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Control of Deflection in Concrete Structures

Reported by ACI Committee 435

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This report presents a consolidated treatment of initial and time-dependent deflection of reinforced and prestressed concrete elements such as simple and continuous beams and one-way and two-way slab systems. It presents the state of the art in practice on deflection as well as analytical methods for computer use in deflection evaluation. The introductory chapter and four main chapters are relatively independent in content. Topics include "Deflection of Reinforced Concrete One-way Flexural Members," "Deflection of Two-way Slab Systems," and "Reducing Deflection of Concrete Members."

One or two detailed computational examples for evaluating the deflection of beams and two-way action slabs and plates are given at the end of Chapters 2, 3, and 4. These computations are in accordance with the current ACI- or PCI-accepted methods of design for deflection.

Keywords: beams; camber; code; concrete; compressive strength; cracking; creep; curvature; deflection; high-strength concrete; loss of prestress; modulus of rupture; moments of inertia; plates; prestressing; pretensioned; post-tensioned; reducing deflection; reinforcement; serviceability; shrinkage; slabs; strains; stresses; tendons; tensile strength; time-dependent deflection.

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CHAPTER 1—INTRODUCTION

Design for serviceability is central to the work of structural engineers and code-writing bodies. It is also essential to users of the structures designed. Increased use of high-strength concrete with reinforcing bars and prestressed reinforcement, coupled with more precise computer-aided limit-state serviceability designs, has resulted in lighter and more material-efficient structural elements and systems. This in turn has necessitated better control of short-term and long-term behavior of concrete structures at service loads.

This report presents consolidated treatment of initial and time-dependent deflection of reinforced and prestressed concrete elements such as simple and continuous beams and one-way and two-way slab systems. It presents current engineering practice in design for control of deformation and deflection of concrete elements and includes methods presented in "Building Code Requirements for Reinforced Concrete (ACI 318)" plus selected other published approaches suitable for computer use in deflection computation. Design examples are given at the end of each chapter showing how to evaluate deflection (mainly under static loading) and thus control it through adequate design for serviceability. These step-by-step examples as well as the general thrust of the report are intended for the non-seasoned practitioner who can, in a single document, be familiarized with the major state of practice approaches in buildings as well as additional condensed coverage of analytical methods suitable for computer use in deflection evaluation. The examples apply AC1 318 requirements in conjunction with PCI methods where applicable.

The report replaces several reports of this committee in order to reflect more recent state of the art in design. These reports include ACI 435.2R, "Deflection of Reinforced Concrete Flexural Members," ACI 435.1R, "Deflection of Prestressed Concrete Members," ACI 435.3R, "Allowable Deflections," ACI 435.6R, "Deflection of Two-Way Reinforced Concrete Floor Systems," and 435.5R, "Deflection of Continuous Concrete Beams."

The principal causes of deflections taken into account in this report are those due to elastic deformation, flexural cracking, creep, shrinkage, temperature and their long-term effects. This document is composed of four main chapters, two to five, which are relatively independent in content. There is some repetition of information among the chapters in order to present to the design engineer a self-contained treatment on a particular design aspect of interest.

Chapter 2, "Deflection of Reinforced Concrete One-Way Flexural Members," discusses material properties and their effect on deflection, behavior of cracked and uncracked members, and time-dependent effects. It also includes the relevant code procedures and expressions for deflection computation in reinforced concrete beams. Numerical examples are included to illustrate the standard calculation methods for continuous concrete beams.

Chapter 3, "Deflection of Prestressed Concrete One-Way Members," presents aspects of material behavior pertinent to pretensioned and post-tensioned members mainly for building structures and not for bridges where more precise and detailed computer evaluations of long-term deflection behavior is necessary, such as in segmental and cable-stayed bridges. It also covers short-term and time-dependent deflection behavior and presents in detail the Branson effective moment of inertia approach (I_e) used in ACI 318. It gives in detail the PCI Multipliers Method for evaluating time-dependent effects on deflection and presents a summary of

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various other methods for long-term deflection calculations as affected by loss of prestressing. Numerical examples are given to evaluate short-term and long-term deflection in typical prestressed tee-beams.

Chapter 4, "Deflection of Two-way Slab Systems," covers the deflection behavior of both reinforced and prestressed two-way-action slabs and plates. It is a condensation of ACI Document 435.9R, "State-of-the-Art Report on Control of Two-way Slab Deflections," of this Committee. This chapter gives an overview of classical and other methods of deflection evaluation, such as the finite element method for immediate deflection computation. It also discusses approaches for determining the minimum thickness requirements for two-way slabs and plates and gives a detailed computational example for evaluating the long-term deflection of a two-way reinforced concrete slab.

Chapter 5, "Reducing Deflection of Concrete Members," gives practical and remedial guidelines for improving and controlling the deflection of reinforced and prestressed concrete elements, hence enhancing their overall long-term serviceability.

Appendix B presents a general method for calculating the strain distribution at a section considering the effects of a normal force and a moment caused by applied loads, prestressing forces, creep, and shrinkage of concrete, and relaxation of prestressing steel. The axial strain and the curvature calculated at various sections can be used to calculate displacements. This comprehensive analysis procedure is for use when the deflections are critical, when maximum accuracy in calculation is desired, or both.

The curvatures and the axial strains at sections of a continuous or simply supported member can be used to calculate the deflections and the change of length of the member using virtual work. The equations that can be used for this purpose are given in Appendix B. The appendix includes examples of the calculations and a flowchart that can be used to automate the analytical procedure.

It should be emphasized that the magnitude of actual deflection in concrete structural elements, particularly in buildings, which are the emphasis and the intent of this Report, can only be estimated within a range of 20-40 percent accuracy. This is because of the large variability in the properties of the constituent materials of these elements and the quality control exercised in their construction. Therefore, for practical considerations, the computed deflection values in the illustrative examples at the end of each chapter ought to be interpreted within this variability.

In summary, this single umbrella document gives design engineers the major tools for estimating and thereby controlling through design the expected deflection in concrete building structures. The material presented, the extensive reference lists at the end of the Report, and the design examples will help to enhance serviceability when used judiciously by the engineer. Designers, constructors, and codifying bodies can draw on the material presented in this document to achieve serviceable deflection of constructed facilities.

CHAPTER 2—DEFLECTION OF REINFORCED CONCRETE ONE-WAY FLEXURAL MEMBERS*

2.1—Notation

A = area of concrete section
A_c = effective concrete cross section after cracking, or area of concrete in compression
A_s = area of nonprestressed steel
A_{sh} = shrinkage deflection multiplier
b = width of the section
c = depth of neutral axis
C_c,(C_T) = resultant concrete compression (tension) force
C_t = creep coefficient of concrete at time t days
C_u = ultimate creep coefficient of concrete
d = distance from the extreme compression fiber to centroid of tension reinforcement
D = dead load effect
E_c = modulus of elasticity of concrete
$\overline{E}_c$ = age-adjusted modulus of elasticity of concrete at time t
E_s = modulus of elasticity of nonprestressed reinforcing steel
EI = flexural stiffness of a compression member
f_c' = specified compressive strength of concrete
f_{ct},f_t' = splitting tensile strength of concrete
f_r = modulus of rupture of concrete
f_s = stress in nonprestressed steel
f_y = specified yield strength of nonprestressed reinforcing steel
h = overall thickness of a member
I = moment of inertia of the transformed section
I_{cr} = moment of inertia of the cracked section transformed to concrete
I_e = effective moment of inertia for computation of deflection
I_g = moment of inertia for gross concrete section about centroidal axis, neglecting reinforcement
K = factor to account for support fixity and load conditions
K_e = factor to compute effective moment of inertia for continuous spans
k_{sh} = shrinkage deflection constant
$K_{(subscript)}$ = modification factors for creep and shrinkage effects
l = span length
L = live load effect
$M_{(subscript)}$ = bending moment
M_a = maximum service load moment (unfactored) at stage deflection is completed
M_{cr} = cracking moment
M_n = nominal moment strength
M_o = midspan moment of a simply supported beam
P = axial force
t = time
T_s = force in steel reinforcement
w_c = specified density of concrete
y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension
α = thermal coefficient
γ_c = creep modification factor for nonstandard conditions
γ_{sh} = shrinkage modification factor for nonstandard

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= conditions
ϕ = cross section curvature
= strength reduction factor
$\phi_{cracked}$ = curvature of a cracked member
ϕ_{mean} = mean curvature
$\phi_{uncracked}$ = curvature of an uncracked member
ϵ_c = strain in extreme compression fiber of a member
ϵ_s = strain in nonprestressed steel
$(\epsilon_{SH})_t$ = shrinkage strain of concrete at time, t days
$(\epsilon_{SH})_u$ = ultimate shrinkage strain of concrete
ρ = nonprestressed tension reinforcement ratio
ρ_b = reinforcement ratio producing balanced strain conditions
ρ' = reinforcement ratio for nonprestressed compression steel
ξ = time dependent deflection factor
δ = elastic deflection of a beam
δ_{cr} = additional deflection due to creep
δ_L = initial deflection due to live load
δ_{LT} = total long term deflection
$\delta_{L\text{-}T}$ = increase in deflection due to long-term effects
δ_{sh} = additional deflection due to shrinkage
δ_{sus} = initial deflection due to sustained load
Δ_y = y-coordinate of the centroid of the age-adjusted section, measured downward from the centroid of the transformed section at t_o
$\Delta f_c(t_o)$ = stress increment at time t_o days
$\Delta f_c(t, t_o)$ = stress increment from zero at time t_o to its full value at time t
$(\Delta\phi)_{creep}$ = additional curvature due to creep
$(\Delta\phi)_{shrinkage}$ = additional curvature due to shrinkage
λ = deflection multiplier for long term deflection
μ = multiplier to account for high-strength concrete effect on long-term deflection
η = correction factor related to the tension and compression reinforcement, CEB-FIP

2.2-General

2.2.1 *Introduction*-Wide availability of personal computers and design software, plus the use of higher strength concrete with steel reinforcement has permitted more material efficient reinforced concrete designs producing shallower sections. More prevalent use of high-strength concrete results in smaller sections, having less stiffness that can result in larger deflections. Consquently, control of short-term and long-term deflection has become more critical.

In many structures, deflection rather than stress limitation is the controlling factor. Deflection computations determine the proportioning of many of the structural system elements. Member stiffness is also a function of short-term and long-term behavior of the concrete. Hence, expressions defining the modulus of rupture, modulus of elasticity, creep, shrinkage, and temperature effects are prime parameters in predicting the deflection of reinforced concrete members.

2.2.2 *Objectives*-This chapter covers the initial and time-dependent deflections at service load levels under static conditions for one-way non-prestressed flexural concrete members. It is intended to give the designer enough basic background to design concrete elements that perform adequately under service loads, taking into account cracking and both short-term and long-term deflection effects.

While several methods are available in the literature for evaluation of deflection, this chapter concentrates on the effective moment of inertia method in *Building Code Requirements for Reinforced Concrete* (ACI 318) and the modifications introduced by ACI Committee 435. It also includes a brief presentation of several other methods that can be used for deflection estimation computations.

2.2.3 *Significance of defection observation*-The working stress method of design and analysis used prior to the 1970s limited the stress in concrete to about 45 percent of its specified compressive strength, and the stress in the steel reinforcement to less than 50 percent of its specified yield strength. Elastic analysis was applied to the design of reinforced concrete structural frames as well as the cross-section of individual members. The structural elements were proportioned to carry the highest service-level moment along the span of the member, with redistribution of moment effect often largely neglected. As a result, stiffer sections with higher reserve strength were obtained as compared to those obtained by the current ultimate strength approach (Nawy, 1990).

With the improved knowledge of material properties and behavior, emphasis has shifted to the use of high-strength concrete components, such as concretes with strengths in excess of 12,000 psi (83 MPa). Consequently, designs using load-resistance philosophy have resulted in smaller sections that are prone to smaller serviceability safety margins. As a result, prediction and control of deflections and cracking through appropriate design have become a necessary phase of design under service load conditions.

Beams and slabs are rarely built as isolated members, but are a monolithic part of an integrated system. Excessive deflection of a floor slab may cause dislocations in the partitions it supports or difficulty in leveling furniture or fixtures. Excessive deflection of a beam can damage a partition below, and excessive deflection of a spandrel beam above a window opening could crack the glass panels. In the case of roofs or open floors, such as top floors of parking garages, ponding of water can result. For these reasons, empirical deflection control criteria such as those in Table 2.3 and 2.4 are necessary.

Construction loads and procedures can have a significant effect on deflection particularly in floor slabs. Detailed discussion is presented in Chapter 4.

2.3-Material properties

The principal material parameters that influence concrete deflection are modulus of elasticity, modulus of rupture, creep, and shrinkage. The following presentation of the expressions used to define these parameters

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as recommended by ACI 318 and its Commentary (1989) and ACI Committees 435 (1978), 363 (1984), and 209 (1982).

2.3.1 *Concrete modulus of rupture*-AC1 318 (1989) recommends Eq. 2.1 for computing the modulus of rupture of concrete with different densities:

$$f_r = 7.5\ \lambda \sqrt{f_c'}, \text{ psi} \qquad (2.1)$$
$$(0.623\ \lambda \sqrt{f_c'}, \text{ MPa})$$

where λ = 1.0 for normal density concrete [145 to 150 pcf (2325 to 2400 kg/m^3)]
= 0.85 for semi low-density [ll0-145 pcf (1765 to 2325 kg/m^3)]
= 0.75 for low-density concrete [90 to 110 pcf (1445 to 1765 kg/m^3)]

Eq. 2.1 is to be used for low-density concrete when the tensile splitting strength, f_{ct}, is not specified. Otherwise, it should be modified by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ should not exceed $\sqrt{f_c'}$.

ACI Committee 435 (1978) recommended using Eq. 2.2 for computing the modulus of rupture of concrete with densities (w_c) in the range of 90 pcf (1445 kg/m^3) to 145 pcf (2325 kg/m^3). This equation yields higher values of f_r.

$$f_r = 0.65 \sqrt{w_c f_c'}, \text{psi} \qquad (2.2)$$
$$(0.013 \sqrt{w_c f_c'}, \text{ MPa})$$

The values reported by various investigators ACI 363, 1984) for the modulus of rupture of both low-density and normal density high-strength concretes [more than 6,000 psi (42 MPa)] range between 7.5 $\sqrt{f_c'}$ and 12 $\sqrt{f_c'}$. ACI 363 (1992) stipulated Eq. 2.3 for the prediction of the modulus of rupture of normal density concretes having compressive strengths of 3000 psi (21 MPa) to 12,000 psi (83 MPa).

$$f_r = 11.7 \sqrt{f_c'}, \text{ psi} \qquad (2.3)$$

The degree of scatter in results using Eq. 2.1, 2.2 and 2.3 is indicative of the uncertainties in predicting computed deflections of concrete members. The designer needs to exercise judgement in sensitive cases as to which expressions to use, considering that actual deflection values can vary between 25 to 40 percent from the calculated values.

