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Deflections of Reinforced Concrete Flexural Members

Reported by ACI Committee 435

DAN E. BRANSON

Chairman

JACK R. BENJAMIN
 ALLEN H. BROWNFIELD
 CARLOS D. BULLOCK
 W. GENE CORLEY
 LAWRENCE N. DALLAM
 RUSSELL S. FLING
 J. A. HANSON

CHARLES M. HERD
 HAROLD J. JOBSE
 GEORGE E. LARGE
 MARVIN A. LARSON
 DONALD R. PEIRCE
 MICHAEL V. PREGNOFF
 A. C. SCORDELIS

DAVID WATSTEIN

Discusses the principal factors affecting short-time and long-time deflections of reinforced concrete flexural members. Several methods for computing deflections are reviewed and a study made of the accuracy of these methods for predicting initial and time-dependent deflections.

Key words: beams (structural); compressive strength; concrete; continuity; cracking; creep; deflections; lightweight concrete; live loads; loading; modulus of elasticity; modulus of rupture; moment; moment of inertia; reinforced concrete; reinforcement; shrinkage; strain; warping.

CONTENTS

Chapter 1—Introduction.....	435.2R-2
101 Object and scope of the report	
102 Notation	
Chapter 2—Behavior of Concrete.....	435.2R-4
201 Behavior under short-time loading	
202 Behavior under long-time loading	
Chapter 3—Calculation of Deflections.....	435.2R-6
301 Introductory remarks	
302 Description of deflection response	
303 Deflections under short-time loading	
303.1 Calculation procedures	
304 Deflections under long-time loading	
304.1 Effect of concrete creep	
304.2 Shrinkage warping of beams	
304.2.1 Equivalent tensile force method	
304.2.2 Empirical methods	
304.2.3 Comparison with test results	
304.3 Calculation procedures	
Chapter 4—Comparison with Test Results and Discussion.....	435.2R-17
401 Comparison with test results	
402 Discussion	
Chapter 5—Summary.....	435.2R-21
Chapter 6—References.....	435.2R-21
Appendix 1—Sample Calculations.....	435.2R-23
Appendix 2—Data Parameters, Comparison with Test Results.....	435.2R-26


Documents

Document Sharing Hub

CHAPTER 1—INTRODUCTION

101—Object and scope of the report

With the present-day tendency toward the use of higher strength concrete and reinforcing steel, and shallower sections, the problem of predicting and controlling deflections of reinforced concrete flexural members under service loads is assuming greater importance. The objectives of this report are to discuss the more significant factors affecting short-time and long-time deflection behavior and to review various methods and their accuracy for predicting these deflections.

Appropriate values of material constants for both normal weight and lightweight concrete are discussed and both simple and continuous beams are considered.

This report is limited to determinative methods for predicting deflections. Actually the deflection of a reinforced concrete beam is a random variable requiring a statistical approach for a rational analysis. Unfortunately, such an approach is beyond the present state of the art, although research has been reported¹ on the modeling of the time variations in the deflection of reinforced concrete beams as a stochastic process.

102—Notation

A_s	= area of tensile steel	D	= total depth of beam
A_s'	= area of compressive steel	d	= effective depth of reinforced concrete beam
b	= width of beam at the compression side	E	= modulus of elasticity
b'	= width of beam at the tension side	E_c	= modulus of elasticity of concrete, short duration of loading
C_i	= creep coefficient defined as the ratio of creep strain to initial (elastic) strain	E_{ct}	= reduced or sustained modulus of elasticity of concrete, long duration of loading
C_u	= ratio of ultimate creep strain to initial (elastic) strain. Used when numerical values are given for ultimate or design coefficients	E_s	= modulus of elasticity of steel
cp	= subscript denoting creep	$\bar{E}_s$	= average effective modulus of elasticity of steel when participation of tensile concrete is taken into account [see Eq. (15)]

