

Report on Indirect Method to Obtain Stress- Strain Response of Fiber- Reinforced Concrete (FRC)

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Report on Indirect Method to Obtain Stress-Strain Response of Fiber-Reinforced Concrete (FRC)

Reported by ACI Committee 544

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Development of proper design procedures for fiber-reinforced concrete (FRC) materials requires use of material tensile and compressive stress strains that reflect the contribution of fibers to the post-cracking behavior. While uniaxial tension tests provide the most fundamental material properties, conducting closed-loop tension tests are difficult to accomplish; therefore, methods based on indirect measurement of tensile properties using flexural tests are typically used.

This report presents the methodologies that are used for data reduction and presentation of the flexural test results in terms of an equivalent tensile stress-strain response for FRC materials. Existing methods for estimating uniaxial tensile stress-strain response of strain-softening and hardening FRCs from flexural beam-test data are introduced. Different approaches applied to beam tests based on elastic equivalent, curve fitting, or back-cal-

ulation of flexural data are introduced. These are divided into two general categories: elastic equivalent approach or inverse analysis method. In the elastic equivalent approach, a summary of available test methods by various code agencies are presented.

Using back-calculation methods, tools based on the finite element method and analytical closed-form solutions are presented. An approach is presented that uses closed-form moment-curvature relationships and obtains load-deflection responses for a beam of three- or four-point loading. The method is used to obtain equivalent parametric tensile stress and strain relationships for a variety of FRC materials. The methods are compared against the available residual strength and also elastically equivalent residual strengths obtained by different specimen geometries.

Results for a range of FRC materials studied show the back-calculated post-peak residual tensile strength is approximately 30 to 37 percent of the elastically equivalent flexural residual strength for specimens with different fiber types and volume fractions.

Keywords: fiber-reinforced concrete; inverse analysis; tensile stress-strain diagram.

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CHAPTER 1—INTRODUCTION AND SCOPE**1.1—Introduction**

This report provides guidelines for obtaining uniaxial stress-strain curves of fiber-reinforced concrete (FRC) from

beam test data. In many FRC systems, the contribution of fibers is apparent after the concrete cracks and the fibers that bridge such cracks start to debond and pullout, thus resisting its opening and generating a force that transfers the loads across the crack. The magnitude of load carried by the fibers depends on the opening of the crack width; it can be normalized with respect to the cracked area and referred to as a residual strength. The role that fibers play in bridging a main tensile crack is therefore characterized as resisting crack opening, also referred to as bridging force and is represented as an average effective stress and described by a tensile stress-crack width relationship. A majority of FRC mixtures exhibit a distinct stress-crack width relationship that can also be integrated with the initial elastic response of the composite into a combined nonlinear stress-strain response.

The stress-crack width relationship can be represented as an equivalent stress-strain response by assuming a characteristic length parameter to smear the crack width into a nominal strain distribution. If mechanical tests that only focus on the tensile strength (ASTM C496/C496M) or flexural strength (ASTM C78/C78M) of the FRC are conducted, this contribution, herein referred to as residual strength, is either inaccurate, not reported, or reported in terms of parameters that may not be useful for design or analysis. Furthermore, accurate measurements of tension tests that capture the post-peak response are difficult to conduct; therefore, many agencies use flexural tests as an indication of tensile response.

While the tension test theoretically shows the true material behavior and the flexural test represents a structural response, a flexural test is often used as a means of property measurement. The difference between the tension and flexural test results of many FRC materials is that in a tension test, the post-peak tensile stress-crack width response does not influence the maximum load obtained by the member. In a flexural test, however, the maximum load can be directly related to the residual stress levels such that the overall behavior can be affected by the post-peak response.

An alternative method is to calculate post-cracking behavior using the experimental flexural results and reduce them into a set of material parameters that are in compliance with the model assumptions. This topic is the subject of this report. The testing methodologies are discussed in detail in committee reports such as ACI 544.3R. The present report addresses procedures to obtain an effective tensile stress-strain curve from the experimental results.