2.3.2 *Concrete modulus of elasticity* -The modulus of elasticity is strongly influenced by the concrete materials and proportions used. An increase in the modulus of elasticity is expected with an increase in compressive strength since the slope of the ascending branch of the stress-strain diagram becomes steeper for higher-strength concretes, but at a lower rate than the compressive strength. The value of the secant modulus of elasticity for normal-strength concretes at 28 days is usually around 4 x 10^6 psi (28,000 MPa), whereas for higher-strength concretes, values in the range of 7 to 8 x 10^6 psi (49,000 to 56,000 MPa) have been reported. These higher values of the modulus can be used to reduce short-term and long-term deflection of flexural members since the compressive strength is higher, resulting in lower creep levels. Normal strength concretes are those with compressive strengths up to 6,000 psi (42 MPa) while higher strength concretes achieve strength values beyond 6,000 and up to 20,000 psi (138 MPa) at this time.

ACI 435 (1963) recommended the following expression for computing the modulus of elasticity of concretes with densities in the range of 90 pcf (1445 kg/m^3) to 155 pcf (2325 kg/m^3) based on the secant modulus at 0.45 f_c' intercept

$$E_c = 33\ w_c^{1.5} \sqrt{f_c'}, \text{ psi} \qquad (2.4)$$
$$(0.043\ w_c^{1.5} \sqrt{f_c'}, \text{ MPa})$$

For concretes in the strength range up to 6000 psi (42 MPa), the ACI 318 empirical equation for the secant modulus of concrete E_c of Eq. 2.4 is reasonably applicable. However, as the strength of concrete increases, the value of E_c could increase at a faster rate than that generated by Eq. 2.4 ($E_c = w_c^{1.5} \sqrt{f_c'}$), thereby underestimating the true E_c value. Some expressions for E_c applicable to concrete strength up to 12,000 psi (83 MPa) are available. The equation developed by Nilson (Carrasquillo, Martinez, Ngab, et al, 1981, 1982) for normal-weight concrete of strengths up to 12,000 psi (83 MPa) and light-weight concrete up to 9000 psi (62 MPa) is:

$$E_c = (40{,}000 \sqrt{f_c'} + 1{,}000{,}000) \left(\frac{w_c}{145}\right)^{1.5}, \text{ psi} \quad (2.5)$$
$$(3.32 \sqrt{f_c'} + 6895)\left(\frac{w_c}{2320}\right)^{1.5}, \text{ MPa}$$

where w_c is the unit weight of the hardened concrete in pcf, being 145 lb/ft^3 for normal-weight concrete and 100-120 lb/ft^3 for sand-light weight concrete. Other investigations report that as f_c' approaches 12,000 psi (83 MPa) for normal-weight concrete and less for lightweight concrete, Eq. 2.5 can underestimate the actual value of E_c. Deviations from predicted values are highly sensitive to properties of the coarse aggregate such as size, porosity, and hardness.

Researchers have proposed several empirical equations for predicting the elastic modulus of higher strength concrete (Teychenne et al, 1978; Ahmad et al, 1982; Martinez, et al, 1982). ACI 363 (1984) recommended the following modified expression of Eq. 2.5 for normal-weight concrete:

$$E_c = 40{,}000 \sqrt{f_c'} + 1{,}000{,}000, \text{ psi} \qquad (2.6)$$

Using these expressions, the designer can predict a modulus of elasticity value in the range of 5.0 to 5.7 x 10^6 psi (35 to 39 x 10^3 MPa) for concrete design strength of up to 12,000 psi (84 MPa) depending on the expression used.

When very high-strength concrete [20,000 psi (140 MPa) or higher] is used in major structures or when deformation is critical, it is advisable to determine the stress-strain relationship from actual cylinder compression test results. In this manner, the deduced secant modulus value of E_c at an $f_c = 0.45\ f_c'$ intercept can be used to predict more accurately the value of E_c for the particular mix and aggregate size and properties. This approach is advisable until an acceptable expression is

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Table 2.1 — Creep and shrinkage ratios from age 60 days to the indicated concrete age (Branson, 1977)

Creep, shrinkage ratios	Concrete age					
	2 months	3 months	6 months	1 year	2 years	> 5 years
C_t/C_u	0.48	0.56	0.68	0.77	0.84	1.00
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$ -M.C.	0.46	0.60	0.77	0.88	0.94	1.00
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$ -S.C.	0.36	0.49	0.69	0.82	0.91	1.00

M.C. = Moist cured
S.C. = Steam curd

available to the designer (Nawy, 1990).

2.3.3 *Steel reinforcement modulus of elasticity*-AC1 318 specifies using the value E_s = 29 x 10^6 psi (200 x 10^6 MPa) for the modulus of elasticity of nonprestressed reinforcing steel.

2.3.4 *Concrete creep and shrinkage*-Deflections are also a function of the age of concrete at the time of loading due to the long-term effects of shrinkage and creep which significantly increase with time. ACI 318-89 does not recommend values for concrete ultimate creep coefficient C_u and ultimate shrinkage strain $(\epsilon_{sh})_u$. However, they can be evaluated from several equations available in the literature (ACI 209, 1982; Bazant et al, 1980; Branson, 1977)ACI 435 (1978) suggested that the average values for C_u and $(\epsilon_{sh})_u$ can be estimated as 1.60 and 400 x 10^{-6}, respectively. These values correspond to the following conditions:

- 70 percent average relative humidity
- age of loading, 20 days for both moist and steam cured concrete
- minimum thickness of component, 6 in. (152 mm)

Table 2.1 includes creep and shrinkage ratios at different times after loading.

ACI 209 (1971, 1982,1992) recommended a time-dependent model for creep and shrinkage under standard conditions as developed by Branson, Christianson, and Kripanarayanan (1971,1977). The term “standard conditions” is defined for a number of variables related to material properties, the ambient temperature, humidity, and size of members. Except for age of concrete at load application, the standard conditions for both creep and shrinkage are

a) Age of concrete at load applications = 3 days (steam), 7 days (moist)
b) Ambient relative humidity = 40 percent
c) Minimum member thickness = 6 in. (150 mm)
d) Concrete consistency = 3 in. (75 mm)
e) Fine aggregate content = 50 percent
f) Air content = 6 percent

The coefficient for creep at time t (days) after load application, is given by the following expression:

$$C_t = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right) C_u \qquad (2.7)$$

where C_u = 2.35 γ_{CR}
$\gamma_{CR} = K_h^c K_d^c K_s^c K_f^c K_{ac}^c K_{to}^c$ = 1 for standard conditions.

Each K coefficient is a correction factor for conditions other than standard as follows:

K_h^c = relative humidity factor
K_d^c = minimum member thickness factor
K_s^c = concrete consistency factor
K_f^c = fine aggregate content factor
K_{ac}^c = air content factor
K_{to}^c = age of concrete at load applications factor

Graphic representations and general equations for the modification factors (K-values) for nonstandard conditions are given in Fig. 2.1 (Meyers et al, 1983).

For moist-cured concrete, the free shrinkage strain which occurs at any time t in days, after 7 days from placing the concrete

$$(\epsilon_{SH})_t = \left(\frac{t}{35 + t}\right)(\epsilon_{SH})_u \qquad (2.8)$$

and for steam cured concrete, the shrinkage strain at any time t in days, after l-3 days from placing the concrete

$$(\epsilon_{SH})_t = \left(\frac{t}{55 + t}\right)(\epsilon_{SH})_u \qquad (2.9)$$

where $(\epsilon_{SH})_{u\ Max}$ = 780 x 10^{-6} γ_{sh}
λ_{sh} = $K_h^s K_d^s K_f^s K_b^s K_{ac}^s$
= 1 for standard conditions

Each K coefficient is a correction factor for other than standard conditions. All coefficients are the same as defined for creep except K_B^s, which is a coefficient for cement content. Graphic representation and general equations for the modification factors for nonstandard conditions are given in Fig. 2.2 (Meyers et al, 1983). The above procedure, using standard and correction equations and extensive experimental comparisons, is detailed in Branson (1977).

Limited information is available on the shrinkage behavior of high-strength concrete [higher than 6,000 psi (41 MPa)], but a relatively high initial rate of shrinkage has been reported (Swamy et al, 1973). However, after drying for 180 days the difference between the shrinkage of high-strength concrete and lower-strength concrete seems to become minor. Nagataki (1978) reported that the shrinkage of high-strength concrete containing high-range water reducers was less than for lower-strength concrete.

On the other hand, a significant difference was reported for the ultimate creep coefficient between high-

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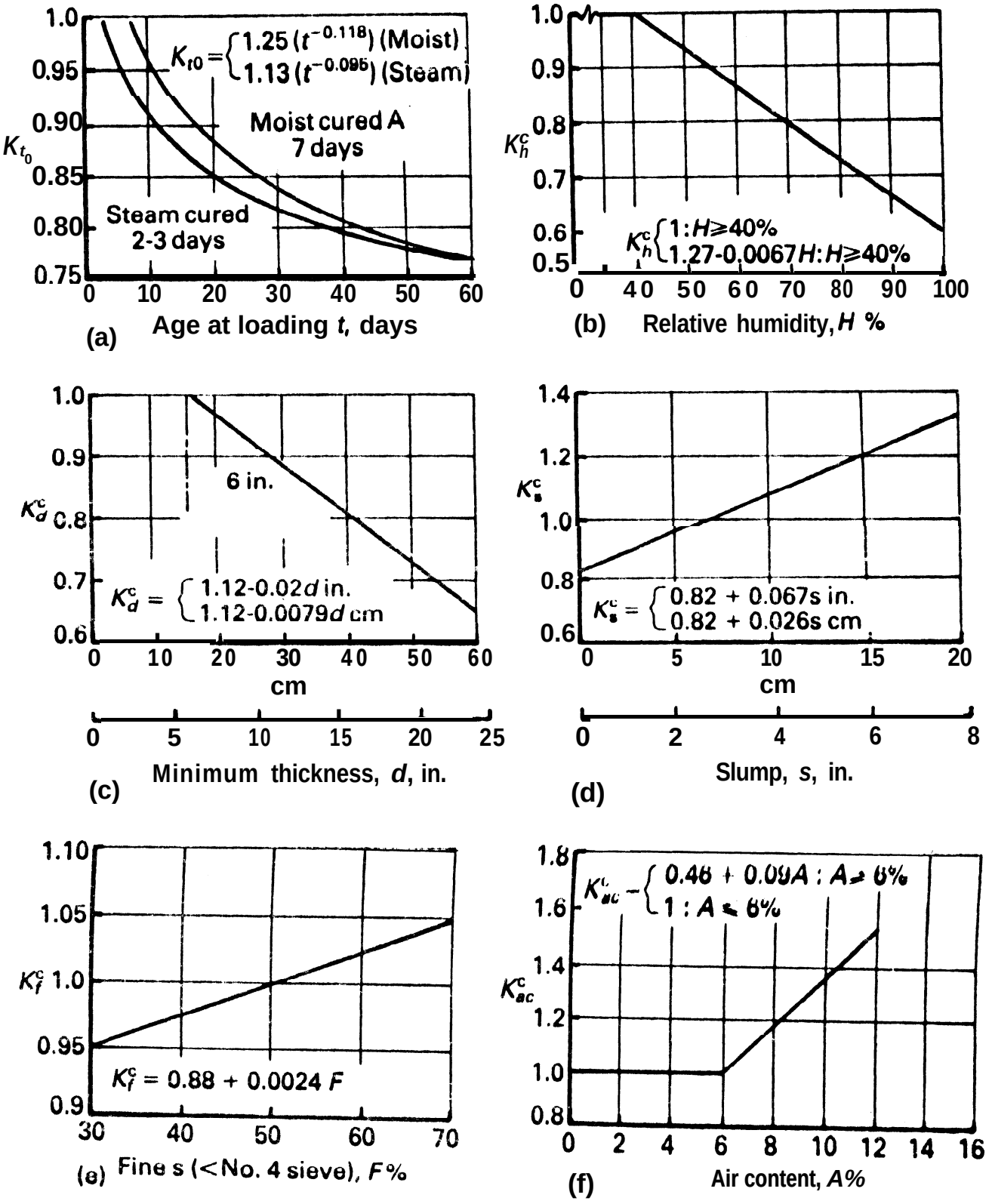


Fig. 2.1-Creep correction factors for nonstandard conditions, ACI 209 method (Meyers, 1983)

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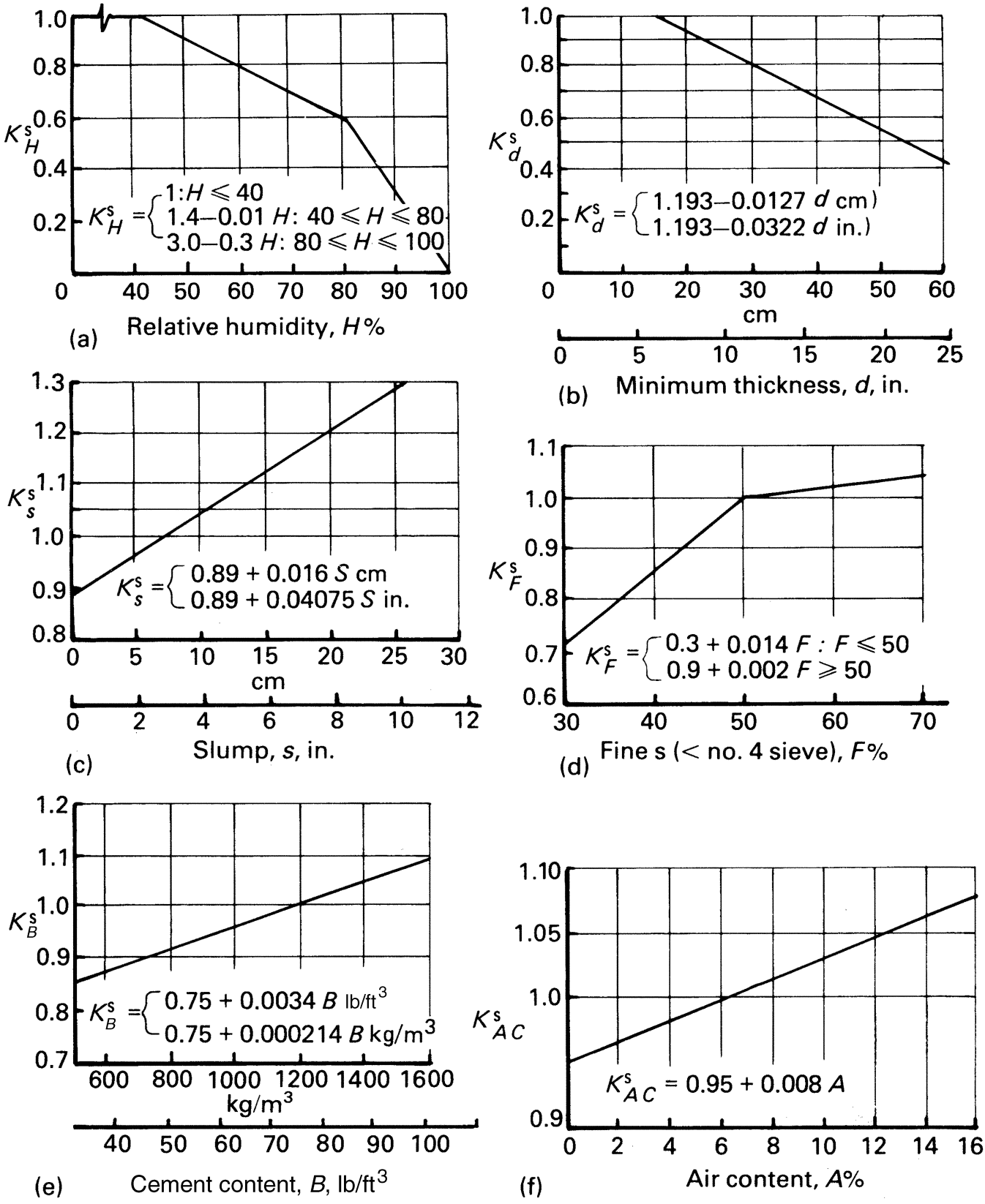


Fig. 2.2—Shrinkage correction factors for nonstandard conditions, ACI 209 method (Meyers, 1983)

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Table 2.2-Recommended tension reinforcement ratios for nonprestressed one-way members so that deflections will normally be within acceptable limits (ACI 435, 1978)

Members	Cross section	Normal weight concrete	Lightweight concrete
Not supporting or not attached to nonstructural elements likely to be damaged by large deflections	Rectangular "T" or box	$\rho \leq 35$ percent ρ_b $\rho_w \leq 40$ percent ρ_b	$\rho \leq 30$ percent ρ_b $\rho_w \leq 35$ percent ρ_b
Supporting or attached to nonstructural elements likely to be damaged by large deflections	Rectangular "T" or box	$\rho \leq 25$ percent ρ_b $\rho_w \leq 30$ percent ρ_b	$\rho \leq 20$ percent ρ_b $\rho_w \leq 25$ percent ρ_b

For continuous members, the positive region steel ratios only may be used. ρ_b: Refers to the balanced steel ratio based on ultimate strength.