e	= steel eccentricity. Used in Eq. (28) to denote the distance between the centroids of the uncracked transformed section (using n_{ct}) and the steel area		
e_g	= distance between the centroids of the gross concrete section and the steel area	k_r	= reduction factor in creep deflection equation [Eq. (36)]
F	= Yu and Winter's multipliers for additional deflection due to shrinkage-plus-creep (see Table 4)	k_1	= constant in Eq. (10) which depends on the type of loading and end restraints
f_c	= maximum compressive stress in concrete	k_2	= constant in Eq. (11), less than unity
f'_c	= ultimate compressive strength of concrete	k_3	= constant in Eq. (12) which depends on loading and end conditions
f'_{cb}	= modulus of rupture of concrete	kd	= distance from extreme compression fiber to neutral axis for cracked transformed section
f_s	= stress in steel reinforcement	L	= span length
f'_t	= ultimate tensile strength of concrete	M	= bending moment
f_y	= yield point of steel	M_{cr}	= cracking moment
H	= relative humidity, percent. ($H = 70$ for 70 percent relative humidity)	M_I, M_{II}	= moments defined in CEB method
I	= moment of inertia	max	= subscript denoting maximum value
I_{cr}	= moment of inertia of cracked transformed section	n	= modular ratio defined as E_s/E_c
I_{ct}	= moment of inertia of cracked or uncracked transformed section computed using n_{ct} . Only referred to generally in this report in Eq. (28)	n_{ct}	= increased modular ratio defined as E_s/E_{ct}
I_{eff}	= effective moment of inertia [Eq. (25) refers to an individual section value and Eq. (20) and (26) refer to average values for the beam]	p	= tensile steel percentage defined as A_s/bd . Also used in percent or $(A_s/bd)100$ as noted. See Eq. (31) through (34), for example
I_g	= moment of inertia of gross concrete section (neglecting all steel)	p'	= compressive steel percentage defined as A'_s/bd . Also used in percent or $(A'_s/bd)100$ as noted. See Eq. (31) through (34), for example
i	= subscript denoting initial value	p_{bal}	= balanced steel percentage
jd	= internal couple lever arm in elastic theory	p_w	= steel percentage in T-beams defined as $A_s/b'd$
K	= flexural rigidity	p_f	= steel percentage in T-beams defined as $A_{sf}/b'd$, where $A_{sf} = (0.85)(f'_c)(b - b')(t)/f_y$
K_I, K_{II}, K'_{II}	= flexural rigidities defined in CEB method [see Eq. (21) through (23)]	q	= $(A_s/bd)(f_y/f'_c)$
K_t	= PCA factor to be multiplied by the short-time (gross section) deflection to obtain the total deflection under sustained loading	R	= concrete cube strength in kg per sq cm [see Eq. (21)]
		s	= subscript denoting steel
		sh	= subscript denoting shrinkage
		T_s	= $(A_s + A'_s) \epsilon_{sh} E_s$ = fictitious compressive force induced in the steel to permit the concrete to shrink uniformly [see Eq. (28) and (29)]

t	= flange thickness for T-beams. Also used in Eq. (35) to denote time in days under sustained loading. Also used as subscript denoting time-dependent and in the case of y_t , tension	Δ	= maximum deflection
u	= subscript denoting ultimate value	Δ_{20}	= 20-year deflection
W	= total load on a span	$\Delta_I, \Delta_{II}, \Delta'_{II}$	= deflections defined in CEB method
w	= unit weight of concrete in pounds per cubic foot when used with Eq. (1). Also used to denote uniformly distributed load	Δ_{sh}	= shrinkage deflection
w_{DL}	= uniformly distributed dead load	δ_t	= unit creep strain defined as creep strain per unit stress (per psi of concrete stress)
w_{SL}	= uniformly distributed superimposed load	ϵ	= unit strain
y_t	= distance from neutral axis to the extreme fiber in tension	ϵ_s	= steel strain
α	= coefficient used in Eq. (27)	ϵ_{sh}	= free shrinkage strain
		σ	= unit stress
		ϕ	= curvature or angle change per unit length of beam
		ϕ_{sh}	= curvature associated with shrinkage warping
		ψ	= coefficient taking into account the participation of concrete in tension [see Eq. (16)]

