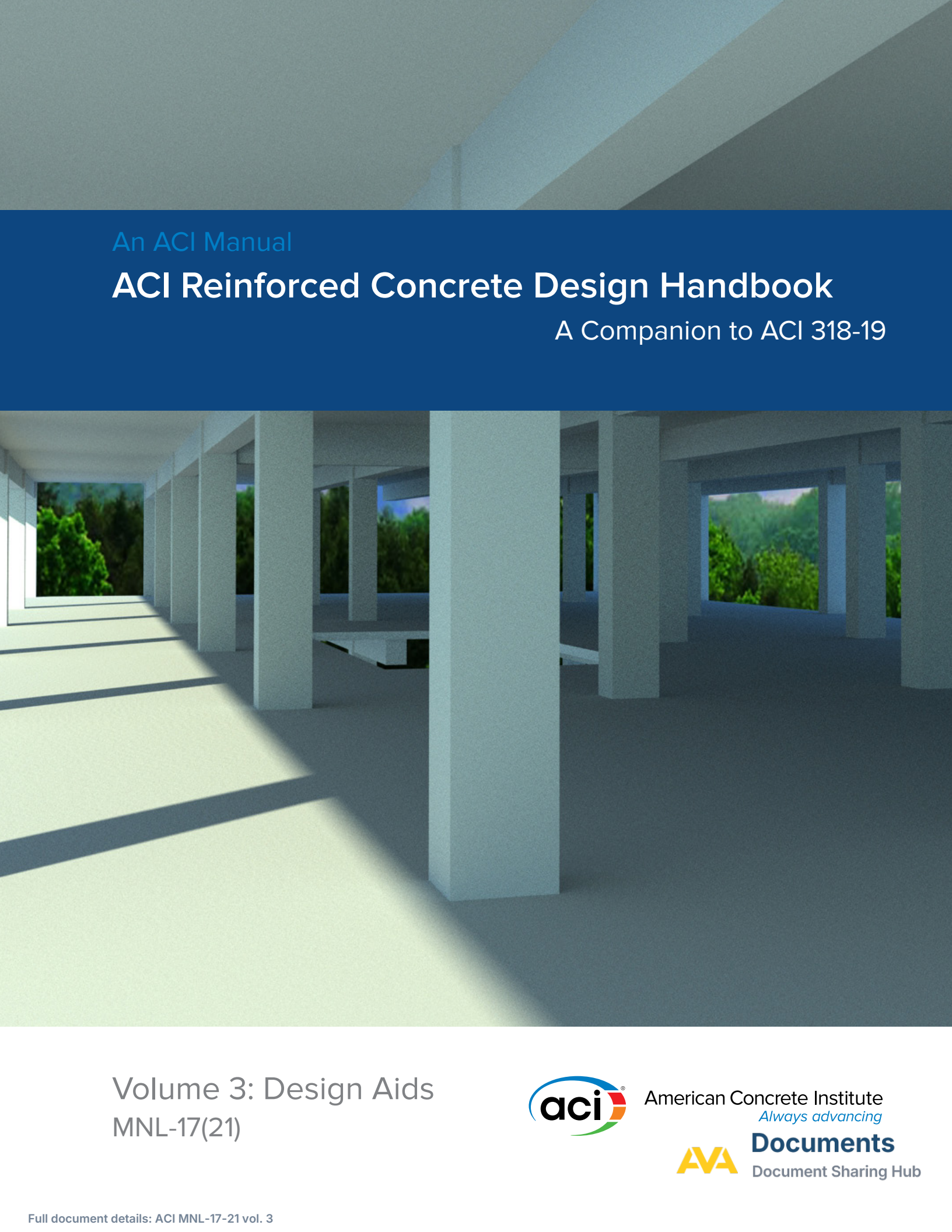




An ACI Manual

ACI Reinforced Concrete Design Handbook

A Companion to ACI 318-19

Volume 3: Design Aids
MNL-17(21)



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Volume 3**

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A Companion to ACI 318-19



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MNL-17(21)—REFERENCE TABLES

Table A-1—Nominal cross section area, weight, and nominal diameter of ASTM standard reinforcing bars

Bar size designation	Nominal cross section area, in. ²	Weight, lb/ft	Nominal diameter, in.	Nominal perimeter, in.
No. 3	0.11	0.376	0.375	1.18
No. 4	0.20	0.668	0.500	1.57
No. 5	0.31	1.043	0.625	1.96
No. 6	0.44	1.502	0.750	2.36
No. 7	0.60	2.044	0.875	2.75
No. 8	0.79	2.670	1.000	3.14
No. 9	1.00	3.400	1.128	3.54
No. 10	1.27	4.303	1.270	3.99
No. 11	1.56	5.313	1.410	4.43
No. 14	2.25	7.650	1.693	5.32
No. 18	4.00	13.600	2.257	7.09

Note: The nominal dimensions of a deformed bar are equivalent to those of a plain bar having the same mass per foot as the deformed bars.

Table A-2—Area of bars in a section 1 ft wide

Spacing, in.	Cross section area of bar A_s (or A_s'), in. ²											Spacing, in.
	Bar size											
	#3	#4	#5	#6	#7	#8	#9	#10	#11	#14	#18	
4.0	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68	—	—	4.0
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16	6.00	—	4.5
5.0	0.26	0.48	0.74	1.06	1.44	1.90	2.40	3.05	3.74	5.40	9.60	5.0
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40	4.91	8.73	5.5
6.0	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12	4.50	8.00	6.0
6.5	0.20	0.37	0.57	0.81	1.11	1.46	1.85	2.34	2.88	4.15	7.38	6.5
7.0	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67	3.86	6.86	7.0
7.5	0.18	0.32	0.50	0.70	0.96	1.26	1.60	2.03	2.50	3.60	6.40	7.5
8.0	0.17	0.30	0.47	0.66	0.90	1.19	1.50	1.91	2.34	3.38	6.00	8.0
8.5	0.16	0.28	0.44	0.62	0.85	1.12	1.41	1.79	2.20	3.18	5.65	8.5
9.0	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08	3.00	5.33	9.0
9.5	0.14	0.25	0.39	0.56	0.76	1.00	1.26	1.60	1.97	2.84	5.05	9.5
10.0	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87	2.70	4.80	10.0
10.5	0.13	0.23	0.35	0.50	0.69	0.90	1.14	1.45	1.78	2.57	4.57	10.5
11.0	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.39	1.70	2.45	4.36	11.0
11.5	0.11	0.21	0.32	0.46	0.63	0.82	1.04	1.33	1.63	2.35	4.17	11.5
12.0	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00	12.0
13.0	0.10	0.18	0.29	0.41	0.55	0.73	0.92	1.17	1.44	2.08	3.69	13.0
14.0	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34	1.93	3.43	14.0
15.0	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.02	1.25	1.80	3.20	15.0
16.0	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17	1.69	3.00	16.0
17.0	0.08	0.14	0.22	0.31	0.42	0.56	0.71	0.90	1.10	1.59	2.82	17.0
18.0	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04	1.50	2.67	18.0

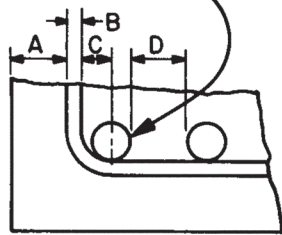
Example: #9 bar spaced 7-1/2 in. apart provides 1.60 in.²/ft of section width.

Table A-3—Minimum beam web widths required for two or more bars in one layer for cast-in-place nonprestressed concrete

Reference: ACI 318-19 Sections 25.3.2, 25.2.1, 20.5.1.3.1, and AASHTO Standard Specifications for Highway Bridges (17th edition, 2002) Division I, Sections 8.17.3.1, 8.21.1, 8.22.1, 8.23.2.2, and Table 8.23.2.1. For use of this Design Aid, see Flexure Example 1.

Minimum beam width = $2(A + B + C) + (n - 1)(D + d_b)$ where $A + B + C - 1/2d_b \geq 2.0$ in. cover required for longitudinal bars and these assumptions are made:

assumed position of bar
nearest side face of beam



for ACI 318

B = 0.375 in. for #3 stirrups
= 0.500 in. for #4 stirrups

D = $1 d_b$
 ≥ 1 in.
 $\geq 1-1/3$ nominal aggregate size

For both ACI and AASHTO

A = 1-1/2 in. concrete cover to stirrup
B = 0.635 in. for #5 stirrups
= 0.750 in. for #6 stirrups

C = stirrup bend radius of 2 stirrup
bar diameter for #5 and smaller
stirrups
= stirrup bend radius of 3 stirrup
bar diameters for #6 stirrups
 $\geq 1/2d_b$ of longitudinal bars

For AASHTO

B = 0.375 in. for #3 stirrups (minimum
stirrup size for #10 and
smaller longitudinal bars)
= 0.500 in. for #4 stirrups (minimum
stirrup size for #11 and larger
longitudinal bars)

D = $1-1/2d_b$
 $\geq 1-1/2$ in.
 $\geq 1-1/2$ nominal aggregate size

Bar size	ACI 318-19 3/4-in. aggregate interior exposure #3 stirrups		ACI 318-19 1-in. aggregate interior exposure #3 stirrups		AASHTO requirements cast-in-place concrete 1-in. aggregate exposed to earth or weather	
	Minimum web width for 2 bars, in.	Increment for each added bar, in.	Minimum web width for 2 bars, in.	Increment for each added bar, in.	Minimum web width for 2 bars, in.	Increment for each added bar, in.
#4	6 3/4	1 1/2	7 1/8	1 7/8	7.25	2.000
#5	6 7/8	1 5/8	7 1/4	2	7.37	2.125
#6	7	1 3/4	7 3/8	2 1/8	7.50	2.250
#7	7 1/8	1 7/8	7 1/2	2 1/4	7.62	2.375
#8	7 1/4	2	7 5/8	2 3/8	7.75	2.500
#9	7 1/2	2 1/4	7 3/4	2 1/2	8.32	2.820
#10	7 7/8	2 1/2	7 7/8	2 5/8	8.68	3.175
#11	8 1/8	2 7/8	8 1/8	2 7/8	9.52	3.525
#14	8 7/8	3 3/8	8 7/8	3 3/8	10.23	4.232
#18	10 1/2	4 1/2	10 1/2	4 1/2	11.90	5.642

Notes

1. **Stirrups:** For stirrups larger than those used for table above, increase web width by the following amounts (in.):

Source	Main reinforcement size	#4 stirrup	#5 stirrup	#6 stirrup
ACI requirements	#4 through #11	3/4	1 1/2	2 1/4
	#14	1/2	1 1/4	2
	#18	1/4	3/4	1 1/2
AASHTO requirements	#4 through #10	0.75	1.50	2.25
	#11 through #14	—	0.75	1.50
	#18	—	0.49	1.24

2. **ACI cover requirements:** For exterior exposure with use of #6 or larger stirrups, add 1 in. to web width.

3. **AASHTO cover requirements:** For interior exposure, 1/2 in. may be deducted from beam widths.

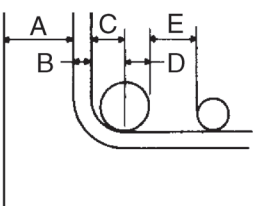
4. **Bars of different sizes:** For beams with bars of two or more sizes, determine from table the beam web width required for the given number of largest size bars; then add the indicated increments for each smaller bar.

5. **Example:** Find the minimum web width for a beam reinforced with two #8 bars; a beam reinforced with three #8 bars; a beam reinforced with three #9 and two #6 bars.