Table 2.3-Minimum thickness of nonprestressed beams and one-way slabs unless deflections are computed (ACI 318, 1989)

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	l/24	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

ℓ = Span length

Values given shall be used directly for members with normal weight concrete (w_c = 145 pcf) and grade 60 reinforcement. For other conditions. the values shall be modified as follows:

a) For structural lightweight concrete having unit weights in the range 90-120 lb per cu ft. the values shall be multiplied by (1.65 - 0.005 w_c) but not less than 1.09, where w_c is the unit weight in lb per cu ft.

b) For f_y other than 60,000 psi, the values shall be multiplied by (0.4 + f_y/100,000).

strength concrete and its normal strength counterpart. The ratio of creep strain to initial elastic strain under sustained axial compression, for high-strength concrete, may be as low as one half that generally associated with low-strength concrete (Ngab et al, 1981; Nilson, 1985).

2.4-Control of deflection

Deflection of one-way nonprestressed concrete flexural members is controlled by reinforcement ratio limitations, minimum thickness requirements, and span/deflection ratio limitations.

2.4.1 *Tension steel reinforcement ratio limitations*-One method to minimize deflection of a concrete member in flexure is by using a relatively small reinforcement ratio. Limiting values of ratio p, ranging from $0.25\rho_b$ to $0.40\rho_b$ are recommended by ACI 435 (1978), as shown in Table 2.2. Other methods of deflection reduction are presented in Chapter 5 of this report.

2.4.2 *Minimum thickness limitations*-Deflections of beams and one way slabs supporting usual loads in buildings, where deflections are not of concern, are normally satisfactory when the minimum thickness provisions in Table 2.3 are met or exceeded. This table (ACI 318, 1989) applies only to members that are not supporting or not attached to partitions or other construction likely to be damaged by excessive deflections. Values in Table 2.3 have been modified by ACI 435 (1978) and expanded in Table 2.4 to include members that are supporting or attached to non-structural elements likely to be damaged by excessive deflections. The thickness may be decreased when computed deflections are shown to be satisfactory. Based on a large number of computer studies, Grossman (1981, 1987) developed a simplified expression for the minimum thickness to satisfy serviceability requirements (Eq. 4.17, Chapter 4).

2.4.3 *Computed deflection limitations*--The allowable computed deflections specified in ACI 318 for one-way systems are given in Table 2.5, where the span-deflection ratios provide for a simple set of allowable deflections. Where excessive deflection may cause damage to nonstructural or other structural elements, only that part of the deflection occurring after the construction of the nonstructural elements, such as partitions, needs to be considered. The most stringent span-deflection limit of l/480 in Table 2.5 is an example of such a case. Where excessive deflection may result in a functional problem, such as visual sagging or ponding of water, the total deflection should be considered.

2.5-Short-term deflection

2.5.1 *Untracked members*-Gross moment of inertia Ig -When the maximum flexural moment at service load in

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Table 2.4-Minimum thickness of beams and one-way slabs used in roof and floor construction (ACI 435, 1978)

Member	Members not supporting or not attached to nonstructural elements likely to be damaged by large deflections				Members supporting or attached to nonstructural elements likely to be damaged by large deflection			
	Simply supported	One end continuous	Both ends continuous	Cantilever	Simply supported	One end continuous	Both ends continuous	Cantilever
Roof slab	*l*/22	*l*/28	*l*/35	*l*/9	*l*/14	*l*/18	*l*/22	*l*/5.5
Floor slab, and roof beam or ribbed roof slab	*l*/18	*l*/23	*l*/28	*l*/7	*l*/12	*l*/15	*l*/19	*l*/5
Floor beam or ribbed floor slab	*l*/14	*l*/18	*l*/21	*l*/5.5	*l*/10	*l*/13	*l*/16	*l*/4

Table 2.5-Maximum permissible computed beflections (ACI 318, 1989)

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\frac{\ell^*}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\frac{\ell}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-time deflection due to all sustained loads and the innediate deflection due to any additional live load)†	$\frac{\ell^‡}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{\ell^§}{240}$

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

† Long-time deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.2 but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§ But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

a beam or a slab causes a tensile stress less than the modulus of rupture, f_r, no flexural tension cracks develop at the tension side of the concrete element if the member is not restrained or the shrinkage and temperature tensile stresses are negligible. In such a case, the effective moment of inertia of the uncracked transformed section, I_t, is applicable for deflection computations. However, for design purposes, the gross moment of inertia, I_g, neglecting the reinforcement contribution, can be used with negligible loss of accuracy. The combination of service loads with shrinkage and temperature effects due to end restraint may cause cracking if the tensile stress in the concrete exceeds the modulus of rupture. In such cases, Section 2.5.2 applies.

The elastic deflection for noncracked members can thus be expressed in the following general form

$$\delta = K \frac{M l^2}{E_c I_g} \qquad (2.10)$$

where K is a factor that depends on support fixity and loading conditions. M is the maximum flexural moment along the span. The modulus of elasticity E_c can be obtained from Eq. 2.4 for normal-strength concrete or Eq. 2.5 for high-strength concrete.

2.5.2 *Cracked members-Effective moment of inertia* I_e -Tension cracks occur when the imposed loads cause bending moments in excess of the cracking moment, thus resulting in tensile stresses in the concrete that are higher than its modulus of rupture. The cracking moment, M_{cr}, may be computed as follows:

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad (2.11)$$

where y_t is the distance from the neutral axis to the tension face of the beam, and f_r is the modulus of rupture of the concrete, as expressed by Eq. 2.1.

Cracks develop at several sections along the member length. While the cracked moment of inertia, I_{cr}, applies to the cracked sections, the gross moment of inertia, I_g, applies to the uncracked concrete between these sections.

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Several methods have been developed to estimate the variations in stiffness caused by cracking along the span, These methods provide modification factors for the flexural rigidity EI (Yu et al, 1960), identify an effective moment of inertia (Branson, 1963), make adjustments to the curvature along the span and at critical sections (Beeby, 1968), alter the M/I ratio (CEB, 1968), or use a section-curvature incremental evaluation (Ghali, et al, 1986, 1989).

The extensively documented studies by Branson (1977, 1982, 1985) have shown that the initial deflections δ_i occurring in a beam or a slab after the maximum moment M_a has exceeded the cracking moment M_{cr} can be evaluated using an effective moment of inertia I_e instead of I_g in Eq. 2.10.

2.5.2.1 *Simply supported beams*-ACI 318-89 requires using the effective moment of inertia I_e proposed by Branson. This approach was selected as being sufficiently accurate to control deflections in reinforced and prestressed concrete structural elements. Branson's equation for the effective moment of inertia I_e, for short term deflections is as follows

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \qquad (2.12)$$

where

M_{cr} = Cracking moment
Ma = Maximum service load moment (unfactored) at the stage for which deflections are being considered
I_g = Gross moment of inertia of section
I_{cr} = Moment of inertia of cracked transformed section

The two moments of inertia I_g and I_{cr} are based on the assumption of bilinear load-deflection behavior (Fig. 3.19, Chapter 3) of cracked section. I_e provides a transition between the upper and the lower bounds of I_g and I_{cr}, respectively, as a function of the level of cracking, expressed as M_{cr}/M_a. Use of I_e as the resultant of the other two moments of inertia should essentially give deflection values close to those obtained using the bilinear approach. The cracking moment of inertia, $I_,$, can be obtained from Fig. 2.3 (PCA, 1984). Deflections should be computed for each load level using Eq. 2.12, such as dead load and dead load plus live load. Thus, the incremental deflection such as that due to live load alone, is computed as the difference between these values at the two load levels. I_e may be determined using M_a at the support for cantilevers, and at the midspan for simple spans. Eq. 2.12 shows that $I,$ is an interpolation between the well-defined limits of I_g and I_{cr}. This equation has been recommended by ACI Committee 435 since 1966 and has been used in ACI 318 since 1971, the PCI *Handbook* since 1971, and the AASHTO *Highway Bridge Specifications* since 1973. Detailed numerical examples using this method for simple and continuous beams, unshored and shored composite beams are available in Branson (1977). The textbooks by Wang and Salmon (1992), and by Nawy (1990) also have an extensive treatment of the subject.

Eq. 2.12 can also be simplified to the following form:

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a}\right)^3 (I_g - I_{cr}) \le I_g \qquad (2.13)$$

Heavily reinforced members will have an I_e approximately equal to I_{cr}, which may in some cases (flanged members) be larger than I_g of the concrete section alone. For most practical cases, the calculated I_e will be less than I_g and should be taken as such in the design for deflection control, unless a justification can be made for rigorous transformed section computations.

2.5.2.2 *Continuous beams*--For continuous members, ACI 318-89 stipulates that I_e may be taken as the average values obtained from Eq. 2.12 for the critical positive and negative moment sections. For prismatic members, I_e may be taken as the value obtained at midspan for continuous spans. The use of midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity including the effect of cracking has the dominant effect on deflections (ACI 435, 1978).

If the designer chooses to average the effective moment of inertia I_e, then according to ACI 318-89, the following expression should be used:

$$I, = 0.5\, I_{e(m)} + 0.25\, (I_{e(1)} + I_{e(2)}) \qquad (2.14)$$

where the subscripts m, 1, and 2 refer to mid-span, and the two beam ends, respectively.

Improved results for continuous prismatic members can, however, be obtained using a weighted average as presented in the following equations:

For beams continuous on both ends,

$$I_e = 0.70\, I_{e(m)} + 0.15\, (I_{e(1)} + I_{e(2)}) \qquad (2.15a)$$

For beams continuous on one end only,

$$I_e = 0.85\, I_{e(m)} + 0.15\, (I_{e(1)}) \qquad (2.15b)$$

When I_e is calculated as indiuated in the previous discussion, the deflection can be obtained using the moment-area method (Fig. 3.9, Chapter 3) taking the moment-curvature (rotation) into consideration or using numerical incremental procedures. It should be stated that the I_e value can also be affected by the type of loading on the member (Al-Zaid, 1991), i.e. whether the load is concentrated or distributed.

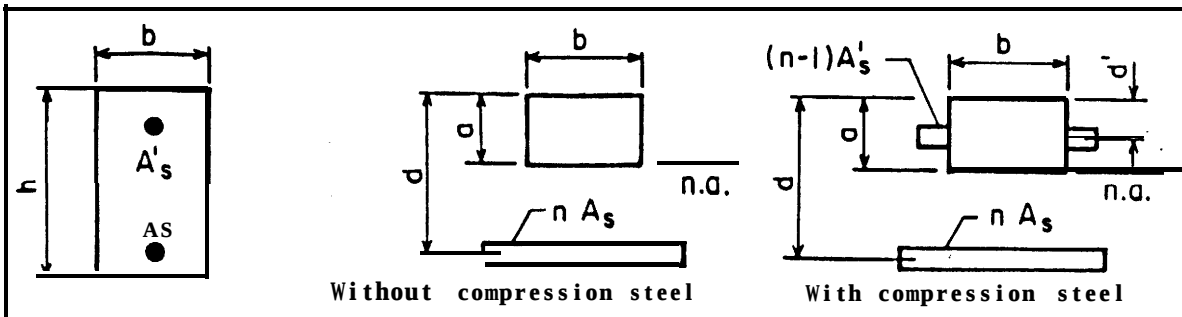
2.5.2.3 *Approximate* I_e *estimation*--An approximation of the I_e value (Grossman, 1981) without the need for calculating I_{cr} which requires a priori determination of the area of flexural reinforcement, is defined by Eq. 2.16. It gives I_e values within 20 percent of those obtained from the ACI 318 Eq. (Eq. 2.12) and could be useful for a trial check of the I_e needed for deflection control of the cracked sections with minimum reinforcement $200/fy$,

For $M_a/M_{cr} \le 1.6$:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^4 I_g \le I_g \qquad (2.16a)$$

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$B = b/(nA_s)$ $r = (n-1)A'_s/(nA_s)$ $I_g = bh^3/12$

Without compression steel

$$a = (\sqrt{2dB+1} - 1)/B$$

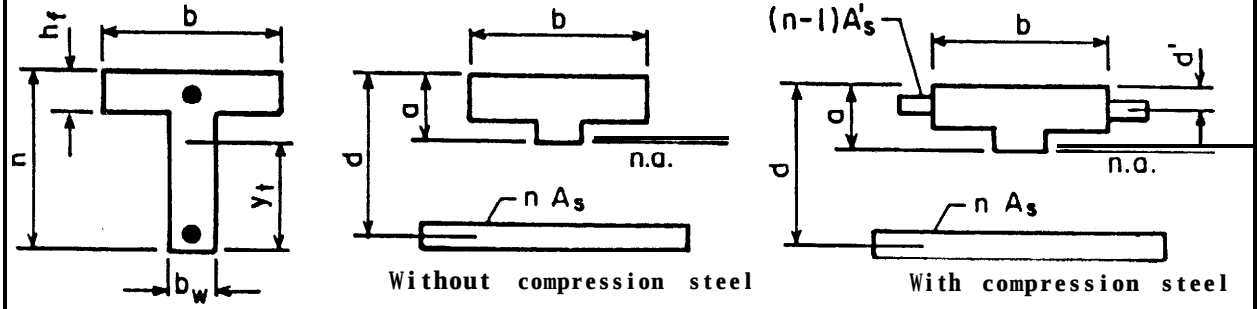
$$I_{cr} = ba^3/3 + nA_s(d-a)^2$$

With compression steel

$$a = [\sqrt{2dB(1+rd'/d) + (1+r)^2} - (1+r)]/B$$

$$I_{cr} = ba^3/3 + nA_s(d-a)^2 + (n-1)A'_s(a-d')^2$$

(a) Rectangular Sections



$$C = b_w/(nA_s),\ f = h_f(b-b_w)/(nA_s),\ y_t = h - 1/2[(b-b_w)h_f^2 + b_w h^2]/[(b-b_w)h_f + b_w h]$$

$$I_g = (b-b_w)h_f^3/12 + b_w h^3/12 + (b-b_w)h_f(h-h_f/2-y_t)^2 + b_w h(y_t-h/2)^2$$