CHAPTER 2—BEHAVIOR OF CONCRETE

201—Behavior under short-time loading

The concrete properties of most importance in the design of reinforced concrete flexural members for short-time loads are compressive strength, modulus of elasticity, and modulus of rupture. Since the rate of load application to concrete is significant, even for very rapid rates of loading, the term short-time loading seems to be preferable to that of instantaneous loading.

The modulus of elasticity of concrete is a rather elusive property because of its significant variation primarily with concrete quality, concrete age, stress level and rate or duration of applied stress. Of primary interest with regard to the design aspects of the concrete modulus of elasticity are the relations between E_c and f'_c , E_c with concrete age (under short-time loading), and E_c with concrete creep (creep and material stiffness are inherently related in a viscoelastic material).

Values of modulus of elasticity and modulus of rupture are not accurate functions of compressive strength alone. Nevertheless, for most practical applications the following approximate formulas are usually satisfactory:

For normal weight and lightweight concrete^{2,3}

$$E_c = 33\sqrt{w^3 f'_c}, \text{ psi; } w \text{ in pcf and } f'_c \text{ in psi} \quad (1)$$

or

$$E_c = 57,600\sqrt{f'_c}, \text{ psi; for concrete weighing 145 pcf} \quad (2)$$

For normal weight concrete and continuously moist lightweight concrete^{4,5}

$$f_{cb} = 7.5\sqrt{f'_c}, \text{ to } 12\sqrt{f'_c}, \text{ psi; } f'_c \text{ in psi} \quad (3)$$

For lightweight concrete which is in a drying condition^{5,6}

$$f_{cb} = 5\sqrt{f'_c}, \text{ to } 11\sqrt{f'_c}, \text{ psi; } f'_c \text{ in psi} \quad (4)$$

The smaller factors apply to higher strength concretes and the larger factors to lower strength concretes. The smaller factors in each case are suggested by this committee for general use.

202—Behavior under long-time loading

Concrete strength, modulus of elasticity, shrinkage and creep continue to increase for long periods. The data of Davis and Troxell⁷ show an



increase in compressive strength and modulus of elasticity of up to 1.3 times the 28-day value at the end of 3 years. More recent results of Klieger,⁸ summarized in Table 1, indicate even greater increases. Average shrinkage and creep curves of Troxell, Raphael, and Davis⁹ show that at the end of 1 year slightly less than 80 percent of the 20-year values have occurred for both shrinkage and creep.

In the case of shrinkage and creep properties, it is only possible to generalize within rather broad limits, and test data which incorporates the effects of local aggregates and conditions should be used when available. Some areas of the country are beginning to collect such information for general use (Reference 10, for example). In the absence of test data, the following shrinkage and creep information is often useful.

Schorer's¹¹ formula has been used for calculating shrinkage strains of hardened, sand-and-gravel concrete (exclusive of plastic shrinkage) for design purposes:

$$\epsilon_{sh} = 12.5 \times 10^{-6} (90 - H) \dots\dots\dots (5)$$

This formula gives an ultimate or design total shrinkage strain as a function of relative humidity, but other variables account for rather wide variations under certain conditions. Shrinkage strains as high as 1200×10^{-6} have been reported in California¹⁰ for a relative humidity of 50 percent, as compared to 500×10^{-6} given by Eq. (5). However, most shrinkage data agree with this equation within usable limits. Shideler,⁴ Hanson,¹² and Reichard¹³ have found that drying shrinkage of some lightweight concretes was less than that of some sand-and-gravel concrete, and as much as 38 percent greater than others.