	2 #8	3 #8	3 #9 + 2 #6
ACI (3/4 in. aggregate)	7 1/4	9 1/4	13 1/4
ACI (1 in. aggregate)	7 5/8	10	14 1/2
AASHTO	7.75	10.25	15.64

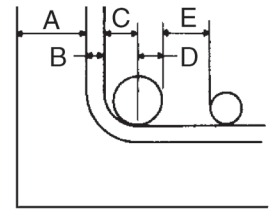
Table A-4—Minimum beam web widths for various bar combinations (interior exposure)

Reference: ACI 318-19 Sections 25.3.2, 25.2.1, and 20.5.1.3.1

No. of bars	ACI min. b_w , in.		Columns at left headed 1-5 and 6-10 bars are for minimum web width b_w of beam having bars of one size only. Remaining columns are for combinations of 1 to 5 bars of each of two sizes. Calculated values of beam web width b_w rounded upward to nearest half inch. Where bars of two sizes are used, larger bar(s) assumed to be placed along outside face(s) of beam. Aggregate size assumed $\leq 3/4$ in.																		
	Bar size	1 to 5 bars	6 to 10 bars																		
1	#3	5.5	12.5	Size of smaller bars	ACI min. b_w , in.					Size of smaller bars						Size of smaller bars	ACI min. b_w , in.				
2		7.0	13.5		No. of smaller bars	1	2	3	4		5	1	2	3	4		5				
3		8.0	15.0																		
4		9.5	16.5																		
5		11.0	18.0																		
1	#4	5.5	13.0	#3	7.0	8.5	9.5	11.0	12.5	#3	7.0	8.5	9.5	11.0	12.5	#3	7.0	8.5	9.5	11.0	12.5
2		7.0	14.5		8.5	9.5	11.0	12.5	14.0		8.5	9.5	11.0	12.5	14.0						
3		8.5	16.0		10.0	11.0	12.5	14.0	15.5		10.0	11.0	12.5	14.0	15.5						
4		10.0	17.5		11.5	12.5	14.0	15.5	17.0		11.5	12.5	14.0	15.5	17.0						
5		11.5	19.0		13.0	14.0	15.5	17.0	18.5		13.0	14.0	15.5	17.0	18.5						
1	#5	5.5	13.5	#4	7.0	8.5	10.0	11.5	13.0	#3	7.0	8.5	9.5	11.0	12.5	#3	7.0	8.5	9.5	11.0	12.5
2		7.0	15.0		8.5	10.0	11.5	13.0	14.5		8.5	10.0	11.5	13.0	14.5						
3		8.5	17.0		10.0	11.5	13.0	14.5	16.0		10.0	11.5	13.0	14.5	16.0						
4		10.5	18.5		12.0	13.5	15.0	16.5	18.0		11.5	13.0	14.5	16.0	17.0						
5		12.0	20.0		13.5	15.0	16.5	18.0	19.5		13.5	14.5	16.5	17.5	19.0						
1	#6	5.5	14.0	#5	7.0	9.0	10.5	12.0	13.5	#4	7.0	8.5	10.0	11.5	13.0	#3	7.0	8.5	10.0	11.5	13.0
2		7.0	16.0		9.0	10.5	12.0	13.5	15.5		8.5	10.0	11.5	13.0	14.5						
3		9.0	17.5		10.5	12.0	14.0	15.5	17.0		10.5	12.0	13.5	15.0	16.5						
4		10.5	19.5		12.5	14.0	15.5	17.0	19.0		12.0	13.5	15.0	16.5	18.0						
5		12.5	21.0		14.0	15.5	17.5	19.0	20.5		14.0	15.5	17.0	18.5	20.0						
1	#7	5.5	15.0	#6	7.5	9.0	11.0	12.5	14.5	#5	7.0	9.0	10.5	12.0	13.5	#4	7.0	8.5	10.0	11.5	13.0
2		7.5	16.5		9.0	11.0	12.5	14.5	16.0		9.0	10.5	12.0	14.0	15.5						
3		9.0	18.5		11.0	12.5	14.5	16.0	18.0		11.0	12.5	14.0	15.5	17.5						
4		11.0	20.5		13.0	14.5	16.5	18.0	20.0		12.5	14.5	16.0	17.5	19.0						
5		13.0	22.5		14.5	16.5	18.0	20.0	21.5		14.5	16.0	18.0	19.5	21.0						
1	#8	5.5	15.5	#7	7.5	9.5	11.0	13.0	15.0	#6	7.5	9.0	11.0	12.5	14.5	#5	7.5	9.0	10.5	12.0	14.0
2		7.5	17.5		9.5	11.0	13.0	15.0	17.0		9.0	11.0	12.5	14.5	16.0						
3		9.5	19.5		11.5	13.0	15.0	17.0	19.0		11.0	13.0	14.5	16.5	18.0						
4		11.5	21.5		13.5	15.0	17.0	19.0	21.0		13.0	15.0	16.5	18.5	20.0						
5		13.5	23.5		15.5	17.0	19.0	21.0	23.0		15.0	17.0	18.5	20.5	22.0						
1	#9	5.5	17.0	#8	7.5	9.5	11.5	13.5	15.5	#7	7.5	9.5	11.5	13.0	15.0	#6	7.5	9.0	11.0	12.5	14.5
2		8.0	19.0		10.0	12.0	14.0	16.0	18.0		9.5	11.5	13.5	15.5	17.0						
3		10.0	21.5		12.0	14.0	16.0	18.0	20.0		12.0	14.0	15.5	17.5	19.5						
4		12.5	23.5		14.5	16.5	18.5	20.5	22.5		14.0	16.0	18.0	20.0	21.5						
5		14.5	26.0		16.5	18.5	20.5	22.5	24.5		16.5	18.5	20.0	22.0	24.0						
1	#10	5.5	18.0	#9	8.0	10.0	12.5	14.5	17.0	#8	8.0	10.0	12.0	14.0	16.0	#7	7.5	9.5	11.5	13.5	15.0
2		8.0	20.5		10.5	12.5	15.0	17.0	19.5		10.0	12.0	14.0	16.0	18.0						
3		10.5	23.5		13.0	15.0	17.5	19.5	22.0		12.5	14.5	16.5	18.5	20.5						
4		13.0	26.0		15.5	17.5	20.0	22.0	24.5		15.0	17.0	19.0	21.0	23.0						
5		15.5	28.5		18.0	20.0	22.5	24.5	27.0		17.5	19.5	21.5	23.5	25.5						
1	#11	5.5	19.5	#10	8.0	10.5	13.0	15.5	18.0	#9	8.0	10.5	12.5	15.0	17.0	#8	8.0	10.0	12.0	14.0	16.0
2		8.5	22.5		11.0	13.5	16.0	18.5	21.0		10.5	13.0	15.0	17.5	19.5						
3		11.0	25.0		13.5	16.0	19.0	21.5	24.0		13.5	15.5	18.0	20.0	22.5						
4		14.0	28.0		16.5	19.0	21.5	24.0	26.5		16.0	18.5	20.5	23.0	25.0						
5		17.0	31.0		19.5	22.0	24.5	27.0	29.5		19.0	21.5	23.5	26.0	28.0						
1	#14	5.5	22.5	#11	8.5	11.5	14.5	17.0	20.0	#10	8.5	11.0	13.5	16.0	18.5	#9	8.5	10.5	13.0	15.0	17.5
2		9.0	26.0		12.0	14.5	17.5	20.5	23.0		11.5	14.0	16.5	19.0	22.0						
3		12.5	29.5		15.5	18.0	21.0	23.5	26.5		15.0	17.5	20.0	22.5	25.0						
4		16.0	33.0		18.5	21.5	24.5	27.0	30.0		18.5	21.0	23.5	26.0	28.5						
5		19.0	36.0		22.0	25.0	27.5	30.5	33.5		22.0	24.5	27.0	29.5	32.0						
1	#18	6.5	29.0	#14	10.0	13.5	16.5	20.0	23.5	#11	9.5	12.5	15.0	18.0	21.0	#10	9.5	12.0	14.5	17.0	19.5
2		11.0	33.5		14.0	17.5	21.0	24.5	27.5		13.5	16.5	19.0	22.0	25.0						
3		15.5	38.0		18.5	22.0	25.5	29.0	32.0		18.0	21.0	23.5	26.5	29.5						
4		20.0	42.5		23.0	26.5	30.0	33.5	36.5		22.5	25.5	28.5	31.0	34.0						
5		24.5	47.0		27.5	31.0	34.5	38.0	41.0		27.0	30.0	33.0	35.5	38.5						

Examples: For 2 #6 bars, minimum $b_w = 7.0$ in. For 8 #6 bars, minimum $b_w = 17.5$ in. For 2 #7 bars plus 3 #6 bars, minimum $b_w = 12.5$ in.
 For 3 #6 bars plus 5 #4 bars, minimum $b_w = 16.5$ in.

Columns at left headed 1-5 and 6-10 bars are for minimum web width b_w of beam having bars of one size only. Remaining columns are for combinations of 1 to 5 bars of each of two sizes. Calculated values of beam web width b_w rounded upward to nearest half inch. Where bars of two sizes are used, larger bar(s) assumed to be placed along outside face(s) of beam. Aggregate size assumed $\leq 3/4$ in.



A = clear cover of 1-1/2 in.
 B = 3/4 in. diameter of #3 stirrups.
 C = for #11 and smaller bars: twice diameter of #3 stirrups; for #14 and #18 bars: 1/2 diameter of bar
 D = 1/2 diameter of larger bar
 E = 1/2 spacing for larger bar plus 1/2 spacing for smaller bar (spacing is d_b for #9 and larger bars, 1 in. for #8 and smaller bars)

REF. TABLES

Table A-5—Properties of bundled bars

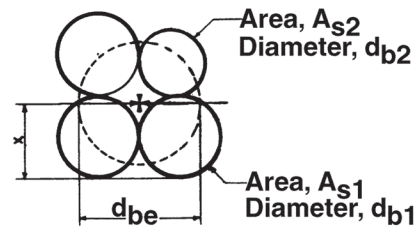
Reference: ACI 318-19 Section 25.6.1.6

$$\text{Equivalent diameter, } d_{be} = \sqrt{\frac{4}{\pi} A_s}$$

Centroidal distance, x

For the bundled bars configuration shown here, the centroidal distance is calculated by the following equation:

$$x = \frac{\frac{5}{2} A_{s1} d_{b1} + A_{s2} (d_{b1} + d_{b2}/2)}{\Sigma A_{si}}$$



Bars				Combination of bars							
											
Equivalent diameters d_{be} , in.											
#8	1.42	1.74	2.01	#7	#6	1.15	1.37	1.45	1.56	1.63	1.69
9	1.60	1.95	2.26	7	5	1.08	1.25	1.39	1.40	1.52	1.64
10	1.80	2.20	2.54	8	7	1.33	1.59	1.67	1.82	1.88	1.94
11	1.99	2.44	2.82	8	6	1.25	1.46	1.60	1.64	1.77	1.89
				9	8	1.51	1.81	1.88	2.07	2.13	2.20
				9	7	1.43	1.67	1.82	1.89	2.02	2.14
				10	9	1.70	2.04	2.12	2.33	2.40	2.47
				10	8	1.62	1.90	2.06	2.15	2.29	2.42
				11	10	1.90	2.28	2.36	2.61	2.68	2.75
				11	9	1.81	2.13	2.29	2.41	2.55	2.69
Centroidal distance x , from bottom of bundle, in.											
#4	0.25	0.39	0.50	#4	#3	0.23	0.38	0.33	0.43	0.40	0.46
5	0.31	0.49	0.62	5	4	0.29	0.47	0.43	0.55	0.53	0.58
6	0.37	0.59	0.75	5	3	0.28	0.46	0.37	0.49	0.44	0.55
				6	5	0.35	0.57	0.53	0.67	0.66	0.71
7	0.44	0.69	0.87	6	4	0.34	0.55	0.47	0.61	0.57	0.67
8	0.50	0.79	1.00								
9	0.56	0.89	1.13	7	6	0.41	0.67	0.62	0.80	0.78	0.83
				7	5	0.39	0.65	0.56	0.73	0.69	0.79
10	0.63	1.00	1.27	8	7	0.47	0.77	0.72	0.93	0.90	0.95
11	0.70	1.11	1.41	8	6	0.46	0.75	0.66	0.86	0.81	0.92
				9	8	0.54	0.86	0.82	1.05	1.03	1.08
				9	7	0.52	0.85	0.75	0.98	0.94	1.04
				10	9	0.60	0.98	0.92	1.19	1.16	1.22
				10	8	0.58	0.95	0.86	1.11	1.07	1.18
				11	10	0.67	1.08	1.03	1.32	1.31	1.36
				11	9	0.65	1.06	0.96	1.24	1.20	1.31

Example: Find the equivalent diameter of a single bar for 4 #9 bars. For 4 #9 bars, read $d_{be} = 2.26$ in., and the centroidal distance x equals 1.13 in.

