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Strut-and-Tie Method Guidelines for ACI 318-19— Guide

Reported by Joint ACI-ASCE Committee 445

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Strut-and-Tie Method Guidelines for ACI 318-19—Guide

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Strut-and-tie models (STMs) were first used at the end of the nineteenth century as a concrete design method. The method was added to ACI 318 in 2002 as Appendix A. In 2014, STM provisions were moved into the main body of the code as Chapter 23, Strut-and-Tie Method. This document focuses on the ACI 318-19 implementation of strut-and-tie modeling. The main objectives of this document are to: 1) explain the intent and application of ACI 318 STM provisions; 2) provide additional design guidance for the STM based on other design codes, specifications, and committee documents; and 3) present design recommendations from recent research publications. This document provides practical guidance to the structural design community.

Keywords: D-regions; design; disturbed regions; model; node; strut; tie.

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CHAPTER 1—INTRODUCTION**1.1—Background**

While the origins of strut-and-tie models date back to the end of the nineteenth century (Ritter 1899; Considère 1900; Mörsch 1909), the strut-and-tie design method was not formally introduced into ACI 318 until 2002. Up until the 2014 version of ACI 318, strut-and-tie model (STM) provisions were included in Appendix A of ACI 318. With the reorganization of ACI 318 in 2014, STM provisions were moved into the main body of the code as Chapter 23, in part to recognize the importance of this method in structural design practice. Subsequently, significant changes to the STM provisions occurred in ACI 318-19. Chapter 23 was renamed to “Strut-and-Tie Method.” The abbreviation of STM only stands for strut-and-tie model.

In an effort to provide information on the use of strut-and-tie models in structural design, explanatory notes are included with the ACI 318 provisions. Additionally, two ACI special publications—ACI SP-208 and ACI SP-273—were developed by Subcommittee A, Strut & Tie, of Joint ACI-ASCE Committee 445, Shear and Torsion, in which the application of STM was illustrated through various design examples. Nevertheless, there is a need to provide additional guidance on the use of STM techniques.

1.2—Objectives

This document focuses on the ACI 318-19 implementation of strut-and-tie modeling. The main objectives of this document are to: 1) explain the intent and application of ACI 318 STM provisions; 2) provide additional design guidance for the STM based on other design codes, specifications, and committee documents; and 3) present design recommendations from recent research publications. This document is intended to provide practical guidance to the structural design community.

CHAPTER 2—NOTATION AND DEFINITIONS**2.1—Notation**

This section defines notations used in this guide.

A_1	= loaded area for consideration of bearing strength, in. ² (mm ²)	F_{nn}	= nominal strength at face of a nodal zone, lb (N)
A_2	= area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having its upper base equal to the loaded area. The sides of the pyramid, cone, or tapered wedge should be sloped one vertical to two horizontal, in. ² (mm ²)	F_{ns}	= nominal strength of a strut, lb (N)
A_b	= area of an individual bar or wire, in. ² (mm ²)	F_{nt}	= nominal strength of a tie, lb (N)
A_{core}	= cross-sectional area of a member measured to the outside edges of transverse reinforcement, in. ² (mm ²)	F_u	= factored force acting in a strut, tie, or nodal zone in a strut-and-tie model, lb (N)
A_{cs}	= cross-sectional area at one end of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut, in. ² (mm ²)	f_c	= compressive stress in concrete, psi (MPa)
A_{cv}	= gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered in the case of walls, and gross area of concrete section in the case of diaphragms. Gross area is total area of the defined section minus area of any openings, in. ² (mm ²)	f'_c	= specified compressive strength of concrete, psi (MPa)
A_g	= gross area of concrete section, in. ² (mm ²)	f_{ce}	= effective compressive strength of concrete in a strut or a nodal zone, psi (MPa)
A_{nz}	= area of a face of a nodal zone or a section through a nodal zone, in. ² (mm ²)	f_{pu}	= specified tensile strength of prestressing reinforcement, psi (MPa)
A_{ps}	= area of prestressing reinforcement in flexural tension zone, in. ² (mm ²)	f_{py}	= specified yield strength of prestressing reinforcement, psi (MPa)
A_s	= area of nonprestressed longitudinal tension reinforcement, in. ² (mm ²)	f_r	= modulus of rupture of concrete, psi (MPa)
A'_s	= area of compression reinforcement, in. ² (mm ²)	f_s	= calculated tensile stress in reinforcement at service loads, psi (MPa)
A_{tp}	= area of prestressing reinforcement in a tie, in. ² (mm ²)	f'_s	= stress in compression reinforcement under factored loads, psi (MPa)
A_{ts}	= area of nonprestressed reinforcement in a tie, in. ² (mm ²)	f_{se}	= effective stress in prestressing reinforcement (after allowance for all prestress losses), psi (MPa)
A_v	= area of shear reinforcement within spacing s , in. ² (mm ²)	f_{si}	= stress in the i -th layer of surface reinforcement, psi (MPa)
A_{vd}	= total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam, in. ² (mm ²)	f_y	= specified yield strength of reinforcement, psi (MPa)
$A_{v,min}$	= minimum area of shear reinforcement within spacing s , in. ² (mm ²)	f_{yt}	= specified yield strength of transverse reinforcement, psi (MPa)
a	= shear span to near support, in. (mm)	h	= overall thickness or height of member, in. (mm)
b	= shear span to far support, in. (mm)	I	= moment of inertia of section about centroidal axis, in. ⁴ (mm ⁴)
b_s	= effective width of strut, in. (mm)	I_{cr}	= moment of inertia of cracked section transformed to concrete, in. ⁴ (mm ⁴)
b_w	= web width or wall thickness, in. (mm)	I_e	= effective moment of inertia for computation of deflection, in. ⁴ (mm ⁴)
C	= compression force acting on a nodal zone, lb (N)	I_g	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. ⁴ (mm ⁴)
D	= effect of service dead load	L	= effect of service live load
d	= distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in. (mm)	ℓ	= span length of beam or one-way slab; clear projection of cantilever, in. (mm)
d'	= distance from extreme compression fiber to centroid of longitudinal compression reinforcement, in. (mm)	ℓ_{anc}	= length along which anchorage of a tie should occur, in. (mm)
E	= effect of horizontal and vertical earthquake-induced forces	ℓ_b	= width of bearing, in. (mm)
E_c	= modulus of elasticity of concrete, psi (MPa)	ℓ_c	= segment length of curved reinforcement, in. (mm)
E_s	= modulus of elasticity of reinforcement and structural steel, psi (MPa)	ℓ_d	= development length in tension of deformed bar, deformed wire, plain and deformed welded wire reinforcement, or pretensioned strand, in. (mm)
F_n	= nominal strength of a strut, tie, or nodal zone, lb (N)	ℓ_{dc}	= development length in compression of deformed bar and deformed wire, in. (mm)
		ℓ_{dh}	= development length in tension of deformed bar or deformed wire with a standard hook, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook [point of tangency] plus inside radius of bend and one bar diameter), in. (mm)
		ℓ_{dt}	= development length in tension of headed deformed bar, measured from the critical section to the bearing face of the head, in. (mm)

ℓ_n = length of clear span measured face-to-face of supports, in. (mm)
 ℓ_s = effective length of node, in. (mm)
 N_u = factored axial force normal to cross section; to be taken as positive for compression and negative for tension, lb (N)
 R = reaction, lb (N)
 r_b = bend radius at the inside of a bar, in. (mm)
 s = center-to-center spacing of items, such as longitudinal reinforcement, transverse reinforcement, prestressing tendons, or wires, in. (mm)
 s_i = center-to-center spacing of reinforcement in the i -th layer adjacent to the surface of the member, in. (mm)
 T = tension force acting on a nodal zone, lb (N)
 W = effect of wind load
 w_a = effective height of concrete compressive stress region, used to dimension nodal zone, in.
 w_c = density (unit weight) of normalweight concrete or equilibrium density of lightweight concrete, lb/ft³ (kg/m³)
 w_s = width of a strut perpendicular to the axis of the strut and in the plane of the strut-and-tie model, in. (mm)
 w_t = effective height of concrete concentric with a tie, used to dimension nodal zone, in.
 $w_{t,max}$ = maximum effective height of concrete concentric with a tie, in. (mm)
 w_u = factored load per unit length of beam or one-way slab, lb/in. (kg/m)
 α = angle defining the orientation of reinforcement
 α_i = angle between the axis of a strut and the bars in the i -th layer of reinforcement crossing that strut
 β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
 β_c = confinement modification factor for struts and nodes in a strut-and-tie model
 β_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone
 β_s = factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut
 Δf_p = increase in stress in prestressing reinforcement due to factored loads, psi (MPa)
 ϵ_1 = principal tensile strain in cracked concrete due to factored loads, in./in. (mm/mm)
 θ = angle between axis of strut, compression diagonal, or compression field and the tension chord of the member
 θ_c = the smaller of the two angles between the strut (or the resultant of two or more struts) and ties extending from a curved-bar node
 θ_{cr} = angle of diagonal cracking relative to the longitudinal axis of a member
 θ_v = difference between angle of diagonal cracking and compression
 λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative

to normalweight concrete of the same compressive strength

λ_s = factor used to modify shear strength based on the effects of member depth, commonly referred to as the size effect factor
 ν = Poisson's ratio (may be assumed to be 0.17 for concrete with f'_c up to 10,000 psi [69 MPa])
 ϕ = strength reduction factor

2.2—Definitions

Please refer to the latest version of ACI Concrete Terminology for a comprehensive list of definitions. Definitions provided herein complement that resource.

B-region—portion of a member in which it is reasonable to assume that strains due to flexure vary linearly through the section.

cover, concrete—distance between the outermost surface of embedded reinforcement and the closest outer surface of the concrete.

D-region—portion of a member within approximately a distance h of a force discontinuity or a geometric discontinuity, where nonlinear strain distributions occur.

discontinuity—abrupt change in geometry or loading.

effective depth of section—distance measured from extreme compression fiber to centroid of longitudinal tension reinforcement.

headed deformed bars—deformed bars with heads attached at one or both ends.

nodal zone—volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node.

node—point in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect.

node, curved bar—the bend region of a continuous reinforcing bar (or bars).

strength, design—nominal strength multiplied by a strength reduction factor ϕ .

strength, nominal—strength of a member or cross section calculated in accordance with provisions and assumptions of the strength design method before application of any strength reduction factors.

strength, required—strength of a member or cross section required to resist factored loads or related internal moments and forces in such combinations as required by codes.

strut, bottle-shaped—strut that is wider at midlength than at its ends.

strut, boundary—strut located along the boundary of a member or discontinuity region.

strut, interior—strut not located along the boundary of a member or discontinuity region.

strut-and-tie model—a framework or truss model of a structural member or of a D-region in such a member made up of struts and ties connected at nodes, capable of transferring the factored loads to the supports or to adjacent B-regions.

CHAPTER 3—OVERVIEW OF THE STRUT-AND-TIE MODEL DESIGN PROCEDURE

3.1—Introduction

The strut-and-tie method is an approach to evaluate or design structural concrete by simplifying complex states of stress within a structural member into simple load paths. These load paths are idealized as two-dimensional or three-dimensional networks of straight, pin-connected, axially loaded elements that link all applied loads with supports or with adjacent portions of the structure, such as shown in Fig. 3.1. The arrangement of struts, ties, and nodes is collectively known as a strut-and-tie model (STM) and resembles a truss or framework within the body of the member.

The process of approximating the load path in a structure, or portion thereof, entails visualizing the flow of forces from the applied loads to the reactions while maintaining equilibrium of all forces. External and internal equilibrium conditions should be satisfied. The strut-and-tie method complies with the lower-bound theorem of plasticity (Baker et al. 1956; Nielsen 1971; Thürlimann et al. 1983)—the concept that a structure can resist a set of applied loads as long as there is at least one stress distribution that satisfies equilibrium without violating the yield condition. Accordingly, the STM should achieve equilibrium with the applied loads without exceeding the capacity of any model component (strut, tie, or node). Thus, for design purposes, the internal force in each element of the STM is limited to that element's design strength as prescribed by the corresponding ACI 318 strut-and-tie modeling provisions. The nominal strength of each strut, tie, or node is based on the material strength as well as the geometry and other characteristics of the STM that should fall within the bounds defined by the code provisions. Strut or overall member dimensions may be controlled by sectional shear force limits. These provisions and limits are described in Chapter 4.

The strut-and-tie method is most useful when applied to regions of a structure near localized changes in geometry or applications of external loads. In these instances, the stress and strain distributions may be complex with nonlinear variation over the cross section. Because these regions are defined by the presence of one or more *discontinuities*, they are denoted D-regions, sometimes referred to as disturbed regions. Common examples include corbels, non-slender or deep beams, anchorage zones for post-tensioned tendons, and regions near openings in structural walls. In addition to these regions of complex load paths, the strut-and-tie method can be used to model simpler stress states or load paths, including those with a linear variation of strain over the cross section. B-regions, sometimes referred to as beam regions, are those portions of the structure where the assumption that plane sections remain plane is reasonable, and Bernoulli's classical beam theory is appropriate to approximate the actual behavior. Because simplified methods for the design of B-regions are available, strut-and-tie modeling is most commonly applied to the D-regions of members.

One additional consideration of the strut-and-tie method is compatibility. In the strut-and-tie method, load paths are

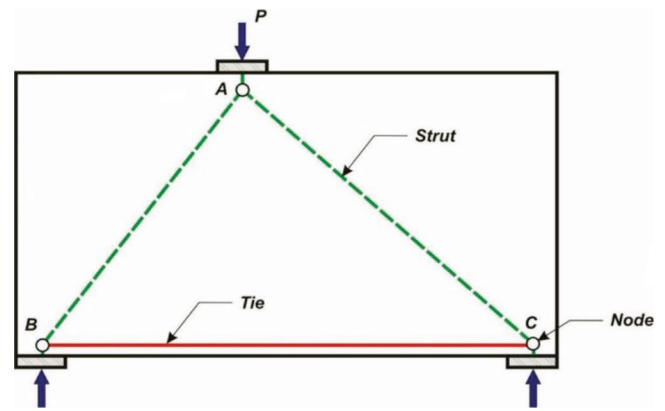


Fig. 3.1—Basic elements of a strut-and-tie model for a deep beam.

discretized into struts and ties connected at nodes, where these elements are assigned finite dimensions. The method is based on the assumption that struts carry compressive stresses, while the areas immediately adjacent to, but outside of, the strut boundaries are considered unstressed. A similar scenario exists with both ties and nodes. This simplification of the load paths leads to apparent strain discontinuities at element boundaries. These discontinuities are permissible: kinematic compatibility is not required to satisfy the lower-bound theorem of plasticity, on which the method is based. Nonetheless, compatibility considerations can be addressed by aligning the STM elements with the linear-elastic stress field to improve efficiency and service-load performance as described in Section 5.3.

