

COMPUTER USE for STATISTICAL ANALYSIS of CONCRETE TEST DATA

P. Balaguru
V. Ramakrishnan
Editors



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Documents

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Printed in the United States of America

Editorial production: Patricia J. Kost

LIBRARY OF CONGRESS CATALOG CARD NUMBER 87-71693

PREFACE

The mission of ACI Committee 214, Evaluation of Results of Tests Used to Determine the Strength of Concrete, is to "Analyze the variations which occur in the results of tests used to evaluate the strength of concrete; present tools of statistics which are useful in interpretation of these variations, and report on measures of reliability which can be used in establishing specifications and design criteria for the strength of concrete." Within this mission, one of the goals of the committee is to write a state-of-the-art report on the use of computers (including micro and mini computers) in concrete quality control and quality evaluation.

In 1985, at the ACI annual convention in Denver, Committee 214 sponsored an international symposium on the Use of Computers for Statistical Analysis of Test Data, which brought together researchers, producers, and users. Seven papers were presented during this symposium. This volume includes six of the seven papers that were presented and an additional paper written specially for this volume.

The first paper deals with the analysis of in-place test data using spreadsheet software. The next two papers deal with the analysis of pullout test data and cover meter data. Pullout tests provide an indication of in-place strength and cover meters provide data regarding the location of the reinforcement. The fourth paper presents a computer code to compute the required average strength for a specified compressive strength. The next two papers deal with the prediction equations of cylinder strengths. The final paper deals with the analysis and plotting of concrete data.

Thanks are extended to all the authors for their contributions. The help of reviewers and the ACI staff who helped to organize the symposium and prepare this volume are also gratefully acknowledged. Special thanks are due to Professor Tarun Naik who organized and chaired the symposium.

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Analysis of In-Place Test Data with Spreadsheet Software

by N. Carino and W. Stone

Synopsis: In-place testing offers the contractor an alternative to testing field-cured cylinders for assessing concrete strength during critical phases of construction. A relationship between cylinder compressive strength and the quantity measured by in-place tests is needed in order to use these methods; this requires statistical analysis of test data. The personal computer and "electronic spreadsheet" software provide a cost-effective means of performing such analyses. Available spreadsheet software can be used to perform the required calculations automatically, thus providing the user with the benefits of automated computation without requiring knowledge of a scientific computing language. Data from a study of the pullout test method are used to illustrate the use of spreadsheet software to perform linear regression and determine the required correlation relationship. In addition, a simplified technique is proposed for using the results of in-place tests to make a reliable estimate of the characteristic strength of structural concrete. The in-place characteristic strength permits an evaluation of structural safety during construction.

Keywords: compressive strength; computer programs; concrete construction; microcomputers; pullout tests; safety; statistical analysis

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INTRODUCTION

The advent of the personal computer will probably be recorded as one of the most significant developments of our time. For a very low cost, an extraordinary amount of computing power is readily available for enhancing productivity. Paralleling the developments in hardware have been similar developments in low-cost, easy to use, yet powerful software (programs) for the personal computer. As a result one may use the personal computer for a wide variety of applications without knowing any of the formal computer languages. This paper discusses how the personal computer and existing software can be used to analyze data from in-place tests for concrete strength.

In concrete construction, the contractor needs to be able to measure the in-place concrete strength before performing critical operations such as removing forms or reshores, applying post-tensioning forces, or terminating cold weather protection. Historically, the recommended practice has been to test field-cured cylindrical specimens as specified in ASTM Method C 31 [1]. This approach has its drawbacks: 1) Field-cured cylinders are often subjected to abuse at the job-site and during transport which may lead to erroneous test results. 2) There is no guarantee that the curing history of the specimens is identical to that of the structure. 3) The procedure is inconvenient since the specimens must usually be shipped to a testing laboratory, and the contractor must wait for a report of the strength test results. Recognizing these deficiencies, the latest version of the ACI Code [2] permits, for the first time, the use of alternative methods. Paragraph 6.2.1.1 states that:

Concrete strength data may be based on tests of field-cured cylinders or, when approved by the Building Official, on other procedures to evaluate concrete strength.

The Commentary to the Code lists some of the alternative procedures. The Commentary also states that these alternative methods "require sufficient data using job materials to demonstrate correlation of measurements on the structure with compressive strength of molded cylinders or drilled cores."

Hence, in order to use any of the alternative methods, a correlation curve must be developed to translate the in-place test results to compressive strength values. In addition, a procedure is needed for analyzing the results of in-place tests so that one can estimate the in-place compressive strength with a high degree of confidence. Both of these steps require statistical analysis of test data. The personal computer provides a cost-efficient means for routinely performing such analyses. A variety of computer programs, known as "electronic spreadsheets", are available which are well suited for doing the required calculations.

This paper describes the essential features of the spreadsheet software used to perform the data analyses mentioned above and illustrates an application using the pullout test method [3]. A simple approach is proposed for using the results of in-place tests to estimate the in-place cylinder strength. The suitability of the proposed procedure is evaluated by comparing the results with a more rigorous statistical method recently developed at the National Bureau of Standards (NBS).

SPREADSHEET SOFTWARE

A brief description is given of the general features of electronic spreadsheet programs for the reader who is unfamiliar with the subject. An electronic spreadsheet is a blank page, residing in the computer's memory, composed of vertical columns (usually identified by letters) and horizontal rows (usually identified by numbers) much like the blank forms used for accounting purposes or for recording engineering data. The intersection of a column and a row defines a "cell" having a specific identification name. For example, "cell C3" is the intersection of column C and row 3. A cell may contain a number, a label or a formula. It is this last feature that gives the spreadsheet program its power, for it permits the user to perform calculations using the contents of cells in the spreadsheet. For data analysis, the user must first enter the experimental data into the appropriate cells. Then, by using the formula capability, the data are reduced and the results are displayed.

