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Report on Deflection of Nonprestressed Concrete Structures

Reported by ACI Committee 435

ACI 435R-20



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Report on Deflection of Nonprestressed Concrete Structures

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Report on Deflection of Nonprestressed Concrete Structures

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This report presents a consolidated treatment of initial and time-dependent deflection of nonprestressed reinforced concrete members such as simple and continuous beams and one-way and two-way slab systems. It presents the current state of practice of deflection prediction as well as analytical methods for computer use in deflection estimation. Topics include material properties, deflection of reinforced concrete one-way flexural members, deflection of two-way slab systems, and reducing deflection of concrete members.

Keywords: camber; cracking; creep; curvature; deflection; modulus of rupture; moments of inertia; serviceability; shrinkage; time-dependent deflection.

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ACI 435R-20 supersedes ACI 435R-95(03) and became effective November 2020.
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CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Introduction

Design for serviceability is central to the work of structural engineers and code-writing bodies. It is also essential to users of the designed structures. Increased use of high-strength concrete and higher-strength reinforcing bars, coupled with more detailed computer-aided designs, has resulted in lighter and more material-efficient and, thus, more flexible structural members and systems. This in turn has necessitated better prediction and control of short-term and long-term behavior of concrete structures at service loads.

This report presents a consolidated treatment of initial and time-dependent deflection of nonprestressed reinforced concrete members such as simple and continuous beams and one- and two-way slab systems. It presents current engineering practice in design for control of deformation and deflection of concrete members and includes methods presented in ACI 318 plus selected other approaches suitable for computer-based use in deflection computation. Design examples are given at the end of one- and two-way framing chapters showing how to evaluate deflection and, thus, control it through adequate design for serviceability. The content of the report as well as the step-by-step examples are intended to familiarize practitioners with the current methods for estimating deflections in buildings as well as analytical methods suitable for computer-based application. The examples apply ACI 318 requirements and a recommended alternative approach with a lower cracking moment (to account for shrinkage restraint). Methods for predicting initial and time-dependent deflections

of prestressed concrete are not addressed in this document, although prestressing can be an effective tool for controlling both short- and long-term deflections.

1.2—Scope

The principal causes of deflections taken into account in this report are those due to elastic deformation, flexural cracking, creep, shrinkage, and temperature effects. This document is composed of two introductory chapters and four main chapters that provide information on calculating and controlling deflections of members constructed using reinforced concrete. The organization of the report is:

- a) **Chapter 1—Introduction and Scope** provides background information on the document.
- b) **Chapter 2—Notation and Definitions** provides a listing of the notation used throughout the document.
- c) **Chapter 3—Material Properties** discusses material properties that affect deflections.
- d) **Chapter 4—Deflection of Reinforced Concrete One-Way Flexural Members** discusses behavior of uncracked and cracked members, and time-dependent effects. It also includes the relevant code procedures and expressions for deflection computation in reinforced concrete beams. Numerical examples are included to illustrate the standard calculation methods for simply supported and continuous concrete beams.
- e) **Chapter 5—Deflection of Two-Way Slab Systems** covers the deflection behavior of reinforced two-way-action slabs and plates. This chapter gives an overview of classical and other methods of deflection estimation, such as the crossing beam analogy and the finite element method for immediate deflection computation. It also discusses approaches for determining the minimum thickness requirements for two-way slabs and plates and gives a detailed computational example for evaluating the long-term deflection of a two-way reinforced concrete slab. The chapter emphasizes the uncertainties inherent in estimating deflections of two-way slab systems.
- f) **Chapter 6—Reducing Deflection of Concrete Members** gives practical and remedial guidelines for improving and controlling the deflection of reinforced concrete members, hence enhancing their overall long-term serviceability.

It should be emphasized that the magnitude of actual deflection in concrete structural members, particularly in buildings, which are the emphasis and the intent of this report, can only be estimated with limited accuracy. This is because of the large variability in the properties of the constituent materials of these members, the quality control exercised in their construction, and the construction methods used. Therefore, for practical considerations, the computed deflection values in the illustrative examples at the end of each chapter should be interpreted with this in mind.

In summary, this single document gives design engineers the key tools for estimating, and thereby controlling through design, the expected deflection in nonprestressed reinforced concrete building structures. The material presented and the design examples will help to enhance serviceability when

used judiciously by the engineer. Designers, constructors, and codifying bodies can draw on the material presented in this document to achieve serviceable deflection of constructed facilities.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

A_s	= area of nonprestressed tension steel, in. ² (mm ²)
A_s'	= area of nonprestressed steel in compression zone, in. ² (mm ²)
b	= width of the section, in. (mm)
b_w	= web width, in. (mm)
C_t	= creep coefficient of concrete at time t , days
C_u	= ultimate creep coefficient of concrete
c	= depth of centroidal axis, in. (mm)
d	= distance from the extreme compression fiber to centroid of tension reinforcement, in. (mm)
d'	= distance from the extreme compression fiber to centroid of compression reinforcement, in. (mm)
d_b	= bar diameter, in. (mm)
E	= modulus of elasticity, psi (MPa)
E_c	= modulus of elasticity of concrete, psi (MPa)
f_c	= stress in concrete, psi (MPa)
f_c'	= specified compressive strength of concrete, psi (MPa)
f_{cr}	= stress to cause cracking in concrete, psi (MPa)
f_r	= modulus of rupture of concrete, psi (MPa)
f_{res}	= stress from restraint to shrinkage, psi (MPa)
f_s	= stress in nonprestressed steel, psi (MPa)
f_y	= specified yield strength of nonprestressed reinforcing steel, psi (MPa)
h	= overall thickness of a member, in. (mm)
h_f	= flange thickness, in. (mm)
I	= moment of inertia, in. ⁴ (mm ⁴)
I_{cr}	= moment of inertia of the cracked section transformed to concrete, in. ⁴ (mm ⁴)
I_e	= effective moment of inertia for computation of deflection, in. ⁴ (mm ⁴)
I_g	= moment of inertia for gross concrete section about centroidal axis, neglecting reinforcement, in. ⁴ (mm ⁴)
k	= depth of compression zone divided by d
ℓ	= span length, in. (mm)
ℓ_n	= distance from the inside of the support to the inside of support, clear span, in. (mm)
M	= bending moment, lb-in. (N-mm)
M_1	= moment at End 1 of a continuous member, lb-in. (N-mm)
M_2	= moment at End 2 of a continuous member, lb-in. (N-mm)
M_a	= maximum service load moment (unfactored) at stage deflection is computed, lb-in. (N-mm)
M_{cr}	= cracking moment, lb-in. (N-mm)
M_D	= moment due to dead load, lb-in. (N-mm)
M_L	= moment due to live load, lb-in. (N-mm)
M_m	= midspan moment, lb-in. (N-mm)
N	= axial member load, lb (N)
n	= modular ratio E_s/E_c
T	= temperature, °F (°C)

t	= time, s
w	= uniformly distributed load (load per unit length), lb/in. (N/mm)
w_c	= unit weight of normalweight concrete or equilibrium density of lightweight concrete, lb/ft ³ (kg/m ³)
y_t	= distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in. (mm)
Δ	= elastic deflection of a beam or slab, in. (mm)
Δ_{cr}	= additional deflection due to creep, in. (mm)
Δ_{inc}	= incremental deflection that occurs after attachment on nonstructural elements (includes long-term deflection Δ_{LT} from sustained loads and immediate deflection from the remaining part of live load that is not sustained, in. (mm))
Δ_L	= initial (immediate) deflection due to live load, in. (mm)
Δ_{LT}	= deflection from long-term effects, in. (mm)
Δ_{sh}	= additional deflection due to shrinkage, in. (mm)
Δ_{sus}	= initial (immediate) deflection due to sustained load, in. (mm)
ϵ_{cf}	= strain due to stress in the concrete
ϵ_o	= free strain, such as unrestrained shrinkage
ϵ_{sf}	= strain due to stress in nonprestressed steel
ϵ_{sh}	= shrinkage strain of concrete
$(\epsilon_{sh})_t$	= shrinkage strain of concrete at time t , days
$(\epsilon_{sh})_u$	= ultimate shrinkage strain of concrete
ϵ_t	= total strain
ζ	= distribution coefficient
κ	= cross section curvature, in. ⁻¹ (mm ⁻¹)
κ_{sh}	= shrinkage curvature, in. ⁻¹ (mm ⁻¹)
λ_c	= creep multiplier for long-term deflection
λ_{sh}	= shrinkage warping multiplier for long-term deflection
λ_t	= total multiplier for long-term deflection
λ_Δ	= time-dependent multiplier for long-term deflection
ν	= Poisson's ratio
ρ	= nonprestressed tension reinforcement ratio (A_s/bd)
ρ'	= reinforcement ratio for nonprestressed compression steel (A_s'/bd)
ξ	= time-dependent multiplier for deflection

2.2—Definitions

Please refer to the latest version of ACI Concrete Terminology for a comprehensive list of definitions.

CHAPTER 3—MATERIAL PROPERTIES

3.1—Objective

Deflections in reinforced concrete structures are affected significantly by numerous material properties, including concrete and reinforcement moduli of elasticity, concrete modulus of rupture, creep, and shrinkage. The purposes of this chapter are to: 1) briefly address how each of these material properties affects deflection; and 2) provide brief guidance on the most common expressions recommended by various ACI committees for estimation of these parameters during the design process. This chapter is intended to provide a comprehensive review of all the material prop-

erty expressions that have been suggested by researchers, committees, and design codes.

3.2—Material properties affecting deflection

The primary material properties that affect the deflection of reinforced concrete structures are identified in the following.