Many structural systems that use FRC, such as structural floors or indeterminate structures, can exhibit an increase in strength values in proportion to the residual strength, which is a direct contribution of fibers; however, this parameter is only measured in a qualitative way using flexural tests such as ASTM C1609/C1609M.

To develop and apply design procedures for FRC materials, simplified equations are needed to account for the fiber's contribution to the tensile response, especially after cracking has occurred. This report addresses methods to compute the stress transfer after cracking is initiated in a concrete section.

1.2—Scope

This report presents existing methods for estimating characteristic tensile stress-strain or tensile stress crack width response of strain-softening fiber-reinforced concrete (FRC) using flexural beam test data. Methods are proposed for strain-softening FRCs that do not exhibit distributed or parallel microcracking when tested in flexural loading conditions, and strain-softening FRCs that do exhibit distributed or parallel microcracking when tested in flexural loading conditions.

A set of definitions for an equivalent stress-strain diagram (Naaman and Reinhardt 2006; Noghabai 1998) are presented first and followed by calculation procedures for obtaining flexural tensile and residual flexural strengths from beam test data. Specific coefficient values for notched beams (RILEM TC 162-TDF 2003) and third-point beams (NBN B 15 238:1992) are validated. Because the coefficient values are not directly reported for beam types tested per ASTM C1609/C1609M and ASTM C1399/C1399M, this report proposes an approach to do so and compares the results with other methods. The relationship is presented in terms of parameter-based stress coefficients that are determined using a step-by-step inverse analysis procedure in Appendix A. The report concludes with the relationship between the parameters that define the stress-strain diagram and the experimental flexural residual strengths.

The proposed approach has several drawbacks, as it is for a one-dimensional material behavior model, and yields an effective stress-strain response based on the model assumptions of trilinear tension and bilinear compression. Furthermore, because the results of back-calculation are size- and geometry-dependent, the procedure is presented for test data obtained from ASTM C1609/C1609M specimens. Other geometries and specimen dimensions may need to be correlated with standard-sized specimens. The method can be applied to any size specimen and the results can be used for comparative basis. Different beam sizes may yield different stress-strain values because of the size effect. The subject of the size effect is not addressed in this document because the majority of the flexural test specimens are in the narrow range of 4 x 4 to 6 x 6 in. (100 x 100 to 150 x 150 mm) size. Comprehensive sets of data with different fiber types and specimen sizes have been analyzed using back-calculation procedures, and a comparative evaluation of the results are discussed in recent publications (Bakhshi et al. 2014; Mobasher et al. 2014).

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

b = beam width, in. (mm)
 C_1, C_2, C_3, C_i = constants for definition of tensile stress strain curve
 d = beam depth, in. (mm)
 E = Young's modulus, psi (MPa)
 E_c = Young's modulus, psi (MPa)
 E_{cr} = equivalent Young's modulus of a cracked tension member, psi (MPa)