Table A-6—Minimum beam web widths b_w for various combinations of bundled bars (interior exposure)

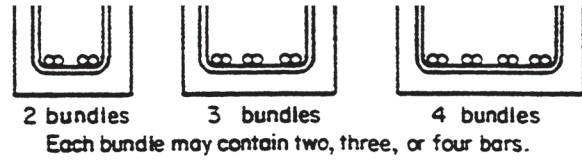
Reference: ACI 318-19 Sections 25.3.2, 25.6.1.1, 25.6.1.2, 25.6.1.3, 25.6.1.6, and 20.5.1.3.1

Calculated values of beam width b_w rounded upward to nearest half-inch.

Assumptions:Aggregate size: $\leq 3/4$ in.

Clear cover of 1-1/2 in.

No. 3 stirrups



Minimum beam web width b_w , in.*			
Bar size	∞	⊞	⊞⊞
Two bundles			
#8	10.0	10.0	10.5
#9	10.5	11.0	11.0
#10	11.0	11.5	12.0
#11	11.5	12.0	12.5
Three bundles			
#8	13.5	14.0	14.5
#9	14.5	15.0	15.5
#10	15.5	16.0	17.0
#11	16.5	17.5	18.0
Four bundles			
#8	17.0	17.5	18.5
#9	18.0	19.0	20.0
#10	20.0	21.0	22.0
#11	21.5	22.5	24.0
*For beams conforming to AASHTO specifications, add 1 in. to tabulated beam web width. For example, for two bundles of three #10, minimum $b_w = 11.5$ in.			

Table A-7—Basic development length ratios of bars in tension

Reference: ACI 318-19 Sections 25.4.2.2 and 25.4.2.3

Development length ratios:

$$\frac{\ell_d}{d_b} = \alpha \frac{\psi_t \psi_e \psi_g}{\lambda} \frac{f_y}{\sqrt{f'_c}}$$

For $\psi_t = 1.0$; $\psi_e = 1.0$, and $\lambda = 1.0$ (see notes below).

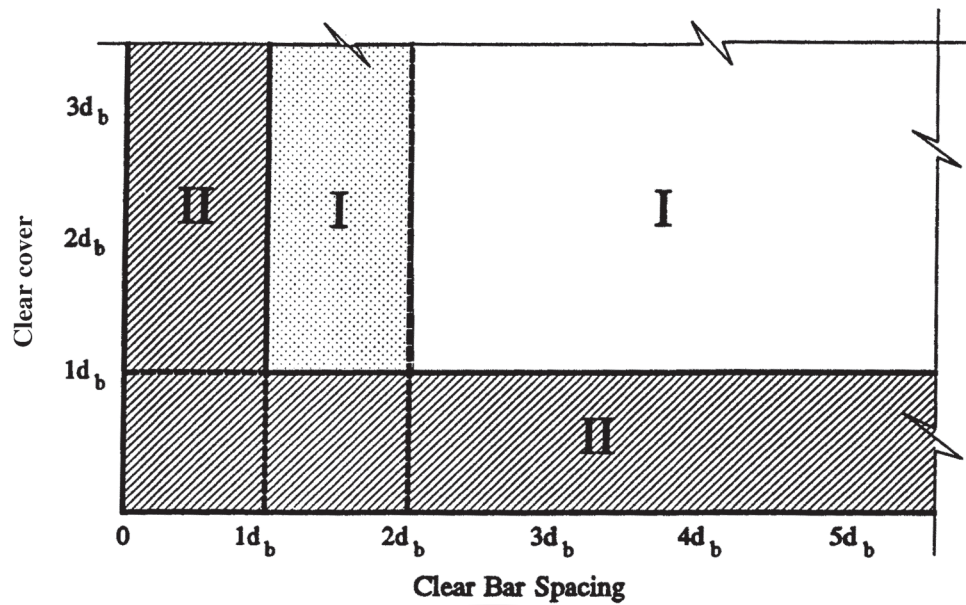
			Basic development length ratios of bars in tension																	
Bar size	Category	f_y	60,000 psi						80,000 psi						100,000 psi					
		f'_c	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi
#3 to #6	I	1/25	44	38	34	31	27	24	67	58	52	48	41	37	95	82	74	67	58	52
	II	3/50	66	57	51	46	40	36	101	87	78	71	62	55	142	123	110	101	87	78
#7 to #18	I	1/20	55	47	42	39	34	30	84	73	65	59	51	46	119	103	92	84	73	65
	II	3/40	82	71	64	58	50	45	126	109	98	89	77	69	178	154	138	126	109	98

Notes:

1. See category chart for Categories I and II.

2. ψ_t = casting position (1.3 for bars placed such that more than 12 in. of fresh concrete is cast below the development length or splice; 1.0 for other bars). ψ_e = coating factor (1.5 = epoxy-coated reinforcement with cover $< 3d_b$ or clear spacing $< 6d_b$; 1.2 = all other epoxy-coated reinforcement; and 1.0 = uncoated and zinc-coated [galvanized] reinforcement). λ = lightweight-aggregate concrete factor (0.75 for lightweight concrete, and 1.0 for normalweight concrete).3. Minimum development length $\ell_d \geq 12$ in.4. ψ_g = 1.0, 1.15, and 1.3 for f_y = 60 ksi, 80 ksi, and 100 ksi, respectively

Table A-7—Basic development length ratios of bars in tension (cont.)



CATEGORY CHART

Category I: { Clear spacing $\geq d_b$
 Clear cover $\geq d_b$
 Code minimum stirrups or ties throughout ℓ_d

or

{ Clear spacing $\geq 2d_b$
 Clear cover $\geq d_b$

Category II: All other cases

Table A-8—Basic development length ℓ_{dh} of standard hooks in tension

Reference: ACI 318-19 Sections 25.3.1, 25.4.3.1, 25.4.3.2, and 25.4.10.1.

STANDARD HOOK GEOMETRY FOR DEVELOPMENT OF DEFORMED BARS IN TENSION				
TYPE OF STANDARD HOOK	BAR SIZE	MINIMUM INSIDE BEND DIAMETER, IN.	STRAIGHT EXTENSION [1] L _{ext} , IN.	TYPE OF STANDARD HOOK
90-DEGREE HOOK	#3 THROUGH #8	6db	12db	
	#9 THROUGH #11	8db		
	#14 AND #18	10db		
180-DEGREE HOOK	#3 THROUGH #8	6db	GREATER OF 4db AND 2.5"	
	#9 THROUGH #11	8db		
	#14 AND #18	10db		

[1] A STANDARD HOOK FOR DEFORMED BARS IN TENSION INCLUDES THE SPECIFIC INSIDE BEND DIAMETER AND STRAIGHT EXTENSION LENGTH. IT SHALL BE PERMITTED TO USE A LONGER STRAIGHT EXTENSION AT THE END OF A HOOK. A LONGER EXTENSION SHALL NOT BE CONSIDERED TO INCREASE THE ANCHORAGE CAPACITY OF THE HOOK.

$$\ell_{dh} = \left(\frac{\psi_e \psi_r \psi_o \psi_c}{55\lambda \sqrt{f'_c}} \right) d_b^{1.5}$$

This table is calculated with $\psi_e = 1.0$, $\psi_t = 1.0$, and $\lambda = 1.0$.

		Basic development length ℓ_{dh} , in., of standard hooks in tension																		$8d_b$, in.
Bar size	f_y	60,000 psi						80,000 psi						100,000 psi						
	f'_c d_b , in.	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi	3000 psi	4000 psi	5000 psi	6000 psi	8000 psi	10,000 psi	
#3	3/8	3.7	3.4	3.3	3.2	2.8	2.5	4.9	4.6	4.1	4.3	3.7	3.3	6.1	5.7	5.5	5.4	4.7	4.2	3
#4	1/2	5.6	5.3	5.1	5.0	4.3	3.9	7.5	7.0	6.3	6.6	5.7	5.1	9.4	8.8	8.5	8.3	7.2	6.4	4
#5	5/8	7.9	7.4	7.1	7.0	6.0	5.4	10.5	9.8	8.8	9.3	8.0	7.2	13.1	12.3	11.9	11.6	10.0	9.0	5
#6	3/4	10.3	9.7	9.4	9.1	7.9	7.1	13.8	12.9	11.6	12.2	10.6	9.4	17.2	16.2	15.6	15.2	13.2	11.8	6
#7	7/8	13.0	12.2	11.8	11.5	10.0	8.9	17.4	16.3	14.6	15.4	13.3	11.9	21.7	20.4	19.6	19.2	16.6	14.9	7
#8	1	15.9	14.9	14.4	14.1	12.2	10.9	21.2	19.9	17.8	18.8	16.3	14.5	26.6	24.9	24.0	23.5	20.3	18.2	8
#9	1.128	19.1	17.9	17.3	16.9	14.6	13.1	25.5	23.9	21.4	22.5	19.5	17.4	31.8	29.8	28.8	28.1	24.4	21.8	9
#10	1.27	22.8	21.4	20.6	20.2	17.5	15.6	30.4	28.5	25.5	26.9	23.3	20.8	38.0	35.7	34.3	33.6	29.1	26.0	10
#11	1.41	26.7	25.0	24.1	23.6	20.4	18.3	35.6	33.4	29.8	31.4	27.2	24.4	44.5	41.7	40.2	39.3	34.0	30.4	11
#14	1.693	35.1	32.9	31.7	31.0	26.9	24.0	46.8	43.9	39.3	41.4	35.8	32.0	58.5	54.9	52.9	51.7	44.8	40.1	14
#18	2.257	54.0	50.7	48.8	47.8	41.4	37.0	72.0	67.6	60.4	63.7	55.1	49.3	90.0	84.5	81.4	79.6	68.9	61.7	18

Note 1: To compute development length ℓ_{dh}^* for a standard hook in tension, multiply basic development length ℓ_{dh} from table above by applicable modification factors. For ψ_e , ψ_r , ψ_o , ψ_c , and λ values, refer to ACI 318-19 Table 25.4.3.2.

Note 2: Values of basic development length ℓ_{dh} above the heavy line are less than the minimum development length of 6 in. Development length ℓ_{dh} shall not be less than 8d_b, nor less than 6 in., whichever is greater.

