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Deflections of Continuous Concrete Beams

Reported by ACI Committee 435*

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This report discusses the necessary considerations and outlines the procedure for a theoretically exact method of computing the deflection of continuous reinforced concrete beams. It describes the approximate methods currently in use and a proposed new method. Each approximate method is compared for accuracy with certain necessary characteristics and also with pertinent test data. Advantages and disadvantages are cited, and the committee's recommendations are given.

Keywords: beams (supports); continuous beams; cracking (fracturing); deflection; flexural strength; reinforced concrete; structural analysis; structural design; warpage.

■ THE DISCUSSION OF APPROXIMATE METHODS is directed toward the computation of short-term deflections. It is assumed that the procedures of obtaining long-term deflections apply equally well in the case of continuous beams. Also, throughout this report, the discussion pertains to the computations of deflections under service loads.

THEORETICALLY EXACT METHOD

A theoretically correct method for computing deflections of continuous members must use the actual moment pattern in the member. The moment pattern can be determined by an iterative procedure in which the moments in the frame are computed from an assumed set of flexural rigidities. The actual flexural rigidities, taking into account cracking at all points^{1,2} in the frame, are then computed from the moments determined in the previous step. Moments and flexural rigidities are recomputed a sufficient number of cycles to allow both quantities to converge to within ac-

ceptable margins of error. The deflection is then computed from the moment pattern and flexural rigidities obtained in the final iteration cycle, using any of the classical techniques such as moment area, conjugate beam, energy techniques or the differential equation approach.

For long-term deflection, the above procedure must be carried out at each change in load or ambient conditions and at sufficient intervals in between to present a realistic picture of the time-dependent behavior of the member. Creep and shrinkage strain calculations should be based on the actual stress conditions computed in the previous cycle.

Obviously, the solution of deflections by this method is suitable only for electronic computers. However, because of the large amount of data required for the input parameters it is useful only for research work. The required data includes: modulus of elasticity, modulus of rupture, creep and shrinkage strains and the variation with re-

*Roster of the committee when the final submission and balloting of the report was done. Jack R. Benjamin, W. Gene Corley, Bernard L. Gabrielsen, and Helmut G. Gevrmayer served on the committee earlier, while Robert A. Crist and Robert G. Drysdale have become members of the committee since then.

†Members of Subcommittee 7 who prepared this report.

‡Chairman from 1968 to 1973 during which time the report was developed.

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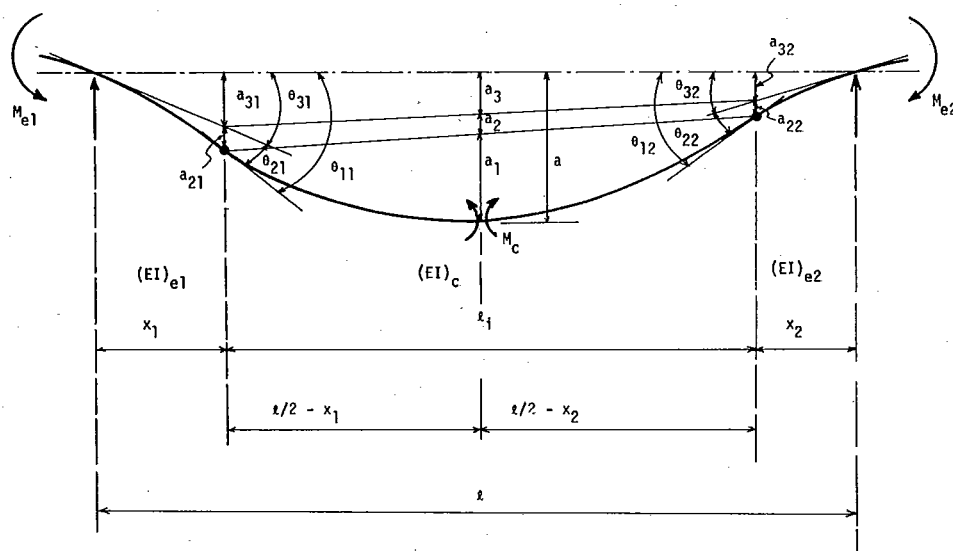


Fig. 1 — Beam divided into three parts at inflection points

spect to time of these parameters, as well as the temperature, humidity and loading history of the structure. Such information is rarely available to designers.

