

Recent Approaches to Shear Design of Structural Concrete

Reported by Joint ACI-ASCE Committee 445



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Recent Approaches to Shear Design of Structural Concrete

Reported by Joint ACI-ASCE Committee 445

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Truss model approaches and related theories for the design of reinforced concrete members to resist shear are presented. Realistic models for the design of deep beams, corbels, and other nonstandard structural members are illustrated. The background theories and the complementary nature of a number of different approaches for the shear design of structural concrete are discussed. These relatively new procedures provide a unified, intelligible, and safe design framework for proportioning structural concrete under combined load effects.

Keywords: beams (supports); concrete; design; detailing; failure; models; shear strength; structural concrete; strut and tie.

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CHAPTER 1—INTRODUCTION

1.1—Scope and objectives

Design procedures proposed for regulatory standards should be safe, correct in concept, simple to understand, and should not necessarily add to either design or construction costs. These procedures are most effective if they are based on relatively simple conceptual models rather than on complex empirical equations. This report introduces design engineers to some approaches for the shear design of one-way structural concrete members. Although the approaches explained in the subsequent chapters of this report are relatively new, some of them have reached a sufficiently mature state that they have been implemented in codes of practice. This report builds upon the landmark state-of-the-art report by the ASCE-ACI Committee 426 (1973), *The Shear Strength of Reinforced Concrete Members*, which reviewed the large body of experimental work on shear and gave the background to many of the current American Concrete Institute (ACI) shear design provisions. After reviewing the many different empirical equations for shear design, Committee 426 expressed in 1973 the hope that “the design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner.”

The purpose of this report is to answer that challenge and review some of the new design approaches that have evolved since 1973 (CEB 1978, 1982; Walraven 1987; IABSE 1991a,b; Regan 1993). Truss model approaches and related theories are discussed and the common basis for these new approaches are highlighted. These new procedures provide a unified, rational, and safe design framework for structural concrete under combined actions, including the effects of axial load, bending, torsion, and prestressing.

Chapter 1 presents a brief historical background of the development of the shear design provisions and a summary of the current ACI design equations for beams. Chapter 2 discusses a sectional design procedure for structural-concrete one-way members using a compression field approach. Chapter 3 addresses several approaches incorpo-

rating the “concrete contribution.” It includes brief reviews of European Code EC2, Part 1 and the Comité Euro-International du Béton–Fédération International de la Précontrainte (CEB-FIP) Model Code, both based on strut-and-tie models. The behavior of members without or with low amounts of shear reinforcement is discussed in Chapter 4. An explanation of the concept of shear friction is presented in Chapter 5. Chapter 6 presents a design procedure using strut-and-tie models (STM), which can be used to design regions having a complex flow of stresses and may also be used to design entire members. Chapter 7 contains a summary of the report and suggestions for future work.

1.2—Historical development of shear design provisions

Most codes of practice use sectional methods for design of conventional beams under bending and shear. ACI Building Code 318M-95 assumes that flexure and shear can be handled separately for the worst combination of flexure and shear at a given section. The interaction between flexure and shear is addressed indirectly by detailing rules for flexural reinforcement cutoff points. In addition, specific checks on the level of concrete stresses in the member are introduced to ensure sufficiently ductile behavior and control of diagonal crack widths at service load levels.

In the early 1900s, truss models were used as conceptual tools in the analysis and design of reinforced concrete beams. Ritter (1899) postulated that after a reinforced concrete beam cracks due to diagonal tension stresses, it can be idealized as a parallel chord truss with compression diagonals inclined at 45° with respect to the longitudinal axis of the beam. Mörsch (1920, 1922) later introduced the use of truss models for torsion. These truss models neglected the contribution of the concrete in tension. Withey (1907, 1908) introduced Ritter’s truss model into the American literature and pointed out that this approach gave conservative results when compared with test evidence. Talbot (1909) confirmed this finding.

Historically, shear design in the United States has included a concrete contribution V_c to supplement the 45 degree sectional truss model to reflect test results in beams and slabs with little or no shear reinforcement and ensure economy in the practical design of such members. ACI Standard Specification No. 23 (1920) permitted an allowable shear stress of $0.025f'_c$, but not more than 0.41 MPa, for beams without web reinforcement, and with longitudinal reinforcement that did not have mechanical anchorage. If the longitudinal reinforcement was anchored with 180 degree hooks or with plates rigidly connected to the bars, the allowable shear stress was increased to $0.03f'_c$ or a maximum of 0.62 MPa (Fig. 1.1). Web reinforcement was designed by the equation

$$A_v F_v = V' s \sin \alpha / j d \quad (1-1)$$

where

A_v = area of shear reinforcement within distance s ;

f_v = allowable tensile stress in the shear reinforcement;

$j d$ = flexural lever arm;

- V' = total shear minus $0.02f'_c bjd$ (or $0.03f'_c bjd$ with special anchorage);
 b = width of the web;
 s = spacing of shear steel measured perpendicular to its direction; and
 α = angle of inclination of the web reinforcement with respect to the horizontal axis of the beam.

The limiting value for the allowable shear stresses at service loads was $0.06f'_c$ or a maximum of 1.24 MPa, or with anchorage of longitudinal steel $0.12f'_c$ or a maximum of 2.48 MPa. This shear stress was intended to prevent diagonal crushing failures of the web concrete before yielding of the stirrups. These specifications of the code calculated the nominal shear stress as $v = V/bjd$.

This procedure, which formed the basis for future ACI codes, lasted from 1921 to 1951 with each edition providing somewhat less-conservative design procedures. In 1951 the distinction between members with and without mechanical anchorage was omitted and replaced by the requirement that all plain bars must be hooked, and deformed bars must meet ASTM A 305. Therefore, the maximum allowable shear stress on the concrete for beams without web reinforcement (ACI 318-51) was $0.03f'_c$ and the maximum allowable shear stress for beams with web reinforcement was $0.12f'_c$.

ACI 318-51, based on allowable stresses, specified that web reinforcement must be provided for the excess shear if the shear stress at service loads exceeded $0.03f'_c$. Calculation of the area of shear reinforcement continued to be based on a 45 degree truss analogy in which the web reinforcement must be designed to carry the difference between the total shear and the shear assumed to be carried by the concrete.

The August 1955 shear failure of beams in the warehouse at Wilkins Air Force Depot in Shelby, Ohio, brought into question the traditional ACI shear design procedures. These shear failures, in conjunction with intensified research, clearly indicated that shear and diagonal tension was a complex problem involving many variables and resulted in a return to forgotten fundamentals.

Talbot (1909) pointed out the fallacies of such procedures as early as 1909 in talking about the failure of beams without web reinforcement. Based on 106 beam tests, he concluded that

It will be found that the value of v [shear stress at failure] will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam.... In beams without web reinforcement, web resistance depends upon the quality and strength of the concrete.... The stiffer the beam the larger the vertical stresses which may be developed. Short, deep beams give higher results than long slender ones, and beams with high percentage of reinforcement [give higher results] than beams with a small amount of metal.

Unfortunately, Talbot's findings about the influence of the percentage of longitudinal reinforcement and the length-to-depth ratio were not reflected in the design equations until much later. The research triggered by the 1956 Wilkins

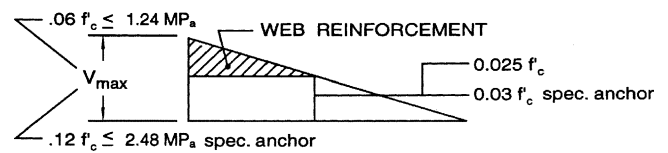


Fig. 1.1—American Specification for shear design (1920-1951) based on ACI Standard No. 23, 1920.

warehouse failures brought these important concepts back to the forefront.

More recently, several design procedures were developed to economize on the design of the stirrup reinforcement. One approach has been to add a concrete contribution term to the shear reinforcement capacity obtained, assuming a 45 degree truss (for example, ACI 318-95). Another procedure has been the use of a truss with a variable angle of inclination of the diagonals. The inclination of the truss diagonals is allowed to differ from 45 degree within certain limits suggested on the basis of the theory of plasticity. This approach is often referred to as the "standard truss model with no concrete contribution" and is explained by the existence of aggregate interlock and dowel forces in the cracks, which allow a lower inclination of the compression diagonals and the further mobilization of the stirrup reinforcement. A combination of the variable-angle truss and a concrete contribution has also been proposed. This procedure has been referred to as the modified truss model approach (CEB 1978; Ramirez and Breen 1991). In this approach, in addition to a variable angle of inclination of the diagonals, the concrete contribution for nonprestressed concrete members diminishes with the level of shear stress. For prestressed concrete members, the concrete contribution is not considered to vary with the level of shear stress and is taken as a function of the level of prestress and the stress in the extreme tension fiber.

As mentioned previously, the truss model does not directly account for the components of the shear failure mechanism, such as aggregate interlock and friction, dowel action of the longitudinal steel, and shear carried across uncracked concrete. For prestressed beams, the larger the amount of prestressing, the lower the angle of inclination at first diagonal cracking. Therefore, depending on the level of compressive stress due to prestress, prestressed concrete beams typically have much lower angles of inclined cracks at failure than nonprestressed beams and require smaller amounts of stirrups.

Traditionally in North American practice, the additional area of longitudinal tension steel for shear has been provided by extending the bars a distance equal to d beyond the flexural cutoff point. Although adequate for a truss model with 45 degree diagonals, this detailing rule is not adequate for trusses with diagonals inclined at lower angles. The additional longitudinal tension force due to shear can be determined from equilibrium conditions of the truss model as $V \cot \theta$, with θ as the angle of inclination of the truss diagonals. Because the shear stresses are assumed uniformly distributed over the depth of the web, the tension acts at the section middepth.