F = force component in stress diagram, lb (N)
 f'_c = uniaxial compressive strength, psi (MPa)
 f_{eq} = equivalent flexural tensile strength parameter, psi (MPa)
 f_i = tensile strength, psi (MPa)
 f_R = residual flexural tensile strength parameter, psi (MPa)
 h_n = net section height, in. (mm)
 h_{unc} = distance between notch tip and cross-section top
 k = neutral axis depth ratio
 L = clear span, in. (mm)
 L_f = fiber length, in. (mm)
 L_p = plastic length for crack localization in beams, in. (mm)
 M = moment, in.-lb (N.mm)
 M_{cr} = moment at first cracking, in.-lb (N.mm)
 M_e = moment in elastic range, in.-lb (N.mm)
 M_{it} = moment at an intersection point, in.-lb (N.mm)
 M_n = nominal flexural strength, lb-ft (N.mm)
 M_u = nominal flexural strength at maximum deflection/curvature, in.-lb (N.mm)
 P_{cr} = total load at first cracking, lb (N)
 V_f = volume fraction of fiber
 α = normalized transitional strain ($\epsilon_{trn}/\epsilon_{cr}$)
 β_{crit} = normalized critical tensile strain
 β_{tu} = normalized tensile strain at maximum stress ($\epsilon_{tu}/\epsilon_{cr}$)
 δ_u = maximum deflection, in. (mm)
 ϵ_c = compressive strain
 ϵ_{cr} = first cracking tensile strain
 ϵ_{cu} = maximum compressive strain
 ϵ_{cy} = compressive strain at yielding
 ϵ_i = strain value at a point along the stress strain response, in./in. (mm/mm)
 ϵ_{peak} = strain value corresponding to tensile strength, in./in. (mm/mm)
 ϵ_t = tensile strain
 ϵ_{trn} = transitional tensile strain ($\alpha \times \epsilon_{cr}$)
 ϵ_{tu} = maximum tensile strain
 ϕ = curvature, 1/in. (1/mm)
 ϕ_{cr} = curvature at first cracking, 1/in. (1/mm)
 ϕ_{it} = curvature at an intersection point, 1/in. (1/mm)
 ϕ_u = maximum curvature, 1/in. (1/mm)
 η = ratio of E_{cr}/E_c
 λ = normalized top compressive strain (ϵ_c/ϵ_{cr})
 λ_{cu} = normalized maximum compressive strain ($\epsilon_{cu}/\epsilon_{cr}$)
 λ_{tu} = normalized compressive strain when bottom fiber reaches maximum tensile strain
 μ = normalized post-peak residual tensile strength
 μ_{crit} = normalized post-peak residual tensile strength for elastic perfectly plastic moment-curvature response
 σ_c = compressive stress, psi (MPa)
 σ_{cy} = compressive yield strength, psi (MPa)
 σ_i = stress value at a point along the stress strain response, psi (MPa)
 σ_t = tensile stress, psi (MPa)
 ω = compressive-to-tensile strength ratio ($\epsilon_{cy}E/\epsilon_{cr}E$)



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2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, “ACI Concrete Terminology,” <https://www.concrete.org/store/productdetail.aspx?ItemID=CT13>. Definitions provided herein complement that resource.

bend-over point—tensile stress measured at the onset of first cracking (in a uniaxial tension test); this point is considered as tensile yield point or limit of proportionality.

crack bridging—ability of the fibers to extend from one face of a crack to the opposite side and transfer load during crack opening to the opposite face.

crack mouth opening displacement—measure of crack length extension beyond the tip of a notch created at the center of a beam specimen subjected to flexural loads in a three-point bending configuration.

deflection hardening—multiple cracking accompanied by an increase in flexural strength after first cracking due to strain hardening of the bridging material.

deflection softening—flexural response with a primarily linear elastic ascending branch up to the first flexural cracking, followed by an immediate reduction in load with increasing deflection.

first crack strength—stress corresponding to the load at first crack for a fiber-reinforced concrete composite in bending or tension.

flexural toughness—area under the flexural load-deflection curve obtained from a static test of a specimen up to a specified deflection, which is an indication of the energy absorption capability of a material.

limit of proportionality—flexural stress measured at the onset of first cracking (in a bending test).

microcrack coalescence—process by which microcracks grow sufficiently to join together and form a larger, perhaps visible, macrocrack.

microcracking—formation of localized, microscopic cracks within concrete, approximately 0.04 in. (0.1 mm) wide up to a few mm (less than 0.16 to 0.20 in. [4 to 5 mm]) long that may not be visible.

progressive collapse—failure of a primary structural element that results in the failure of adjoining structural elements, leading to further structural failure.

residual strength—apparent strength in the post-peak load region of a static load-deflection curve, calculated by the ratio of the bending moment in the post-cracking stage at a given deflection by the section modulus of original section.

serviceability limit state—performance criterion for serviceability of a structure, corresponding to conditions beyond which the structure might fail to meet technical requirements such as stress limits, deformation limits, and crack control and, therefore, declared unfit for routine loading conditions.

