

ACI 447R-18

Design Guide for Twisting Moments in Slabs

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Design Guide for Twisting Moments in Slabs

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Design Guide for Twisting Moments in Slabs

Reported by Joint ACI-ASCE Committee 447

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This guide assists practitioners in understanding: 1) twisting moments in two-way slabs, when twisting moments are an essential consideration; 2) methods that can be used to account for twisting moments in design; and 3) the options available for each method of the various system geometries. Descriptions of twisting moments are provided theoretically and visually in the guide, and six methods of accounting for twisting moments in design are discussed. Applicability of the various methods is evaluated through a comparison of designs resulting from each method for a variety of two-way slab types and geometries. The theories described in the guide also apply to the design of two-way wall and two-way dome systems.

Keywords: finite element analysis; shell design; slab design; torsion; twist; twisting moments; wall design.

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CHAPTER 1—INTRODUCTION AND SCOPE**1.1—Introduction**

Section 8.2.1 of **ACI 318-14** allows slabs to be designed by any procedure that satisfies equilibrium and geometric compatibility, and requires that, at each section, the design strength exceeds the required strength and serviceability requirements are fulfilled.

Traditional strip design methods for slabs are based on approximate analysis and provide neither a complete equilibrium load path or satisfy geometric compatibility. Nonetheless, these methods have been used successfully for many years to design slabs with supports arranged in a rectangular grid.

From 1995 to 2015, design engineers transitioned from predominantly using traditional slab analysis methods to using finite element analysis (FEA). More recently, engineers use FEA to assist in the structural design of two-way concrete members. Twisting moments in two-way slabs can require additional reinforcement from those proportioned for bending moments, yet they are often misunderstood and sometimes ignored, neglected, or both, by practitioners in design. This is most likely due to their lack of being discussed comprehensively in design codes and frequent exclusion from college concrete design course curricula.

Although FEA solutions provide a full equilibrium load path and satisfy geometric compatibility, they determine load paths that require twisting moments for equilibrium (**Shin et al. 2009**). Many designers using FEA have ignored these twisting moments—a possible unconserva-

tive assumption where twisting moments are high (Park and Gamble 2000). To provide designers with guidance related to this issue, methods for explicitly incorporating twisting moments determined from FEA in the design of slabs are discussed in this guide.

The purpose of this design guide is to provide advice to design engineers who analyze slab systems with finite element methods and who need to ensure their designs are satisfactory for the twisting moments predicted by the analysis. This guide provides background information regarding twisting moments and describes multiple approaches for consideration of twisting moments in design. It also provides advice for designers of walls and shells with twisting moment conditions similar to those in slabs.

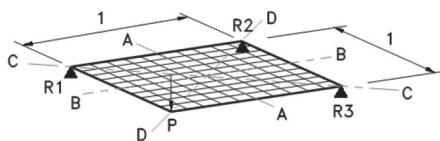
1.2—Scope

This design guide applies to slabs of both uniform and nonuniform thicknesses, including drop caps and drop panels, except where noted in the text. This guide does not apply to waffle slabs, or the beams of beam-and-slab floor systems. **Chapters 3 through 6** address slabs and walls in which the response is determined purely by bending. Chapter 7 addresses shells for which the response is determined by bending and membrane action. Chapter 6 and the theory sections of this guide are applicable to walls. **Chapter 7** and the theory sections of this guide are applicable to shells, with the caveat that equations presented in Chapter 3 are not valid for curved shells.

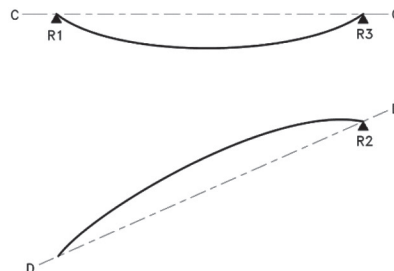
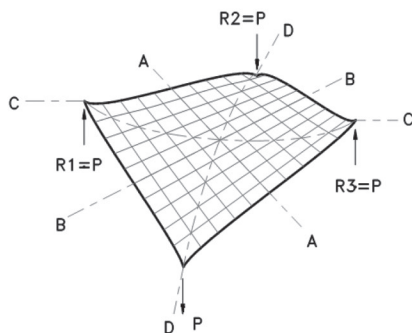
CHAPTER 2—NOTATION AND DEFINITIONS**2.1—Notation**