Without compression steel

$$a = [\sqrt{C(2d+h_f f) + (1+f)^2} - (1+f)]/C$$

$$I_{cr} = (b-b_w)h_f^3/12 + b_w a^3/3 + (b-b_w)h_f(a-h_f/2)^2 + nA_s(d-a)^2$$

With compression steel

$$a = [\sqrt{C(2d+h_f f+2rd') + (f+r+1)^2} - (f+r+1)]/C$$

$$I_{cr} = (b-b_w)h_f^3/12 + b_w a^3/3 + (b-b_w)h_f(a-h_f/2)^2 + nA_s(d-a)^2 + (n-1)A'_s(a-d')^2$$

(b) Flanged Sections

Fig. 2.3-Moments of inertia of uncracked and cracked transformed sections (PCA 1984)

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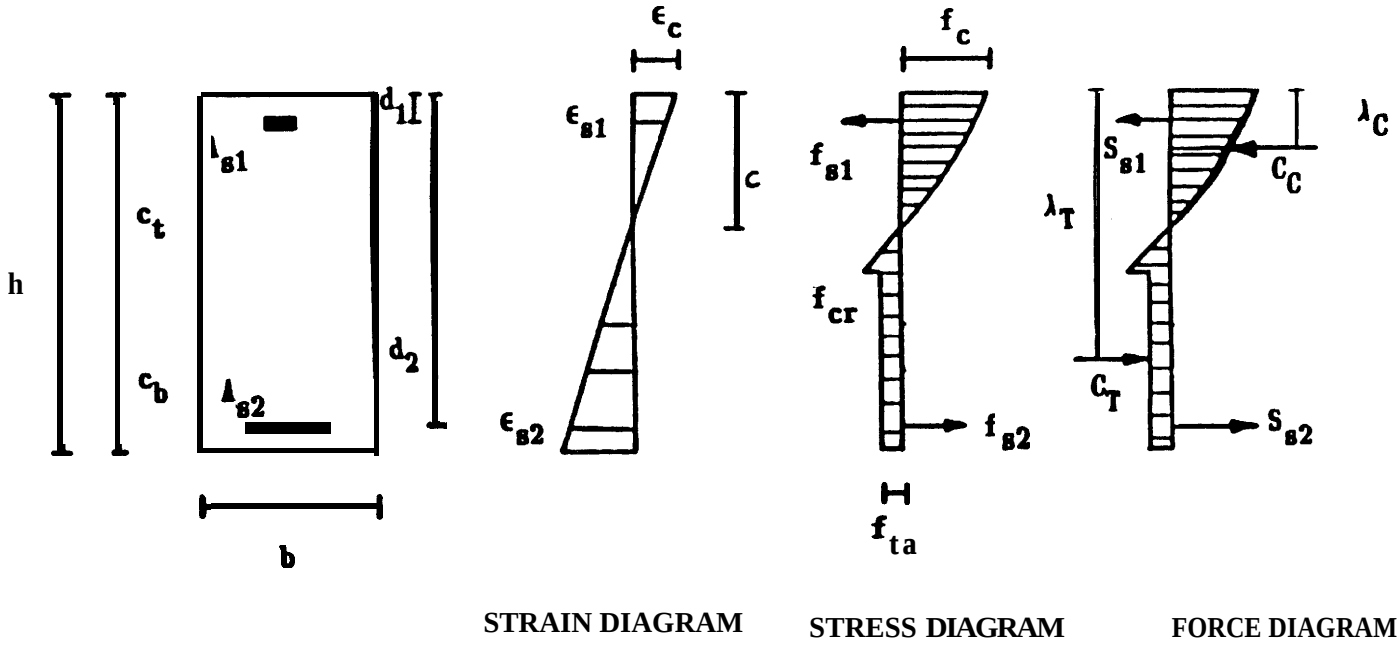


Fig. 2.4-Bending behavior of cracked sections

For $1.6 \le M_a/M_{cr} \le 10$:

$$I_e = 0.1\, K_e \left(\frac{M_a}{M_{cr}}\right) I_g \le I_g \qquad (2.16b)$$

where

$$K_e = \frac{d}{0.9h}\left|\frac{\sqrt{145/w_c}}{0.4 + \left(\frac{1.4M_a}{\phi M_n}\right)\left(\frac{f_y}{100{,}000}\right)}\right. \qquad (2.16c)$$

but, I_e computed by Eq. 2.16a and 2.16b should not be less than

$$I_e = 0.35\, K_e\, \text{I-} \qquad (2.16d)$$

nor less than the value from Eq. 2.16b, 2.16c, and 2.16d, where M_a is the maximum service moment capacity, computed for the provided reinforcement.

2.5.3 *Incremental moment-curvature method*-Today with the easy availability of personal computers, more accurate analytical procedures such as the incremental moment-curvature method become effective tools for computing deflections in structural concrete members [Park et al, 1975]. With known material parameters, a theoretical moment-curvature curve model for the cracked section can be derived (see Fig. 2.4). For a given concrete strain in the extreme compression fiber, ϵ_c, and neutral axis depth, c, the steel strains, $\epsilon_{s1}, \epsilon_{s2}, \ldots$, can be determined from the properties of similar triangles in the strain diagram. For example:

$$\epsilon_{s1} = \frac{c - d_1}{c}\,\epsilon_c \qquad (2.17)$$

The stresses, $f_{s1}, f_{s2}, \ldots$, corresponding to the strains, ϵ_{s1}, $\epsilon_{s2}, \ldots$, may be obtained from the stress-strain curves. Then, the reinforcing steel forces, $T_{s1}, T_{s2}, \ldots$, may be calculated from the steel stresses and areas. For example:

$$T_{s1} = f_{s1} * A_{s1} \qquad (2.18)$$

The distribution of concrete stress, over the compressed and tensioned parts of the section, may be obtained from the concrete stress-strain curves. For any given extreme compression fiber concrete strain, ϵ_c, the resultant concrete compression and tension forces, C_C and C, are calculated by numerically integrating the stresses over their respective areas.

Eq. 2.19 to 2.21 represent the force equilibrium, the moment, and the curvature equations of a cracked section, respectively:

$$T_{s1} + T_{s2} + \ldots + C_C + C_T = 0 \qquad (2.19)$$

$$M = \Sigma\,(A_s)_i\,(f_s)_i\,[c - (d)_i] + C_T\,\lambda_T + C_C\,\lambda_C \qquad (2.20)$$

and

$$\phi = \frac{\epsilon_c}{C} \qquad (2.21)$$

The complete moment-curvature relationship may be determined by incrementally adjusting the concrete strain, ϵ_c, at the extreme compression fiber. For each value of ϵ_c the neutral axis depth, c, is determined by satisfying Eq. 2.19.

Analytical models to compute both the ascending and descending branches of moment-curvature and load-deflection curves of reinforced concrete beams are presented in Hsu (1974, 1983).

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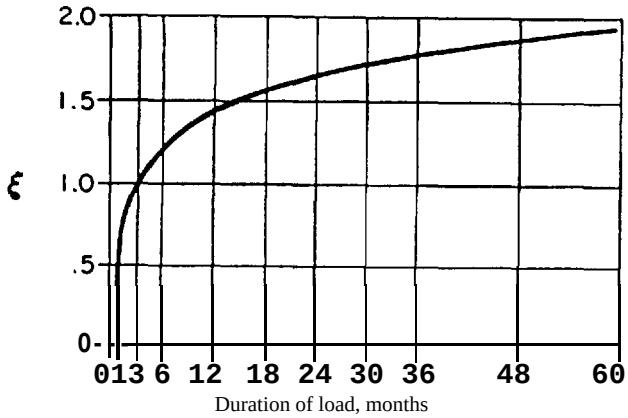


Fig. 2.5-ACI code multipliers for long-term deflections

2.6--Long-term deflection

2.6.1 *ACI method*-Time-dependent deflection of one-way flexural members due to the combined effects of creep and shrinkage, is calculated in accordance with ACI 318-89 (using Branson's Equation, 1971, 1977) by applying a multiplier, λ, to the elastic deflections computed from Equation 2.10:

$$\lambda = \frac{\xi}{1 + 50\,\rho'} \qquad (2.22)$$

where ρ' = reinforcement ratio for non-prestressed compression steel reinforcement
ξ = time dependent factor, from Fig. 2.2 (ACI 318, 1989)

Hence, the total long-term deflection is obtained by:

$$\delta_{LT} = \delta_L + \lambda_t\,\delta_{sus} \qquad (2.23)$$

where
δ_L = initial live load deflection
δ_{sus} = initial deflection due to sustained load
λ_t = time dependent multiplier for a defined duration time t

Research has shown that high-strength concrete members exhibit significantly less sustained-load deflections than low-strength concrete members (Luebkeman et al, 1985; Nilson, 1985). This behavior is mainly due to lower creep strain characteristics. Also, the influence of compression steel reinforcement is less pronounced in high-strength concrete members. This is because the substantial force transfer from the compression concrete to compression reinforcement is greatly reduced for high-strength concrete members, for which creep is lower than normal strength concrete. Nilson (1985) suggested that two modifying factors should be introduced into the ACI Code Eq. 2.22. The first is a material modifier, μ_m, with values equal to or less than 1.0, applied to ξ to account for the lower creep coefficient. The second is a section modifier, μ_s, also having values equal to or less than 1.0, to be applied to ρ' to account for the decreasing importance of compression steel in high-strength concrete members. Comparative studies have shown that a single modifier, μ, can be used to account satisfactorily for both effects simultaneously, leading to the following simplified equation

$$\lambda = \frac{\mu\,\xi}{1 + 50\,\mu\,\rho'} \qquad (2.24)$$

where $0.7 \le \mu = 1.3 - 0.00005\,f'_c \le 1.0$.

This equation results in $\mu = 1.0$ for concrete strength less than 6000 psi (42 MPa), and provides a reasonable fit of experimental data for higher concrete strengths. However, more data is needed, particularly for strengths between 9000 to 12,000 psi (62 MPa to 83 MPa) and beyond before a definitive statement can be made.

2.6.2 *ACI Committee 435 modified method (Branson, 1963, 1977)*-For computing creep and shrinkage deflections separately, Branson's (1963,1977) Eq. 2.25 and 2.26 are recommended by ACI 435 (1966, 1978).

$$\delta_{cr} = \lambda_c\,(\delta_{sus}) \qquad (2.25)$$

$$S_{sh} = k_{sh}\,\phi_{sh}\,l^2 = k_{sh}\left(A_{sh}\,\frac{(\epsilon_{sh})_t}{h}\right)1^2 \qquad (2.26)$$

where

$$\lambda_c = \frac{0.85\,C_t}{1 + 50\,\rho'}$$

C_t and (e,h), may be determined from Eq. 2.7 through 2.9 and Table 2.1.

A_{sh} = $0.7(\rho - \rho')^{1/3}\left(\frac{\rho - \rho'}{\rho}\right)^{1/2}$ for $\rho - \rho' \le 3.0$ percent
= $0.7\,\rho^{1/3}$ for $\rho' = 0$
= 1.0 for $\rho - \rho' > 3.0$ percent

ρ and ρ' are computed at the support section for cantilevers and at the midspan sections for simple and continuous spans.

The shrinkage deflection constant k_{sh} is as follows:

Cantilevers	= 0.50
Simple beams	= 0.13
Spans with one end continuous (multi spans)	= 0.09
Spans with one end continuous (two spans)	= 0.08
Spans with both ends continuous	= 0.07

Separate computations of creep and shrinkage are preferable when part of the live load is considered as a sustained load.

2.63 *Other methods*-Other methods for time-dependent deflection calculation in reinforced concrete beams and one-way slabs are available in the literature. They include several methods listed in ACI 435 (1966), the CEB-FIP Model Code (1990) simplified method, and other methods described in Section 3.8, Chapter 3, including the section curvature method (Ghali-Favre, 1986). This section highlights the CEB-FIP Model Code method (1990) and describes the Ghali-Favre approach, referring the reader to the literature for details.

2.6.3.1 *CEB-FIP Model Code simplified method*-On the basis of assuming a bilinear load-deflection relationship, the time-dependent part of deflection of cracked concrete members can be estimated by the for-

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lowing expression [CEB-FIP, 1990]:

$$\delta_{L\text{-}T} = (h/d)^3 \eta (1 - 20\,\rho_{cm})\delta_g \qquad (2.27)$$

where

δ_g = elastic deformation calculated with the rigidity $E_c I_g$ of the gross cross section (neglecting the reinforcement)

η = correction factor (see Fig. 2.6), which includes the effects of cracking and creep

ρ_{cm} = geometrical mean percentage of the compressive reinforcement

The mean percentage of reinforcement is determined according to the bending moment diagram (Fig. 2.6) and Eq. (2.28):

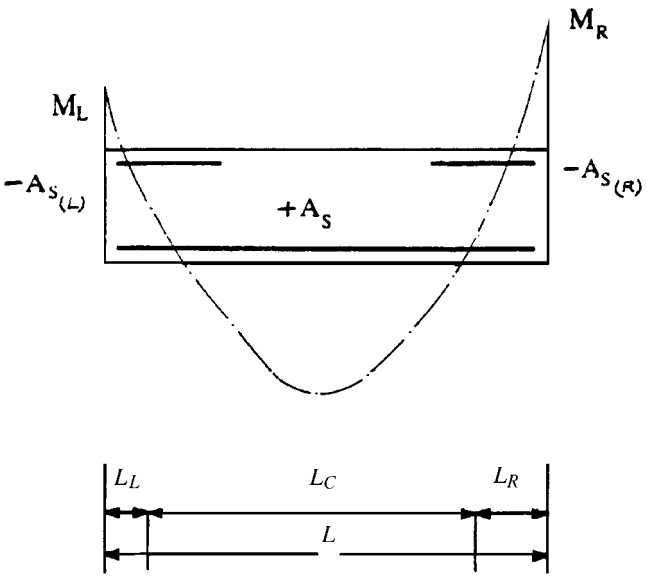
$$\rho_m = \rho_L(l_L/l) + \rho_c\,(l_C/l) + \rho_R(l_R/l) \qquad (2.28)$$

where

ρ_L, ρ_R = percentage of tensile reinforcement at the left and right support, respectively

ρ_C = percentage of tensile reinforcement at the maximum positive moment section

l_L,l_C, and l_R = length of inflection point segments as indicated in Fig. 2.6 (an estimate of lengths is generally sufficient)



(a) Bending moment diagram defining L_R L_C L_L

ρ_m [%]	0.15	0.2	0.3	0.5	0.75	1.0	1.5
η	10	8	6	4	3	2.5	2

(b) Mean percentage ρ_m of the tensile reinforcement

Fig. 2.6—CEB-FIP simplified deflection calculation method (CEB-FIP, 1990)

2.6.3.2 *Section curvature method (Ghali, Favre, and Elbadry 2002)*—Deflection is computed in terms of curvature evaluation at various sections along the span, satisfying compatibility and equilibrium throughout the analysis. Appendix B gives a general procedure for calculation of displacements (two translation components and a rotation) at any section of a plane frame. The general method calculates strain distributions at individual sections considering the effects of a normal force and a moment caused by applied loads, prestressing, creep and shrinkage of concrete, relaxation of prestressed steel, and cracking. The axial strains and the curvatures thus obtained can be used to calculate the displacements.