3.2.1 Concrete modulus of elasticity—The elastic modulus of concrete (E_c) has a significant impact on deflections in reinforced concrete structures, as it is a direct measure of material stiffness. Based on simple mechanics principles developed for elastic theory, deflection is inversely related to the elastic modulus of the concrete. However, the concrete elastic modulus also affects the sectional moment of inertia at the cracked section (I_{cr}) and, thus, the relationship between concrete elastic modulus and deflection is far more complex.

In indeterminate structures, the member forces and moments themselves (and thus resulting deflections) will be affected by the modulus of elasticity if it varies across the different components of the structure.

3.2.2 Concrete modulus of rupture—The concrete modulus of rupture (f_r) affects deflection primarily in that it establishes the stress threshold at which a member cracks. Upon reaching the modulus of rupture, the member (or a portion of the member) transitions from an uncracked to a cracked state and there is a significant reduction in stiffness accompanied by an increase in deflection. While most reinforced concrete members are designed to be cracked at service loads, the extent of cracking along the length of the member will have an impact on the member deflection.

For indeterminate structures, the extent of cracking has a significant effect on member stiffness, which in turn affects internal member forces and moments used in evaluating deflections.

3.2.3 Time-dependent concrete properties—Creep and shrinkage of concrete have a pronounced effect on the deflection behavior of reinforced concrete members. Creep of concrete will cause additional deformation over time, resulting in increased deflection of flexural members under sustained load. In many cases, the additional deflection will exceed the instantaneous deflection. The consideration of creep is not as simple as using a multiplier based solely on the creep coefficient of the concrete material because creep also causes a redistribution of internal stresses over time. This redistribution of internal stresses can be considered to cause a variation in effective sectional properties (at any point in the member) with time because the neutral axis and moment of inertia will change as the compressive strain in the concrete increases with age due to creep.

Similarly, shrinkage of concrete will directly lead to time-dependent deflections whenever warping is present due to differential shortening within the depth of the member. Differential shortening may occur due to an unsymmetric arrangement of internal reinforcement or external environmental effects. Restraint to shrinkage, either due to external restraint or internal restraint of embedded reinforcement, can also have a significant effect on deflection by inducing tensile stresses in the concrete that effectively reduce the external load required to cause cracking and thereby increase deflec-

tion. The most common approach for calculating deflection is to use a lower cracking moment computed with a reduced modulus of rupture to account for the tensile stresses that develop from restraint to shrinkage. For indeterminate structures, the member forces and moments themselves (and thus resulting deflections) may be significantly affected by creep and shrinkage. Creep effects can lead to a redistribution of forces among the members in a structure. Shrinkage restraint can also develop among the members in a structure, leading to cracking and a subsequent loss in stiffness.

3.2.4 Reinforcement modulus of elasticity—The effect of reinforcement modulus of elasticity on deflections can be significant but is limited primarily to the effect on sectional properties and the restraint of concrete shrinkage. The use of a lower modulus reinforcement will lead to smaller sectional moments of inertia for a constant reinforcement ratio, particularly for the cracked section, which in turn will lead to larger deflections. On the other hand, a lower modulus reinforcement will offer less internal restraint to concrete shrinkage and thus will not have as significant an effect in reducing the effective cracking moment.

3.3—Concrete material properties

Expressions recommended by ACI committees for estimation of concrete material properties during the design process are presented in this section. In general, these properties vary widely among different concretes or even among different batches of concrete produced using a single set of mixture proportions. Therefore, these expressions should only be expected to provide rough estimates for these concrete material properties, with a relatively low degree of accuracy. When a higher level of accuracy in deflection calculations is desired, concrete material properties should be measured using appropriate ASTM standards on test specimens produced using field trial batches.

3.3.1 Concrete modulus of elasticity—The modulus of elasticity is strongly influenced by the constitutive materials and proportions used and is especially affected by the properties and quantity of the coarse aggregate. An increase in the modulus of elasticity is commonly associated with an increase in compressive strength because the slope of the ascending branch of the stress-strain diagram becomes steeper for higher-strength concretes, but at a lower rate than the compressive strength. The value of the secant modulus of elasticity at $0.45f_c'$ for normal-strength concretes at 28 days is usually approximately 4×10^6 psi (28,000 MPa), whereas for higher-strength concretes with compressive strengths in excess of 8000 psi (56 MPa), values as high as 8×10^6 psi (56,000 MPa) have been reported (Myers and Yang 2004).

ACI 318 recommends Eq. (3.3.1) for computing the modulus of elasticity of concretes with densities in the range of 90 to 150 lb/ft³ (1445 to 2325 kg/m³) based on the secant modulus at the $0.45f_c'$ intercept.

$$E_c = 33w_c^{1.5}\sqrt{f_c'} \quad (\text{psi})$$

$$E_c = 0.043w_c^{1.5}\sqrt{f_c'} \quad (\text{MPa}) \quad (3.3.1)$$

Table 3.3.3—Creep and shrinkage ratios from age 60 days to the indicated concrete age (Branson 1977)

Creep, shrinkage ratios	Concrete age					
	2 months	3 months	6 months	1 year	2 years	>5 years
C_t/C_u	0.48	0.56	0.68	0.77	0.84	1.00
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$ —MC	0.46	0.60	0.77	0.88	0.94	1.00
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$ —SC	0.36	0.49	0.69	0.82	0.91	1.00

Notes: MC is moist-cured; and SC is steam-cured.

Although Eq. (3.3.1) is considered to provide reasonable predictions for concretes in the strength range up to 6000 psi (42 MPa), significant variation is still expected between predicted and measured values. ACI 318 commentary suggests that differences of up to 20 percent between predicted and measured values is to be expected.

3.3.2 Concrete modulus of rupture—ACI 318 recommends Eq. (3.3.2a) for computing the modulus of rupture of concrete with different densities

$$\begin{aligned} f_r &= 7.5\lambda\sqrt{f'_c} \quad (\text{psi}) \\ f_r &= 0.62\lambda\sqrt{f'_c} \quad (\text{MPa}) \end{aligned} \quad (3.3.2a)$$

where

$\lambda = 1.0$ for normal-density concrete (145 to 150 lb/ft³ [2325 to 2400 kg/m³])

$= 0.85$ for normalweight fine aggregate and lightweight coarse aggregate concrete

$= 0.75$ for all-lightweight aggregate concrete

Alternatively, ACI Committee 435 (1978) recommended using Eq. (3.3.2b) for computing the modulus of rupture of concrete with densities w_c in the range of 90 to 145 lb/ft³ (1445 to 2325 kg/m³). This equation provides a more continuous consideration of concrete density than Eq. (3.3.2a) and yields slightly higher values of f_r .

$$\begin{aligned} f_r &= 0.65\sqrt{w_c f'_c} \quad (\text{psi}) \\ f_r &= 0.013\sqrt{w_c f'_c} \quad (\text{MPa}) \end{aligned} \quad (3.3.2b)$$

A large degree of scatter can be expected in modulus of rupture test results, so only limited improvement in the accuracy should be expected when using the aforementioned equations in design calculations. Furthermore, these expressions are based on small prism specimens and do not reflect the presence of tensile stresses due to shrinkage restraint caused by embedded reinforcing bars and supporting elements.

3.3.3 Time-dependent concrete properties—ACI 318 does not specify explicit values for the concrete ultimate creep coefficient C_u and ultimate shrinkage strain $(\epsilon_{sh})_u$. However, they can be evaluated from several equations available in the literature (ACI Committee 209 1982; Bažant and Panula 1980; Branson 1977). ACI Committee 435 (1978) suggested that the average values for C_u and $(\epsilon_{sh})_u$ can be estimated as 1.60 and 400×10^{-6} , respectively. These values correspond to the following conditions:

a) 70 percent average relative humidity

b) Age of loading: 20 days for both moist and steam-cured concrete

c) Minimum thickness of component: 6 in. (150 mm)

Table 3.3.3 includes creep and shrinkage ratios at different times after loading, where the ratio indicates the percentage of ultimate creep or shrinkage that can be expected to have occurred at a certain age. Most of the time-dependent deformation can be expected to occur within the first year.

ACI Committee 209 (1971, 1982) and ACI 209R recommend a time-dependent model for creep and shrinkage under standard conditions as developed by Branson (1971, 1977). The term “standard conditions” is defined for a number of variables related to material properties, the ambient temperature, humidity, and size of members. The standard conditions for both creep and shrinkage are:

a) Age of concrete at load application = 3 days (steam), 7 days (moist)

b) Ambient relative humidity = 40 percent

c) Minimum member thickness = 6 in. (150 mm)

d) Concrete slump = 3 in. (75 mm)

e) Fine aggregate content = 50 percent

f) Air content = 6 percent

The coefficient for creep at time t (days) after load application is given by Eq. (3.3.3a)

$$C_t = \left(\frac{t^{0.6}}{10 + t^{0.6}} \right) C_u \quad (3.3.3a)$$

where $C_u = 2.35K_h^c K_d^c K_s^c K_f^c K_{ac}^c K_{to}^c = 2.35$ for standard conditions.

Each K coefficient is a correction factor for conditions other than standard as follows:

K_h^c = relative humidity factor

K_d^c = minimum member thickness factor

K_s^c = concrete consistency factor

K_f^c = fine aggregate content factor

K_{ac}^c = air content factor

K_{to}^c = age of concrete at load application factor

Graphic representations and general equations for the modification factors (K -values) for nonstandard conditions are given in Fig. 3.3.3a (Meyers and Thomas 1983).