- c_{ij} = fraction for consideration of sections partially crossing element to apply to forces in local node j in element i
- D = flexural rigidity of plate, in.-lb (N·mm)
- E = Young's modulus, psi (MPa)
- F = force vector, lb (N)
- f_{ij} = nodal force vector for local node j in element i
- h = thickness of slab or plate, in. (mm)
- L = width of design section, in. (mm)
- M = bending moment, or moment vector, in.-lb (N·mm)
- M_d = design bending moment, in.-lb (N·mm)
- M_i = bending moment from isotropic analysis, in.-lb (N·mm)
- M_{tf} = bending moment from twist-free analysis, in.-lb (N·mm)
- M_u = design moment for slab cross section, in.-lb (N·mm)
- m_{ij} = nodal moment vector for local node j in element i , in.-lb/in. (N·mm/mm)
- m_r = bending moment causing stresses parallel to r -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{rs} = twisting moment relative to r - s -axes per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_s = bending moment causing stresses parallel to s -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)

LOADING & BOUNDARY CONDITIONS:



DEFLECTED SHAPE AND REACTIONS:



SUM OF MOMENTS ABOUT AXIS A-A = 0
 SUM OF MOMENTS ABOUT AXIS B-B = 0

Fig. 3.1—Twisting moment example free body diagram.

- m_{ux} = design moment causing stresses parallel to x -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{ux}^+ = positive design moment causing stresses parallel to x -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{ux}^- = negative design moment causing stresses parallel to x -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{uy} = design moment causing stresses parallel to y -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{uy}^+ = positive design moment causing stresses parallel to y -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{uy}^- = negative design moment causing stresses parallel to y -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_x = bending moment causing stresses parallel to x -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_{xy} = twisting moment relative to x - y -axes per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- m_y = bending moment causing stresses parallel to y -axis, per unit length of slab or plate, in.-lb/in. (N·mm/mm)
- n_x = membrane tension in x -axis direction per unit length, lb/in. (N/mm)
- n_{xy} = membrane in plane-shear in x - y -axes direction per unit length, lb/in. (N/mm)
- n_y = membrane tension in y -axis direction per unit length, lb/in. (N/mm)
- q = transverse load per unit area, lb/in.² (N/mm²)
- T = torsional moment, in.-lb (N·mm)
- V = shear force, lb (N)
- V_d = design shear force, lb (N)

- v_x = transverse shear on x -face per unit length, lb/in. (N/mm)
- v_y = transverse shear on y -face per unit length, lb/in. (N/mm)
- w = transverse deflection, in. (mm)
- x_{ij} = distance vector from section centroid to local node j in element i , in. (mm)
- ν = Poisson's ratio

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, ACI Concrete Terminology. Definitions provided herein complement that source.

anticlastic bending—curvature caused by the Poisson effect and curvature about a perpendicular axis.

strip design method—a method of designing slabs by dividing them into two sets of approximately perpendicular strips, with each strip analyzed and designed independently from each other.

CHAPTER 3—BACKGROUND

3.1—Qualitative introduction to twisting moments in slabs

Twist exists in most every slab, except those theoretical-case-only slabs whose moments at any point are identical about any axis. Figure 3.1 illustrates an extreme case of twist; a square slab with supports at three corners and a load at the fourth corner.

From equilibrium, it can be shown that the bending moment about the A-A and B-B axes in the figure is zero, although this slab is clearly supporting a load and needs to be reinforced. Looking at the C-C axis, whose bending moment is nonzero, gives us insight to the load-carrying

mechanism of the slab. What appears as twist about A-A and B-B is bending about C-C and D-D, as shown in the deflected shapes along C-C and D-D.

3.2—Behavior of linear-elastic isotropic slabs

Like other components in concrete structures, the design forces for slabs are typically determined using some form of a linear-elastic analysis. For slabs that are not thick (span-to-thickness ratio of more than 10), linear-elastic slab behavior can be predicted by well-known plate theory.

The deflection of a thin, linear-elastic, isotropic plate subjected to loads perpendicular to its plane can be expressed by a fourth-order partial differential equation (Timoshenko and Woinowski-Krieger 1959)

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (3.2a)$$

where $w = w(x,y)$ is the transverse (out-of-plane) displacement field in the direction of loading; $q = q(x,y)$ per unit area; and D is the flexural rigidity of the plate, which is expressed as

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (3.2b)$$

in which E is the modulus of elasticity, h is the thickness of the plate, and ν is Poisson's ratio.

The deformations of the plate can be interpreted as two

curvatures $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$, and one twist $\frac{\partial^2 w}{\partial x \partial y}$. Corresponding to these three deformations, there are two bending moments and one twisting moment as shown in Eq. (3.2c).

$$\begin{Bmatrix} m_x \\ m_y \\ m_{xy} \end{Bmatrix} = - \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & (1-\nu)D/2 \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (3.2c)$$

From Eq. (3.2c), note that unless $\frac{\partial^2 w}{\partial x \partial y}$ is zero, there will

be twisting moments in the plate. In general, $\frac{\partial^2 w}{\partial x \partial y}$ is zero in only a small subset of locations in a slab, so twisting moments will exist almost everywhere in a linear-elastic isotropic slab.