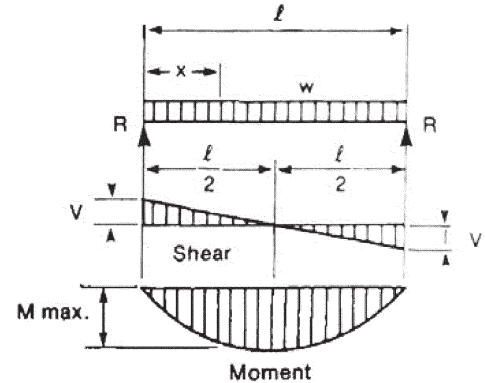
MNL-17(19)—ANALYSIS TABLES

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Table B-1 Beam Diagrams

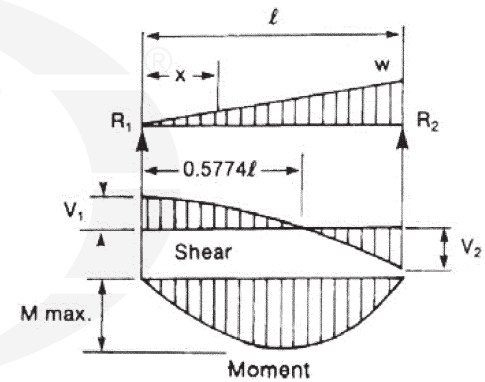
Simple Beam — uniformly distributed load

$$\begin{aligned}
 R = V & \dots\dots\dots = \frac{w\ell}{2} \\
 V_x & \dots\dots\dots = w\left(\frac{\ell}{2} - x\right) \\
 M \text{ max. (at centre)} & \dots\dots\dots = \frac{w\ell^2}{8} \\
 M_x & \dots\dots\dots = \frac{wx}{2}(\ell - x) \\
 \Delta \text{ max. (at centre)} & \dots\dots\dots = \frac{5w\ell^4}{384EI} \\
 \Delta_x & \dots\dots\dots = \frac{wx}{24EI}(\ell^3 - 2\ell x^2 + x^3)
 \end{aligned}$$



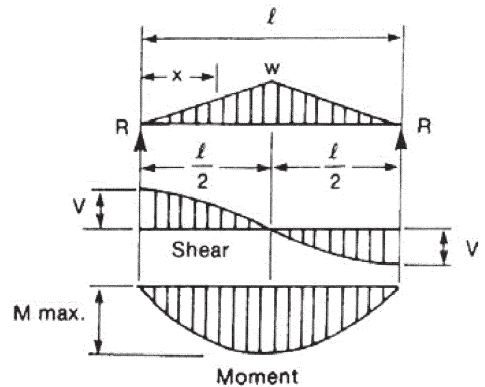
Simple Beam — load increasing uniformly to one end

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{w\ell}{6} \\
 R_2 = V_2 & \dots\dots\dots = \frac{w\ell}{3} \\
 V_x & \dots\dots\dots = \frac{w\ell}{3} - \frac{wx^2}{2\ell} \\
 M \text{ max. (at } x = \frac{\ell}{\sqrt{3}} = 0.5774\ell) & \dots\dots\dots = \frac{w\ell^2}{9\sqrt{3}} = 0.06415w\ell^2 \\
 M_x & \dots\dots\dots = \frac{wx}{6\ell}(\ell^2 - x^2) \\
 \Delta \text{ max. (at } x = \ell\sqrt{1 - \sqrt{\frac{8}{15}}} = 0.5193\ell) & \dots\dots\dots = 0.00652 \frac{w\ell^4}{EI} \\
 \Delta_x & \dots\dots\dots = \frac{wx}{360EI}(3x^4 - 10\ell^2x^2 + 7\ell^4)
 \end{aligned}$$



Simple Beam — load increasing uniformly to centre

$$\begin{aligned}
 R = V & \dots\dots\dots = \frac{w\ell}{4} \\
 V_x \left(\text{when } x < \frac{\ell}{2}\right) & \dots\dots\dots = \frac{w}{4\ell}(\ell^2 - 4x^2) \\
 M \text{ max. (at centre)} & \dots\dots\dots = \frac{w\ell^2}{12} \\
 M_x \left(\text{when } x < \frac{\ell}{2}\right) & \dots\dots\dots = \frac{w\ell x}{2} \left(\frac{1}{2} - \frac{2x^2}{3\ell^2}\right) \\
 \Delta \text{ max. (at centre)} & \dots\dots\dots = \frac{w\ell^4}{120EI} \\
 \Delta_x & \dots\dots\dots = \frac{wx}{960EI\ell}(5\ell^2 - 4x^2)^2
 \end{aligned}$$



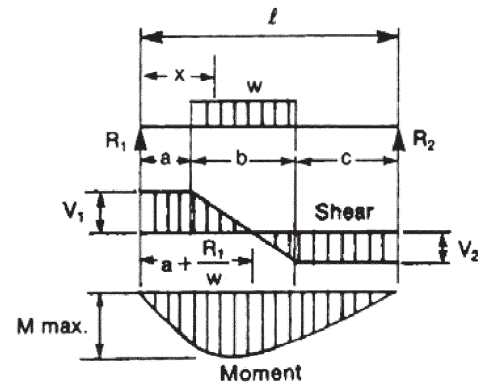
Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.
P: Concentrated load.

Table B-1 Beam Diagrams (Cont'd)

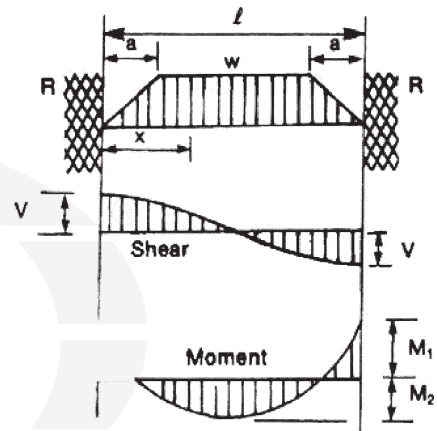
Simple Beam — uniform load partially distributed

$$\begin{aligned}
 R_1 = V_1 \text{ (max. when } a < c) & \dots\dots\dots = \frac{wb}{2\ell} (2c + b) \\
 R_2 = V_2 \text{ (max. when } a > c) & \dots\dots\dots = \frac{wb}{2\ell} (2a + b) \\
 V_x \text{ (when } x > a \text{ and } < (a + b)) & \dots\dots\dots = R_1 - w(x - a) \\
 M \text{ max. (at } x = a + \frac{R_1}{w}) & \dots\dots\dots = R_1 \left(a + \frac{R_1}{2w} \right) \\
 M_x \text{ (when } x < a) & \dots\dots\dots = R_1 x \\
 M_x \text{ (when } x > a \text{ and } < (a + b)) & \dots\dots\dots = R_1 x - \frac{w}{2} (x - a)^2 \\
 M_x \text{ (when } x > (a + b)) & \dots\dots\dots = R_2 (\ell - x)
 \end{aligned}$$

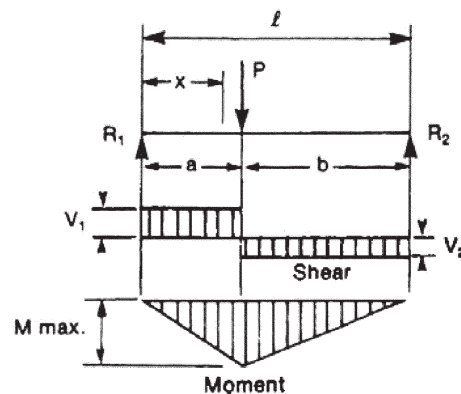
**Beam fixed at both ends — symmetrical trapezoidal load**

$$\begin{aligned}
 R = V & \dots\dots\dots = \frac{w\ell}{2} \left(1 - \frac{a}{\ell} \right) \\
 M_1 & \dots\dots\dots = -\frac{w}{12} \left(1 - 2\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3} \right) \\
 M_2 & \dots\dots\dots = \frac{w\ell^2}{24} \left(1 - 2\frac{a^3}{\ell^3} \right) \\
 M_x \text{ (when } x < a) & \dots\dots\dots = M_1 + R_1 x - \frac{wx^3}{6a} \\
 M_x \text{ (when } a < x < \ell - a) & \dots\dots\dots = M_1 + R_1 x - \frac{wa}{2} \left(x - \frac{2}{3}a \right) - \frac{w}{2} (x - a)^2
 \end{aligned}$$

Note: When $a = \ell/2$ loading is triangular

**Simple Beam — concentrated load at any point**

$$\begin{aligned}
 R_1 = V_1 \text{ (max. when } a < b) & \dots\dots\dots = \frac{Pb}{\ell} \\
 R_2 = V_2 \text{ (max. when } a > b) & \dots\dots\dots = \frac{Pa}{\ell} \\
 M \text{ max. (at point of load)} & \dots\dots\dots = \frac{Pab}{\ell} \\
 M_x \text{ (when } x < a) & \dots\dots\dots = \frac{Pbx}{\ell} \\
 \Delta \text{ max. (at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b) & \dots\dots\dots = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell} \\
 \Delta a \text{ (at point of load)} & \dots\dots\dots = \frac{Pa^2b^2}{3EI\ell} \\
 \Delta_x \text{ (when } x < a) & \dots\dots\dots = \frac{Pbx}{6EI\ell} (\ell^2 - b^2 - x^2)
 \end{aligned}$$

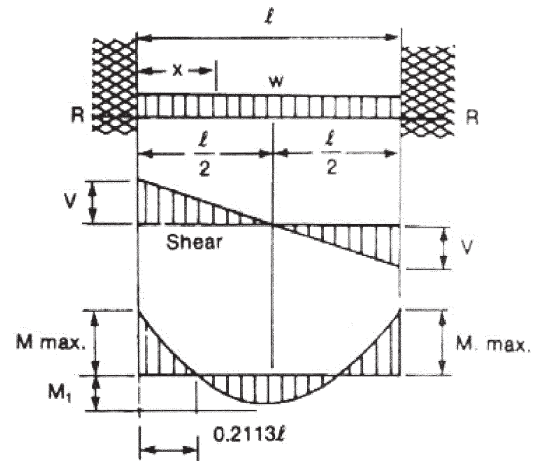
**Note:**

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.
P: Concentrated load.