3.2—Overview of the strut-and-tie method

An overview of the general strut-and-tie method for modeling and design of a structural concrete member is provided in this section. The process can be adapted to a wide variety of structure and model types. Some of the concepts described will be used later in this chapter to illustrate the application of the method through the development of an example STM. These are the basic steps of model development for each load case:

1. Determine all applied loads and reactions. This step may need to be repeated to suit the geometric layout of the STM in later steps.
2. Identify portions of a structure where the use of the strut-and-tie method may be necessary, including regions near load or geometric discontinuities. These regions are called D-regions and are discussed in more detail in Section 3.3.2.
3. Calculate all forces that should be transferred at the boundaries of the region modeled and the locations where these forces are expected to act on the STM.
4. Determine if minimum distributed reinforcement is warranted in accordance with ACI 318-19 Section 23.5. Use of distributed reinforcement may influence the allowable strut strength (Step 10).
5. Check that the member dimensions are adequate to satisfy any applicable sectional shear force limits and those prescribed in ACI 318-19 Sections 23.4.4 or 9.9.2.

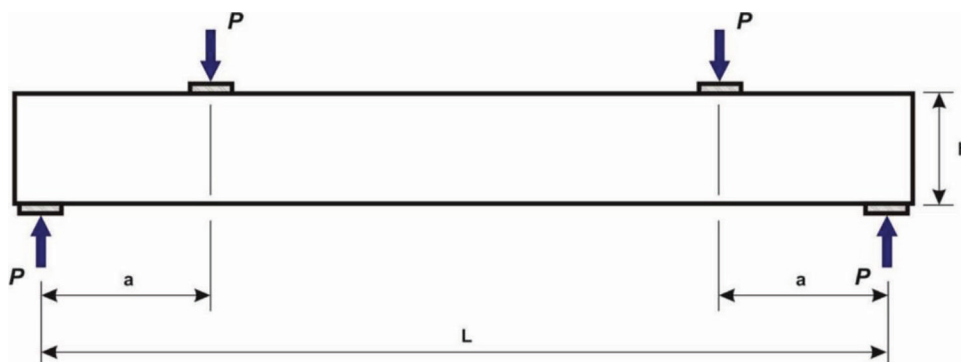


Fig. 3.3.1—Elevation view of example structure with actions.

6. Develop an STM consisting of struts and ties intersecting at nodes that is based on the flow of forces through the region to either supports or adjacent B-regions. Model development should include consideration of possible effects of cracking. In some situations, especially in more complex configurations, a linear-elastic finite element analysis (FEA) can be helpful, but not required, for establishing load paths based on principal stress trajectories. Three-dimensional effects should also be considered.

7. Calculate the internal force effects for each element within the STM to achieve equilibrium with the forces applied at the boundaries of the region modeled.

8. Determine a configuration of reinforcement, prestressed or nonprestressed, to resist all tie forces from the STM. Often, ties are sized first, as they can affect the sizes of nodes and struts and may necessitate modification of the model geometry. This process is discussed in Chapter 4.

9. Determine the geometry of the nodes and struts based on the overall model layout and the configuration of the ties chosen in the previous step. This process is discussed in Chapter 4.

10. Calculate the design strength of each strut and node by considering the relevant effectiveness factors and compare these capacities to the factored load demands at each node/strut interface.

11. Consider revisions to the STM configuration, member dimensions, or specified concrete strength as necessary to satisfy the strut or nodal strength requirements. The arrangement of reinforcement should be consistent with the geometric layout of the model. A suitable configuration of longitudinal or confinement reinforcement to enhance the strength of struts and nodes may also be considered. Recheck the design strength of all ties, struts, and nodes relative to factored forces after revising the STM.

12. Determine a configuration for additional distributed reinforcement to address serviceability and ductility considerations, such as that required by ACI 318-19 Section 23.5.1, as well as other requirements that apply to specific member types (for example, corbels). Such distributed reinforcement, if provided in sufficient quantity at appropriate locations, may increase the efficiency factor of interior struts, enhance the deformation capacity of the member, and help control crack widths in struts.

These latter benefits are discussed in detail in Chapter 6 of this guide.

13. Ensure adequate anchorage of all ties. Reinforcement used as ties should be properly anchored to achieve the tie force relative to the geometric layout of each node, in accordance with Section 23.8 and Chapter 25 of ACI 318-19. Adequate anchorage of the distributed reinforcement should also be achieved.

3.3—Conceptual example

3.3.1 Introduction—This section presents an introductory discussion of the strut-and-tie method based on a conceptual example, which is presented to introduce and illustrate general strut-and-tie modeling concepts. Subsequent chapters include details of STM application. The example structure consists of a simply-supported beam shown in Fig. 3.3.1. The self-weight is neglected for simplicity in this conceptual example.

In general, several load combinations may need to be investigated for any given member or structure and each may require a different geometric configuration of the STM to maintain static equilibrium at the nodes. Further, the presence of movable gravity loads or reversing lateral loads may necessitate the use of a different STM for each load pattern or combination. Given the simplicity of this example structure, a single load case is sufficient for discussion.

3.3.2 Identification of B- and D-regions—After the applied loads and corresponding reactions are determined, the structure is divided into B-regions and D-regions. The B-regions can be designed and detailed with cross-sectional strength methods described in Chapter 22 of ACI 318-19. The use of an STM as described in Chapter 23 of ACI 318-19 is also allowed in these regions. In the D-regions, conventional beam theory from Chapter 22 of ACI 318-19 that assumes a linear variation of strains over the cross section is not a reasonable basis for design. Instead, STMs are used to represent idealized load paths in these D-regions.

Based on St. Venant's principle, D-regions are normally identified as regions occurring within one cross-sectional dimension h of a geometric or load discontinuity. A commonly adopted value of h is the cross-sectional dimension perpendicular to the axis of bending. The actual length of the segment selected for analysis can be expanded to larger regions if desired or if required to ensure the adequate

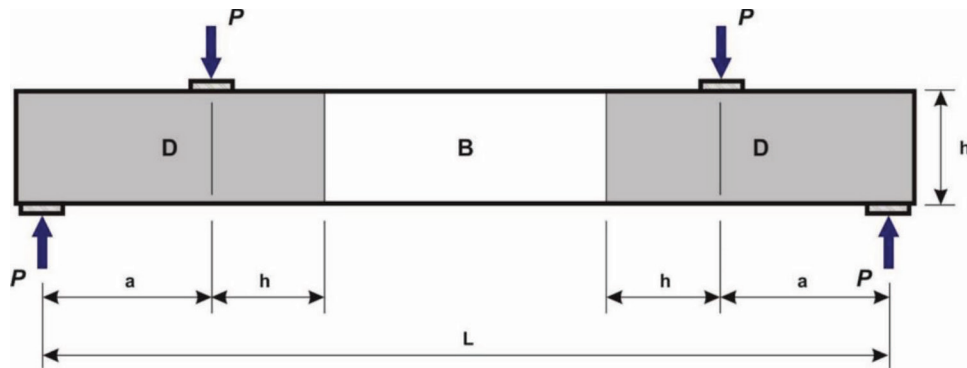


Fig. 3.3.2a—B- and D-regions of example structure.

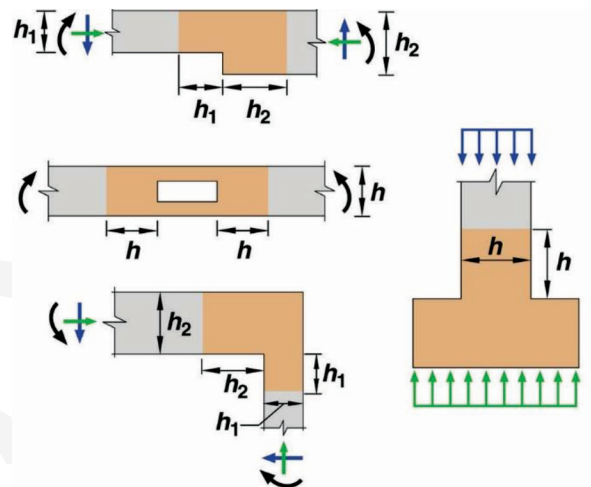
transition of the load path from the D-region to adjacent B-regions.

For the example beam, two D-regions resulting from loading discontinuities are determined as illustrated in Fig. 3.3.2a. Each of these two shaded D-regions represents the overlapping discontinuity zones that result from the applied load and the reaction force near each end of the beam. A single B-region occupies the remaining portion of the beam. Due to the symmetry of the beam and this load case, strut-and-tie modeling will be discussed only for the D-region at the left end of the beam.

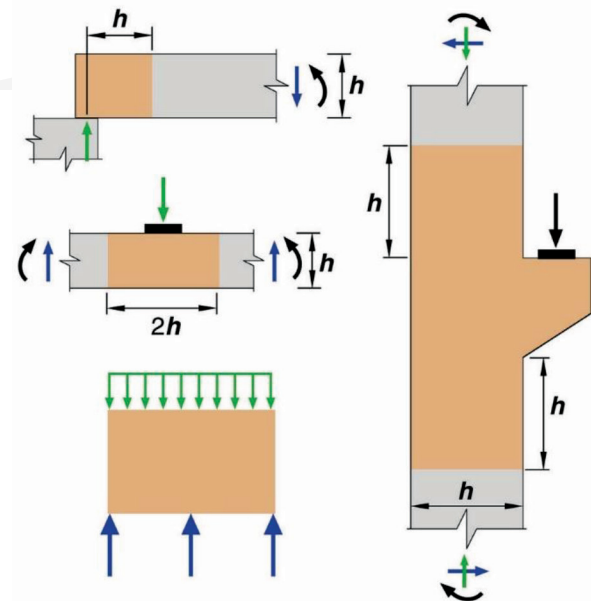
ACI 318-19 contains additional guidance on the determination of D-regions for the common types of geometric or loading discontinuities depicted in Fig. 3.3.2b, along with the forces acting on the D-region boundaries. The geometric discontinuities in Fig. 3.3.2b(a) are so named because the disturbance arises from an abrupt change in the geometric shape of a member. Loading discontinuities shown in the left three cases in Fig. 3.3.2b(b) arise due to the presence of a localized application of load, including support reactions or applied concentrated loads, as is the case in this example. A corbel, as shown at right in Fig. 3.3.2b(b), is influenced by both geometric and loading discontinuities.

3.3.3 Boundary forces and basic strut-and-tie model—The forces acting on each of the boundaries of each D-region are determined next. Figure 3.3.3 indicates the internal sectional load effects (shear force V and bending moment M) computed at the D-region boundary within the beam, as well as how these load effects are resolved into forces acting on the selected STM. These forces should be equilibrated by the load path defined by the model.

This simple direct-strut model represents an intuitive flow of forces in the D-region. Struts are indicated as thick, dashed lines (green); ties are thick, solid lines (red). The lower chord provides tension resistance whereas the upper chord and the diagonal Strut AJ are in compression. The depth of the lower chord is determined by the practical location of tension reinforcement and by the dimensions of nodes described later. The depth of the upper chord is at first estimated; this could be done by employing the equivalent rectangular compressive stress distribution used in B-region design. The diagonal that frames into Node C carries no force because there is no shear force V on the model boundary. Likewise, the vertical CJ carries no force. These zero-force members (indicated



(a) Geometric discontinuities



(b) Loading and geometric discontinuities

Fig. 3.3.2b—Types of D-regions (ACI 318-19 Fig. R23.1).

with thin, dotted lines and labeled 0) are often not explicitly included in the STM. However, they would be necessary to transmit forces if the loading was changed to

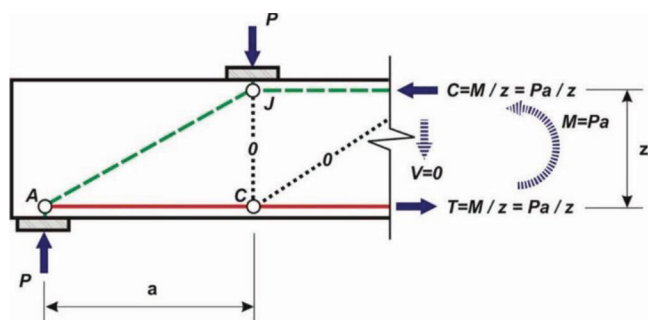


Fig. 3.3.3—D-region: boundary forces and direct-strut model.

an asymmetric case, or if shear forces from self-weight were included. They might also be needed for numerical stability if the STM forces are determined using some commercial software or other calculation techniques. With no shear resisted by the diagonal at the boundary for the symmetric loading in the example, the upper and lower chords resist the boundary bending moment M with equal but opposite forces computed to satisfy equilibrium. The chord positions can be refined after strut stresses and reinforcement quantities are estimated from an initial model analysis.

3.3.4 Model refinement and alternatives—For the remainder of this example, several variants on the basic model introduced in Fig. 3.3.3 are presented to illustrate a range of possibilities that can be considered in developing an STM. In practical applications, it is common to consider only one or a few model geometries. A common alternative model that incorporates transverse reinforcement between the load and the support is shown in Fig. 3.3.4a.

The vertical Tie BH in this indirect load transfer STM is used to incorporate conventionally oriented transverse reinforcement. This model becomes more appropriate as a/d increases (larger than approximately 2) and the slope of the direct strut AJ in Fig. 3.3.3 flattens (less than approximately 25 degrees), indicating a transition away from deep-beam shear behavior as the direct-strut force demand becomes extremely large. Analysis of the STM in Fig. 3.3.4a indicates that the vertical reinforcement in BH should be proportioned and detailed to provide a design tie strength no less than the applied load P . The force magnitudes in the diagonal struts are less than in the direct-strut model shown in Fig. 3.3.3, and the lower-chord tension demand decreases in Tie AB, thus reducing the tie anchorage demand at Node A.

Figure 3.3.4b represents an STM that combines features of the direct-strut and indirect-transfer models. This model is statically indeterminate, but it can be decomposed into a superposition of two statically determinate models—one like Fig. 3.3.3 and one like Fig. 3.3.4a.

The designer is then faced with choosing how to apportion load between the direct-strut STM of Fig. 3.3.3 and the indirect-transfer STM of Fig. 3.3.4a. Thus, Tie BH in the combined STM will be subjected to less tension than Tie BH in the STM of Fig. 3.3.4a. Methods for determining an appropriate distribution of loads between the statically determinate STMs are discussed in Section 5.4.