In considering the effects of creep deformation, the use of a unit creep strain δ , (creep per unit stress) or creep coefficient C_t , (ratio of creep strain to initial strain) yields the same results, since the concrete modulus E_c must be included in either case and:

$$C_t = \delta_t E_c \dots\dots\dots (6)$$

This is seen from the relations:

$$\text{Creep strain} = (\sigma_{\text{constant}}) \delta_t = (\epsilon_{\text{initial}}) C_t, \quad E_c = \frac{\sigma_{\text{constant}}}{\epsilon_{\text{initial}}} \dots\dots\dots (7)$$

The choice of δ , or C_t , is a matter of convenience depending on whether it is desired to apply the creep factor to stress or strain.

Approximate ultimate values for the creep coefficient for normal weight and lightweight concrete under average design conditions are shown in Table 2, where the smaller values tend to be applicable in each case except when the age of the concrete at the time the sustained load is applied is of the order of a few days to a week or so. For load durations less than "permanent," the percentages of ultimate creep from Reference 9 are approximately 12, 35, and 75 percent for load durations of 1 day, 1 month, and 1 year, respectively. A similar time relation is

TABLE 1—ONE-YEAR AND THREE-YEAR RELATIVE INCREASES IN COMPRESSIVE STRENGTH AND MODULUS OF ELASTICITY OF CONCRETE MADE WITH DIFFERENT TYPES OF CEMENT

Type of cement	Compressive strength			Modulus of elasticity		
	28 days	1 year	3 years	28 days	1 year	3 years
I	1.00	1.25	1.35	1.00	1.53	1.58
II	1.00	1.40	1.48	1.00	1.54	1.63
III	1.00	1.20	1.28	1.00	1.40	1.40

TABLE 2—RATIO OF ULTIMATE CREEP STRAIN TO INITIAL (ELASTIC) STRAIN, C_u , FOR BOTH NORMAL WEIGHT AND LIGHTWEIGHT CONCRETE

Concrete strength	Average relative humidity			Average values for normal conditions
	100 per-cent	70 per-cent	50 per-cent	
Ordinary (3000 psi $\pm$)	1-2	1.5-3	1.5-2	2
High (5000 psi $\pm$)	0.7-1.5	1-2.5	1.5-3.5	1.5

given in Table 4 for combined shrinkage and creep effects and by Eq. (35) for initial-plus-creep deflections. Although Shank's formula,^{14,15} which provides a time relation for unit creep strain, δ_t , predicts creep values that are generally much too high, it is sometimes useful in a qualitative sense because it includes both duration-of-load and age-of-concrete-when-loaded parameters in addition to a concrete quality consideration. The methods of References 16 and 17 may be more applicable in this regard, however. Perhaps the best formula for predicting the creep-time relation was given by Lorman.¹⁸

Shideler⁴ found that creep coefficients of some lightweight concretes compare very favorably with those of sand-and-gravel concrete, while those of other lightweight concretes were as much as 60 percent greater than those of sand-and-gravel concrete. Hanson¹² reported creep of one lightweight concrete to be less than that of comparable normal weight concrete. These findings are in general agreement with a recent report by Reichard¹⁸ which showed that relatively high values of creep and shrinkage are not always associated with lightweight concretes and that, in fact, many lightweight concretes have practically the same mechanical properties as some normal weight concretes. This report also suggested that it is possible to obtain a reliable estimate of long-time creep from a relatively short-time test, but that such an estimate appears to be considerably less reliable for shrinkage.

References 12 and 19 show that low pressure steam curing of precast products reduce both creep and shrinkage and tend further to equalize these properties for concrete containing sand-and-gravel, and lightweight aggregates. It should also be noted that recent work^{20,21} clearly indicates that smaller members have greater creep and shrinkage than larger ones.

