in this section.

If the plane under consideration is an existing crack or interface, failure usually involves slippage or relative movement along the crack or plane. If the plane is located in monolithic concrete, a number of diagonal cracks occur across the interface and failure involves a truss action along the plane.

(a) Shear Transfer Behavior of Uncracked Concrete with Reinforcement Normal to Shear Plane

Several researchers have studied shear transfer across monolithic shear planes. A typical specimen is shown in Fig. 2.6. Before cracking shear is transmitted by stress across the shear plane. At higher loads short diagonal cracks form across the shear plane (Fig. 2.7). Failure is prevented by transverse reinforcement. Steel bars and the compression between the diagonal cracks form a truss which resists further loads. Failure occurs when the compression diagonals crush under the combined action of axial and shear forces. For ductile behavior it is necessary to proportion the bars so that they yield before the diagonals crush. Although shear is transferred across a definite plane, this mechanism is not strictly interface shear transfer since appreciable shear is not transferred across a crack. In addition to the truss mechanism, shear is also carried by dowel action. However, the shear transferred by that action is small if the bars yield before failure.

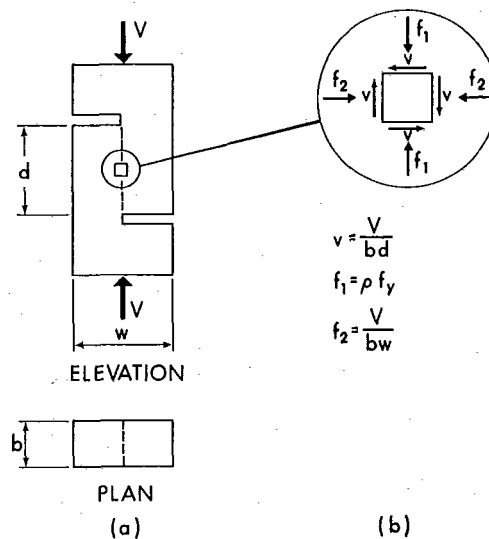


FIG. 2.6.—Shear Transfer Test Specimen (87)

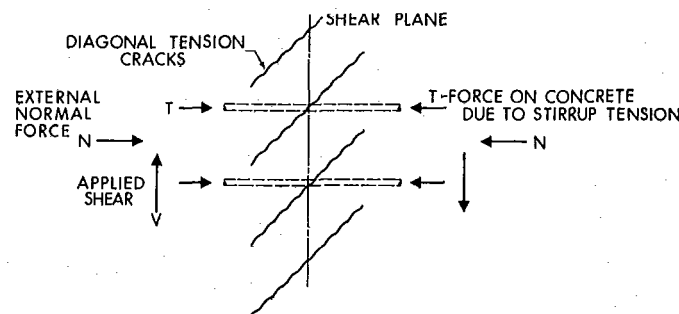


FIG. 2.7.—“Truss” Formed After Diagonal Tension Cracking (134)

(b) Shear Transfer Behavior of Initially Cracked Concrete with Reinforcement Normal to Shear Plane

If an initially cracked specimen is tested, shear can be transmitted only if lateral confinement or transverse steel exists. The irregularities of the surfaces of the two sides of the crack

ride up on each other and this tends to open the crack and create forces in the transverse steel (Fig. 2.8).

At failure the shear capacity was found to be proportional to the amount of average restraining stress ρf_y , equal to the transverse steel percentage multiplied by the yield stress (87). This proportionality is similar to the situation in the case of simple friction, and therefore this approach has been called

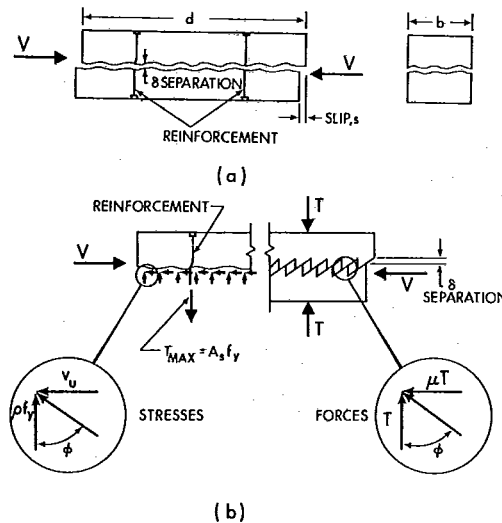


FIG. 2.8.—Shear-Friction Analogy

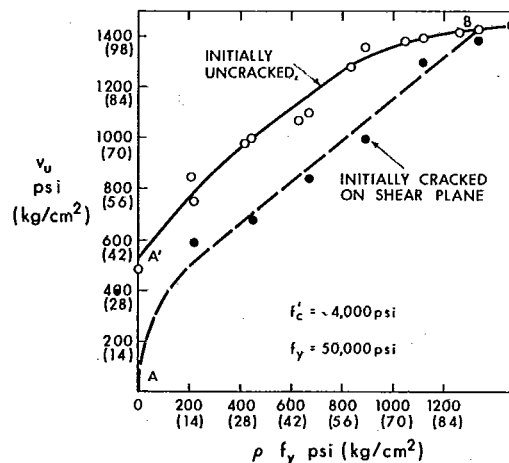


FIG. 2.9.—Variation of Shear Strength with Reinforcement Parameter ρf_y (87)

“shear-friction hypothesis,” although strictly speaking, friction does not play an appreciable role. The shear at failure can be written as (87,134)

$$V_u = A_s f_y \tan \phi \quad (2.2)$$

in which A_s = the total area of transverse steel and $\tan \phi$ = the equivalent coefficient of friction.

In addition, some shear is transferred by dowel action of the reinforcement crossing the crack.

Because the “coefficient of friction” along the crack is independent of the concrete strength, so also is the shearing strength, V_u , provided the shearing stresses do not exceed some limiting value. Similarly direct stresses parallel to the shear plane will not affect either the friction or dowel action and, as a result, the shearing strength is the same whether the shearing force, V , in Fig. 2.6(a) are due to compression forces as shown or tension forces.

In a heavily reinforced shear plane or one subjected to a normal compressive stress, the shear resistance due to friction and dowel action may reach the shear corresponding to failure of an initially uncracked specimen having the same characteristics. In such a case the crack locks and the behavior and strength are similar to those for an initially uncracked section.

(c) Test Results

Test results obtained by Mattock and his co-workers (87,134) for specimens similar to Fig. 2.6(a) are shown in Fig. 2.9. The ultimate shear stress increases almost linearly with the index, ρf_y , from a finite value for ρf_y equal to zero to a limit dependent on the concrete strength for high ρf_y values. With a monolithic shear plane, strengths are consistently greater than with a precracked shear plane. For a precracked shear plane the strength decreases rapidly with decreasing ρf_y for ρf_y values less than 200 psi (14 kg/cm²). For ρf_y lying between A and A' and B in Fig. 2.9, failure is relatively gentle and is due to a breakdown of the concrete after the reinforcement crossing the shear plane yields. For ρf_y values lying between B and C, failure occurs abruptly before the reinforcement yields. In this region the failure loads are similar for uncracked and precracked specimens, as explained previously.

Hanson (79) used push-off specimens similar to those in Fig. 2.6(a) to examine the effects of the roughness of a bonded surface and the length of the surface. For low ρf_y values and a rough bonded surface, the ultimate shear stresses were similar to those measured by Mattock (134) for a monolithic shear plane. For a smooth bonded surface and a low ρf_y value, the stresses were similar to those for precracked specimens. For both types of specimen the ultimate shear strength decreased about 100 psi (7 kg/cm²) for an increase in the bonded length from 6 in. to 24 in. (15 cm to 60 cm). This represented a 20% to 50% reduction for the rough bonded and smooth bonded surfaces, respectively. The addition of shear keys stiffened the connection at low slips but had no significant effect on its strength.

Fenwick and Paulay (60) and Loeber (122) studied shear transfer in precracked unreinforced push-off specimens. In one group of tests the crack width was held constant and variations in the size of the aggregate from 3/8 in. to 3/4 in. (1 cm to 2 cm) had no effect on the shear stress-slip curve or the ultimate strength. For shear stresses between 200 psi and 1,000 psi (14 kg/cm² and 70 kg/cm²) the shear stress-slip curve was practically linear, and the stiffness increased as the width of the crack decreased. In a second group of tests the crack width was increased at 0.002 in. (0.05 mm) per 100 psi (7 kg/cm²) of shear stress. There was a continuous degradation of the aggregate interlock mechanism on the shear plane with increasing load and for a 4,000 psi (280 kg/cm²) concrete, failure occurred at a shear stress of about 900 psi (63 kg/cm²).

To simulate the deformations at an inclined crack in a beam, Taylor (179) tested blocks of concrete in a device that maintained a constant ratio between the opening normal to the crack, ΔN , and the shearing movement along the crack, ΔS . The ultimate shearing stresses and the deformations developed both decreased linearly as $\Delta N/\Delta S$ increased. The shear transfer strength was a function of the roughness of the fracture surface. In some very high strength concretes, fractures crossed the aggregates, leaving a relatively smooth fracture surface, and consequently a relatively low shear transfer at failure.