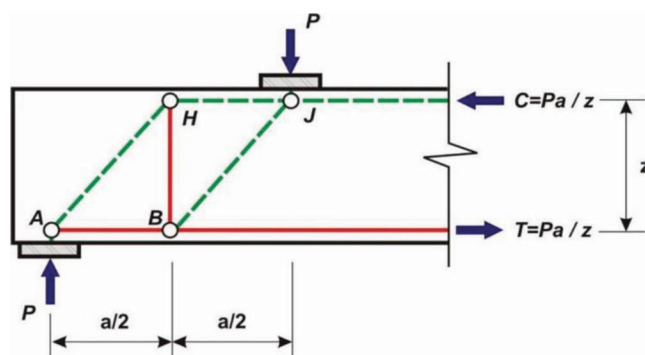


Fig. 3.3.4a—D-region: model with vertical transverse reinforcement.

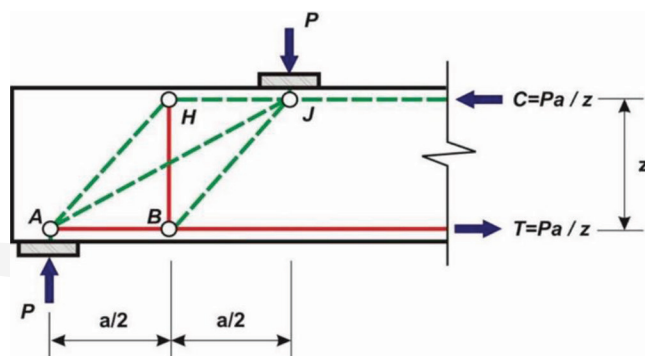


Fig. 3.3.4b—D-region: combined statically indeterminate model.

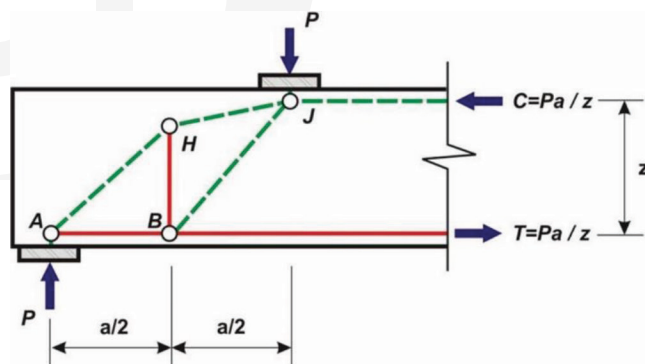


Fig. 3.3.4c—D-region: adjusting node location.

If the designer prefers adjusting the geometry of a single STM rather than distributing loads between superimposed models, the model shown in Fig. 3.3.4c offers a single, statically determinate, indirect-transfer STM for selectively proportioning the longitudinal and transverse reinforcement.

The forces in AB, BH, AH, HJ, and BJ depend on the selected location of Node H, which is the only change from the STM of Fig. 3.3.4a. As Node H approaches the line connecting Nodes A and J, this indirect-transfer STM transforms into the direct-strut STM. The optimum position of Node H could be estimated by visualizing compressive stress trajectories or by minimizing the strain energy in the model (as discussed in 5.4.3), which would theoretically require the least reinforcement and exhibit the best performance (least post-cracking deformation).

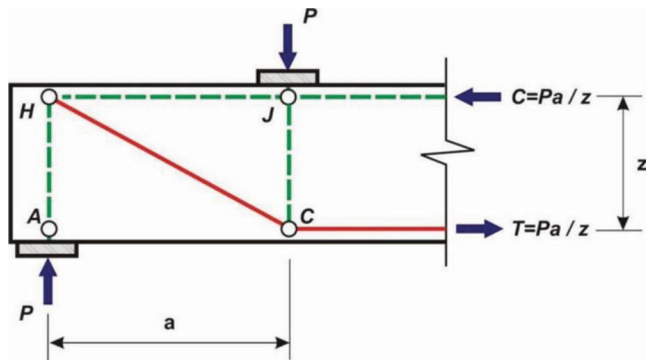


Fig. 3.3.4d—D-region: poor model.

Figure 3.3.4d represents another STM that might be considered for this example but is a poor choice for several reasons. Although the STM theoretically provides a solution that is in equilibrium with the given loading, the diagonal tension force in CH and the lack of tie force between Nodes A and C are problematic as presented in Fig. 3.3.4d. The slightest (in-plane) horizontal load or end-region cracking will result in instability of Strut AH. The tie reinforcement could be difficult to anchor adequately due to limited space beyond Node H. Even if these issues were resolved, this STM could exhibit wide cracking under service loads. In effect, the tension that was resisted by the lower Chord AC in the other STMs is now resisted by the inclined Tie CH. However, cracking of the concrete across CH is necessary before the tie reinforcement becomes fully engaged. For this to occur, very wide cracks will open in the unreinforced region between Nodes A and C, resulting in unacceptable service-level performance. This poor performance could be quantified via the strain energy in the model, which is greater than the strain energy in the models presented previously. In addition, the use of inclined nonprestressed reinforcement in Tie CH can be difficult to construct and adequately anchor, especially when better-alternative STMs exist.

3.4—Historical development of the strut-and-tie method

3.4.1 Early development and codification—The strut-and-tie method has its origin in the parallel-chord truss models proposed by Ritter (1899), Considère (1900), and Mörsch (1909) to rationally determine the amount of reinforcement necessary to transmit applied loads to the supports of concrete beams. This modeling approach was subsequently refined by other researchers (Kupfer 1964; Leonhardt 1965) and became the basis for shear design provisions of European codes. Three-dimensional truss models were also proposed by various researchers as a rational approach for the design of beams for torsion (Rausch 1929; Lampert and Thürlimann 1971; Collins and Mitchell 1980; Hsu 1988).

Further development of these modeling concepts included the adoption of limit analysis in the plasticity theory as the scientific basis for the strut-and-tie method (Nielsen 1971; Thürlimann et al. 1983). However, application of the method was still mainly limited to a selection of typical D-regions,

such as deep beams, dapped-end beams, corbels, and walls (IABSE 1978, 1979).

Works by Marti (1985a,b), Collins and Mitchell (1986), Schlaich et al. (1987), Schlaich and Schäfer (1991, 2001), and Reineck (1996) were instrumental to the expanded use of strut-and-tie modeling as a rational design method. Strut-and-tie model provisions were added to various major North American and European codes or standards, including CAN3-A23.3M-84, CEB-FIP Model Code 1990, AASHTO LRFD Bridge Design Specifications (AASHTO 1994), DIN 1045-1, and ACI 318-02.

3.4.2 Research efforts

3.4.2.1 Struts and nodes—Considerable research has focused on understanding factors influencing the strength of struts and nodes within STMs. Researchers have quantified and recommended efficiency factors for strut and node strengths that are appropriate for use in design or analysis (MacGregor 1997; Marti 1985a; Nielsen et al. 1978; Schlaich et al. 1987; Tuchscherer et al. 2011a; Yun and Ramirez 1996, 2016; Yun 2006).

Experimental investigations have been conducted to understand the behavior of concrete under biaxial stress conditions (Kupfer et al. 1969; Liu et al. 1972; Vecchio and Collins 1986). Effective strut strengths in uniform compression fields that reflect strain compatibility and the resulting softened response of cracked reinforced concrete have been proposed (Collins and Mitchell 1986).

Analytical failure criteria have also been studied, with corresponding proposals used as a basis for nodal zone stress limits (Kupfer and Gerstle 1973). Bearing strengths of nodal zones with different configurations of intersecting struts and ties have been studied experimentally (Jirsa et al. 1991; Polla 1992), and effective strengths of the concrete within nodes have been proposed (Bergmeister et al. 1991; Tuchscherer et al. 2011b). Schäfer reviewed nodes for the *fib Structural Concrete* textbook (fib 2010a).

3.4.2.2 Configuration of struts and ties—In an indeterminate structure, the location and quantity of the tie reinforcement based on the selected configuration of struts and ties can substantially change the flow of forces (Rogowsky and MacGregor 1986; Maxwell and Breen 2000; Kuchma et al. 2008; Garber et al. 2014; Chae and Yun 2015). Schlaich et al. (1987) recommended arranging the struts and ties within the STM in accordance with the principal stress directions obtained from a linear-elastic solution of the region. By adopting this approach, a design can provide adequate strength and service-load performance with a statically indeterminate STM (Barnes 2002). The resulting design requires little or no redistribution of internal forces in the indeterminate STM after cracking; therefore, the ductility demand is minimized.

The development of STM techniques to implement three-dimensional (3-D) truss models has been investigated by several researchers (Adebar and Zhou 1996; Nguyen 2002; Widianto and Bayrak 2010).

3.4.3 Design applications and examples—Marti (1985a,b) provided introductory overviews of STM applications. Collins and Mitchell (1988) demonstrated the use and effectiveness

of the strut-and-tie method for design of common D-regions. **Joint ACI-ASCE Committee 445 (1998)** included the method in a report on shear design and published an extensive STM bibliography (**Joint ACI-ASCE Committee 445 1997**). Design with STM was summarized by Schäfer for *fib* Bulletin 3 (*fib* 1999). **FIP (1999)**, based on the **CEB-FIP Model Code 1990**, formed a foundation for STM application examples in two *fib* reports (*fib* 2002, 2011). **ACI SP-208** included many examples for the application of strut-and-tie models. The same volume included a presentation and justification of the original ACI 318 STM provisions by **MacGregor (2002)**, as well as comparisons to tests by **Mitchell et al. (2002)**. A further set of examples was provided in **ACI SP-273**.

Application of the method has been demonstrated for the design of precast concrete connections (**Pensiero 1989**), design of prestressed concrete members (**Ramirez 1994**; **Schlaich et al. 1987**; **Tan and Naaman 1993**; **Tan and Tong 1999**), and in anchorage zones (**Breen et al. 1994**; **Yun 2005**). **Reineck (2005)** provided numerical examples for the design of D-regions in prefabricated members, including corbels, dapped-end beams, and beam webs with openings. **Birkle et al. (2002)** used the method to design double corbels reinforced using T-headed bars. **Adebar et al. (1990)** and **Nguyen (2002)** applied 3-D STMs to the design of pile caps. The application of strut-and-tie principles for the evaluation of seismic actions, such as in beam-column joints (**Sritharan et al. 2001**), walls with openings (**Yanez et al. 1992**), and diaphragms (**Blandón and Rodríguez 2005**), has also been introduced.

3.4.4 Validation and improvement of code provisions—Experimental evaluation of the ultimate load capacity of deep beams designed in accordance with **ACI 318-02** was completed by several researchers (**Aguilar et al. 2002**; **Chen et al. 2002**; **Maxwell and Breen 2000**). Experimental validation of the strength of corbels, dapped-ended beams, and deep beams designed according to ACI 318-02, **CSA A23.3-94**, and CEB-FIP Model Code 1990 was also completed (**Mitchell et al. 2002**). **Polla (1992)** assessed the nodal zone strength recommendations of **CAN3-A23.3-M84**.

Other testing programs on deep beams and inverted-tee beams have provided information on the accuracy and conservatism of **ACI 318-11**, **AASHTO LRFD (AASHTO 2010)**, and **CSA A23.3-04** STM provisions (**Tuchscherer et al. 2010b, 2011a, 2016**; **Bircher et al. 2013, 2014**) and recommended improvements to those ACI 318 and AASHTO LRFD STM design provisions (**Tuchscherer et al. 2014**). In general, these research studies as well as others (**Kuchma et al. 2008**) validated the conservativeness of the ACI 318 STM provisions while highlighting areas where improvement is possible, such as the effect of triaxial confinement on node strength, the role of distributed reinforcement, and the influence of bond stresses outside the nodal zone. The AASHTO LRFD Bridge Design Specifications were revised in 2016 to implement some of these findings, and these changes are described in **Section 8.3.2** of this guide.

Bazant (1997) proposed a size effect factor for strut strength in beams without stirrups. Based on a review of test results

from the ACI-DAfStb Shear Database (**ACI-DAfSTB 617**), **Reineck and Todisco (2014)** concluded that **ACI 318-14** STM provisions were unconservative for deep beams without transverse reinforcement and recommended a reduction in the design strength of unreinforced struts. **Laughery and Pujol (2015)** reported no significant difference in the strength of prismatic unreinforced struts and planar bottle-shaped unreinforced struts. **Rezaei et al. (2019)** showed that interior struts in deep beam specimens are weaker than the struts in comparable truss-like specimens because interior struts in a beam pass through a region of diagonal tension. Based on this testing and analysis of the ACI-DAfStb Shear Database, **Klein et al. (2019)** developed design requirements for maximum allowable shear strength in D-regions without distributed reinforcement. These issues were addressed by changes to strut location definitions, limits on allowable shear force, and revised strut strength coefficients in ACI 318-19 Section 23.4.

3.4.5 Recent developments—Strut-and-tie model code provisions include limitations on material strengths, geometry, or other criteria. Recent research has investigated the use of strut-and-tie analysis and design approaches that extend these limits. For example, researchers have examined the use of STMs for deep beams containing high-strength steel reinforcement (**Garay and Lubell 2016**; **Hassan et al. 2008**). **Andermatt and Lubell (2013a,b)** investigated the application of STM provisions to predict the strength of deep beams with fiber-reinforced polymer reinforcing bars and suggested practical limits of STM use due to the limited ductility available with this type of reinforcement. The extension of STMs to design with higher-strength concrete has been investigated (**Foster and Gilbert 1998**; **Warwick and Foster 1993**; **Tan et al. 1997**; **Watanabe and Kabeyasawa 1998**; **Ghoneim 2001**; **Nielsen 2011**), including members with concrete compressive strengths up to 20,300 psi (140 MPa) (**Von Ramin and Matamoros 2004, 2006**). Section 6.4 of **ACI ITG-4.3R-07** contains a summary of recommendations related to strut strength in high-strength concrete.

CHAPTER 4—DESIGN STRENGTH OF STRUTS, TIES, AND NODES

4.1—Overview

Chapter 4 presents the **ACI 318-19** strut-and-tie methodology and related design considerations. Criteria in ACI 318-19 include limitations on the stresses applied to struts, ties, and nodes relative to their ultimate strengths, as well as geometric design limitations. This chapter provides guidance for proportioning the elements of a strut-and-tie model (STM) and determining their respective capacities. In addition to the prescriptive provisions in ACI 318-19, this chapter also discusses design conditions where the Code does not provide guidance. In these cases, recommendations are presented from other codes, standards of practice, and research studies.

