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Variability of Deflections of Simply Supported Reinforced Concrete Beams

Reported by ACI Committee 435

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The variability of actual deflection of simply supported reinforced concrete beams with respect to deflection calculated by accepted methods is discussed. The probability of the deflection of a particular beam falling within a given level of uncertainty is evaluated by analyzing existing data and fitting a probability distribution to the data. Using deflection criteria of ACI 318-71, there is approximately a 90 percent chance that deflections of a particular beam will be within the range of 20 percent less than to 30 percent more than the calculated value. This represents a slight improvement over ACI 318-63.

Keywords: beams (supports); building codes; deflection; loads (forces); probability distribution functions; probability theory; reinforced concrete; structural design; variability.

■ A VARIETY OF EQUATIONS and relations¹ as well as the new recommendations formulated by ACI 318-71² based on an effective moment of inertia (I_{eff}) procedure developed by Branson³ are available to forecast the deflections of simply supported reinforced concrete members. All of these procedures yield a single number which is either smaller or larger than the deflection that would be likely to occur in a real structure. If the variability of actual deflections with respect to calculated deflections was sufficiently small, the engineer could use calculated values with a high

degree of confidence. However, the variability of actual deflections under nominally identical conditions is often large, rather than small. For example, a beam that is cambered 2 in. (5.1 cm) to offset a calculated dead load deflection, may only deflect 1 in. (2.54 cm) in actuality, leaving the beam with unsatisfactory camber; conversely it might deflect as much as 3 in. (7.6 cm) producing undesirable sag.

Even though deflections of nominally identical members may vary substantially, the average or mean observed laboratory deflection can be

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accurately predicted by the available relationships. This is because the parameters in the forecasting relations can be adjusted to fit the mean laboratory data.

The problem then, is *not* how to forecast the average deflection of a large number of identical beams, but, to establish the probability of predicting a specific value of deflection for a particular beam. This is the problem that is discussed in this report. The theory of probability is used to model the basic problem of forecasting under conditions of uncertainty. While it is entirely possible to develop new probabilistic relations to forecast both the mean and variability characteristics of deflection, this report deals entirely with the variability of deflections that are calculated using ACI Committee 318 methods.

The report first describes the variability of measured deflections. Fitting of probabilistic models to the data is then discussed and illustrated. Finally, specific recommendations are made for estimating the likelihood of the variability of deflection both for short and long term service loading. Short term loading includes all live loads that are removed immediately after application, and long term loading includes all dead load and live loads of permanent duration.

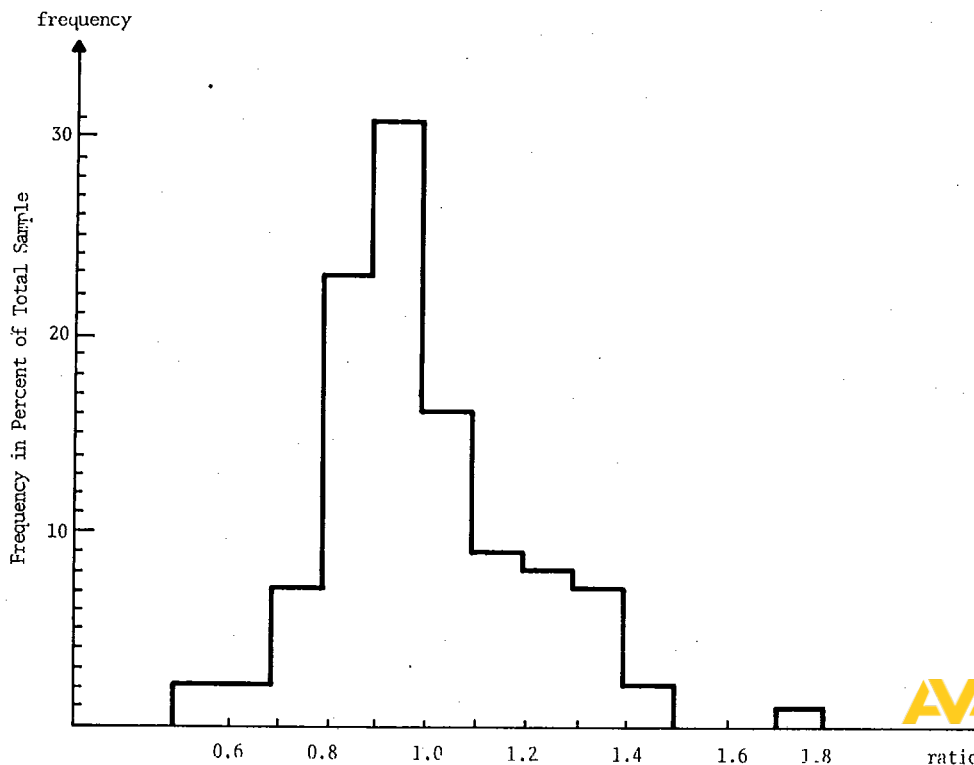
EXPERIMENTAL EVIDENCE

Although the deflections of many beams in actual structures have proven unsatisfactory (dif-

ferent from design deflection), the reports of measurements on such beams are meager and inadequate to draw statistical conclusions. Actual measurements on beams which perform satisfactorily are also lacking. Because of the lack of deflection measurements on actual structures, the evidence from laboratory tests was used in this study. An unevaluated level of uncertainty exists between these results and actual field conditions because of the differences between these conditions as well as differences in the amount of information related to service conditions available for laboratory tests as compared to field conditions. That is, a variety of factors may be known or estimated in the laboratory that are unknown in the design situation. Because of these uncertainties, the variability of deflection between actual field and calculated deflection can be expected to be greater than the variability of deflection between actual laboratory and calculated deflection. Therefore, the results of the analysis reported in this report must be viewed as minimums rather than average or expected values. Greater variability would most likely be obtained if field data were used instead of laboratory data.