Figure 1 illustrates how the electronic spreadsheet can be used for data reduction. In this example, cylinder strength test results are analyzed. Columns B and C contain the values of measured ultimate loads for cylinder pairs. The user entered these numbers into the cells of columns B and C. The remainder of the values shown in the spreadsheet were generated automatically. Columns D and E contain the stress values corresponding to the loads. These values were obtained by entering formulas into cells

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"D4" and "E4" for calculating stress from load. The formula entered into cell D4, for example, expresses the following relationship: "take the value in the second cell to the left of D4 (value of ultimate load) and divide it by 28.27 (area of standard test cylinder)." After the formula has been entered, it is transparent to the user; only the result of the calculation is displayed, that is, $119220/28.27 = 4217$ (see Fig. 1). The formula entered into cell D4 was copied into the remaining cells of column D using the "copy" feature of the program. A similar procedure was used to create the values in column E. Column F contains the average strength of the two cylinders. The formula entered into cell F4 instructed the computer to add the values in the first and second cells to the left of F4 and divide the sum by 2. Again, only the result of the calculation is displayed. Column G contains the average of the three previous tests (a test is the average of two cylinders). The formula entered into cell G6 instructed the computer to add the values in the first cell to the left and in the two cells above it and to divide the sum by three.

The important point is that to create the entire spreadsheet shown in Fig. 1 the user only had to enter the ultimate cylinder loads and four formulas. Figure 1 is the actual appearance of the printed copy obtained with the program and using a letter-quality printer.

The spreadsheet program offers great advantages over a calculator for data analysis. One has a permanent record of the data and the calculated values, which can be incorporated directly into a report for the client. When multiple calculations must be performed with the same data, the original data has to be entered only once. There are also certain advantages over writing an ordinary computer program to analyze the data. First, the user does not need programming skills. The spreadsheet program is interactive, thus allowing one to see the results of the calculations immediately. This latter feature enables the user to examine efficiently alternative analysis procedures. Spreadsheet programs are available that have the full range of scientific functions commonly found on calculators. There are also statistical functions for computing the average and standard deviation of a row or a column of numbers. Finally, some programs have plotting capabilities to permit a pictorial presentation of data. Figure 2, which is a plot of the averages in column G of Fig. 1, illustrates the type of plots that can be obtained. It was created with a low-cost, 2-pen plotter.

REGRESSION ANALYSIS

Data from a study [4] of the pullout test are used to illustrate how spreadsheet software can be used to determine the relationship between cylinder strength and the results of an in-place test. In addition, a method is proposed for predicting in-place cylinder strength based on in-place test results.

Computer Analysis 5

The analysis procedure carried out with the spreadsheet program is divided into three steps.

- 1) Input of the individual results from the correlation testing;
- 2) Determination of the correlation relationship by means of linear regression; and
- 3) Prediction of the in-place strength based on the results of the in-place tests.

First, a "template" spreadsheet was created which contained blank cells for data input and the formulas required for the analysis. The template was stored as a file on a floppy disk. To analyze a particular set of data, the template spreadsheet was retrieved, the data were entered, and the formulas were evaluated. The following discussion outlines the tasks performed by various portions of the spreadsheet.

Figure 3 shows the input portion of the spreadsheet, which was designed to analyze the data reported in Reference [4]. Eleven pullout tests and 5 cylinder compression tests were performed at each of 8 test ages. (Recommendations for the number of tests to use in a correlation testing program are given in Reference [4].) The 8 columns at the top of Fig. 3 contain the individual values of pullout load. At the bottom of each column are shown the average (AVG), the sum of squared residuals (SSR), the standard deviation (S.D.), and the coefficient of variation (C.V.) expressed as a fraction. These statistical parameters were computed with functions built into the spreadsheet program. The bottom portion of Fig. 3 shows the corresponding values for the cylinder strength tests.

At this point, the usual procedure is to determine the correlation between cylinder strength and pullout load by performing a linear regression using the average values of pullout strength and cylinder strength. Routine linear regression is based on two assumptions: 1) the standard deviation of Y-values (cylinder strength) is constant with increasing X-values (pullout load) and 2) the X-values are known without error. However, a study [4] of the standard deviations and coefficients of variation of cylinder tests and pullout tests showed that a better description of the data is that they have constant coefficients of variation. In Fig. 3, the coefficients of variation of the different groups of pullout loads and cylinder strength are not identical. However, it can be shown that the differences are not statistically significant and can be attributed to random error.

It can be shown by propagation of error formulas [5] that if groups of test data have a constant coefficient of variation, the standard deviations of the natural logarithms of the data are constant and approximately equal to the coefficient of variation of the untransformed data. For example, if the coefficient of variation (expressed as a fraction) of a data group equals 0.10, the standard deviation of the natural logarithms of the data is

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also equal to about 0.10. Thus, to satisfy the first of the above assumptions, a better procedure is to perform the regression analysis after transforming the data by computing their the natural logarithms.

When the X-variable has a large variability, as is the case for pullout load, a procedure other than ordinary linear regression is needed to compute the best unbiased estimators of the coefficients of the line. The procedures for such a rigorous analysis are presented in Ref [13]. However, it was found that the differences between the correlation curves obtained from the rigorous analysis and the method presented here are not significant for the purposes of concrete strength prediction. Hence, for this study, the data were analyzed using routine linear regression with the logarithms of the tests results.

