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State-of-the-Art Report on Control of Two-Way Slab Deflections

Reported by ACI Committee 435



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State-of-the-Art Report on Control of Two-Way Slab Deflections

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The current state of the art for controlling deflections of reinforced concrete two-way slabs is reviewed. Computation procedures, loads to be considered, use of minimum thickness relationships, and the variability that can be expected in deflections of actual slab systems are covered.

Keywords: concrete construction; cracking (fracturing); deflection; flat concrete plates; formwork (construction); loads (forces); multistory buildings; shoring; structural analysis; two-way slabs.

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CHAPTER 1—INTRODUCTION

Excessive deflections, when they occur, present serious serviceability problems in reinforced concrete building structures. Visual sagging of floors, damage to such nonstructural elements as partitions and windows, and jamming of doors are examples of the types of problems that can occur.

ACI 318 specifies minimum thickness requirements for control of two-way slab deflections. If the slab thickness equals or exceeds the specified minimum thickness, deflections need not be computed, and serviceability in terms of deflection control is deemed to be satisfied.

It should be recognized that the primary concern of a building code is public safety; thus, serviceability considerations cannot be treated in detail in it. Experience has shown that using ACI minimum-thickness equations produces slabs that behave in a satisfactory manner under service loads in most cases. The Building Code does permit a smaller than minimum thickness to be used if deflections are calculated and shown to be within limits specified by the Building Code; however, the calculation of slab deflections requires careful con-

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sideration of a number of factors if realistic estimates of deflections are to be obtained.

The objective of this report is to review the current state-of-the-art for control of two-way slab deflections. Methods of calculating slab deflections are presented. Effects of two-way action, cracking, creep, shrinkage, and construction loads are considered. The current ACI Building Code minimum-thickness equations are examined, and recent proposals for alternative methods of specifying minimum thickness are reviewed.

CHAPTER 2—DEFLECTION CALCULATION METHODS FOR TWO-WAY SLABS

2.1—Immediate deflection of uncracked slabs

In this section, calculation procedures are given for immediate deflections based on classical solutions, simplified crossing-beam analogies, and finite element analysis.

2.1.1 Classical solutions—Immediate deflection of uncracked two-way slab systems can be determined using plate-bending theory for elastic thin plates. Load-deflection response is governed by the plate equation:

$$\frac{\partial^4 a}{\partial x^4} + 2 \frac{\partial^4 a}{\partial x^2 \partial y^2} + \frac{\partial^4 a}{\partial y^4} = \frac{w}{D} \quad (2-1)$$

where

- x, y = orthogonal coordinate axes of the middle surface
- a = deflection of the plate
- w = transverse load per unit area
- D = flexural rigidity per unit width, $Eh^3/12(1 - \nu^2)$
- E = modulus of elasticity
- h = plate thickness
- ν = Poisson's ratio

The relationship between moments and curvatures is given by

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \frac{Eh^3}{12(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1 + \nu) \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \\ \phi_{xy} \end{Bmatrix} \quad (2-2)$$

For rectangular plates with uniformly distributed loads, the solution of Eq. (2-1) leads to an expression for maximum deflection in the form

$$a = \frac{\alpha w \ell^4}{D} = \frac{\alpha w \ell^4}{Eh^3/12(1 - \nu^2)} \quad (2-3)$$

where

- ℓ = longer span length
- w = uniform transverse load
- α = coefficient depending on the boundary conditions and aspect ratio

Note from Eq. (2-3) that the influence of Poisson's ratio ν on deflections is quite small. Typical values of ν for concrete fall in the range between 0.15 and 0.25. The term $(1 - \nu^2)$ in the flexural rigidity D falls in the range 0.94 to 0.98. Therefore, the error involved in neglecting Poisson's ratio is approximately 2 to 6 percent.