The STM provisions contained within ACI 318-19 are applicable to most structural concrete members regardless of their configuration. As such, applying the STM provisions to a design result that satisfies the strength requirements at

the ultimate limit state (ULS). This is one of the strengths of adopting the strut-and-tie method for design or analysis. However, it is important to note that even if structural members are designed per the STM provisions, the design professional should also be aware of other prescriptive provisions within ACI 318-19 for specific member types. These prescriptive provisions can often govern aspects of geometry, reinforcement details, and strength limitations for materials. For example, Sections 9.9, 11.7, and 16.5 of ACI 318-19 contain prescriptive detailing requirements specific to deep beams, walls, and corbels, respectively.

4.2—Strut and node types and factors influencing strength

4.2.1 Overview—ACI 318-19 provisions indicate the stress limit for struts, ties, and nodes. The resistance of struts is influenced by their geometric shape, dimensions of surrounding concrete, and the relationship to reinforcement or tensile stress fields. Section 4.2.2 of this document discusses strut types and the factors that influence their strength. The strength of nodes is influenced by similar factors, as discussed in Section 4.2.3.

4.2.2 Struts

4.2.2.1 Strut types—Struts are used to represent the resultant of compressive stress fields. Beginning with ACI 318-19, strut types are classified based on location within the member as either: 1) boundary struts; or 2) interior struts. Figure 4.2.2.1 provides an example of these strut types as they occur within a beam.

A boundary strut occurs within a uniform compressive stress field, or regions where the compressive stress is not traversed by tensile stress fields. Interior struts spread laterally and are crossed by tensile stress fields in the transverse direction. As shown in Fig. 4.2.2.1, boundary struts and interior struts are often approximated as prismatic having the same cross section size at each end. However, they can also be non-prismatic.

In ACI 318-14 and prior versions of the code, strut types are defined by shape, namely prismatic, and bottle-shaped, where the overall concrete geometry allows the compressive force to spread to a larger strut size along the length than occurs at either end. The prismatic strut was renamed as a boundary strut to more accurately define its location and associated uniform stress field. Bottle-shaped struts were renamed as interior struts to emphasize the fact that their strength is determined by the presence of transverse tension, rather than the spreading of the compression. Treatment of these strut definitions by other international codes is presented in Chapter 8.

4.2.2.2 Factors influencing strut strength—The strength of concrete in a strut depends on the multi-axial state of stress as well as disturbances from cracks. Struts can fail due to axial crushing or from lack of restraint to the presence of transverse diagonal tension. Major factors that affect the strut axial strength include: 1) the effective strength of concrete; 2) presence of cracking and diagonal tension; 3) confinement provided by surrounding concrete or reinforcement; and 4) distributed reinforcement across

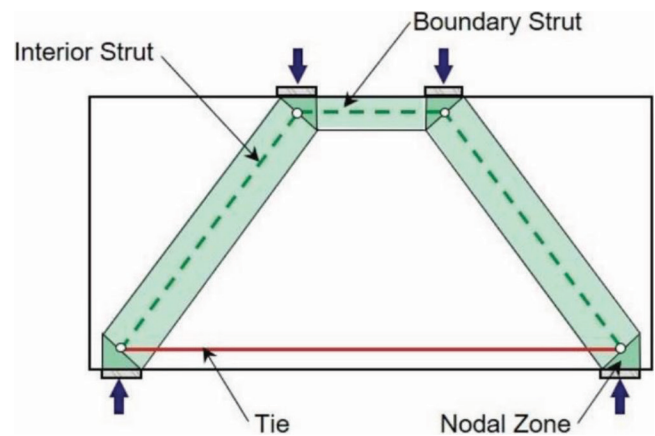


Fig. 4.2.2.1—Example of strut types as they occur in a strut-and-tie model.

the strut. Experimental tests on deep beams indicate that diagonal tension can be the governing failure mode, especially if distributed reinforcement is not provided (Klein et al. 2019).

4.2.2.2.1 Effective strength of concrete—The effective strength of concrete in a boundary strut is limited to $0.85f'_c$. This reduction relative to f'_c accounts for load duration effects, the difference in stress fields between cylinders and struts, and the influence of vertical migration of bleed water (MacGregor 2002).

The effective strength of concrete in a strut compared to f'_c may be further reduced as the cylinder compressive strength is increased because the quality of aggregate has a greater influence on its ability to transfer shear and tension (Bergmeister et al. 1993; MacGregor 2002; Bahen and Sanders 2007; Martin and Sanders 2007; Bahen and Sanders 2009; Andermatt and Lubell 2013b; Tuchscherer et al. 2014). For this reason, strut strength coefficients specified within some codes of practice or guidelines (for example, fib 1999) are determined as a function of the cylinder compressive strength. However, strut strength coefficients specified in ACI 318-19 are independent of the cylinder compressive strength because these provisions have been calibrated to available test data which include a wide range of compressive strengths.

4.2.2.2.2 Struts crossing cracks or diagonal tension fields—The effective strength of concrete in compression is further reduced when the compression stress field crosses regions of cracking, transverse tension, or both (Vecchio and Collins 1986). The location, spacing, orientation, and widths of cracking at struts can be attributed to numerous factors, including: 1) diagonal tension and the presence of tensile strains crossing the idealized strut (Fig. 4.2.2.2.2a); 2) lateral spreading of compression in an interior strut; 3) cracking from prior load cases, including cyclic or reverse loading; 4) biaxial or triaxial loading or confinement; 5) reinforcement configuration and spacing; and 6) member depth. One source of tension transverse to a strut is diagonal tension from shear, as illustrated in Fig. 4.2.2.2a, which shows the elastic principal stress trajectories for the deep beam shown in Fig. 4.2.2.1.

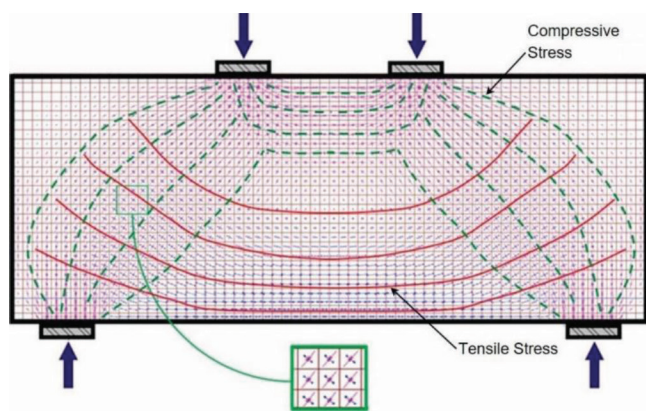


Fig. 4.2.2.2a—Principal stress trajectories in a deep beam result in tension transverse to interior strut.

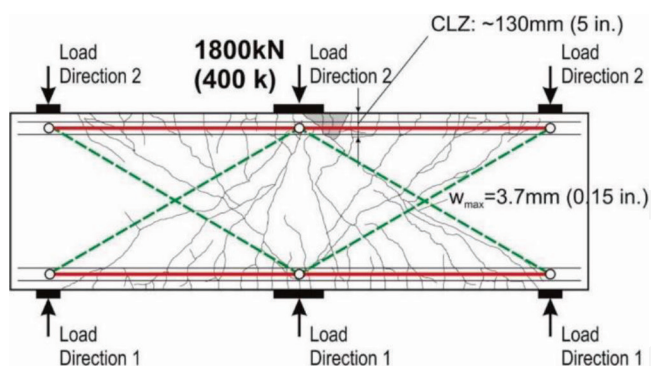


Fig. 4.2.2.2b—Strut crossing a crack caused by reverse loading (adapted from Mihaylov et al. [2010]).

The lateral spreading of compression introduces tensile stress transverse to the strut resulting in splitting. This tensile stress tends to cause longitudinal cracks near the ends of the strut. According to MacGregor (2002) and Schlaich et al. (1987), this cracking will occur when the compressive stress on the end of the strut exceeds approximately 55 percent of the concrete compressive strength.

In members without distributed reinforcement, shear strength decreases with member depth due to the size effect. The size effect factor is included in the sectional shear design equations in ACI 318-19 as well as Eq. (4.4.5) of this document, which applies to discontinuity regions without distributed reinforcement.

Finally, strut strength is reduced when structures are loaded in reverse or multiple directions, as illustrated in Fig. 4.2.2.2b. For this reason, Section 23.11.2.1 of ACI 318-19 requires a multiplier of 0.8 on strut strength for struts located in regions of the seismic-force-resisting system subject to reverse loading from earthquakes because struts will cross wide preexisting cracks that formed during earlier loading cycles.

4.2.2.2.3 Confinement—Whereas transverse tension adversely affects the strut strength, compression that is transverse to a strut can increase its strength if it confines the strut in the biaxial or triaxial directions. The beneficial effects of concrete perpendicular to the STM or reinforcement placed to confine the strut may be used to increase the

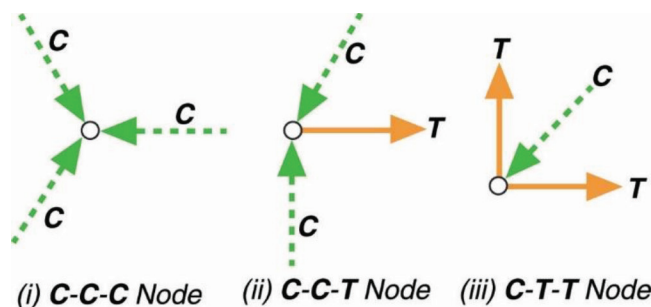


Fig. 4.2.3.1a—Classification of nodes (ACI 318-19).

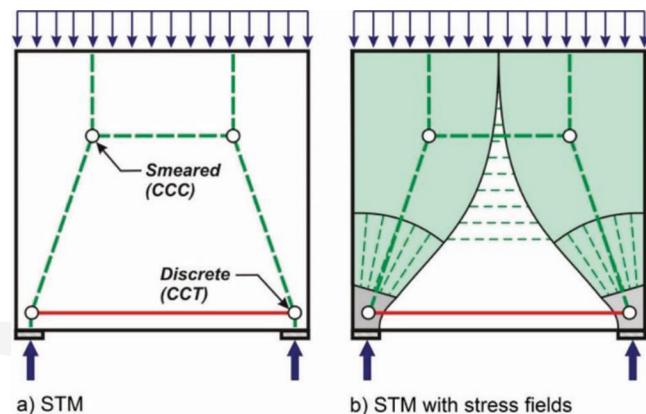


Fig. 4.2.3.1b—STM with discrete and smeared node noted (Schlaich et al. 1987).

strut strength. Refer to Section 4.4.6 of this document for more information.

4.2.3 Nodes

4.2.3.1 Node types—A nodal zone is the region where three or more struts or ties or external forces intersect. Nodes are classified based on the types of forces (that is, element types) that meet at the node. As illustrated in Fig. 4.2.3.1a, a CCC node is used to denote a node bounded by struts only. A CCT node anchors one tie and a CTT node anchors two ties.

If one of the struts or ties adjoining a node represents a concentrated stress or force, which typically occurs at an external load or support, then the node is referred to as a *singular* or *discrete* node. On the other hand, a node bounded by wide stress fields, which typically occurs at an internal location along a structural member, is referred to as a *smeared* or *continuous* node because the distribution of force is smeared (or spread) over an indistinct length. Figure 4.2.3.1b presents an STM with both discrete and smeared nodes identified.

4.2.3.2 State of stress in node

4.2.3.2.1 Hydrostatic nodes—Originally, STM provisions were developed assuming that nodal zones would have equal stresses on all sides (MacGregor 2002). In this case, the node is termed a hydrostatic node. In the hydrostatic approach, the idealized shape of nodal zones is arranged such that the stresses on all sides of the node, from the truss member forces as well as from the boundary forces meeting at the node, are equal. As the name implies, the resulting state of stress inside the node is hydrostatic—that is, the stresses

are isotropic, uniform, and equal in all directions. For plane stress problems, the term “hydrostatic” may be misleading because only the in-plane stresses are isotropic; the out-of-plane stresses are negligible. The condition of equal stresses on all node sides is achieved by selecting the sides such that their lengths are proportional and perpendicular to the

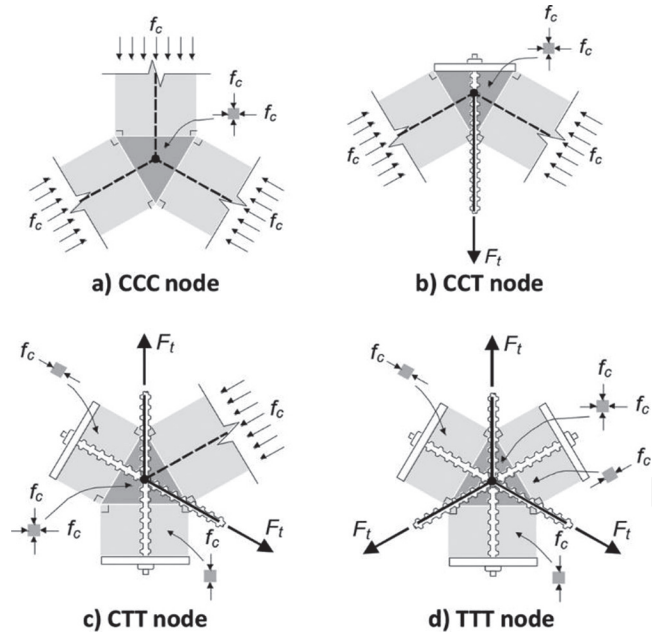


Fig. 4.2.3.2.1—Examples of hydrostatic nodes.

forces acting on them; refer to Fig. 4.2.3.2.1(a). In defining the effective widths of ties intersecting at the node, the tie forces are assumed to be anchored by fictitious bearing plates behind the node; refer to Fig. 4.2.3.2.1(b) through (d).

4.2.3.2.2 Non-hydrostatic node—The requirement of equal stresses on all faces of a node often results in impractical construction details. Therefore, nodal regions are often proportioned to be non-hydrostatic. Figure 4.2.3.2.2a illustrates the states of stress associated with hydrostatic and non-hydrostatic nodes for a two-dimensional node.

The shape of a non-hydrostatic node can be simple to construct, but the state of stress within the node can be difficult to determine. The presence of shear stress within a non-hydrostatic node and its effect on node strength is a topic that requires further investigation. Departing from a hydrostatic state of stress could potentially result in compatibility stresses not covered by typical strength values (Schlaich et al. 1987). Accordingly, Schlaich et al. (1987) recommended designers limit the ratio of maximum to minimum stresses on the node faces to no more than 2. Finite element analyses have been used to obtain the stress distribution in nodal zones (Alshegeir and Ramirez 1992; Yun 2000; Yun and Ramirez 1996). Alternatively, Mohr’s circle can provide insight into the idealized state of stress in these zones as described in Section 5.7.