The next step in the procedure is to compute the natural logarithms of the individual test results. Figure 4 shows that portion of the spreadsheet giving the logarithms of the test values previously shown in Fig. 3. These values were generated automatically using the "log" and "copy" functions of the program. It was not necessary to enter any additional data. Comparing the standard deviations of the transformed data with the coefficients of variation of the original data, one sees that the values are about the same. Also shown in Fig. 4 are the values of the "pooled" standard deviations for the logarithms of the pullout and cylinder tests. These are calculated by adding the values of the sum of squared residuals, dividing by the corresponding degrees of freedom, and taking the square root of the quotient. Since the standard deviation of "n" samples is based on "n-1" degrees of freedom, the appropriate degrees of freedom for the pooled estimates are 80 for the pullout tests and 32 for the cylinder tests. These data suggest that for this particular concrete, which was made with river gravel aggregate, the variability of the pullout strengths is about three times that of the cylinder strengths.

The above procedure implies that the individual test results have a lognormal distribution rather than a normal distribution as is commonly assumed. It is felt that any errors introduced by this assumption are not significant as there is evidence that concrete strength can be represented by a lognormal distribution [6].

The average values of the logarithms of the pullout and cylinder strengths at each test age are used in the linear regression analysis. The formula for the straight line is

$$\ln(C) = m \ln(P) + b \quad (1)$$

where $\ln(C)$ = average of the natural logarithms of cylinder strength (C),

$\ln(P)$ = average of the natural logarithms of pullout strength (P),
 m = slope of the line,
 b = intercept of the line on the $\ln(C)$ -axis

By taking the anti-logarithm of Eq. (1), we obtain the following equation

$$C = B P^m \quad (2)$$

where $B = e^b$,
 e = base of the natural logarithm

Thus performing linear regression with the log-values of pullout and cylinder strengths is equivalent to finding the coefficients of a power function (Eq. (2)). The power function passes through the origin and the value of the exponent, m , determines the degree of non-linearity. For the case where m equals 1, the power function is a straight line with a slope equal to B .

The procedure for performing linear regression is given in Appendix A. This procedure was adapted from that given in Chapter 5 of Reference [7], which was developed for data analysis using a calculator. The basic steps of the analysis involve finding the sums of various columns of numbers -- a simple procedure for the spreadsheet program. These sums are used to calculate the coefficients of the best-fit line and the estimates of error.

Figure 5 shows the portion of the spreadsheet which performs the regression analysis. The column labeled "X" contains the average of the logarithms of the pullout loads and the column labeled "Y" contains the average of the logarithms of cylinder strengths. These values were copied from the appropriate rows of the portion of the spreadsheet shown in Fig. 4. The X- and Y-values have been arranged in increasing numerical order (using the "sort" function of the program) to facilitate subsequent plotting. The top half of Fig. 5 gives the results of the calculations outlined in Appendix A. Of particular interest are the values of the slope, the intercept, and their estimated errors. The correlation coefficient, which in itself is not a very useful statistic [8], is commonly reported as a qualitative indicator of the fit of the data to the regression line. A more useful statistic is the standard error (shown as "STERROR" on the spreadsheet), which is a measure of the average deviation of the data points about the fitted line and is used for determining the lower confidence limit of the correlation curve.

Having established the correlation equation, one may plot the data and the best-fit curve. The bottom portion of Fig. 5 gives the information for plotting the test data and correlation curve in terms of logarithms or real units. The values in the columns

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labeled "Ln(P)" and "Ln(C)" are copied from the "X" and "Y" columns in the top part of this portion of the spreadsheet. The values in the column "Ln(C)est" are the logarithms of the estimated average cylinder strength for the values in the "Ln(P)" column, that is, they are values on the correlation curve. The values in the column labeled "Ln(C)low" define the lower confidence limit for the correlation curve and are computed according to the equation given in step 11 of Appendix A. Here, the lower limit is set at the 95% confidence level, which means that for any pullout strength there is a 95% probability that the true value of the average cylinder strength is above this limit. The four other columns in the lower half of Fig. 5 are obtained by taking the anti-logarithms of the values in the previous columns. Figure 6, which was generated with the plotting capabilities of the program, shows the average values of the test results, the correlation curve and the lower confidence limit. The value of m equals 1.021 for this particular case, thus the power function is nearly a straight line.

PREDICTION OF IN-PLACE STRENGTH

The correlation equation may now be incorporated into a procedure to estimate in-place cylinder strength based on the results of in-place tests. No standard practice presently exists for making a reliable estimate of in-place strength. The following paragraphs discuss the nature of the problems associated with this aspect of in-place testing. A simple procedure was developed to address these problems and was implemented with the spreadsheet program.

In designing a structure to safely resist service loads, the designer uses the specified compressive strength of the concrete. However, the strength of concrete in a structure is variable and can be described by some type of probability density function. The specified strength used for design is generally taken to represent the strength that is expected to be exceeded with 90% probability, that is, if 10 random samples were taken, 9 of them would be expected to exceed the specified strength. To ensure that this condition is achieved, the contractor is required to use a concrete mixture having an average cylinder strength well above the specified strength, as prescribed in Chapter 4 of ACI 318-83 [2].