Tests at the Portland Cement Association have shown that interface shear transfer is a viable mechanism even under fatigue loading for the transfer of shear between concrete paving slabs (40,144). The shear transferred for a given slip decreases with cycling and with increasing load and increasing crack

width. However, 90% of the decrease occurs during the first 5×10^5 loading cycles and the decrease diminished with increasing aggregate size and hardness.

For a shear stress ranging from 0 psi to 820 psi (57 kg/cm²) repeated 33 times for a 4,000 psi (280 kg/cm²) concrete, Loeber (122) found there was no sudden breakdown of the shear transfer mechanism. Although the permanent slip increased with cycling, the stiffness increased also.

White and Holley (193) applied cyclic shear forces to 1.5-ft by 2.0-ft by 3.0-ft (45-cm by 60-cm by 90-cm) specimens precracked by wedging them in two. These tests were designed to simulate seismic conditions in a nuclear containment vessel. Even after 50 cycles, more than 120 psi (8.5 kg/cm²) shear stress could be carried for the crack width and the size of restraining rod used. The shear stiffness decreased during cycling even though the external clamping rods did not reach yield.

In tests in which the parameter, ρ , was varied by varying both the bar spacing and the bar size, Mattock, et al. (87,134) found no marked variation in the effect of ρf_y . Similarly, the effect of the yield strength of the stirrups was found to be properly reflected by the parameter, ρf_y .

As shown in Fig. 2.9, a limiting shear strength of about $0.3 f'_c$ was observed for both precracked and monolithic specimens with concrete strengths of 4,000 psi (280 kg/cm²). A similar limit was observed for strengths varying from 2,500 psi to 4,000 psi (175 kg/cm² to 280 kg/cm²). In tests with an applied compressive stress normal to the shear plane, the shear strength was found to increase in proportion to the sum of the normal stress and ρf_y and to reach a limiting shear strength of about $0.45 f'_c$.

No tests have been carried out on specimens having a tensile stress normal to the shearing surface.

2.2.3 Dowel Shear

If reinforcing bars cross a crack, shearing displacements along the crack will be resisted, in part, by a dowelling force in the bar. The dowel force gives rise to tensions in the surrounding concrete and these in combination with the wedging action of the bar deformations produce splitting cracks along the reinforcement (14, 69). This in turn decreases the stiffness of the concrete around the bar and therefore the dowel force. Following splitting, the doweling force is a function of the stiffness of the concrete directly under the bars and the distance from the point where the doweling shear is applied to the first stirrups supporting the dowel (14). Compression failure under the bar sometimes affects this behavior, especially in slabs or in mass concrete.

Although dowel action has been studied by a number of investigators (14,69,107,149,178), dowel tests are characterized by the large number of variables that have to be considered to cover all likely bar layouts. So far, reliable results are available only for a few practical cases.

Relative to other shear transfer mechanisms, the dowel shear force is generally not dominant in beams. On the other hand, its role in the failure mechanisms of some types of structural elements is important. In beams, splitting cracks develop along the tension reinforcement at inclined cracks as a result of the dowel effects. This allows the inclined cracks to open, which in turn reduces the interface shear transfer along the diagonal crack and thus leads to failure.

In slabs, some investigators (10,102) credit dowel action with about 30% of the total shear.

2.2.4 Arch Action

In deep beams and slabs part of the load is transmitted to the supports by arch action. This is not a shear mechanism in the sense that it does not transmit a tangential force to a nearby parallel

plane. However, arch action does permit the transfer of a vertical concentrated force to a reaction in a deep member and thereby reduces the contribution of the other types of shear transfer.

For arch action to develop, a horizontal reaction component is required at the base of the arch. In beams this is usually provided by the tie action of the longitudinal bars. Frequently deep beams will fail due to a failure of the anchorage of these bars. In beams arch action occurs not only outside the outermost cracks but also between diagonal tension cracks. Part of the arch compression is resisted by dowel forces and therefore splitting cracks may develop along the bars. It was found that stirrups close to the base of diagonal cracks can provide support to the arches (69,99).

In slabs, arching action may occur around interior columns. Here the horizontal reaction is provided by the in-plane stiffness of the slab adjacent to the punching region (131).

2.2.5 Shear Reinforcement

It has been customary to view the action of web reinforcement in beams as part of a truss. Although this analogy helps to simplify design concepts and methods, it does not consider the various types of shear transfer methods properly.

It is also helpful to examine the action of shear reinforcement from the viewpoint of how it aids the several kinds of shear transfer action. In addition to any shear carried by the stirrup itself, when an inclined crack crosses shear reinforcement, the steel may contribute significantly to the capacity of the member by increasing or maintaining the shear transferred by interface shear transfer, dowel action, and arch action. Thus, shear reinforcement restricts the widening of inclined cracks in beams and thus slows the decrease of interface shear transfer quite effectively. Its action is similar in other situations, such as in corbels or other types of members. The effects of web reinforcement in beams and deep members are discussed in Chapters 3 and 4.

CHAPTER 3

3. Behavior and Strength of Beams Failing in Shear

3.1 Modes of Inclined Cracking and Shear Failure

Shear failures of beams are characterized by the occurrence of inclined cracks. In some cases inclined cracking is immediately followed by member failure and in other cases, the inclined cracks stabilize and substantially more shear force may be applied before the member fails.

Inclined cracks in the web of a beam may develop either before a flexural crack occurs in their vicinity or as an extension of a previously developed flexural crack. The first type of inclined crack is often referred to as a "web-shear crack" [Fig. 3.1(a)]; the second type is often identified as a "flexure-shear crack;" and the flexural crack causing the inclined crack is referred to as the "initiating flexural crack" [128,174, and Fig. 3.1(b)].

In addition to the primary cracks (flexural and the two types of inclined cracks), secondary cracks often result from splitting forces developed by the deformed bars when slip between concrete and steel reinforcement occurs, or from dowel action forces in the longitudinal bars transferring shear across the crack.

The manner in which inclined cracks develop and grow and the type of failure that subsequently develops is strongly affected by the relative magnitudes of the shearing stress, v , and flexural stress, f_x . As a first approximation, these stresses may be defined as:

$$v = \alpha_1 \frac{V}{b_w d} \text{ and } f_x = \alpha_2 \frac{M}{b d^2} \dots \dots \dots (3.1)$$

in which α_1 and α_2 are coefficients depending on several variables, including geometry of the beam, the type of loading, the amount and arrangement of reinforcement, the type of steel, and the interaction between steel and concrete. The values, V and M , are the shear and moment at a given section, respectively; b = the width of the flange; b_w = the

width of the web; and d = the effective depth of the beam. The ratio f_x/v is therefore

$$\frac{f_x}{v} \frac{\alpha_2 M b_w}{\alpha_1 V d b} = \alpha_3 \frac{M b_w}{V d b} \quad (3.2)$$

in which $\alpha_3 = \alpha_2/\alpha_1$. The shear, V , is a measure of moment gradient; $V = dM/dx$; and for beams subjected to concentrated loads may be expressed by $V = M/a$, in which a is called the "shear span." Then

$$\frac{f_x}{v} = \alpha_3 \left(\frac{a}{d} \right) \left(\frac{b_w}{b} \right) \quad (3.3)$$

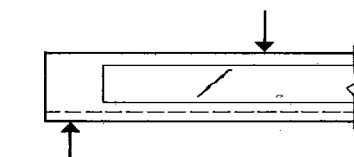
and the variation in the inclined cracking load and shear capacity of rectangular beams may be conveniently considered as a function of varying a/d , i.e., the ratio of shear-span to depth (24).

In short deep members the vertical normal stresses, f_v , become significant as the value of a/d decreases below 2 to 2.5 and this leads to an additional increase in the shear strength.

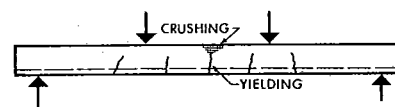
When all other things are kept constant, the influence of the a/d ratio on the cracking of a rectangular simply supported reinforced concrete beam can be illustrated by considering beams of varying slenderness with two symmetrically placed concentrated loads placed a distance, a , from the supports.