Table B-1 Beam Diagrams (Cont'd)

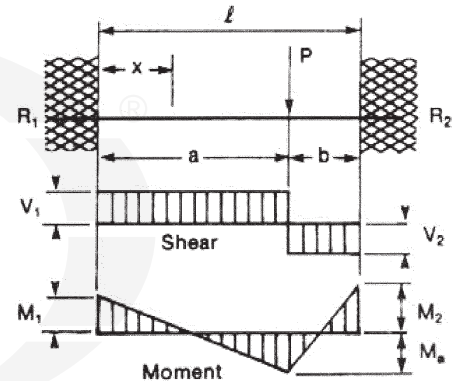
Beam fixed at both ends — uniformly distributed loads

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{w\ell}{2} \\
 R_2 & \dots\dots\dots = w\left(\frac{\ell}{2} - x\right) \\
 M_{\text{max. (at ends)}} & \dots\dots\dots = -\frac{w\ell^2}{12} \\
 M_{\text{(at centre)}} & \dots\dots\dots = \frac{w\ell^2}{24} \\
 M_x & \dots\dots\dots = \frac{w}{12}(6\ell x - \ell^2 - 6x^2) \\
 \Delta_{\text{max. (at centre)}} & \dots\dots\dots = \frac{w\ell^4}{384 EI} \\
 \Delta_x & \dots\dots\dots = \frac{wx^2}{24 EI}(\ell - x)^2
 \end{aligned}$$



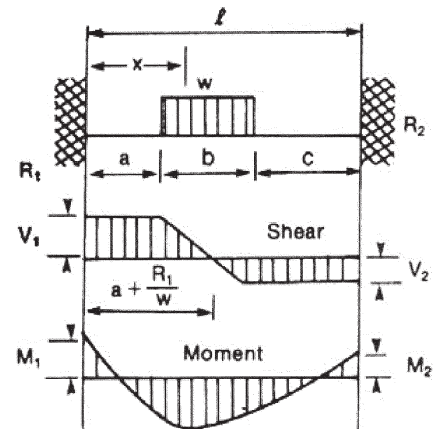
Beam fixed at both ends — concentrated load at any point

$$\begin{aligned}
 R_1 = V_1 \text{ (max. when } a < b) & \dots\dots\dots = \frac{Pb^2}{\ell^3}(3a + b) \\
 R_2 = V_2 \text{ (max. when } a > b) & \dots\dots\dots = \frac{Pa^2}{\ell^3}(a + 3b) \\
 M_{\text{(max. when } a < b)} & \dots\dots\dots = -\frac{Pab^2}{\ell^2} \\
 M_{\text{(max. when } a > b)} & \dots\dots\dots = -\frac{Pa^2b}{\ell^2} \\
 M_{\text{(at point of load)}} & \dots\dots\dots = \frac{2Pa^2b^2}{\ell^3} \\
 M_x \text{ (when } x < a) & \dots\dots\dots = R_1x - \frac{Pab^2}{\ell^2} \\
 \Delta_{\text{max. (when } a > b \text{ at } x = \frac{2a\ell}{3a+b})} & \dots\dots\dots = \frac{2Pa^3b^2}{3EI(3a+b)^2} \\
 \Delta_{\text{(at point of load)}} & \dots\dots\dots = \frac{Pa^3b^3}{3EI\ell^3} \\
 \Delta_x \text{ (when } x < a) & \dots\dots\dots = \frac{Pb^2x^2}{6EI\ell^3}(3a\ell - 3ax - bx)
 \end{aligned}$$



Beam fixed at both ends — uniform load partially distributed

$$\begin{aligned}
 M_1 & \dots\dots\dots = \frac{w}{12\ell^2}[(\ell - a)^3(\ell + 3a) - c^3(4\ell - 3c)] \\
 M_2 & \dots\dots\dots = \frac{w}{12\ell^2}[(\ell - c)^3(\ell + 3c) - a^3(4\ell - 3a)] \\
 R_1 = V_1 & \dots\dots\dots = \frac{1}{\ell}[M_1 - M_2 + wc\left(c + \frac{b}{2}\right)] \\
 R_2 = V_2 & \dots\dots\dots = \frac{1}{\ell}[M_2 - M_1 + wc\left(a + \frac{b}{2}\right)] \\
 M_x \text{ (when } x < a) & \dots\dots\dots = R_1x - M_1 \\
 M_x \text{ (when } a < x < (a + c)) & \dots\dots\dots = R_1x - M_1 - \frac{w}{2}(x - a)^2 \\
 M_x \text{ (when } x > (a + c)) & \dots\dots\dots = R_2(\ell - x) - M_2
 \end{aligned}$$



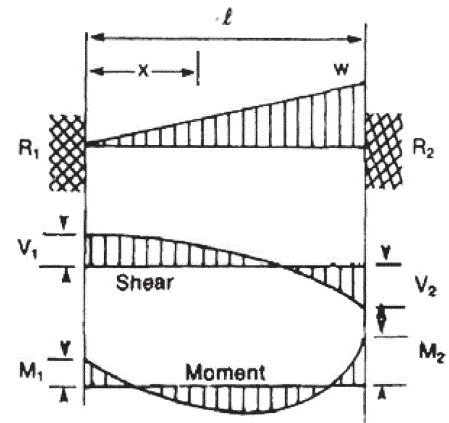
Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.
 P: Concentrated load.

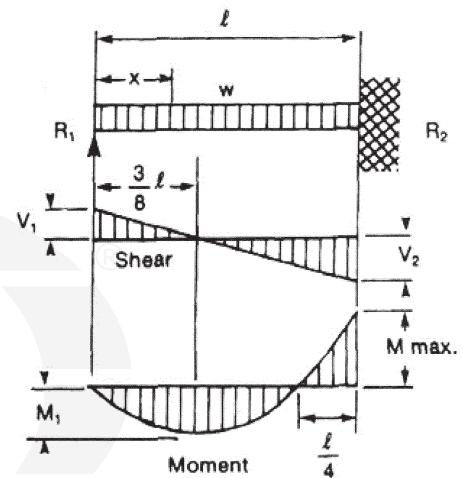
Table B-1 Beam Diagrams (Cont'd)

Beam fixed at both ends — load increasing uniformly to one end

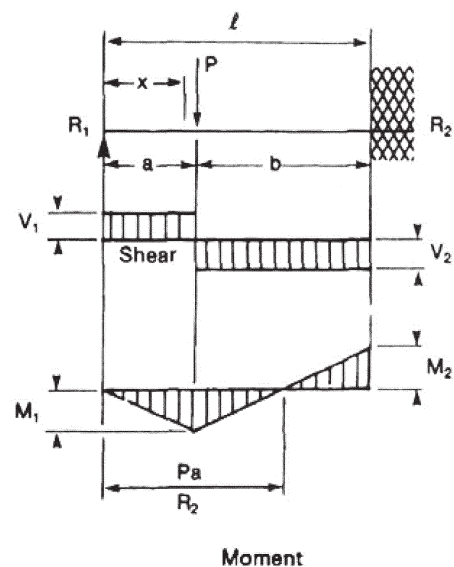
$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{3wl}{20} \\
 R_2 = V_2 & \dots\dots\dots = \frac{7wl}{20} \\
 V_x & \dots\dots\dots = \frac{3wl}{20} - \frac{wx^2}{2l} \\
 M_1 & \dots\dots\dots = -\frac{wl^2}{30} \\
 M_2 & \dots\dots\dots = -\frac{wl^2}{20} \\
 M_x & \dots\dots\dots = \frac{3wlx}{20} - \frac{wl^2}{30} - \frac{wx^3}{6l}
 \end{aligned}$$

**Beam fixed at one end, supported at other — uniformly distributed load**

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{3wl}{8} \\
 R_2 = V_2 \text{ max.} & \dots\dots\dots = \frac{5wl}{8} \\
 V_x & \dots\dots\dots = R_1 - wx \\
 M \text{ max.} & \dots\dots\dots = -\frac{wl^2}{8} \\
 M_1 \left(\text{at } x = \frac{3}{8}l \right) & \dots\dots\dots = \frac{9}{128}wl^2 \\
 M_x & \dots\dots\dots = R_1x - \frac{wx^2}{2} \\
 \Delta \text{ max. (at } x = \frac{l}{16}(1 + \sqrt{33}) = 0.4215l) & \dots\dots\dots = \frac{wl^4}{185EI} \\
 \Delta_x & \dots\dots\dots = \frac{wx}{48EI} (l^3 - 3lx^2 + 2x^3)
 \end{aligned}$$

**Beam fixed at one end, supported at other — concentrated load at any point**

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{Pb^2}{2l^3} (a + 2l) \\
 R_2 = V_2 & \dots\dots\dots = \frac{Pa}{2l^3} (3l^2 - a^2) \\
 M_1 \text{ (at point of load)} & \dots\dots\dots = R_1a \\
 M_2 \text{ (at fixed end)} & \dots\dots\dots = -\frac{Pab}{2l^2} (a + l) \\
 M_x \text{ (when } x < a) & \dots\dots\dots = R_1x \\
 M_x \text{ (when } x > a) & \dots\dots\dots = R_1x - P(x - a) \\
 \Delta \text{ max. (when } a < 0.414l \text{ at } x = l \frac{l^2 + a^2}{3l^2 - a^2}) & \dots\dots\dots = \frac{Pa}{3EI} \frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2} \\
 \Delta \text{ max. (when } a > 0.414l \text{ at } x = l \sqrt{\frac{a}{2l + a}}) & \dots\dots\dots = \frac{Pab^2}{6EI} \sqrt{\frac{a}{2l + a}} \\
 \Delta a \text{ (at point of load)} & \dots\dots\dots = \frac{Pa^2b^3}{12EI l^3} (3l + a) \\
 \Delta x \text{ (when } x < a) & \dots\dots\dots = \frac{Pb^2x}{12EI l^3} (3al^2 - 2lx^2 - ax^2) \\
 \Delta x \text{ (when } x > a) & \dots\dots\dots = \frac{Pa}{12EI l^3} (l - x)^2 (3l^2x - a^2x - 2a^2l)
 \end{aligned}$$

**Note:**

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

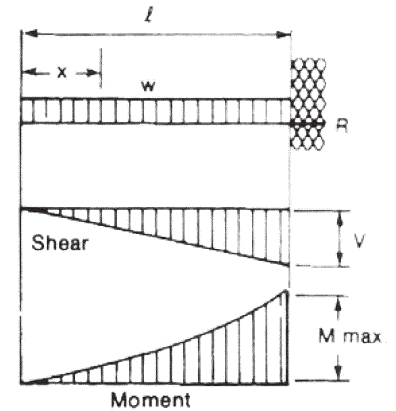
P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

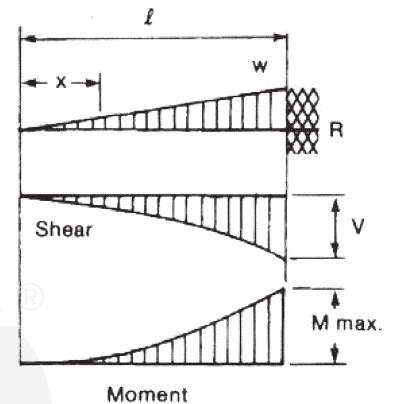
Table B-1 Beam Diagrams (Cont'd)

Cantilever Beam — uniformly distributed load

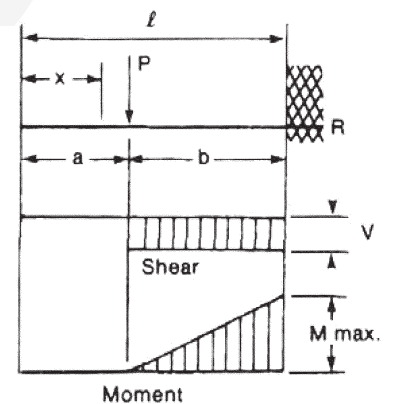
$$\begin{aligned}
 R = V & \dots\dots\dots = w\ell \\
 V_x & \dots\dots\dots = wx \\
 M \text{ max. (at fixed end)} & \dots\dots\dots = -\frac{w\ell^2}{2} \\
 M_x & \dots\dots\dots = -\frac{wx^2}{2} \\
 \Delta \text{ max. (at free end)} & \dots\dots\dots = -\frac{w\ell^4}{8EI} \\
 \Delta_x & \dots\dots\dots = -\frac{w}{24EI} (x^4 - 4\ell^3x + 3\ell^4)
 \end{aligned}$$

**Cantilever Beam — load increasing uniformly to fixed end**

$$\begin{aligned}
 R = V & \dots\dots\dots = \frac{w\ell}{2} \\
 V_x & \dots\dots\dots = \frac{wx^2}{2\ell} \\
 M \text{ max. (at fixed end)} & \dots\dots\dots = -\frac{w\ell^2}{6} \\
 M_x & \dots\dots\dots = -\frac{wx^3}{6\ell} \\
 \Delta \text{ max. (at free end)} & \dots\dots\dots = -\frac{w\ell^4}{30EI} \\
 \Delta_x & \dots\dots\dots = -\frac{w}{120EI\ell} (x^5 - 5\ell^4x + 4\ell^5)
 \end{aligned}$$