The idealized shape of a non-hydrostatic nodal zone is formed by the intersection of the effective widths of struts and ties (Fig. 4.2.3.2.2b). In this example, the length of the

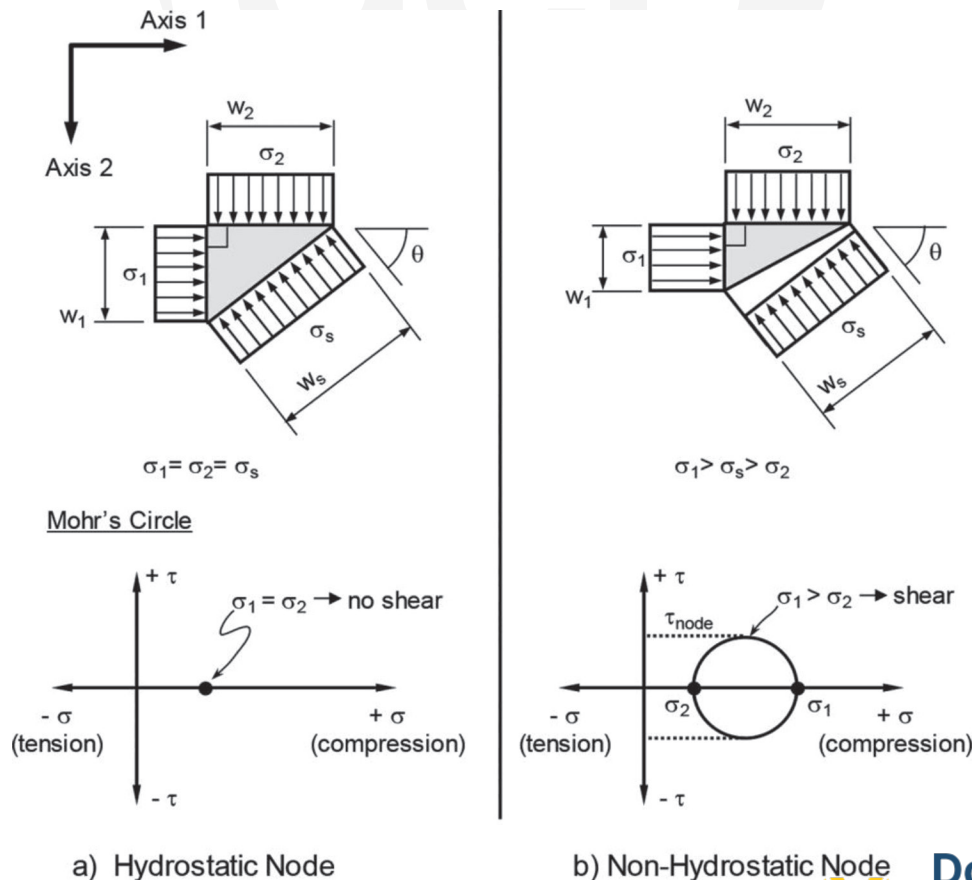


Fig. 4.2.3.2.2a—Mechanics of hydrostatic and non-hydrostatic node (Thompson 2002)

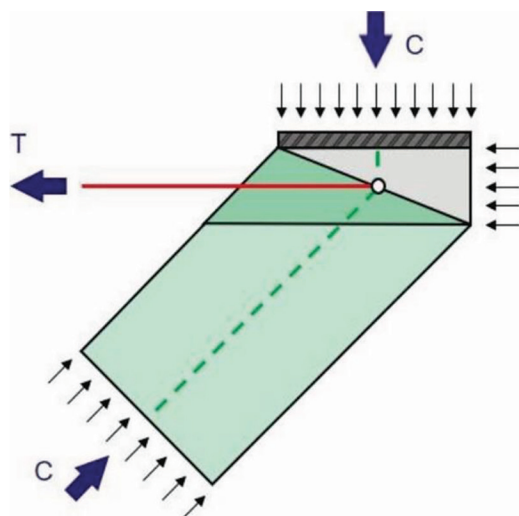


Fig. 4.2.3.2.2b—Construction of non-hydrostatic nodal zone.

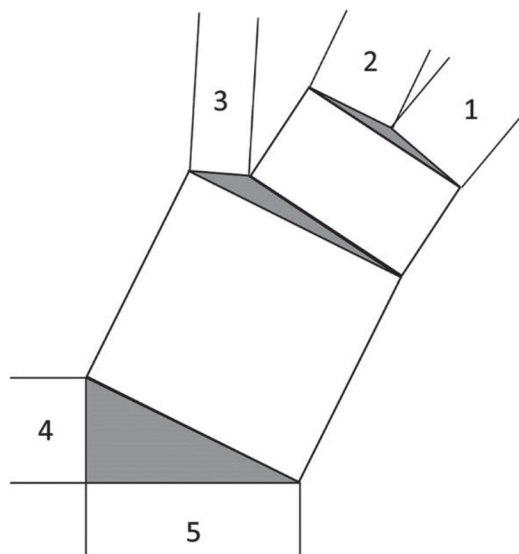


Fig. 4.2.3.2.2c—Example of a transition zone for a node formed by five truss elements (Schlaich and Anagnostou 1990).

bearing plate is fixed, as is the effective height of the tie based on the reinforcement placement.

If more than three members frame into a node, they are often resolved into three equivalent vector components. As a conceptual alternative, Schlaich and Anagnostou (1990) suggest using transition stress fields to subdivide a node into several intermediate nodes (Fig. 4.2.3.2.2c). Both approaches reduce the number of nodal forces to three.

4.3—STM strength requirements

The strength of an internally and externally statically determinate structural member is determined by the strength of the STM, based on the element with the least strength (that is, the weakest link). If the internal force applied to one of these elements exceeds its strength, potential failure modes can include the crushing of concrete in the strut or at the interface between the strut and node, fracture (yielding)

of tie reinforcement, or anchorage failure of the tie reinforcement. Thus, the design strength of each element should be greater than or equal to the factored load effects on the element, or

$$\phi F_n \geq F_u \quad (4.3)$$

The nominal strengths F_n of struts, ties, or nodal regions are described in Sections 4.4, 4.5, and 4.6, respectively.

It is noted that because the strut-and-tie method relies on the lower-bound theorem of plasticity, for statically indeterminate systems, the assessment of the weakest link should be made after any and all redistribution of forces occurs. Common industry practice is to achieve redistribution through yielding of selected ties, but some redistribution from the deformation strength of concrete undergoing localized crushing may be possible. In either case, redistribution requires local and/or global deformation of the member, and the design professional should ensure detailing of the primary and distributed reinforcement is provided to allow for this deformation.

4.4—Strength and requirements for struts

4.4.1 Strut strength—The nominal compressive strength of a strut is taken as

$$F_{ns} = f_{ce} A_{cs} + A_s' f_s' \quad (4.4.1a)$$

where A_{cs} is the cross-sectional area of the strut; f_{ce} is the maximum effective stress limit for the strut; A_s' is the area of compression reinforcement along the length of the strut; and f_s' is the stress in the compression reinforcement at the nominal axial strength of the strut. F_{ns} shall be evaluated at each end of the strut (that is, at the strut-to-node interface).

The effective compressive strength of the concrete in the strut is

$$f_{ce} = 0.85 \beta_s \beta_c f_c' \quad (4.4.1b)$$

where the value of the strut efficiency factor β_s is dependent on the type of strut and quantity of transverse reinforcement crossing its axis, as listed in Table 4.4.1.

The strut efficiency factor β_c is based on the area of concrete surrounding and confining the discrete node adjoining the strut and is equal to $\sqrt{A_2/A_1} \leq 2$. For other cases, β_c is equal to 1 (Section 4.6.1). The ACI 318-19 Code has substantively updated the strut efficiency factor β_s compared with the ACI 318-14 Code and previous versions. These changes provide consistency with the new definitions for strut types as described above in 4.2.2.1. In addition, for interior struts that do not satisfy the minimum reinforcement requirements of 4.4.4, the strut efficiency is taken as either 0.75 or 0.40, depending on the depth of the member (Section 4.4.5). This limitation is intended to preclude diagonal tension failure and may be either more or less restrictive than the former ACI 318-14 factor of 0.60, depending on the configuration of the strut.

4.4.2 Strut orientation—To avoid incompatibilities between the shortening of struts and lengthening of adjacent ties, or a convergence of high compressive and tensile stress, the orientation of a strut should not cause an excessively acute angle with any tie. Accordingly, ACI 318-19 requires a strut to meet a tie at an angle θ_s greater than or equal to 25 degrees.

4.4.3 Cross-sectional dimensions for struts—The width of a strut is normally defined by the dimensions of the nodes at each end which are, in turn, limited by the width of the member, column, bearing plate, or tie geometry. Examples of common geometries are given in Fig. 4.4.3.

Table 4.4.1—Strut efficiency factor β_s

Condition	β_s
Boundary struts	1.0
Interior struts with reinforcement satisfying requirements in Section 4.4.4	0.75
Interior struts with diagonal tension strength satisfying requirements in Section 4.4.5	
Struts satisfying beam-column joint requirements in ACI 318-19 Chapter 15	
Interior struts with less reinforcement than specified in Section 4.4.4	0.40
Struts in tension members, or the tension zones of members	
All other cases	

When a strut frames into an internal or smeared node, the nodal dimensions and, thus, the effective width of strut (b_s) perpendicular to the planar model, are less clear. Leonhardt and Walther (1962) originally theorized the strut forces to be concentrated at the stirrup legs. Thus, the strength or required width of the strut would be directly related to the spacing of stirrups across the width of the web. As a result, Leonhardt and Walther (1962) recommend limiting the maximum spacing of stirrups across the web to 8 in. (200 mm) for beams with *high shear stress* (approximately 4 to $8\sqrt{f'_c}$ [psi] or 0.33 to $0.66\sqrt{f'_c}$ [MPa]) and 16 in. (410 mm) for beams with *low shear stress* (approximately 2 to $4\sqrt{f'_c}$ [psi] or 0.17 to $0.33\sqrt{f'_c}$ [MPa]) in order to consider the full beam width as effective; their recommendations are based on tests of beams with 12 in. (300 mm)-wide webs. CSA A23.3-14 recommends that the effective width of a strut perpendicular to the STM planar model should be limited to the concrete within eight times the longitudinal bar diameters at a stirrup. By this approach, a wide member with wide transverse spacing of stirrups in Fig. 4.4.3(a) could be considered multiple parallel STMs. AASHTO LRFD (AASHTO 2017) provisions no longer contain specific requirements for transverse widths of struts but encourage the use of multiple stirrup legs in wider bridge elements. ACI 318-19 Chapter 9 requires a maximum spacing of stirrups legs across the beam width of d , or 24 in. (610 mm) maximum, for nonprestressed beams with low shear stress and $(d/2)$, or 12 in. (305 mm) maximum, for nonprestressed beams with high shear stress, similar to the

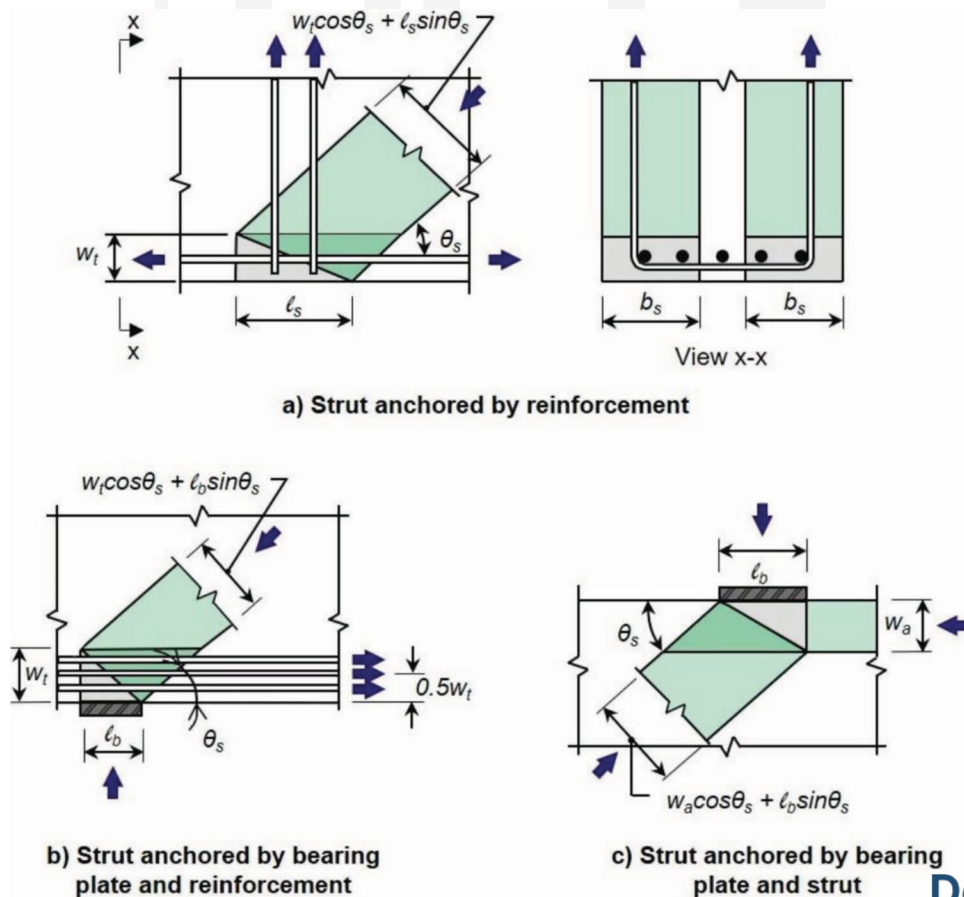


Fig. 4.4.3—Determination of projected strut width for strut anchored by reinforcement

recommendations of Lubell et al. (2009). Tuchscherer et al. (2011a) similarly note that stirrup leg spacing less than d had no significant effect on the experimental shear strength and crack widths of deep beams with 21 x 44 in. and 36 x 48 in. (530 x 1120 mm and 910 x 1220 mm) cross sections ($b_w/h \leq 0.75$).