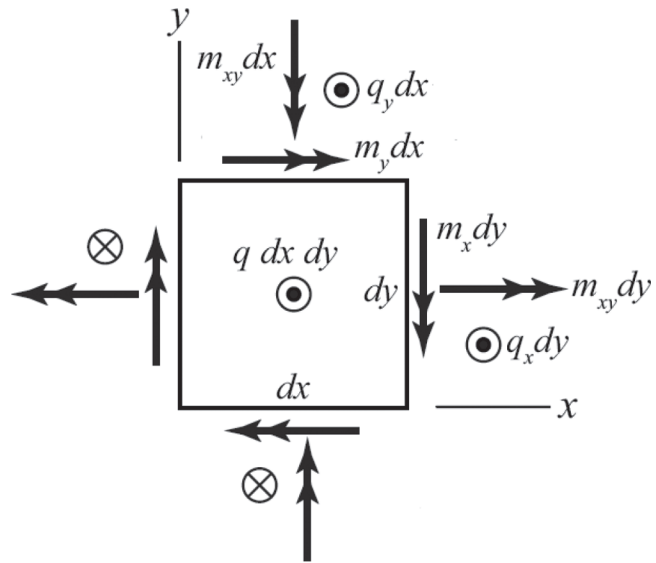


Fig. 3.3—An infinitesimal plate element shown with resulting shear forces and moments due to transverse loading.

3.3—Equilibrium in slabs

By considering the rotational equilibrium of the slab element shown in Fig. 3.3, Eq. (3.3), describing the inter-relationship of m_x , m_y , and m_{xy} , can be derived.

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -q \quad (3.3)$$

Equation (3.3), which holds for all slabs regardless of material behavior, reinforcement, or cracking, shows that any change in m_{xy} must be offset by some change in m_x , m_y , or both. Hence, m_{xy} can only be reduced to zero by changing the load path of the slab.

3.4—Principal axes

The values of m_x , m_y , and m_{xy} are related via Mohr's circle (Timoshenko and Woinowski-Krieger 1959); the moments m_r , m_s , and m_{rs} for any set of perpendicular r - s axes can be determined from m_x , m_y , and m_{xy} , as shown in Eq. (3.4a) through (3.4c).

$$m_r = \frac{1}{2}(m_x + m_y) + \frac{1}{2}(m_x - m_y) \cos 2\theta + m_{xy} \sin 2\theta \quad (3.4a)$$

$$m_s = \frac{1}{2}(m_x + m_y) - \frac{1}{2}(m_x - m_y) \cos 2\theta - m_{xy} \sin 2\theta \quad (3.4b)$$

$$m_{rs} = -\frac{1}{2}(m_x - m_y) \sin 2\theta + m_{xy} \cos 2\theta \quad (3.4c)$$

For every point in a plate, there is one set of principal axes, where m_{rs} is zero and about which the slab is in pure bending. The orientation of these axes changes from point to point in the slab, and the principal axes are typically not valuable for general understanding, then for determining

design forces. Figure 3.4 illustrates the construction of Mohr's circle for Eq. (3.4a) through (3.4c).

3.5—Orthogonal reinforcement and equilibrium for twisting moments

How can reinforcement be used to resist twisting moments? This question is most conveniently addressed by considering the reinforcement required to resist the moments about the principal axes. Figure 3.5 illustrates the conceptual approach.

Figures 3.5(a) and 3.5(b) illustrate the slab differential element to be reinforced, and its principal moments m_1 and

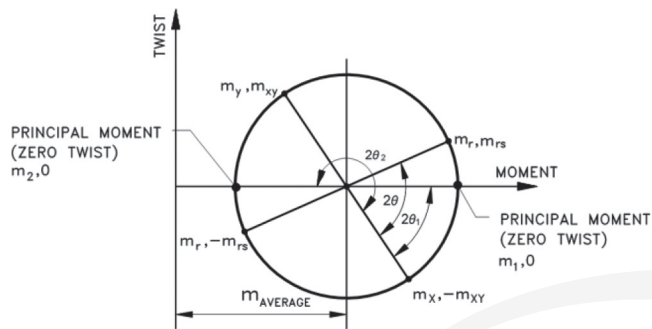
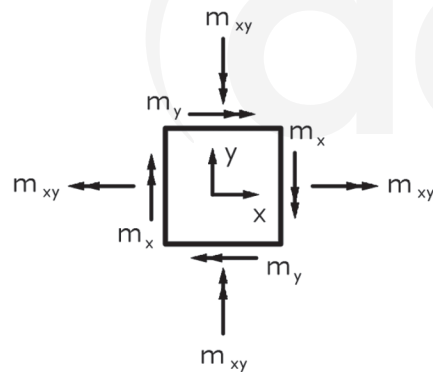
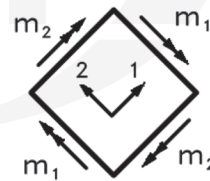


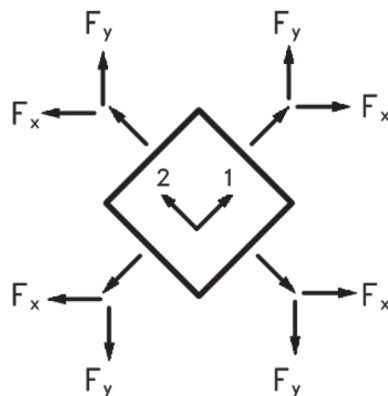
Fig. 3.4—Mohr's circle for situation where m_x and m_y are both positive and $m_x > m_y$.