**Cantilever Beam — concentrated load at any point**

$$\begin{aligned}
 R = V & \dots\dots\dots = P \\
 M \text{ max. (at fixed end)} & \dots\dots\dots = -Pb \\
 M_x \text{ (when } x > a) & \dots\dots\dots = P(x - a) \\
 \Delta \text{ max. (at free end)} & \dots\dots\dots = \frac{Pb^2}{6EI} (3\ell - b) \\
 \Delta a \text{ (at point of load)} & \dots\dots\dots = \frac{Pb^3}{3EI} \\
 \Delta_x \text{ (when } x < a) & \dots\dots\dots = \frac{Pb^2}{6EI} (3\ell - 3x - b) \\
 \Delta_x \text{ (when } x > a) & \dots\dots\dots = \frac{P(\ell - x)^2}{6EI} (3b - \ell + x)
 \end{aligned}$$

**Note:**

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

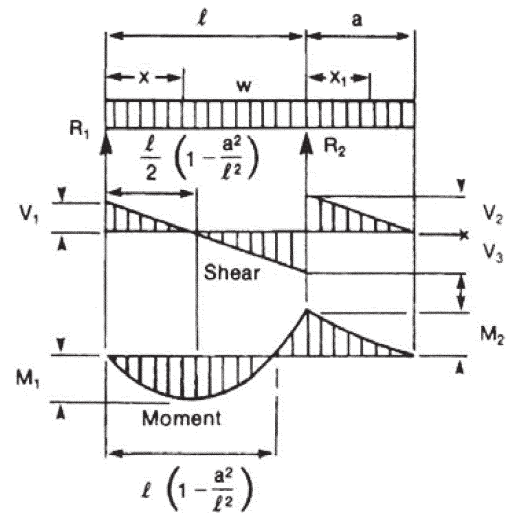
P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-1 Beam Diagrams (Cont'd)

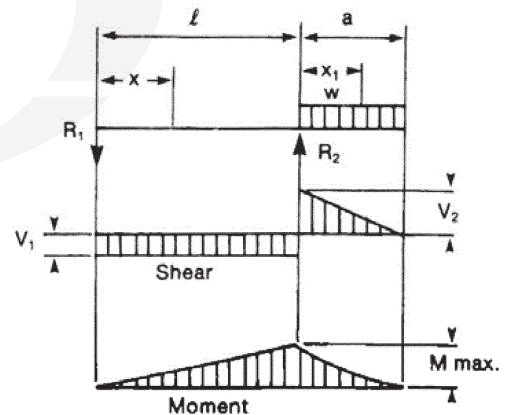
Beam overhanging one support — uniformly distributed load

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{w}{2\ell} (\ell^2 - a^2) \\
 R_2 = V_2 + V_3 & \dots\dots\dots = \frac{w}{2\ell} (\ell + a)^2 \\
 V_2 & \dots\dots\dots = wa \\
 V_3 & \dots\dots\dots = \frac{w}{2\ell} (\ell^2 + a^2) \\
 V_x \text{ (between supports)} & \dots\dots\dots = R_1 - wx \\
 V_{x_1} \text{ (for overhang)} & \dots\dots\dots = w(a - x_1) \\
 M_1 \left(\text{at } x = \frac{\ell}{2} \left[1 - \frac{a^2}{\ell^2} \right] \right) & \dots\dots\dots = \frac{w}{8\ell^2} (\ell + a)^2 (\ell - a)^2 \\
 M_2 \text{ (at } R_2) & \dots\dots\dots = \frac{wa^2}{2} \\
 M_x \text{ (between supports)} & \dots\dots\dots = \frac{wx}{2\ell} (\ell^2 - a^2 - x\ell) \\
 M_{x_1} \text{ (for overhang)} & \dots\dots\dots = \frac{w}{2} (a - x_1)^2 \\
 \Delta_x \text{ (between supports)} & \dots\dots\dots = \frac{wx}{24EI\ell} (\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2) \\
 \Delta_{x_1} \text{ (for overhang)} & \dots\dots\dots = \frac{wx_1}{24EI} (4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)
 \end{aligned}$$



Beam overhanging one support — uniformly distributed load on overhang

$$\begin{aligned}
 R_1 = V_1 & \dots\dots\dots = \frac{wa^2}{2\ell} \\
 R_2 = V_1 + V_2 & \dots\dots\dots = \frac{wa}{2\ell} (2\ell + a) \\
 V_2 & \dots\dots\dots = wa \\
 V_{x_1} \text{ (for overhang)} & \dots\dots\dots = w(a - x_1) \\
 M \text{ max. (at } R_2) & \dots\dots\dots = \frac{wa^2}{2} \\
 M_x \text{ (between supports)} & \dots\dots\dots = \frac{wa^2x}{2\ell} \\
 M_{x_1} \text{ (for overhang)} & \dots\dots\dots = \frac{w}{2} (a - x_1)^2 \\
 \Delta \text{ max. (between supports at } x = \frac{\ell}{\sqrt{3}}) & \dots\dots\dots = \frac{wa^2\ell^2}{18\sqrt{3}EI} = 0.03208 \frac{wa^2\ell^2}{EI} \\
 \Delta \text{ max. (for overhang at } x_1 = a) & \dots\dots\dots = \frac{wa^3}{24EI} (4\ell + 3a) \\
 \Delta_x \text{ (between supports)} & \dots\dots\dots = \frac{wa^2x}{12EI\ell} (\ell^2 - x^2) \\
 \Delta_{x_1} \text{ (for overhang)} & \dots\dots\dots = \frac{wx_1}{24EI} (4a^2\ell + 6a^2x_1 - 4ax_1^2 + x_1^3)
 \end{aligned}$$



Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length

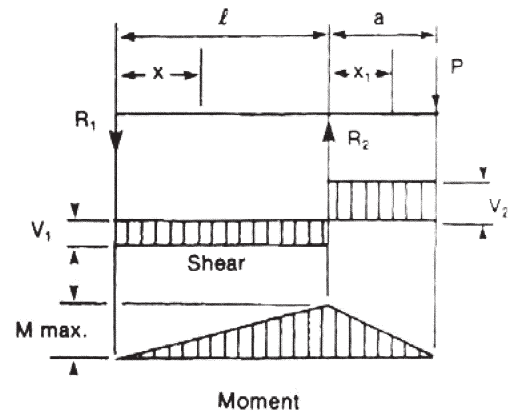
P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-1 Beam Diagrams (Cont'd)

Beam overhanging one support — concentrated load at end of overhang

$$\begin{aligned}
 R_1 = V_1 &= \frac{Pa}{\ell} \\
 R_2 = V_1 + V_2 &= \frac{P}{\ell} (\ell + a) \\
 V_2 &= P \\
 M \text{ max. (at } R_2) &= Pa \\
 M_1 \text{ (between supports)} &= \frac{Pax}{\ell} \\
 M_{1,1} \text{ (for overhang)} &= P(a - x_1) \\
 \Delta \text{ max. (between supports at } x = \frac{\ell}{\sqrt{3}}) &= \frac{Pa\ell^2}{9\sqrt{3}EI} = 0.06415 \frac{Pa\ell^2}{EI} \\
 \Delta \text{ max. (for overhang at } x_1 = a) &= \frac{Pa^2}{3EI} (\ell + a) \\
 \Delta_1 \text{ (between supports)} &= \frac{Pax}{6EI\ell} (\ell^2 - x^2) \\
 \Delta_{1,1} \text{ (for overhang)} &= \frac{Px_1}{6EI} (2a\ell + 3ax_1 - x_1^2)
 \end{aligned}$$

**Note:**

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-2 Moments and Reactions in Continuous Beams under Uniformly Distributed Loads

Moment : $\text{Coefficient} \times w\ell^2$ Reaction: $\text{Coefficient} \times w\ell$		w: Uniform load per unit length ℓ : Length of one span	
Two Equal Spans		Four Equal Spans	
Three Equal Spans			

Table B-3 Moments and Reactions in Continuous Beams under Central Point Loads

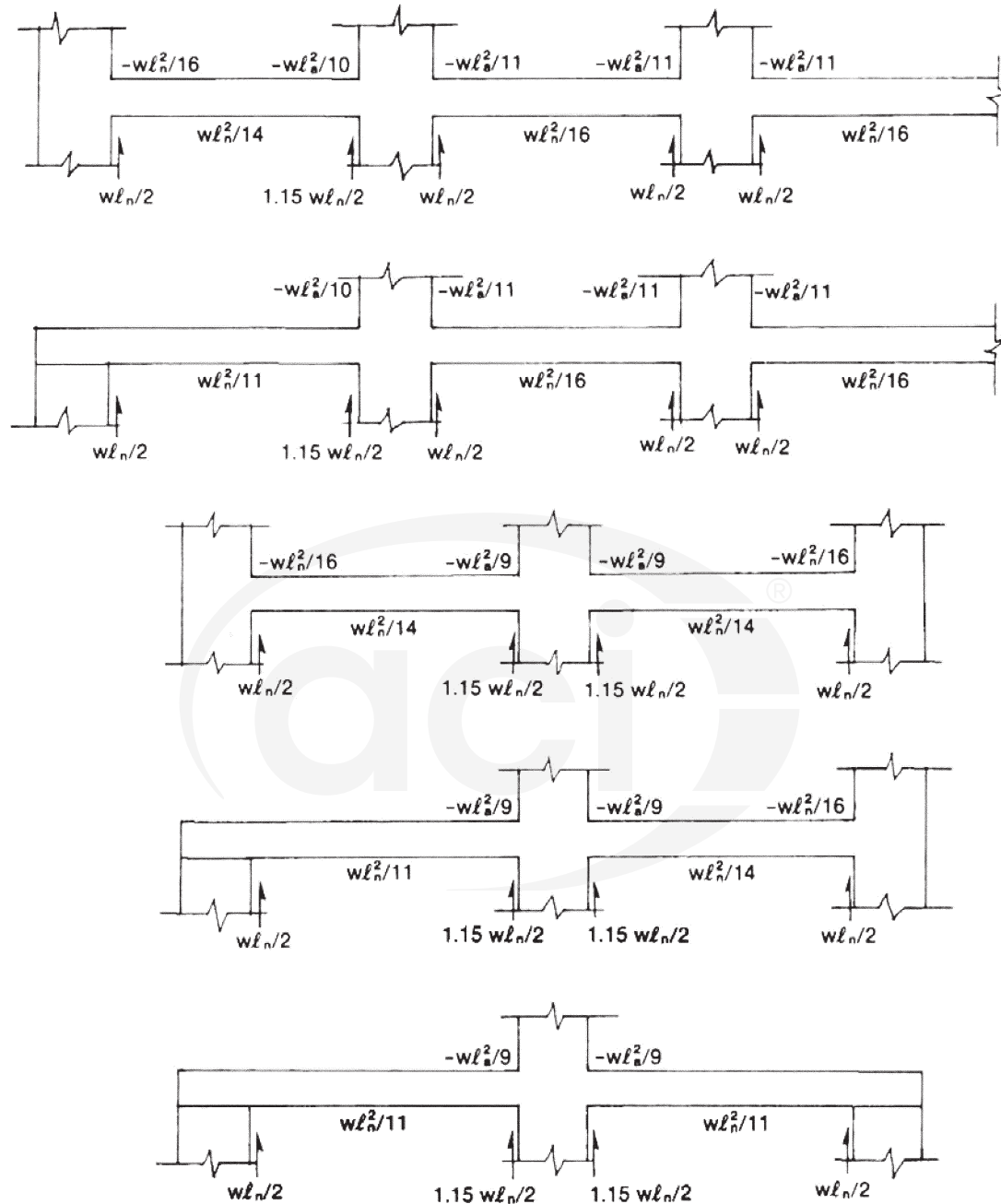
Moment : Coefficient $\times Pl$ Reaction: Coefficient $\times P$		P: Concentrated load l : Length of one span	
Two Equal Spans		Four Equal Spans	
Three Equal Spans			