4.4.4 Distributed crack-control reinforcement—Per ACI 318-19 Commentary, “the strut-and-tie method is derived from the lower bound theorem of plasticity; therefore, a member designed using this method requires sufficient reinforcement to promote redistribution of the internal forces in the cracked state (Marti 1985a).” Also, as discussed in Section 4.2.2.2, interior struts may be subjected to transverse tension, or may split due to the spreading of compression. Thus, the strength of an interior strut is reduced when it contains less distributed reinforcement than specified, with two exceptions—a strut may attain its upper strength without distributed reinforcement: 1) when laterally restrained; or 2) when member dimensions satisfy a diagonal tension limit (Section 4.4.5). A continuous ledge supporting a discrete bearing pad is an example of lateral restraint.

ACI 318-19 does not require distributed reinforcement for a laterally restrained strut. For unrestrained struts, a grid of reinforcement equal to 0.25 percent of the cross section, or $0.0025b_w s$, in each of the two orthogonal directions meets the minimum requirement. For reinforcement placed in one orthogonal direction, its component transverse to the strut should be equal to 0.25 percent of the cross-sectional area. Table 23.5.1 in ACI 318-19 prescribes the required distributed reinforcement, where α_1 is the minimum angle between the strut and reinforcement (Fig. 4.4.4).

While ACI 318-19 allows placement of the transverse reinforcement in a single direction, providing all the reinforcement in one direction is undesirable in large members. This is because there is the potential for cracks to orient to avoid intersection with the provided steel. Thus, the distributed crack-control reinforcement should be placed equally in two orthogonal directions to the extent possible and the spacing limited between bars in each direction. However, as is the case for corbels (Fig. 4.4.4), instances may exist where it is acceptable to place the reinforcement in a single direction. Per Section 23.5.2 of ACI 318-19, the spacing of distributed reinforcement shall not exceed 12 in. (305 mm) and the orientation of the strut axis (α_1) shall not be less than 40 degrees.

4.4.5 Shear force limit—As described in Section 4.4.1, a reduced strut factor of 0.4 should be taken for discontinuity regions without minimum reinforcement unless member dimensions are selected such that

$$\begin{aligned} V_u &\leq \phi 5 \tan \theta \lambda_s \sqrt{f'_c} b_w d \quad (\text{lb}) \\ V_u &\leq \phi 0.42 \theta \tan \lambda_s \sqrt{f'_c} b_w d \quad (\text{N}) \end{aligned} \quad (4.4.5)$$

For prestressed members, d shall be taken as the distance from the extreme compression fiber to the centroid of the prestressed reinforcement and any nonprestressed reinforcement but need not be taken less than $0.8h$. This limitation

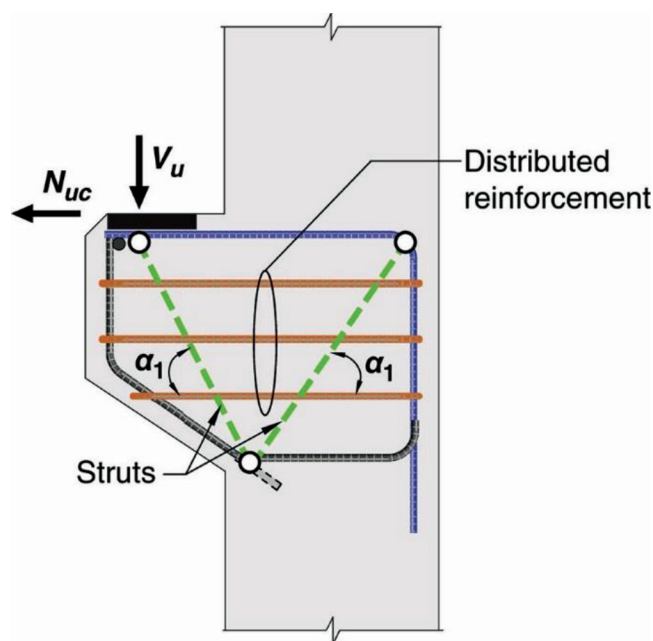


Fig. 4.4.4—Reinforcement crossing a strut (ACI 318-19).

is intended to preclude diagonal tension failure in members without minimum distributed reinforcement. Note that the equation includes adjustment factors for lightweight concrete (λ) and member depth (λ_s) per ACI 318-19 Sections 19.2.4 and 22.5.5.1.3, respectively.

4.4.6 Confined struts—The strength of the strut-to-node interface may be increased when the node connected to the strut includes a bearing surface surrounded by concrete; that is, when the adjoining node is confined. A strut can also be confined with closely spaced hoops or spiral reinforcement. In this case, ACI 318-19 does not provide explicit guidance. Bergmeister et al. (1993) proposed an expression for determining the effective strength of a confined compression field. Section 7.9 presents a similar application for a coupling beam.

4.4.7 Struts located in tension regions—Struts may be located in tension members or in tension zones of members. In the presence of transverse tension, the effective compressive strength of concrete is reduced (Section 4.4.1). Some common cases of a strut occurring in a tension zone include: a) an inverted T-beam (Fig. 4.4.7(a)), where the load on the ledger is carried to the top of the beam with hanger reinforcement, and b) around a large opening located in the tension flange of a beam, such as a box girder bridge (Fig. 4.4.7(b)). Example models for inverted T-shape sections are provided in Section 7.7.

4.4.8 Struts crossing interfaces—Struts may cross an interface such as a construction joint or existing crack. For example, consider the strut crossing the construction joint in the corbel shown in Fig. 4.4.8. Section 23.2.11 of ACI 318-19 explicitly requires an independent check of the shear friction requirements of ACI 318-19 Section 22.9 “where it is appropriate to consider shear transfer across any given plane, such as an existing or potential crack or interface.”

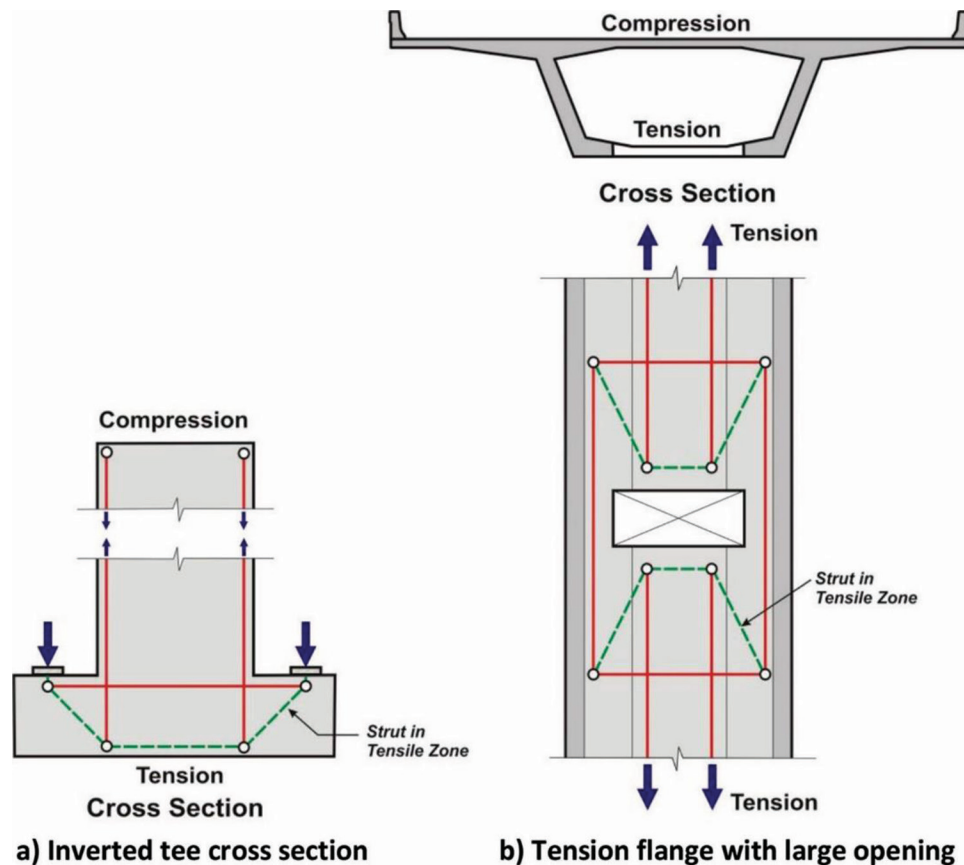


Fig. 4.4.7—Examples of struts located in tension regions of members.

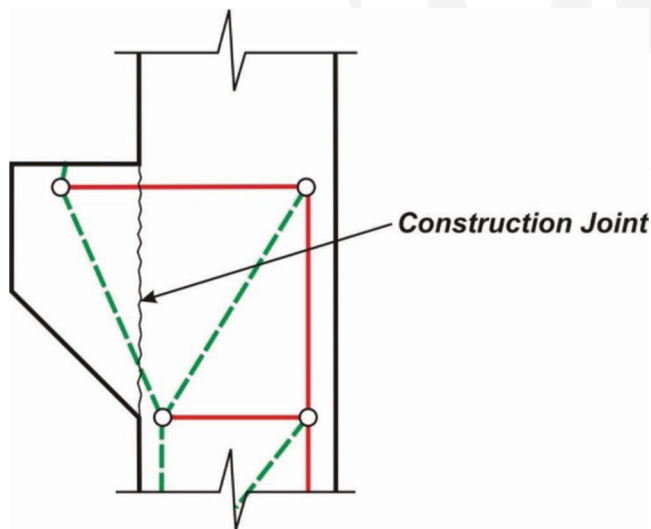


Fig. 4.4.8—Strut crossing an interface between two concretes cast at different times.

between dissimilar materials, or an interface between two concretes cast at different times.”

4.5—Strength and requirements for ties

4.5.1 Strength of ties—Calculating the design strength of a tie is usually straightforward. Details such as bar spacing and anchorage are critical to the tie strength and deserve careful consideration when detailing the tie reinforcement. The centroidal axis of the reinforcement in a tie should align

with the axis of the tie in the STM. In addition, ACI 318-19 Section 20.2.2.4 limits the maximum value of f_y permitted for design calculations to 80,000 psi (550 MPa) for longitudinal ties and 60,000 psi (410 MPa) for other reinforcement.

In ACI 318, the effects of prestressing shall be included in the strut-and-tie model as an external load in accordance with Section 5.3.11 of ACI 318-19. Thus, the nominal strength of a tie is

$$F_{nt} = A_{ts}f_y + A_{tp}(\Delta f_p) \quad (4.5.1)$$

where the resistance is provided by the yield strength of the mild reinforcement and the remaining incremental stress increase in the prestressing due to the application of factored loads. Refer to Section 5.12 of this document for additional discussion of the considerations of prestressing.

4.5.2 Effective tie height—Because the centroid of a tie should coincide with the centroid of reinforcement, the effective height of a horizontal tie in a beam is often taken as twice the distance from the edge of the beam to the centroid of reinforcement (refer to Fig. 4.5.2a(a)). A similar analogy can be used for ties in other member types. Depending on reinforcement placement, an upper limit recommended for the tie height dimension is the width of a hydrostatic nodal zone (ACI 318-19 Section R23.8.1), or

$$w_{t,max} = \frac{F_{nt}}{f_c b_s} \quad (4.5.2)$$

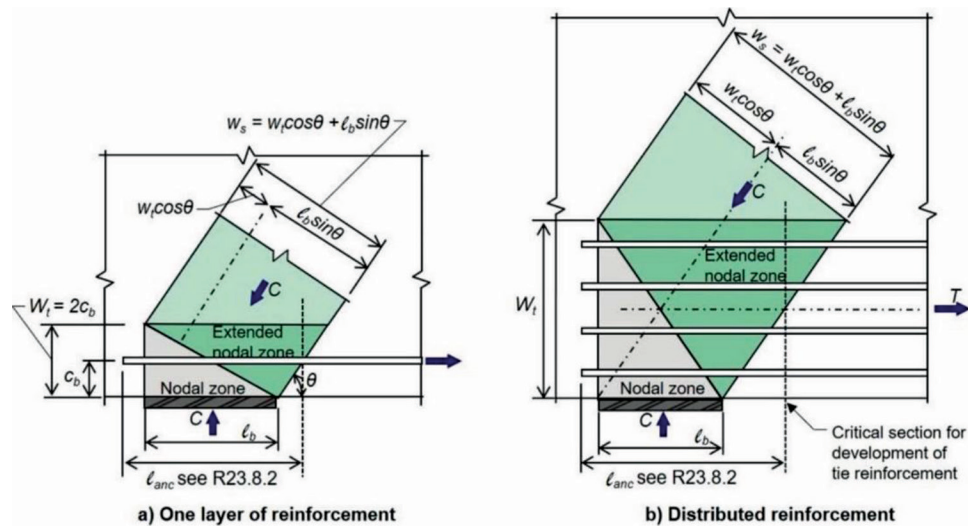


Fig. 4.5.3a—Extended nodal zone showing the effect of the distribution of the force (ACI 318-19).

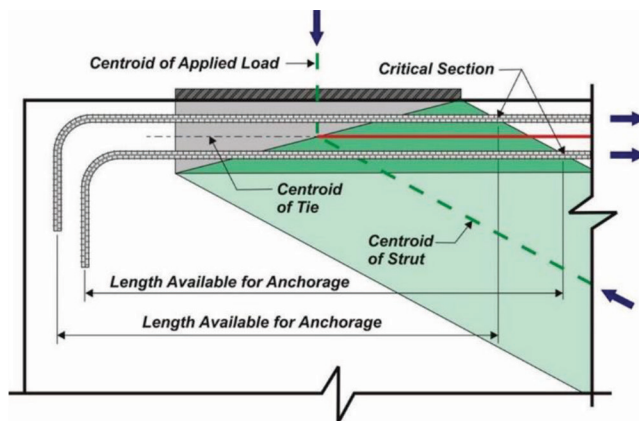


Fig. 4.5.3b—Length available to anchor a tie consisting of multiple layers of reinforcement (after Tuchscherer et al. [2010a]).

4.6—Strength and requirements for nodes

4.6.1 Strength of nodes—The nominal compression strength of the surface of a nodal zone (F_{nn}) is

$$F_{nn} = f_{ce} A_{nz} \quad (4.6.1a)$$

where f_{ce} is the effective compressive strength of the concrete in the node, and A_{nz} is the area of the boundary surface or section taken perpendicular to the resultant force applied to the node.

