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Deflections of Prestressed Concrete Members

Reported by Subcommittee 5, ACI Committee 435

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This report discusses the factors affecting the short-time and long-time deflection behavior of prestressed concrete members. Analytical methods are presented for calculating these deflections taking into account prestress, transverse loading, creep, shrinkage, and relaxation of steel stress.

Key words: ACI committee report; beam; creep; deflection; design; prestress; prestressed concrete; relaxation; shrinkage; slab; transverse load.

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CHAPTER 1 — INTRODUCTION

101 — Objectives

The objectives of this report are to discuss the factors affecting the short-time and long-time deflection behavior of prestressed concrete members and to recommend both approximate and more precise analytical methods for calculating these deflections.

In the design of prestressed concrete structures the deflections under short-time or long-time service loads may often be the governing criteria in the determination of the required member sizes and amounts of prestress. The variety of possible situations that can arise are too numerous to be covered by a single set of fixed rules for calculating deflections. However, it is felt that a thorough understanding of the basic factors contributing to these deflections, which are discussed in this report, will enable a competent designer to make a reasonable estimate of the deflections in most of the cases encountered in prestressed concrete design.

102 — Scope

Both short-time and long-time transverse deflections of beams, girders, and slabs involving prestressing with high strength steel are considered. Specific values of material constants given in this report, such as modulus of elasticity, creep coefficients, and shrinkage coefficients, refer to normal weight concrete. Bonded or unbonded prestressing steel is included, but the effect of unprestressed reinforcement is not considered.

103 — Notation

A	= area of section	M	= bending moment
B, b	= subscript denoting bottom fiber of section	P	= prestress force
C_i	= creep coefficient defined as ratio of creep strain to initial strain at any time t	sh	= subscript denoting shrinkage
C_u	= ratio of ultimate creep strain to initial strain	T, t	= subscript denoting top fiber of section
c	= subscript denoting concrete, as f'_c and E_c	t	= subscript denoting variable time measured from the time of application of prestress force
E	= modulus of elasticity	u	= subscript denoting ultimate value
e	= eccentricity of prestress steel	x	= variable distance along beam
H	= relative humidity ($H = 70$ for 70 percent relative humidity)	y	= variable deflection along beam
h	= over-all depth of cross section	y	= distance from centroid of section to fiber under consideration
I	= moment of inertia (second moment of the area) of section	δ	= unit creep strain defined as creep per unit stress
i	= subscript denoting initial value (immediately following application of prestress or transverse load)	ϵ	= unit strain, tensile strains are positive and compressive strains are negative
L	= beam span	ϕ	= curvature or angle change per unit length of beam = d^2y/dx^2
		σ	= unit stress

CHAPTER 2 — GENERAL FEATURES OF BEHAVIOR

201 — Introductory remarks

This chapter is concerned with the transverse deflections of prestressed concrete members caused by the application of prestressing and external forces. Especially in indeterminate structures, the axial changes in length of prestressed concrete members caused by the prestressing force or by external forces may also be critical. Although direct reference will be made only to transverse deflections, it is also possible in general to estimate the effects of axial displacements by means of the methods outlined in this report.

The definition of the deflection of a prestressed concrete beam can be ambiguous. In this report, the deflection will refer to the position of the beam before the prestressing operation unless specified otherwise.

A simply supported beam deflects upwards under the action of the prestressing force and downward under the action of the transverse loads. The deflection may be defined with respect to two different reference lines: the position of the beam before the release of prestress or the position of the beam just before the application of load. The situation may become complicated if the load or prestress is applied in stages. Any definition may be justified in relation to the critical quantity being sought. For roof beams, the critical deflection would be that with respect to the original position of the beam in the forms. For bridges with cast-in-place riding surfaces, the critical deflection would be that occurring after the riding surface is placed.

Only uncracked prestressed concrete sections will be considered; cracked prestressed concrete sections can also be analyzed on the basis of the methods described here. Unless noted otherwise, the beams considered will be assumed to be cast in a single operation and have bonded reinforcement.

The deflections of prestressed concrete beams are considered under two engineering definitions: short-time and long-time deflections.

Short-time deflections are defined as those occurring instantaneously under the application of any internal or external force. The time element is assumed to be unimportant, no matter what the rate of loading, provided the load is applied within a matter of hours.