For concrete moist-cured for 7 days, the free shrinkage strain can be estimated using Eq. (3.3.3b) at any time t , where t is the time in days from the end of initial curing

$$(\epsilon_{sh})_t = \left(\frac{t}{35 + t} \right) (\epsilon_{sh})_u \quad (3.3.3b)$$



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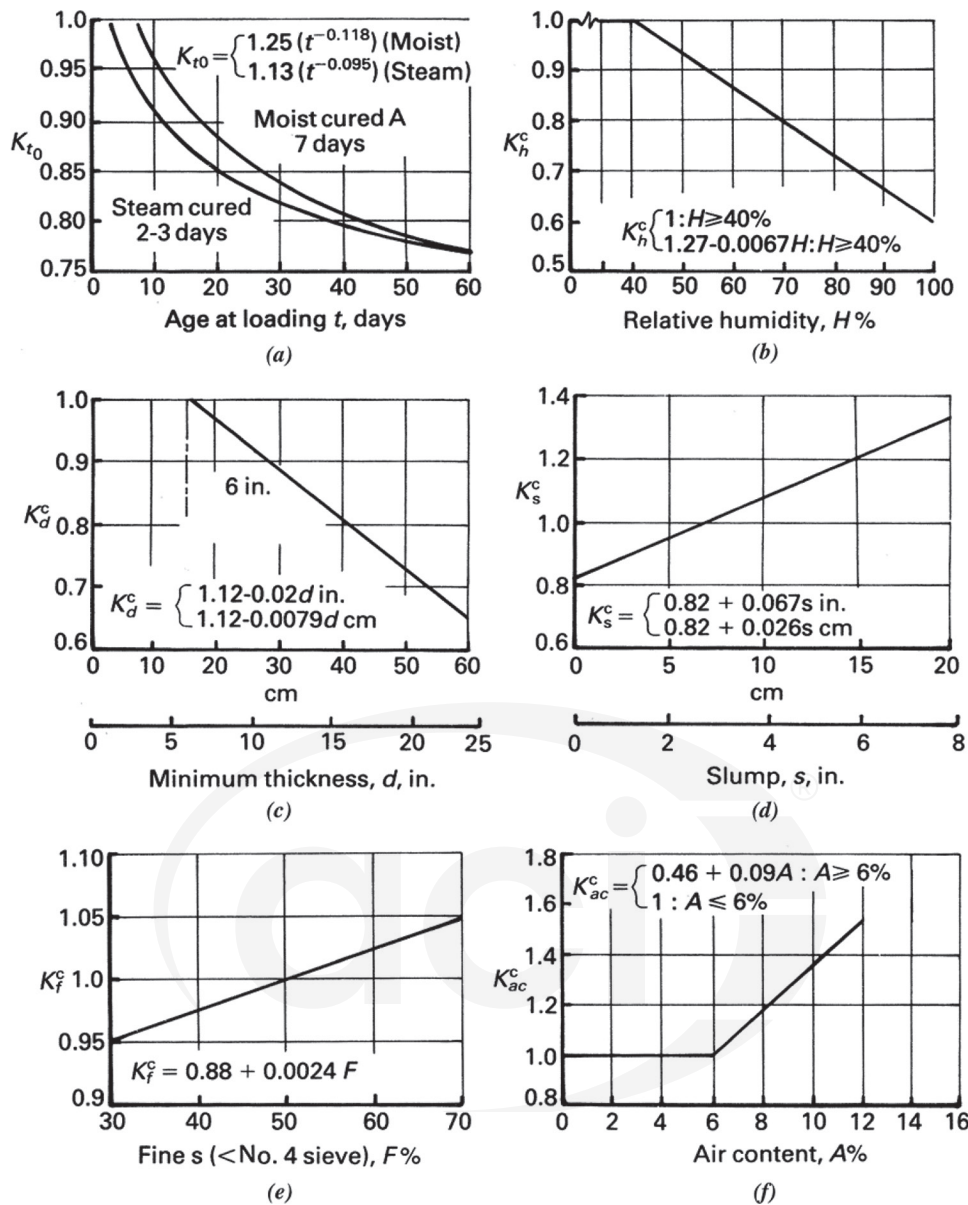


Fig. 3.3.3a—Creep correction factors for nonstandard conditions, ACI 209R method (Meyers and Thomas 1983).

and for concrete steam-cured for 1 to 3 days, the shrinkage strain can be estimated using Eq. (3.3.3c) at any time t , where t is the time in days from the end of steam curing

$$(\epsilon_{sh})_t = \left(\frac{t}{55+t} \right) (\epsilon_{sh})_u \quad (3.3.3c)$$

where $(\epsilon_{sh})_u = 780 \times 10^{-6} K_h^s K_d^s K_f^s K_{ac}^s = 780 \times 10^{-6}$ for standard conditions.

Each K coefficient is a correction factor for other than standard conditions. All coefficients are the same as defined for creep except K_B^s , which is a coefficient for cement content. Graphic representation and general equations for the modification factors for nonstandard conditions are given in Fig. 3.3.3b (Meyers and Thomas 1983). The aforementioned procedure, using standard and correction equations and extensive experimental comparisons, is detailed in Branson

(1977). The presence of chemical admixtures, specifically water reducers, has been shown to decrease creep and shrinkage (Brooks and Al-Qarra 1999), a factor that was not included in the development of the ACI Committee 209 (1971, 1982) and ACI 209R prediction models. ACI 209.2R outlines other creep and shrinkage functions in addition to the ACI 209R function based on Branson's work.

One approach that is employed to include the influence of creep in deflection calculations is to use an age-adjusted modulus of elasticity of the concrete, $\bar{E}_c(t, t_0)$, which can be calculated using Eq. (3.3.3d).

$$\bar{E}_c(t, t_0) = \frac{E_c(t_0)}{1 + \chi C_t(t, t_0)} \quad (3.3.3d)$$

where $E_c(t_0)$ is the modulus of elasticity of concrete at time t_0 . $C_t(t, t_0)$ is the creep coefficient, which is equal to the creep

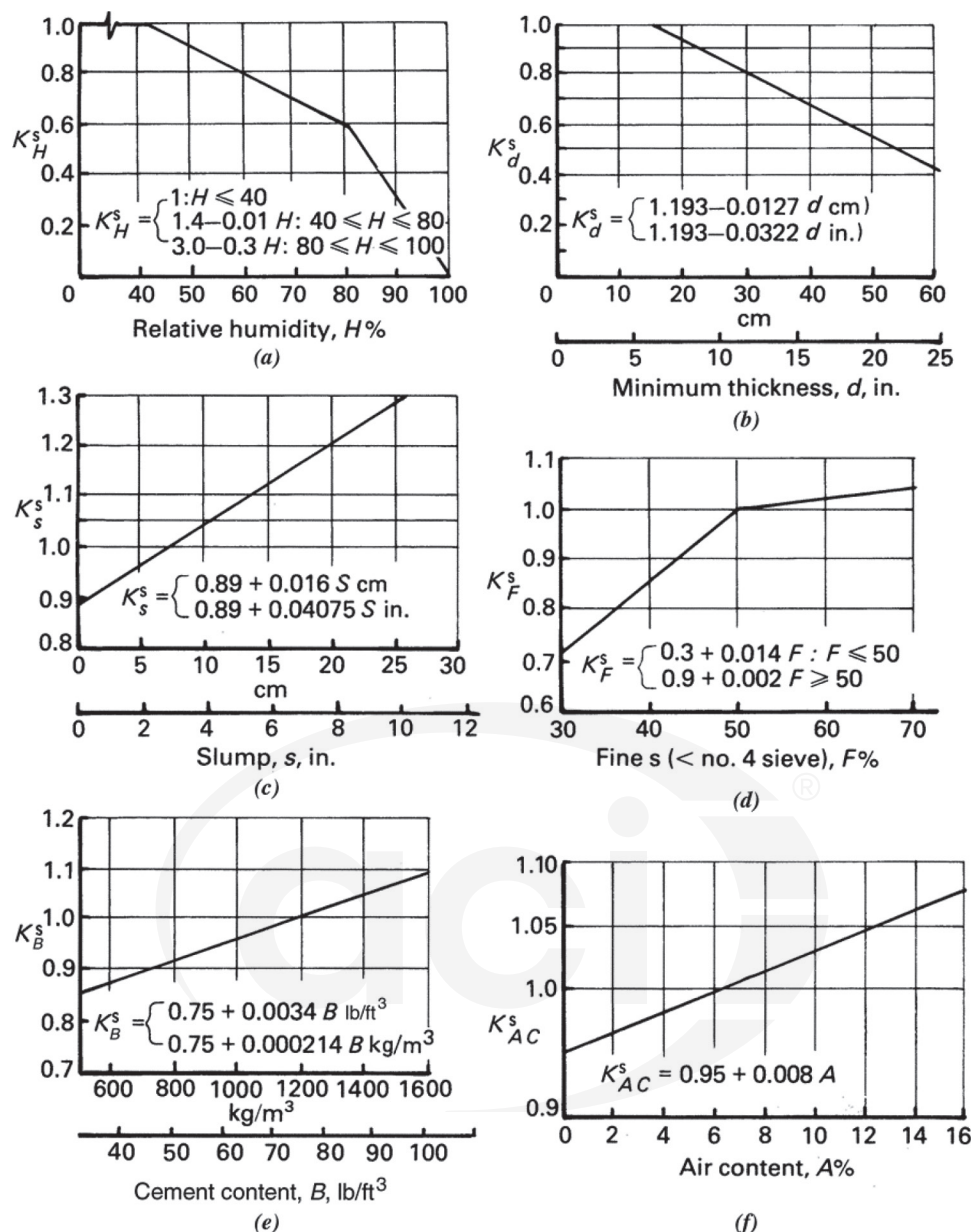


Fig. 3.3.3b—Shrinkage correction factors for nonstandard conditions, ACI 209R method (Meyers and Thomas 1983).

strain in the period t_0 to t divided by the instantaneous strain. The aging coefficient χ was introduced by **Trost (1967)** and **Bažant and Kim (1979)** and is mainly a function of t_0 and t (**Ghali et al. 2011**). A value of 0.8 can be used in most cases because χ varies between 0.7 and 0.9. Tabulated values of χ are given in **ACI 209R**.