The objective of in-place testing is to assure, with a high degree of confidence, that the strength of concrete is sufficient for the structure to safely resist construction loads. To provide a safety margin during construction consistent with that under service conditions, one needs to know the in-place "characteristic strength" of the concrete in the structure; that is, the strength expected to be exceeded with 90% probability. Using the results of in-place tests and the appropriate correlation curve, one obtains information about the average in-place strength. However, this is insufficient for a safety evaluation because one must also account for the variation of the strength in the structure. Presently, no standard practice exists for making this determination based on

in-place tests. For the case of pullout testing, a suggested approach [9] assumes that the coefficient of variation of the in-place compressive strength is the same as the coefficient of variation of the in-place pullout tests. This does not seem to be a reasonable assumption because the reported results from correlation testing programs indicate that the within-batch coefficient of variation of the pullout test is greater than that for the compression test of cylinders [4,10,11]. Thus the approach in Reference [9], which is based on computing the lower tolerance limit, overestimates the variability of in-place cylinder strength and gives overly conservative estimates of in-place characteristic strength [4]. An alternative approach for estimating in-place variation of cylinder strength has been proposed [4] and is discussed below.

When an estimate of in-place compressive strength is desired, one should perform in-place tests and compute the average and standard deviation of the test results. However, because of the sampling process, the sample average based on a finite number of tests may not be the true average value of the concrete. Hence, to have a high degree of confidence in the in-place test results, allowance must be made for the uncertainty of the sample average.

The correlation equation is developed from a finite number of tests in the correlation testing program. Thus there is uncertainty associated with the relationship obtained from the regression analysis. This uncertainty must also be considered when using the correlation curve to convert in-place test results to cylinder strength.

Based on the above discussion it is concluded that, to assure structural safety during construction, it is insufficient to rely simply upon the correlation curve and the average value of the in-place tests. A rigorous statistical procedure has been developed to estimate the in-place characteristic strength at any desired confidence level [13]. The procedure, which is referred to as the "NBS Method", is rather involved and is best-suited for programming using a computer language such as FORTRAN or BASIC. Therefore, a simpler alternative procedure was developed which can be incorporated into the present analysis using the spreadsheet program. This simplified approach is compared with the NBS Method in a subsequent part of the paper.

The simplified procedure is similar, in principle, to that proposed in Ref. [12], and attempts to account for the uncertainties mentioned in the above discussion. The pullout test is used as a specific example to illustrate the approach, but the procedure is equally applicable to other methods that use a correlation equation.

Figure 7 illustrates the steps of the proposed procedure, which are as follows:

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- 1) Determine the average, P_a , and the standard deviation, S_{pf} , of the natural logarithms of the in-place pullout tests.
- 2) Compute the 90% lower confidence limit, P_{low} , for the average of the natural logarithm of the in-place pullout strength using the following equation:

$$P_{low} = P_a - \frac{t S_{pf}}{\sqrt{n_{pf}}} \quad (3)$$

where t = the Student t -value for $(n_{pf}-1)$ degrees of freedom at the 90% confidence level

n_{pf} = number of in-place pullout tests

- 3) Compute the 95% lower confidence limit for the estimated average of the logarithm of the in-place cylinder strength, $(C_a)_{low}$, by using the value of, P_{low} , and the formula in step 11 of Appendix A.
- 4) Compute the logarithm of the in-place characteristic cylinder strength, CS , using the following formula

$$CS = (C_a)_{low} - Z S_{cf} \quad (4)$$

where $Z = 1.282$, which is the value of the standard normal variable for the 10th percentile point of the normal distribution

S_{cf} = the estimated standard deviation of the logarithm of the in-place cylinder strength

It is proposed [4] that S_{cf} can be estimated by assuming that the ratio of the standard deviation of cylinder strength to the standard deviation of pullout strength has the same value in the field as in the laboratory during the correlation testing. Thus S_{cf} is obtained as follows:

$$S_{cf} = \frac{S_{pf} S_{cl}}{S_{pl}} \quad (5)$$

where S_{cl} = the pooled standard deviation of the logarithms of cylinder strength from the laboratory correlation tests

S_{pl} = the pooled standard deviation of the logarithms of pullout strength from the laboratory correlation tests

The final step is to take the anti-logarithm of the value obtained in Eq. (4). Thus one obtains an estimate of the in-place characteristic strength which can be compared with the strength required to safely resist construction loads.

Figure 8 shows the portion of the spreadsheet which estimates the in-place characteristic strength according to the proposed procedure. For the sake of illustration, 10 hypothetical in-place pullout strengths are shown on the left side of the spreadsheet. These values are entered into the cells of the spreadsheet by the user. The program computes the logarithms of the pullout values and computes the average and standard deviation. These values are automatically substituted into the appropriate equations which have been entered into the template spreadsheet, and the in-place characteristic strength is computed and displayed. Figure 9 shows a portion of the correlation curve and its lower confidence limit, and it illustrates the sequence of steps in going from the average of the logarithm of pullout strengths to the logarithm of characteristic strength. If the number of in-place pullout tests were not equal to 10, the user enters into the appropriate cell of the spreadsheet the correct t-value for the number of tests.

In this example the estimated characteristic strength (rounded off to the nearest 10 psi) is 1650 psi. Notice in Fig. 8 that the value of average cylinder strength, C_a , is 2140 psi (also rounded off to nearest 10 psi). This difference of 490 psi arises because the proposed method accounts for: 1) the variability of in-place cylinder strength; 2) the variability of in-place pullout tests; and 3) the variability of the correlation equation. To assess structural safety during critical stages of construction, the estimated characteristic strength of 1650 psi would be compared with the required strength.

The proposed procedure is conservative as assures with high confidence that the true in-place characteristic strength will be as least as large as the estimated value. The next section compares the present procedure with the rigorous NBS Method.