Long-time deflections refer to those existing at some time interval after the prestressing or loading operation.

202 — Short-time deflections

In general, the variables affecting the short-time deflections of a prestressed concrete beam are the magnitude and distribution of the load, the length of the span, the size and configuration of the cross section, and the quality of the concrete. More specifically, the effect of critical variables may be summarized by the magnitude of the strain or stress gradient or the curvature at a section and the variation of this quantity along the span.

The curvature at a particular section (see Fig. 1) is defined by

$$\phi_t = \frac{\epsilon_{bt} - \epsilon_{tt}}{h} = \frac{M}{E_t I} \quad (1)$$

in which tensile strains are positive and compressive strains are negative.

In most cases the variations in the amount of the prestressing steel affect the short-time deflections due to transverse loads negligibly. As long as the beam remains uncracked, variations in the prestressing force do not affect the short-time changes in deflection at all, provided the concrete and the steel strains increase linearly with stress, as will be assumed throughout this chapter.

In effect, the short-time deflections of uncracked prestressed concrete elements can be calculated in accordance with the usual methods of calculating deflections applied to linearly elastic members. The modulus of elasticity to be assumed in these calculations is discussed in the following chapter. As indicated above, the calculations may be carried out on the basis of the gross concrete section.

203 — Long-time deflections

Even if the external loads remain constant, the deformations and displacements of a prestressed concrete beam will change with time as a result of creep and shrinkage of the concrete and relaxation of the prestressing reinforcement.

Shrinkage strain is defined as deformation of the concrete which occurs without stresses attributable to forces external to the concrete. Creep is defined as the time-dependent deformation occurring under stress over and above that which would have been caused by shrinkage.

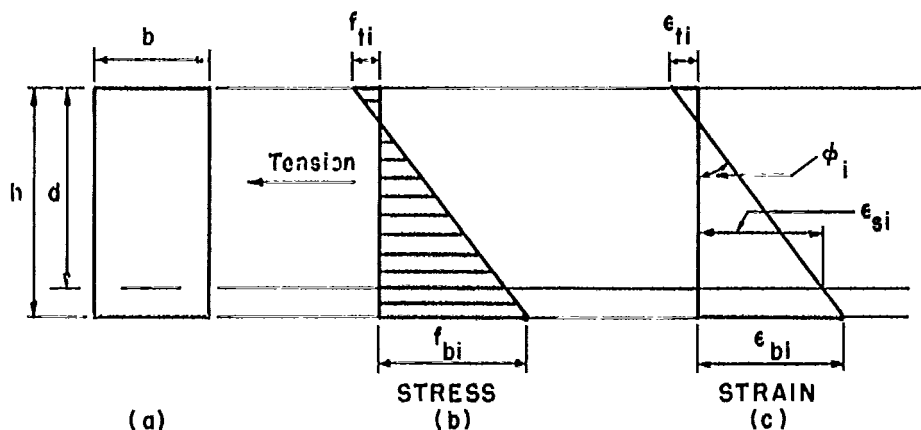


Fig. 1—Stress and strain distribution immediately after application of the prestressing force

Actually, these two types of strains are rather difficult to separate from each other and more elaborate definitions have to be stated if the quest of the analysis were these strains alone. Nevertheless, in considering the deflections of reinforced or prestressed concrete elements, the above simple definitions are satisfactory.

Relaxation loss of the prestressing steel is defined as the stress loss occurring at constant strain, a phenomenon related to creep.

In Chapter 3, all of these effects are discussed in detail.

To bring out the effects of the critical variables on time-dependent deflections, two simple cases will be considered in the following sections: a prestressed concrete beam without any transverse load and a prestressed concrete beam with all the transverse load applied at once.

203.1 — Prestressed concrete beam without transverse loads

Stress and strain distributions over the depth of a cross section of a rectangular bonded beam immediately after application of the prestressing force are shown in Fig. 1. It is assumed that there is a linear distribution of strain over the depth of the section and that there is a linear relationship between concrete stress and strain. Under ordinary conditions, both of these assumptions are reasonably correct. The stress at any level is given by the well-known relationship:

$$f = \pm \frac{P}{A} \pm \frac{Mc}{I} \quad (2)$$

and the curvature can be expressed as

$$\phi = \frac{f_{ti} - f_{bi}}{h} = \frac{Pc}{E.I} = \frac{M}{E.I} \quad (3)$$

where P is the prestressing force.