3.3.4 Special concretes—In this section, brief guidance is provided on the material properties for special concretes.

3.3.4.1 Lightweight concrete—Lightweight concrete is structural concrete made with lightweight aggregates with bulk density less than 70 lb/ft³ (1120 kg/m³) (**ACI 213R**). Extensive information on the material properties of lightweight concrete is provided in **ACI 213R**.

The modulus of elasticity of lightweight concrete is lower than for normal-density concrete because the moduli of lightweight aggregates are significantly lower than the moduli of sand, stone, and gravel used in normal-density

concretes (**ACI 213R**). The modulus of elasticity for lightweight concrete may be estimated using Eq. (3.3.1), which directly considers the weight difference via the parameter w_c . It is important to note however, that the simplified form of Eq. (3.3.1) often used in practice: $E_c = 57,000 \sqrt{f'_c}$ (psi) ($4700 \sqrt{f'_c}$ [MPa]) is only valid for normal-density concrete and should never be used for lightweight concrete.

The modulus of rupture is also proportionally lower for lightweight concrete as compared to normal-density concrete of the same compressive strength. Equations (3.3.2a) or (3.3.2b) may be used to estimate the modulus of elasticity of lightweight concrete. In the case of Eq. (3.3.2a), the equation should be used as presented when the tensile splitting strength f_{ct} is not specified. Otherwise, the equation should be modified by substituting $f_{ct}/6.7$ for $\sqrt{f'_c}$ (psi) ($1.8 f_{ct}$ for $\sqrt{f'_c}$ [MPa]) but the value of $f_{ct}/6.7$ (psi) ($1.8 f_{ct}$ [MPa])

should not exceed $\sqrt{f'_c}$. Work by Nassar (2002) indicates that Eq. (3.3.2a) may be unconservative for lightweight concretes with higher compressive strengths.

ACI 213R reports that while there is a large variation in measured values of drying shrinkage for lightweight concrete, it appears that low-strength lightweight concrete generally has greater drying shrinkage than comparable normal-density concretes. At higher strengths, some lightweight concrete has been found to exhibit lower shrinkage. Creep coefficients reported in the literature are generally higher for lightweight concrete as compared to comparable normal-density concrete.

3.3.4.2 High-strength concrete—High-strength concrete is defined by ACI 363R as concrete with specified compressive strengths exceeding 8000 psi (56 MPa). Extensive information on the material properties of high-strength concrete is provided in ACI 363R.

The modulus of elasticity is generally higher for high-strength concrete as compared to normal-strength concrete. Several equations have been proposed by various researchers for the modulus of elasticity of high-strength concrete, but a study by ACI 363R found that no single empirical expression estimated the modulus of elasticity to a high degree of accuracy for all concretes considered in the study. However, Eq. (3.3.4.2a) was found by ACI 363R to be a relatively reliable lower-bound expression for normal-density high-strength concrete based on most of the test data considered

$$\begin{aligned} E_c &= 40,000\sqrt{f'_c} + 1,000,000 \quad (\text{psi}) \\ E_c &= 3320\sqrt{f'_c} + 6900 \quad (\text{MPa}) \end{aligned} \quad (3.3.4.2a)$$

The values reported by various investigators (ACI 363R) for the modulus of rupture of both low-density and normal-density high-strength concretes range between $7.5\sqrt{f'_c}$ and $12\sqrt{f'_c}$ (psi) ($0.62\sqrt{f'_c}$ and $1.0\sqrt{f'_c}$ [MPa]). Therefore, Eq. (3.3.2a) can underestimate the modulus of rupture of normal-density high-strength concrete, and ACI 363R recommends the use of Eq. (3.3.4.2b) for high-strength concrete instead

$$\begin{aligned} f_r &= 11.7\sqrt{f'_c} \quad (\text{psi}) \\ f_r &= 0.97\sqrt{f'_c} \quad (\text{MPa}) \end{aligned} \quad (3.3.4.2b)$$

For high-strength concretes with low water-cementitious materials ratio (w/cm) and high binder content, the mechanisms of shrinkage may be more complex than for normal-strength concretes, with chemical and autogenous shrinkage supplementing the effects of drying shrinkage. There are considerably less data available on these mechanisms of shrinkage due to the difficulty in making such measurements. According to ACI 363R, there is no clear trend with respect to drying shrinkage of high-strength concrete, except that it is generally similar to the shrinkage of normal-strength concretes. Some researchers have reported a relatively high initial rate of shrinkage for high-strength concrete (Swamy and Anand 1973), but ACI 363R reports that there is general

agreement that the drying rates in high-strength concrete will be slower when only drying shrinkage is considered. Nagataki and Yonekura (1978) reported that the shrinkage of high-strength concrete containing high-range water reducers was less than for lower-strength concrete.

On the other hand, there is general agreement that the creep coefficients for high-strength concretes are lower than for normal-strength concrete (ACI 363R). The creep coefficient (ratio of creep strain to initial elastic strain under sustained axial compression) for high-strength concrete may be as low as one-half that generally associated with low-strength concrete (Ngab et al. 1981; Nilson 1985). However, high-strength concretes are often exposed to a higher level of sustained stress such that the total creep strain may be similar to that for normal strength concrete.

3.3.4.3 Self-consolidating concrete—Self-consolidating concrete is concrete proportioned such that it may be consolidated without mechanical vibration. Many self-consolidating concrete mixtures use admixtures and, in some cases, a reduced quantity of coarse aggregate to maximize stability and minimize segregation and bleeding, which can lead to differences in hardened material properties relative to conventionally-placed concrete. However, self-consolidating mixtures can also be engineered to have properties similar to, or better than, those of conventionally placed concrete (ACI 237R). Information on the material properties of self-consolidating concrete is provided in ACI 237R.

In some studies, self-consolidating concrete mixtures with high sand-to-total aggregate ratios (s/a) have been observed to have a modulus of elasticity of approximately 10 to 15 percent lower than that of conventional concrete of similar compressive strength, whereas, in other studies, there has been little effect observed on the modulus of elasticity (ACI 237R). ACI 237R recommends the use of Eq. (3.3.1) for the modulus of elasticity of self-consolidating concrete in applications where the modulus is not critical, and recommends testing to determine the modulus of elasticity when greater accuracy is required.

ACI 237R does not recommend any unique considerations for the estimation of the modulus of rupture for self-consolidating concrete. As such, Eq. (3.3.2a) or (3.3.2b) may be used to estimate the modulus of rupture. A study by Sonebi and Bartos (2001) found that the modulus of rupture of self-consolidating concrete may actually be higher than that of conventional concrete with similar proportions.

ACI 237R notes that several studies have reported the drying shrinkage of self-consolidating concrete to be similar to or lower than that of conventional concrete with similar compressive strength. When mixture proportions are similar to those for conventionally placed concrete, self-consolidating concrete has been found to have similar creep characteristics to those for conventional concrete. When a greater paste volume is used for stability considerations, a higher creep in comparison to conventionally placed concrete may occur (ACI 237R). ACI 237R recommends testing to determine the creep coefficient when creep is an important parameter in design.

3.3.4.4 Fiber-reinforced concrete—Fiber-reinforced concrete consists primarily of hydraulic cements, aggregates, and discrete reinforcing fibers. Common materials used as fiber reinforcement include steel, glass, and synthetic materials, including polypropylene and polyethylene. **ACI 544.1R** provides detailed information on the characteristics and behaviors of the common types of fiber reinforcement, and their effect on concrete properties.

The primary contribution of fibers to concrete is to improve the pre- and post-cracking behavior of the concrete in tensile regions. Some fibers have the ability to increase the tensile capacity of the concrete, effectively increasing the modulus of rupture. No general equation has been proposed to quantify this increase as the effect is highly dependent on the fiber type, shape, and application. In addition to affecting the precracking behavior, fibers also allow for stress transfer across a crack. This typically results in a decrease in crack width, and a decrease in crack spacing. This behavior serves to decrease the immediate deflections of the structural member by reducing the rise of the neutral axis upon cracking, thereby reducing the change in the post-cracking moment of inertia relative to the gross moment of inertia. As with the modulus of rupture, no general equation has been proposed to quantify this response due to the variation in performance of various fiber types and geometries.

Fibers can also affect the creep properties of concrete. Compression creep is not significantly affected; however, long-term flexural creep is affected by the presence of fibers (**ACI 544.5R**). The long-term deflection response is dependent on the characteristics of the fibers used in the concrete. Longer fibers with higher moduli of elasticity tend to improve long-term deflection behavior. Specifically, fiber-reinforced concrete beams with steel fibers have shown less long-term deflections than reinforced concrete beams without fibers (**Tan et al. 1994**). However, fibers with lower moduli of elasticity tend to exhibit more creep behavior over time (**Bernard 2004**).

The shrinkage response of concrete is generally not affected by the presence of fibers. However, the potential increase in the apparent modulus of rupture and the ability of the fibers to distribute cracking may decrease the effect of shrinkage on the deflection behavior of the concrete.

3.4—Reinforcement material properties

Deflections are not as sensitive to variation in reinforcement material properties in comparison to concrete properties, so the use of specified or manufacturer-supplied values is usually adequate and additional testing on field specimens is not required. The property of interest is the reinforcement modulus of elasticity. Typical values are provided in the following.

3.4.1 Steel reinforcement—**ACI 318** specifies using the value $E_s = 29 \times 10^6$ psi (200,000 MPa) for the modulus of elasticity of nonprestressed reinforcing steel.