Solutions of the plate equation for various geometries and support conditions have been given by Timoshenko and Woinowsky-Krieger (1959) and by Jensen (1938). Since closed-form solutions of the plate equation are available for only a limited number of cases, alternative solution procedures are required for most practical situations.

2.1.2 Crossing beam methods—Several approaches have been developed in which the two-way slab system is considered as a system of orthogonal one-way systems allowing methods for calculating beam deflections to be applied. Some of the earlier approaches were summarized in ACI 435.6R.

More recently, Rangan (1976) and Scanlon and Murray (1982) described calculation procedures in which the column and middle strips are treated as continuous beams, with the middle strip considered to be supported by adjacent column strips running perpendicular to the middle strip.

For two-way systems supported on a rectangular layout of columns, the ACI 318 method of design for strength involves dividing the slab into column strips and middle strips in each of the two orthogonal directions. The total static moment for each span, $M_o = W_u \ell_2 \ell_n^2 / 8$, is divided between positive and negative moment regions and then between column and middle strips, using either the direct-design method or the equivalent-frame method. The distribution of moments approximates the elastic distribution for the given loading and therefore can be used to obtain an estimate of immediate deflections. However, the applied moments for strength design use factored loads, while the moments for calculating immediate deflections use service loads (unfactored).

Fig. 1 shows a rectangular panel in a column-supported two-way slab system. The shaded areas represent a set of crossing beams from which column strip deflection a_c and middle strip deflection a_m can be obtained. Each beam can be treated as a strip of unit width for which end moments, midspan moment, and flexural rigidity properties can be obtained. Note that end moments are defined at the faces of supports (column or column capital faces) and the beam span is the clear span between such supports.

Once the end moments and midspan moment have been obtained for a column or middle strip, the deflection for the strip can be calculated using the elastic beam deflection equation