CONFIDENCE LEVEL OF STRENGTH PREDICTION

The rigorous analysis determines the value of the in-place characteristic strength for any desired confidence level. For example, if the confidence level is 0.95, there is a 95% probability that the true characteristic strength is as least as large as the computed value. The details of the method are presented in Reference [13]. The key step involves computing the variance of the estimated in-place characteristic strength. This variance and the appropriate Student t-value are used to calculate the lower confidence limit of the true characteristic strength. The rigorous method accounts for the variability of pullout strength in the regression analysis, and it also uses Eq. (5) to calculate the standard deviation of in-place cylinder strengths.

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The proposed simplified procedure simultaneously uses the lower 90% confidence limit of the average in-place pullout strength and the lower 95% confidence limit of the predicted average cylinder strength. In addition, regression analysis does not account for the uncertainty in the pullout strength. Thus the confidence level of the computed in-place characteristic strength is not entirely clear. To determine whether the proposed method is reliable, the characteristic strengths computed by this method were compared with the values obtained by the NBS Method.

For this comparison, four different correlation relationships were used along with different simulated in-place pullout test results. The four correlation relationships are for the test series described in Ref. [4]. Test series "G70" and "G54" refer to tests on concrete made with river gravel coarse aggregate and using apex angles of 70 and 54 degrees, respectively. Test series "LS" and "LW" refer to tests using crushed limestone and lightweight aggregate with a 70-degree apex angle.

Table 1 compares the results of the regression analyses of the correlation test data obtained with the approximate procedures proposed here. Note that the best-fit lines have standard errors ranging from 0.032 to 0.093. The reader will recall that the standard error is a measure of the average deviation of the data points about the regression line. Also note that for the correlation tests with lightweight aggregate concrete the pooled standard deviation of the logarithms of pullout strength was lower than for the concretes made with gravel and crushed limestone. The significance of this fact is discussed later.

Three nominal levels of simulated average in-place pullout strength were used (3000, 5000, and 8000 lbs) along with two levels of coefficient of variation (10 % and 20%). In addition, two cases were studied: in one case there were 10 in-place pullout tests, and in the second case there were 15 tests. For the lightweight concrete the 8000-lb pullout strengths were not used because they are greater than the maximum pullout strength obtained during the correlation testing. A correlation equation should not be used for extrapolating beyond the limits of the correlation testing program. Therefore, when developing the correlation curve, one should be certain to cover the complete range of strengths expected at the job site.

Table 2 summarizes the results of the calculations. Column 5 gives the average predicted cylinder strength corresponding to the average pullout strength based on the correlation curve obtained from the procedure presented here. Columns 6 and 7 give the estimated characteristic strengths based on the NBS Method and on the method proposed here. The characteristic strength for the NBS Method represents the lower 95% confidence limit. The last column in Table 2 shows the percent differences between the values in columns 7 and 6. The differences tend to be smaller when the pullout tests have greater variability. The average of the percent differences is -2.6%, which means that the proposed method results

in a characteristic strength which is, on the average, smaller than the lower 95% confidence limit of the NBS Method. This difference is on the conservative side and is considered to be acceptable for predicting in-place concrete strength.

Figure 10 shows the percentage differences given in Table 2 plotted as a function of the standard error of the correlation curves. It is seen that the number of in-place tests does not have a significant effect on the differences. However, the standard error of the correlation curve affects the difference between the two methods. With increasing standard error, the proposed method tends to be more conservative than the NBS Method. Note that the percent differences for the lightweight concrete series appear to deviate from the trend of the normalweight concretes. This is probably due to the lower coefficient of variation of the pullout strengths obtained during the correlation testing with the lightweight concrete. When the same hypothetical in-place pullout test data are used with the lightweight concrete, the predicted variability of the in-place cylinder strength is greater than with the normal weight concretes (see Eq. (5)), and this apparently reduces the difference between values obtained by the two methods.

In summary, it has been shown that the simplified method is reliable and gives values of in-place characteristic strength that are, on the average, slightly less than the lower 95% confidence limit obtained from the rigorous NBS Method. The degree of conservatism of the proposed method increases with increasing standard error of the correlation curve, and it decreases with increasing variability of the in-place strength. The proposed method is a significant improvement over the tolerance limit approach [9], which has been shown to be overly conservative [4].

SUMMARY

The use of in-place testing to assess concrete strength during the application of critical construction loads requires a previously established relationship between cylinder strength and the quantity measured by the in-place test. In addition, the in-place test results need to be evaluated in a manner that assures high confidence in the estimate of the in-place characteristic strength. It is proposed that the evaluation of in-place test results should account for the following major sources of variability or uncertainty:

- 1) the variability of in-place concrete strength;
- 2) the uncertainty in the sample average of the in-place test results; and
- 3) the uncertainty in the correlation relationship derived from a limited set of laboratory tests.

This paper has described how a personal computer, combined with available spreadsheet software, can be used to automate the computations required for establishing the correlation curve. A

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procedure is proposed for making statistically valid estimates of in-place characteristic strength. The proposed method, while simple to implement, has been shown to give strength estimates similar to those of a more rigorous analysis. The rigorous analysis, being more involved, is better suited for coding with a computer language such as FORTRAN or BASIC. Efforts are underway to prepare an interactive computer program¹ which is based on the rigorous analysis and which will assist the user in interpreting in-place test results.

To use a spreadsheet program for the data analysis procedure described in this paper, the user must first prepare a "template" spreadsheet containing blank cells for data input and formulas for the required computations. A detailed procedure for preparing the template spreadsheet used in this study is available on request from the first author². After it is prepared, the template spreadsheet is stored as a file on a disk.