3.4.2 Fiber-reinforced polymer (FRP) reinforcement—The modulus of elasticity of FRP reinforcement (E_f) varies widely, depending on the type and volume fraction of the fiber used in the composite. In general, elastic modulus

values provided by the reinforcement manufacturer should be used when available. Ranges of elastic moduli for different commercially-available FRPs are provided by **ACI 440.1R**:

- a) Glass-fiber-reinforced polymer (GFRP): $E_f = 5.1$ to 7.4×10^6 psi (35,000 to 51,000 MPa)
- b) Carbon-fiber-reinforced polymer (CFRP): $E_f = 15.9$ to 84.0×10^6 psi (110,000 to 580,000 MPa)
- c) Aramid-fiber-reinforced polymer (AFRP): $E_f = 6.0$ to 18.2×10^6 psi (41,000 to 125,000 MPa)

CHAPTER 4—DEFLECTION OF REINFORCED CONCRETE ONE-WAY FLEXURAL MEMBERS

4.1—General

4.1.1 Introduction—In many structures, deflection rather than strength is the controlling factor. For these cases, deflection computations will determine the proportions and configuration of the structural system members. Member stiffness is a function of short- and long-term mechanical properties of the concrete and the extent of flexural cracking. Hence, expressions defining the modulus of rupture, modulus of elasticity, creep, shrinkage, and temperature effects are important parameters in predicting the deflection of reinforced concrete members (refer to **Chapter 3** for a detailed explanation of these properties). The stiffness and amount of reinforcement also control the member stiffness after cracking.

The designer may choose to calculate deflections and check that these computed values are less than specified permissible limits. Alternatively, design codes such as **ACI 318** give specified maximum span/depth ratios, with limits of applicability, for which serviceability is assumed to be satisfied and deflections do not need to be explicitly calculated.

4.1.2 Objectives—This chapter covers the initial (immediate) and time-dependent deflections at service load levels under static conditions for one-way nonprestressed flexural concrete members. Details are also provided on deemed-to-comply limiting span-thickness ratios for which deflections do not need to be calculated. This chapter is intended to give the designer enough basic background to design concrete members that perform adequately under service loads, taking into account cracking and both short- and long-term effects.

Information is presented on the effective moment of inertia method prescribed in **ACI 318** to evaluate deflection. **ACI 318** permits use of an expression for the effective moment of inertia (I_e) to compute deflection, unless a more comprehensive analysis is carried out to determine the member stiffness. The existing approach for I_e used by **ACI 318-14** and earlier editions can underestimate deflection in certain conditions (**Bischoff and Scanlon 2007**); hence, an alternative expression for the effective moment of inertia that works well over a wide range of conditions is proposed. Other modifications and more refined methods that require detailed calculations for estimating deflection are also discussed. This includes the effects of preloading from construction loads and reduced flexural stiffness that can result when tensile stresses develop in the concrete from restraint to shrinkage.

Table 4.2.1—Recommended tension reinforcement ratios for nonprestressed one-way members so that deflection will normally be within acceptable limits (ACI Committee 435 1978)

Members	Cross section	Normalweight concrete	Lightweight concrete
Not supporting or not attached to nonstructural elements likely to be damaged by large deflections	Rectangular, T, or box	$\rho \leq 35$ percent ρ_b $\rho_w \leq 40$ percent ρ_b	$\rho \leq 30$ percent ρ_b $\rho_w \leq 35$ percent ρ_b
Supporting or attached to nonstructural elements likely to be damaged by large deflections	Rectangular, T, or box	$\rho \leq 25$ percent ρ_b $\rho_w \leq 30$ percent ρ_b	$\rho \leq 20$ percent ρ_b $\rho_w \leq 25$ percent ρ_b

Notes: For continuous members, the positive region steel ratios only may be used. ρ_b refers to the balanced steel ratio based on ultimate strength.

Table 4.2.2a—Minimum thickness of nonprestressed beams and one-way slabs unless deflections are computed (from ACI 318)

Member	Simply supported	Minimum thickness, h		Cantilever
		One end continuous	Both ends continuous	
	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes: Values given should be used directly for members with normalweight concrete and Grade 60 (Grade 420) reinforcement. For other conditions, the values shall be modified as follows:

a) For f_y other than 60,000 psi (420 MPa), the values shall be multiplied by $0.4 + f_y/100,000$ ($0.4 + f_y/700$).

b) For lightweight concrete having equilibrium density w_c in the range 90 to 115 lb/ft³ (1440 to 1840 kg/m³), the values should be multiplied by $1.65 - 0.005w_c$ ($1.65 - 0.0003w_c$) but not less than 1.09.

Table 4.2.2b—Minimum thickness of beams and one-way slabs used in roof and floor construction (ACI Committee 435 1978)

Member	Members not supporting or not attached to nonstructural elements likely to be damaged by large deflections				Members supporting or attached to nonstructural elements likely to be damaged by large deflection			
	Simply supported	One end continuous	Both ends continuous	Cantilever	Simply supported	One end continuous	Both ends continuous	Cantilever
Roof slab	$\ell/22$	$\ell/28$	$\ell/35$	$\ell/9$	$\ell/14$	$\ell/18$	$\ell/22$	$\ell/5.5$
Floor slab and roof beam or ribbed roof slab	$\ell/18$	$\ell/23$	$\ell/28$	$\ell/7$	$\ell/12$	$\ell/15$	$\ell/19$	$\ell/5$
Floor beam or ribbed floor slab	$\ell/14$	$\ell/18$	$\ell/21$	$\ell/5.5$	$\ell/10$	$\ell/13$	$\ell/16$	$\ell/4$

4.2—Control of deflection

Beams and one-way slabs are rarely built as isolated members but are part of a monolithic integrated system. Excessive deflection of a floor slab may cause dislocations in the partitions it supports or difficulty in leveling furniture or fixtures. Excessive deflection of a beam can damage a partition below, and excessive deflection of a spandrel beam or perimeter slab above a window opening could crack the glass panels. In the case of roofs or open floors, such as top floors of parking garages, ponding of water can result.

During design, deflection is controlled indirectly through reinforcement ratio limitations or minimum thickness requirements, or is controlled directly through calculating a deflection and limiting its value to a maximum based on functional requirements usually expressed as a fraction of the span length.

4.2.1 Tension steel reinforcement ratio limitations—One method to minimize deflection of a concrete member in flexure is by using a relatively small reinforcement ratio. When smaller reinforcement ratios are used, larger (and stiffer) sections are required to meet strength requirements. Limiting values of the reinforcement ratio ρ , ranging from $0.25\rho_b$ to $0.40\rho_b$ were recommended by ACI Committee 435

(1978), as shown in Table 4.2.1. Other methods for reducing deflection are presented in Chapter 6.

4.2.2 Minimum thickness limitations—Deflections of beams and one-way slabs of normal proportions and supporting typical loads in buildings are normally satisfactory when the minimum thickness provisions in Table 4.2.2a are met or exceeded. This table (ACI 318) applies only to members that are not supporting or not attached to partitions or other construction likely to be damaged by excessive deflections. Values in Table 4.2.2a were modified by ACI Committee 435 (1978) and are expanded in Table 4.2.2b to include members that are supporting or attached to nonstructural elements likely to be damaged by excessive deflections. The thickness limits in Tables 4.2.2a and 4.2.2b may be ignored when computed deflections are shown to be satisfactory. The values in Tables 4.2.2a and 4.2.2b should be used with care, as they may not be appropriate for certain situations such as sensitive window systems, long spans, and high live loads.

Due to limitations on the code provisions for minimum thickness, several authors have proposed alternative minimum thickness approaches based on computer simulations or engineering beam equations for deflection calculation (Grossman 1981, 1987; Rangan 1982; Gander 2011; Scanlon and Lee 2006). Grossman (1981, 1987) carried out

Table 4.2.2c—Maximum permissible computed deflections (ACI 318)

Member	Condition	Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to maximum of roof live load, snow and rain	$\ell/180^*$
Floors		Immediate deflection due to live load	$\ell/360$
Roof or floors	Supporting or attached to nonstructural elements	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load ^f	$\ell/480^{\ddagger}$
			$\ell/240^{\S}$

*Limit not intended to safeguard against ponding. Ponding should be checked by calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[†]Time-dependent deflection should be determined in accordance with 24.2.4 of ACI 318-14 but should be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount should be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[‡]Limit should be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[§]Limit should not exceed tolerance provided for nonstructural elements.

many computer studies to develop a simplified expression for the minimum thickness needed to satisfy serviceability requirements (Eq. (5.3e) in **Chapter 5**). Work by **Bischoff and Scanlon (2009)** indicates that heavily loaded slabs (with a service load greater than approximately 200 lb/ft² [9.6 kPa]) designed using the **ACI 318** minimum thickness requirements in Table 4.2.2a are not likely to satisfy the corresponding ACI 318 deflection requirements outlined in Table 4.2.2c for that part of the total (incremental) deflection occurring after attachment of the nonstructural elements; while beams with the minimum thickness requirements from Table 4.2.2a are also not likely to satisfy the deflection requirements in Table 4.2.2c. Subsequent work by **Bischoff and Veysey (2011)** provides recommended values of minimum member thickness needed to satisfy the $\ell/240$ and $\ell/480$ deflection limits for one-way slabs and beams. As the ACI 318 minimum thickness requirements are not always conservative, it is recommended that deflections be computed and compared with the appropriate deflection limits.

Other codes and standards such as **BS EN 1992-1-1** give limiting span/effective depth ratios for flexural members and provide limits on computed values of deflection. **Gardner (2011)** provides an extensive comparison of requirements from numerous code or standard authorities.

4.2.3 Computed deflection limitations—The permissible computed deflections specified in ACI 318 for one-way systems are given in Table 4.2.2c, where the span-deflection ratios provide for a simple set of permissible deflections. Where excessive deflection may cause damage to the nonstructural or other structural elements, only that part of the deflection occurring after construction of the affected elements (such as partitions) needs to be considered. The most stringent deflection limit of $\ell/480$ in Table 4.2.2c is an example of such a case. Where excessive deflection may result in a functional problem, such as visual sagging or ponding of water, the total deflection should be considered. Project specific requirements, such as for sensitive equipment or curtain wall systems, may require more stringent limits than those listed in Table 4.2.2c.