The upper limit of shear strength is established by limiting the stress in the compression diagonals f_d to the concrete cylinder strength f'_c . The concrete in the cracked web

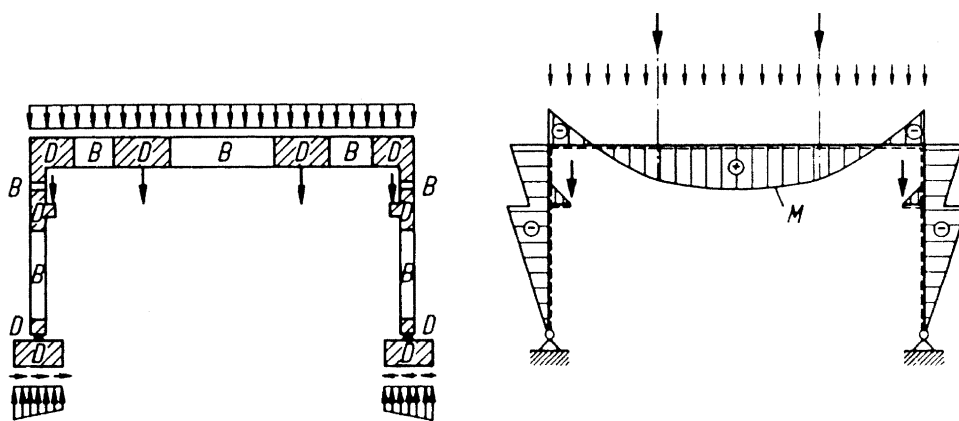


Fig. 1.2—Frame structure containing substantial part of B regions, its statical system, and bending moments (Schlaich et al. 1987).

of a beam is subjected to diagonal compressive stresses that are parallel or nearly parallel to the inclined cracks. The compressive strength of this concrete should be established to prevent web-crushing failures. The strength of this concrete is a function of 1) the presence or absence of cracks and the orientation of these cracks; 2) the tensile strain in the transverse direction; and 3) the longitudinal strain in the web. These limits are discussed in Chapters 2, 3, and 6.

The pioneering work from Ritter and Morsch received new impetus in the period from the 1960s to the 1980s, and therefore, in more recent design codes, modified truss models are used. Attention was focused on the truss model with diagonals having a variable angle of inclination as a viable model for shear and torsion in reinforced and prestressed concrete beams (Kupfer 1964; Cafilisch et al. 1971; Lampert and Thurlimann 1971; Thurlimann et al. 1983). Further development of plasticity theories extended the applicability of the model to nonyielding domains (Nielsen and Braestrup 1975; Muller 1978; Marti 1980). Schlaich et al. (1987) extended the truss model for beams with uniformly inclined diagonals, all parts of the structure in the form of STM. This approach is particularly relevant in regions where the distribution of strains is significantly nonlinear along the depth. Schlaich et al. (1987) introduced the concept of D and B regions, where D stands for discontinuity or disturbed, and B stands for beam or Bernoulli. In D regions the distribution of strains is nonlinear, whereas the distribution is linear in B regions. A structural-concrete member can consist entirely of a D region; however, more often D and B regions will exist within the same member or structure [see Fig. 1.2, from Schlaich et al. (1987)]. In this case, D regions extend a distance equal to the member depth away from any discontinuity, such as a change in cross section or the presence of concentrated loads. For typical slender members, the portions of the structure or member between D regions are B regions. The strut-and-tie approach is discussed in detail in Chapter 6.

By analyzing a truss model consisting of linearly elastic members and neglecting the concrete tensile strength, Kupfer (1964) provided a solution for the inclination of the

diagonal cracks. Collins and Mitchell (1980) abandoned the assumption of linear elasticity and developed the compression field theory (CFT) for members subjected to torsion and shear. Based on extensive experimental investigation, Vecchio and Collins (1982, 1986) presented the modified compression field theory (MCFT), which included a rationale for determining the tensile stresses in the diagonally cracked concrete. Although the CFT works well with medium to high percentages of transverse reinforcement, the MCFT provides a more realistic assessment for members having a wide range of amounts of transverse reinforcement, including the case of no web reinforcement. This approach is presented in Chapter 2. Parallel to these developments of the truss model with variable strut inclinations and the CFT, the 1980s also saw the further development of shear friction theory (Chapter 5). In addition, a general theory was developed for beams in shear using constitutive laws for friction and by determining the strains and deformations in the web. Because this approach considers the discrete formation of cracks, the crack spacing and crack width should be determined and equilibrium checked along the crack to evaluate the crack-slip mechanism of failure. This method is presented in Chapter 3. The topic of members without transverse reinforcement is dealt with in Chapter 4.

1.3—Overview of current ACI design procedures

The ACI 318M-95 sectional design approach for shear in one-way flexural members is based on a parallel truss model with 45-degree constant inclination diagonals supplemented by an experimentally obtained concrete contribution. The contribution from the shear reinforcement V_s for the case of vertical stirrups (as is most often used in North American practice), can be derived from basic equilibrium considerations on a 45-degree truss model with constant stirrup spacing s , and effective depth d . The truss resistance is supplemented with a concrete contribution V_c for both reinforced and prestressed concrete beams. Appendix A presents the more commonly used shear design equations for the concrete contribution in normalweight concrete beams, including effects of axial loading and the contribution from vertical stirrups V_s .

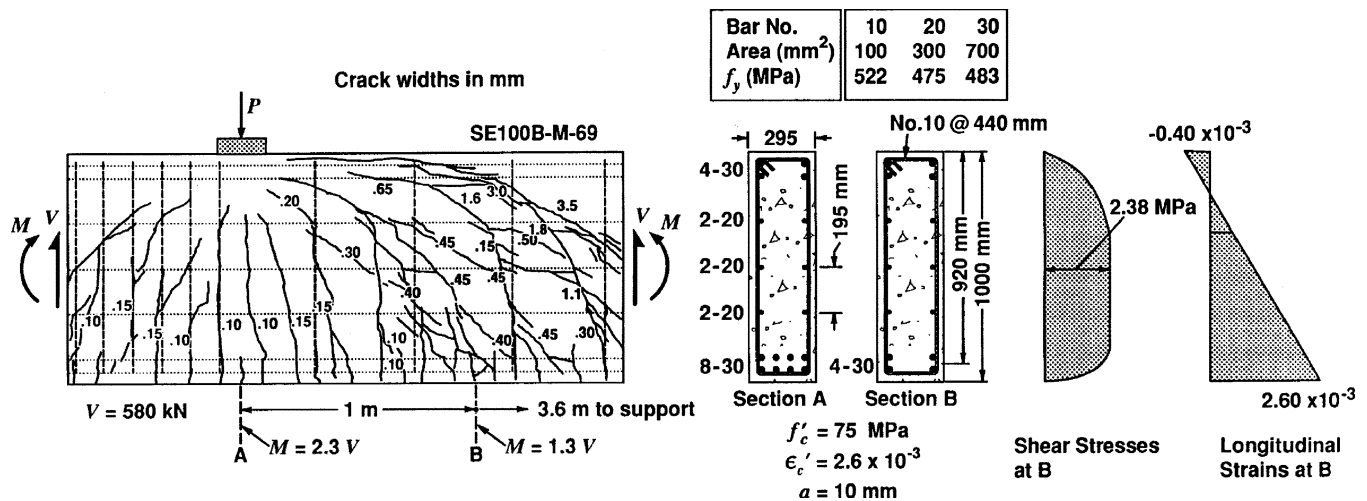


Fig. 2.1—Example of cracked web of beam failing in shear.

1.4—Summary

ACI 318 procedures have evolved into restricted, semiempirical approaches. The primary shortcomings of ACI 318M-95 are the many empirical equations and rules for special cases, and particularly the lack of a clear model that can be extrapolated to cases not directly covered. This situation would be improved if code approaches were based on clear and transparent physical models. Several of such models are discussed in subsequent chapters.

In Chapter 2, a sectional design approach using the MCFT is described. Chapter 3 discusses other truss models incorporating a concrete contribution and provides a brief review of some European code approaches.

The special case of members with no transverse shear reinforcement is addressed in Chapter 4. This chapter also presents an overview of the way the concrete contribution V_c is determined for beams. Chapter 5 presents a method of limit analysis in the form of a shear-friction mechanism. In Chapter 6, the generalized full member truss approach in the form of strut-and-tie systems for one-way flexural members is illustrated. Particular attention is given to the design approach in B and D regions, including the detailing of reinforcement ties, individual struts, and nodal zones.

The aim of this report is to describe these recent approaches to shear design and point out their common roots and complementary natures. This report does not endorse any given approach but provides a synthesis of these truss model-based approaches and related theories. The final goal of this report is to answer the challenge posed by Committee 426 over 20 years ago.

CHAPTER 2—COMPRESSION FIELD APPROACHES

2.1—Introduction

The cracked web of a reinforced concrete beam transmits shear in a relatively complex manner. As the load is increased, new cracks form while preexisting cracks spread and change inclination. Because the section resists moment as well as shear, the longitudinal strains and the crack inclinations vary over the depth of the beam (Fig. 2.1).

The early truss models of Ritter (1899) and Mörsch (1920, 1922) approximated this behavior by neglecting tensile stresses in the diagonally cracked concrete and assuming that the shear would be carried by diagonal compressive stresses in the concrete inclined at 45 degree to the longitudinal axis. The diagonal compressive concrete stresses push apart the top and bottom faces of the beam, while the tensile stresses in the stirrups pull them together. Equilibrium requires that these two effects be equal. According to the 45 degree truss model, the shear capacity is reached when the stirrups yield and will correspond to a shear stress of

$$v = \frac{A_v f_y}{b_w s} = \rho_v f_y \quad (2-1)$$

For the beam shown in Fig. 2.1, this equation would predict a maximum shear stress of only 0.80 MPa. As the beam actually resisted a shear stress of about 2.38 MPa, it can be seen that the 45 degree truss equation can be very conservative.

One reason why the 45 degree truss equation is often very conservative is that the angle of inclination of the diagonal compressive stresses measured from the longitudinal axis θ is typically less than 45 degrees. The general form of Eq. (2-1) is

$$v = \rho_v f_y \cot \theta \quad (2-2)$$

With this equation, the strength of the beam shown in Fig. 2.1 could be explained if θ was taken equal to 18.6 degrees. Most of the inclined cracks shown in Fig. 2.1 are not this flat.