The comprehensive analysis presented in Appendix B requires more calculations than the simplified methods. It also requires more input parameters related to creep, shrinkage, and tensile strength of concrete and relaxation of prestressing steel. With any method of analysis, the accuracy in the calculation of deflections depends upon the rigor of the analysis and the accuracy of the input parameters. The method presented in Appendix B aims at improving the rigor of the analysis, but it cannot eliminate any inaccuracy caused by the uncertainty of the input parameters.

The comprehensive analysis can be used to study the sensitivity of the calculated deflections to variations in the input parameters. The method applies to the reinforced concrete members, with or without prestressing, having variable cross sections.

2.6.4 *Finite element method*—Finite element models have been developed to account for time-dependent deflections of reinforced concrete members (ASCE, 1982). Such analytical approaches would be justifiable when a high degree of precision is required for special structures and only when substantially accurate creep and shrinkage data are available. In special cases, such information on material properties is warranted and may be obtained experimentally from tests of actual materials to be used and inputting these in the finite element models.

2.7—Temperature-induced deflections

Variations in ambient temperature significantly affect deformations of reinforced concrete structures. Deflections occur in unrestrained flexural members when a temperature gradient occurs between its opposite faces. It has been standard practice to evaluate thermal stresses and displacements in tall building structures. Movements of bridge superstructures and precast concrete elements are also computed for the purpose of design of support bearings and expansion

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joint designs. Before performing an analysis for temperature effects, it is necessary to select design temperature gradients. Martin (1971) summarizes design temperatures that are provided in various national and foreign codes.

An ACI 435 report on temperature-induced deflections (1985) outlines procedures for estimating changes in stiffness and temperature-induced deflections for reinforced concrete members. The following expressions are taken from that report.

2.7.1 *Temperature gradient on unrestrained cross section*—With temperature distribution $t(y)$ on the cross section, thermal strain at a distance y from the bottom of the section can be expressed by

$$\in_t(y) = \alpha t(y) \tag{2.33}$$

To restrain the movement due to temperature $t(y)$, a stress is applied in the opposite direction to $\in_t(y)$:

$$f(y) = E_c \alpha t(y) \tag{2.34}$$

The net restraining axial force and moment are obtained by integrating over the depth:

$$P = \int_A f dA = \int_0^h [\alpha E_c t(y) b(y)] dy \tag{2.35}$$

$$M = \int_A f(y-n) dA = \int_0^h [\alpha E_c t(y) b(y)(y-n)] dy \tag{2.36}$$

In order to obtain the total strains on the unrestrained cross section, P and M are applied in the opposite direction to the restraining force and moment. Assuming plane sections remain plane, axial strain $\in_a$ and curvature ϕ are given by:

$$\in_a = \frac{P}{AE_c} = \frac{\alpha}{A}\int_0^h [t(y) b(y)] dy \tag{2.37}$$

$$\phi = \frac{M}{E_c I} = \frac{\alpha}{I}\int_0^h [t(y) b(y)(y-n)] dy \tag{2.38}$$

The net stress distribution on the cross section is given by:

$$f_n(y) = \frac{P}{A} \pm \frac{M(y-n)}{I} - E_c \alpha t(y) \tag{2.39}$$

For a linear temperature gradient varying from 0 to Δt, the curvature is given by:

$$\phi = \frac{\alpha \Delta t}{h} \tag{2.40}$$

In the case of a uniform vertical temperature gradient constant along the length of a member, deflections for simply supported (δ_{ss}) and cantilever beams (δ_{cont}) are calculated as:

$$\delta_{ss} = \frac{\phi l^2}{8} = \frac{\alpha \Delta t}{h}\frac{l^2}{8} \tag{2.41}$$

$$\delta_{cont} = \frac{\phi l^2}{2} = \frac{\alpha \Delta t}{h}\frac{l^2}{h} \tag{2.42}$$

The deflection-to-span ratio is given by:

$$\frac{\delta}{l} = \frac{\alpha \Delta t}{k}\frac{l}{h} \tag{2.43}$$

where $k = 8$ for simply supported beams and 2 for cantilever beams.

2.7.2 *Effect of restraint on thermal movement*—If a member is restrained from deforming under the action of temperature changes, internal stresses are developed. Cracking that occurs when tensile stresses exceed the concrete tensile strength reduces the flexural stiffness of the member and results in increased deflections under subsequent loading. Consequently, significant temperature effects should be taken into account in determining member stiffness for deflection calculation. The calculation of the effective moment of inertia should be based on maximum moment conditions.

In cases where stresses are developed in the member due to restrain of axial deformations, the induced stress due to axial restraint has to be included in the calculation of the cracking moment in a manner analogous to that for including the prestressing force in prestressed concrete beams.

APPENDIX A2

Example A2.1: Deflection of a four-span beam

A reinforced concrete beam supporting a 4-in. (100 mm) slab is continuous over four equal spans 1 = 36 ft (10.97 m) as shown in Fig. A2.1 (Nawy, 1990). It is subjected to a uniformly distributed load w_D = 700 lb/ft (10.22 kN/m), including its self-weight and a service load w_L = 1200 lb/ft (17.52 kN/m). The beam has the dimensions b = 14 in. (355.6 mm), d = 18.25 in. (463.6 mm) at midspan, and a total thickness h = 21.0 in. (533.4 mm). The first interior span is reinforced with four No. 9 bars

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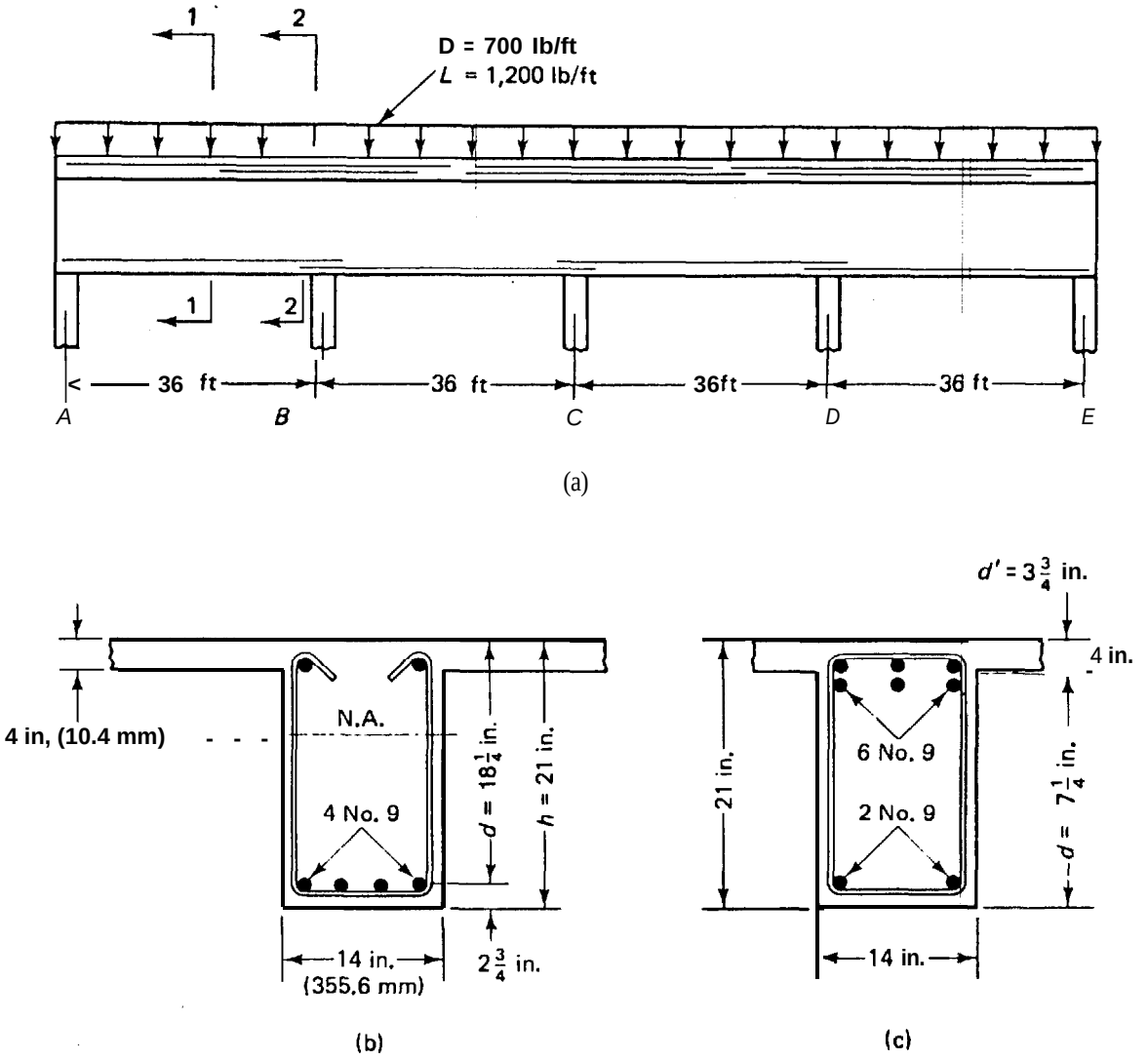


Fig. A2.1-Details of continuous beam in Ex. A2.1 (Nawy, 1990, courtesy Prentiss Hall)

at midspan (28.6 mm diameter) at the bottom fibers and six No. 9 bars at the top fibers of the support section.

Calculate the maximum deflection of the continuous beam using the ACI 318 method.

Given:

f'_c = 4000 psi (27.8 MPa), normal weight concrete
f_y = 60,000 psi (413.7 MPa)
50 percent of the live load is sustained 36 months on the structure.

Solution-ACI Method

Note: All calculations are rounded to three significant figures.

Material properties and bending moment values

$E_c = 57{,}000 = \sqrt{f'_c} = 57{,}000\sqrt{4000} = 3.6 \times 10^6$ psi (24,800 MPa)

E_s = 29 x 10^6 psi (200,000 MPa)

modular ratio $n = E_s/E_c = \dfrac{29\times10^6}{3.6\times10^6} = 8.1$

modulus of rupture $f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000}$ = 474 psi (3.3 MPa)

For the first interior span, the positive moment = 0.0772 wl^2

$+M_D$ = 0.0772 x $700(36.0)^2$ x 12 = 840,000 in.-lb

$+M_L$ = 0.0772 x $1200(36.0)^2$ x 12 = 1,440,000 in.-lb

$+(M_D + ML)$ = 0.0772x $1900(36.0)^2$ x 12 = 2,280,000 in.-lb

negative moment = 0.107 wl^2

$-M_D$ = 0.1071 x $700(36.0)^2$ x 12 = 1,170,000 in.-lb

$-M_L$ = 0.1071 x $1200(36.0)^2$ x 12 = 2,000,000 in.-lb

$-(M_D + M_L)$ = 0.1071 x $1900(36.0)^2$ x 12 = 3,170,000 in.-lb

Effective moment of inertia I_e

Fig. A2.2 shows the theoretical midspan and support

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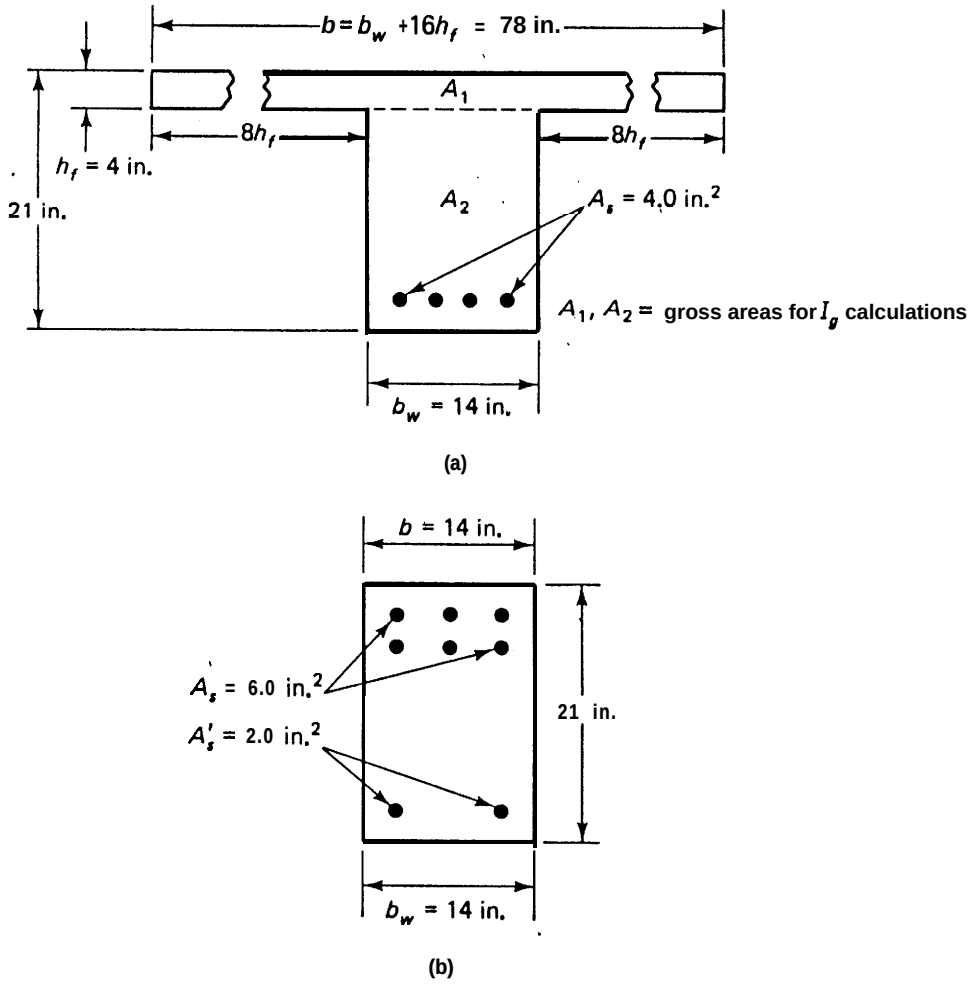


Fig. A2.2-Gross moment of inertia I_g *cross sections in Ex. A2.1*

cross sections to be used for calculating the gross moment of inertia I_g.

1. Midspan section:

Width of T-beam flange = $b_w + 16h_f$ = 14.0 + 16 x 4.0 = 78 in. (1981 mm)

Depth from compression flange to the elastic centroid is:

$$y' = (A_1y_1 + A_2y_2)/(A_1 + A_2)$$

$$= \frac{78(4x2) + 14x(21-4)x12.5}{78x4 + 14 x 17} = 6.54 \text{ in.}$$

$$y_t = h - y' = 21.0 - 6.54 = 14.5 \text{ in.}$$

$$I_g = \frac{78(4)^3}{12} + 78 \text{ x } 4(6.54 - 4/2)^2 + \frac{14(21-4)^3}{12} + 14(21-4)\left(14.46 - \frac{21-4}{2}\right)^2$$

$$= 21.000 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.3 \times 21{,}000}{14.46} = 690{,}000 \text{ in.-lb}$$

Depth of neutral axis:

(A_s = four No. 9 bars = 4.0 in^2).