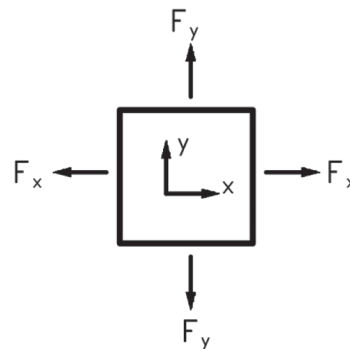
(a) Moments and twists about x-y axes



(b) Principle axes moments



(c) Principle axes bottom reinforcement



(d) The x-y axes bottom reinforcement

Fig. 3.5—Orthogonal reinforcement from twisting moments.

m_2 . The values of m_1 and m_2 cause tension in the bottom of the slab. Figure 3.5(c) shows bottom reinforcement forces aligned with the principal axes, selected using standard bending design approaches, and the x- and y-axis force vectors the reinforcement provides. Figure 3.5(d) shows the reinforcement forces along the 1- and 2-axes replaced by reinforcement forces along the x- and y-axes that, when combined, provide vector forces equivalent to the 1- and 2-axes reinforcement forces.

3.6—Effects of slab geometry on twisting moments

The slab in Fig. 3.6a illustrates how different slab regions and geometries affect the importance of the consideration of twisting moments when designing a slab. The figure highlights three slab regions that are discussed as follows. The slab is assumed to be designed with reinforcing bars parallel to the x- and y-axes.

Region 1 of the slab is regular with all columns arranged in a rectangular grid. Regions similar to this are not twist-sensitive. Ignoring twisting moments in these regions can be proved safe through yield line theory. Although yield line theory is an upper-bound approach, for a slab with regular supports such as those in Region 1, the number of yielding patterns to consider is small. If top reinforcement is clustered at the columns, only the folded-plate pattern in each

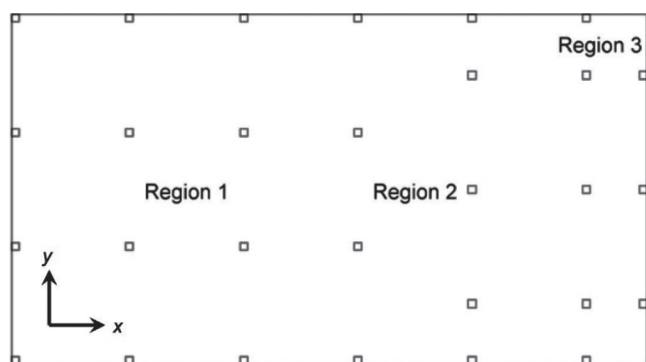


Fig. 3.6a—Representative regions of a slab floor plan. Region 1 is twist-sensitive, Region 2 moderately twist-sensitive, and Region 3 highly twist-sensitive.

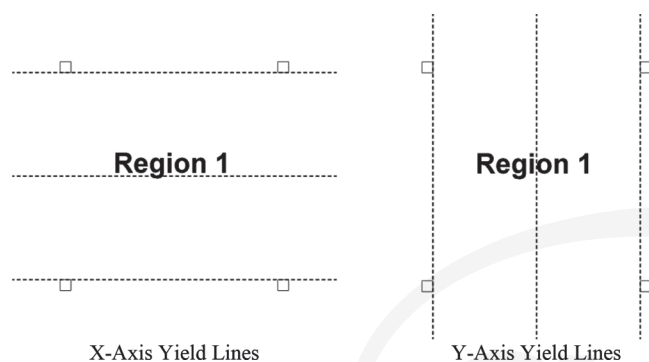


Fig. 3.6b—Folded plate yield lines.

direction shown in Fig 3.6b need be considered (Kennedy and Goodchild 2003). The folded plate yield line patterns show that any reinforcement design that supports the total span static moment ($wL^2/8$ for a uniform load) in each direction is safe.