The effective compressive strength of the concrete at the boundary of the nodal zone is dependent on the number of tie forces anchored within the node. The effective compressive strength of the node decreases as the number of tie forces anchored within the node increases; this is the result of the increasing incompatibility between the tensile strains in the ties and compressive strains in the struts (ACI 318-19).

$$f_{ce} = 0.85\beta_n\beta_c f'_c \quad (4.6.1b)$$

Table 4.6.1—Node efficiency factor β_n

Condition	β_n	Type
Nodal zones bounded by struts, bearing areas, or both	1.0	CCC
Nodal zone anchoring one tie	0.80	CCT
Nodal zone anchoring two or more ties	0.60	CTT

Values for the node efficiency factor β_n are listed in Table 4.6.1.

The node efficiency factor β_c accounts for the effects of concrete confinement on the effective compressive strength of a node containing a bearing surface and is equal to $\sqrt{A_2/A_1} \leq 2$. For other cases, β_c is equal to 1.

Node types are discussed in Section 4.2.3.1.

4.6.2 Confined nodes—In addition to surrounding a node with concrete, confinement may also be provided with transverse hoop or spiral reinforcement. Increasing node strength through the use of spiral reinforcement is a common practice for the design of post-tensioned anchorage zones.

4.6.3 CCC node—A CCC node is shown in Fig. 4.2.3.1a. Application of these guidelines including treatment of various node types is discussed in ACI SP-208 and ACI SP-273. To illustrate a typical approach for determining nodal dimensions, consider the asymmetrically loaded beam shown in Fig. 4.6.3a.

When calculating the strength of the rightmost D-region, it is first necessary to determine the nodal dimensions. The CCC node (Node I) can be subdivided into two nodes based on the proportion of load that is being transferred to the right and left supports. The faces of the non-hydrostatic node can be proportioned as illustrated in Fig. 4.6.3b. The forces applied the top surface of the node, as shown, are resultants of the respective uniform stress.

4.6.4 CCT node

4.6.4.1 Typical dimensions—Consider the beam and STM shown in Fig. 4.6.3a. The three faces of the non-hydrostatic CCT node at the right support (Node E) can be proportioned as illustrated in Fig. 4.6.4.1.

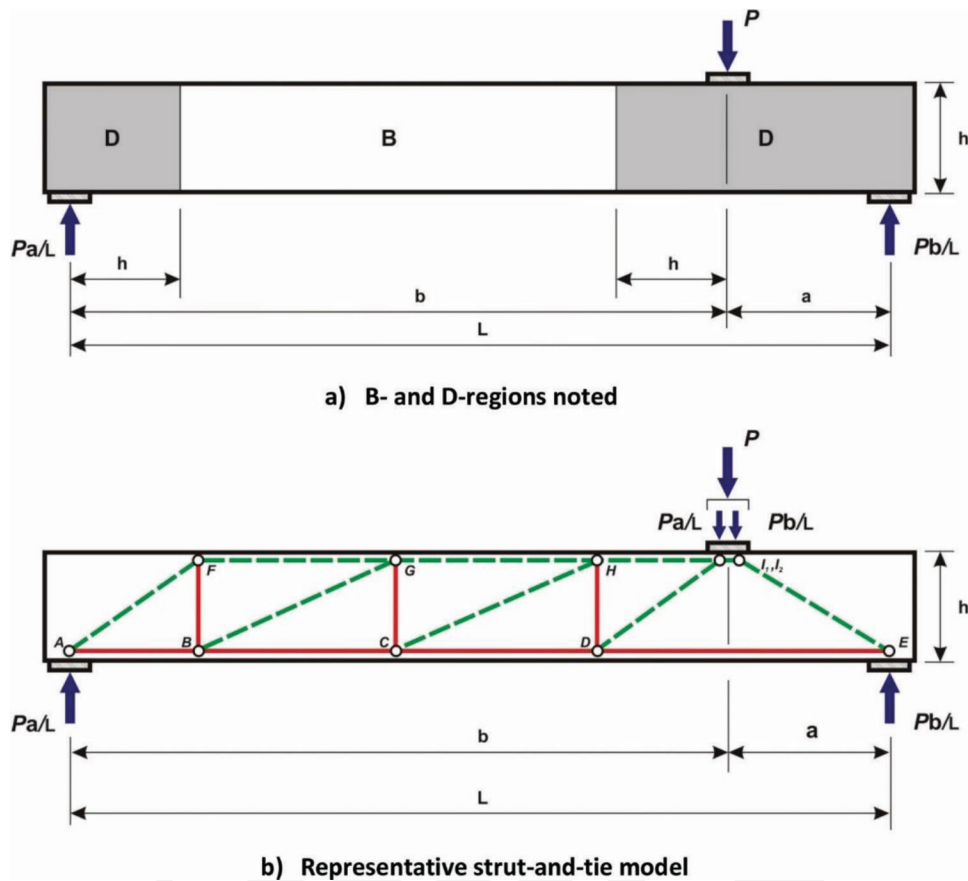


Fig. 4.6.3a—Asymmetrically loaded beam.

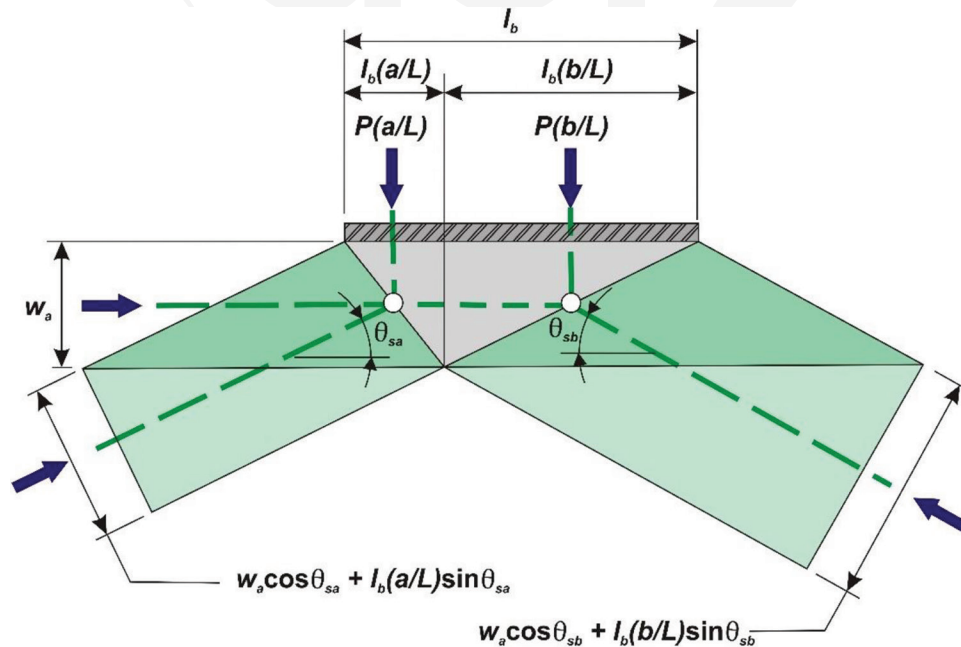


Fig. 4.6.3b—Proportions of a CCC node.

Given the **ACI 318-19** requirement for determining the effective height of a tie (Section 4.5.1 of this document), the force in the tie is visualized as being resisted entirely at the back face of the node. However, if tie forces are resisted by

adequately anchored reinforcement, then it is unlikely that crushing of concrete at the back face of a node will occur (*fib* 1999; *Bircher et al.* 2009). In this case, the allowable stress at the back face of the node is not critical. However,

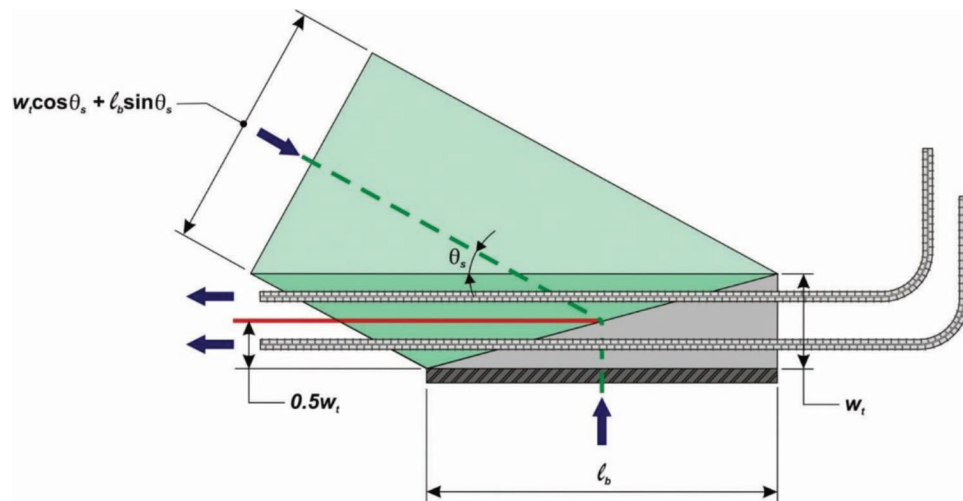


Fig. 4.6.4.1—Proportions of a CCT node.

it is important to point out this exception only applies to straight, hooked, and headed reinforcement that meets the anchorage requirements of ACI 318-19 Section 25.4. For other cases (for example, when the force in the bar is transferred to concrete entirely through bearing at the head), the designer should consider anchor failure modes that include the crushing of concrete at the face of the node or side-face blowout.

4.6.4.2 Interior node over continuous support—A unique type of CCT node is an interior node over a continuous support, as shown in Fig. 4.6.4.2.

ACI 318 does not provide explicit guidance for this type of node. The most conservative interpretation of the Code is to treat this as a CTT node, or *nodal zone anchoring two or more ties*. Although two ties exist in the node, only one horizontal force (that is, the difference between the horizontal forces at each side) acts through the node. Thus, the state of stress is more representative of a CCT nodal region. For that reason, Schlaich and Schäfer (1991) and fib (1999) recommended the node be treated as two adjacent CCT nodal regions.

4.6.5 CTT node (discrete)—CTT nodes can be classified as either smeared or discrete, as described in Section 4.2.3.1. One example of a discrete CTT node is that which may occur near the dap of a dapped-end beam. This node type is critical and should be carefully evaluated. The proportions of this type of CTT node are illustrated in Section 7.5 of this document.

4.6.6 CTT node (smeared)—Provided reinforcement is anchored properly, the strength of a smeared node will generally not govern the strength of an STM. According to fib (1999), “If... bars are distributed over a great length of the main reinforcement, as is normally the case in beams; the node is of the ‘smeared’ type and needs not be checked in detail.” Nonetheless, for the purpose of illustration, consider Node D of the beam and STM shown Fig. 4.6.3a. The proportions of this node are illustrated in Fig. 4.6.6a.

Any stirrup fully anchored at each end within the node that intersects an adjacent strut at an angle greater than 25 degrees from each end may be included as part of the

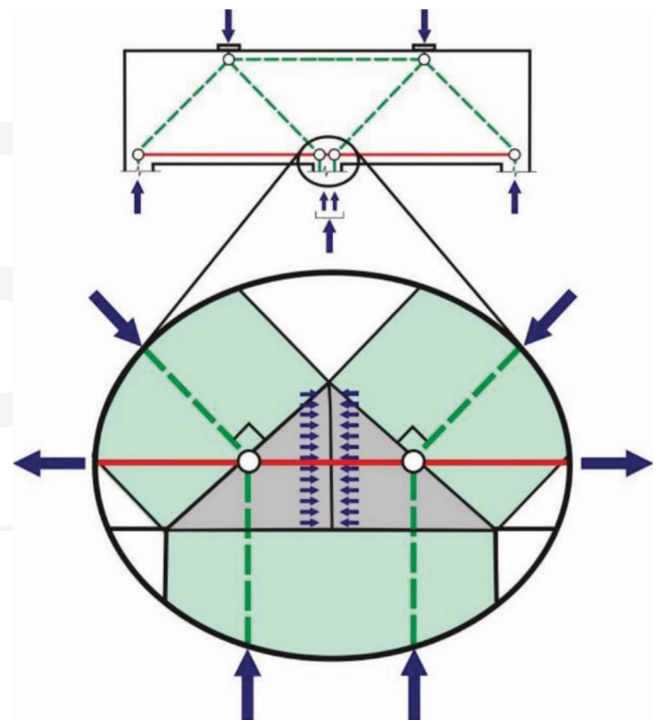


Fig. 4.6.4.2—Interior node over a continuous support.

vertical tie (Wight and Parra-Montesinos 2003). In other words, Tie HD shown in Fig. 4.6.3a may include all applicable stirrups to the left of Node I₁ and intersecting a line projecting at 25 degrees from the vertical and horizontal axis at Node I₁ (Fig. 4.6.6b). Tie HD should be located at the centroid of the applicable stirrups, as will Tie CG be at the centroid of the stirrups it represents. Stirrups included in Tie HD cannot also be used for Tie CG.

4.6.7 Curved-bar nodes

4.6.7.1 Overview—ACI 318-19 Section 23.10 includes provisions for a curved-bar node, a type of discrete CTT and CCT node that deserves special attention. A curved-bar node is defined as the bend region of a continuous reinforcing bar (or bars) whose straight extensions form two ties in a STM. The deviation of the tie for a curved-bar node should

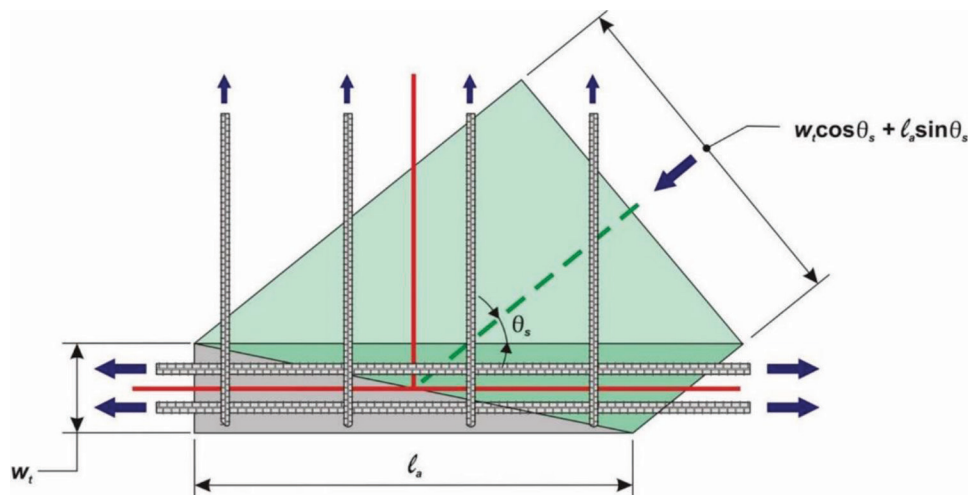


Fig. 4.6.6a—Proportions of a smeared CTT node.