To locate the position, c, of the neutral axis, take moment of area of the transformed flanged section, namely

$$b_w(c - h_f)^2 - 2nA_s(d - c) + bh_f(2c - h_f) = 0$$

or

$$14(c - 4.0)^2 - 2 \text{ x } 8.1 \text{ x } 4.0(18.25 - c) + 78 \text{ x } 4(2c - 4.0) = 0$$

or $c^2 + 4.17c - 157.0 = 0$

to give c = 3.5 in. Hence the neutral axis is inside the flange and the flange section is analyzed as a rectangular section.

For rectangular sections,

$$\frac{78c^2}{2} + 8.1 \text{ x } 4 \text{ x } c - 8.1 \text{ x } 4 \text{ x } 18.25 = 0$$

Therefore, c = 3.5 in.

$$I_{cr} = \frac{78(3.5)^3}{3} + 0.8 \text{ x } 4\ (18.25 - 3.5)^2 = 8160 \text{ in.}^4$$

Ratio M_{cr}/M_a:

$$D \text{ ratio} = \frac{690{,}000}{840{,}000} = 0.82$$

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D + 50 percent L ratio = $\frac{690,000}{840,000 + 0.5 \times 1,440,000}$ = 0.44

$D + L$ ratio = $\frac{690,000}{2,280,000}$ = **0.30**

Effective moment of inertia for midspan sections:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 + \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

I_e for dead load = 0.55 x 21,000 + 0.45 x 8160
= 15,200 **in.**^{4}

I_e for $D + 0.5L$ = 0.086 x 21,000 + 0.914 x 8160
= 9276 **in.**^{4}

I_e for $D + L$ = 0.027 x 21,000 + 0.973 x 8160
= 8500 **in.**^{4}

If using the simplified approach to obtain I_e (Section 2.5.2.3) values of 14,200 in^4(7 percent smaller), 9200 **in.**^{4} (1 percent smaller), and 8020 **in.**^{4} (6 percent smaller), are obtained respectively.

2. Support section:

$$I_g \quad \frac{bh^3}{12} \quad \frac{14(21)^3}{12} = 10,800 \text{ in.}^4$$

$$y_t = \frac{21}{2} = 10.5 \text{ in.}$$

$$M_{cr} = f_r I_g / y_t = \frac{470 \times 10,800}{10.5} = 483,000 \text{ in.-lb}$$

Depth of neutral axis:

A_s = six No. 9 bars = 6.0 **in.**^{2} (3870 **mm**^{2})
A_s' = two No. 9 bars = 2.0 **in.**^{2} (1290 **mm**^{2})
d = 21.0 - 3.75 = 17.30 in. (438 mm)

Similar calculation for the neutral axis depth c gives a value c = 7.58 in.

Hence,

$$I_{cr} = \frac{bc^3}{3} + nA_s(d - c)^2 + (n - 1)A_s'(c - d')^2$$

Ratio M_{cr}/M_a:

$$D \text{ ratio} = \frac{483,000}{1,170,000} = 0.41$$

$$D + 50 \text{ percent } L \text{ ratio} = \frac{483,000}{1,170,000 + 0.5 \times 2,000,000} = 0.22$$

$$D + L \text{ ratio} = \frac{483,000}{3,170,000}$$

Effective moment of inertia for support section:

I_e for dead load = 0.07 x 10,800 + 0.93 x 6900
= 7170 **in.**^{4}

I_e for $D + 0.5L$ = 0.01 x 10,800 + 0.99 x 6900
= 6940 in.^{4}

I_e for $D + L$ = 0.003 x 10,800 + 0.99 x 6900
= 6910 **in.**^{4}

Average effective I_e *for continuous span*

average $I_e = 0.85 I_m + 0.15 I_{ec}$

dead load: I_e = 0.85 x 15,200 + 0.15 x 7170
= 14,000 **in.**^{4}

$D + 0.5L$: I_e = 0.85 x 9260 + 0.15 x 6940
= 8900 **in.**^{4}

$D + L$: I_e = 0.85 x 8500 + 0.15 x 6910 = 8260 **in.**^{4}

Short-term deflection

The maximum deflection for the first interior span is:

$$\delta = \frac{0.0065wl^4}{EI}$$

l assumed = l_n for all practical purposes

$$\delta = \frac{0.0065(36 \times 12)^4}{36 \times 10^6} \times \frac{w}{I_e} \times \frac{1}{12} = 5.240 \frac{w}{I_e} \text{ in.}$$

Initial dead-load deflection:

$$\delta_D = \frac{5.240(700)}{14.000} = 0.26 \text{ in., say } 0.3 \text{ in.}$$

Initial live-load deflection:

$$\delta_L = \delta_{L+D} \quad \delta_D$$

$$\delta_L = \frac{5.240(1900)}{8260} - \frac{5.240(700)}{14,000}$$

= 1.21 - 0.26 = 0.95 in., say 1 in.

Initial 50 percent sustained live-load deflection:

$$\rho' = \frac{A_s'}{bd} = 0 \text{ (at midspan in this case)}$$

multiplier $\lambda = \xi / (1 + 50\rho')$

From Fig. 2.5

T = 1.75 for 36-month sustained load
T = 2.0 for 5-year loading

Therefore,

λ_∞ = 2.0 and λ_t = 1.75

The total long-term deflection is

$$\delta_{LT} = \delta_L + \lambda_\infty \delta_D + \lambda_t \delta_{LS}$$

$$\delta_{LT} = 0.95 + 2.0 \times 0.26 + 1.75 \times 0.5 = 0.95 + 1.41$$
$$= 2.34 \text{ in. (60 mm), say } 2.4 \text{ in.}$$

Deflection requirements (Table 2.5)

$\frac{l}{180} = \frac{36 \times 12}{180}$ = 2.4 in. > δ_L = 1.0 in., O.K

$\frac{l}{360}$ = 1.2 in. > δ_L = 1.0 **in.**, O.K.

$\frac{l}{480}$ = 0.9 in. < δ_{LT} = 2.4 in., N.G.

$\frac{l}{240}$ = 1.8 in. < δ_{LT} = 2.4 in., N.G.

Hence, the continuous beam is limited to floors or roofs not supporting or attached to nonstructural elements such as partitions.

Application of CEB-FIP method to obtain long-term deflection due to sustained loads:

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midspan $\rho = \frac{+A_s}{bd} = \frac{4 \times 1.0}{78 \times 18.25} = 0.0028 = \rho_c$

support $\rho = \frac{-A_s}{b_w d} = \frac{6 \times 1.0}{14 \times 18.25} = 0.0235 = \rho_L = \rho_R$

$$\left(\frac{h}{d}\right)^3 = \left(\frac{21}{18.25}\right)^3 = 1.52$$

$$\rho_m = \rho_L\left(\frac{l_L}{l}\right) + \rho_c\left(\frac{l_c}{l}\right) + \rho_R\left(\frac{l_R}{l}\right)$$

Assuming that the location of the inflection points as defined by l_L and l_R for negative moment region, and l_c for the positive moment region in Figure 2.16 are as follows:

$L_l/L = L_R/L = 0.21$ and $L_c/L = (1\text{-}0.21 \times 2) = 0.58$

Also, assume $\rho_L = \rho_R$

Hence,

$\rho_m = 2(0.0235 \times 0.2) + 0.0028 \times 0.58$
$= 0.0094 + 0.0017 = 0.0111 = 1.11$ percent

From Fig. 2.6, $\eta = 2.4$

From ACI Method Solution: $I_g = 21{,}000$ in.^{4}

Short-term deflection,

$$\delta = \frac{0.0069wl^4}{E_c I_g} = \frac{5.240\ w}{I_g}$$

$$+ \frac{5.240(700 + 1200)}{21{,}000} = 0.47 \text{ in., say } 0.5 \text{ in.}$$

Long-Term increase in deflection due to sustained load:

$$\delta_{L\text{-}T} = \left(\frac{h}{d}\right)^3 \eta(1 - 20\rho_c)\delta$$

$= 1.52 \times 2.4(1 - 20 \times 0.0111)0.5$
$= 1.35$ in., say 1.4 in. (35 mm).
(1.41 in. by the ACI procedure solution)

Example A2.2: Temperature-induced deflections

These design examples illustrate the calculation procedures for temperature induced deflections.

Example (a): Simply supported vertical wall panel — Linear temperature gradient

$\Delta t = 40$ F (4.4 C)
$\alpha = 0.0000055$ in./in./F
$h = 4$ in. (101 mm)

a) Single story span: $L = 12$ ft. (3.66 m)
$\delta = (0.0000055 \times 40 \times 144^2)/(4 \times 8)$
$= 0.14$ in. (3.6 mm), say 0.2 in.

b) Two story span: $L = 24$ ft. (7.32 m)
$\delta = (0.0000055 \times 40 \times 288^2)/(4 \times 8)$
$= 0.57$ in. (14.5 mm), say 0.6 in.

Example (b): Simply supported tee section — Linear Temperature gradient over depth

$\Delta t = 40$ F (4.4 C)
$\alpha = 0.0000055$ in./in./F
$h = 36$ in. (914 mm)
$L = 60$ ft. (18.4 m) - simply supported
$\delta = (0.0000055 \times 40 \times 720^2)/(36 \times 8)$
$= 0.40$ in. (10 mm), say 0.5 in.

Example (c): Simply supported tee section — Constant temperature over flange depth

$I = 69319$ in.^{4} (2.88 x 10^{10} mm^4)
$n = 26.86$ in (682 mm)
$\Delta t = 40$ F (4.4 C)
$\alpha = 0.0000055$ in./in./F
$h = 36$ in. (914 mm)
$L = 60$ ft. (18.4 m)

$$\phi = (\alpha/I)\int_0^{36} t(y)\,b(y)\,(y - n)dy$$

$$= (\alpha/I)\int_{33}^{36} (40 \times 96)\,(y - 26.86)dy$$

$= (88{,}013 \times 0.0000055)/69319$
$= 0.00000698$
$\delta = (\phi L^2)/8 = (0.00000698 \times 720^2)/8$
$= 0.45$ in. (11.4 mm), say 0.5 in.

CHAPTER 3-DEFLECTION OF PRESTRESSED CONCRETE ONE-WAY FLEXURAL MEMBERS*

3.1-Notation

A_c = area of section
A_g = gross area of concrete section
A_s = area of nonprestressed reinforcement
A_{ps} = area of prestressed reinforcement in tension zone
b = width of compression face of member
b_w = web width
c = depth of compression zone in a fully-cracked section
cgc = center of gravity of concrete section
cgs = center of gravity of reinforcement
C = creep coefficient, defined as creep strain divided by initial strain due to constant sustained stress
C_1 = PCI multiplier for partially prestressed section
C_2 = PCI multiplier for partially prestressed section
C_t = creep coefficient at a specific age
C_u = ultimate creep coefficient for concrete at loading equal to time of release of prestressing
d = distance from extreme compression fiber to centroid of prestressing steel
d' = distance from extreme compression fiber to centroid of compression reinforcement
d_p = distance from extreme compression fiber to centroid of prestressed reinforcement
e_c = eccentricity of prestress force from centroid of section at center of span
e_{cr} = eccentricity of prestress force from centroid of cracked section
e_e = eccentricity of prestress force from centroid of section at end of span
E_c = modulus of elasticity of concrete
E_{ci} = modulus of elasticity of concrete at time of initial prestress
E_s = modulus of elasticity of nonprestressed reinforcement

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E_{ps} = modulus of elasticity of prestressed reinforcement
ES = stress loss due to elastic shortening of concrete
f'_c = specified compressive strength of concrete
f_d = concrete stress at extreme tensile fibers due to unfactored *dead* load when tensile stresses and cracking are caused by external load
f'_t = strength of concrete in tension
f_l = calculated stress due to live load
f_{max} = stress in extreme tension fibers due to effective prestress, if any, plus maximum unfactored load, using uncracked section properties
f_{ce} = compressive stress in concrete due to effective prestress only after losses when tensile stress is caused by applied external load
f_{pe} = effective prestress in prestressing reinforcement after losses
f_{pi} = stress in prestressing reinforcement immediately prior to release
f_{pJ} = stress in pretensioning reinforcement at jacking (5-10 percent higher than f_{pi})
f_{pu} = specified tensile strength of prestressing tendons
f_{py} = yield strength of the prestressing reinforcement
f_r = modulus of rupture of concrete $7.5\sqrt{f'_c}$
f_{TL} = final calculated total stress in member
f_y = specified yield strength of nonprestressed reinforcement
h = overall thickness of member
h_f = depth of flange
I_{cr} = moment of inertia of cracked section transformed to concrete
I_e = effective moment of inertia for computation of deflection
I_g = moment of inertia of gross concrete section about centroid axis
I_t = moment of inertia of transformed section
K_{CR} = coefficient for creep loss in Eq. 3.7
L = span length of beam
M_a = maximum service unfactored *live load* moment
M_{cr} = moment due to that portion of applied *live load* that causes cracking
M_L = moment due to service live load
M_n = nominal flexural strength
M_{SD} = moment due to superimposed dead load
n = modular ratio of normal reinforcement (= E_s/E_c)
n_p = modular ratio of prestressing reinforcement (= E_{ps}/E_c)
P_e = effective prestressing force after losses
P_i = initial prestressing force prior to transfer
r = radius of gyration = $\sqrt{I_g/A_g}$
REL = stress loss due to relaxation of tendons
SH = stress loss due to shrinkage of concrete
t_o = initial time interval
t = time at any load level or after creep or shrinkage are considered
y_g = distance from extreme compression fibers to centroid of A_g
y_t = distance from centroid axis of gross section, neglecting reinforcement to extreme tension fibers
α = length parameter that is a function of tendon profile used
δ = deflection or camber
ϵ_c = maximum usable strain in the extreme compression fiber of a concrete element (0.003 in./in.)
ϵ_{cr} = strain at first cracking load
ϵ_{ps} = strain in prestressed reinforcement at ultimate flexure
ϵ_{SH} = unit shrinkage strain in concrete
$(\epsilon_{SH})_t$ = shrinkage strain at any time t
$(\epsilon_{SH})_u$ = average value of ultimate shrinkage strain
ϵ_u = ultimate strain
ϕ = curvature (slope of strain diagram)
ϕ_c = curvature at midspan
ϕ_e = curvature at support
γ = correction factor for shrinkage strain in nonstandard conditions (see also Sec. 2.3.4)
Δf_{pCR} = stress loss due to creep in concrete
Δf_{pSH} = stress loss due to concrete shrinkage
Δf_{pR} = stress loss due to relaxation of tendons

3.2-General

3.2.1 *Introduction*-Serviceability behavior of prestressed concrete elements, particularly with regard to deflection and camber, is a more important design consideration than in the past. This is due to the application of factored load design procedures and the use of high-strength materials which result in slender members that may experience excessive deflections unless carefully designed. Slender beams and slabs carrying higher loads crack at earlier stages of loading, resulting in further reduction of stiffness and increased short-term and long-term deflections.

3.2.2 *Objectives*-This chapter discusses the factors affecting short-term and long-term deflection behavior of prestressed concrete members and presents methods for calculating these deflections.