Table B-4 Moments and Reactions in Continuous Beams, Point Loads at Third Points of Span

Moment : Coefficient $\times P\ell$ Reaction: Coefficient $\times P$		P: Total concentrated load on one span ℓ : Length of one span	
Two Equal Spans		Four Equal Spans	
Three Equal Spans			

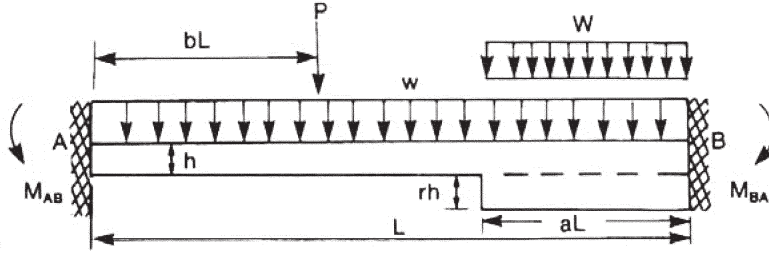
Table B-5 Approximate Moments and Shears for Continuous Beams and One-way Slabs

w : Factored load per unit length
 l_n : Clear span length
 l_a : Average length of adjacent clear spans

**Notes:**

- 1 This figure is applicable to prismatic members loaded with uniformly distributed load where the ratio of factored live load to factored dead load is not greater than 3.0, and the span lengths approximately equal, with the longer of the two adjacent spans not greater than the shorter by more than 20 per cent.
- 2 If the exterior support is a spandrel or girder, the negative moment at the interior face of exterior support is $wl_n^2/24$.
- 3 For slabs with spans not exceeding 3 m or beams with the ratio of the sum of the column stiffnesses to the beam stiffness exceeds eight at each end, the negative moment at all supports can be taken equal to $wl_n^2/12$.

Table B-6 Beams with Prismatic Haunch at One End



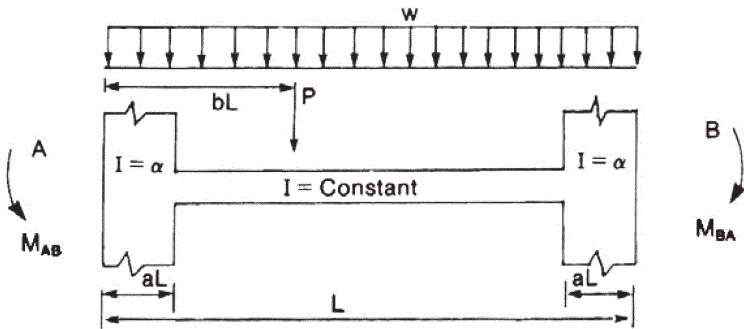
Right haunch	Carry-over factors	Stiffness factors	Unif. load		Concentrated load F.E.M. — coef. $\times PL$												Haunch load	
			F.E.M. coef. $\times wL^2$		b												F.E.M. coef. $\times WL^2$	
					0.1		0.3		0.5		0.7		0.9					
a	r		C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
0.1	0.4		0.593	0.491	4.24	5.12	0.0749	0.1016	0.0799	0.1113	0.1397	0.0788	0.1110	0.1553	0.0478	0.1798	0.0042	0.0911
	0.6		0.615	0.490	4.30	5.40	0.0727	0.1062	0.0797	0.1119	0.1378	0.0828	0.1074	0.1630	0.0439	0.1881	0.0029	0.0937
	1.0		0.639	0.488	4.37	5.72	0.0703	0.1114	0.0794	0.1125	0.1358	0.0873	0.1035	0.1716	0.0396	0.1974	0.0016	0.0966
	1.5		0.652	0.487	4.40	5.89	0.0690	0.1143	0.0792	0.1129	0.1346	0.0898	0.1012	0.1764	0.0373	0.2026	0.0008	0.0982
	2.0		0.658	0.487	4.42	5.97	0.0684	0.1156	0.0791	0.1131	0.1341	0.0910	0.1002	0.1786	0.0361	0.2050	0.0005	0.0990
0.2	0.4		0.677	0.469	4.42	6.37	0.0706	0.1126	0.0791	0.1134	0.1345	0.0925	0.1020	0.1788	0.0409	0.1975	0.0050	0.0890
	0.6		0.730	0.463	4.56	7.18	0.0664	0.1225	0.0785	0.1149	0.1302	0.1025	0.0942	0.1972	0.0335	0.2148	0.0037	0.0917
	1.0		0.793	0.458	4.74	8.22	0.0610	0.1353	0.0777	0.1168	0.1248	0.1154	0.0843	0.2207	0.0242	0.2368	0.0022	0.0951
	1.5		0.831	0.455	4.86	8.88	0.0576	0.1434	0.0772	0.1180	0.1214	0.1235	0.0781	0.2355	0.0182	0.2507	0.0012	0.0973
	2.0		0.849	0.453	4.91	9.20	0.0559	0.1473	0.0769	0.1186	0.1197	0.1276	0.0750	0.2429	0.0153	0.2576	0.0007	0.0984
0.3	0.4		0.741	0.439	4.52	7.63	0.0698	0.1155	0.0787	0.1149	0.1319	0.1013	0.0987	0.1899	0.0420	0.1929	0.0056	0.0868
	0.6		0.831	0.427	4.75	9.24	0.0642	0.1296	0.0777	0.1175	0.1255	0.1182	0.0877	0.2185	0.0338	0.2130	0.0045	0.0893
	1.0		0.954	0.415	5.09	11.69	0.0559	0.1511	0.0762	0.1215	0.1158	0.1440	0.0711	0.2621	0.0217	0.2436	0.0028	0.0930
	1.5		1.036	0.409	5.34	13.53	0.0497	0.1673	0.0751	0.1245	0.1085	0.1633	0.0587	0.2948	0.0128	0.2665	0.0017	0.0959
	2.0		1.078	0.407	5.48	14.54	0.0464	0.1762	0.0745	0.1262	0.1045	0.1740	0.0520	0.3129	0.0080	0.2792	0.0010	0.0974
0.4	0.4		0.774	0.405	4.55	8.70	0.0703	0.1117	0.0786	0.1156	0.1315	0.1035	0.0992	0.1855	0.0445	0.1773	0.0059	0.0849
	0.6		0.901	0.386	4.83	11.28	0.0646	0.1269	0.0774	0.1192	0.1240	0.1254	0.0875	0.2182	0.0377	0.1932	0.0049	0.0869
	1.0		1.102	0.367	5.33	16.03	0.0549	0.1548	0.0752	0.1257	0.1105	0.1658	0.0671	0.2780	0.0267	0.2222	0.0034	0.0904
	1.5		1.260	0.357	5.79	20.46	0.0462	0.1807	0.0732	0.1319	0.0982	0.2035	0.0485	0.3339	0.0173	0.2491	0.0022	0.0938
	2.0		1.349	0.352	6.09	23.32	0.0407	0.1975	0.0719	0.1358	0.0903	0.2278	0.0367	0.3699	0.0113	0.2664	0.0014	0.0959
0.5	0.4		0.768	0.371	4.56	9.45	0.0700	0.1048	0.0786	0.1154	0.1312	0.0993	0.0983	0.1679	0.0442	0.1663	0.0059	0.0836
	0.6		0.919	0.343	4.84	12.94	0.0651	0.1176	0.0774	0.1193	0.1240	0.1218	0.0884	0.1935	0.0386	0.1769	0.0051	0.0849
	1.0		1.200	0.316	5.42	20.61	0.0561	0.1451	0.0749	0.1280	0.1096	0.1709	0.0706	0.2486	0.0299	0.1993	0.0038	0.0877
	1.5		1.470	0.301	6.10	29.74	0.0466	0.1777	0.0720	0.1384	0.0934	0.2290	0.0516	0.3137	0.0215	0.2255	0.0027	0.0909
	2.0		1.647	0.295	6.63	37.04	0.0393	0.2036	0.0698	0.1466	0.0807	0.2755	0.0370	0.3655	0.0153	0.2463	0.0019	0.0934
0.6	0.4		0.726	0.341	4.62	9.84	0.0675	0.0986	0.0782	0.1146	0.1280	0.0916	0.0923	0.1519	0.0419	0.1603	0.0056	0.0829
	0.6		0.872	0.305	4.88	13.97	0.0630	0.1072	0.0771	0.1183	0.1214	0.1096	0.0835	0.1664	0.0368	0.1666	0.0048	0.0837
	1.0		1.196	0.267	5.43	24.35	0.0560	0.1277	0.0748	0.1274	0.1092	0.1537	0.0705	0.1999	0.0299	0.1804	0.0038	0.0854
	1.5		1.588	0.247	6.18	39.79	0.0482	0.1572	0.0718	0.1408	0.0939	0.2183	0.0572	0.2478	0.0237	0.1997	0.0030	0.0878
	2.0		1.905	0.237	6.92	55.51	0.0412	0.1870	0.0688	0.1544	0.0792	0.2839	0.0455	0.2960	0.0186	0.2189	0.0023	0.0901
0.7	0.4		0.657	0.321	4.86	9.96	0.0631	0.0954	0.0770	0.1138	0.1175	0.0846	0.0844	0.1461	0.0392	0.1582	0.0053	0.0827
	0.6		0.770	0.275	5.14	14.39	0.0580	0.1006	0.0758	0.1167	0.1097	0.0955	0.0745	0.1543	0.0335	0.1621	0.0045	0.0832
	1.0		1.056	0.224	5.62	26.45	0.0516	0.1122	0.0738	0.1243	0.0992	0.1213	0.0626	0.1710	0.0269	0.1694	0.0035	0.0841
	1.5		1.491	0.196	6.24	47.48	0.0463	0.1304	0.0714	0.1371	0.0890	0.1633	0.0537	0.1959	0.0223	0.1796	0.0028	0.0854
	2.0		1.944	0.183	6.95	73.85	0.0417	0.1523	0.0687	0.1530	0.0793	0.2149	0.0468	0.2255	0.0191	0.1915	0.0024	0.0869
0.8	0.4		0.583	0.319	5.46	9.97	0.0585	0.0951	0.0741	0.1137	0.1040	0.0837	0.0793	0.1456	0.0380	0.1580	0.0053	0.0826
	0.6		0.645	0.263	5.89	14.44	0.0516	0.0990	0.0721	0.1160	0.0921	0.0907	0.0667	0.1520	0.0311	0.1614	0.0043	0.0831
	1.0		0.818	0.196	6.47	27.06	0.0435	0.1053	0.0696	0.1211	0.0781	0.1025	0.0521	0.1615	0.0232	0.1660	0.0031	0.0838
	1.5		1.128	0.155	6.98	50.85	0.0385	0.1130	0.0676	0.1296	0.0692	0.1175	0.0432	0.1715	0.0184	0.1705	0.0024	0.0844
	2.0		1.533	0.135	7.47	84.60	0.0355	0.1222	0.0658	0.1412	0.0638	0.1357	0.0384	0.1824	0.0159	0.1750	0.0020	0.0849
0.9	0.4		0.524	0.356	6.87	10.10	0.0604	0.0948	0.0674	0.1157	0.1031	0.0835	0.0844	0.1439	0.0418	0.1568	0.0059	0.0824
	0.6		0.542	0.295	7.95	14.58	0.0497	0.0991	0.0623	0.1184	0.0866	0.0913	0.0691	0.1510	0.0339	0.1605	0.0048	0.0830
	1.0		0.594	0.206	9.44	27.16	0.0372	0.1052	0.0553	0.1226	0.0642	0.1023	0.0484	0.1609	0.0231	0.1656	0.0032	0.0837
	1.5		0.695	0.142	10.48	51.25	0.0289	0.1098	0.0506	0.1266	0.0492	0.1105	0.0346	0.1680	0.0159	0.1692	0.0021	0.0842
	2.0		0.842	0.107	11.07	86.80	0.0245	0.1147	0.0481	0.1306	0.0414	0.1159	0.0274	0.1723	0.0121	0.1714	0.0016	0.0845

Table B-7 Beams with Prismatic Haunch at Both Ends

The diagram shows a beam of total length L with fixed supports at both ends, labeled A and B . The beam has prismatic haunches at both ends, each of length aL and height h . The central section of the beam has a length bL . A concentrated load P is applied at the center of the beam. The beam is subjected to a uniformly distributed load W over the entire length L . The moments at the supports are M_{AB} and M_{BA} .