(a) Normal and Long Beams of Rectangular Cross Section ($a/d > 2.5$)

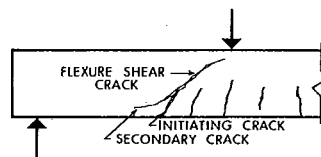
Very shallow beams will usually fail in flexure [Fig. 3.2(a)]. The first crack will form due to flexural tension at the cross section of maximum moment. As the beam load increases prior to failure, the tensile cracking may spread to regions of lesser moment, but



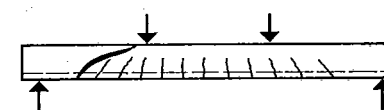
(a) WEB SHEAR CRACK



(a) FLEXURAL FAILURE



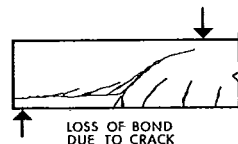
(b) FLEXURE SHEAR CRACK



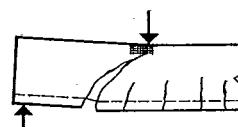
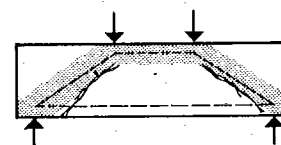
(b) DIAGONAL TENSION FAILURE

FIG. 3.1.—Types of Inclined Cracks

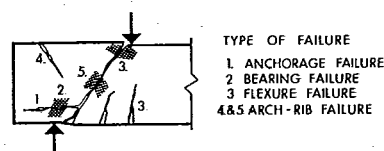
FIG. 3.2.—Failures of Slender Beams



(a) SHEAR - TENSION FAILURE



(b) SHEAR - COMPRESSION FAILURE



- TYPE OF FAILURE
- 1 ANCHORAGE FAILURE
 - 2 BEARING FAILURE
 - 3 FLEXURE FAILURE
 - 4&5 ARCH-RIB FAILURE

$a/d = 1$ TO 2.5

$a/d = 0$ TO 1

FIG. 3.3.—Typical Shear Failures in Short Beams

FIG. 3.4.—Modes of Failure of Deep Beams

failure occurs in flexure near the section of maximum moment.

For a beam with somewhat smaller value of a/d , the fatal crack may well be a flexure-shear crack, as shown in Fig. 3.1(b). Such a crack may cause the beam to fail before its full flexural capacity is developed.

In moderately slender beams one of the cracks may continue to propagate through the beams with further load until at some stage it becomes unstable and extends through the beam as shown in Fig. 3.2(b). This type of failure is called a "diagonal tension failure."

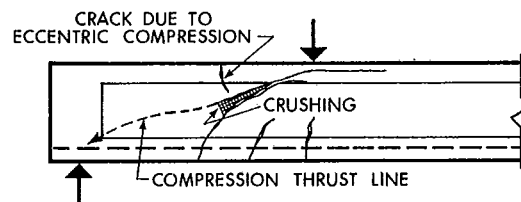
(b) Short Beams: $1 < a/d < 2.5$

A curved tensile crack in a region of combined moment and shear may also trigger one of two additional modes of failure. A secondary crack may propagate backward along the longitudinal reinforcement from the inclined crack, perhaps because of dowel action in the longitudinal reinforcement [Fig. 3.3(a)]. This crack will cause a loss of bond. As the main reinforcement begins to slip, the wedging action of the bar deformations contributes to a splitting of the concrete and a further propagation of the crack, resulting in an anchorage failure of the longitudinal reinforcement, called a "shear-tension" failure (111). Prior to such a failure, the beam acts as a tied arch.

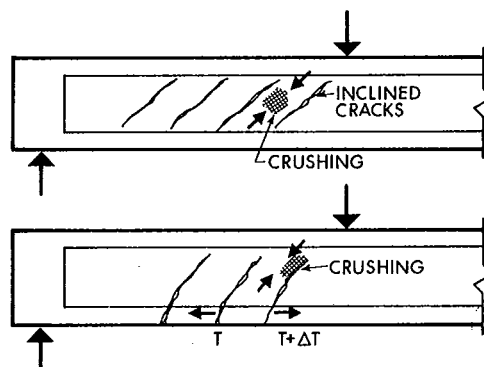
Alternatively, the concrete above the upper end of the inclined crack may fail by crushing [Fig. 3.3(b)] resulting in a "shear-compression" failure.

(c) Deep Beams: $0 < a/d < 1$

In the design of deep beams, shearing stresses and vertical normal stresses require more consideration than flexural stresses. Significant principal compression and tensions, respectively, exist along and across the line joining the load and the reaction, and inclined cracks occur immediately inside the reaction plates (Fig. 3.4). In some cases the cracks appear to be initiated by flexural cracks originating in



(a) ARCH - RIB FAILURE



(b) WEB - CRUSHING FAILURE

FIG. 3.5.—Typical Shear Failures in I-Beams

this region, and in other cases by the principal tensions across the line joining the load and reaction, as in an indirect tension test.

After inclined cracking occurs, a deep beam without web reinforcement transforms almost immediately into a tied-arch which can fail in a number of ways (50,54). The numbers in Fig. 3.4 correspond

to the following modes of failure: (1) Anchorage failure of the tension reinforcement, usually combined with a dowel splitting effect; (2) crushing failure at the reactions; (3) flexural failure—either of the steel reinforcement due to yielding or fracture, or of the “crown of the arch” when the concrete crushes; or (4) tension failure of the “arch-rib” by cracking over the support; followed by (5) crushing along the crack. The tension at Point 4 is produced by the eccentricity of the thrust which acts essentially along the inclined crack.

The strength of deep beams will be discussed more fully in Chapter 4.

(d) I-Beams

In a beam with a narrow web, such as an I-beam, the shearing stresses in the web are much larger in relation to the flexural stresses than they are in a rectangular beam, as is shown by Eq. 3.3. Consequently a tensile crack may begin in the web due to principal tensile stresses associated with the shearing stresses. Such a crack has been called a “web-shear” crack [Fig. 3.1(a)] and is inclined nearer to 45° than flexure-shear cracks because of the dominance of the shearing stresses over the flexural stresses in the web. Additional cracks may also begin in flexural tension.

The I-beams may fail in flexure, diagonal tension, shear-tension, shear-compression, or in the various modes of failure already described for deep beams. Fig. 3.5(a) shows an “arch-rib” failure similar to that of a deep rectangular beam. Fig. 3.5(b) shows a “web-crushing” failure which can occur in a thin-webbed beam with web reinforcement when a thin web element, isolated by diagonal cracks, crushes under the diagonal compression forces (116,153). It should be noted that the mechanics of failure will differ somewhat in the two beams shown in Fig. 3.5(b) depending on whether the lower flange is cracked or not (11). The European Concrete Committee (41) distinguishes between these two cases for design purposes.

3.2 Development of Inclined Cracks in Reinforced and Prestressed Concrete Beams

Because all types of “shear-failure” are affected by or result from inclined cracks due to diagonal tensile stresses, it is essential to understand, at least empirically, the factors affecting the formation of such cracks.

(a) Web-Shear Cracks

Web-shear cracks of the type shown in Fig. 3.1(a) form in a part of the beam that is as yet uncracked in flexure. Generally this type of cracking only occurs in highly prestressed beams with thin webs.

A web-shear crack occurs when the stresses at some point in the uncracked beam reach a limiting value. Based on a limiting principal tensile stress theory of failure, it has been shown that for those portions of beams actually susceptible to this type of cracking, the web-shear cracking load may be predicted with reasonable accuracy by calculating the shearing force required to cause principal tensile stresses equal to the tensile strength of the concrete at or near the neutral axis of the beam (128). More recently it has been shown that a slightly better prediction of the cracking load results from the use of Mohr's Failure Theory for concrete subjected to combined shear and normal stress (100).

From a Mohr's Circle it can be shown that the shear stress, v_{cw} , corresponding to reaching the tensile strength of the concrete, f_t , at the neutral axis of a beam with an average axial stress of f_{pc} is:

$$v_{cw} = f_t \sqrt{1 + \frac{f_{pc}}{f_t} + \frac{V_p}{b_w t}} \dots \dots \dots (3.4)$$

In Eq. (11.12) of the ACI Code this is approximated by the straight line function (5):

$$v_{cw} = 3.5 \sqrt{f'_c} + 0.3 f_{pc} + \frac{V_p}{b_w d} \dots \dots \dots (3.5)$$

in which V_p = the vertical component of the prestressing force.

The latest revisions to the Soviet design regulations and prestressed concrete follow a similar approach except that the tensile strength of the concrete, f_t , is defined considering the effect of the principal compression force using a diagram similar to the upper left quadrant of Fig. 2.2 (197).

(b) Flexure-Shear Cracks

Flexure-shear cracks of the type shown in Fig. 3.1(b) form in a part of a beam already cracked due to flexure. Although these cracks are the most common type of crack observed in reinforced and prestressed concrete beams, the mechanism by which they form is not entirely understood.

In a beam containing flexural cracks, the variation in steel stress from crack to crack sets up forces on the cantilever elements or "teeth" between the flexural cracks [see Fig. 3.2(b)] which tend to cause bending and shearing deflections of the teeth (60,96,129). As the teeth deflect prior to inclined cracking the dowelling forces and aggregate interlock stresses developed between the teeth tend to prevent relative deflections of the teeth, thus delaying the formation of further cracks. The rate of propagation of the flexural cracks and the stage at which the inclined cracks develop in a given beam is a function of the magnitudes of the flexural and shearing stresses at the head of the crack (7,105,129). These, in turn, are a function of the M/Vd and b_w/b ratios, as shown in Eq. 3.3, the height of the flexural crack and the amount of shear transferred by interface shear transfer and dowelling.

In tests of rectangular beams Taylor (180) has shown that approximately one third of the total applied shear was transferred by the compression zone above flexural cracks in the shear spans prior to the formation of inclined cracks. The balance was presumably transferred by interface shear transfer and dowel action. These measurements suggest that a significant shear stress can exist at the upper ends of flexural cracks in the shear spans. Broms (28) has shown that the nonuniform stresses between cracks lead to "secondary shear stresses" which are particularly severe near the top of flexural cracks in the shear spans. These secondary shear stresses, combined with the stresses necessary to balance the applied loads, cause major diagonal principal tensile stresses at the head of the flexural cracks.