Before the general truss equation can be used to determine the shear capacity of a given beam or to design the stirrups to resist a given shear, it is necessary to know the angle θ . Discussing this problem, Mörsch (1922) stated, "it is absolutely impossible to mathematically determine the slope of the secondary inclined cracks according to which one can design the stirrups." Just seven years after Mörsch made this statement, another German engineer, **Documents** **Document Sharing Hub**, solved an analogous problem while dealing with the shear

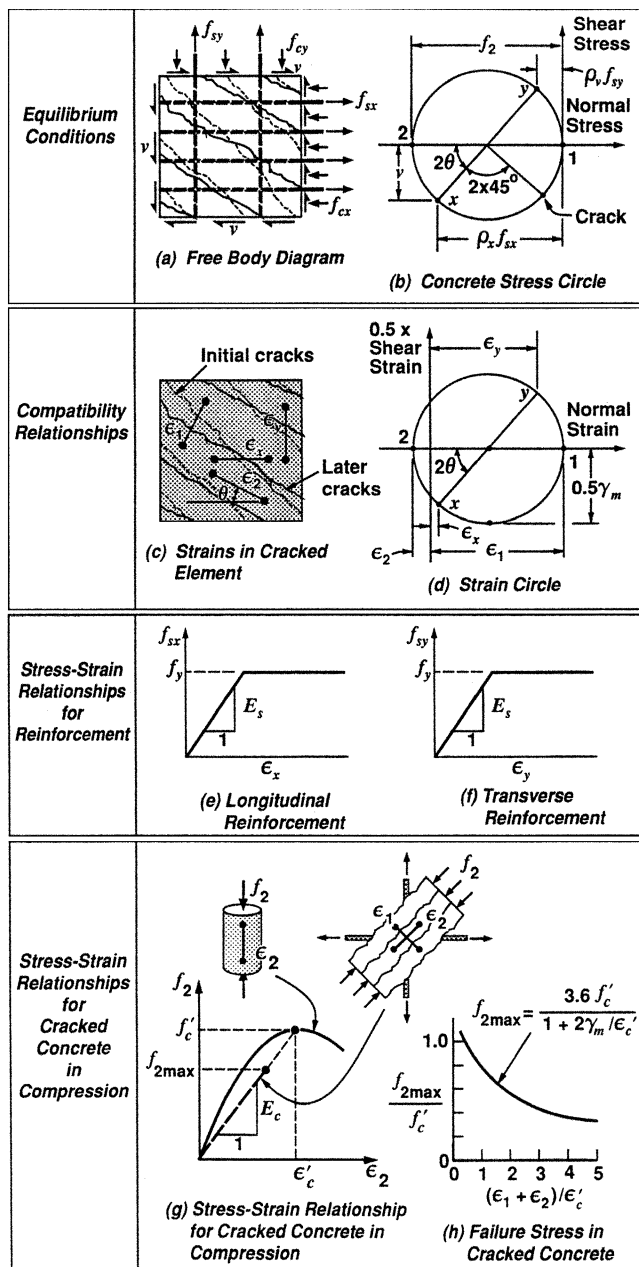


Fig. 2.2—Compression field theory (Mitchell and Collins 1974).

design of “stressed-skin” aircraft. Wagner assumed that after the thin metal skin buckled, it could continue to carry shear by a field of diagonal tension, provided that it was stiffened by transverse frames and longitudinal stringers. To determine the angle of inclination of the diagonal tension, Wagner considered the deformations of the system. He assumed that the angle of inclination of the diagonal tensile stresses in the buckled thin metal skin would coincide with the angle of inclination of the principal tensile strain as determined from the deformations of the skin, the transverse frames, and the longitudinal stringers. This approach became known as the tension field theory.

Shear design procedures for reinforced concrete that, like the tension field theory, determine the angle θ by considering the deformations of the transverse reinforcement, the longitu-

dinal reinforcement, and the diagonally stressed concrete have become known as compression field approaches. With these methods, equilibrium conditions, compatibility conditions, and stress-strain relationships for both the reinforcement and the diagonally cracked concrete are used to predict the load-deformation response of a section subjected to shear.

Kupfer (1964) and Baumann (1972) presented approaches for determining the angle θ assuming that the cracked concrete and the reinforcement were linearly elastic. Methods for determining θ applicable over the full loading range and based on Wagner’s procedure were developed by Collins and Mitchell (1974) for members in torsion and were applied to shear design by Collins (1978). This procedure was known as CFT.

2.2—Compression field theory

Figure 2.2 summarizes the basic relationships of the CFT. The shear stress v applied to the cracked reinforced concrete causes tensile stresses in the longitudinal reinforcement f_{sx} and the transverse reinforcement f_{sy} and a compressive stress in the cracked concrete f_2 inclined at angle θ to the longitudinal axis. The equilibrium relationships between these stresses can be derived from Fig. 2.2 (a) and (b) as

$$\rho_v f_{sy} = f_{cy} = v \tan \theta \quad (2-3)$$

$$\rho_x f_{sx} = f_{cx} = v \cot \theta \quad (2-4)$$

$$f_2 = v(\tan \theta + \cot \theta) \quad (2-5)$$

where ρ_x and ρ_v are the reinforcement ratios in the longitudinal and transverse directions.

If the longitudinal reinforcement elongates by a strain of ϵ_x , the transverse reinforcement elongates by ϵ_y , and the diagonally compressed concrete shortens by ϵ_2 , then the direction of principal compressive strain can be found from Wagner’s (1929) equation, which can be derived from Mohr’s circle of strain (Fig. 2.2(d)) as

$$\tan^2 \theta = \frac{\epsilon_x + \epsilon_2}{\epsilon_y + \epsilon_2} \quad (2-6)$$

Before this equation can be used to determine θ , however, stress-strain relationships for the reinforcement and the concrete are required. It is assumed that the reinforcement strains are related to the reinforcement stresses by the usual simple bilinear approximations shown in Fig. 2.2(e) and (f). Thus, after the transverse strain ϵ_y exceeds the yield strain of the stirrups, the stress in the stirrups is assumed to equal the yield stress f_y , and Eq. (2-3) becomes identical to Eq. (2-2).

Based on the results from a series of intensively instrumented beams, Collins (1978) suggested that the relationship between the principal compressive stress f_2 and the principal compressive strain ϵ_2 for diagonally cracked concrete would differ from the usual compressive stress-strain curve derived from a cylinder test (Fig. 2.2(g)). He postulated that as the strain circle becomes larger, the compressive stress required

to fail the concrete f_{2max} becomes smaller (Fig. 2.2(h)). The relationships proposed were

$$f_{2max} = \frac{3.6f'_c}{1 + 2\gamma_m/\epsilon'_c} \quad (2-7)$$

where

γ_m = diameter of the strain circle (that is, $\epsilon_1 + \epsilon_2$); and

ϵ'_c = strain at which the concrete in a cylinder test reaches the peak stress f'_c .

For values of f_2 less than f_{2max}

$$\epsilon_2 = \frac{f_2}{f'_c \epsilon'_c} \quad (2-8)$$

It was suggested that the diagonally cracked concrete fails at a low compressive stress because this stress must be transmitted across relatively wide cracks. If the initial cracks shown in Fig. 2.2(a) formed at 45 degrees to the longitudinal reinforcement, and if θ is less than 45 degrees, which will be the case if ρ_v is less than ρ_x , then significant shear stresses should be transmitted across these initial cracks (Fig. 2.2(b)). The ability of the concrete to transmit shear across cracks depends on the width of the cracks, which, in turn, is related to the tensile straining of the concrete. The principal tensile strain ϵ_1 can be derived from Fig. 2.2(d) as

$$\epsilon_1 = \epsilon_x + (\epsilon_x + \epsilon_2)\cot^2\theta \quad (2-9)$$

For shear stresses less than that causing first yield of the reinforcement, a simple expression for the angle θ can be derived by rearranging the previous equations to give

$$\tan^4\theta = \left(1 + \frac{1}{n\rho_x}\right) / \left(1 + \frac{1}{n\rho_v}\right) \quad (2-10)$$

where

n = modular ratio E_s/E_c ; and

E_c is taken as f'_c/ϵ'_c .

For the member shown in Fig. 2.1, ρ_x is 0.0303, ρ_v is 0.00154, and n = 6.93; therefore, Eq. (2-10) would give a θ value of 26.4 degrees. This would imply that the stirrups would yield at a shear stress of 1.62 MPa.

After the stirrups have yielded, the shear stress can still be increased if θ can be reduced. Reducing θ will increase the tensile stress in the longitudinal reinforcement and the compressive stress in the concrete. Failure will be predicted to occur either when the longitudinal steel yields or when the concrete fails. For the member shown in Fig. 2.1, failure of the concrete is predicted to occur when θ is lowered to 15.5 degrees, at which stage the shear stress is 2.89 MPa and ϵ_x is 1.73×10^{-3} . Note that these predicted values are for a section where the moment is zero. Moment will increase the longitudinal tensile strain ϵ_x , which will reduce the shear capacity. For example, if ϵ_x was increased to 2.5×10^{-3} , concrete

failure would be predicted to occur when θ is 16.7 degrees and the shear stress is 2.68 MPa.

2.3—Stress-strain relationships for diagonally cracked concrete

Since the CFT was published, a large amount of experimental research aimed at determining the stress-strain characteristics of diagonally cracked concrete has been conducted. This work has typically involved subjecting reinforced concrete elements to uniform membrane stresses in special-purpose testing machines. Significant experimental studies have been conducted by Aoyagi and Yamada (1983), Vecchio and Collins (1986), Kollegger and Mehlhorn (1988), Schlaich et al. (1987), Kirschner and Collins (1986), Bhide and Collins (1989), Shirai and Noguchi (1989), Collins and Porasz (1989), Stevens et al. (1991), Belarbi and Hsu (1991), Marti and Meyboom (1992), Vecchio et al. (1994), Pang and Hsu (1995), and Zhang (1995). A summary of the results of many of these studies is given by Vecchio and Collins (1993).

These experimental studies provide strong evidence that the ability of diagonally cracked concrete to resist compression decreases as the amount of tensile straining increases (Fig. 2.3). Vecchio and Collins (1986) suggested that the maximum compressive stress f_{2max} that the concrete can resist reduces as the average principal tensile strain ϵ_1 increases in the following manner

$$f_{2max} = \frac{f'_c}{0.8 + 170\epsilon_1} \leq f'_c \quad (2-11)$$

The Norwegian concrete code (1989) recommended a similar relationship except the coefficient of 170 was reduced to 100. Belarbi and Hsu (1995) suggested

$$f_{2max} = \frac{0.9f'_c}{\sqrt{1 + 400\epsilon_1}} \quad (2-12)$$

The various relationships for the reduction in compressive strength are compared with the experimental results from 73 element tests in Fig. 2.3. It can be seen that Eq. (2-11) lies near the middle of the data scatter band. For larger strains, Eq. (2-12) gives higher values to better fit some data at strains of up to 4%.

The compression field approach requires the calculation of the compressive strain in the concrete ϵ_2 associated with the compressive stress f_2 [Eq. (2-6)]. For this purpose, Vecchio and Collins (1986) suggested the following simple stress-strain relationship

$$f_2 = f_{2max} \left[2 \left(\frac{\epsilon_2}{\epsilon'_c} \right) - \left(\frac{\epsilon_2}{\epsilon'_c} \right)^2 \right] \quad (2-13)$$

where f_{2max} is given by Eq. (2-11).