It should be recognized that the above "theoretically exact" method will not necessarily predict the exact actual deflection. This is because theory and the mathematical niceties may overlook other considerations such as statistical variations in concrete properties even though test results on the concrete cast in the structure are used in the computations.

APPROXIMATE METHODS

The major computational difficulty in applying the "theoretically exact" method manually is the iteration process required to determine the correct flexural rigidities and moments in the frame. All approximate methods considered in this report eliminate this process. The computation of moments is based on elastic frame analysis using gross uncracked concrete flexural rigidities. Based on such moments, reasonably accurate flexural rigidities are determined and used for computations of deflections. It has been shown³ that the effect of cracking on the distribution of moment in the frame is small. However, the effect of cracking is usually taken into account in the evaluation of the flexural rigidities to be used in the computation of deflections.^{1,2,3,5*}

Approximate methods also usually assume shrinkage warping is a function of the elastic deflection. This assumption may lead to gross errors, especially for composite members or unusual beam configurations.

There are two categories of variation in flexural rigidity. In beams with a prismatic gross cross section, the variation in flexural rigidity arises primarily from different extents of cracking. In nonprismatic members, the variation arises from

the change in gross cross section as well as from different extents of cracking.

Most approximate methods currently in use represent the variation in flexural rigidity in some convenient form for manual computations. These methods may be classified under two categories:

1. Approximate methods using constant flexural rigidity segments
2. Approximate methods using a single value of flexural rigidity for each span

The application of each of the methods described under these two classifications, will be illustrated in this report by a graph, in which β_r (ratio of the computed midspan deflection of the continuous span to the midspan deflection of the corresponding simple span) is plotted versus β_m (ratio of midspan moment in the continuous span to the midspan moment of the corresponding simple span, i.e., the total static moment). This relationship may also be visualized as β_r versus the location of the point of inflection, since location of the inflection point is directly related to β_m .

Approximate methods using constant flexural rigidity segments

Each span of the continuous beam is divided into a number of segments within which the flexural rigidity is assumed constant. The most common practice (for beams with prismatic gross

*The following procedures^{1,2,3} can be used to incorporate the effect of load level and degree of cracking in both the theoretically exact and the approximate methods, respectively:

For each individual section as in an exact or numerical solution

$$I_{eff} = (M_{cr}/M)^4 I_g + [1 - (M_{cr}/M)^4] I_{cr}$$

where:

I_{eff} is the effective moment of inertia

I_g is the gross section (neglecting steel) moment of inertia

I_{cr} is the cracked transformed section moment of inertia

M is the moment at the section

M_{cr} is the cracking point

For an average value as in the case of each segment in an approximate solution

$$I_{eff} = (M_{cr}/M_{max})^3 I_g + [1 - (M_{cr}/M_{max})^3] I_{cr}$$

where:

M_{max} is the maximum moment in the segment

The above equation is used in the 1971 ACI Code.²

cross section) is to divide the span in three parts separated by inflection points as shown in Fig. 1. The flexural rigidity of each part is assumed constant for the full length of that part.

This three part division is appropriate for prismatic gross cross sections where variation in flexural rigidity, EI , arises entirely from different degrees of cracking. It does not necessarily apply to nonprismatic members, in which the segments may be separated at locations other than inflection points.

The deflection can then be computed using any of the classical techniques such as moment-area, conjugate beam, energy techniques or the differential equation approach.

To illustrate such a procedure, using a three part division, consider the span shown in Fig. 1. The deflection at midspan may be calculated as the sum of a_1 , a_2 , and a_3 where

a_1 = deflection of the center span as a simple span between the inflection points

a_2 = deflection due to the end spans as cantilever spans

a_3 = deflection due to the rotation of the end spans over the supports

Therefore, the total deflection of midspan is:

$$a = a_1 + a_2 + a_3 \quad (1)$$

The computation of a_1 , a_2 , and a_3 depends upon the type of loading on the span. Assuming uniformly distributed load of intensity w and using the principle of superposition, each can be calculated as follows:

$$a_1 = \frac{w \left(\frac{l}{2} - x_1 \right)}{24(EI)_c} \left[l_i^3 - 2l_i \left(\frac{l}{2} - x_1 \right)^2 + \left(\frac{l}{2} - x_1 \right)^3 \right] \quad (2)$$