In the design of prestressed concrete structures, the deflections under short-term or long-term service loads may often be the governing criteria in the determination of the required member sizes and amounts of prestress. The variety of possible conditions that can arise are too numerous to be covered by a single set of fixed rules for calculating deflections. However an understanding of the basic factors contributing to these deformations will enable a competent designer to make a reasonable estimate of deflection in most of the cases encountered in prestressed concrete design. The reader should note that the word *estimate* should be taken literally in that the properties of concrete which affect deflections (particularly long term deflections) are variable and not determinable with precision. Some of these properties have

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values to which a variability of ± 20 percent or more in the deflection values must be considered. Deflection calculations cannot then be expected to be calculated with great precision.

3.2.3 *Scope*-Both short-term and long-term *transverse* deflections of beams and slabs involving prestressing with high-strength steel reinforcement are considered. Specific values of material properties given in this chapter, such as modulus of elasticity, creep coefficients, and shrinkage coefficients, generally refer to normal weight concrete although the same calculation procedures apply to lightweight concrete as well. This chapter is intended to be self-contained.

Finally several of the methods described in this chapter rely solely on computer use for analysis. They do not lend themselves to any form of hand calculation or approximate solutions. The reader should not be deluded into concluding that such computer generated solutions from complex mathematical models incorporating use of concrete properties, member stiffness, extent of cracking and effective level of prestress somehow generate results with significantly greater accuracy than some of the other methods presented. This is because of the range of variability in these parameters and the difficulty in predicting their precise values at the various loading stages and load history. Hence, experience in evaluating variability of deflections leads to the conclusion that satisfying basic requirments of detailed computer solutions using various values of assumed data can give upper and lower bounds that are not necessarily more rational than present code procedures.

3.3-Prestressing reinforcement

3.3.1 *Types of reinforcement*-Because of the creep and shrinkage which occurs in concrete, effective prestressing can be achieved only by using high-strength steels with strength in the range of 150,000 to 270,000 psi (1862 MPa) or more. Reinforcement used for prestressed concrete members is therefore in the form of stress-relieved or low-relaxation tendons and high-strength steel bars. Such high-strength reinforcement can be stressed to adequate prestress levels so that even after creep and shrinkage of the concrete has occurred, the prestress reinforcement retains adequate remaining stress to provide the required prestressing force. The magnitude of normal prestress losses can be expected to be in the range of 25,000 to 50,000 psi (172 MPa to 345 MPa).

Wires or strands that are not stress-relieved, such as straightened wires or oil-tempered wires, are often used in countries outside North America.

3.3.1.1 *Stress-relieved wires and strands*-Stress-relieved strands are cold-drawn single wires conforming to ASTM A 421 and stress-relieved tendons conform to ASTM A 416. The tendons are made from seven wires by twisting six of them on a pitch of 12 to 16 wire diameters around a slightly larger, straight control wire. Stress-relieving is done after the wires are twisted into the strand. Fig. 3.1 gives a typical stress-strain diagram for wire and tendon prestressing stegl reinforcement,

3.3.1.2 *High-tensile-strength prestressing bars*—High-tensile-strength alloy steel bars for prestressing are either smooth or deformed to satisfy ASTM A 722 requirements and are available in nominal diameters from ⅝ in. (16 mm) to 13/8 in. (35 mm). Cold drawn in order to raise their yield strength, these bars are stress relieved to increase their ductility. Stress relieving is achieved by heating the bar to an appropriate temperature, generally below 500 C. Though essentially the same stress-relieving process is employed for bars as for strands, the tensile strength of prestressing bars has to be a minimum of 150,000 psi (1034 MPa), with a minimum yield strength of 85 percent of the ultimate strength for smooth bars and 80 percent for deformed bars

3.3.2 *Modulus of elasticity*-In computing short-term deflections, the cross-sectional area of the reinforcing tendons in a beam is usually small enough that the deflections may be based on the gross area of the concrete. In this case, accurate determination of the modulus of elasticity of the prestressing reinforcement is not needed. However, in considering time-dependent deflections resulting from shrinkage and creep at the level of the prestressing steel, it is important to have a reasonably good estimate of the modulus of elasticity of the prestressing reinforcement.

In calculating deflections under working loads, it is sufficient to use the modulus of elasticity of the prestressing reinforcement rather than to be concerned with the characteristics of the entire stress-strain curve since the reinforcement is seldom stressed into the inelastic range. In most calculations, the assumption of the modulus value as 28.5 x 10^6 psi (PCI *Design Handbook*, Fourth Edition) can be of sufficient accuracy considering the fact that the properties of the concrete which are more critical in the calculation of deflections are not known with great precision. The ACI Code states that the modulus of elasticity shall be established by the manufacturer of the tendon, as it could be less than 28.5 x 10^6 psi.

When the tendon is embedded in concrete, the freedom to twist (unwind) is lessened considerably and it thus is unnecessary to differentiate between the modulus of elasticity of the tendon and that of single-wire reinforcement (AC1 Committee 435, 1979).

3.3.3 *Steel relaxation*-Stress relaxation in prestressing steel is the loss of prestress that occurs when the wires or strands are subjected to essentially constant strain over a period of time. Fig. 3.2 relates stress relaxation to time t in hours for both stress-relieved and low-relaxation tendons.

The magnitude of the decrease in the prestress depends not only on the duration of the sustained prestressing force, but also on the ratio f_{pi}/f_{py} of the initial prestress to the yield strength of the reinforcement. Such a loss in stress is termed intrinsic stress relaxation.

If f_{pR} is the remaining prestressing stress in the steel tendon after relaxation, the following expression defines f_{pR} for stress-relieved steel:

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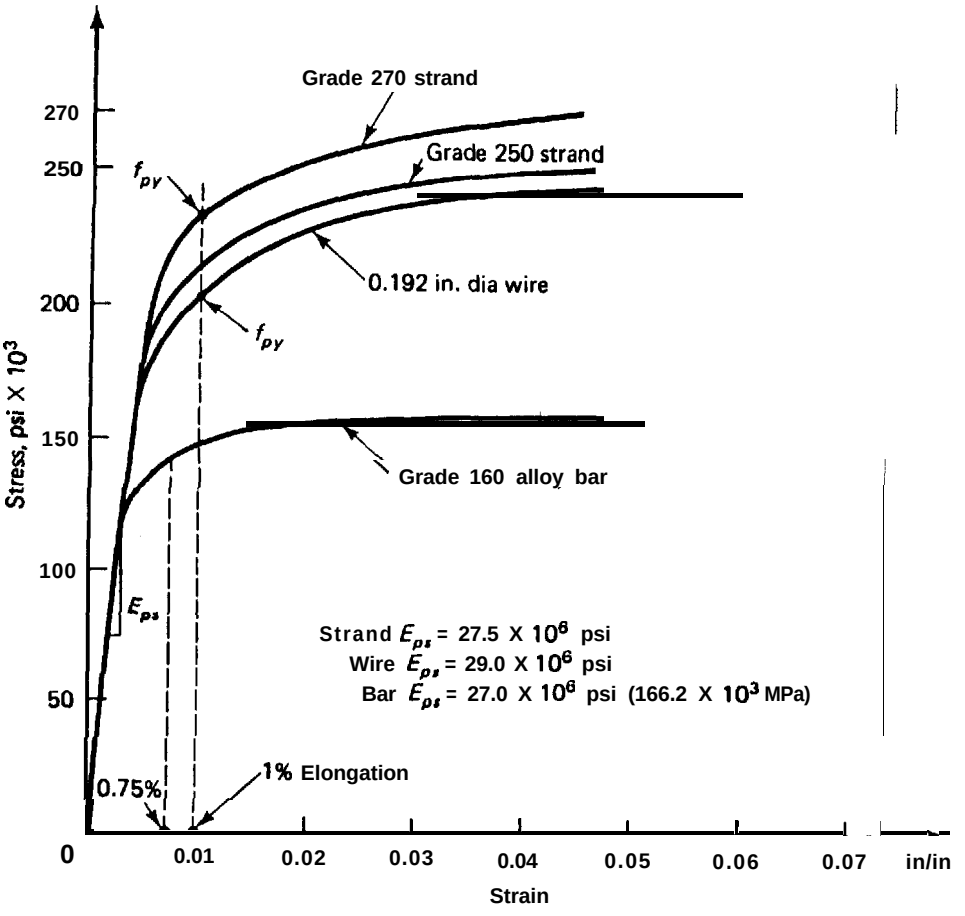


Fig. 3.1-Typical stress-strain diagram for prestressing steel reinforcement

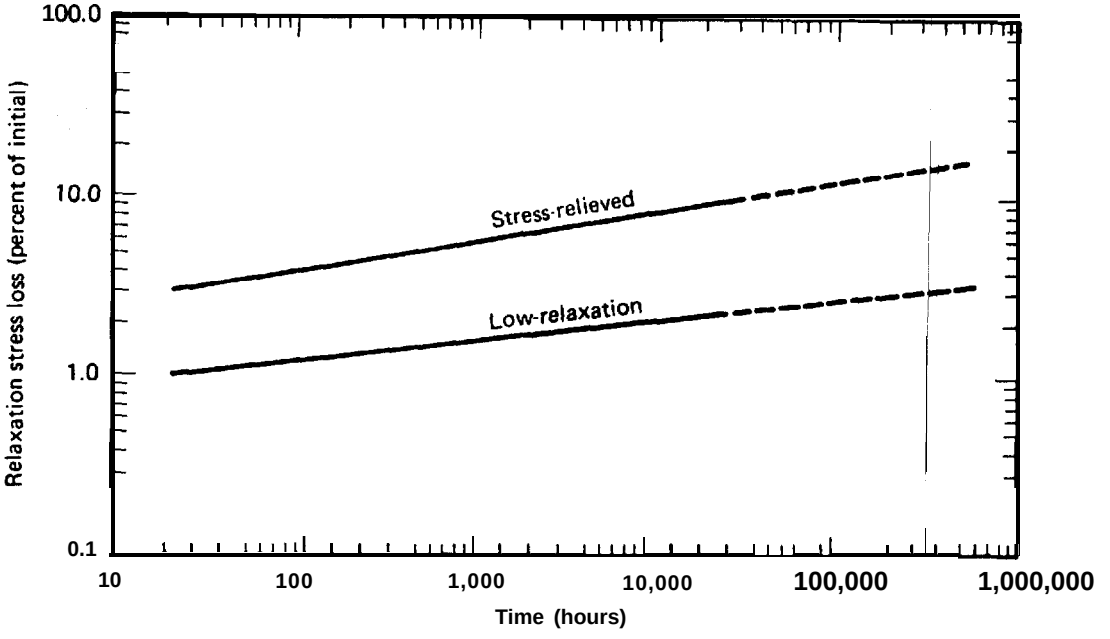


Fig. 3.2-Relaxation loss versus time for stress-relieved low-relaxation strands at 70 percent of the ultimate (Post-Tensioning Institute Manual, fourth edition)

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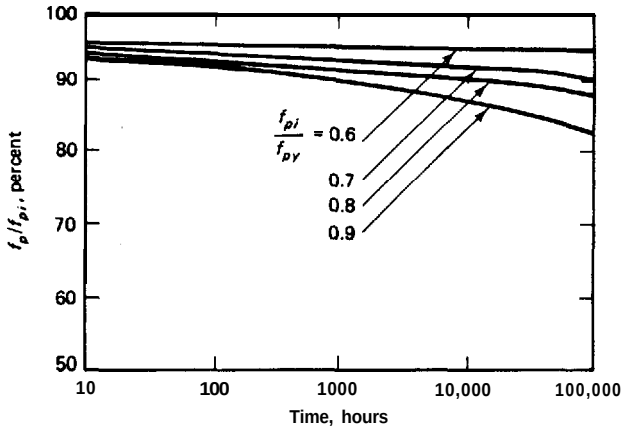


Fig. 3.3-Stress relaxation relationship in stress-relieved strands (Post Tensioning Manual, fourth edition)

$$\frac{f_{pR}}{f_{pi}} = 1 - \left(\frac{\log t}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad (3.1)$$

In this expression, log$t$ in hours is to the base 10, and the ratio f_{pi}/f_{py} must not be less than 0.55. Also, for low-relaxation steel, the denominator of the log term in the equation is divided by 45 instead of 10. A plot of Eq. 3.1 is given in Fig. 3.3. In that case, the intrinsic stress-relaxation loss becomes

$$\Delta f_{pR} = f_{pi}\,\frac{\log t}{45}\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad (3.2)$$

where f_{pi} is the initial stress in steel.

If a step-by-step loss analysis is necessary, the loss increment at any particular stage can be defined as

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad (3.3)$$

where t_1 is the time at the beginning of the interval and t_2 is the time at the end of the interval from jacking to the time when the loss is being considered. Therefore, the loss due to relaxation in stress-relieved wires and strands can be evaluated from Eq. 3.3, provided that $f_{pi}/f_{py} \geq 0.55$, with $f_{py} \approx 0.85 f_{pu}$ for stress-relieved strands and $0.90 f_{pu}$ for low-relaxation tendons.

It is possible to decrease stress relaxation loss by subjecting strands that are initially stressed to 70 percent of their ultimate strength f_{pu} to temperatures of 20 C to 100 C for an extended time in order to produce a permanent elongation, a process called *stabilization.* The prestressing steel thus produced is termed *low-relaxation* steel and has a relaxation stress loss that is approximately 25 percent of that of normal stress-relieved steel.

Fig. 3.2 gives the relative relaxation loss for stress-relieved and low-relaxation steels for seven-wire tendons held at constant length at 29.5 C. Fig. 3.4 shows stress relaxation of stabilized strand at various tension and temperature levels.

It should be noted that relaxation losses may be critically affected by the manner in which a particular wire is manufactured. Thus, relaxation values change not only from one type of steel to another but also from manufacturer to manufacturer. Factors such as reduction in diameter of the wire and its heat treatment may be significant in fixing the rate and amount of relaxation loss that may be expected. Nevertheless, sufficient data exists to define the amount of relaxation loss to be expected in ordinary types of prestressing wires or strands currently in use.

3.4-Loss of prestress

3.4.1 *Elastic shortening loss*-A concrete element shortens when a prestressing force is applied to it due to the axial compression imposed. As the tendons that are bonded to the adjacent concrete simultaneously shorten, they lose part of the prestressing force that they carry.

In pretensioned members, this, force results in uniform longitudinal shortening. Dividing the reduction in beam length by its initial length gives a strain that when multiplied by the tendon modulus of elasticity gives the stress loss value due to elastic shortening. In post-tensioned beams, elastic shortening varies from zero if all tendons are simultaneously jacked to half the value in the pretensioned case if several sequential jacking steps are applied.