Somewhat more complex expressions relating f_2 and ϵ_2 were suggested by Belarbi and Hsu (1995). They are

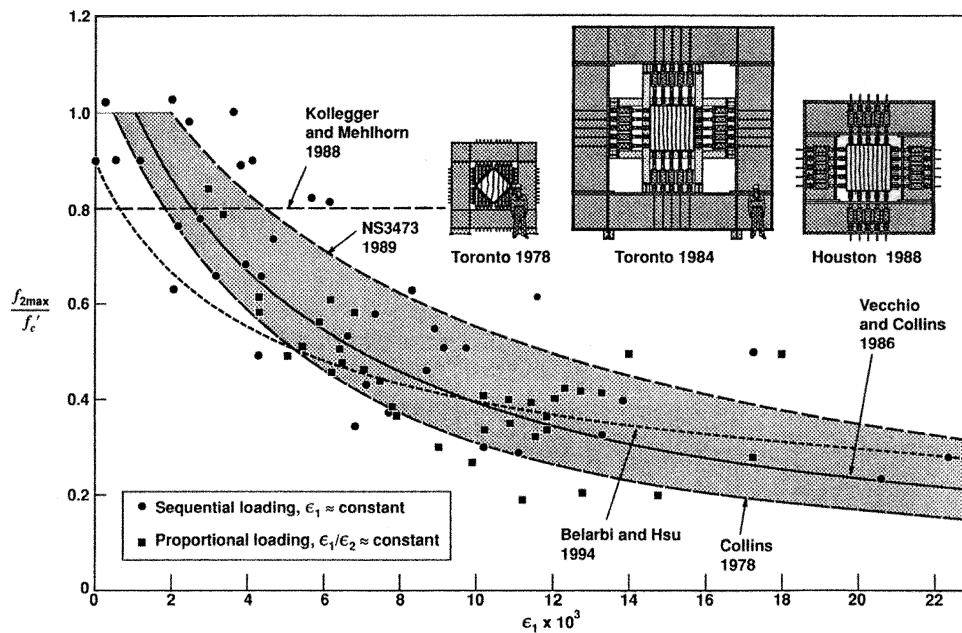
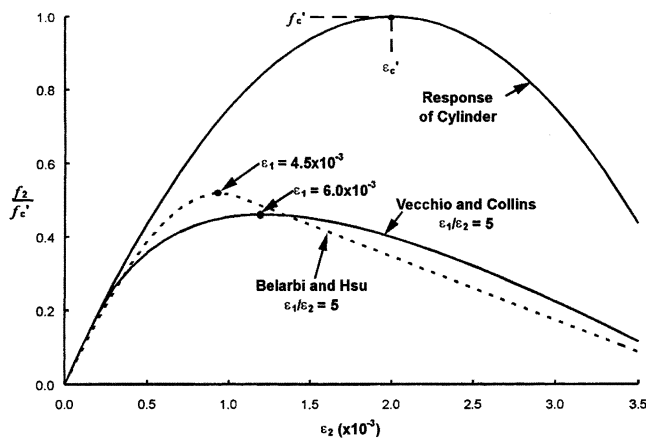
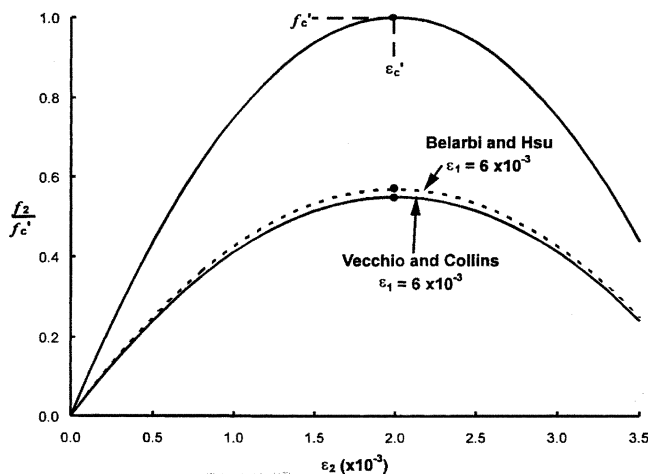


Fig. 2.3—Maximum concrete compressive stress as function of principal tensile strain.



(a) "Proportional Loading": ϵ_1 and ϵ_2 increased simultaneously



(b) "Sequential Loading": ϵ_1 applied first, then ϵ_2 increased

Fig. 2.4—Compressive stress-compressive strain relationships for diagonally cracked concrete: (a) proportion loading, ϵ_1 and ϵ_2 increased simultaneously; and (b) sequential loading ϵ_1 applied first then ϵ_2 increased.

$$f_2 = \zeta_{\sigma 0} f_c' \left[2 \left(\frac{\epsilon_2}{\zeta_{\epsilon 0} \epsilon_c'} \right) - \left(\frac{\epsilon_2}{\zeta_{\epsilon 0} \epsilon_c'} \right)^2 \right] \text{ if } \frac{\epsilon_2}{\zeta_{\epsilon 0} \epsilon_c'} \leq 1 \quad (2-14a)$$

$$f_2 = \zeta_{\sigma 0} f_c' \left[1 - \left(\frac{\epsilon_2 / \zeta_{\epsilon 0} \epsilon_c' - 1}{2 / \zeta_{\epsilon 0} - 1} \right)^2 \right] \text{ if } \frac{\epsilon_2}{\zeta_{\epsilon 0} \epsilon_c'} > 1 \quad (2-14b)$$

where, for "proportional loading"

$$\zeta_{\sigma 0} = \frac{0.9}{\sqrt{1 + 400\epsilon_1}} \quad \text{and} \quad \zeta_{\epsilon 0} = \frac{1}{\sqrt{1 + 500\epsilon_1}} \quad (2-14c)$$

and for "sequential loading"

$$\zeta_{\sigma 0} = \frac{0.9}{\sqrt{1 + 250\epsilon_1}} \quad \text{and} \quad \zeta_{\epsilon 0} = 1 \quad (2-14d)$$

As the cracked web of a reinforced concrete beam is subjected to increasing shear forces, both the principal compressive strain ϵ_2 and the principal tensile strain ϵ_1 are increased. Before yield of the reinforcement, the ratio ϵ_1/ϵ_2 remains reasonably constant. Figure 2.4(a) shows that the compressive stress-compressive strain relationships predicted by Eq. (2-13) and (2-14) for the case where the ratio ϵ_1/ϵ_2 is held constant at a value of 5 are similar. Figure 2.4(b) compares the relationship for the less realistic situation of holding ϵ_1 constant while increasing ϵ_2 . The predicted stress-strain relationships depend on the sequence of loading. Once again, the predictions of Eq. (2-13) and (2-14) are very similar.

For typical reinforced concrete beams, the percentage of longitudinal reinforcement ρ_x will greatly exceed the percentage of stirrup reinforcement ρ_y . In this situation there

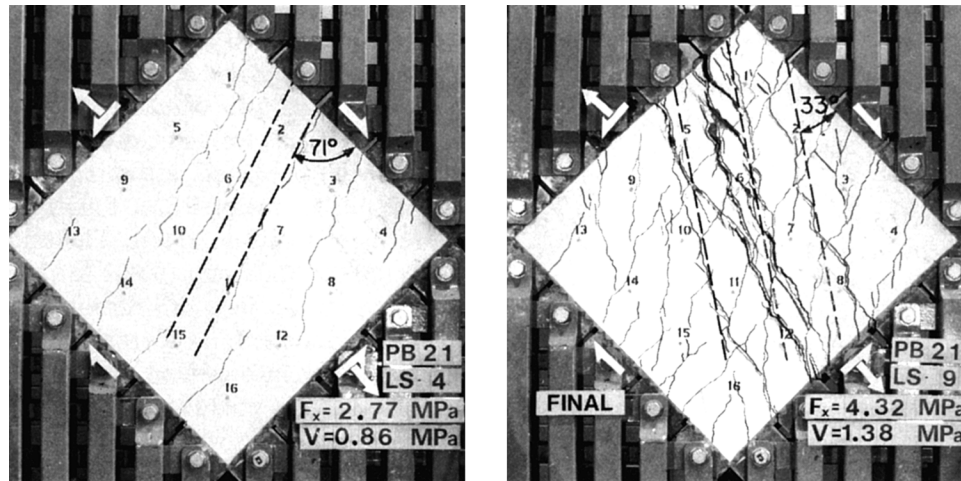


Fig. 2.5—Change of inclination of crack direction with increase in load.

will be a substantial reduction in the inclination θ of the principal compressive stresses after cracking. Figure 2.5 shows the observed crack patterns for a reinforced concrete element that contained reinforcement only in the direction of tension (x -direction) and was loaded in combined tension and shear. The first cracks formed at about 71 degrees to the x -axis. These initial cracks were quite narrow and remained reasonably constant in width throughout the test. As the load was increased, new cracks formed in directions closer to the reinforcement direction, and the width of these new cracks increased gradually. Failure was characterized by the rapid widening of the cracks that formed at about 33 degrees to the x -axis. For this extreme case, the direction of principal stress in the concrete differed by up to 20 degrees from the direction of principal strain (Bhide and Collins 1989). The predicted angle, based on Wagner's assumption that the principal stress direction coincides with the principal strain direction, lay about halfway between the observed strain direction and the observed stress direction. For elements with both longitudinal and transverse reinforcement, the directions of principal stress in the concrete typically deviated by less than 10 degrees from the directions of the principal strain (Vecchio and Collins 1986). Based on these results, it was concluded that determining the inclination of the principal stresses in the cracked concrete by Wagner's equation was a reasonable simplification.

The CFT assumes that after cracking there will be no tensile stresses in the concrete. Tests on reinforced concrete elements, such as that shown in Fig. 2.5, demonstrated that even after extensive cracking, tensile stresses still existed in the cracked concrete and that these stresses significantly increased the ability of the cracked concrete to resist shear stresses. It was found (Vecchio and Collins 1986; Belarbi and Hsu 1994) that after cracking the average principal tensile stress in the concrete decreases as the principal tensile strain increases. Collins and Mitchell (1991) suggest that a suitable relationship is

$$f_1 = \frac{0.33\sqrt{f'_c}}{1 + \sqrt{500\varepsilon_1}} \quad (\text{MPa}) \text{ units} \quad (2-15)$$

while Belarbi and Hsu (1994) suggest

$$f_1 = \frac{0.31\sqrt{f'_c}}{(12,500\varepsilon_1)^{0.4}} \quad (\text{MPa}) \text{ units} \quad (2-16)$$

Equation (2-16) predicts a faster decay for f_1 with increasing ε_1 than does Eq. (2-15). For example, for a 35 MPa concrete and an ε_1 value of 5×10^{-3} , Eq. (2-15) would predict an average tensile stress of 0.76 MPa, whereas Eq. (2-16) would predict 0.35 MPa.