Deflection limits can vary from code to code and represent the consensus of experience for typical structures. **BS EN 1992-1-1** limits the total deflection to $\ell/250$, which should

ensure that the part of deflection occurring after construction of finishes and partitions will be limited to $\ell/500$ or 0.78 in. (20 mm), whichever is less, for spans up to 32.8 ft (10 m). Provision is made in the code for longer spans and for the amount of tension and compression reinforcement. Serviceability requirements of **AS 3600** limit the calculated deflection to a value appropriate to the structure and its intended use, which cannot exceed $\ell/250$ for the total deflection and $\ell/500$ for the incremental deflection that occurs after the addition or attachment of partitions when provision is made to minimize the effect of movement; otherwise the limit is $\ell/1000$. **BS EN 1992-1-1** limits deflections under quasi-permanent (sustained) loads to $\ell/250$ to avoid impairment of the appearance or general utility of a structure. A degree of precamber is permitted. For deflections after construction, a limit of $\ell/500$ is deemed appropriate for quasi-permanent loads and this should avoid damage occurring to adjacent parts of the structure.

4.3—Short-term deflection calculation

The short-term or immediate deflection Δ_i from flexural curvatures can be computed using a generalized elastic equation for a prismatic member under uniformly distributed load w .

$$\Delta_i = C \frac{5}{384} \frac{w \ell^4}{E_c I_e} = K \frac{5}{48} \frac{M \ell^2}{E_c I_e} \quad (4.3)$$

where M is the moment at the critical section due to service loads at the stage deflection is computed. The critical section is usually taken at midspan for simple and continuous members and at the support face for a cantilever. K and C are restraint factors that depend on the support fixity and loading conditions. For continuous members with uniform distributed loading, $K = (1.2M_m - 0.2M_o)/M_m$, where M_m is the moment at midspan and M_o is the total static moment $w\ell^2/8$. Similarly, $C = 1.2M_o/M_m - 0.2$. K equals 2.4 and C equals 9.6 for cantilevers. Restraint factor values are given in Table 4.3 for a flexural member subjected to a uniformly distributed load w . Effects of cracking and reinforcement on member stiffness are taken into consideration with an effective moment of inertia I_e . The modulus of elasticity

Table 4.3—Restraint factors K and C for a uniformly distributed load

Member type (support conditions)	K	C
Cantilever (fixed end)*	2.4	9.6
Simple span	1.0	1.0
One end continuous with the discontinuous end unrestrained ($K = 1.2 - 0.2M_o/M_m$)	0.925 for $M_m = w\ell^2/11$	0.673
One end continuous with the discontinuous end integral with the end support ($K = 1.2 - 0.2M_o/M_m$)	0.85 for $M_m = w\ell^2/14$	0.486
Fixed-hinged	0.80	
Both ends continuous ($K = 1.2 - 0.2M_o/M_m$)	0.80 for $M_m = w\ell^2/16$	0.40
Fixed-fixed	0.60	0.20

*Additional deflection from rotation at support of cantilever should be included.

(E_c) can be obtained from Eq. (3.3.1) for normal-strength concrete or Eq. (3.3.4.2a) for high-strength concrete.

The beam deflection Eq. (4.3) can also be written in terms of the midspan and member end moments (or midspan and member end curvatures), which is a convenient form for calculating deflection of two way-slabs. For example, $\Delta_i = (10M_m - M_1 - M_2) \times \ell^2/(96E_cI_e)$, where M_1 and M_2 are the two end moments. Textbooks on reinforced concrete (Nawy 2009; Wight and MacGregor 2009) also provides information on the different methods used to calculate deflection.

The approach described previously using Eq. (4.3) to compute deflection is most suitable for hand calculations. Use of Eq. (4.3) implies that deflection of continuous members depends on the member stiffness at the critical section, with no consideration given to the stiffness at the end supports unless a weighted average is taken of stiffness at the midspan and end supports. Other approaches involving more detailed calculations that may be amenable for implementation into computer software include integration of section curvature M/E_cI_e (4.3.3) to account for changes in stiffness along the member span.

4.3.1 Uncracked members: gross moment of inertia I_g —When the maximum flexural moment at service load in a beam or a slab causes a tensile stress less than the modulus of rupture (f_r), no flexural tension cracks develop at the tension side of the concrete element if the member is not restrained or the shrinkage and temperature tensile stresses are negligible. In such a case, the effective moment of inertia of the uncracked transformed section (I_t) is applicable for deflection computations. For design purposes, however, the gross moment of inertia I_g , neglecting the reinforcement contribution, can be used with negligible loss of accuracy. The combination of service loads with shrinkage and temperature effects due to end restraint or restraint from the internal reinforcement may cause cracking if the tensile stress in the concrete exceeds the modulus of rupture. In such cases, 4.3.2 applies.

4.3.2 Cracked members: effective moment of inertia I_e —Tension cracks occur when the imposed loads cause bending moments in excess of the cracking moment, thus resulting in tensile stresses in the concrete that are higher than its modulus of rupture. The cracking moment M_{cr} may be computed as follows

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (4.3.2)$$

where y_t is the distance from the centroidal axis to the tension face of the member, and f_r is the modulus of rupture of the concrete, as expressed by Eq. (3.3.2a) or (3.3.2b). Use of a lower cracking moment to account for shrinkage induced tension is discussed in 4.3.2.3 and in 5.2.3 for two-way slabs.

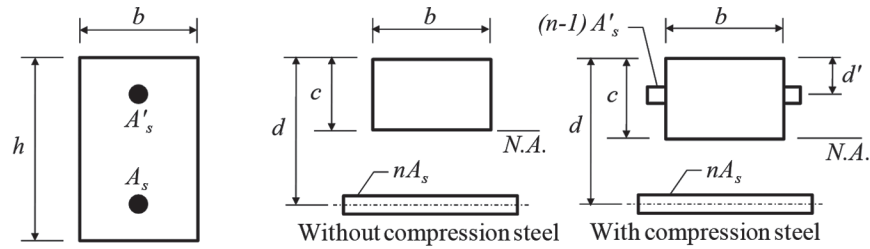
Cracks develop at several sections along the member length. While the cracked moment of inertia I_{cr} applies to the cracked sections, the gross moment of inertia I_g applies to the uncracked concrete between these sections. Several methods have been developed to estimate the variations in stiffness caused by cracking along the span. These methods either modify the flexural rigidity of the member by using an average or effective moment of inertia (Yu and Winter 1960; Branson 1963), make adjustments to the curvature along the span and at critical sections (Beeby 1968), compute an average curvature or the ratio M/EI at each location along the member span (fib MC2010), or use an incremental evaluation of section-curvature (Ghali 1989; Ghali et al. 2011). Deflection can be conservatively estimated by using the effective stiffness or curvature at the critical section where the member stiffness is lowest.

Studies by Branson (1977, 1985) have shown that the initial deflections Δ_i occurring in a beam or a slab after the maximum moment M_a has exceeded the cracking moment M_{cr} can be evaluated using an effective moment of inertia I_e that models a gradual transition from the gross (uncracked) moment of inertia I_g to the cracked transformed moment of inertia I_{cr} .

4.3.2.1 Branson's I_e —With the exception of slender walls (as defined in 11.8 of ACI 318-14), ACI 318 provides an expression for the effective moment of inertia I_e originally proposed by Branson (1963). This approach was selected as being sufficiently accurate to control deflections in reinforced concrete unless stiffness values are obtained by a more comprehensive analysis. Branson's approach is known to work well for beams that typically have a reinforcement ratio greater than 1 percent, but significantly overestimates I_e for lightly reinforced slabs where the in-service moment is often only just greater than the cracking moment (Bischoff and Scanlon 2007; Gilbert 2007).

Branson's equation for the effective moment of inertia I_e for short-term deflections is as follows

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \quad (4.3.2.1a)$$



$$I_g = bh^3/12; \quad c = kd; \quad \rho = A_s/bd; \quad \rho' = A'_s/bd$$

Without compression steel:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

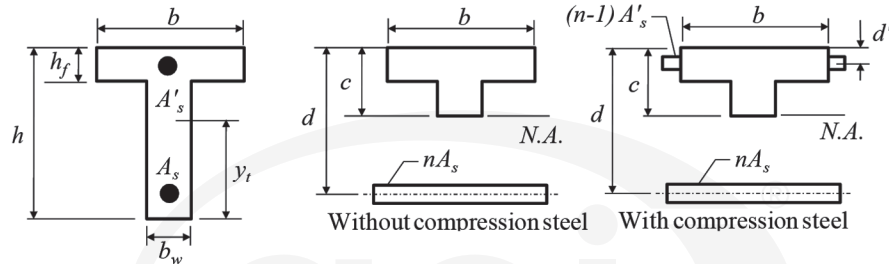
$$I_{cr} = bc^3/3 + nA_s(d-c)^2$$

With compression steel:

$$k = \sqrt{[n\rho + (n-1)\rho']^2 + 2[n\rho + (n-1)\rho'(d'/d)]} - [n\rho + (n-1)\rho']$$

$$I_{cr} = bc^3/3 + nA_s(d-c)^2 + (n-1)A'_s(c-d')^2$$

(a) Rectangular Sections



$$y_t = h - \frac{1}{2} \frac{\zeta h_f^2 + b_w h^2}{\zeta h_f + b_w h}; \quad \zeta = b_f/b_w - 1; \quad \rho_w = A_s/b_w d; \quad \rho'_w = A'_s/b_w d \text{ and } c = kd$$

$$I_g = (b_f - b_w)h_f^3/12 + b_w h^3/12 + (b_f - b_w)h_f(h - h_f/2 - y_t)^2 + b_w h(y_t - h/2)^2$$