The modulus of elasticity employed in the analysis of concrete structures should be used with caution and properly reduced (one way to include creep effects) for loads other than those of very short duration. The resulting reduced modulus is given by the equation:

$$E_{ct} = E_c / (1 + C_t) \dots\dots\dots (8)$$

where the creep coefficient C_t is a function primarily of relative humidity, concrete quality, duration of applied loading, and age of the concrete when loaded. Effects of variable stress history can be considered with the use of the rate-of-creep method¹⁶ or the superposition method¹⁷ when creep-time data and load-history information are available.

CHAPTER 3—CALCULATION OF DEFLECTIONS

301—Introductory remarks

Discussed in this chapter are the principal factors affecting short-time and long-time deflections of reinforced concrete beams and various methods which have been proposed for calculating initial, shrinkage and creep deflections.

The deflection of a beam from a reference base of zero stress, strain, and deflection can be evaluated at any time if the magnitude and the longitudinal distribution of the curvatures, ϕ , (angle change per unit length) for the beam span are known for that instant. Any suitable integration procedure or equivalent method, such as conjugate beam or moment area, may be used to obtain the deflections from the curvatures.

For an elastic homogeneous member subjected to flexure:

$$\phi = \frac{M}{EI} \dots\dots\dots (9)$$

and the deflection at a particular point can be expressed as:

$$\Delta = k_1 \frac{WL^3}{EI} \dots\dots\dots (10)$$

in which k_1 is a constant depending on the type of loading and end restraints, W is the total load on the span, L , and EI is a reference flexural rigidity. Some form of Eq. (10) is commonly used in the calculation of deflections.

The principal factors that influence the short-time deflection behavior of a member are the magnitude and distribution of the load, span and conditions of restraint, section properties including steel percentages, material properties, and amount and extent of flexural cracking.

Long-time deflections, which generally are of primary importance, are due principally to the effects of shrinkage and creep. Values for these effects must usually be approximated, since they occur under a constantly changing strain and stress distribution over the depth and span of the beam. The principal factors which affect long-time deflections are stresses in the concrete, amount of tensile and compressive reinforcement, size of member, curing conditions, temperature, relative humidity, age of concrete at time of loading, and duration of loading.

302—Description of deflection response

A reinforced concrete beam is a member whose various cross sections along the span are subjected to distinctly different stress configurations during bending (Fig. 1). At places of small bending moments the concrete works both in tension and compression. At places of greater moments the concrete in tension fails at the outer fibers and minute cracks are formed at random intervals. At places of still greater moments the tensile failure in the concrete is more extensive, cracking extends closer to the neutral axis and the cracks widen. However, the concrete between the cracks still carries some tension. Due to this fact the tensile stress in the steel between the cracks is less than that at the cracks. The ratio