2.4—Modified compression field theory

The MCFT (Vecchio and Collins 1986) is a further development of the CFT that accounts for the influence of the tensile stresses in the cracked concrete. It is recognized that the local stresses in both the concrete and the reinforcement vary from point to point in the cracked concrete, with high reinforcement stresses but low concrete tensile stresses occurring at crack locations. In establishing the angle θ from Wagner's equation, Eq. (2-6), the compatibility conditions relating the strains in the cracked concrete to the strains in the reinforcement are expressed in terms of average strains, where the strains are measured over base lengths that are greater than the crack spacing (Fig. 2.2(c) and (d)). In a similar manner, the equilibrium conditions, which relate the concrete stresses and the reinforcement stresses to the applied loads, are expressed in terms of average stresses; that is, stresses averaged over a length greater than the crack spacing. These relationships can be derived from Fig. 2.6(a) and (b) as

$$\rho_v f_{sy} = f_{cy} = v \tan \theta - f_1 \quad (2-17)$$

$$\rho_x f_{sx} = f_{cx} = v \cot \theta - f_1 \quad (2-18)$$

$$f_2 = v(\tan \theta + \cot \theta) - f_1 \quad (2-19)$$

These equilibrium equations, the compatibility conditions from Fig. 2.2(d), the reinforcement stress-strain relationships

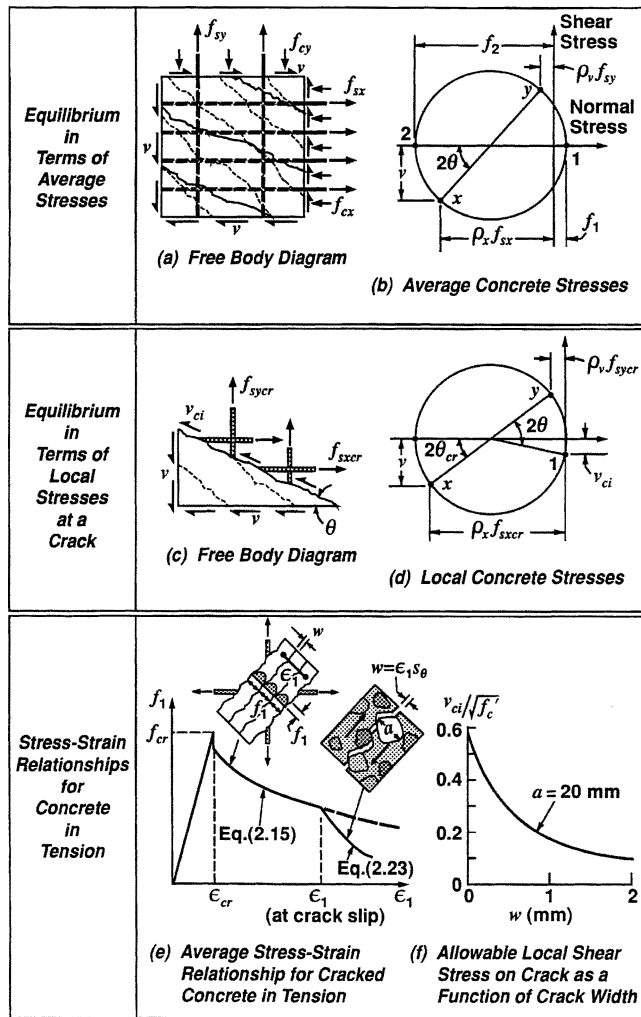


Fig. 2.6—Aspects of modified compression field theory.

from Fig. 2.2(e) and (f), and the stress-strain relationships for the cracked concrete in compression (Eq. (2-13)) and tension (Fig. 2.6(e)) enable the average stresses, the average strains, and the angle θ to be determined for any load level up to the failure.

Failure of the reinforced concrete element may be governed not by average stresses, but rather by local stresses that occur at a crack. In checking the conditions at a crack, the actual complex crack pattern is idealized as a series of parallel cracks, all occurring at angle θ to the longitudinal reinforcement and space a distance s_θ apart. From Fig. 2.6(c) and (d), the reinforcement stresses at a crack can be determined as

$$\rho_v f_{syer} = v \tan \theta - v_{ci} \tan \theta \quad (2-20)$$

$$\rho_x f_{sxxr} = v \cot \theta - v_{ci} \cot \theta \quad (2-21)$$

It can be seen that the shear stress v_{ci} on the crack face reduces the stress in the transverse reinforcement but increases the stress in the longitudinal reinforcement. The maximum possible value of v_{ci} is taken (Bhide and Collins 1989) to be related to the crack width w and the maximum

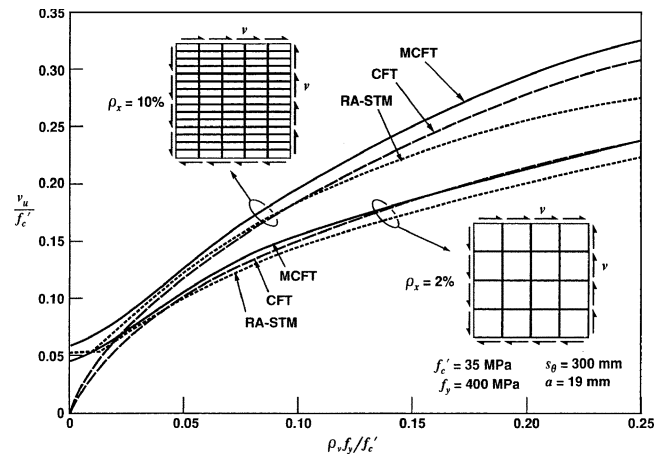


Fig. 2.7—Comparison of predicted shear strengths of two series of reinforced concrete elements.

aggregate size a by the relationship illustrated in Fig. 2.6(f) and given by

$$v_{ci} \leq 0.18 \sqrt{f_c'} \left(0.3 + \frac{24w}{a+16} \right) \quad (\text{MPa, mm}) \quad (2-22)$$

The crack width w is taken as the crack spacing times the principal tensile strain ϵ_1 . At high loads, the average strain in the stirrups ϵ_y will typically exceed the yield strain of the reinforcement. In this situation, both f_{sy} in Eq. (2-17) and f_{syer} in Eq. (2-20) will equal the yield stress in the stirrups. Equating the right-hand sides of these two equations and substituting for v_{ci} from Eq. (2-22) gives

$$f_1 \leq 0.18 \sqrt{f_c'} \tan \theta \left(0.3 + \frac{24w}{a+16} \right) \quad (2-23)$$

Limiting the average principal tensile stress in the concrete in this manner accounts for the possibility of failure of the aggregate interlock mechanisms, which are responsible for transmitting the interface shear stress v_{ci} across the crack surfaces.

Figure 2.7 illustrates the influence of the tensile stresses in the cracked concrete on the predicted shear capacity of two series of reinforced concrete elements. In this figure, RA-STM stands for rotating-angle softened-truss model. If tensile stresses in the cracked concrete are ignored, as is done in the CFT, elements with no stirrups ($\rho_v = 0$) are predicted to have no shear strength. When these tensile stresses are accounted for, as is done in the MCFT, even members with no stirrups are predicted to have significant postcracking shear strengths. Figure 2.7 shows that predicted shear strengths are a function not only of the amount of stirrup reinforcement, but also of the amount of longitudinal reinforcement. Increasing the amount of longitudinal reinforcement increases the shear capacity. Increasing the amount of longitudinal reinforcement also increases the difference between the CFT prediction and the MCFT prediction for the elements with 2% of longitudinal reinforcement and with

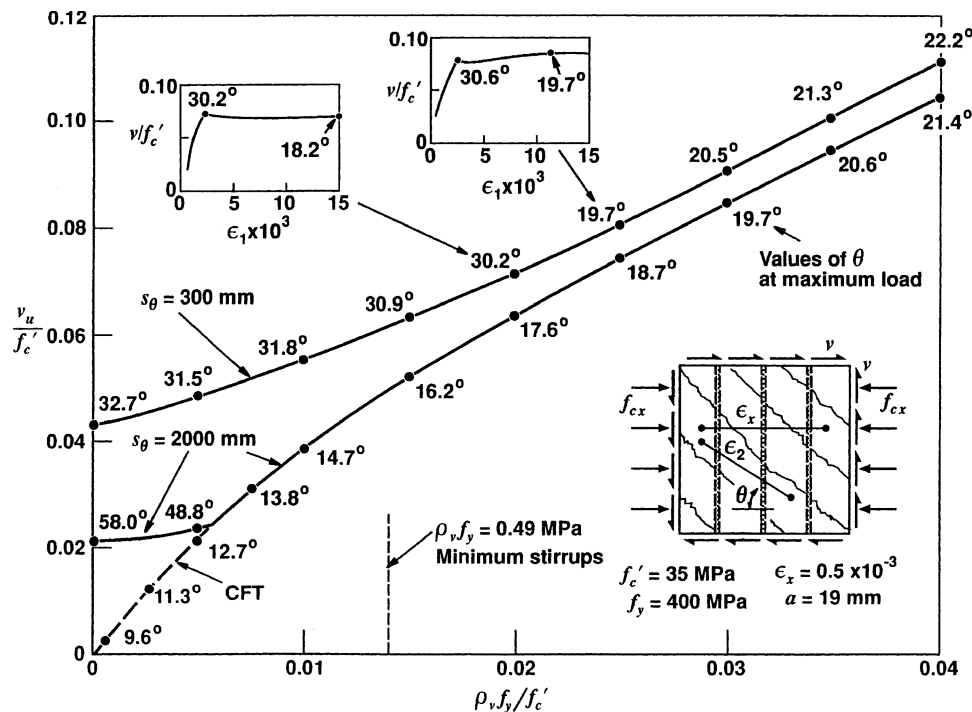


Fig. 2.8—Influence of crack spacing on predicted shear capacity.