Without compression steel:

$$k = \sqrt{[n\rho_w + \zeta(h_f/d)]^2 + 2[n\rho_w + 0.5\zeta(h_f/d)^2]} - [n\rho_w + \zeta(h_f/d)]$$

$$I_{cr} = (b_f - b_w)h_f^3/12 + b_w c^3/3 + (b_f - b_w)h_f(c - h_f/2)^2 + nA_s(d - c)^2$$

With compression steel:

$$k = \sqrt{[n\rho_w + (n-1)\rho'_w + \zeta(h_f/d)]^2 + 2[n\rho_w + (n-1)\rho'_w(d'/d) + 0.5\zeta(h_f/d)^2]} - [n\rho_w + (n-1)\rho'_w + \zeta(h_f/d)]$$

$$I_{cr} = (b_f - b_w)h_f^3/12 + b_w c^3/3 + (b_f - b_w)h_f(c - h_f/2)^2 + nA_s(d - c)^2 + (n-1)A'_s(c - d')^2$$

(b) Flanged Sections

Fig. 4.3.2.1—Moments of inertia of uncracked and cracked transformed sections.

or in a slightly rearranged form as

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 (I_g - I_{cr}) \leq I_g \quad (4.3.2.1b)$$

where M_{cr} is the cracking moment; M_a is the maximum (unfactored) service load moment at the stage for which deflections are being considered; I_g is the gross moment of inertia of section; and I_{cr} is the moment of inertia of cracked transformed section.

The value I_e provides a transition between the upper and the lower bounds of I_g and I_{cr} , respectively, as a function of

the level of cracking, expressed as M_{cr}/M_a . The use of I_e as a weighted average of I_g and I_{cr} is assumed to give deflection values close to those obtained using a bilinear load-deflection response after cracking. Details for computing the cracked moment of inertia I_{cr} are provided in Fig. 4.3.2.1.

Deflections have traditionally been computed for each load level using either form of Eq. (4.3.2.1a) or (4.3.2.1b), such as dead load and dead load plus live load. Thus, the incremental deflection such as that due to live load alone is computed as the difference between these values at the two load levels. Another school of thought, however, assumes the structural member has been preloaded under the full (dead plus live) service load such that subsequent calculations of

both the dead and live load deflection are computed using the same value of I_e corresponding to full dead plus live load (Grossman 1981). The latter approach is generally assumed to be more realistic for slab construction and provides a conservative estimate of long-term deflection. The former approach, on the other hand, provides a conservative estimate of immediate deflection from live load, although this requirement typically does not control design.

The value of I_e may be determined using M_a at the support for cantilevers, and at the midspan for simple spans. Equation (4.3.2.1a) shows that I_e is an interpolation between the well-defined limits of I_g and I_{cr} . This equation was recommended by ACI Committee 435 (1966) and has been used in ACI 318 since 1971 and the AASHTO Bridge Design Specifications since 1973. Detailed numerical examples using this method for simple and continuous beams and unshored and shored composite beams are available in Branson (1977). Textbooks on reinforced concrete such as those by Wang and Salmon (1992) and Nawy (2009) also provide an extensive treatment of the subject.

Heavily reinforced members will have an I_e approximately equal to I_{cr} , which may in some cases (flanged members) be larger than I_g of the concrete section alone. For most practical cases, the calculated I_e will be less than I_g . In both cases, I_e should be taken as the lesser of the calculated value or I_g when designing for deflection control, unless a justification can be made for rigorous transformed section computations.

Equation (4.3.2.1a) or (4.3.2.1b) is intended to represent an averaged value of I_e that accounts for the variation in stiffness along the member. Replacing the cubic power in Eq. (4.3.2.1a) with a fourth power is meant to give the effective moment of inertia at a section and is useful when integrating curvatures at multiple sections along the span length.

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^4 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^4 \right] I_{cr} \leq I_g \quad (4.3.2.1c)$$

Branson's equation (Eq. (4.3.2.1a) or (4.3.2.1b)) is an empirically derived formula that was calibrated to work well for typical steel-reinforced concrete beams having reinforcing ratios greater than approximately 1 percent (Bischoff 2005). However, this equation underestimates deflection for most members reinforced with FRP bars (ACI 440.1R), for certain types of slender tilt-up wall panels (Bischoff and Scanlon 2007), and for steel-reinforced concrete beams and slabs with reinforcing ratios below 1 percent (Bischoff 2005; Gilbert 2006). ACI 440.1R-06 recommends applying a correction factor to Eq. (4.3.2.1a) or (4.3.2.1b) for computing deflection of FRP-reinforced concrete members, while the problem with lightly reinforced slabs has not yet been addressed by ACI 318. However, ACI 318 recognizes the problem with Branson's equation for tilt-up wall panels, and now requires use of a bilinear equation taken from the *Uniform Building Code* (ICBO 1997).

The accuracy of Branson's equation for I_e is dependent on the ratio of I_g to I_{cr} (in addition to the level of loading with respect to the cracking moment), and only works well for

flexural members when $I_g / I_{cr} < 3$ (corresponding to steel reinforced concrete members with a reinforcing ratio greater than approximately 1 percent). Member stiffness using Branson's approach is overestimated when $I_g / I_{cr} > 3$. The ratio I_g / I_{cr} for lightly reinforced concrete slabs (0.2 percent $< \rho < 0.6$ percent) can be as high as 12, varies between 7 to 25 for FRP-reinforced concrete beams and slabs, and ranges from 15 to 25 for walls with a central layer of reinforcement (Bischoff and Scanlon 2007). Hence, Branson's equation (Eq. (4.3.2.1a) or (4.3.2.1b)) can significantly underestimate deflection in these three cases.

Both the member (cubic power) and section (fourth power) based expressions for I_e were calibrated from tests (Branson 1963) of uniformly loaded beams where the value of I_e was within 12 percent of I_{cr} for nearly 85 percent of the specimens assessed. The assumed level of service load ranged from two to four times the cracking moment. There was practically no difference (3 to 6 percent at most) between the member I_e and section based I_e for nearly all the 56 specimens considered. Hence, it is difficult to draw any definite conclusions about the applicability of using either equation for other types of members and loading conditions.

4.3.2.2 Bischoff's I_e —Bischoff (2005, 2007) applied basic concepts of tension stiffening to flexural members to develop a rational expression for I_e given by

$$I_e = \frac{I_{cr}}{1 - \beta \left(\frac{M_{cr}}{M_a} \right) \left(1 - \frac{I_{cr}}{I_g} \right)} = \frac{I_{cr}}{1 - \left(\frac{M_{cr}}{M_a} \right)^2 \left(1 - \frac{I_{cr}}{I_g} \right)} \leq I_g \quad (4.3.2.2a)$$

with the tension stiffening factor $\beta = M_{cr}/M_a$ that varies between 1 (at cracking) and 0 when $M_a \gg M_{cr}$. Setting β equal to 1 (instead of M_{cr}/M_a) gives a bilinear response with constant tension stiffening that represents an upper bound on member stiffness as shown in Fig. 4.3.2.2a, whereas β set equal to 0 (instead of M_{cr}/M_a) provides a lower bound on member stiffness with no tension stiffening to give the I_{cr} response. Bischoff's expression for I_e gives a reasonable estimate of deflection provided the correct cracking moment is used (Scanlon and Bischoff 2008).

Equation (4.3.2.2a) can be rearranged into Eq. (4.3.2.2b) that incorporates similar ratios to Eq. (4.3.2.1a). However, comparing the arrangement of terms in the two equations highlights the fundamental difference between the two approaches. Equation (4.3.2.2b) uses a weighted average of flexibilities whereas Eq. (4.3.2.1a) uses a weighted average of stiffnesses. Because deflection is proportional to flexibility, Bischoff's approach (Eq. (4.3.2.2a) or (4.3.2.2b)) is applicable over a wider range of situations than Branson's approach (Eq. (4.3.2.1a)). The other primary difference is that the cubic power in Branson's equation (Eq. (4.3.2.1a)) has been replaced by a squared power in Bischoff's equation (Eq. (4.3.2.2a and b)) (refer to Bischoff [2005] for motivations for this).

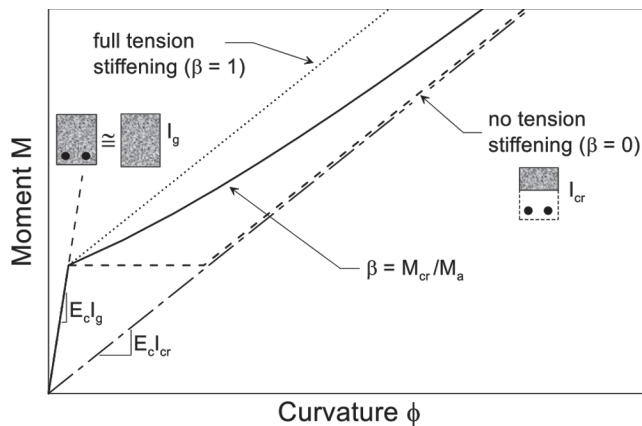


Fig. 4.3.2.2a—Moment-curvature response for different tension stiffening values with Eq. (4.3.2.2a).