smear crack—formation of multiple cracks spread over a distinct area.

stable crack propagation—incremental and sequential growth of a crack as a result of addition of load on the specimen

strain softening—process where unstable crack propagation directly follows crack initiation.

strain softening fiber-reinforced—fiber-reinforced concrete composite with a primarily linear elastic ascending tensile stress-strain curve up to the same point representing both the first cracking and peak stress, followed by a reduction in stress as the strain is increased.

strain hardening fiber-reinforced—fiber-reinforced concrete composite with a primarily linear elastic ascending tensile stress-strain curve up to the first cracking, microcrack coalescence, and a post-elastic portion characterized by an increase in the stress with a much lower effective stiffness (representing tension stiffening) up to the maximum stress.

unstable crack propagation—uncontrolled and dynamic extension and widening of a crack that is not solely dependent on additional load on the specimen.

CHAPTER 3—TENSILE AND FLEXURAL TESTING AND MECHANICAL PROPERTIES

3.1—Tension and flexural testing

Fiber-reinforced concrete (FRC) materials have been extensively studied under direct tension loading conditions. Due to brittleness of the material, problems in conducting stable tension tests that must be addressed include:

- Inherent instability of crack propagation from one side of the specimen to the other that changes the uniformity of applied deformation
- Difficulty in control testing protocol and control mode due to brittleness and sudden crack growth in the material
- Test results that are significantly influenced by the support boundary conditions of fixed or free, specimen size, and notched versus unnotched specimens (Van Mier 1996)
- Maintaining objectivity in the closed-loop control conditions, such as cracking symmetry for notched specimens (Mobasher 2011; Hordijk 1991)

3.2—Strain softening and hardening

Many of the physical, durability, and mechanical properties of fiber-reinforced concrete (FRC) are affected by the amount, type, bond, and nature of anchorage (straight, hooked, twisted, or fibrillated) of fibers used. Other criteria that are affected by the interaction of the fiber on the stress-strain response include, but are not limited to, strength, strain capacity, first crack strain, elastic modulus, crack width, durability, permeability, total energy absorption, fatigue life, thermal conductivity, and shrinkage and creep characteristics. Therefore, characterization of the tensile stress-strain response is an important parameter of the analysis and design of FRC structures.

Strain-softening behavior in FRC represents tensile response with a primarily elastic ascending stress-strain curve up to the point that is both the first cracking bend-over point and also ultimate tensile strength. Response is followed by unloading due to unstable cracking, and opening of a crack with partial resistance offered by the debonding and sliding pullout of fibers. All subsequent stresses remain smaller than the first cracking (maximum) stress. In a strain-softening material, therefore, formation of a peak load is directly followed by its sudden and self-driven propa-

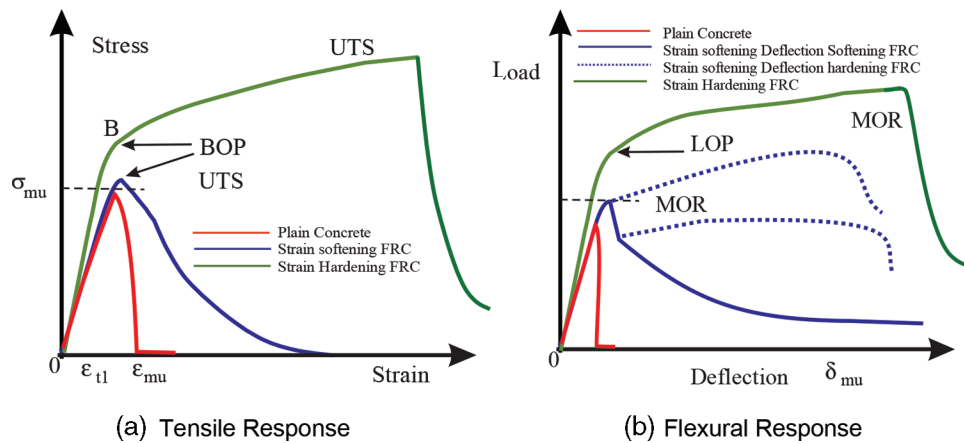


Fig. 3.2—Classification of fiber-reinforced concretes (FRCs) based on their tensile response (after Naaman and Reinhardt [2006]). (Notes: BOP is bend over point; UTS is ultimate tensile strength; MOR is modulus of rupture; and LOP is limit of proportionality.)