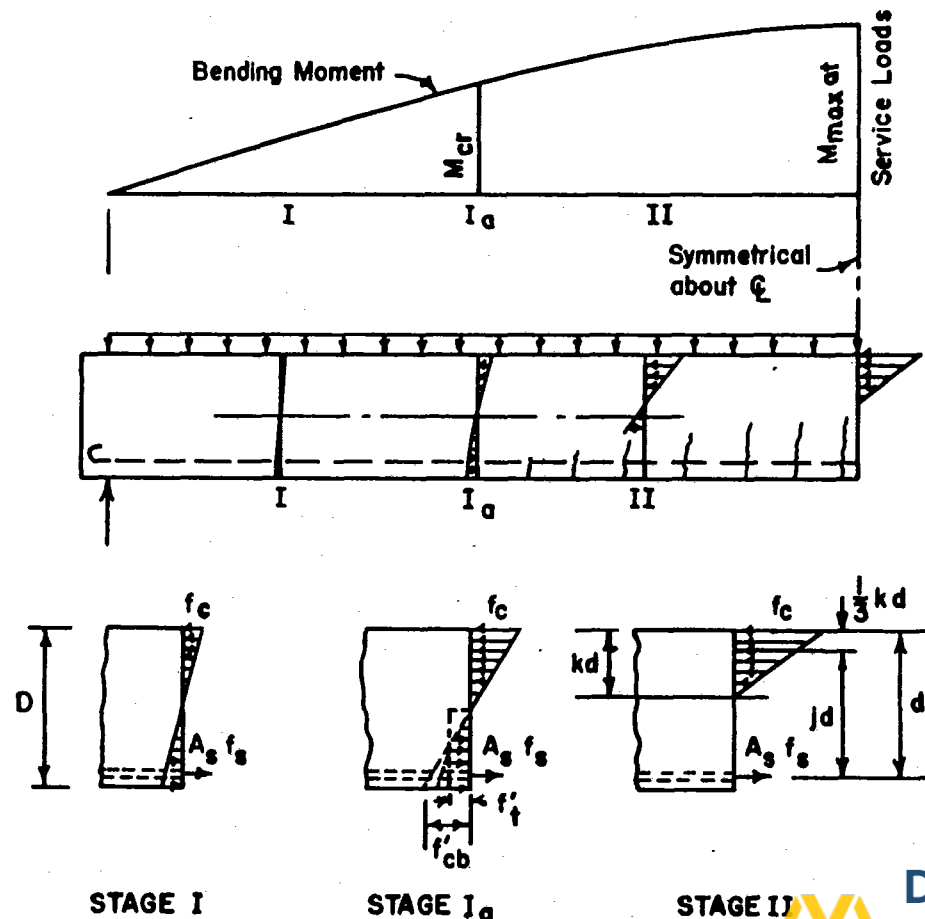


Fig. 1 — Various stages of stress distribution in a reinforced concrete beam carrying service loads

of these stresses may be as low as 60 percent depending on a number of factors.

The short-time deflection response of a typical beam is idealized in Fig. 2. Two distinct phases are seen; the first where an uncracked section behavior predominates and the second at higher loads in which the cracked section dominates. It should be noted that the curve shown in Fig. 2 would not exhibit the same uncracked section behavior on subsequent loading cycles.

Under sustained long-time loads the formation of new cracks, the widening of earlier cracks and the effect of shrinkage and creep cause additional deflections which may be two or three times as large as the short-time deflections.

Concrete shrinkage produces a shortening of the beam which is resisted by the reinforcing steel, inducing compressive stresses in the steel and tensile stresses in the concrete. The magnitude of the shrinkage curvature depends on the amount of nonsymmetry of reinforcement and on the relative areas of concrete and steel in the reinforced concrete beam. These shrinkage curvatures, which are greatest for beams with tension reinforcement only, generally have the same sign as those due to moments produced by transverse loads, and thus they magnify the deflections due to load.

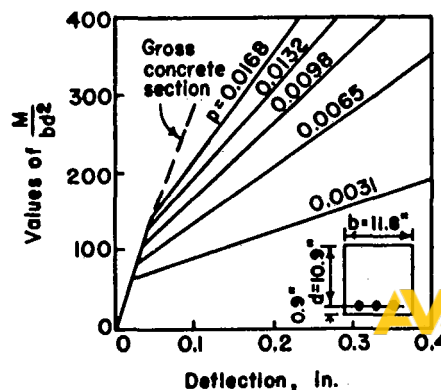
Considering creep alone, the increase in strain with time will be as shown by Line B in Fig. 3b, while Line A indicates the strain distribution immediately after the application of the load. Strain for the extreme fiber in compression increases considerably while the strain at the level of the steel changes very little. As this occurs, the compressive stresses in the concrete are reduced (Fig. 3c), since the neutral axis moves toward the reinforcement, and the steel stresses increase since the internal lever arm shortens. It is seen from Fig. 3b that the relative increase in curvature caused by creep is less than the relative increase in strain due to creep such that:

$$\frac{\phi_{cp}}{\phi_i} = k_2 \frac{\epsilon_{cp}}{\epsilon_i} \quad (11)$$

where k_2 is less than 1. Thus the relative increase in the deflection is less than the increase in strain due to creep.

