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Deflection of Two-Way Reinforced Concrete Floor Systems: State-of-the-Art Report

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Synopsis: This state-of-the-art paper summarizes practical method for calculating the deflection of two-way slab systems, including flat slabs, flat plates, beam-supported slabs, and wall-supported slabs. A selected list of references is included.

Keywords: beams (supports); cracking (fracturing); creep properties; deflection; flat concrete plates; flat concrete slabs; flexural strength; floors; modulus of elasticity; reinforced concrete; reviews; structural analysis; two-way slabs.

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CHAPTER 1 - INTRODUCTION

Present trends toward the use of higher strength materials, refined methods of design, and shallow members produce reinforced concrete systems in which requirements of serviceability may not be met, even though adequate strength is provided. Excessive deflection of structural members and systems must be included in a broad definition of structural failure. Such failure may be accompanied by visually apparent sagging of floors, cracking of partitions, ill-fitting windows and doors, or difficulty with alignment of sensitive machinery.

Prediction of deflection of reinforced concrete structures in general is complicated by the non-homogeneous nature of the construction, the effects of cracking, and the time-dependent nature of material response. In spite of these factors, simple and reasonably accurate methods have been devised by which designers may predict the deflection of reinforced concrete beams (2,6).

Unfortunately, the same cannot be said for two-way reinforced concrete systems such as wall- or beam-supported slabs, flat slab, and flat plates. In such cases, further complications are present as a consequence of the three-dimensional nature of the problem, the less-well-defined influence of cracking, and the effect of deflection and rotation of supporting members.

The building codes of many nations have dealt with deflections in one of two ways. Deflections may be limited indirectly by specifying minimum depth-to-span ratios, based on experience. Alternatively, deflections may be calculated explicitly, and the results compared with deflection limits which are established considering the consequence of excessive flexibility.

The ACI Building Code (4) includes both methods of deflection control. Chapter 9 of the Code contains a table of minimum thickness for beams and one-way slabs, with depths expressed as a simple fraction of the span. For two-way construction, analogous expressions are included. These are complicated by the need to consider such factors as the aspect ratio of the slab panels, the boundary conditions on all sides of the panels, and the flexural stiffness of the supporting beams. For cases where deflection is calculated explicitly, Chapter 9 of the Code contains a table of maximum allowable computed deflections (see also Ref. 3).

The important question of beam moment of inertia to be used for deflection calculations is treated in the Code through use of an effective moment of inertia as suggested by Branson (6), being the weighted average of the moment

of inertia of the gross concrete section and that of the cracked transformed section. The influence of creep and shrinkage is incorporated through use of a multiplying factor, to be applied to calculated immediate deflection.

For two-way systems, it is stated in the Code that these same devices may be used, but it is noted further that deflection calculations for two-way systems shall take into account the size and shape of the panel, the conditions of support, and the nature of restraint at the panel edges. Little guidance is given in either the Code (4) or the accompanying Code Commentary (5) regarding the means by which this may be done.

The present report will review both classical and contemporary methods by which the deflection of two-way systems may be calculated. Some of these methods are mainly of historical interest and are summarized only briefly. Others of greater practical importance are treated more fully. In all cases, reference material is given where more complete details may be found.

CHAPTER 2 - CLASSICAL METHODS OF ELASTICITY

If stress levels are low, so that the normal assumptions made for the solution of thin plates with small deflections are satisfied, the classical methods of analysis can be used to predict the deflections of concrete slabs carrying distributed or concentrated loads. A comprehensive review of the response of slabs with various support conditions and loads is given by Timoshenko and Woinowsky-Krieger (28).

The deflection of a thin, rectangular plate of uniform thickness can be expressed by the Lagrange equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (2.1)$$

where x, y = orthogonal coordinate axes of the middle surface

w = deflection of the plate

q = transverse load

D = flexural rigidity of the plate, $Eh^3/12(1-\nu^2)$

E = modulus of elasticity of the plate material

h = thickness of plate

ν = Poisson's ratio of the plate material

For a given loading condition $q = f_1(x, y)$ and boundary conditions, the deflection $w = f_2(x, y)$ can be found by integrating Eq. (2.1). Once the deflection is known, the moments, shears and reactions can be evaluated by combining

appropriate derivatives of the deflection function. Stresses can be found from the known moments and shears.

Solutions for isolated plates subjected to various loading conditions and with various support conditions are given in detail in Ref. 28 (see also Ref. 14). In addition to the usual loading conditions (distributed, partially distributed, or concentrated), the effects of moments distributed along the edges are also treated; and in addition to the usual support conditions (built in, simply supported and free edges), elastically supported and elastically built-in edges are discussed. With the aid of these solutions, the realistic problem of multiple panel two-way floor slabs can be solved using ordinary techniques employed for indeterminate structures (e.g., consistent deformations and superposition).