$$a_2 = a_{21} \left(\frac{\frac{l}{2} - x_2}{l_i} \right) + a_{22} \left(\frac{\frac{l}{2} - x_1}{l_i} \right) \quad (3)$$

where:

$$a_{21} = \frac{\left(\frac{wl_i}{2} \right) x_1^3}{3(EI)_{c1}} + \frac{wx_1^4}{8(EI)_{c1}} \quad (4)$$

$$a_{22} = \frac{\left(\frac{wl_i}{2} \right) x_2^3}{3(EI)_{c2}} + \frac{wx_2^4}{8(EI)_{c2}} \quad (5)$$

Therefore:

$$a_2 = \left(\frac{wl_i x_1^3}{6(EI)_{c1}} + \frac{wx_1^5}{8(EI)_{c1}} \right) \left(\frac{\frac{l}{2} - x_2}{l_i} \right) + \left(\frac{wl_i x_2^3}{6(EI)_{c2}} + \frac{wx_2^5}{8(EI)_{c2}} \right) \left(\frac{\frac{l}{2} - x_1}{l_i} \right) \quad (6)$$

$$a_3 = a_{31} \left(\frac{\frac{l}{2} - x_2}{l_i} \right) + a_{32} \left(\frac{\frac{l}{2} - x_1}{l_i} \right) \quad (7)$$

where:

$$a_{31} = \theta_{31} x_1 \quad (8)$$

$$a_{32} = \theta_{32} x_2 \quad (9)$$

θ_{31} and θ_{32} are the rotations of ends 1 and 2 over the supports and are computed as:

$$\theta_{31} = \theta_{11} - \theta_{21}, \quad \theta_{32} = \theta_{12} - \theta_{22} \quad (10)$$

or

$$\theta_{31} = \frac{wl_i^3}{24(EI)_c} - \left(\frac{wl_i x_1^2}{4(EI)_{c1}} + \frac{w x_1^3}{6(EI)_{c1}} \right) \quad (11)$$

Therefore:

$$a_{31} = \frac{wl_i^3 x_1}{24(EI)_c} - \left(\frac{wl_i x_1^3}{4(EI)_{c1}} + \frac{w x_1^4}{6(EI)_{c1}} \right) \quad (12)$$

Similarly:

$$a_{32} = \frac{wl_i^3 x_2}{24(EI)_c} - \left(\frac{wl_i x_2^3}{4(EI)_{c2}} + \frac{w x_2^4}{6(EI)_{c2}} \right) \quad (13)$$

Thus:

$$a_3 = \left[\frac{wl_i^3 x_1}{24(EI)_c} - \left(\frac{wl_i x_1^3}{4(EI)_{c1}} + \frac{w x_1^4}{6(EI)_{c1}} \right) \right] \left(\frac{\frac{l}{2} - x_2}{l_i} \right) + \left[\frac{wl_i^3 x_2}{24(EI)_c} - \left(\frac{wl_i x_2^3}{4(EI)_{c2}} + \frac{w x_2^4}{6(EI)_{c2}} \right) \right] \left(\frac{\frac{l}{2} - x_1}{l_i} \right) \quad (14)$$

Again, the total midspan deflection of uniformly loaded span in Fig. 1 is equal to:

$$a = a_1 + a_2 + a_3 \quad (1)$$

where a_1 , a_2 , and a_3 are given by Eq. 2, 6, and 14, respectively.

The parameter β_a is then equal to:

$$\beta_a = \frac{a}{\frac{5}{384} \frac{wl^4}{(EI)_c}} \quad (15)$$

For the symmetrical case, i.e., $x_1 = x_2 = x$ and $(EI)_{c1} = (EI)_{c2} = (EI)_c$

$$\beta_a = \frac{1}{5wl^4} \left[\underbrace{\frac{5wl_i^4}{384(EI)_c}}_{a_1} + \underbrace{\frac{wl_i x^3}{6(EI)_c} + \frac{wx^4}{8(EI)_c}}_{a_2} + \underbrace{\frac{w}{24(EI)_c} (l_i^3 x - 6l_i x^3 \alpha - 4x^4 \alpha)}_{a_3} \right] \quad (16)$$

where:

$$\alpha = \frac{(EI)_c}{(EI)_e}$$