Region 2 of the slab is where there is a major shift in the column layout. Regions similar to this are moderately twist-sensitive. The slab tends to span along a 45-degree line between the closest columns. This diagonal spanning reduces the bending moments along the x - and y -axes while increasing twisting moments. Ignoring twisting moments in design of similar regions could lead to a slab load capacity that is approximately 20 percent less than calculated. This percentage is calculated by comparing integrated slab strip bending moments using isotropic plates and using orthotropic plates with zero twist stiffness. This calculation assumes that the slab has no reliable twist capacity if twist has not been considered in the design of reinforcement, and is performed by dividing the bending moment determined by a twist-free analysis, M_{tf} , by the bending moment determined by an isotropic analysis, M_i . For Region 2, $M_{tf}/M_i = 1.2$.

Region 3 of the slab is a cantilevered corner. Regions similar to this are highly twist-sensitive. The slab tends to span between, and cantilever off, the line between the two columns. Although this diagonal cantilevering reduces the bending moments along the x - and y -axes, it adds large twisting moments. Ignoring twisting moments in design of similar regions could lead to a slab load capacity that

is approximately 40 percent less than calculated. This percentage is calculated by comparing integrated slab strip bending moments using isotropic plates and using orthotropic plates with zero-twist stiffness. This calculation assumes that the slab has no reliable twist capacity if twist has not been considered in the design of reinforcement ($M_{tf}/M_i = 1.4$). The strip design methods referred to in this document are those commonly used by practicing engineers, not the more sophisticated strip approaches discussed in academic literature.

3.7—Traditional slab design methods

Traditionally, slab design is performed using design forces determined by simplified analysis methods that idealize the slab as a set of intersecting strips (effectively supports and wide beams) in two perpendicular directions. The required reinforcement is calculated using the strip bending moments and standard beam bending design approaches.

These methods have two primary flaws. The first is that equilibrium is not fully satisfied because there is no consistency between the load paths of the two sets of strips. The second flaw is that deformation compatibility between both parallel and perpendicular strips is ignored.

Despite these flaws, slabs designed using these methods have generally performed well. For slabs with supports arranged in a rectangular grid it is often shown, using yield-line theory or lower-bound methods, that the traditional strip methods provide adequate capacity when reinforcement is distributed appropriately. (Burgoyne 2004; Kennedy and Goodchild 2003). For those slabs with supports not arranged in a rectangular grid, engineering judgment is necessary to determine if the traditional methods will produce a safe design.

3.8—Finite element analysis (FEA)-based slab design resultants

To complete slab design, an engineer should determine the quantity of reinforcement required for each design cross section. Although design cross section locations and lengths are often guided by code rules, in general, sections are needed at peak stress locations and their lengths based on the extent of the slab that can be assumed to act as a unit in resisting internal forces. The width of each section should be chosen so that the moment distribution along the section is reasonably uniform, does not change sign, and can be resisted by uniformly distributed reinforcements.

When using FEA to support slab design, the engineer should convert the slab analysis element results to resultant forces and moments acting on these sections. At this stage, the engineer should first transform the results of FEA into a coordinate system orthogonal to the section, and then integrate/sum all forces and moments acting on the section to determine the design moment. Consideration of twisting moments is the most difficult aspect of this conversion. ACI 318 does not explicitly address twisting moments, nor does the commentary provide guidance on their consideration. However, Section 6.1.1 of ACI 318-19, which addresses shell reinforcement, states that reinforcement

must be designed to resist tension from bending and twisting moments.

Design can also be performed considering the FEA results at each point in the slab, independently and foregoing the integration along a section. A zero-length section should still be considered to provide the correct axes for the design resultants and reinforcement. Design code requirements, such as those in [ACI 318](#), can be difficult to apply to these point results, as the codes are written for section-based results. Another potential difficulty with using FEA results at a point for design is that the results at certain locations, such as under a point load, will approach infinity as the mesh is refined. In general, the approaches described in this guide apply to either point-based or section-based resultants.

CHAPTER 4—AVAILABLE DESIGN METHODS

This chapter discusses various options to consider for twisting moments. [Chapters 5](#) through [7](#) evaluate some of these options in sample structures. Design methods discussed in this chapter are typically used with the results from linear-elastic analyses.

4.1—Finite element analysis (FEA)-based design ignoring twist

A simple, commonly-used approach to determine the design moment for a slab cross section is to integrate the FEA-predicted moment about the axis of the section along the length of the section.

$$M_u = \int m_r ds \quad (4.1)$$

In Eq. (4.1), m_r is the moment per unit length along the section. This approach ignores twisting moments in the equilibrium load path and may significantly underestimate the design demand.

The effect of twisting moments can be safely ignored in this manner when sections are approximately aligned orthogonally to the principal bending directions at all locations, as the twist on the section vanishes in this case. An instructive rule of thumb is that twist can be ignored if it is smaller than 10 percent of the primary bending moment ([Deaton 2005](#)). If this criterion is not satisfied, neglecting the effect of twist could lead to unconservative results.