$\rho_v f_y/f_c'$ greater than about 0.10, yielding of the longitudinal reinforcement at a crack (Eq. (2-21)) limits the maximum shear capacity. In this situation, the influence of the tensile stresses in the cracked concrete on the predicted shear strength is negligibly small. On the other hand, when the total longitudinal reinforcement is 10% of the web area, this longitudinal reinforcement remains well below yield stress, and the failure, for larger amounts of stirrups, is then governed by crushing of the concrete. The tensile stresses in the cracked concrete stiffen the element, reduce the concrete strains, and make it possible to resist larger shear stresses before failure. The predicted shear strength of elements that contain relatively small amounts of stirrups are influenced by the spacing of the diagonal cracks s_θ . If this spacing is increased, the crack width, w , associated with a given value of ϵ_1 increases, and the tension transmitted through the cracked concrete decreases (Eq. (2-23)). This aspect of behavior is illustrated in Fig. 2.8, which compares the shear capacities predicted by the MCFT for two series of elements. In one series, the crack spacing is taken as 300 mm, whereas in the other, it is taken as 2000 mm. It is assumed that the amount of longitudinal reinforcement and the axial loading of the elements is such that the longitudinal strain, ϵ_x , is held constant at 0.5×10^{-3} . It can be seen that the predicted shear capacity becomes more sensitive to crack spacing as the amount of stirrup reinforcement, ρ_v , is reduced. When ρ_v is zero, the element with a 2000 mm crack spacing is predicted to have only about half the shear capacity of the element with a 300 mm crack spacing.

2.5—Rotating-angle softened-truss model

A somewhat different procedure to account for tensile stresses in diagonally cracked concrete has been developed by

Hsu and his coworkers at the University of Houston (Belarbi and Hsu 1991, 1994, 1995; Pang and Hsu 1995; Hsu 1993). This procedure is called the rotating-angle softened-truss model (RA-STM). Like the MCFT, this method assumes that the inclination of the principal stress direction θ in the cracked concrete coincides with the principal strain direction. For typical elements, this angle will decrease as the shear is increased, hence the name rotating angle. Pang and Hsu (1995) limit the applicability of the rotating-angle model to cases where the rotating angle does not deviate from the fixed angle by more than 12 degrees. Outside this range, they recommend the use of a fixed-angle model, which is discussed in Section 3.5.

The method formulates equilibrium equations in terms of average stresses (Fig. 2.6(b)) and compatibility equations in terms of average strains (Fig. 2.2(d)). The softened stress-strain relationship of Eq. (2-14) is used to relate the principal compressive stress in the concrete f_2 to the principal compressive strain ϵ_2 , whereas Eq. (2-16) is used to relate the average tensile stress in the concrete f_1 to the principal tensile strain ϵ_1 .

Instead of checking the stress conditions at a crack, as is done by the MCFT, the RA-STM adjusts the average stress-average strain relationships of the reinforcement to account for the possibility of local yielding at the crack. The relationships suggested are

$$f_s = E_s \epsilon_s \text{ if } \epsilon_s \leq \epsilon_n \quad (2-24a)$$

$$f_s = f_y \left[(0.91 - 2B) + (0.02 + 0.25B) \frac{E_s}{f_y} \epsilon_s \right] \quad (2-24b)$$

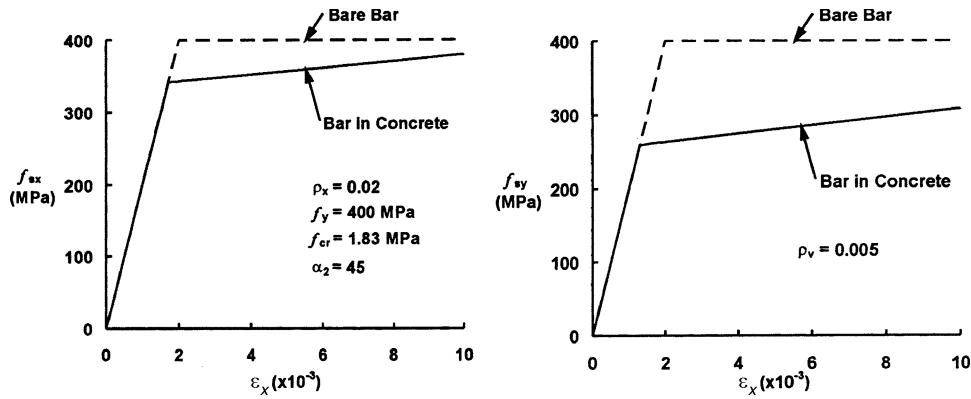


Fig. 2.9—Average-reinforcement-stress/average-reinforcement-strain relationships used in rotating-angle softened-truss model.

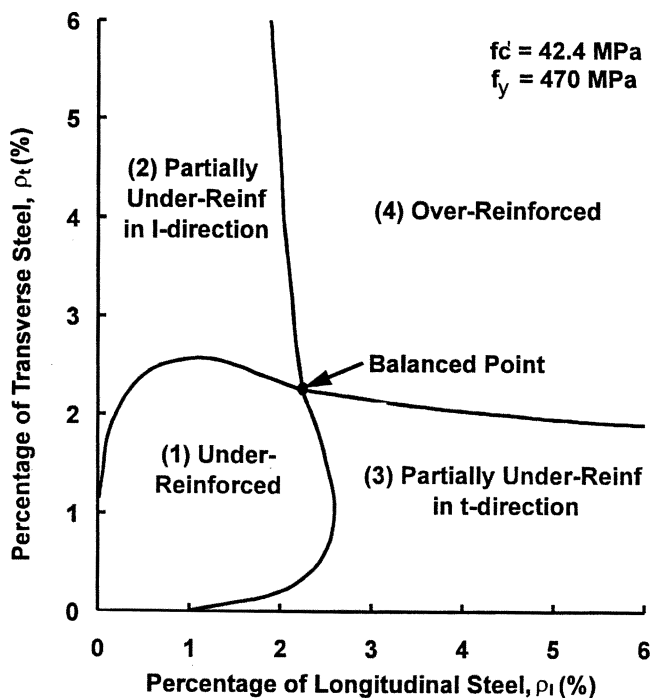


Fig. 2.10—Four failure modes predicted by rotating-angle softened-truss model for element loaded in pure shear (adapted from Pang and Hsu [1995]).

$$\epsilon_n = \frac{f_y}{E_s} (0.93 - 2B) \left[1 - \frac{2 - \alpha_2/45}{1000\rho} \right] \quad (2-24c)$$

$$B = \left(\frac{f_{cr}}{f_y} \right)^{1.5} / \rho \quad (2-24d)$$

where

ρ = reinforcement ratio; and
 α_2 = angle between the initial crack direction and the longitudinal reinforcement.

The resulting relationships for a case where the longitudinal reinforcement ratio is 0.02 and the transverse reinforcement ratio is 0.005 are illustrated in Fig. 2.9.

Figure 2.7 shows the shear strength-predictions for the RA-STM for the two series of reinforced concrete elements. Although the MCFT and the RA-STM give similar predictions for low amounts of reinforcement, the predicted strengths using the RA-STM are somewhat lower than those given by the MCFT for elements with higher amounts of reinforcement.

Both the MCFT and the RA-STM are capable of predicting not only the failure load but also the mode of failure. Thus, as shown in Fig. 2.10, a reinforced concrete element loaded in pure shear can fail in four possible modes. Both the longitudinal and transverse reinforcement can yield at failure (Mode 1), only the longitudinal reinforcement yields at failure (Mode 2), only the transverse reinforcement yields at failure (Mode 3), or neither reinforcement yields at failure (Mode 4).

2.6—Design procedure based on modified compression field theory

The relationships of the MCFT can be used to predict the shear strength of a beam such as that shown in Fig. 2.11. Assuming that the shear stress in the web is equal to the shear force divided by the effective shear area $b_w d_v$, and that, at failure, the stirrups will yield, equilibrium equations (2-17) can be rearranged to give the following expression for the shear strength V_n of the section

$$V_n = V_c + V_s + V_p \quad (2-25a)$$

$$V_n = f_1 b_w d_v \cot \theta + \frac{A_v f_y}{s} d_v \cot \theta + V_p \quad (2-25b)$$

$$V_n = \beta \sqrt{f'_c} b_w d_v + \frac{A_v f_y}{s} d_v \cot \theta + V_p \quad (2-25c)$$

where

V_c = shear strength provided by tensile stresses in the cracked concrete;
 V_s = shear strength provided by tensile stresses in the stirrups;
 V_p = vertical component of the tension in prestressing tendon;

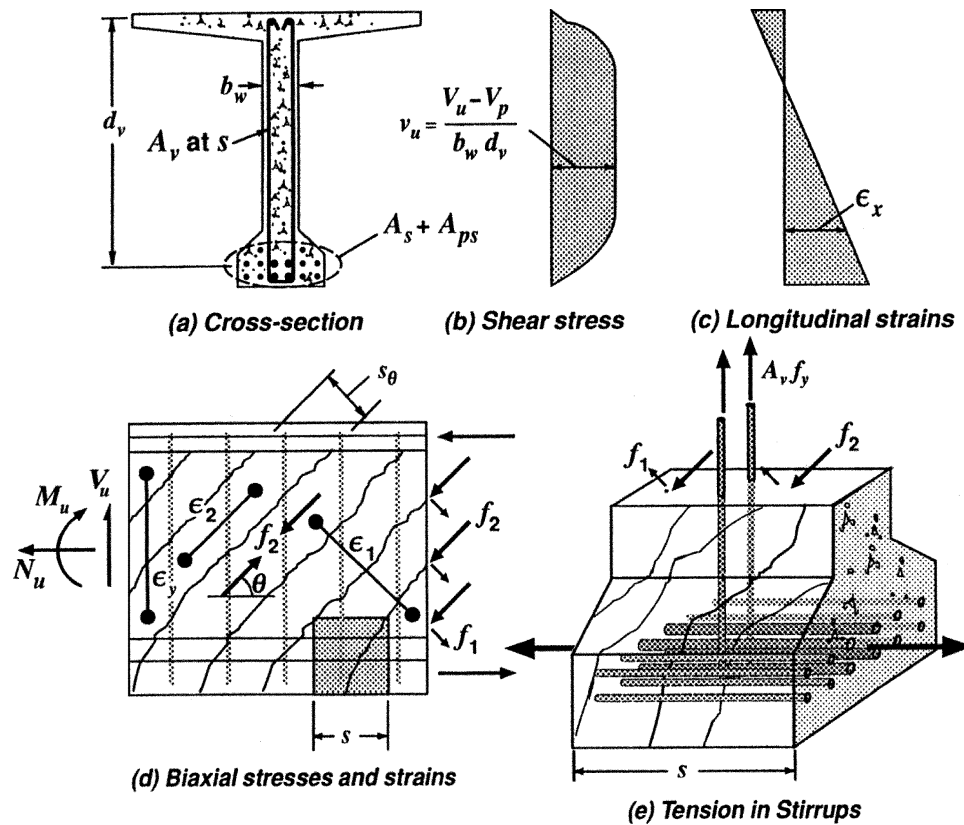


Fig. 2.11—Beam subjected to shear, moment, and axial load.