The stress and strain distributions in Fig. 2 depict the conditions existing after a given time. The normal stresses on the section decrease as a result of a reduction in the prestressing force while there is a general shift to the right in the strain distribution accompanied by an increase in the strain gradient or angle change ϕ .

These changes are caused by an interaction between creep and shrinkage of the concrete and relaxation of the reinforcement. All of these effects occur continuously with time and affect each other continuously. However, it is preferable to treat these individually and as step-functions to simplify the discussion.

Consider first the effect of shrinkage strains. It is assumed that each element of concrete in the cross section shrinks equally. Thus, the shrinkage strain distribution after a time t is given in Fig. 3b. This distribution of shrinkage strain causes a reduction in the reinforcement strain which corresponds to a reduction in the prestress. The loss in

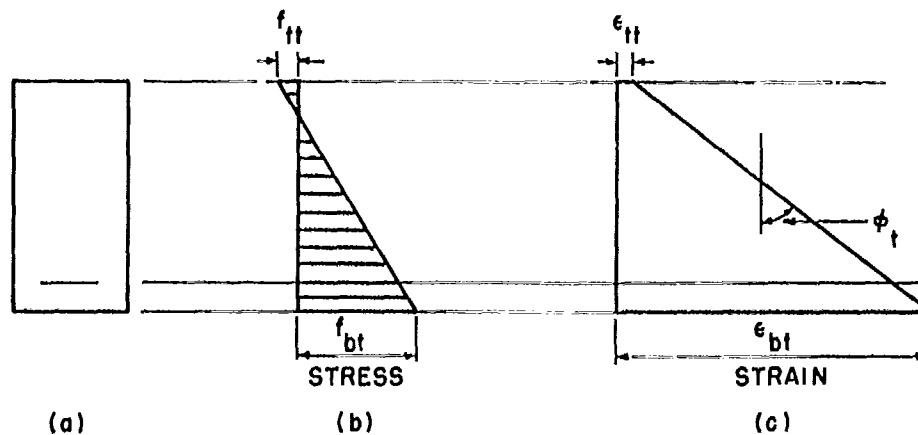


Fig. 2—Stress and strain distribution at a time t after initial application of prestressing force

prestress causes a change in the stress distribution over the depth of the section as indicated in Fig. 3c and the corresponding change in the strain distribution, Fig. 3d. Thus, the change in curvature is

$$\Delta\phi = \frac{\Delta\epsilon_b - \Delta\epsilon_t}{h} \quad (4)$$

The effect of the relaxation losses in the steel is quite similar to that of shrinkage. At a time t there is a certain loss in the prestress force which creates a reduction in the curvature as explained above.

The effects of the creep of the concrete are not as simple, since the reduction in stress causes changes in the rate of creep strain.

It is assumed that the amount of creep strain at a given time is proportional to the stress. Thus, the change in strain caused by creep is directly proportional to the instantaneous strain distribution (Fig. 1c), which is related directly to the stress distribution. This change in the strain distribution involves a contraction at the level of the steel and, therefore, a reduction in prestress. The reduction in prestress caused by creep, shrinkage, and relaxation decreases the normal stress. This decrease in normal stress reduces the rate of creep.

Thus, after a finite interval of time the increase in curvature caused by creep is a multiple of the instantaneous curvature. However, this multiple is less than the ratio of creep to instantaneous strain obtained for the same time interval from a comparable concrete subjected to constant stress.