When a correlation relationship needs to be determined, the template spreadsheet is retrieved and the test data from the correlation testing program are entered into the appropriate blank cells. After all data are entered, the formulas are evaluated and the results are displayed. The completed spreadsheet may be stored as another file, thereby creating a complete record of the data and the correlation relationship. The template spreadsheet remains available for analysis of other correlation data.

After in-place tests are performed, the completed spreadsheet, which is stored on disk, is retrieved and the test results are entered into the appropriate cells. The user also enters the Student t-value corresponding to the number of tests. The program computes the in-place characteristic strength according to the proposed method. This value can then be compared with the strength required for safe application of construction loads.

By using spreadsheet software for this type of data analysis, one may take advantage of the speed and consistency of automated computation with no background in computer programming. A further benefit of automated analysis is that it promotes uniform data analysis among different organizations.

¹Stone, W. C., "NBS-INSITU: An Interactive Microcomputer Program for Analysis of In-Place Test Data," Manuscript in preparation

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ACKNOWLEDGEMENTS

The authors extend their appreciation to Charles P. Reeve of the Statistical Engineering Division of the National Bureau of Standards for his role in developing procedures for the rigorous analysis of correlation data for in-place tests. Having this rigorous method enabled the development of the simplified procedure described in the paper. The contribution of Bruce J. Giza in obtaining the test data used in this paper is also acknowledged.

REFERENCES

1. "Standard Method of Making and Curing Concrete Test Specimens in the Field," ASTM C 31-84, 1985 Annual Book of ASTM Standards, Vol. 04.02, American Society for Testing and Materials, Philadelphia, PA.
2. "Building Code Requirements for Reinforced Concrete," ACI 318-83, American Concrete Institute, Detroit, MI.
3. "Standard Test Method for Pullout Strength of Hardened Concrete," ASTM C 900-82, 1985 Annual Book of ASTM Standards, Vol. 04.02, American Society for Testing and Materials, Philadelphia, PA.
4. Stone, W.C., Carino, N.J. and Reeve, C.P., "Statistical Methods for In-Place Strength Predictions by the Pullout Test," To be published in the ACI Journal, 1986.
5. Ku, H.H., "Notes on the Use of Propagation of Error Formulas," in Precision Measurement and Calibration - Statistical Concepts and Procedures, National Bureau of Standards SP 300, Vol. 1, pp 331-341.
6. Mirza, S.A., Hatzinikolas, M. and MacGregor, J.G., "Statistical Descriptions of Strength of Concrete," Journal of the Structural Division, ASCE, Vol. 105, No. ST6, June 1979, pp 1021-1037.
7. Natrella, M.G., Experimental Statistics, National Bureau of Standards Handbook 91, 1963.
8. Arni, H.T., "The Significance of the Correlation Coefficient for Analyzing Engineering Data," Materials Research Standards, Vol. 11, No.5, May 1971, pp 16-18.
9. Hindo, K. R. and Bergstrom, W. R., "Statistical Evaluation of the In-Place Strength of Concrete," Concrete International, Vol. 7, No. 2, February 1985, pp 44-48.

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10. Carrete, G.G. and Malhotra, V.M., "In-Situ Tests: Variability and Strength Prediction of Concrete at Early Ages," ACI Special Publication SP-82, In Situ/Nondestructive Testing of Concrete, American Concrete Institute, Detroit, 1984, pp 111-141.
11. Khoo, L.M., "Pullout Technique - An Additional Tool for In Situ Concrete Strength Determination," ACI Special Publication SP-82, In Situ/Nondestructive Testing of Concrete, American Concrete Institute, Detroit, 1984, pp 143-159.
12. Moore, J.K. and Taylor, M.A., "Statistical Properties of Techniques for Predicting Concrete Strength and Examples of Their Use," ACI Special Publication SP-56, Accelerated Strength Testing, American Concrete Institute, Detroit, 1978, pp 259-283.
13. Stone, W.C. and Reeve, C.P., "A New Statistical Method for Prediction of Concrete Strength from In-Place Tests," ASTM Journal of Cement, Concrete, and Aggregates, Summer 1986, Vol. 8, No. 1, pp 3-12.

TABLE 1

Comparison of Regression Analyses of Data
From Correlation Testing Program [4]

SERIES	AGGREGATE TYPE	APEX ANGLE	POOLED Ln(C)	S.D. Ln(P)	REGRESSION COEFFICIENTS		STANDARD ERROR
					m	B	
G70	River Gravel	70	0.037	0.112	1.021	0.606	0.0546
G54	River Gravel	54	0.037	0.093	1.024	0.474	0.0931
LS	Crushed Limestone	70	0.045	0.096	0.860	2.239	0.0321
LW	Exp. Shale Lightweight	70	0.051	0.064	1.095	0.342	0.0531

TABLE 2
Comparison of In-Place Characteristic Strengths
Using Proposed Method and NBS Method [13]