A number of semirational expressions have been developed to predict the flexure-shear cracking load of reinforced concrete beams. The best known is the ACI Code equation for reinforced concrete, developed by ACI-ASCE Committee 326 (7):

$$\frac{V_c}{b_w d} = 1.9 \sqrt{f'_c} + \frac{2,500 \rho_w V d}{M} \leq 3.5 \sqrt{f'_c} \quad (3.6)$$

in which V_c = the shear carried by the concrete, taken equal to the inclined cracking shear; $\sqrt{f'_c}$ is a measure of the tensile strength of the concrete; M and V = the moment and shear at the point under consideration; and ρ_w = the amount of longitudinal reinforcement at that point related to the web thickness. This equation was derived considering the principal tensile stresses at the top of a flexural crack in the shear span.

For flexure-shear cracking in prestressed concrete beams, the inclined cracking load can be expressed in terms of the shear necessary to cause a flexural crack at a point located $d/2$ to d toward the reaction from the section under consideration, plus an increment of shear assumed to be necessary for this flexural crack to develop into an inclined crack (128,174). The 1963 ACI Code (5,6) equation for flexure-shear cracking in prestressed beams was expressed in this form:

$$V_{ci} = \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + 0.6 \sqrt{f'_c} b_w d \quad (3.7)$$

in which the first term predicts the shear necessary to cause the initiating flexural crack and the second term expresses the additional increment of shear. The external moment required to cause a flexural crack at the point under consideration is M_{cr} . This equation was slightly simplified in the 1971 ACI Code (126). Since the effect of inclined prestressing tendons is included in the flexural cracking moment, M_{cr} , it is not necessary to add the additional term, V_p , to this equation.

Recently Mattock (133) extended Eq. 3.7 to reinforced concrete beams with or without axial loads. To do this, the steel percentage, ρ , and the modular ratio, n , were included in the expression as shown in Eq. 3.8

$$V_{ci} = \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + \left(1.75 - \frac{0.036}{n\rho} + 4.0 n\rho \right) b_w d \sqrt{f'_c} \quad (3.8)$$

Statistical studies of tests of reinforced concrete beams suggest that the most significant variables affecting the shear at flexure-shear cracking are the tensile strength of the concrete, f_t or $\propto \sqrt{f'_c}$; the longitudinal reinforcement ratio, ρ ; the ratio of flexural tension to shearing stresses measured by the (M/Vd) or $(M/Vd\rho)$ ratio; the effective depth of the beam, d ; and perhaps the ratio of compression to tension reinforcement (101,199). Perhaps the best known statistically derived equation was presented by Zsutty (199).

$$v_c = 59 \sqrt[3]{f'_c \rho \frac{d}{a}} \text{ (psi)} \quad (3.9)$$

Placas and Regan derived a similar equation starting from a failure theory for concrete (153).

3.3 Forces in Beams with Diagonal Tension Cracks

Traditionally, the transfer of shear across a section in a cracked beam has been explained using a "truss analogy" in which the beam is represented by a truss with parallel chords and a web composed of diagonal concrete compression struts and vertical or inclined stirrups acting in tension (7,88). In the 70 yr since this analogy was first proposed, it has formed the principal basis for the interpretation of forces in beams and for the design of reinforced concrete beams for shear. On the other hand, recent studies have shown that several significant components of shear force shown in Fig. 2.5 are not included in the basic truss analogy. These will be examined in the following paragraphs.

(a) Beams without Web Reinforcement

A number of experimental investigations have been carried out on beams without stirrups to assess the relative magnitude of the forces shown in Fig. 2.5. Acharya and Kemp (1) showed that only 40% of the total shear on a section can be carried by the compression zone if the stress conditions at the head of the crack are to be compatible with an accepted failure criterion for concrete subjected to combined shear and normal force (Section 2.2.1). Experimental work has subsequently been carried out by Fenwick and Paulay (60), Taylor (179), and Gergely (69) in which a variation of compression zone shear between 20% and 40% of the total shear on beams was found. This range depended mainly on the shape and nature of the cracks in the beams.

A number of investigations by Krefeld and Thurston (107), Parmelee (149), Fenwick and Paulay (60), Gergely (69), Taylor (178), Baumann (14), and many others have been carried out on dowel action indicating that the dowel shear force is between 15%-25% of the total shear force (see Section 2.2.3).

As discussed in Section 2.2.2, interface shear transfer tests have been carried out by Fenwick and Paulay (60), and Taylor (179), and Mattock, et al. (134), among others. Gergely (69) conducted tests in which interface shear transfer was eliminated by preparing smooth sided cracks in beams. These tests and those by Taylor indicate that between 33%-50% of the total force on a beam may be carried by interface shear transfer.

The figures given in the preceding were measured on rectangular beams without web reinforcement at loads near their failure load. Different distributions of shear force apply for lesser loads on the beams. Thus, before cracking, a parabolic distribution of shear stress across a beam can be expected and after cracking, as the shear displacements across the cracks increase, dowel and interface shear transfer action become increasingly significant.

(b) Beams with Web Reinforcement

Web reinforcement has three primary effects on the strength of a beam: (1) It carries part of the shear, V_s ; (2) it restricts the growth of the diagonal tension crack width and thereby helps maintain the interface shear transfer, V_{ay} ; and (3) it holds the longitudinal bars and increases their dowel capacity, V_d . In addition to these three actions, stirrups may transfer a small force across the crack by dowel action and they tend to enhance the strength of the compression zone by confining the concrete.

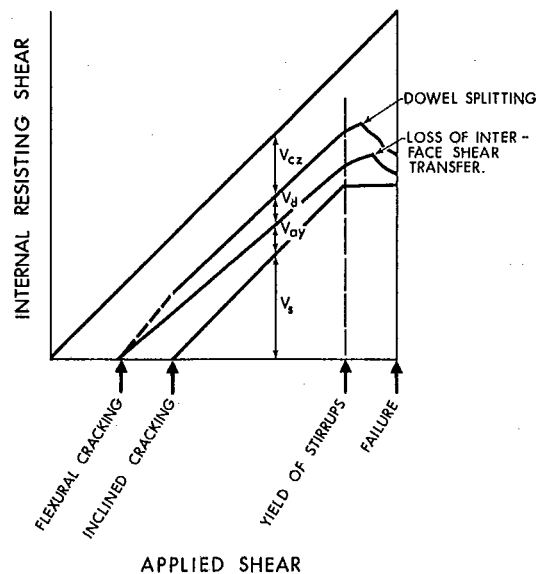


FIG. 3.6.—Distribution of Internal Shears in Beam with Web Reinforcement

If a stirrup happens to be near the bottom of a major diagonal crack, it is very effective in maintaining the dowel force and restraining the splitting failure, provided that the stirrups are of sufficient size, are well anchored, and are spaced close enough together that each potential diagonal crack reaches the tension steel near a stirrup (99).

The mechanism of a shear failure is not yet completely understood, although some careful tests have demonstrated it quantitatively in specific cases (1,60,69,179). The internal shearing force components at a crack such as the one shown in Fig. 2.5 are related to the applied shear in Fig. 3.6. Prior to flexural cracking all the shear is carried by the uncracked concrete. Between flexural and inclined cracking, the external shear is resisted by the concrete, V_{cz} ; the interface shear transfer, V_{ay} ; and by dowel action, V_d . Following the formation of inclined cracks, a portion of the shear is carried by the web reinforcement, V_s . When the shear carried by the web reinforcement can no longer increase due to its yielding, any additional shear must be carried by V_{cz} , V_d , and V_{ay} . As the inclined crack widens, the interface transfer shear, V_{ay} , decreases, forcing V_d and V_{cz} to increase at an accelerated rate until either splitting (dowel) failure occurs or the compression zone fails due to combined shear and compression.

In the case of beams without stirrups, beam failure may be caused by the breakdown of any of the components of force transfer across the section with different mechanisms dominating in beams of dif-

ferent types. For example, in some tests it is possible to see dowel splitting at the tension steel level before the beam collapses and in other tests, dowel failure and beam collapse occur simultaneously. In some cases it is also possible to see spalling from the sides of inclined cracks before beam failure, indicating that failure of the interlock mechanism is imminent.

Each of the ways in which force is carried across beams, apart from the force carried by stirrups, has a load deformation curve which initially rises rapidly, followed by a falling portion. In the case of the force carried in the compression zone, the falling portion of the curve follows that of concrete under a combined shear and compressive stress system. Dowel failure has a very steep falling portion, except where the dowel is restrained from splitting by stirrups enclosing the longitudinal bars. The failure of the interface shear transfer can be very abrupt, particularly if a crack is inclined over most of its length and the predominant movement across the crack tends to open the crack rather than to shear it (179). Because of this it is very difficult to say which mechanism of force transfer breaks down to cause beam failure from the more ductile behavior of beams with stirrups.