Long-time deflections due to shrinkage and creep can be reduced substantially by the addition of compressive reinforcement. The compressive reinforcement tends to equalize the restraint to concrete shrinkage at the top and bottom of the beam section and thus reduces the shrinkage curvatures. With respect to creep, while strains in the concrete increase with time, the compressive stresses are gradually shifted more and more to the compressive reinforcement, which in turn tends to limit the increase in beam curvatures due to creep in the concrete.

Fig. 2 — Short-time deflection response of a reinforced concrete beam (taken from Reference 26)



303—Deflections under short-time loading

Because of the varying amounts of reinforcement and extent of cracking along the span length of the beam, the effective flexural rigidity is not a constant. One could evaluate the zones (Fig. 1) for which the theoretical cracking moment (at which the concrete fails in tension) had been exceeded and then calculate the corresponding transformed section moments of inertia along the beam length based on the appropriate cracked or uncracked sections. With EI known along the span length, Eq. (9) could be used to evaluate deflections. This procedure would underestimate the flexural rigidity of the beam, however, since it would neglect the participation of tensile concrete between the cracks. Concrete tensile strength may be even more important with regard to slab deflections since large portions of a slab are frequently uncracked.

303.1 Calculation procedures

Due to the complex nature of the flexural behavior of reinforced concrete beams and the desirability of having a simplified method for calculating deflections due to short-time loads, various procedures have been proposed. These procedures will be described in order of their chronological appearance in the literature.

In 1914 Maney²² proposed a formula:

$$\Delta = k_3 L^2 \frac{M_{max}}{EI} = k_3 L^2 \left(\frac{f_s + n f_c}{E_s d} \right) \quad (12)$$

where k_3 is a constant depending on loading and end conditions and M_{max} is the maximum bending moment in the span ($k_3 = 5/48$ and $M_{max} = WL/8$ for a uniformly loaded simple beam, etc.). Eq. (12) is based on a cracked transformed section throughout the length of the span.

In 1924 Swain²³ presented a useful expression for EI based on a cracked section as follows:

$$\frac{M}{EI} = \frac{\epsilon_s}{(1-k)d} \quad (13)$$

but $\epsilon_s = f_s/E_s$ and $M = f_s A_s j d$ which when substituted into Eq. (13) gives:

$$EI = E_s A_s (1-k) j d^2 \quad (14)$$

In 1931 Myrlea,²⁴ reporting as chairman of ACI Committee 307, suggested the adoption of Eq. (12) which is based on a cracked section, for the calculation of deflections.

In 1940 Murashev²⁵ proposed that the cracked transformed section be used for calculating I , but that an increased effective steel modulus of elasticity, $\bar{E}_s$, be used to account for the participation of tensile concrete between cracks:

$$\bar{E}_s = E_s / \psi \quad \text{where } \psi \leq 1.0 \quad (15)$$

$$\psi = 1 - (2/3) (M_{cr}/M)^2 \quad (16)$$

in which M is the moment at service load, and M_{cr} is the cracking mo-

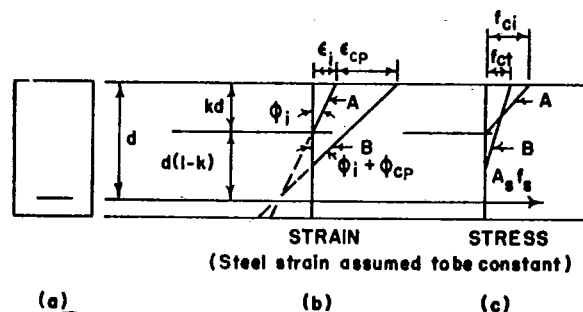


Fig. 3 — Strain Distribution in a reinforced concrete beam subjected to long-time creep