The effects of shrinkage, creep, and relaxation add up to the stress and strain distributions shown in Fig. 2b and 2c. Although the stress in

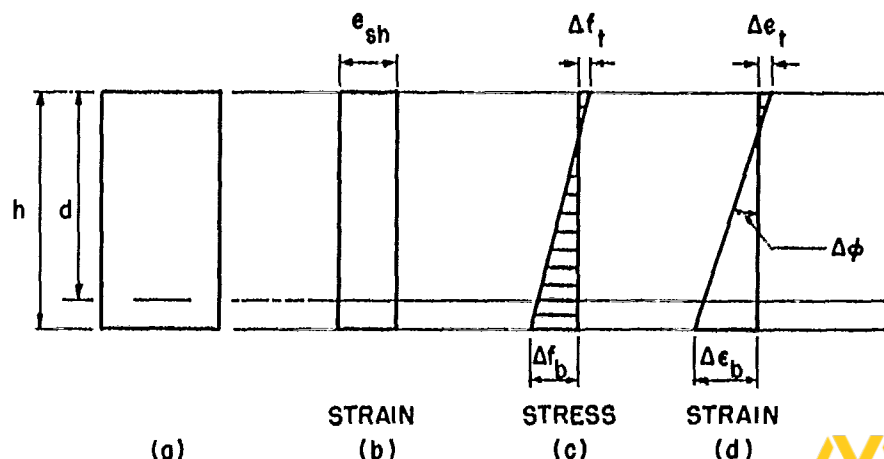


Fig. 3—Stress and strain distributions due to shrinkage

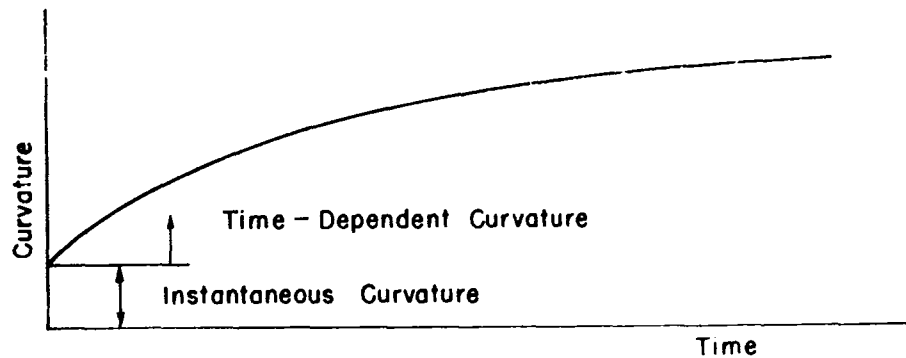


Fig. 4—Plot of curvature versus time for a beam subjected to prestressing force only

the top fiber remains in tension, the total strain may be a shortening if the shrinkage strain is large.

Creep strains affect the curvature almost directly while the shrinkage and relaxation affect it indirectly through losses in prestress. The beams subjected to prestress alone may be considered to be subjected to two different effects: the increase in curvature caused by creep and the decrease in curvature caused by the prestress loss due to relaxation, shrinkage, and creep. In general, these add up to an increase in the curvature.

A qualitative curvature versus time curve is shown in Fig. 4. The magnitude of the time-dependent deflection depends on the particular characteristics of the beam involved. The deflection of the beam depends on the distribution of the curvature which is influenced primarily by the profile of the steel.

As in the case of short-time deflections, the magnitude of the deflection may be estimated by the magnitude of the stress gradient over the depth of the section after release of prestress. If the stress gradient is very small, then shrinkage and relaxation are bound to dominate, in which case the beam may deflect downward. However, under usual circumstances the stress gradient is large and creep dominates the deflection thus causing the beam to move upward in a simply supported case.

203.2 — Prestressed concrete beam subjected to transverse loading

If the beam considered in the preceding paragraphs is subjected to a transverse load, the stress distribution across the section at a given point along the span may be as indicated in Fig. 5d. Provided neither the concrete nor the steel is strained into the inelastic range, the stress distribution caused by the prestressing force (Fig. 5b) can be superposed on the stress distribution caused by the transverse load on the uncracked transformed section (Fig. 5c) to obtain the stress distribution shown in Fig. 5d.

Naturally, the magnitude and gradient of the stress distribution varies depending on the combinations of the stress distributions shown in Fig.