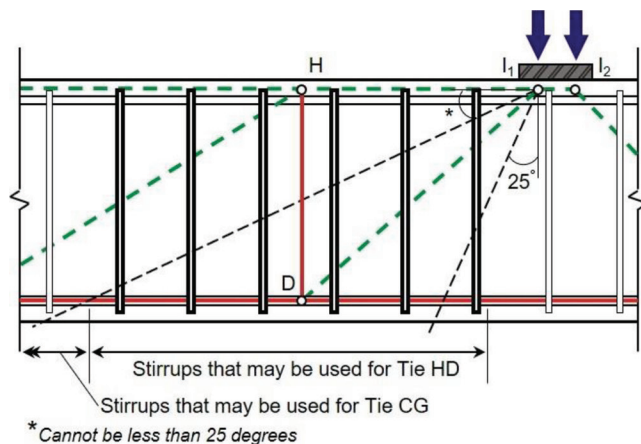


Fig. 4.6.6b—Stirrups forming Tie HD (refer to Fig. 4.6.3a).

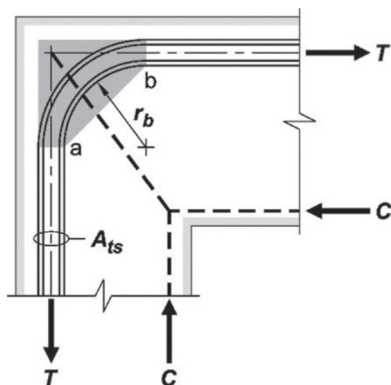


Fig. 4.6.7.1a—Curved-bar CTT node at a frame corner.

be equilibrated by one or more struts. Examples of CTT and CCT curved-bar nodes are shown below in Fig. 4.6.7.1a and 4.6.7.1b, respectively.

Design guidelines for curved-bar nodes were adopted within ACI 318-19. More specific designs of frame corners, corbels, and dapped-end connections using strut-and-tie models with curved-bar nodes are discussed in Sections 7.3, 7.4, and 7.5, respectively.

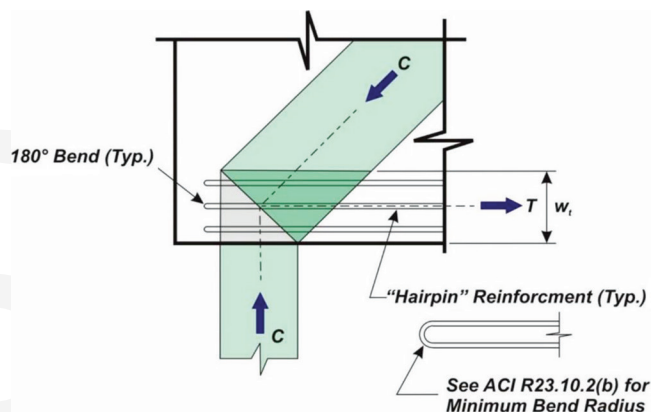


Fig. 4.6.7.1b—Curved-bar CCT node using ties anchored by 180-degree bends.

4.6.7.2 Bend radius and node stress—Bend radii of the reinforcing bars should be within the limits of the following equations.

$$\text{CTT nodes: } r_b \geq \frac{2A_{ts}f_y}{b_s f'_c} \quad (4.6.7.2a)$$

$$\text{CCT nodes: } r_b \geq \frac{1.5A_{ts}f_y}{w_t f'_c} \quad (4.6.7.2b)$$

Equation (4.6.7.2a) ensures that the node stress does not exceed that allowed for a CTT node ($\beta_n = 0.6$). Equation (4.6.7.2b) applies to the higher node stress at CCT nodes ($\beta_n = 0.80$) formed by two struts and a bar (or bars) with a 180-degree bend. Generally, a larger bend radius will result in lower nodal stress and provide for a smoother flow of forces. For optimum performance, the bend radius should be as large as possible, but the center of curvature should fall within the limits of the member or joint as defined by the geometry of the STM (refer to Section 7.3).

4.6.7.3 Multi-layer curved-bar nodes—Where more than one layer of reinforcement is used in the plane of the STM, nodal zone stresses are increased in proportion to the number

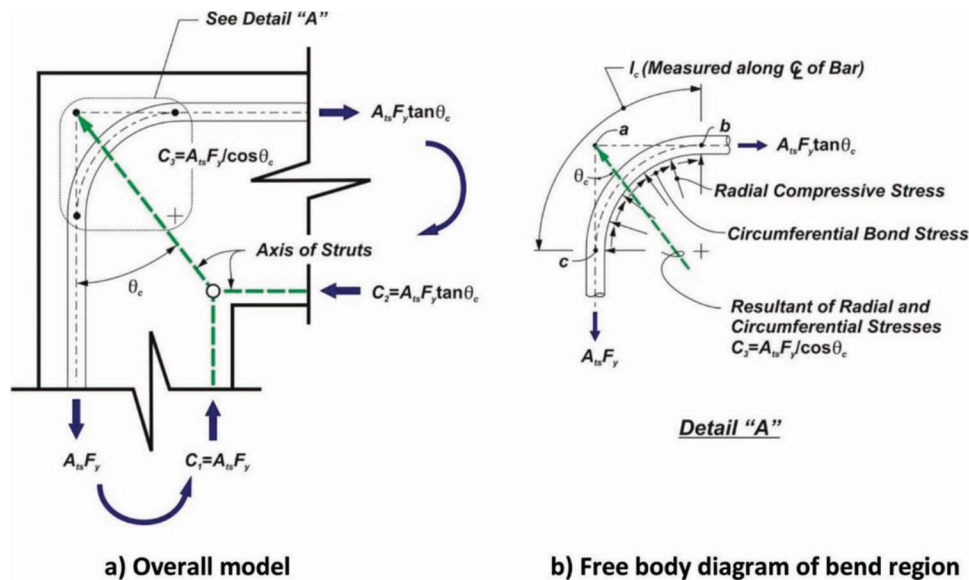


Fig. 4.6.7.4—Forces acting on a curved-bar node where there is a difference in tie forces.

of layers. An example of a multi-layer curved-bar node is illustrated in Fig. 4.6.7.1a. The aforementioned equations may be used, provided A_{ts} is taken as the area of tie reinforcement in all layers and r_b is taken as the bend radius at the inside layer.

4.6.7.4 Bond stress in curved-bar nodes—In some cases—for example, a wall/slab junction with differing effective depths of the two members joined—the tie forces are not equal. Such a case is similar to the detail shown in Fig. 4.6.7.4. ACI 318-19 requires that the difference in tie force be developed within the nodal zone. In cases where the strut (or the resultant of multiple struts) does not bisect the angle formed by the ties extending from the curved-bar node, the length of the curve should be sufficient to develop the difference in tie force. For 90-degree bends, the bond force that should be developed along the curve length (ℓ_c) is $A_{ts} f_y (1 - \tan \theta_c)$. Integration of the linearly varying normal stress and uniform bond stress over the length of the curve gives a resultant identical to the resultant of the two ties extending from the curved-bar node, with respect to magnitude and direction as well as point of origin.

For 90-degree bends, the minimum radius r_b required for sufficient development length through the curve may be checked using the following equation

$$r_b \geq \frac{2\ell_d(1 - \tan \theta_c)}{\pi} - d_b/2 \quad (4.6.7.4)$$

4.6.7.5 Side cover and effective width of curved-bar nodes—A clear side cover of at least two bar diameters is recommended to avoid side splitting. Where this side cover cannot be provided, the bend radius should be increased in inverse proportion to the decrease in clear cover (Östlund 1963).

The effective width of a curved-bar node, transverse to the plane of the truss model, is usually taken as the width of the member transverse to the plane of the truss model, provided that reinforcement is evenly distributed entirely

across the width. Where the centroid of the tie is not located in the middle of the member, the effective width of the node should be taken as twice the distance from the centroid of the tie to the nearest face of the member (Fig. 4.5.2a).

CHAPTER 5—MODELING AND DESIGN CONSIDERATIONS

5.1—Introduction

Chapter 5 presents and discusses modeling and design considerations that are most relevant to complex D-regions, such as when the geometry of the strut-and-tie model (STM) is less apparent and/or when the performance of the designed region under service, ultimate, and overloads is significantly uncertain. The content of this chapter will assist the design professional in selecting suitable models for primarily complex D-regions, in calculating truss-member forces in indeterminate models, in using the STM for capacity evaluation, and in taking advantage of computer-based design and analysis tools.

5.2—Influence of model topology on performance of complex D-regions

5.2.1 Four different STM design solutions for the same D-region—The selected geometry of a model for design of a complex D-region can significantly affect the performance of the built structure under load. A detailed example is presented herein to illustrate that there can be more than one code-compliant STM design solution for a given problem and that the outcomes can be quite different.

Figure 5.2.1 presents four different STMs used to design a deep beam member that had a dapped-end and an opening; these members were 6 ft long and 6 in. thick. The structure supported a top point load near its mid-length and had vertical supports at the base on the left and right sides of the structure. While the structure was externally statically determinate, each of the four different truss models was internally indeterminate. A full design that still satisfies the ACI 318-19

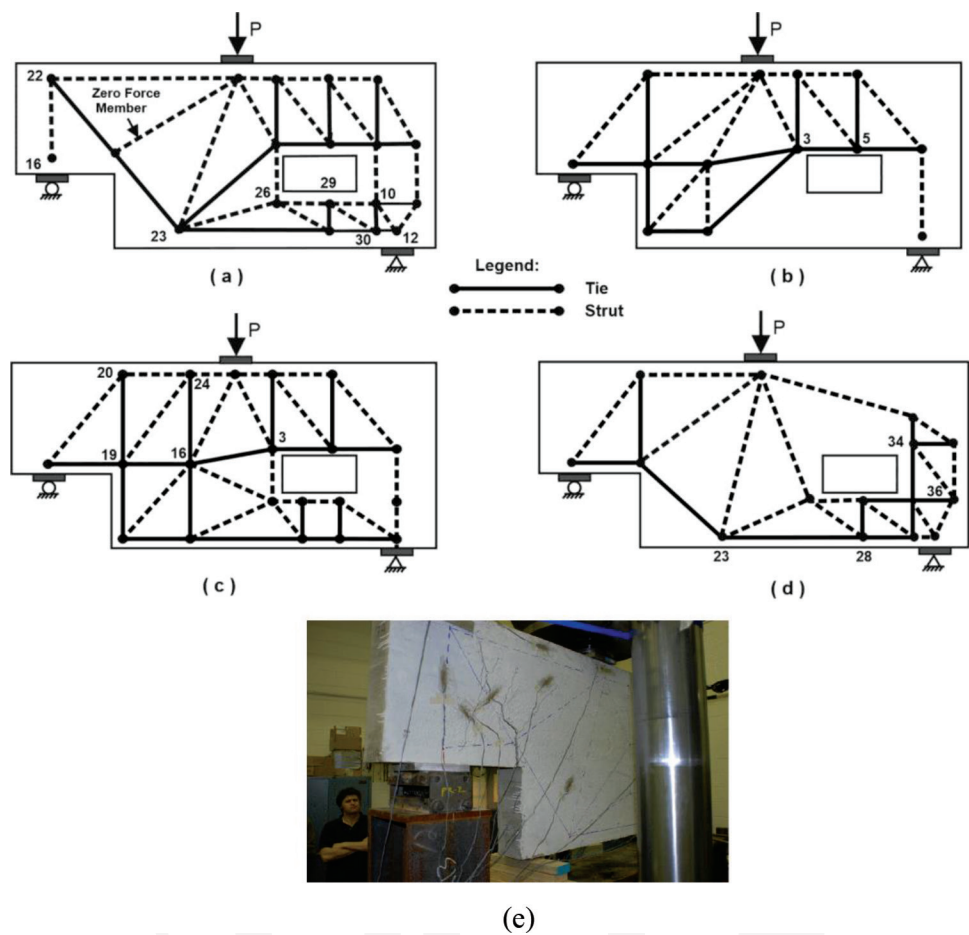


Fig. 5.2.1—Multiple strut-and-tie models for deep member with dapped-end and opening.

STM provisions was completed for each of these four model geometries, and then these structures were constructed and load-tested to failure. (Kuchma et al. 2008)

The four models were deliberately selected to provide quite different load path solutions. Of the group, Model (c) had the most regular truss in which the load to the right support is carried by an upper and lower truss past the opening, and a similar Howe truss carried the load to the left support. The ties in Model (c) were essentially vertical and horizontal, with two horizontal ties, one being at the base and the other being at mid-depth. The shape of Model (a) is similar to that of Model (c), but the loading to the left support was considered to run all the way to the bottom of the member and then be lifted up by a single diagonal tie over the dapped end. The shape of Model (b) is similar to Model (c) in the left part of the member, but on the right half of the member the entire load is carried above the opening in a determinate truss to the right support. While both Models (a) and (b) may be suspected to not perform as well as Model (c), they are both acceptable design solutions by ACI 318-19. In Model (d), the struts radiated out from the point of loading in a compressive fan toward the supports as far as possible before ties were used. This resulted in the reinforcement being concentrated at the bottom, lower right, and left sides of the structure. All these models had a welded wire mesh with a reinforcement ratio of 0.0027 in both directions in addition to the reinforcement for the primary ties from the models.

Table 5.2.2—Comparison of tested (measured) and ACI 318-19 Code-calculated nominal capacity

Design	P_{test} , kip (kN)	$P_{n,ACI}$, kip (kN)	$P_{test}/P_{n,ACI}$
(a)	252 (1120)	146 (651)	1.72
(b)	183 (814)	140 (624)	1.30
(c)	290 (1292)	193 (860)	1.50
(d)	194 (863)	127 (563)	1.53
(a) No mesh	61 (270)	146 (651)	0.40

5.2.2 Comparison of capacities and modes of failure of four designs—Table 5.2.2 compares the experimentally measured capacities of these designs with the ACI 318-19 calculated nominal strengths. As shown, the measured capacities were always well above the nominal strengths, which may be expected because the strut-and-tie method is consistent with the lower-bound plasticity theorem. However, this theorem is only applicable if the member is sufficiently ductile to transfer the load in the manner selected by the designer. To examine the necessity of using distributed reinforcement to provide the needed ductility, a repeat of Model (a) was built without the $0.0027A_g$ of welded wire reinforcement and load tested. This specimen failed at only 40 percent of the calculated nominal capacity because of a brittle failure that occurred after a crack extended from the inside corner of the dapped end along the line of the diagonal