(c) T-Beams and Prestressed Concrete Beams

In T-beams the interface shear transfer is probably less important than in rectangular beams due to the proportionately smaller width of cracked concrete. In addition, V_{cz} is concentrated over the web while the flexural compressive stresses are distributed over a greater width. This leads to a higher ratio of shear to compression than in a rectangular beam. Fig. 2.3 and 2.4 suggest that the magnitude of V_{cz} and the way in which the compression zone fails may change as a result (153).

The role of interface shear transfer and dowel effects has not yet been studied in the case of prestressed concrete beams having flexure-shear cracks. Prestressing strands carry less dowel force and produce smaller wedging action, therefore splitting should occur later than in a reinforced concrete beam. Furthermore, inclined cracks also occur later because of the prestressing effect. On the other hand, once started, the inclined crack tends to develop more rapidly than in a reinforced concrete beam because of the smaller steel percentage, and tends to widen fairly quickly, leading to a rapid decrease in V_{ay} . Once splitting initiates, the strand can bend at the crack and does not inhibit the rotation that accompanies the opening of the crack. As a result, V_d is small. The shear capacity of a prestressed beam is approximately equal to the sum of V_s and V_{cz} . The term, V_{cz} , appears to depend on the depth of the compression zone, which in turn is a function of the load at which inclined cracking occurs.

3.4 Factors Affecting the Shear Strength and Serviceability of Beams

Most current building codes express the shear strength of beams in terms of an equation of the form:

$$V_u = V_c + V_s \dots \dots \dots (3.10)$$

in which V_c = the shear "carried by the concrete" at ultimate including V_{cz} , V_d , and V_{ay} in Eq. 2.1; and V_s = the shear carried by transverse reinforcement. For simplicity, the factors affecting the shear strength will be discussed in terms of their effect on V_c or on V_s , assuming these can be separated.

3.4.1 Effect of Cross Section

(a) Effect of Size

Leonhardt and Walther (116) studied the effect of beam size by testing two series of similar specimens without web reinforcement. The first series consisted of four completely similar specimens in which the cross section varied from 2 in. $\times$ 3.1 in. (5 cm $\times$ 8 cm) to 8 in. $\times$ 12.6 in. (20 cm $\times$ 32 cm). In this series, the bar diameter was proportional to the external dimensions and the number of bars was constant. In the second series, the cross section varied from 4 in. $\times$ 7 in. (10 cm $\times$ 18 cm) to 9 in. $\times$ 21.3 in. (22.5 cm $\times$ 67 cm), the ratios

of depth were different from the ratios of widths, the bar diameter was constant, and the number of bars was varied to maintain the same steel percentage. The shear stress at failure decreased 37% between the smallest and largest specimens in the first series and decreased 21% in the second series. The greater strength decrease in the first series was explained on the basis of poorer bond quality with increasing bar diameter. The high shear strength of small scale beams has also been reported by Swamy (176) who attributed it to the influence of the high value of the modulus of rupture of the material and the increase in strength produced by extreme strain gradients in small specimens.

Kani (98) tested beams of various depths and the same concrete strength, steel percentage, and a/d ratio. The shear stress at failure decreased with increasing beam depth. In these tests the beam width was held constant at 6 in. (15 cm) as the depth was varied. As the depth increased the splitting forces due to the wedging bond action of the reinforcement increased and as a result, the failures of the two deeper series of beams involved splitting along the reinforcement while those in the shallower beams did not.

In all three test series mentioned in the foregoing, one mix design was used for all beam sizes. It is possible that part of the loss of strength can be accounted for by the fact that although the cracks were wider in the deep beams than in the shallow beams, the surface of the cracks had the same roughness in both cases so that the contribution of aggregate interlock to their shear capacity was different. Similar size effects have been observed in punching shear tests of model slabs when the same mix was used for various size models (141).

Tests by Taylor (181) have shown much less size effect if the size of the coarse aggregate is changed in the same proportion as the beam size. Taylor also showed that if large beams with normal b/d ratios ($d/b < 4$) are tested, the loss of strength is not as serious as that reported by Kani. For beams with $d/b > 4$ Taylor has proposed that the design value of v_c , the shear stress carried by the concrete, should be reduced by 40%.

The beams tested by Leonhardt and Walther, Kani and Taylor to study the effect of size on shear strength did not have web reinforcement. It appears reasonable to expect that while V_c has a scale effect, V_s has not, so that the effect of size on beams with web reinforcement is small. Statistical analysis (101) of test results confirms that the beam depth has no significant effect on the ultimate strength of beams with web reinforcement. This is confirmed in part by tests of two 4 ft (122-cm) deep "wall beams" with web reinforcement (37). In these tests the shear at inclined cracking was about 50% of that predicted by the ACI Building Code, but the early inclined cracking did not lead to a reduction in the shear capacity of the beams.

(b) Effect of Shape

(i) b/d Ratio of Rectangular Beams

The effect of the b/d ratio in rectangular beams has been studied by Diaz de Cossio (55), Leonhardt and Walther (116), Kani (98), and Iyengar, et al. (91). Diaz de Cossio found that shear strength of beams without web reinforcement increases with b/d ratio. Leonhardt and Walther tested slab strips and also found slightly higher nominal ultimate shear stresses than in normal beams, perhaps due to anti-elastic bending effects. However, Kani did not find any significant change in strength when the beam width was increased from 6 in. to 24 in. (15 cm to 60 cm) and statistical studies by Iyengar,

et al. of a large amount of data showed no significant effect of b/d in the range from $1/4 \leq b/d \leq 1$. In slabs, the critical range of b/d is about 4 to 10 and in this range, some strength enhancement may occur.

(ii) Circular Sections

Farodji and Diaz de Cossio (58) tested circular beams with and without transverse reinforcement. In general, the cross sections resembled those of spiral columns. The maximum value of the ratio between concrete cover and external diameter was about $1/10$ in the specimens.

It was found that the usual equations for rectangular sections are applicable to circular sections if the external diameter is used for the effective depth, and the gross area of the circular section for the product, bd . These findings are reflected in Ref. 6.

(iii) T-Beams and I-Beams

Tests of 24 simply-supported T-beams were reported by Placas and Regan (153). In one series the web thickness was kept constant at 6 in. (15 cm) and the flange widths varied from 6 in. to 42 in. (15 cm to 109 cm). All the beams had the same longitudinal and web reinforcement except that the rectangular beam had compression reinforcement to replace the flange. As shown in Fig. 3.7 the beams with 12-in. (30-cm) or wider flanges had about 20% greater ultimate shear strength than the rectangular beam. Placas (153) and Leonhardt (114) have suggested that only the portion of the flange immediately adjacent to the web can transmit a component of the shear in the compression zone. As a result, the compression is distributed over the width of the flange while the shear is concentrated above the web, leading to a critical state of stress in this area. Placas and others (72) have observed that the compression zone of a T-beam will frequently fail due to shearing in the region over the web rather than crushing.

Zsutty (201) has proposed that Eq. (3.11) be used to calculate the shear, V_c , carried by the concrete:

$$V_c = v_c (b_w d + 2h_f) \dots \dots \dots (3.11)$$

in which h_f = the thickness of the flange.

For design purposes, however, it seems reasonable to ignore the strengthening effect of the flange and to continue to compute $V_c = v_c b_w d$.

In pan-joint floors the webs generally taper from top to bottom. Although this is not serious in the positive moment region, the section of least width occurs in the compression zone in the negative moment region. In tests of beams simulating the negative moment region of a continuous pan-joint, Hanson (75) found that V_c should be based on the width at the bottom of the web.

Leonhardt and Walther (116) tested a series of beams with constant flange width but varying web thicknesses, with the sections shown near the top of Fig. 3.8. All the beams for each loading had the same stirrup capacity, V_s . As shown in this figure the failure loads decreased as the web width was reduced. The rectangular and thick web T-beams failed by shear compression and shearing of the compression flange while the thinnest web T-beams failed by crushing of the web due to oblique compression stresses in the manner discussed in Section-3.1(d) of this report. The inclined cracking load, V_{cr} , and the stresses in the stirrups also depended on the web width. At any shear V , the stresses in the stirrups were approximately a function of $V - V_{cr}$.

The crushing of thin webs of I- or T-beams due to diagonal compression has been studied by a number of

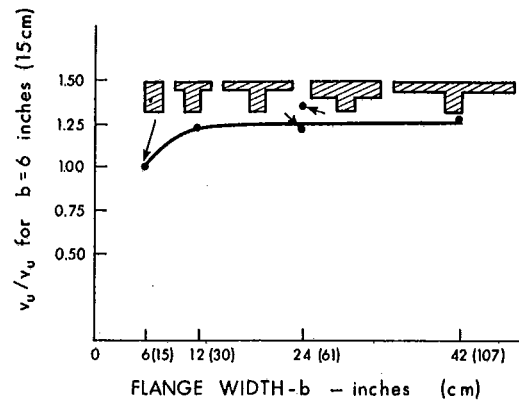


FIG. 3.7.—Effect of Flange Width (153)

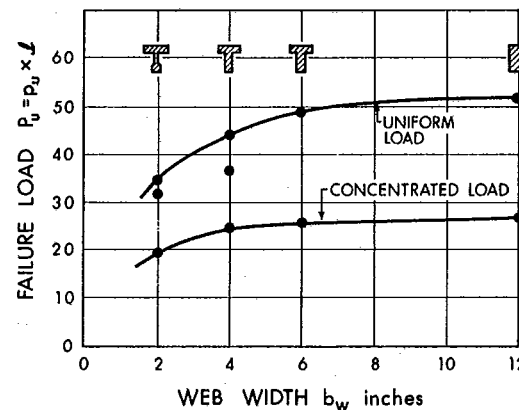


FIG. 3.8.—Effect of Web Thickness on Shear Strength (116)

authors (114,116,153) who have used the truss analogy to suggest that the inclined compression stress in the web varies from $f_c = V/b_w d$ for 45° stirrups to twice this value for vertical stirrups.