VARIABILITY OF MEASURED DEFLECTIONS

Fig. 1 and 2 contain histograms of the ratio of observed short term deflection for laboratory test beams to deflection calculated using ACI 318-63⁴ and ACI 318-71,² respectively. In both figures, note that the mode or most frequently found ratio



is unity. The data show some skew toward larger deflections and the ratio of largest to smallest value is approximately 3 to 1.

Fig. 3, 4, and 5 contain histograms of the ratio of actual long term deflection to that calculated according to procedures recommended by Winter and Yu,⁵ ACI 318-63,⁴ and ACI 318-71.² In all cases an analysis of the data results in similar mean characteristic and standard deviations.

PROBABILISTIC INTERPRETATION OF DATA

If an evaluation of the probability of the deflection of a particular beam falling within a given level of uncertainty is to be made the data must be analyzed using a probability model. Some of the models that could be used are the normal and the log-normal models.

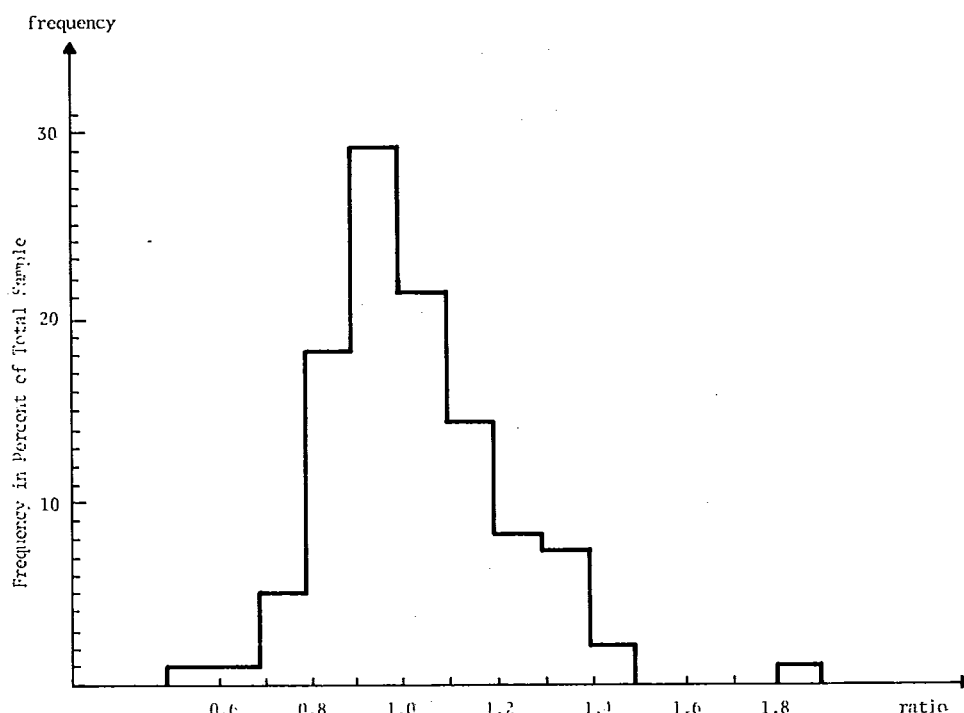


Fig. 2—Histogram of ratio of actual short term deflection calculated by ACI 318-71 method

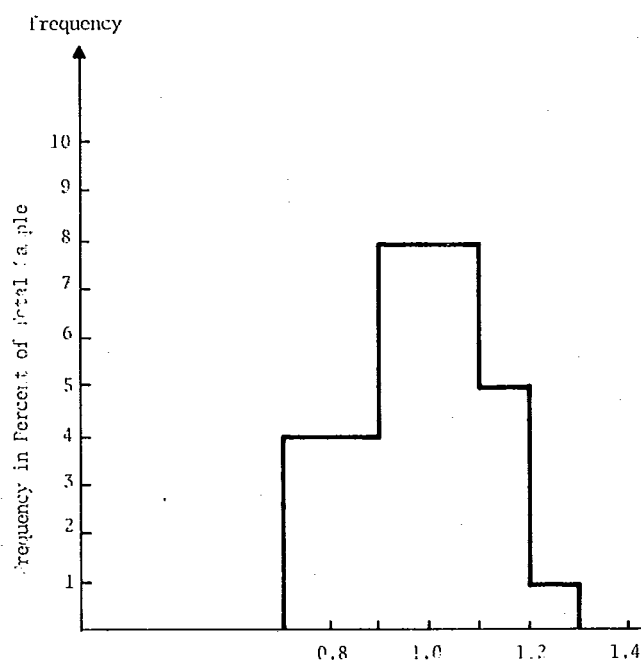


Fig. 3—Histogram of ratio of actual long term deflection to deflection calculated by method of Winter and Yu