Because of the tedious nature of such methods, approximate means for the analysis of continuous rectangular plates have been proposed (28) whereby the moments at the centers of the plates and at the middle of the supporting edges can be found using tabulated values of coefficients. These are functions of aspect ratio and support conditions.

In addition to the various plate problems discussed in Ref. 28, a comprehensive study of the response of rectangular slabs with different support conditions and various load conditions was made by Jensen (17) using the classical methods of thin plate analysis.

Deflection of flat-plate slabs supported on columns without drop panels or capitals have also been treated in Ref. 28. Assuming that the column cross-sectional dimensions are negligible compared with the column spacing, the deflection at the center of a typical interior panel can be evaluated from the expression:

$$w = \alpha \frac{qb^4}{D} \quad (2.2)$$

where b = longer span

α = coefficient varying with aspect ratio

q = uniform transverse load

D = flexural rigidity of the plate, $\frac{Eh^3}{12(1-\nu^2)}$

Eq. (2.2) can be written

$$w = k \frac{qb^4}{Eh^3} \quad (2.3)$$

where $k = 12(1-\nu^2)\alpha$.

Eq. (2.3) has been proposed to determine the deflection of all two-way flat slabs as well as waffle slabs (11).

For flat slabs with drop panels or for waffle slabs, an equivalent average thickness l_e is used in Eq. (2.3).

CHAPTER 3 - CROSSING BEAM ANALOGIES

3.1 General Concepts

The complexities and limitations of the classical methods led very early to attempts to devise simple, yet reasonably accurate, methods to obtain slab moments and deflections. In 1904 Marsh (21) suggested the replacement of a uniformly loaded continuous slab panel by a gridwork of crossing beams, as shown in Fig. 1(a). The applied load was assumed to be divided between the beams spanning in the short direction A and the long direction B. By applying the requirement of equal vertical deflection at the center of the panel, Marsh concluded that the fraction of the uniformly distributed load q to be carried by the beam-strips in the short direction was

$$r_s = \frac{B^4}{A^4 + B^4} \quad (3.1)$$

while the remaining load $(1 - r_s) q$ was carried by the beam-strips spanning in the long direction.

This result is approximate because it neglects the presence of torsional moments in the slab. Referring to Fig. 1(b), it will be seen that the two central strips S_1 and L_1 bend in a manner similar to that of Fig. 1(a). The outer strips S_2 and L_2 , however, are not only bent but twisted. The load on the slab is resisted not only by the bending moments, but also by the twisting moments; consequently the bending moments are smaller than would be computed for a set of unconnected strips loaded by q_s and q_l . For a simply-supported square plate, the error in bending moments is about 25 percent. Corresponding errors are introduced in the calculation of deflection, and actual center deflection of a slab panel is significantly less than predicted by Marsh's analysis.

Marsh's approach was later modified by Marcus (20) in an attempt to take torsional moments into account. Marcus introduced modifying factors in the calculation of moments and deflections; these factors were a function of the aspect ratio of the slab panel and the conditions of support at the boundaries. Since the method was applicable only to uniformly loaded rectangular panels on unyielding supports, for which theoretical solutions are available, there appears to be little value in the approach.

A variation of the crossing beam analogy has been proposed in which the deflection at the center of a uniformly loaded interior panel is estimated by considering a beam of unit width having a span equal to the diagonal length of the slab. The beam is assumed to have fixed ends, and to carry a load per unit length equal to the load per unit area acting on the slab. This represents a cruder modeling of actual behavior than earlier proposals and has little merit other than simplicity.

3.2 Analogous Gridwork Method

A related method known as the "analogous gridwork method" was presented by Ewell, Okubo, and Abrams in 1952 (12). It represents a significant improvement over the crossing beam method described above in that the influence of torsional moments can be accounted for directly.

The slab is first divided into strips in each direction as shown in Fig. 2(a). These slab strips are then replaced by equivalent beams as in Fig. 2(b). The beam strips are assumed to be interconnected at their crossing points; the flexural and torsional properties of the beams are those of the corresponding slab strips. At each joint of the grid of beams, flexural and torsional stiffness factors are found in the usual way, finding the bending or torsional moment associated with a unit rotation of the near end of the member, the far end being fixed. The total stiffness at joint o , Fig. 2(c), is the sum of the stiffnesses (flexural and torsional) at that joint contributed by the individual members. Distribution factors for each member are defined in the usual sense, as the ratio of member stiffness (flexural or torsional) to total stiffness at the joint. An applied moment M acting in the vertical plane $l-o-m$ is distributed to the four members shown. It will produce bending moments in members $l-o$ and $o-m$ and twisting moments in members $n-o$ and $o-p$.