Recent tests by Demorieux (51) on panels loaded in compression in one direction and loaded in the transverse direction by tensile forces in transverse reinforcement to simulate the conditions in the web of a beam, suggest that the crushing strength of the concrete in the web will be about 2/3 of that in a cylinder test. This agrees with strengths obtained from biaxially loaded concrete (Fig. 2.2). Placas (153) has suggested the web crushing is also affected by the stiffness of the web reinforcement.

For design purposes the CEB-FIP (41) limits the shear in thin webs as a function of the principal tension and compression stresses in the web. For the normal case of reinforced or prestressed concrete beams without transverse prestressing their limit can be reduced to 0.2 times the design compressive strength with vertical stirrups or 0.25 times the design compressive strength with 45° stirrups to prevent crushing of the webs. Accounting for differences in load factors and design strengths, these correspond to $v_u = V_u/\phi b_w d = 0.18 f'_c$ and $0.22 f'_c$, respectively, in ACI practice. For the practical range of web reinforcement ratios, the equation suggested by Placas and Regan corresponds to $v_u = (15 \text{ to } 20) \sqrt{f'_c}$ for vertical stirrups and $v_u = (25 \text{ to } 50) \sqrt{f'_c}$ for 45° stirrups.

3.4.2 Effect of Reinforcement Details

(a) Percentage of Longitudinal Reinforcement

Tests (97,153,158) and analyses (133,153) have shown that the shear strength of reinforced concrete beams drops signifi-

cantly if the longitudinal reinforcement ratio is decreased below 1.2% to 1.5% based on the web thickness as shown in Fig. 3.9. For the analysis of the strength of beams a number of equations are proposed for v_c .

Mattock (133) and Zsutty (199) have proposed Eqs. 3.8 and 3.9, respectively, while Rajagopalan and Ferguson (158) have fitted the following equation to test data:

$$v_c = (0.8 + 100\rho) \sqrt{f'_c} \leq 2.0 \sqrt{f'_c} \quad (3.12)$$

The 1971 British Standard Code of Practice (27) presents a table of values of v_c that are a function of $\rho_w = A_s/b_w d$ and the 1971 CEB Recommendations (41) reduce the allowable shear stress if ρ_w is less than 1.5% at $2d$ from the support. The various design equations are compared to tests in Fig. 3.9.

The effect of steel percentage may be explained in two ways:

- (1) The dowel shear, V_d , will be smaller if ρ is reduced; and
- (2) as ρ is reduced, the flexural cracks extend higher into the beam and are wider, reducing both the shear capacity of the compression zone and the interface shear transfer.

(b) Yield Strength of Longitudinal Reinforcement

Mathey and Watstein (132) have reported tests on beams with high yield strength longitudinal reinforcement. For a/d ratios from 1.5 to 3.8 they found that the shear strength was independent of the yield point. If, however, the increase in yield strength was offset by a reduction in ρ to give a constant moment capacity, they found a reduction in shear strength. This finding was corroborated by Taylor (183) who suggested that the shear strength decreased as the steel stress at inclined cracking increased. The reasons presented in section 3.4.2(a) to explain the effect of ρ also can be used to explain the effect of the yield strength on V_c when ρ is constant or varied.

(c) Cut-Off and Bent-Up Reinforcement

Major inclined cracks frequently develop near the ends of reinforcing bars cut off in a zone of tension. Adjacent to a cutoff point the stresses and deformations are decreasing in the cutoff bars and increasing in the remaining bars. This, combined with the eccentric pull in the cutoff bars, leads to

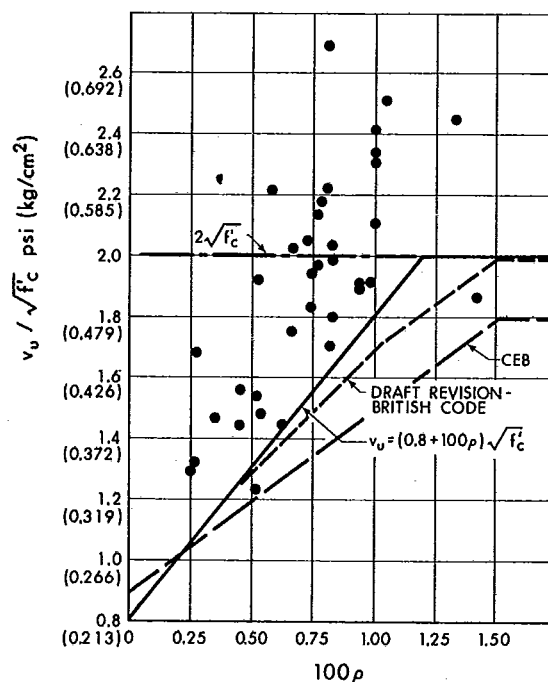


FIG. 3.9.—Effect of Longitudinal Reinforcement Ratio, ρ , on v_c (158)

a state of high shearing and diagonal tensile stresses in the vicinity of the cutoff. Baron (13) has shown that the changes in moment arm in this region lead to increased shears in the cutoff zone.

Tests by Ferguson and Matloob (65), Baron (13), and Bresler and Scordelis (26) show that:

1. Cutting off bars in a tension zone will lower the shear strength of a beam.
2. Closely spaced stirrups in the cutoff zone will prevent premature failure. The ACE-381-71 rules seem adequate in this regard.
3. Bending bars up instead of cutting them off will largely nullify these ill effects, probably because the bend discontinuity is less severe. On the other hand, a major inclined crack frequently occurs at the bend point.

(d) Web Reinforcement

The ACI procedure for designing stirrups assumes that the shear not carried by the concrete after inclined cracking is resisted by stirrups or bent bars acting essentially according to the truss analogy. Recent studies (26,72,85) have shown, however, that small amounts of web reinforcement have a significantly larger effect on the shear strength than predicted by the modified truss analogy. This is shown in Fig. 3.10 (72). While no adequate explanation of this effect has been presented, it has been noted that the horizontal projection of the inclined cracks tends to decrease as the web reinforcement ratio, ρ_n , increases, so that the increase in the number of stirrups crossed by a crack is less than the increase in ρ_n . Web reinforcement near the bottom of diagonal cracks is very effective in preventing dowel splitting cracks and in increasing the bond strength by providing confinement. Kani (99) and Gergely (69) claim that isolated stirrups at the bottom of diagonal cracks would prevent shear failure in most cases, but of course, the position of cracks is not known in design. However, their tests indicate that the primary role of web reinforcement

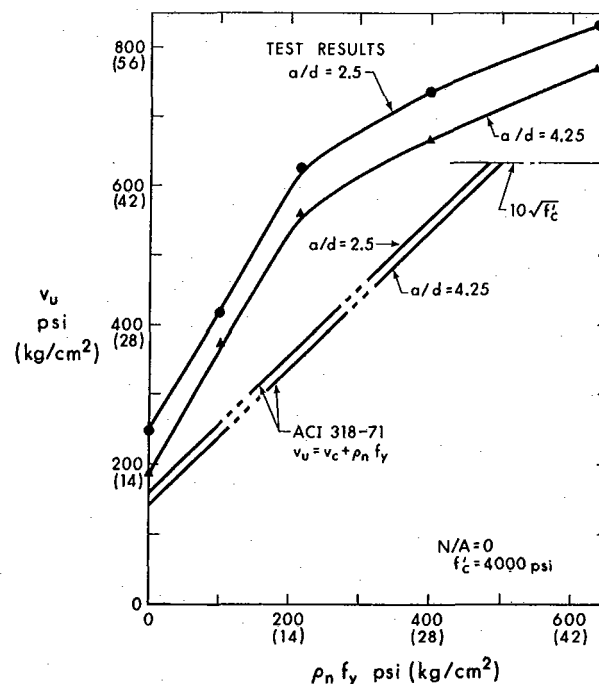


FIG. 3.10.—Effect of Variation in a/d on Relationship Between v and $\rho_n f_y$, (72)

is the restriction of the width of diagonal tension cracks. These authors believed that the interface shear transfer does not decrease appreciably until some splitting occurs at the bar and therefore additional stirrups crossing diagonal cracks are not necessary.

The maximum yield strength of web reinforcement was empirically set at 60,000 psi (4200 kg/cm²) in the 1971 ACI Building Code because difficulties are encountered in bending higher strength stirrups and also to prevent excessively wide inclined cracks.