- b_w = effective web width, taken as the minimum web width within the shear depth;
- d_v ; d_v = effective shear depth, taken as the flexural lever arm, but which need not be taken less than $0.9d$; and
- β = concrete tensile stress factor indicating the ability of diagonally cracked concrete to resist shear.

The shear stress that the web of a beam can resist is a function of the longitudinal straining in the web. The larger this longitudinal straining becomes, the smaller the shear stress required to fail the web. In determining the shear capacity of the beam, it is conservative to use the highest longitudinal strain ϵ_x occurring within the web. For design calculations, ϵ_x can be approximated as the strain in the tension chord of the equivalent truss. Therefore

$$\epsilon_x = \frac{(M_u/d_v) + 0.5N_u + 0.5V_u \cot \theta - A_{ps}f_{po}}{E_s A_s + E_p A_{ps}} \quad (2-26)$$

but need not be taken greater than 0.002, where

- f_{po} = stress in the tendon when the surrounding concrete is at zero stress, which may be taken as 1.1 times the effective stress in the prestressing f_{se} after all losses;
- A_s = area of nonprestressed longitudinal reinforcement on the flexural tension side of the member;
- A_{ps} = area of prestressed longitudinal reinforcement on the flexural tension side of the member;
- M_u = moment at the section, taken as positive; and
- N_u = axial load at the section, taken as positive if tensile and negative if compressive.

The determination of ϵ_x for a nonprestressed beam is illustrated in Fig. 2.12.

The longitudinal strain parameter, ϵ_x , accounts for the influence of moment, axial load, prestressing, and amount of longitudinal reinforcement on the shear strength of a section. If ϵ_x and the crack spacing s_θ are known, the shear capacity corresponding to a given quantity of stirrups can be calculated (Fig. 2.8). This is equivalent to finding the values of β and θ in Eq. (2-25).

Values of β and θ determined from the modified compression field theory (Vecchio and Collins 1986) and suitable for members with at least minimum web reinforcement are given in Fig. 2.13. In determining these values, it was assumed that the amount and spacing of the stirrups would limit the crack spacing to about 300 mm. The θ values given in Fig. 2.13 ensure that the tensile strain in the stirrups, ϵ_v , is at least equal to 0.002 and that the compressive stress, f_2 , in the concrete does not exceed the crushing strength f_{2max} . Within the range of values of θ that satisfy these requirements, the values given in Fig. 2.13 result in close to the smallest amount of total shear reinforcement being required to resist a given shear.

2.6.1 Minimum shear reinforcement—A minimum amount of shear reinforcement is required to control diagonal cracking and provide some ductility. In ACI 318M-95, this amount is specified as

$$\frac{A_v f_y}{b_w s} > 0.33 \text{ MPa} \quad (2-27)$$

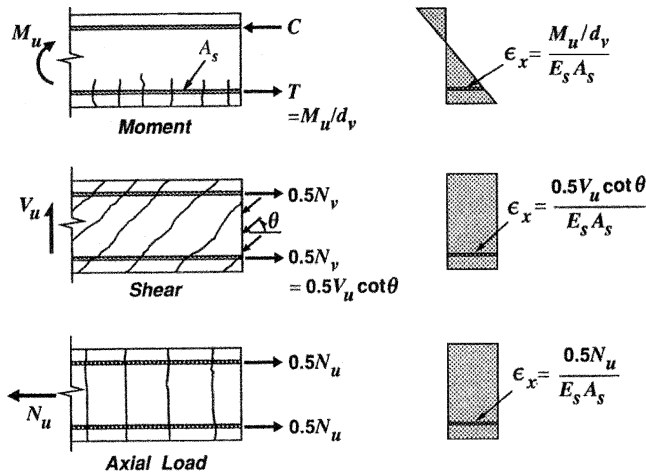


Fig. 2.12—Determination of strains, ϵ_x , for nonprestressed beam.

The AASHTO specifications (1994) relate the minimum reinforcement required to the concrete strength and require a larger quantity of stirrups for high-strength concrete. The AASHTO specifications require that

$$\frac{A_v f_y}{b_w s} > 0.083 \sqrt{f'_c} \quad \text{MPa} \quad (2-28)$$

This type of relationship was found to give reasonable estimates compared with experiments conducted by Yoon et al. (1996). There is some concern, however, that the equation may not be conservative enough for large reinforced concrete members that contain low percentages of longitudinal reinforcement (ASCE-ACI 426 1973).

2.6.2 Example: Determine stirrup spacing in reinforced concrete beam—To illustrate the method, it will be used to determine the stirrup spacing at Section B for the member tested as shown in Fig. 2.1, which will result in a predicted shear strength of 580 kN with a capacity-reduction factor of 1.0

$$\frac{v_u}{f'_c} = \frac{V_u}{b_w d_v f'_c} = \frac{580,000}{295 \times 0.9 \times 920 \times 75} = 0.032$$

From Eq. (2-26)

$$\epsilon_x = \frac{(M_u/d_v) + 0.5 V_u \cot \theta}{E_s A_s}$$

$$\epsilon_x = \frac{(580,000 \times 1300/828) + 0.5 \times 580,000 \cot \theta}{200,000 (4 \times 700 + 3 \times 300)}$$

$$\epsilon_x = 1.23 \times 10^{-3} + 0.329 \times 10^{-3} \cot \theta$$

Figure 2.13 shows that if v_u/f'_c is less than 0.05 and ϵ_x is between 1.5×10^{-3} and 2×10^{-3} , θ is about 42 degrees. With this value of θ , the calculated value of ϵ_x is 1.67×10^{-3} and β is approximately 0.155, according to Fig. 2.13. Equation (2-25) then becomes

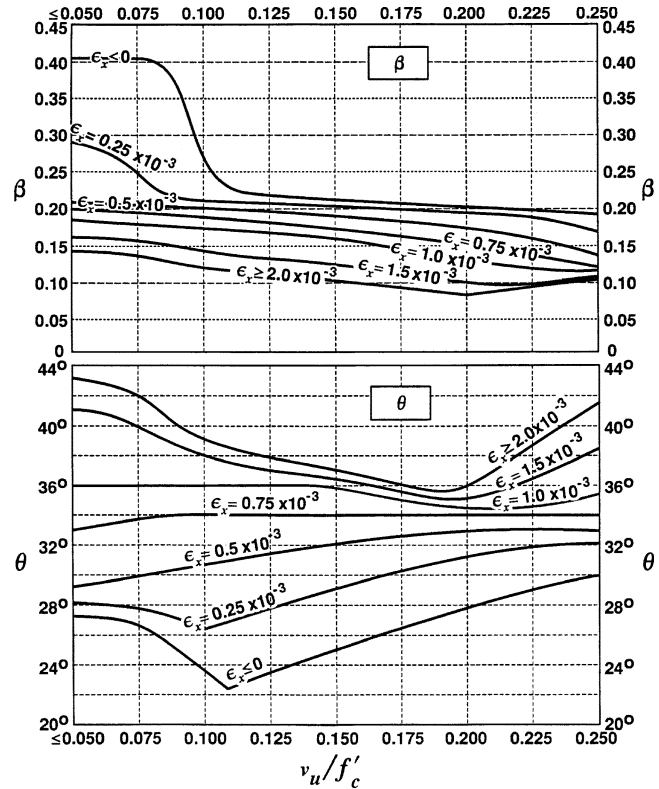


Fig. 2.13—Values of β and θ for members containing at least the minimum amount of stirrups.

$$580,000 = 0.155 \sqrt{75} \times 295 \times 828 + \frac{200 \times 522}{s}$$

$$\times 828 \cot 42 \text{ degrees}$$

which gives a required stirrup spacing of 381 mm. Note that because the tested beam contained stirrups at a spacing of 440 mm, the design method is a little conservative.

2.6.3 Example: Determine stirrup spacing in a prestressed concrete beam—If the member had also contained a straight post-tensioned tendon near the bottom face consisting of six 13 mm strands with an effective stress of 1080 MPa, the calculations for the required stirrup spacing would change in the following manner. Equation (2-26) becomes

$$\epsilon_x \times \{[(580,000 \times 1300/828) + 0.5 \times 580,000 \cot \theta - 6 \times 99 \times 1.1 \times 1080]/[200,000(4 \times 700 + 3 \times 300 + 6 \times 99)]\}$$

Figure 2.13 shows that if v_u/f'_c is less than 0.05 and ϵ_x is approximately 0.75×10^{-3} , θ is about 33 degrees. With this value of θ , the calculated value of ϵ_x is 0.76×10^{-3} . For this value of ϵ_x , the value of β is approximately 0.155, according to Fig. 2.13. Equation (2-25) then becomes

$$580,000 = 0.20 \sqrt{75} \times 295 \times 828 + \frac{200 \times 522}{s}$$

$$\times 828 \cot 33 \text{ degrees}$$

which gives a required stirrup spacing of 848 mm

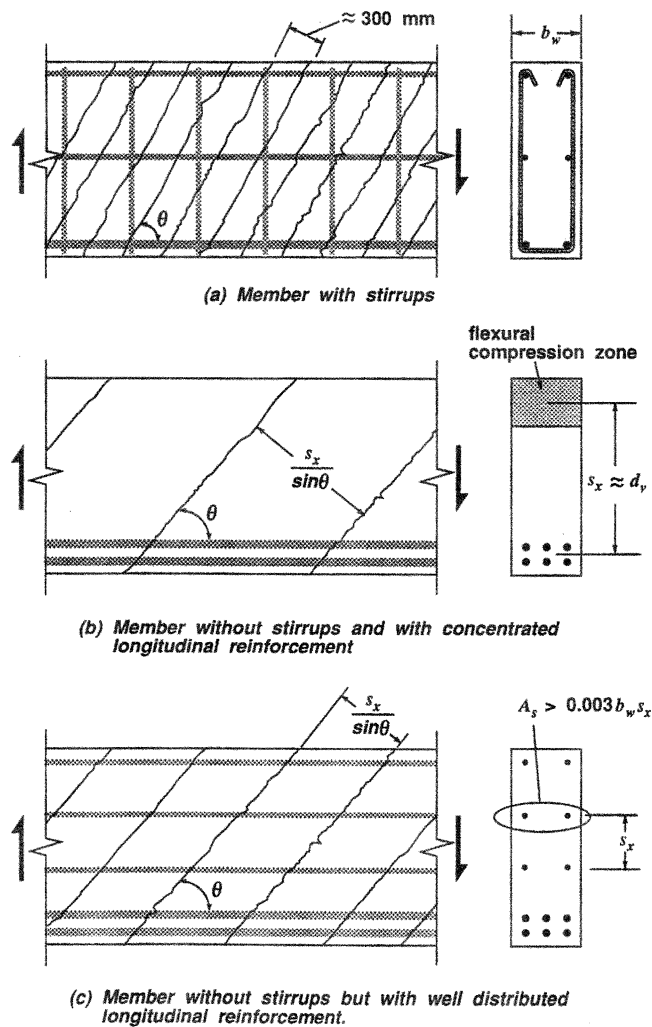


Fig. 2.14—Influence of reinforcement on spacing of diagonal cracks.