4.2—Design using the Wood and Armer method

[Wood \(1968\)](#) and [Armer \(1968\)](#) (referred to in this document as Wood and Armer) proposed one of the most popular methods for explicitly incorporating twisting moments in slab design. The method seeks to prevent yielding in all directions and was developed by considering the normal moment and Johansen's yield criteria ([Park and Gamble 2000](#)). At any point in the slab, for any arbitrary direction, the design moment determined from FEA results must not exceed the ultimate normal resisting moment in that direction. The ultimate normal resisting moments calculated from the reinforcement in the x - and y -directions are typically compared to adjusted design moments m_{ux} and m_{uy} . Design moments based on the Wood and Armer method are computed

$$m_{ux} = m_x \pm |m_{xy}| \quad (4.2a)$$

$$m_{uy} = m_y \pm |m_{xy}| \quad (4.2b)$$

where all plus signs are used to compute required bottom reinforcement, all minus signs are used to compute required top reinforcement, and m_{ux} and m_{uy} will be negative when the top reinforcement is in tension. This assumes a sign convention where positive moment causes tension on the bottom surface of the slab and negative moment causes tension on the top surface of the slab. For non-slab systems such as walls and domes, top and bottom should be defined such that they are consistent throughout and consistent with loading assumptions.

Where m_{ux} or m_{uy} is found to have the opposite sign from what is expected (negative for bottom reinforcement or positive for top reinforcement), the design moment can be conservatively set to zero. A less conservative calculation option for this case is shown in Table 4.2.

The Wood and Armer approach requires that a fine regular mesh be used to produce accurate slab moment and twist predictions. Also, because moments, twists, or both, can be theoretically infinite at concentrated loads or reactions (such as slab column connections), integration of the moments or twists at these locations is difficult to perform accurately. Previous studies have indicated that the Wood and Armer method could produce unconservative results for slabs with high reinforcement ratios (approximately more than 0.75 percent) at regions of significant twisting moments, especially near restrained slab corners ([May and Lodi 2005](#)). Note also that this method cannot be directly applied to slabs with beams or drop panels. The Wood and Armer method has been modified and implemented in more conservative forms in both [CSA-A23.3-04](#) and [EN 1992-1-1:2004](#).

4.3—Design using the sandwich model

Designs based on the normal moment yield criterion, such as the Wood and Armer method, do not account for transverse shear or membrane forces within the slab. The sandwich model approach allows for the computation of reinforcement to resist not only bending and twisting moments, but also the effects of shear and membrane stresses. In this approach, membrane effects (n_x , n_y , and n_{xy}) and bending and twisting moments (m_x , m_y , and m_{xy}) are resisted by the sandwich exterior layers, whereas the shear effects (v_x and v_y) are resisted via the sandwich core. Figure 4.3 ([Marti 1990](#)) shows this concept.

To design by using the sandwich model, a slab section is divided into three layers; the depth of each layer can be determined such that the middle planes of the outer layers coincide with the center of the top and bottom reinforcement layers. Then, bending moments are decomposed into a couple of tensile and compressive normal forces, and twisting moments are decomposed into a couple of in-plane shear forces acting at top and bottom layers of the slab. The sandwiched inner layer is used to resist transverse shear. These normal and shear forces due to moments and twists are resisted with membrane (in-plane) forces. Finally, slab reinforcement

Table 4.2—Less conservative design moments (Wood 1968; Armer 1968)

	$m_x > m_{xy} $	$m_x \leq m_{xy} $	$m_x < - m_{xy} $
$m_y > m_{xy} $	$m_{ux}^{bot} = m_x + m_{xy} $ $m_{ux}^{top} = 0$ $m_{uy}^{bot} = m_y + m_{xy} $ $m_{uy}^{top} = 0$	$m_{ux}^{bot} = m_x + m_{xy} $ $m_{ux}^{top} = m_x - \left \frac{m_{xy}^2}{m_y} \right $ $m_{uy}^{bot} = m_y + m_{xy} $ $m_{uy}^{top} = 0$	$m_{ux}^{bot} = 0$ $m_{ux}^{top} = m_x - \left \frac{m_{xy}^2}{m_y} \right $ $m_{uy}^{bot} = m_y + \left \frac{m_{xy}^2}{m_x} \right $ $m_{uy}^{top} = 0$
$m_y \leq m_{xy} $	$m_{ux}^{bot} = m_x + m_{xy} $ $m_{ux}^{top} = 0$ $m_{uy}^{bot} = m_y + m_{xy} $ $m_{uy}^{top} = m_y - \left \frac{m_{xy}^2}{m_x} \right $	$m_{ux}^{bot} = m_x + m_{xy} $ $m_{ux}^{top} = m_x - m_{xy} $ $m_{uy}^{bot} = m_y + m_{xy} $ $m_{uy}^{top} = m_y - m_{xy} $	$m_{ux}^{bot} = 0$ $m_{ux}^{top} = m_x - m_{xy} $ $m_{uy}^{bot} = m_y + \left \frac{m_{xy}^2}{m_x} \right $ $m_{uy}^{top} = m_y - m_{xy} $
$m_y < - m_{xy} $	$m_{ux}^{bot} = m_x + \left \frac{m_{xy}^2}{m_y} \right $ $m_{ux}^{top} = 0$ $m_{uy}^{bot} = 0$ $m_{uy}^{top} = m_y - \left \frac{m_{xy}^2}{m_x} \right $	$m_{ux}^{bot} = m_x + \left \frac{m_{xy}^2}{m_y} \right $ $m_{ux}^{top} = m_x - m_{xy} $ $m_{uy}^{bot} = 0$ $m_{uy}^{top} = m_y - m_{xy} $	$m_{ux}^{bot} = 0$ $m_{ux}^{top} = m_x - m_{xy} $ $m_{uy}^{bot} = 0$ $m_{uy}^{top} = m_y - m_{xy} $