The 1971 ACI Building Code requires a minimum area of web reinforcement in most members when V_u exceeds $1/2$ of the allowable V_c . This is intended to prevent or restrain shear failures in members where the sudden formation of inclined cracking may lead to distress. Such reinforcement may add valuable ductility where members are subjected to unexpected tensile forces, settlements or catastrophic loadings. The amount required corresponds to a nominal shearing stress resisted by stirrups of 50 psi ($\rho_n f_y = 50$ psi or 3.5 kg/cm²) and a minimum ratio of web reinforcement, ρ_n , between 0.00083 and 0.00125.

The European Concrete Committee (41) relates the minimum to the compressive strength of the concrete and the yield strength of the stirrups with a minimum of 0.0014 for deformed stirrups and 0.0025 for smooth mild steel stirrups. The Swiss Code (11) requires a minimum web reinforcement, $\rho_n f_y$, equal to half the nominal shear stress carried by the concrete. This corresponds to about 0.0010 for 2,200 psi (154 kg/cm²) concrete up to 0.00175 for 5,800 psi (410 kg/cm²) concrete. The increase in the minimum web reinforcement in this code is intended to reflect the fact that better concrete cracks at a higher stress and therefore at a higher load.

Placas (153) found that the horizontal projection of inclined cracks averaged $1.5d$ for beams with vertical stirrups and tended to be even greater in the case of inclined stirrups. The beneficial effect of the latter observation was offset by the inclination of the force in the stirrups. As a result he found vertical and inclined stirrups had equal efficiency. Leonhardt (116) and Bruce (29) also noted similar effects.

In view of its relative simplicity and applicability it appears that the modified truss analogy used in the ACI Building Code is the best design method available at the present time for members subjected to statically applied loads, and this report advocates its continued usage for beams with $M/V_d > 2.5$. More study should be given, however, to the Soviet design procedure that involves both shear and moment equilibrium on inclined planes as outlined later in this chapter (20,21,138). This procedure gives the designer a better understanding of the behavior of the finished member than does the ACI procedure.

In Leonhardt and Walther's tests (116) beams with bent-up bars as shear reinforcement tended to have a lower shear strength and much wider cracks than similar beams with stirrups for several reasons: (1) Bent-up bars have more tributary area loading each bar; (2) they tend to cause longitudinal cracking or crushing at the bend points; (3) they do not confine the concrete in the shear region; and (4) they are less efficient in tying the compression flange and web together. As a result, the 1970 CEB Recommendations (41) require the use of vertical stirrups in addition to bent-up bars. This problem is also discussed by Robinson (163). The ACI Building Code (6) also limits the shear that can be resisted by bent bars.

The principal conclusions of tests carried out in Stuttgart on welded wire mesh stirrups were (117)

1. Mats with 2-in. to 4-in. (5-cm to 10-cm) spacing of the stirrup bar were best with respect to crack widths and compressive stresses in the web.

2. An equivalent yield strength of 60,000 psi (4200 kg/cm²) can be used in load factor design.

The Commentary to the 1971 ACI Building Code provides similar anchorage details for welded wire mesh stirrups.

In some cases it is convenient to use pairs of U-shaped stirrups, lapped in the web of the beam to form a rectangular stirrup. In such a stirrup the lap occurs in a region of diagonal cracking. The relatively few tests of this type of stirrup carried out to date suggest that the capacities of the stirrups are a function of the lap length, stirrup diameter, and cover (77). Allen and Huggins (4) suggest that a lapped stirrup will have the same capacity as a rectangular stirrup if the lap is long enough to develop the yield strength of the stirrup bars. Section 12.13.4 of ACI 318-71 has similar requirements.

(e) Reinforcement of Flanges

Studies made by J. R. Robinson (163) suggest that the behavior of the tension chord is affected by the mechanics of the transmission of forces between the longitudinal reinforcement and the concrete web. In that respect the importance of the reinforcement details cannot be overemphasized. The studies of these details cannot be separated from the study of the bond and anchorage of the longitudinal and web bar including effects of transverse "binding" reinforcement used to prevent longitudinal splitting of the concrete along the main reinforcement. Robinson uses a truss analogy to design transverse reinforcement in both flanges. Leonhardt also presents these rules (114).

In tests of simply supported T-beams, Placas (153) observed that cracks may develop in the flange along the edge of the web. These cracks were parallel to the web rather than at a 45° angle, as frequently assumed. The model shown in Fig. 3.11 was used to explain this cracking. This figure shows the top of a portion of flange bounded by the web and midspan.

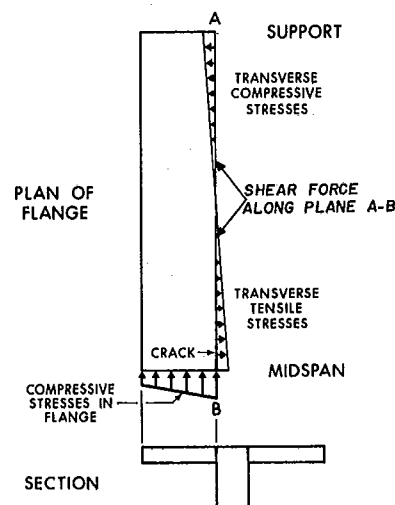


FIG. 3.11.—Flange Cracking in T-Beam

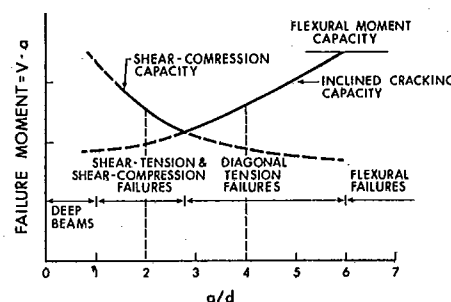


FIG. 3.12.—Variation in Shear Capacity with a/d for Rectangular Beams

Placas found that shear friction reinforcement across the surface, AB, would prevent failure. Johnson (93) has reported similar cracks and based on a shear-friction analysis recommends that the total amount of transverse reinforcement in the slab over the beam web should be:

$$\rho_t f_y \geq 1.26 v_u - 3.8 \sqrt{f'_c} \geq 80 \text{ (psi)} \quad (3.13)$$

Half of this reinforcement should be at the bottom face.

3.4.3 Effect of M/Vd Ratios and Types of Supports

(a) Span to Depth Ratios M/Vd

A number of authors (24) have presented figures similar to Fig. 3.12 showing that a/d or M/Vd is an important variable in defining the shear strength of a beam. This effect is explained in a conceptual fashion by Eqs. 3.2 and 3.3 and is discussed in Ref. 7. This chapter will deal primarily with beams having a/d or M/Vd ratios greater than about two. Deeper beams will be considered in Chapter 4.

It is generally assumed that the M/Vd concept can be used equally well in simply supported and continuous beams. This is not entirely true, however, especially for the common range of M/Vd ratios for negative moment regions ($M/Vd = 2$ to 3). The initial development of flexure-shear cracking is similar in the various portions of a continuous beam and the equivalent simple beams (22). Once the inclined cracks have developed, the load carrying mechanism of a beam loaded on its top face and supported on the bottom changes from pure beam action to a type of tied arch action with a diagonal compression strut from the load to the reaction as shown in Fig. 3.13(b). As a result, the regions of tensile force in the longitudinal bars frequently extend past the point of inflection, as shown in Fig. 3.14(c), leading to possible anchorage failures in these regions (30,112,119,151,164). The meaning of the terms, a/d and M/Vd , becomes difficult to define in such a case. Ferguson (64) has proposed that the ratio, a_3/d , shown in Fig. 3.13(a) be used in place of M/Vd in Eq. 3.6 when computing v_c or that v_c be taken equal to $2\sqrt{f'_c}$ in all cases for continuous reinforced concrete beams.

For continuous prestressed concrete beams the inclined thrust shown in Fig. 3.13(b) appears to delay the development of inclined cracking in the region of contraflexure (84). Once inclined cracks occur in that region, however, the ultimate shear strength of prestressed concrete beams with web reinforcement is best calculated using the ACI Code formulas (84, 168).

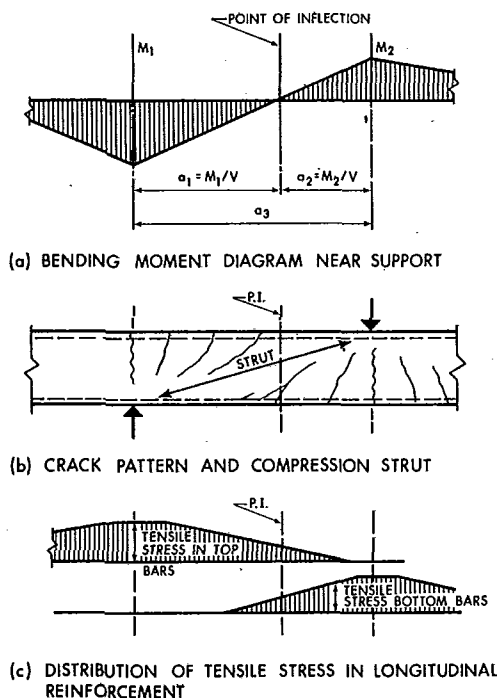


FIG. 3.13.—Behavior of Continuous Beams

(b) Directly and Indirectly Loaded Beams

In most beams tests the loads and reactions are applied on the top and bottom of the beam, respectively. Such beams are said to be "directly loaded." For directly loaded beams with a/d ratios less than 2.5 this leads to a significant vertical compressive stress component between the load and reaction and as a result the shear strength is increased. Thus, a portion of the increase shown in Fig. 3.12 between the inclined cracking and shear compression capacities of beams with a/d less than 2.5 is due to vertical stresses or arch action. For directly loaded beams having a/d ratios less than 2.5 the 1971 ACI Building Code Eq. (11-22) allows an increase in v_c . Zsutty (200) has proposed Eq. 3.14 to account for the direct loading effects in short beams:

$$v_c = (\text{Eq. 3.9}) \left(\frac{2.5}{a/d} \right) \dots \dots \dots (3.14)$$

Leonhardt (112) has proposed a similar equation.