$$a = \frac{5}{48} \frac{\ell_n^2}{EI} [M_m - 0.1 (M_1 + M_2)] \quad (2-4)$$

where

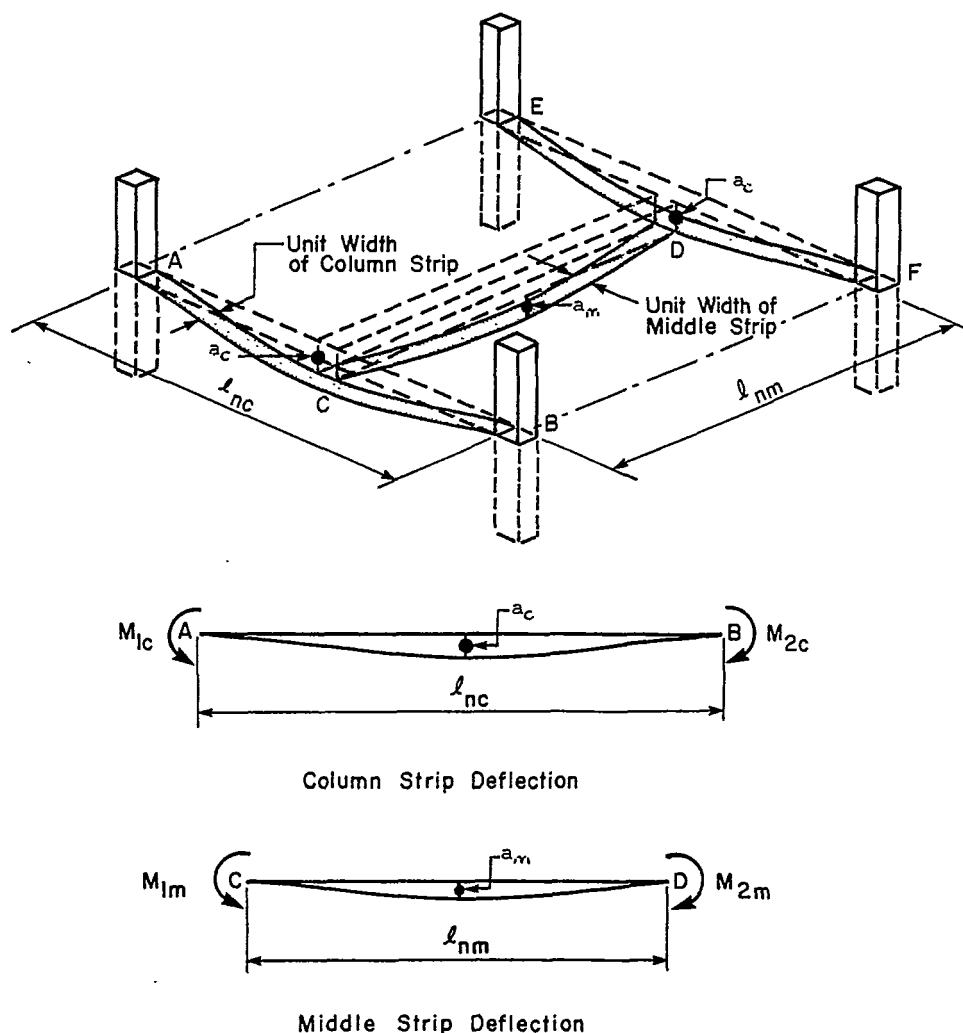


Fig. 1—Crossing beam approach

M_1, M_2 = end moments per unit width (positive directions, as shown in Fig. 1)

M_m = midspan moment per unit width

Using this procedure, the deflection of each column strip a_c and of each middle strip a_m can be calculated. The midpanel deflection a_{mp} is obtained by adding the column and middle strip deflections

$$a_{mp} = a_c + a_m \quad (2-5)$$

The rotation at the support must be included for cantilever slabs.

An earlier version of the equivalent-frame method for calculating deflections proposed by Vanderbilt, Sozen, and Siess (1965) considers the midpanel deflection as the sum of a column strip deflection, cantilever deflection of a portion of the middle strip extending from the column strip, and the midpanel deflection of a simply supported rectangular plate. The procedure developed by Nilson and Walters (1975) based on the equivalent-frame method is similar to the method just outlined except that a reference deflection is calculated

for the total panel width. Deflections for column and middle strips are then obtained from this reference deflection using lateral distribution factors based on relative M/EI values.

Ghali* calculates the deflection at midspan of a column or middle strip from values of curvature calculated at the midspan and supports using the relationship

$$a = \frac{\ell^2}{96} (\phi_L + 10\phi_c + \phi_R) \quad (2-6)$$

where ϕ_L, ϕ_c, ϕ_R = curvatures calculated from analysis of sections at the left support, center, and right support, respectively, and ℓ = span length. This relationship is exact if the true curvature distribution is parabolic in shape. Effects of cracking, creep, and shrinkage are accounted for in the analysis of each section.

2.1.3 Finite element method—The finite element method can be used to analyze plates with irregular support and loading conditions. Effects of beams and

*Ghali, A., "Prediction of Deflections of Two-Way Floor Systems," presented at 1987 ACI Fall Convention, Seattle, Washington.

columns can be included, and a number of general purpose computer programs are available for elastic analysis of plate systems.

The plate is divided into a number of subregions, or elements. Within each element, the transverse displacement is expressed in terms of a finite number of degrees of freedom (displacements, slopes, etc.) specified at element nodal points. In other words, the continuous displacement function $w(x,y)$ is approximated by a function with a finite number of degrees of freedom. Based on the assumed displacement function and given stress-strain or moment-curvature relationships [such as Eq. (2-2) for elastic plates], the element-stiffness matrix can be derived. The stiffness matrix of the entire slab is then assembled.

The solution for displacements and internal moments proceeds using standard matrix-analysis techniques for solving the equilibrium equations as outlined in a number of textbooks (e.g., Cook 1974; Gallagher 1975; and Zienkiewicz 1977). Although the method is becoming increasingly popular in engineering practice, some skill is required to select an appropriate finite element model, develop an appropriate mesh, prepare computer input data, and interpret the results.