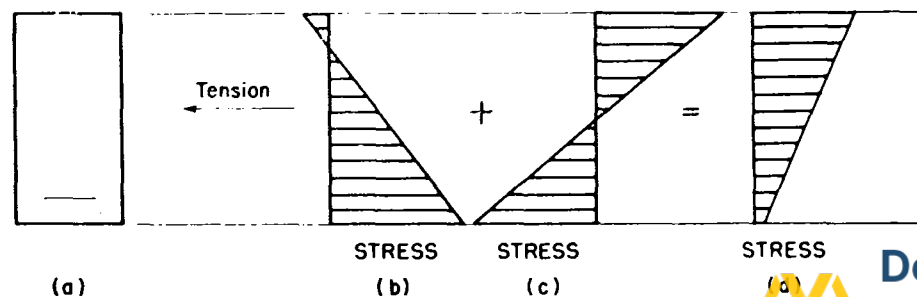


Fig. 5—Stress distributions due to prestressing force and transverse loading

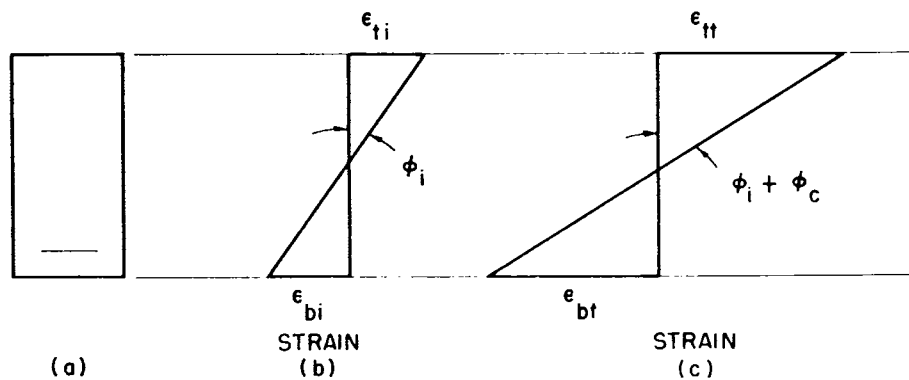


Fig. 6—Strain distribution due to transverse loading only

5b and 5c. For example, in a simply supported uniformly loaded beam with straight cables, the stress distribution over the end reaction may be assumed to be essentially that caused by the prestress. The stress distribution at midspan is the one influenced most by the transverse loads. While the curvature near the reaction may be increasing with time so that the curvature is concave downward, the curvature at midspan may be changing so that the additional curvature is concave upward.

The phenomena occurring at a section under the combined influence of the prestressing force and the transverse load may be described in a simpler manner if the effects of the prestress and the transverse load are treated separately. The effect of the prestressing force was discussed in the preceding section along with the effects of shrinkage and relaxation. The effect of the stresses caused by the transverse load are discussed below.

The strain distribution shown in Fig. 6b corresponds to the stress distribution in Fig. 5c. It depicts the strains that would occur in an uncracked section under the influence of only the transverse load. The short-time curvature is

$$\phi_i = \frac{\epsilon_{ti} - \epsilon_{bi}}{h} \quad (5)$$

If it is assumed that the creep characteristics for this imaginary beam are equal in tension and compression, it follows that after a given time the strain distribution will be as shown in Fig. 6c. Depending on the amount of steel and creep strain at the level of the steel, the neutral axis will shift towards the steel and the stresses in the concrete will be reduced. However, this is a minor effect and may be neglected. Consequently, the relative increase in curvature after a given time will be

the same as the ratio of the creep to instantaneous strain in a comparable specimen of concrete under constant stress.

The changes in the curvature or in the deflection of the beam caused by the prestress and the transverse load may be determined by superposition. For example, in the uniformly loaded beam considered, the curvature distribution on application of the load is as shown in Fig. 7b. Assuming that the prestress force is developed so that the corresponding strain distribution is constant throughout the span, the

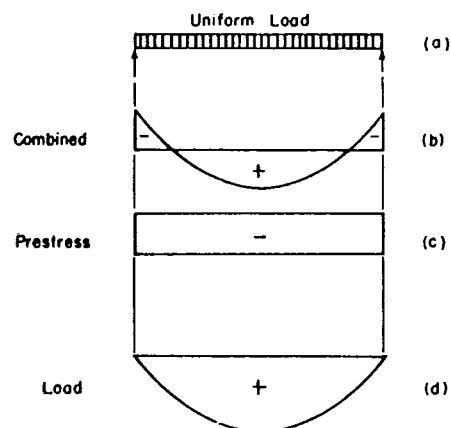


Fig. 7—Variation in curvature with span