Frequently in practice, beams are loaded or supported by intersecting beams so that the load transfer is by shear rather than by bearing on the top and bottom surface. These are referred to as "indirectly" loaded beams. A number of investigators have reported tests of such beams (15,61,119,177,182). For beams having a/d ratios greater than 2.5 to 3, there was no reduction in shear strength in such beams due to indirect loading provided that there was sufficient reinforcement in the intersection region to transfer the load from the supported member to the supporting member. Leonhardt, et al. (119) recommended the use of "hanger" reinforcement of stirrups or bent bars sufficient to transfer 80% of the shear force at the joint when beams of equal depth are connected and 100% of the shear force in the case in which the supported beam is connected only to the tension side of the supporting beam. Baumann and Rüschi (15) recommend hanger reinforcement for 100% of the load transferred. These stirrups should be provided within $d/2$ of the center of the joint in the members joined (115). In addition, it is good practice for the longitudinal reinforcement in the supported beam to pass over that in the supporting beam where possible.

For a/d less than 2.5, however, indirectly loaded beams are weaker than directly loaded beams because arch action cannot develop. Such beams tend to fail in diagonal tension at the inclined cracking load. Zsutty (200) shows that Eq. 3.9, developed to predict the shear strength of slender beams without web reinforcement, also applies for indirectly loaded short beams.

3.4.4 Effect of Type of Loading

(a) Repeated Loadings

From tests of 39 small beams without web reinforcement, Chang and Kesler (38) found that the inclined cracking shears and ultimate shears having a 50% probability at 2,000,000 cycles were 50% and 63% of the corresponding static strength. These beams failed either by crushing of the compression zone over the inclined crack or by a diagonal tension failure.

Price and Edwards (156) found the tensile fatigue strength of concrete at 3,000,000 cycles was about 60%-75% of the static tensile strength. This strength reduction may explain the reduced inclined cracking load in Chang and Kesler's beams.

Kokubu and Higai (103) report that beams with a/d ratios of 2 to 4 and no stirrups may fail due to fatigue fracture of the longitudinal reinforcement at the point where the inclined crack intersects the bar. In their tests, localized flexural

stresses due to the dowel action in the reinforcement in this region led to fatigue failures at 51% to 70% of the static failure load. Similar beams developed shear compression failures in static tests. For beams with a/d of 5 the inclined cracking and diagonal tension failure loads were each reduced by about 35%. On the basis of their tests, Kokubu and Higai have suggested that the shear strength corresponding to 10^6 cycles of load in each shear failure made is about 60% of the static failure load.

In tests of the cracking behavior of T-, I-, and rectangular reinforced concrete beams Kaar and Mattock (95) found that the stirrup stresses increased approximately 50% during the first 300,000 cycles of service load ($D + L$) and did not increase appreciably during another 700,000 cycles at the same load.

Prestressed concrete beams tested by Hanson, et al. (77,78) had an excellent shear fatigue resistance. Fatigue fractures of the web reinforcement were due primarily to flexural stresses in the stirrups due to dowelling. The range in the crack widths during repeated loading was an important factor in determining whether a shear fatigue failure would occur.

Burton and Hognestad (32) have shown that tack welding of stirrups to longitudinal reinforcement can significantly reduce the fatigue strength of the longitudinal bars. In their tests the stress range corresponding to failure at 5,000,000 cycles was 35% lower in beams with tack welded stirrups than in beams with tied stirrups.

(b) Dynamic Loadings

Extensive tests of simply supported reinforced concrete beams under dynamic loads indicate that the shear, moment, shear strength, and flexural strength all increase under a dynamic load with respect to the same load applied statically (166). However, the usable ultimate shear strength and the flexural yield strength increase in different proportions as do the contributions of the concrete and web reinforcement to the shear strength. In some cases the compression zones of beams failed prior to yield of the stirrups suggesting a need for an upper limit on shear strength under dynamic loads. For design, Seabold (166) has recommended the use of the modified truss analogy using Eq. 3.12 to compute V_c and using an amplified stirrup yield strength to account for the increase in yield strength under rapidly applied loads.

(c) Reversed Loadings

When a beam is subjected to reversals of loading, inclined cracks develop across the cracking caused by the preceding loading. Tests in which reversals of loading have been applied have shown that cross-inclined-cracking occurring before yielding of shear reinforcement does not destroy the integrity of the member (2,23), but uncertainty exists regarding the effect of reversals of loading on ultimate shear strength.

Alatorre and Casillas (2) tested 52 beams with a/d equal to 4.4 under reversed cycles of shear and moment. They report that the behavior was not significantly altered by the application of a few cycles of relatively high loads, except that there was a trend to a lower stiffness with increasing loads. The ACI design equations were found to be conservative for these beams.

On the other hand, Japanese investigators (146) have proposed the use of a reduced capacity of stirrups in beams and girders subjected to seismic loadings as given by:

$$v_{u_{min}} = v_c + 0.5f_y(\rho_n - 0.002) \quad (3.15)$$

in which v_c is about equal to the ACI value. Thus the shear assumed to be carried by the web reinforcement is less than half that computed using the ACI recommendations.

Paulay (151) applied reversed cycles of high shearing force to deep members restrained at each end. In the first few cycles the beams behaved as predicted by the ACI Code Sections 11.4.2 and 11.6.1 but as load cycling continued the shear carried by the concrete decreased. Following yielding of the web reinforcement, a high load was carried by arch action. Failure generally occurred by crushing and shearing of the ends of the arch. Significant decreases in stiffness were observed as the test progressed.

The apparent discrepancies may be due to the severity of the loading cycles considered by the various investigators. In a beam loaded statically to failure without load cycle reversals, Eq. 2.1 suggests that portions of the shear are carried by the uncracked concrete above the inclined cracks, the dowel action of the reinforcement, and interface shear transfer across the crack. For statically loaded beams and beams subjected to nonreversing cyclic loads ACI 318-71 Chapter 11 gives conservative strength values. In a member containing x-shaped cracks due to load reversals, however, the compression zone is cracked and due to residual tension strains in the precompression reinforcement following a load reversal, the cracks may not close completely. Thus the shear transferred by the compression zone may be smaller than in a normal beam. Interface shear transfer is unreliable since it may be reduced by a "working back and forth" action along the cracks as the load is cycled. Finally, dowel action will only be dependable under cyclic loads if splitting along the longitudinal bars is restrained by closely spaced stirrups. Based on this reasoning, Bresler (23) has proposed that stirrups be provided for the full shear in beams where full reversal takes place.

3.4.5 Effect of Axial Loading

(a) Behavior of Axially Loaded Beams in Tests

In a group of three otherwise identical beams subjected to axial tension, no axial force, and axial compression, the initial flexural cracks would occur earliest in the beam with axial tension forces and would extend farther and be more nearly vertical in this beam. In all three beams, however, approximately the same additional increment of shear is required between flexural cracking in the shear span and inclined cracking. The inclined cracks would have essentially the same slope in all three beams, cutting across or growing out of pre-existing flexural cracks (72,133). As shown in Fig. 3.14, the effect of axial forces on the shear at inclined cracking may not be too significant (72) especially for slender beams or for tensile loadings. Axial tensile cracks which had formed prior to the application of the shear loading had very little effect on the behavior and strength of beams tested by MacGregor (125) but reduced

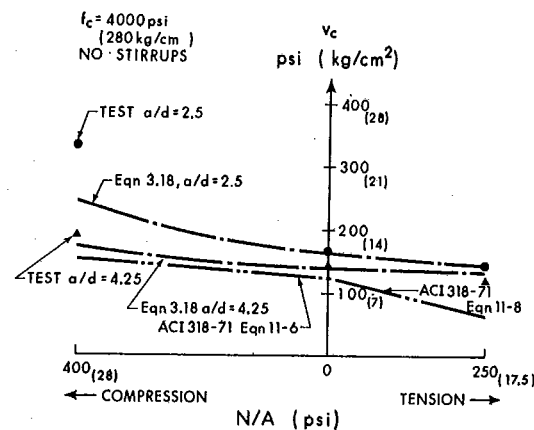


FIG. 3.14.—Variation of v_c with Axial Stress for Different Values of a/d (133)