Stirrups spaced at 848 mm in the member shown in Fig. 2.1 would not provide adequate crack control, and the assumption that the crack spacing was about 300 mm would no longer be valid. The AASHTO specifications (1994) require that the stirrup spacing not exceed $0.8d_v$ or 600 mm.

To satisfy the minimum stirrup amount requirement of Eq. (2-28), the stirrup spacing for the beam shown in Fig. 2.1 should be

$$s < \frac{200 \times 522}{295 \times 0.083 \sqrt{75}} = 492 \text{ mm}$$

2.6.4 Design of member without stirrups—For members without stirrups or with less than the minimum amount of stirrups, the diagonal cracks will typically be more widely spaced than 300 mm. For these members, the diagonal cracks will become more widely spaced as the inclination of the cracks θ is reduced (Fig. 2.5 and 2.14). The crack spacing when θ equals 90 degrees is called s_x , and this spacing is primarily a function of the maximum distance between the longitudinal reinforcing bars (Fig. 2.14). As shown in Fig. 2.5 the values of β and θ for members with less than the

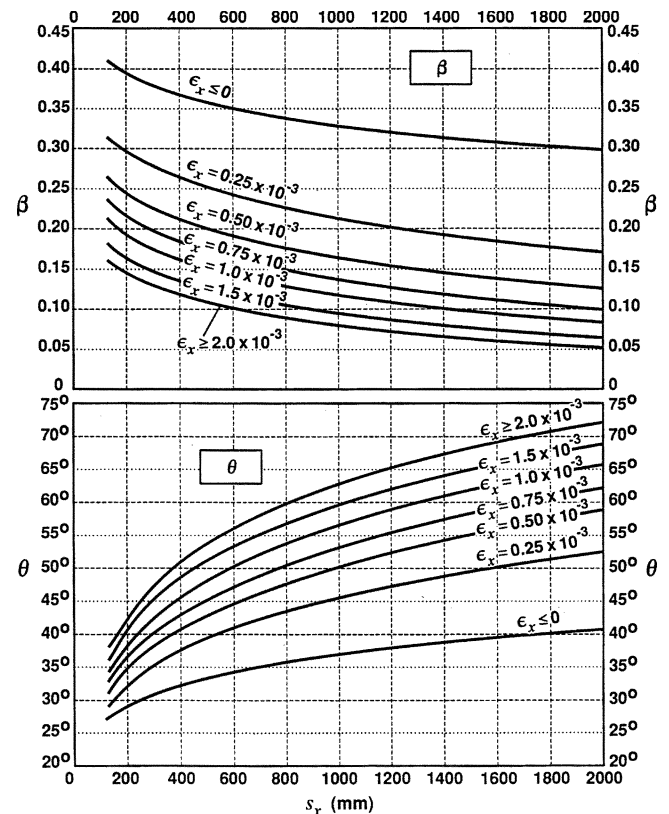


Fig. 2.15—Values of β and θ for members containing less than the minimum amount of stirrups.

minimum amount of stirrups depend on the longitudinal strain parameter, ϵ_x , and the crack spacing parameter, s_x , where s_x need not be taken as greater than 2000 mm. As s_x increases, β decreases, and the predicted shear strength decreases.

The β and θ values given in Fig. 2.15 were calculated assuming that the maximum aggregate size a is 19 mm. The values can be used for other aggregate sizes by using an equivalent crack spacing parameter

$$s_{xe} = s_x \frac{35}{a + 16} \quad (2-29)$$

If the member shown in Fig. 2.1 did not contain any stirrups, the shear strength at Section B could be predicted from Fig. 2.15 in the following manner. As the longitudinal bars are spaced 195 mm apart and the maximum aggregate size is 10 mm, the crack spacing parameter is

$$s_{xe} = 195 \frac{35}{10 + 16} = 263 \text{ mm}$$

If ϵ_x is estimated to be 1.0×10^{-3} , then from Fig. 2.15, θ is about 41 degrees and β is about 0.18. Equation (2-25) then gives

$$V_n = 0.18 \sqrt{75} \times 295 \times 828 = 381 \text{ kN}$$

whereas Eq. (2-26) gives  **Documents**
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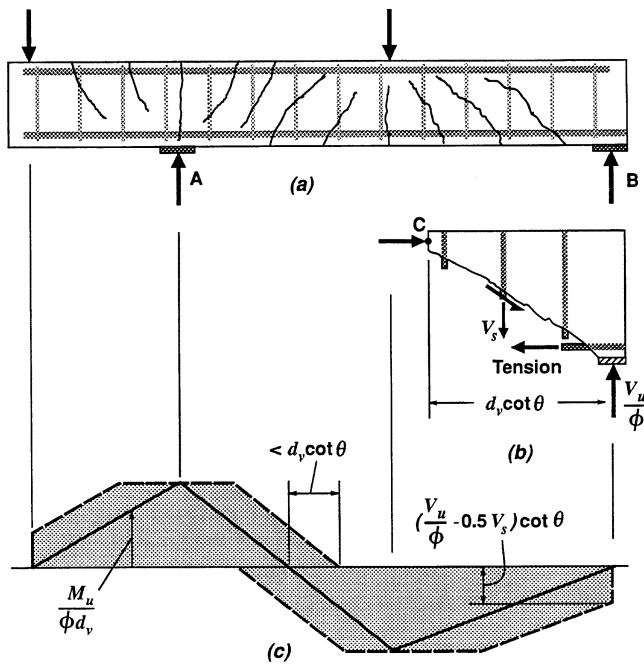


Fig. 2.16—Influence of shear on tension in longitudinal reinforcement.

$$\epsilon_x = \frac{(381,000 \times 1300/828) + 0.5 \times 381,000 \cot 41^\circ}{200,000(4 \times 700 + 3 \times 300)}$$

$$= 1.10 \times 10^{-3}$$

Using the calculated value of ϵ_x of 1.10×10^{-3} , β would be about 0.17, and a more accurate estimate of the failure shear would be 360 kN.

If the member did not contain the six 20 mm diameter skin-reinforcement bars, the crack spacing parameter would increase to

$$s_{xe} = 828 \frac{35}{10 + 16} = 1115 \text{ mm}$$

If ϵ_x is estimated to be 0.75×10^{-3} , then from Fig. 2.15, θ is about 55.5 degrees and β is about 0.13. With these values, V_n would be reduced to 275 kN and ϵ_x would be calculated as 0.71×10^{-3} . A second iteration would give the predicted shear strength as 283 kN.

2.6.5 Additional design considerations—In the design approach based on the MCFT, the stirrups required at a particular section can be determined from Eq. (2-25) as

$$V_s \geq \frac{V_u}{\phi} - V_c - V_p \quad (2-30)$$

given the shear, moment, and axial load acting at the section. Although this calculation is performed for a particular section, shear failure caused by yielding of the stirrups involves yielding this reinforcement over a length of beam of about $d_v \cot \theta$. Therefore, a calculation for one section can be taken as representing a length of beam $d_v \cot \theta$ long with the calcu-

lated section being in the middle of this length. Near a support, the first section checked is the section $0.5d_v \cot \theta$ from the face of the support. In addition, near concentrated loads, sections closer than $0.5d_v \cot \theta$ to the load need not be checked. As a simplification, the term $0.5d_v \cot \theta$ may be approximated as d_v . The required amount of stirrups at other locations along the length of the beam can be determined by calculating sections at about every tenth point along the span, until it is evident that shear is no longer critical.

Shear causes tensile stresses in the longitudinal reinforcement as well as in the stirrups (Eq. (2-21)). If a member contains an insufficient amount of longitudinal reinforcement, its shear strength may be limited by yielding of this reinforcement. To avoid this type of failure, the longitudinal reinforcement on the flexural tension side of the member should satisfy the following requirement

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M}{\phi d_v} + \frac{0.5 N_u}{\phi} \quad (2-31)$$

$$+ \left(\frac{V_u}{\phi} - 0.5 V_s - V_p \right) \cot \theta$$

Figure 2.16 illustrates the influence of shear on the tensile force required in the longitudinal reinforcement. Whereas the moment is zero at the simple support, there still needs to be considerable tension in the longitudinal reinforcement near this support. The required tension T at a simple support can be determined from the free-body diagram in Fig. 2.16 as

$$T = \left(\frac{V_u}{\phi} - 0.5 V_s - V_p \right) \cot \theta \quad (2-32)$$

but

$$T \geq 0.5 \left(\frac{V_u}{\phi} - V_p \right) \cot \theta \quad (2-33)$$

The reinforcement provided at the support should be detailed in such a manner that this tension force can be safely resisted and that premature anchorage failures do not occur.

At the maximum moment locations shown in Fig. 2.16, the shear force changes sign, causing fanning of the diagonal compressive stresses as θ passes through 90 degrees. See also Fig. 2.1. Because of this, the maximum tension in these regions need not be taken as larger than that required for the maximum moment. The traditional North American procedure for accounting for the influence of shear on the longitudinal reinforcement involves extending the longitudinal reinforcement a distance d beyond the point at which it is no longer required to resist flexure, in addition to other detailing requirements. As can be seen from Fig. 2.16, the requirements of Eq. (2-31) can be satisfied in a conservative manner by simply extending the longitudinal reinforcement a