gation because the material is quasi-brittle with low bridging stress. Hence, the sample loses its ability to carry load and the stress-strain response goes through a descending branch, as shown in Fig. 3.2. The post-cracking stress remains below the peak stress in subsequent loading (Naaman and Reinhardt 2006); therefore, the softening behavior does not affect tensile strength, but increases tensile ductility.

Strain hardening behavior in FRC is a behavior represented by an elastic ascending tensile stress-strain curve up to the bend-over point, followed by an increasing load as the strain is increased, as shown in Fig. 3.2a. The effective stiffness in the response after the bend-over point level is much lower than the elastic stiffness (representing tension stiffening). The response continues at this stiffness up to the nominal tensile strength (ultimate tensile strength). In this case, the magnitudes of first cracking and nominal tensile strength may be distinctly different. Response after the ultimate tensile strength is followed by gradual unloading.

The strain hardening occurs under condition with a sufficiently high fiber content, stiffness, and bond properties such that the composite tensile strength exceeds the matrix cracking strength. In this process, after initial formation, a stable crack propagation stage takes place and the material could behave as perfectly-plastic or exhibit hardening behavior. Because the sample is capable of supporting additional load, multiple parallel cracks may also be formed and the tensile and flexural load deformation responses after cracking are characterized by an ascending load-deformation branch up to the nominal strength (Fig. 3.2b).

3.3—Deflection softening and hardening

Flexural behavior may vary significantly depending on whether fiber-reinforced concretes (FRCs) exhibit tension strain-softening or strain-hardening response. An approach to classify FRC based on its tensile mechanical response from the uniaxial tension can also be extended to flexural behavior. This classification depends on the efficiency and amount of fibers that covers a range from strain softening to hardening (Naaman and Reinhardt 2006).

It is noted that many of the characteristics of strain-softening and strain-hardening responses in tension testing are also observed when the specimen is tested in flexure. The responses in these conditions are referred to as deflection softening and deflection hardening; however, there is no one-to-one correlation between the two modes. Deflection hardening occurs for samples that have a sufficiently high level of fibers to exhibit strain-softening behavior in tension, but hardening behavior under flexural testing. The post-crack response in flexure exhibits ascending load with increasing deflection after the first crack point. For example, if the volume of the fibers is increased in excess of a level designated as critical fiber content, then the entire stress-strain response is also affected because the manner of load and strain distribution change in the sample. This is because, as the plain matrix cracks, there are sufficient fibers to carry the load that is being released by the matrix.

Because the flexural design of FRC elements is normally governed by the tensile stress-strain response, the post-cracking behavior has a direct effect on the results (Soranakom and Mobasher 2009b). Proper characterization of the load-deformation response and computation of valid material behavior data from these tests is essential to successful design applications (Banthia and Trottier 1995a,b; Mobasher 2011; Cunha 2010). Many structural systems with strain-softening materials, such as structural floors or indeterminate structures, can exhibit deflection-hardening behavior and an increase in strength values in proportion to the residual strength.

The difference between the tension and flexural test results of strain-softening materials is that, in a tension test, the post-peak tensile stress-crack width response does not influence the maximum load obtained. In a flexural test, the maximum load attained is related to the softening stress magnitudes. While a strain-softening FRC can develop deflection softening or deflection hardening based on the residual stress level present, FRC with tensile strain-hardening behavior always exhibits a deflection-hardening response.