IN-PLACE PULLOUT TESTS				CHARACTERISTIC STRENGTH			
TEST	No PO	AVG. LOAD	C.V.	PRED	NBS METHOD	PROPOSED	% DIFF
SERIES	TESTS	(1b)	(%)	AVG STR	95% Prob	METHOD	(7-6)/6
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
G70	10	3010	10.9	2140	1880	1810	-3.7
G70	10	3030	20.7	2130	1700	1650	-2.9
G70	10	4970	11.9	3590	3130	3060	-2.2
G70	10	5060	20.3	3610	2890	2860	-1.0
G70	10	8120	9.9	5960	5260	5040	-4.2
G70	10	8110	21.9	5820	4400	4320	-1.8
G54	10	3010	10.9	1690	1420	1320	-7.0
G54	10	3030	20.7	1680	1300	1200	-7.7
G54	10	4970	11.9	2860	2450	2330	-4.9
G54	10	5060	20.3	2880	2270	2170	-4.4
G54	10	8120	9.9	4790	4160	3930	-5.5
G54	10	8110	21.9	4680	3500	3360	-4.0
LS	10	3010	10.9	2170	1890	1850	-2.1
LS	10	3030	20.7	2160	1680	1670	-0.6
LS	10	4970	11.9	3350	2880	2870	-0.3
LS	10	5060	20.3	3370	2620	2660	1.5
LS	10	8120	9.9	5120	4510	4440	-1.6
LS	10	8110	21.9	5020	3680	3750	1.9
LW	10	3010	10.9	2180	1760	1720	-2.3
LW	10	3030	20.7	2160	1470	1470	0.0
LW	10	4970	11.9	3780	2990	2970	-0.7
LW	10	5060	20.3	3810	2590	2640	1.9
G70	15	3070	10.1	2200	1950	1880	-3.6
G70	15	3005	19.6	2120	1760	1690	-4.0
G70	15	4970	9.1	3600	3270	3180	-2.8
G70	15	5180	19.8	3690	3030	2980	-1.7
G70	15	7990	9.7	5840	5240	5020	-4.2
G70	15	8430	19.8	6080	4980	4800	-3.6
G54	15	3070	10.1	1760	1460	1380	-5.5
G54	15	3005	19.6	1700	1330	1230	-7.5
G54	15	4970	9.1	2880	2540	2440	-3.9
G54	15	5180	19.8	2960	2380	2270	-4.6
G54	15	7990	9.7	4680	4140	3910	-5.6
G54	15	8430	19.8	4870	3950	3730	-5.6
LS	15	3070	10.1	2220	1960	1910	-2.6
LS	15	3005	19.6	2150	1740	1710	-1.7
LS	15	4970	9.1	3360	3030	2990	-1.3
LS	15	5180	19.8	3430	2730	2750	0.7
LS	15	7990	9.7	5040	4510	4430	-1.8
LS	15	8430	19.8	5220	4160	4150	-0.2
LW	15	3070	10.1	2230	1840	1800	-2.2
LW	15	3005	19.6	2150	1540	1530	-0.6
LW	15	4970	9.1	3790	3220	3150	-2.2
LW	15	5180	19.8	3900	2730	2750	0.7

APPENDIX A

PROCEDURE FOR LINEAR REGRESSION

Definition of Variables

X_i = Average of natural logarithms of Pullout Load in Group i

Y_i = Average of natural logarithms of Cylinder Strength in Group i

n = Number of data groups

Procedure

1. List the X-values and Y-values and compute their products.
Compute the sums of the columns.

X_1	Y_1	$(X_1)^2$	$(Y_1)^2$	$X_1 Y_1$
X_2	Y_2	$(X_2)^2$	$(Y_2)^2$	$X_2 Y_2$
X_3	Y_3	$(X_3)^2$	$(Y_3)^2$	$X_3 Y_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
X_n	Y_n	$(X_n)^2$	$(Y_n)^2$	$X_n Y_n$
$\hline \Sigma X$	$\hline \Sigma Y$	$\hline \Sigma X^2$	$\hline \Sigma Y^2$	$\hline \Sigma XY$

2. Compute the average of X-values and Y-values.

$$X_a = \frac{\Sigma X}{n}$$

$$Y_a = \frac{\Sigma Y}{n}$$

3. Compute the following sum of squares.

$$S_{xx} = \frac{\Sigma X^2 - (\Sigma X)^2}{n}$$

$$S_{yy} = \frac{\Sigma Y^2 - (\Sigma Y)^2}{n}$$

$$S_{xy} = \frac{\Sigma XY - \Sigma X \cdot \Sigma Y}{n}$$

4. Compute the slope of the line.

$$m = \frac{S_{xy}}{S_{xx}}$$

5. Compute the intercept of the line.

$$b = Y_a - m X_a$$

6. Compute the sum of the squares of the residuals of Y-values from the best-fit line.

$$SSR = S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$$

7. Compute the standard error of Y-values about the best-fit line.

$$SE_y = \sqrt{\frac{SSR}{n-2}}$$

8. Compute the standard error of the slope.

$$SE_m = \frac{SE_y}{\sqrt{S_{xx}}}$$

9. Compute the standard error of the intercept.

$$SE_b = SE_y \sqrt{\frac{1}{n} + \frac{(X_a)^2}{S_{xx}}}$$

10. Compute the correlation coefficient.

$$r = m \sqrt{\frac{S_{xx}}{S_{yy}}}$$

11. Compute the lower confidence limit (Y_{low}) for the estimated value of Y (Y_{est}) for any new value of X.

$$Y_{low} = (b + m X) - \sqrt{2F} SE_y \sqrt{\frac{1}{n} + \frac{(X - X_a)^2}{S_{xx}}}$$

where F = value from F-distribution at the desired confidence level and for (2,n-2) degrees of freedom