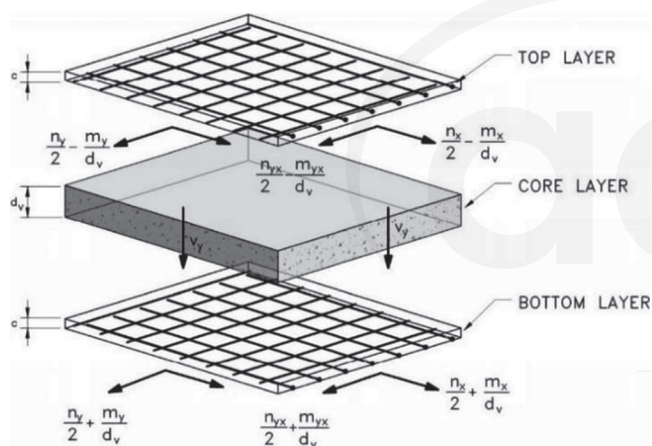


Fig. 4.3—Sandwich model.

ment is proportioned to satisfy equilibrium conditions at each top and bottom layer in the two orthogonal directions. Further descriptions of this method have been presented by *fib* Bulletin 45 (Comité Euro-International du Béton 2008), Brøndum-Nielsen (1985), Lourenço and Figueiras (1995), and the CEB-FIP Model Code (Comité Euro-International du Béton 1993).

The sandwich model is not typically used for slab design for commercial structures or generally available in slab design software. The assumed lever arm between the outer layers is very conservative for thin slabs, as the ratio of true ultimate lever arm to assumed lever arm is significantly greater than 1.0 (although this conservatism is reduced as the slab thickness is increased).

4.4—Design using element nodal forces

4.4.1 Nodal forces approach—The methods discussed in 4.1, 4.2, and 4.3 use slab stresses as the analysis quanti-

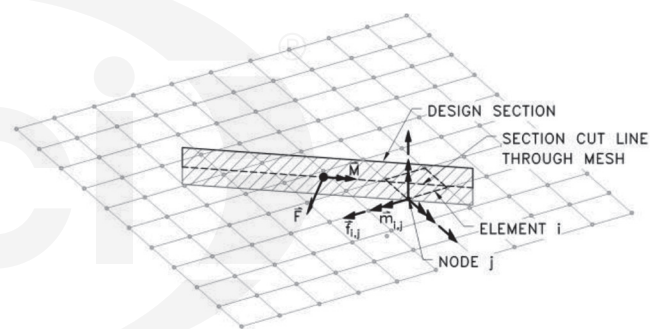


Fig. 4.4.2—Design section forces derived from element nodal forces.

ties from which slab design forces are calculated. Another approach for slab design using FEA results is to compute results for the design cross section from element nodal forces and moments. Using this approach, the section design forces are guaranteed to be in equilibrium with the externally applied nodal loads (Deaton 2005). Slab design methods based on element nodal forces have been implemented in various FEA software. These methods are attractive because results are relatively accurate (even for very coarse or irregular meshes), as they can be used for slabs containing beams or drop panels and are easily extended to design post-tensioned floors.

4.4.2 Calculation of nodal forces—The nodal forces approach transforms all element nodal forces and moments acting at nodes on the design cross section into resultant forces and moments acting at the centroid of the section, as shown in Fig. 4.4.2. Because transformation of nodal forces to the centroid considers the eccentricity of the nodes from the centroid, equilibrium of all acting forces and moments is provided.