3.4—Equivalent tensile stress-strain responses

As an alternative to direct measurement of the stress-strain by uniaxial tension test, a back-calculation approach can be applied to flexural tests. This is attractive, as flexural tests are routinely conducted as quality control tests and, therefore, the data collected can be used to verify the material properties and further used in the design of the next sections. Back-calculation of equivalent tension results from flexural tests, also defined as inverse analysis, is a useful tool because it provides equivalent tension response needed in member design. Approaches have been developed by RILEM TC 162-TDF (2003) and DIN EN 14561:2006-08. The equivalent tensile response can then be used in the design of flexural members for applications such as slabs, panels, and other applications where fiber contribution is introduced using equivalent softening parameters. Different approaches do not always yield unique results, as assumptions regarding the shape of the stress-strain response inherently affect the back-calculated parameters and, therefore, several test methods for calculation of the equivalent tensile properties are summarized. An overview of the existing inverse methods for obtaining uniaxial tensile stress-strain curves from beam test data follows in 3.5.

3.5—Inverse analysis methods

There is no standard specimen geometry for flexural testing among the various test methods of RILEM TC 162-TDF (2003), CEB-FIP (2011), and DIN EN 14561:2006-08. These approaches use notched three-point bending tests loaded along the midspan. ASTM methods such as ASTM C1609/C1609M and C1399/C1399M, however, recommend unnotched four-point beam tests. Based on these recommended standards, tensile stress-strain response can be calculated using prescribed equations. Values of the coefficients of these equations are determined from the experimental data using an equivalent elastic or inelastic method. The inverse-analysis method is optimal for finding values when specific coefficient values are unknown and include:

a) Technical guidelines for FRC, developed by RILEM TC 162-TDF (2003) for steel fiber-reinforced concrete (SFRC) by introducing the concept of equivalent f_{eq} and residual f_R flexural tensile strength parameters. These values were used to derive the stress-strain or the stress-crack width response of an SFRC (Namman and Reinhardt 2006; DIN EN 14561:2006-08; RILEM TC 162-TDF 2000a,b).

b) CEB-FIP (2011) also adopts the concept of residual flexural tensile strength parameters to derive the constitutive law that characterizes the tensile behavior of strain-softening and strain-hardening FRCs. The code proposes two simplified stress-crack width models based on the rigid-plasticity and linear-elasticity tensile behavior of FRCs. Correction factors are applied to scale the equivalent residual flexural strength at serviceability and nominal states, which are determined from three-point bending tests conducted based on BS EN 14651:2005+A1:(2007) (Blanco et al. 2013; Bakhshi et al. 2014).

c) German guidelines similar to RILEM TC 162-TDF (2003) for design of flexural members use the strain compat-

ibility analysis to determine the flexural strength of SFRC members (Teutsch 2004).

d) The UK traditionally followed JCI SF4:1983, but recently shifted toward the RILEM design methodology (Barr and Lee 2004) and methods based on BS EN 14651:2005+A1:(2007).

e) The Italian guidelines (di Prisco et al. 2004) are based on load-crack tip opening displacements from four-point bending tests on notched beams using equivalent stresses.

f) The Scandinavian approach is also based on residual strength parameters (Silfwerbrand 2008).

g) The U.S. design guidelines for flexural members are based on empirical equations of Swamy et al. (1975), ASTM C1609/C1609M, ASTM C1399/C1399M, Banthia and Dubey (1999), and Fischer (2004). The residual strength parameter is obtained using elastic section properties; the effect of fiber type and concrete composition are not specified. ACI 544.6R and ACI 544.7R have initiated the use of residual tensile strength as a parameter in the design.

h) Soranakom and Mobasher (2007c, 2008, 2009a) proposed closed-form solutions to correlate the flexural load deflection to a simplified tensile stress strain response—a procedure applicable to a wide variety of design applications, such as one- and two-way slabs, beams, slabs-on-ground, and earth-retaining structures.