Next each joint i of the equivalent grid is given an arbitrary vertical displacement Δ_i . The fixed end moments produced are distributed according to the distribution factors found as above and moments are carried over to adjacent joints. The solution proceeds as a two-dimensional adaptation of conventional moment distribution.

Using the end moments so obtained, the vertical reactions at each joint are found. These represent "auxiliary forces" required to hold the gridwork in the displaced configuration for which the starting moments were found. A separate force system arises from each joint that is displaced, and as a result of this procedure joint forces are expressed in terms of the arbitrary displacements Δ_i .

The total moments, torques, and vertical reactions at each joint are found by adding the contributions associated with each joint displacement and the total reactions are equated to the actual external joint loads. In this way, a set of simultaneous equations is created, in terms of the arbitrary joint displacements Δ_i ; these equations are solved for these unknown displacements. Then actual bending and twisting moments as well as displacements may easily be found.

Engineers will recognize this approach as directly analogous to the moment distribution analysis of a multi-story plane frame for sidesway. Its familiarity is appealing. As the grid size is refined, the solution agrees increasingly well with theoretically exact solutions. However it must be noted that the amount of work required is considerable. And for practical cases the number of simultaneous equations to be solved is too large for manual solution.

Scordelis has suggested a method of successive shear corrections which eliminates the need to solve a set of simultaneous equations (26). However, for practical cases the calculations are still lengthy.

3.3 Statics Ratio Method

Still another variation of the crossing beam analogy was offered by Furr (15) in 1959. He refers to the method as the statics ratio method. It makes use of an assumed deflection configuration which is modified as necessary to satisfy the requirements of statics. In common with the Marcus modification of the crossing beam analogy, the statics ratio method permits inclusion of the effects of torsional resistance of the beam strips. By this method, the distributed slab load is replaced by statically equivalent concentrated loads acting at the beam strip intersection points. The grid is deflected to an arbitrary position, with joints considered locked against rotation during this displacement. Fixed end moments developed are then distributed in the way described above, and vertical shears at each joint are found. The statics ratio is defined as the ratio of the summation of vertical shears at each joint corresponding to the assumed displacements, to the applied load at the joint. If the slab has a constant statics ratio at all joints, the actual moments and deflections are found by dividing these for the assumed displacement pattern by the constant statics ratio. If the statics ratio differs appreciably over the gridwork the assumed deflection pattern is modified until a constant or nearly constant value is obtained.

The method is attractive in that the convergence can be accelerated by wise initial choice of deflection sur-

face, and by judicious modification considering the results of the previous iteration. However the solution is laborious and the method has not been widely used.

CHAPTER 4 - WIDE BEAM METHOD

An approximate method for calculating slab deflection has been used for flat-plates, or for slabs for which the stiffening effects of column-line beams, drop panels, and column capitals may be neglected. The method considers the deformation of a typical slab panel in one direction separately from that in the other, later adding the two contributions to obtain the total deflection at any point of interest.*

Referring to Fig. 3 (a), the slab in any panel is considered to act as a broad, shallow beam of width equal to the panel dimension l_y and having a span l_x . At this stage, the slab is considered to be resting on unyielding support lines at the two opposite edges. A similar analysis is carried out in the perpendicular direction (Fig. 3(b)). The mid-panel deflection, for example, is obtained as the sum of the center span deflection of the column in a strip direction, and that of the middle strip in the other direction as shown in Fig. 3(c). Note that the full applied loading must be accounted for in the calculations for each direction.

All slab strips spanning in the X direction (for example) will not deform identically. It is a convenient approximation in analysis to divide the slab into a "middle strip" of one-half the full panel width, bounded by two half "column strips" of one-fourth the panel width. Because of the concentration of moments and curvatures in the column strips, those deflections will be greater, as shown in Fig. 3(a). This can be accounted for in an approximate way by calculating an average value of deflection for the entire panel spanning in the X direction, assuming all strips to deform identically, then proportioning these deflections according to the ACI lateral distribution factors for assigning moments to column and middle strips (4,5). Since the greater part of the span is subjected to positive bending, and also considering the approximate nature of the calculation, it is usually considered sufficiently accurate to work with the lateral distribution factors for positive bending moment.

For example, for an interior slab panel without column line beams, the Code contains the recommendation that the column strips be assigned 60 percent of the moment in the positive bending region (instead of 50 percent as for the uniform distribution of moments). Accordingly, the column strip deflection can

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