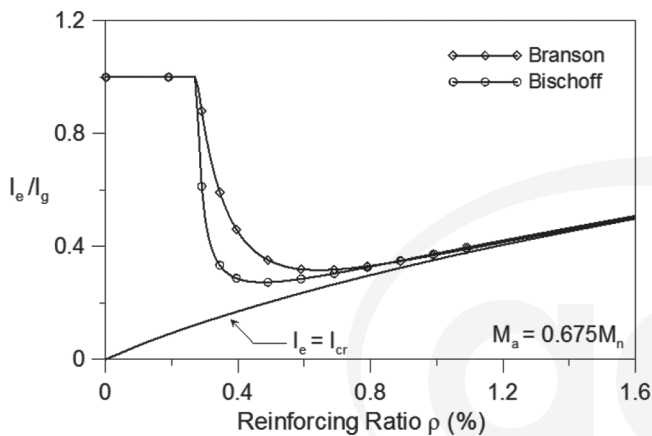


Fig. 4.3.2.2b—Comparison of I_e using Branson and Bischoff's approach with M_{cr} .

$$\frac{1}{I_e} = \left(\frac{M_{cr}}{M_a} \right)^2 \frac{1}{I_g} + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^2 \right] \frac{1}{I_{cr}} \geq \frac{1}{I_g} \quad (4.3.2.2b)$$

Bischoff's I_e (defined by Eq. (4.3.2.2a) or (4.3.2.2b)) is simple and works equally well for concrete beams and slabs reinforced with either steel or FRP bars over a wide range of reinforcing ratios (Bischoff and Scanlon 2007). Bischoff's equation (Eq. (4.3.2.2a)) and Branson's equation (Eq. (4.3.2.1a)) give similar computed values of deflection for beams reinforced with steel at reinforcing ratios greater than 1 percent (where tension stiffening is not significant) as shown in Fig. 4.3.2.2b. Differences only become significant for flexural members with an I_g/I_{cr} greater than 3 (typically for flexural members with $\rho < 1$ percent for steel-reinforced concrete). For the reasons given previously, Bischoff's equation (Eq. (4.3.2.2a)) is recommended for computing deflection.

4.3.2.3 Shrinkage restraint and preloading—Restraint to shrinkage from either the internal reinforcement or adjoining members and supports induces tensile stresses in the concrete that decrease the cracking moment and reduce the flexural

stiffness of the member. Decreasing the cracking moment has a significant effect on computed values of deflection for lightly reinforced members such as slabs. Deflection of moderate to more heavily reinforced members is not significantly affected at full-service load as the effective moment of inertia I_e is closely approximated by I_{cr} at the full service level of load for these members (where $M_a/M_{cr} > 3$).

CSA A23.3 uses a reduced cracking moment for computing deflection (based on the work of Scanlon and Murray [1982]) whereas AS 3600 reduces the cracking moment with a computed value of the shrinkage restraint stress (Gilbert 1999). BS EN 1992-1-1 provisions for computing deflection can also account for a reduced cracking moment. The use of a lower cracking moment also accounts for cases where the service load moment is only slightly less than the unrestrained cracking moment as factors such as shrinkage and temperature can still cause the section to crack over time.

To account for this reduction in cracking load, Gilbert (1999) recommends reducing the modulus of rupture f_r by an amount equal to the restraint stress $f_{res} = 2.5\rho E_s \epsilon_{sh}/(1 + 50\rho)$, where ϵ_{sh} is the shrinkage strain in the concrete. AS 3600 uses a modified version of Gilbert's (1999) expression that includes the effect of compression steel. For design, Scanlon and Bischoff (2008) recommend using a reduced cracking moment equal to two-thirds the value of M_{cr} (computed using Eq. (4.3.2) with Eq. (3.3.2a)) when computing deflections for steel-reinforced concrete using Eq. (4.3.2.2a) for I_e . ACI 318 already uses $(2/3)M_{cr}$ for computing out-of-plane deflection of slender walls. For FRP-reinforced concrete, the cracking moment should only be reduced by 20 percent to reflect a lower degree of internal restraint when using the more flexible FRP reinforcement (Bischoff and Gross 2011b). Further reductions in the cracking moment may be required to account for axial tension induced by restraint to shrinkage from the supports and adjacent parts of the structure. For shrinkage stresses approaching f_r , or other causes of large net axial forces, it may be more appropriate and numerically stable to modify M_a instead of M_{cr} .

Preloading from construction loads prior to installation of the nonstructural elements can cause additional cracking that reduces the member stiffness and increases deflection arising from the sustained portion of loading. Scanlon and Bischoff (2008) recommend computing the immediate deflection under sustained loads with an effective moment of inertia corresponding to the full (dead plus live) service load moment to account for the reduced stiffness. This simplifies the calculation procedure considerably as only one value of I_e is now considered in deflection calculations.

4.3.2.4 Variation in stiffness along member length—Equation (4.3.2.2a) or (4.3.2.2b) effectively provides a section-based expression for the effective moment of inertia. For members with uniform cross section and typical reinforcement patterns, using either of these equations based on the service load moment at the critical section (where the moment is greatest) will provide a conservative estimate of deflection as the member stiffness is lowest at this location.

If warranted, a more accurate calculation of deflection can be carried out by integrating curvatures along the length of

the member based on the calculated I_e value at each section (Bischoff and Gross 2011a). Using this approach gives a modified expression for Eq. (4.3.2.2a) that includes an additional factor γ to account for the variation in stiffness along the length of member. ACI 440.1R has adopted this approach for design of FRP-reinforced concrete.

$$I_e = \frac{I_{cr}}{1 - \gamma \left(\frac{M_{cr}}{M_a} \right)^2 \left(1 - \frac{I_{cr}}{I_g} \right)} \leq I_g \quad (4.3.2.4)$$

The integration factor $\gamma \geq 1$ accounts for the length of the uncracked regions of the member in addition to the change in stiffness along the cracked regions. The value of γ depends on the type of loading, boundary conditions, and reinforcement pattern. Earlier work by Al-Zaid et al. (1991) has also recognized that the I_e value can be affected by the type of loading (concentrated or distributed) on the member. Expressions for the factor γ in Table 4.3.2.4, provided for different types of beams and loading cases, can be used in place of numerical integration of curvature. Closed-form solutions for deflection of continuous members with moments at the end supports are not readily available and would typically require numerical integration unless simplifying assumptions are made.

The increase in member stiffness with Eq. (4.3.2.4) depends on the type of load, boundary conditions, reinforcing pattern, reinforcing material, reinforcement ratio, and level of service load relative to the cracking moment. Setting $\gamma = 1$ results in Eq. (4.3.2.2a), which provides a reasonably conservative estimate of deflection in many cases, particularly for steel reinforced concrete members subjected to a uniformly distributed load (Bischoff and Gross 2011a).

4.3.2.5 Continuous members—For continuous beams and one-way slabs, ACI 318 stipulates that I_e may be taken as the average of values obtained for the critical positive and negative moment sections. For prismatic members, I_e may be taken as the value obtained at midspan for continuous spans. The use of midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity, including the effect of cracking, has a dominant effect on deflections (ACI Committee 435 1978).

ACI 318 suggests a simple averaging of I_e values for the positive and negative moment section if the designer chooses to average the effective moment of inertia I_e .

$$I_e = 0.50I_{e(m)} + 0.25(I_{e(1)} + I_{e(2)}) \quad (4.3.2.5a)$$

where the subscripts m and 1 and 2 refer to midspan and the two beam ends, respectively.

Improved results for continuous prismatic members can, however, be obtained using a weighted average, as presented in the following equations.

For beams continuous on both ends

$$I_e = 0.70I_{e(m)} + 0.15(I_{e(1)} + I_{e(2)}) \quad (4.3.2.5b)$$

For beams continuous on one end only

$$I_e = 0.85I_{e(m)} + 0.15I_{e(1)} \quad (4.3.2.5c)$$

Work by Bischoff (2007), however, indicates that deflection of continuous members can be underestimated when using a weighted average of I_e , for cases where the member stiffness is greater at the end supports than at midspan. Once again, deflection would be reflected more correctly by taking a weighted average of the inverse stiffness (flexibility) values ($1/E_e I_e$). Reasonable values of deflection are obtained by using the midspan value of the effective moment of inertia provided that: 1) the end support moments do not exceed twice the midspan value; and 2) the value of I_e at the end supports is between one-half and three times the value at midspan.

When I_e is calculated as indicated in the previous discussion, the deflection can be obtained using a generalized form of the elastic equation such as Eq. (4.3), or any other approach used to compute elastic deflection such as the moment-area method or conjugate beam method. Numerical integration procedures can also be used. The moment-curvature (rotation) response of the member is taken into consideration with each approach.

4.3.3 Section-based moment-curvature approach—The calculation methods for member deflection described in 4.3.1 and 4.3.2 rely on the use of an average member stiffness between the supports. For uncracked members (4.3.1), the gross moment of inertia, I_g , is used, while an average effective moment of inertia, I_e , can be used for cracked members (4.3.2). In the latter case, the parameter I_e can account for variations in the amount of cracking along the member length as well as the tension stiffening of concrete between the cracks by using integration factors or other simplifications to represent the variation in bending moment. In general, the mathematical complexity in these techniques is intentionally low, using linear-elastic material properties to facilitate hand calculations or simple computer-based techniques, such as spreadsheets. However, with the availability of personal computers in engineering design offices, more accurate analytical procedures can be routinely considered for the calculation of deflections under complex loading scenarios, time dependent effects (4.4.3.2), or to directly consider the nonlinear material behavior.

A numerical section-based approach can directly consider the compressive stress-strain response characteristics of different concrete types within a cross section as well as the calculation of deflections before and after yielding of the reinforcement at different locations along the member. Sectional-based analytical methods can also be used to estimate the influence of nonprismatic geometry or variable reinforcement detailing on the overall deflection of members.

Numerical modeling is performed by first determining the nonlinear moment-curvature response of representative cross sections along the member span that satisfy compatibility of axial strains and equilibrium of section forces. The member deflection is then computed by numerically integrating the curvatures (Park and Paulay 1975; Collins and Mitchell 1997; Ghali et al. 2011). Such calculations can be performed