2.2—Effect of cracking

The procedures outlined in the preceding section are applicable to linear elastic isotropic plate systems and must be modified for concrete slabs to include the effect of cracking on flexural stiffness. An early application of classical anisotropic theory to analysis of two-way reinforced concrete slabs is given in the text by Timoshenko and Woinowsky-Krieger (1959). More recently, procedures have been proposed for including cracking in finite element analysis and in the crossing beam analogies for two-way slabs.

The I_e concept developed originally for beams by Branson (1963) can be applied directly to the column and middle strips in the crossing beam analogies described earlier for elastic uncracked plates. In Eq. (2-4), the cross-sectional stiffness EI , becomes EI_e using the usual averaging procedures given in ACI 318 for I_e calculated at positive and negative moment locations. Krishnarayanan and Branson (1976) presented an extension of the Nilson and Walters equivalent-frame procedure to include cracking using the I_e procedure.

A review of the more sophisticated cracking models proposed for finite element analysis of slabs is given in the report of an ASCE Task Committee (1982). A simple generalization of Branson's effective moment of inertia concept to two-way systems has been suggested by Scanlon and Murray (1982) and implemented in a modified version of a linear elastic plate bending element by Graham and Scanlon (1985). The following is a brief outline of the procedure.

Input for material properties is given in the form of the plane-stress constitutive relation for anisotropic material. Specialized for orthotropy with respect to the x,y directions, the stress-strain relations are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} E_x / (1 - \nu_x \nu_y) & \nu_x E_y / (1 - \nu_x \nu_y) & 0 \\ \nu_y E_x / (1 - \nu_x \nu_y) & E_y / (1 - \nu_x \nu_y) & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (2-7)$$

For linear elastic isotropic material

$$\begin{aligned} E_x &= E_y = E \\ \nu_x &= \nu_y = \nu \\ G_{xy} &= G \frac{E}{2(1 + \nu)} \end{aligned} \quad (2-8)$$

moment-curvature relationships are obtained by integrating over the slab depth to obtain

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E_x I_g}{1 - \nu_x \nu_y} & \frac{\nu_x E_y I_g}{(1 - \nu_x \nu_y)} & 0 \\ \frac{\nu_y E_x I_g}{(1 - \nu_x \nu_y)} & \frac{E_y I_g}{(1 - \nu_x \nu_y)} & 0 \\ 0 & 0 & \frac{G_{xy} h^3}{12} \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \\ \phi_{xy} \end{Bmatrix} \quad (2-9)$$

where

$$\begin{aligned} I_g &= h^3/12 \\ &= \text{moment of inertia per unit} \\ &\quad \text{width of gross concrete section} \\ &\quad \text{about centroidal axis} \end{aligned}$$

Based on an analysis of the uncracked slab, moments in the x and y direction M_x and M_y are calculated for each element. Reduction in flexural stiffness in each direction can be accounted for using the effective moment-of-inertia equation

$$\begin{aligned} I_{ex} &= \left(\frac{M_{cr}}{M_x} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_x} \right)^3 \right] I_{crx} \\ I_{ey} &= \left(\frac{M_{cr}}{M_y} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_y} \right)^3 \right] I_{cry} \end{aligned} \quad (2-10)$$

where I_{crx} and I_{cry} = moments of inertia of cracked section transformed to concrete in x and y directions, respectively.

The effective moment of inertia is calculated for individual cracked elements; hence, the I_e equation should use terms raised to the fourth power, as suggested by Branson (1963, 1977) for one-way beams. However, since the third-power equation is familiar to most engineers and there is little numerical difference between the two results compared to effects of other highly variable parameters, the third-power equation is considered to be adequate.

The moment-curvature relationships can be rewritten as