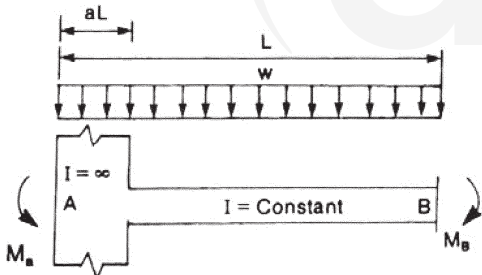
a	r	Carry-over factors $C_{AB} = C_{BA}$	Stiffness factors $k_{AB} = k_{BA}$	Unif. load	Concentrated load F.E.M. — coef. $\times PL$										Haunch load both haunches F.E.M. coef. $\times WL^2$	
				F.E.M. coef. $\times WL^2$	b											
					0.1		0.3		0.5		0.7		0.9			
				$M_{AB} = M_{BA}$	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	$M_{AB} = M_{BA}$	
0.1	0.4	0.583	5.49	0.0921	0.0905	0.0053	0.1727	0.0606	0.1396	0.1396	0.0606	0.1727	0.0053	0.0905	0.0049	
	0.6	0.603	5.93	0.0940	0.0932	0.0040	0.1796	0.0589	0.1428	0.1428	0.0589	0.1796	0.0040	0.0932	0.0049	
	1.0	0.624	6.45	0.0961	0.0962	0.0023	0.1873	0.0566	0.1462	0.1462	0.0566	0.1873	0.0023	0.0962	0.0050	
	1.5	0.636	6.75	0.0972	0.0980	0.0013	0.1918	0.0551	0.1480	0.1480	0.0551	0.1918	0.0013	0.0980	0.0050	
	2.0	0.641	6.90	0.0976	0.0988	0.0008	0.1939	0.0543	0.1489	0.1489	0.0543	0.1939	0.0008	0.0988	0.0050	
0.2	0.4	0.634	7.32	0.0970	0.0874	0.0079	0.1852	0.0623	0.1506	0.1506	0.0623	0.1852	0.0079	0.0874	0.0187	
	0.6	0.674	8.80	0.1007	0.0899	0.0066	0.1993	0.0584	0.1575	0.1575	0.0584	0.1993	0.0066	0.0899	0.0191	
	1.0	0.723	11.09	0.1049	0.0935	0.0046	0.2193	0.0499	0.1654	0.1654	0.0499	0.2193	0.0046	0.0935	0.0195	
	1.5	0.752	12.87	0.1073	0.0961	0.0029	0.2338	0.0420	0.1699	0.1699	0.0420	0.2338	0.0029	0.0961	0.0197	
	2.0	0.765	13.87	0.1084	0.0976	0.0018	0.2410	0.0372	0.1720	0.1720	0.0372	0.2410	0.0018	0.0976	0.0198	
0.3	0.4	0.642	9.02	0.0977	0.0845	0.0097	0.1763	0.0707	0.1558	0.1558	0.0707	0.1763	0.0097	0.0845	0.0397	
	0.6	0.697	12.09	0.1027	0.0861	0.0095	0.1898	0.0700	0.1665	0.1665	0.0700	0.1898	0.0095	0.0861	0.0410	
	1.0	0.775	18.68	0.1091	0.0890	0.0084	0.2136	0.0627	0.1803	0.1803	0.0627	0.2136	0.0084	0.0890	0.0426	
	1.5	0.828	26.49	0.1132	0.0920	0.0065	0.2376	0.0492	0.1891	0.1891	0.0492	0.2376	0.0065	0.0920	0.0437	
	2.0	0.855	32.77	0.1153	0.0943	0.0048	0.2555	0.0366	0.1934	0.1934	0.0366	0.2555	0.0048	0.0943	0.0442	
0.4	0.4	0.599	10.15	0.0937	0.0825	0.0101	0.1601	0.0732	0.1509	0.1509	0.0732	0.1601	0.0101	0.0825	0.0642	
	0.6	0.652	14.52	0.0966	0.0833	0.0106	0.1668	0.0776	0.1632	0.1632	0.0776	0.1668	0.0106	0.0833	0.0668	
	1.0	0.744	26.06	0.1067	0.0847	0.0112	0.1790	0.0835	0.1833	0.1833	0.0835	0.1790	0.0112	0.0847	0.0711	
	1.5	0.827	45.95	0.1131	0.0862	0.0113	0.1919	0.0852	0.1995	0.1995	0.0852	0.1919	0.0113	0.0862	0.0746	
	2.0	0.878	71.41	0.1169	0.0876	0.0108	0.2033	0.0822	0.2089	0.2089	0.0822	0.2033	0.0108	0.0876	0.0766	
0.5	0.0	0.500	4.00	0.0833	0.0810	0.0090	0.1470	0.0630	0.1250	0.1250	0.0630	0.1470	0.0090	0.0810	0.0833	

Table B-8 Prismatic Member with Equal Infinitely Stiff End Regions



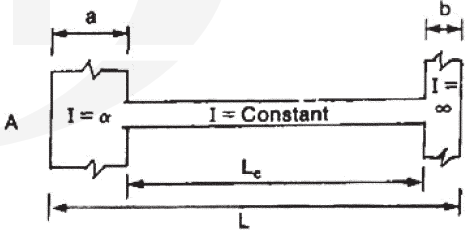
a	Carry-over Factors	Stiffness Factors	Unif. Load F.E.M. Coef. × wL ²	Concentrated Load F.E.M. — Coef. × PL									
				b									
				0.1		0.2		0.3		0.4		0.5	
				M _{AB}	M _{BA}	M _{AB}	M _{BA}	M _{AB}	M _{BA}	M _{AB}	M _{BA}	M _{AB}	M _{BA}
0.05	0.575	5.23	0.0913	0.0940	0.0030	0.1505	0.0245	0.1711	0.0595	0.1640	0.0999	0.1375	0.1375
0.10	0.648	7.11	0.0983	0.1000	0.0000	0.1722	0.0152	0.1968	0.0532	0.1856	0.1019	0.1500	0.1500
0.15	0.719	10.17	0.1046	0.1000	0.0000	0.1909	0.0056	0.2247	0.0431	0.2095	0.1013	0.1625	0.1625
0.20	0.786	15.56	0.1100	0.1000	0.0000	0.2000	0.0000	0.2546	0.0286	0.2369	0.0964	0.1750	0.1750
0.25	0.846	26.00	0.1146	0.1000	0.0000	0.2000	0.0000	0.2830	0.0118	0.2699	0.0851	0.1875	0.1875

Table B-9 Prismatic Member with Infinitely Stiff Region at One End



a	Carry-over Factors		Stiffness Factors		Unif. Load F.E.M. Coef. × WL ²	
	C _{AB}	C _{BA}	k _{AB}	k _{BA}	M _{AB}	M _{BA}
0.05	0.496	0.579	4.91	4.21	0.1002	0.0752
0.10	0.486	0.657	6.00	4.44	0.1175	0.0675
0.15	0.471	0.765	7.64	4.71	0.1352	0.0602
0.20	0.452	0.875	9.60	5.00	0.1533	0.0533
0.25	0.429	1.000	12.44	5.33	0.1719	0.0469

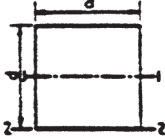
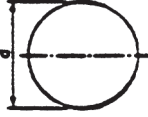
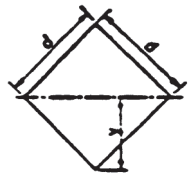
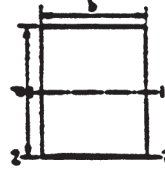
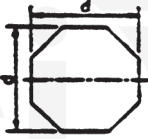
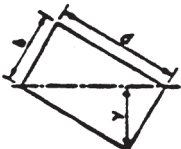
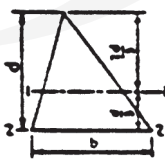
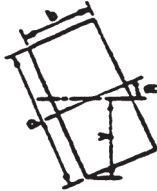
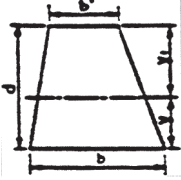
Table B-10 Prismatic Member with Unequal Infinitely Stiff End Regions



$$k' = \left[1 - \frac{3(L_c + 2b)(L_c + 2a)}{L_c^2} \right] \frac{L}{L_c}$$
$$k_{AB} = \left[1 + 3 \left[\frac{L_c + 2a}{L_c} \right]^2 \right] \frac{L}{L_c}$$
$$k_{BA} = \left[1 + 3 \left[\frac{L_c + 2b}{L_c} \right]^2 \right] \frac{L}{L_c}$$
$$C_{AB} = k'/k_{AB}$$
$$C_{BA} = k'/k_{BA}$$

MNL-17(19)—SECTIONAL PROPERTIES

Table C-1—Properties of sections

Dash-and-dot lines are drawn through centers of gravity A = area of section; I = moment of inertia; R = radius of gyration			
	$A = d^2$ $I_1 = \frac{d^4}{12}$ $I_2 = \frac{d^4}{3}$ $R_1 = 0.2887d$ $R_2 = 0.57774d$		$A = \frac{\pi d^2}{4} = 0.7854d^2$ $I = \frac{\pi d^4}{64} = 0.0491d^4$ $R = \frac{d}{4}$
	$A = d^2$ $y = 0.7071d$ $I_1 = \frac{d^4}{12}$ $R = 0.2887d$		$A = 0.8660d^2$ $I = 0.060d^4$ $R = 0.264d$
	$A = bd$ $I_1 = \frac{bd^3}{12}$ $I_2 = \frac{bd^3}{3}$ $R_1 = 0.2887d$ $R_2 = 0.57774d$		$A = 0.8284d^2$ $I = 0.055d^4$ $R = 0.257d$
	$A = bd$ $y = \frac{bd}{\sqrt{b^2 + d^2}}$ $I = \frac{b^3 d^3}{6(b^2 + d^2)}$ $R = \frac{bd}{\sqrt{6(b^2 + d^2)}}$		$A = \frac{bd}{2}$ $I_1 = \frac{bd^3}{36}$ $I_2 = \frac{bd^3}{12}$ $R_1 = 0.236d$ $R_2 = 0.408d$
	$A = bd$ $y = \frac{b \sin \infty + d \cos \infty}{2}$ $R = \frac{\sqrt{b^2 \sin^2(\infty + d^2 \cos^2 \infty)}}{12}$		$A = \frac{d}{2}(b + b')$ $y = \frac{d(2b + b')}{3(b + b')}$ $y = \frac{d(b + 2b')}{3(b + b')}$ $I = \frac{d^2(b^2 + 4bb' + b'^2)}{36(b + b')}$ $R = \frac{d}{6(b + b')} \sqrt{2(b^2 + 4bb' + b'^2)}$