The equilibrium equations in vector form are:

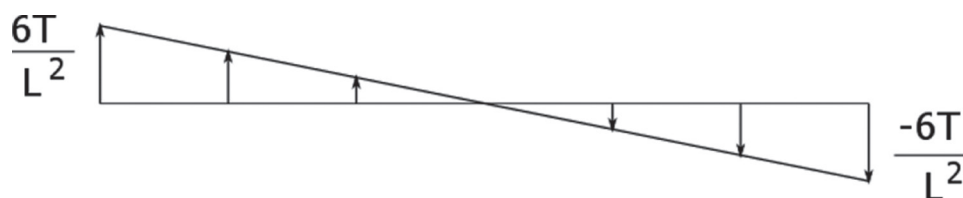


Fig. 4.4.4—Assumed shear stress distribution causing torsion.

$$\vec{F} = \sum_i \sum_j c_{i,j} \cdot \vec{f}_{i,j} \quad (4.4.2a)$$

$$\vec{M} = \sum_i \sum_j c_{i,j} \cdot (\vec{m}_{i,j} + \vec{x}_{i,j} \times \vec{f}_{i,j}) \quad (4.4.2b)$$

where i is the iterator over the elements crossed; j is the iterator over the local nodes in element i (on one side of the section); $\vec{F}$ and $\vec{M}$ are the force and moment vectors at the section centroid, respectively; $\vec{f}_{i,j}$ and $\vec{m}_{i,j}$ are nodal force and moment vectors, respectively; $\vec{x}_{i,j}$ is a vector distance from the section centroid to node ij ; and $c_{i,j}$ is a multiplier that equals 1.0 unless the section does not fully cross element i , in which case it varies between 0.0 and 1.0, typically based on the fraction of the element crossed.

The forces and moments to be used in the slab design are found by transforming the $\vec{F}$ and $\vec{M}$ vectors into the coordinate system of the design cross section. While six resultants are determined, typically only transverse shear V , bending moment M , and torsion T are of interest. There is no resultant quantity that directly corresponds to the twisting moments on the section. Methods available for consideration of the calculated torsion are as follows.

4.4.3 Torsion as bending—One approach to incorporating T is to directly combine M with $\pm T$, as shown in Eq. (4.4.3). Effectively, this method uses T as a proxy for integrated twist and then applies the Wood and Armer approach.

$$M_d = M \pm T \quad (4.4.3)$$

This approach can lead to unconservative design moments in two conditions:

- (1) Condition 1—Twisting moments along the cross section change sign and partially cancel in their integration
- (2) Condition 2—Torsion due to shear forces is of the opposite sign as torsion due to twist

Condition 1 can be detected visually and avoided by breaking the design section into two design sections; however, this can be difficult when considering numerous loading conditions, as the location of the sign change could vary. Condition 2 rarely occurs without Condition 1 also occurring. When neither Condition 1 or 2 occur, the design moments derived from considering torsion as bending can be conservative. For a simple torsional cantilever slab, the design moments are twice those derived from the Wood and Armer approach (Den Hartog 2014).

4.4.4 Torsion as shear—The total torsion T acting on the design cross section can be decomposed into an assumed linearly varying transverse shear per unit length, as shown in

Fig. 4.4.4. This torsional shear stress has a maximum value equal to $6T/L^2$, where L is the width of the section—that is, the width of section cut in Fig. 4.4.2. This is analogous to the bending stress in a rectangular beam being calculated as $6M/bh^2$.

By multiplying this maximum shear stress assumed due to torsion by the width of the section, an equivalent, but very conservative, resultant design shear force due to torsion, which is equal to $6T/L$, can be considered during the shear design of the section, as is shown in Eq. (4.4.4).

$$V_d = V \pm \frac{6T}{L^2} L = V \pm \frac{6T}{L} \quad (4.4.4)$$

Considering torsion as shear is generally not applicable when significant twisting moments exist. However, applying the torsion-as-shear design approach to the results of a twist-free analysis (3.5) has the potential for providing improved vertical shear designs while simultaneously fully considering twisting moments.

4.4.5 Nodal forces with twisting moment integrations—While the nodal forces approach does not provide twisting moments, it can be supplemented with element twisting moment integrations to provide a full set of design forces suitable for the Wood and Armer method or other design methods. The benefit of using such a hybrid approach is to gain accuracy of the nodal forces approach for all quantities to which it applies.

4.5—Design using twist-free analysis

Slab analysis and design software capable of analyzing orthotropic slabs (with different properties in two orthogonal directions) typically allows engineers to separately modify the stiffness corresponding to each of the bending moments about the two orthogonal axes and the twisting moment (Shin et al. 2009). Setting the twist stiffness to zero leads to an analytical solution and load path in which all the twisting moments are zero. The behavior and load path of a twist-free slab is somewhat similar to that of a grillage of beams. The design philosophy behind twist-free analysis is similar to that for the compatibility torsion approach that has long been accepted in ACI 318.

Bending and twisting moments are interrelated through a Mohr's circle equilibrium constraint. Because of this constraint, the twist stiffness can only be set to zero about a single set of perpendicular axes at each location. The most practical axes for this purpose are the axes of the design cross sections, which are perpendicular to the reinforcement. In a typical slab that is reinforced in only two perpendicular direc-