3.4.2 *Loss of prestress due to creep of concrete*-The deformation or strain resulting from creep losses is a function of the magnitude of the applied load, its duration, the properties of the concrete including its mix proportions, curing conditions, the size of the element and its age at first loading, and the environmental conditions. Size and shape of the element also affect creep and subsequent loss of prestress. Since the creep strain/stress relationship is essentially linear, it is feasible to relate the creep strain ϵ_{CR} to the elastic strain ϵ_{EL} such that the ultimate creep coefficient C_u can be defined as

$$C_u = \frac{\epsilon_{CR}}{\epsilon_{EL}} \quad (3.4)$$

The creep coefficient at any time t in days can be taken as

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}}\,C_u$$

(See Eq. 2.7 of Chapter 2; also Branson, et. al, 1971, 1977 and ACI 209, 1971, 1992)

The value of C_u usually ranges between 2 and 4, with an average of 2.35 for ultimate creep. The loss of pre-

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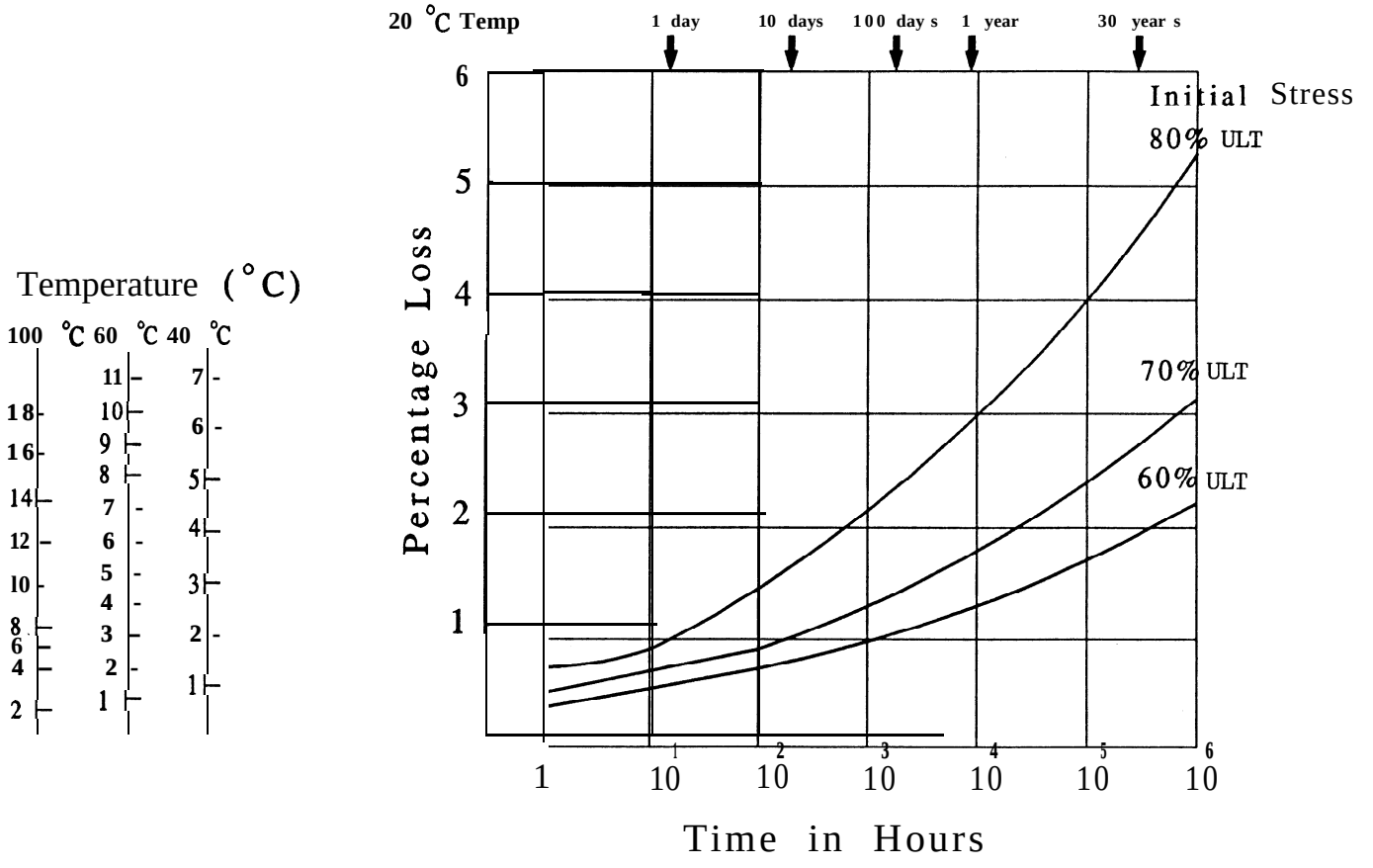


Fig. 3.4-Stress relaxation of stabilized strand at various tensions and temperatures (courtesy STELCO Inc., Canada)

stress for bonded prestressed members due to creep can be defined as

$$\Delta f_{pCR} = C_u \frac{E_{ps}}{E_c} f_{cs} \qquad (3.6)$$

where f_{cs} is the stress in the concrete at the level of the centroid of the prestressing tendon. In general, this loss is a function of the stress in the concrete at the section being analyzed. In post-tensioned, nonbonded draped tendon members, the loss can be considered essentially uniform along the whole span. Hence, an average value of the concrete stress between the anchorage points can be used for calculating the creep in post-tensioned members. A modified ACI-ASCE expression for creep loss can be used as follows:

$$\Delta f_{pCR} = K_{CR} \frac{E_{ps}}{E_c} (\bar{f}_{cs} - f_{csd}) \qquad (3.7)$$

where

K_{CR} = 2.0 for pretensioned members
= 1.60 for post-tensioned members (both for normal weight concrete)

$\bar{f}_{cs}$ = stress in concrete at the *cgs* level of the reinforcement immediately after transfer

f_{csd} = stress in concrete at the *cgs* level of the reinforcement due to all superimposed dead loads applied after prestressing is accomplished

K_{CR} should be reduced by 20 percent for lightweight concrete.

Fig. 3.5 shows normalized creep strain plots versus time for different loading ages while Fig. 3.6 illustrates in a three-dimensional surface the influence of age at loading on instantaneous and creep deformations. Fig. 3.7 gives a schematic relationship of total strain with time excluding shrinkage strain for a specimen loaded at a one day age.

3.4.3 *Loss of prestress due to shrinkage of concrete*-As with concrete creep, the magnitude of the shrinkage of concrete is affected by several factors. They include mix proportions, type of aggregate, type of cement, curing time, time between the end of external curing and the application of prestressing, and the environmental conditions. Size and shape of the member also affect shrinkage. Approximately 80 percent of shrinkage takes place in the first year of life of the structure. The average value of ultimate shrinkage strain in both moist-cured and stream-cured concrete is given as 780 x 10^{-6} in./in. in the ACI 209R-92 Report. This average value is affected by the duration of initial moist curing, ambient relative humidity, volume-surface ratio, temperature, and con-

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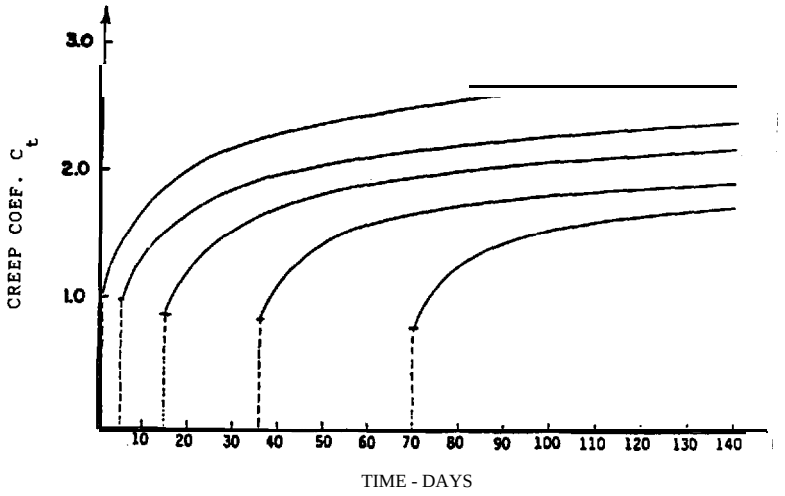


Fig. 3.5-Creep curves for different loading ages at same stress level

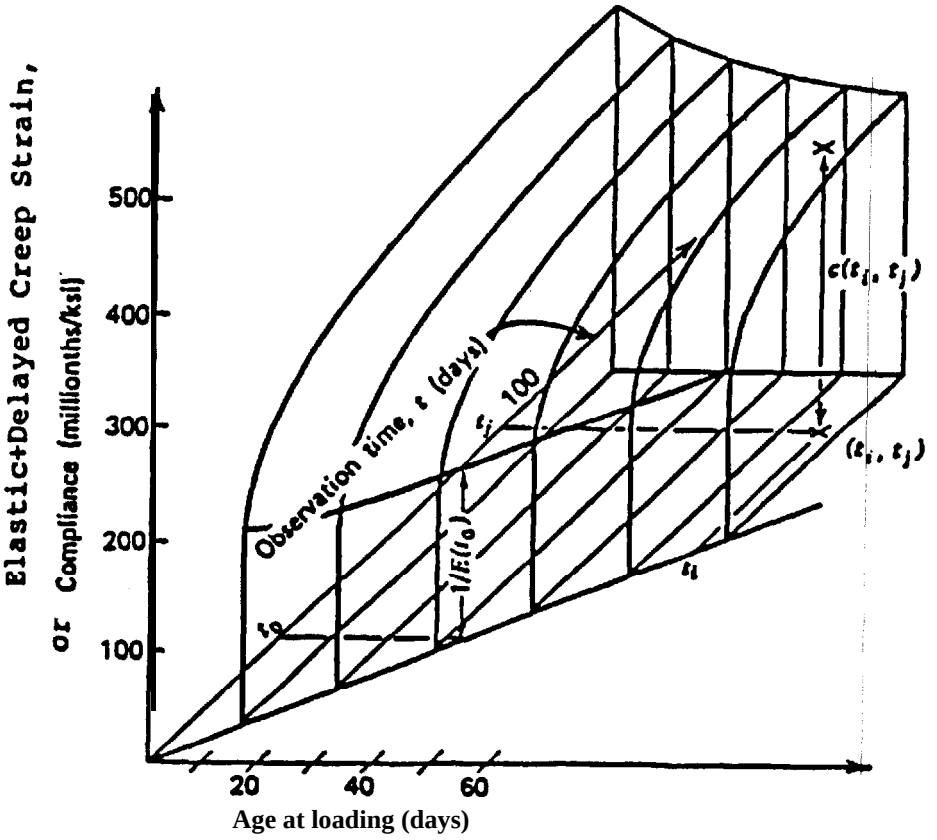


Fig. 3.6-Influence of age at loading on instantaneous and creep deformations (3-D surface)

crete composition. To take such effects into account, the average value of shrinkage strain should be multiplied by a correction factor γ_{SH} as follows

$$\epsilon_{SH} = 780 \times 10^{-6} \gamma_{SH} \quad (3.8)$$

Components of γ_{SH} are given in Sec. 2.3.4.

The Prestressed Concrete Institute stipulates for standard conditions an average value for nominal ultimate shrinkage strain $(\epsilon_{SH})_u$ = 820 x 10" in./in. (mm/mm), (PCI *Handbook,* 1993). If ϵ_{SH} is the shrinkage strain after adjusting for relative humidity at volume-to-surface ratio V/S, the loss in prestressing in pretensioned members is

$$\Delta f_{pSH} = \epsilon_{SH} \times E_{ps} \quad (3.9)$$

For post-tensioned members, the loss in prestressing due to shrinkage is somewhat less since some shrinkage has already taken place before post-tensioning. If the relative humidity is taken as a percent value and the V/S ratio effect is considered, the PCI general expression for loss in prestressing due to shrinkage becomes

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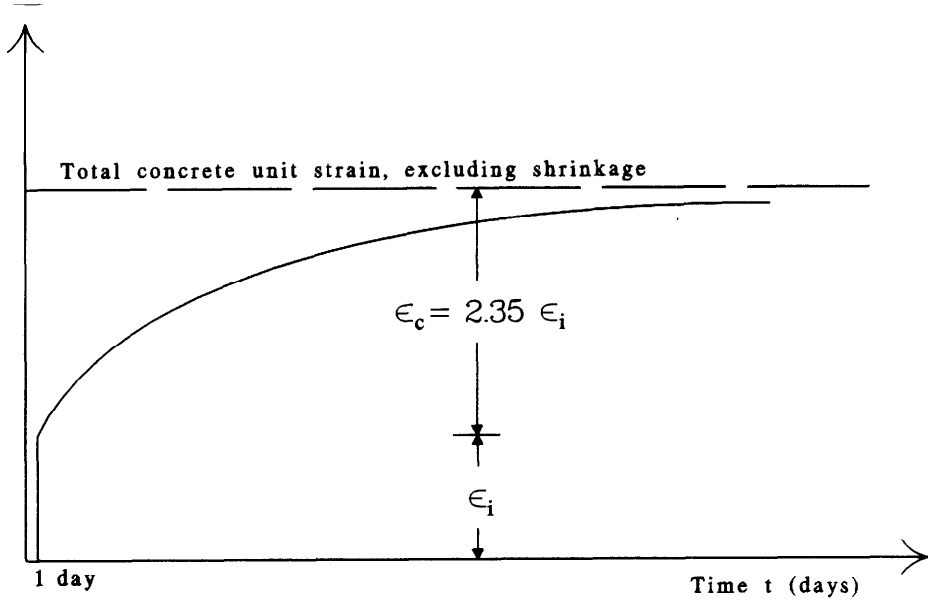


Fig. 3.7-Typical concrete strain versus time curve for constant stress applied at release time

$$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} E_{ps} \left(1 - 0.06 \frac{V}{S}\right)(100 - RH) \quad (3.10)$$

where K_{SH} = 1.0 for pretensioned members. Table 3.1 gives the values of K_{SH} for post-tensioned members.

Adjustment of shrinkage losses for standard conditions as a function of time t in days after seven days for moist curing and three days for steam curing can be obtained from the following expressions (Branson, et.al, 1971):

a) Moist curing, after seven days:

$$(\epsilon_{SH})_t = \frac{t}{35 + t}(\epsilon_{SH})_u \quad (3.11a)$$

where $(\epsilon_{SH})_u$ is the ultimate shrinkage strain, t = time in days after shrinkage is considered.

b) Steam curing, after one to three days:

$$(\epsilon_{SH})_t = \frac{t}{55 + t}(\epsilon_{SH})_u \quad (3.11b)$$

Fig. 3.8 schematically shows shrinkage strain versus time.

It should be noted that separating creep from shrinkage calculations as presented in this chapter is an accepted engineering practice. Also, significant variations occur in the creep and shrinkage values due to variations in the properties of the constituent materials from the various sources, even if the products are plant-produced such as pretensioned beams. Hence, it is recommended that information from actual tests be obtained especially on manufactured products, large span-to-depth ratio cases and/or if loading is unusually heavy (Aswad, 1985, 1989, 1992).

3.4.4 *Friction losses in post-tensioned beams*-Relaxation losses are covered in 3.3.3. Loss of prestressing occurs in post-tensioned members due to friction between the tendons and the surrounding concrete ducts. The magnitude of this loss is a function of the tendon form or alignment, called the curvature effect, and the local deviations in the alignment, called the wobble effect. The values of the loss coefficients are affected by the types of tendons and the duct alignment. Whereas the curvature effect is predetermined, the wobble effect is the result of accidental or unavoidable misalignment, since ducts or sheaths cannot be perfectly held in place. Section 186.2 of ACI 318-89 and Table R18.6.2 of the Commentary to the code give the friction coefficients that apply to the friction loss in the various types of prestressing wires and tendons.

Table 3.1---Values of K_{SH} for post-tensioned members

Time from end of moist curing to application of prestress, days	1	3	5	7	10	20	30	60
K_{SH}	0.92	0.85	0.80	0.7	0.73	0.64	0.58	0.45

Source: Prestressed Concrete Institute

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