In a majority of these cases, the uniaxial compressive response is determined directly from standard compression cylinder tests. Flexural tests conducted by BS EN 14651:2005+A1:(2007) are recommended for characterization of tensile behavior. The CEB-FIP (2011) advises against using uniaxial tensile tests for standard testing of new mixtures due to the associated difficulty of execution and interpretation (Blanco et al. 2013). These procedures can also be modified for a serviceability level design criteria. The long-term behavior of cracked FRC under tension, however, can be developed and required for structural fibers, such as organic or natural fibers, whose long-term performance is influenced by creep. CEB-FIP (2011) specifies if certain requirements about characteristic residual strengths and the limit of proportionality (BS EN 14651:2005+A1:(2007)) are fulfilled, then fiber reinforcement can substitute conventional reinforcement at serviceability limit.

CHAPTER 4—TEST METHODS

4.1—Test and specimen types

Various test methods exist for direct or indirect evaluation of the uniaxial-tensile properties of fiber-reinforced concrete (FRC). These include uniaxial tensile, wedge splitting, and beam tests. Common flexural beam tests are performed under a three- or four-point layout:

a) Three-point load recommended in RILEM TC 162-TDF (2003), CEB-FIP (2011), and ASTM C293/C293M

b) Four-point load recommended in ASTM C78/C78M

c) Four-point load for FRC in ASTM C1399/C1399M, ASTM C1609/C1609M, Teutsch (2004), JCI SF4:1983, and NBN B 15 238:1992

Beam tests can be subdivided by beam type:

- a) Unnotched, such as those used by ASTM
- b) Notched, such as those used with the three-point load test by **RILEM TC 162-TDF (2003)** and **CEB-FIP (2011)**

Controlling the post-peak response is much more dependable using a crack-mouth-opening displacement (CMOD) in a notched beam as opposed to deflection control used in unnotched beams. The CMOD measured in a notched beam is a monotonic increasing variable, which provides stable testing conditions while deflection includes elastic deformation outside the fracture process zone, which may cause unstable tests if extra caution is not considered (**Hordijk 1991**). Further detailed description of each beam type and test procedure is presented in the corresponding standards and, therefore, beyond the scope of this report.

4.2—Stress-strain diagrams

There are various ways to represent the residual strength of a sample after the point of maximum load. In each case, the model should include the localization effect within the crack region where all incremental deformation is concentrated, while the remainder of the specimen undergoes unloading in an elastic or inelastic manner. The average strain over the descending response is affected by the definition of a characteristic length parameter as a material property, as crack opening is represented by a smeared crack. Existing softening responses are subdivided into two main categories:

- 1) Stress-crack width definition
- 2) Stress-strain approach, where the strain is defined in terms of average displacement over a predetermined characteristic length or, alternatively, with a smeared crack approach (**di Prisco et al. 2004**; **CEB-FIP 2011**)

4.3—Stress-strain diagram in RILEM approach

The stress-strain diagram shown in Fig. 3.2 is proposed by **RILEM TC 162-TDF (2003)**. The values that define this constitutive model are based on a piecewise linearized response in the post peak domain. Average or characteristic values are then used later for design purposes.

In Fig. 4.3a, the key points of the compression side of the diagram are taken directly from the standard compressive cylinder test. For the tension side of the diagram, the key points are:

- a) σ_1 and ϵ_1 —Tensile strength and corresponding strain
- b) σ_2 and ϵ_2 —Stress and strain at the onset of the stable strain-softening branch
- c) σ_3 and ϵ_3 —Stress and strain at the end of the softening branch

The values of σ_i and ϵ_i ($i = 1, 2, 3$) can next be directly related to beam test data using the approach presented in **RILEM TC 162-TDF (2000a,b; 2002a,b)** and **Barros et al. (2005)**. In this case, σ_i ($i = 1, 2, 3$) are related to the following characteristic points on the beam-test load displacement curve (Fig. 4.3b):

- a) Average value of the flexural tensile strength, $f_{m,fl}$, for example, is the limit of proportionality of the beam response
- b) Residual flexural tensile strength at deflection of 1/64 in. (0.46 mm), $f_{R,1}$

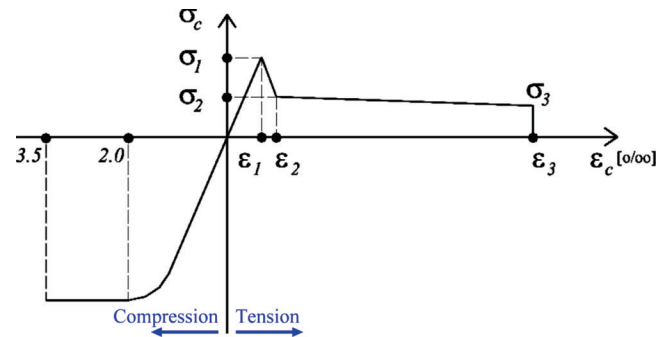


Fig. 4.3a—Stress-strain diagram for FRC in uniaxial tension and compression (**RILEM TC 162-TDF 2003**).

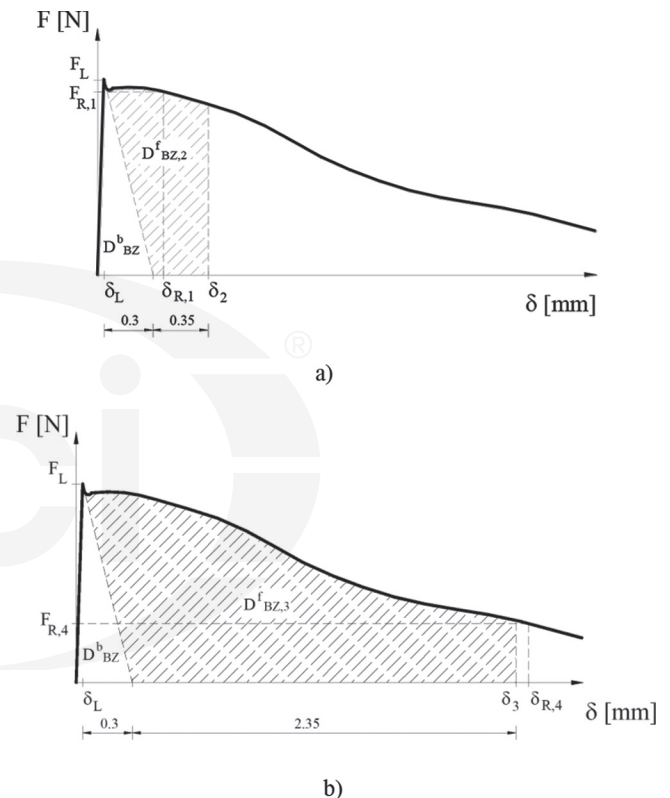


Fig. 4.3b—Load-displacement response of a test beam used to evaluate: (a) $f_{R,1}$ from $F_{R,1}$; and (b) $f_{R,4}$ from $F_{R,4}$ (**RILEM TC 162-TDF 2000a,b; 2002a,b; Barros et al. 2005**).

- c) Residual flexural tensile strength at deflection of 1/8 in. (3.0 mm), $f_{R,4}$

Details on how to evaluate $f_{m,fl}$, $f_{R,1}$, and $f_{R,4}$ are presented in 4.4 and details on σ_i , ϵ_i , $f_{m,fl}$, $f_{R,1}$, and $f_{R,4}$ are provided in 4.5.

4.4—Flexural tensile strength $f_{t,fl}$ and residual flexural tensile strengths $f_{R,1}$ and $f_{R,4}$

The flexural tensile strength $f_{t,fl}$ and residual flexural tensile strengths $f_{R,1}$ and $f_{R,4}$ values are calculated directly from the load displacement response of the test beam, which is schematically represented in Fig. 4.3b ($\delta_{R,1} = 1/64$ in. [0.46 mm] and $\delta_{R,4} = 1/8$ in. [3.0 mm]).

4.4.1 Flexural tensile strength $f_{t,fl}$ —The flexural tensile strength $f_{t,fl}$ is defined as the maximum flexural tensile stress