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	A	B	C	D	E	F	G
1	TEST	LOAD 1	LOAD 2	STRESS 1	STRESS 2	AVG	3-AVG
2	No.	(lbs)	(lbs)	(psi)	(psi)	(psi)	(psi)
3							
4	1	119220	112000	4217	3962	4089	
5	2	119480	138470	4226	4898	4562	
6	3	113910	111150	4029	3932	3981	4211
7	4	103140	128090	3648	4531	4090	4211
8	5	161100	110190	5699	3898	4798	4289
9	6	137740	130730	4872	4624	4748	4545
10	7	141850	132220	5018	4677	4847	4798
11	8	130530	121260	4617	4289	4453	4683
12	9	136620	107780	4833	3813	4323	4541
13	10	136870	136720	4842	4836	4839	4538
14	11	110800	106140	3919	3755	3837	4333
15	12	150090	126900	5309	4489	4899	4525
16	13	147530	131250	5219	4643	4931	4556
17	14	136070	129450	4813	4579	4696	4842
18	15	139570	130920	4937	4631	4784	4804
19	16	129300	119730	4574	4235	4404	4628
20	17	135060	130611	4778	4620	4699	4629
21	18	135290	122850	4786	4346	4566	4556
22	19	105380	123250	3728	4360	4044	4436
23	20	129980	125620	4598	4444	4521	4377
24	21	143500	144900	5076	5126	5101	4555
25	22	113403	129660	4011	4586	4299	4640
26	23	138670	134550	4905	4759	4832	4744
27	24	118040	128050	4175	4530	4352	4495
28	25	129350	133621	4576	4727	4651	4612
29	26	141400	121600	5002	4301	4652	4552
30	27	130180	129050	4605	4565	4585	4629
31	28	127450	124340	4508	4398	4453	4563
32	29	140930	128800	4985	4556	4771	4603
33	30	133150	121520	4710	4299	4504	4576

Fig. 1--Example of spreadsheet used to analyze cylinder test results

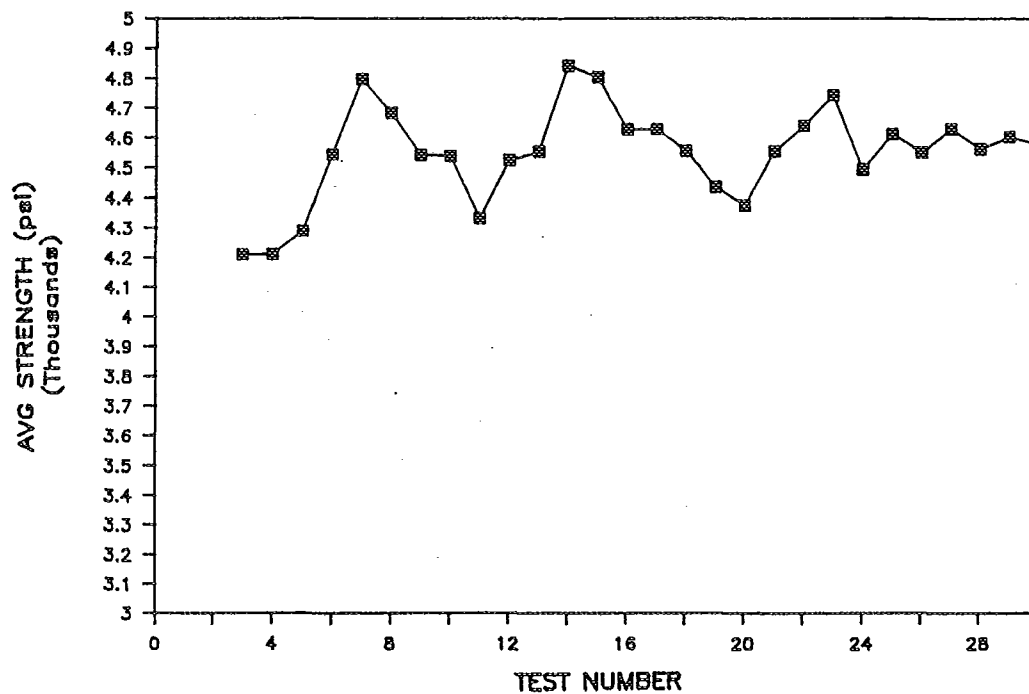


Fig. 2--Example of plot generated by spreadsheet program

CORRELATION DATA								
PULLOUT LOAD (lb)								
1	2580	5772	2892	6392	3940	6647	4022	6971
2	1876	6110	2646	7702	3703	8679	4093	8177
3	2124	6387	2728	6705	3632	7385	3566	8820
4	1988	5227	2754	5388	3685	7351	4178	9027
5	2309	6341	2717	5904	3282	7530	4890	8983
6	2164	4621	2647	5213	3727	6498	4741	5411
7	2164	6663	2783	8268	4218	8057	4694	8163
8	2074	6466	2675	6987	3385	8804	4528	8421
9	2415	6179	2970	7683	3706	7989	4828	9091
10	2481	6830	2792	7330	3954	7846	5042	8872
11	1864	6476	2506	6871	3837	7473	4779	7064
n=	11	11	11	11	11	11	11	11
AVG=	2185	6097	2737	6768	3734	7660	4487	8091
SSR=	566171	4329075	157810	9548259	676403	5246931	2096207	13426171
S.D.=	238	658	126	977	260	724	458	1159
C.V.=	0.109	0.108	0.046	0.144	0.070	0.095	0.102	0.143
CYLINDER STRENGTH (psi)								
1	1610	4090	2110	4580	2705	6070	3295	6335
2	1550	4060	1925	4605	2850	5490	3355	6040
3	1495	4135	2150	4615	2540	5530	3270	6370
4	1430	4330	2020	4560	2660	6120	3335	6375
5	1460	3940	2090	4600	2800	5830	3295	6000
n=	5	5	5	5	5	5	5	5
AVG=	1509	4111	2059	4592	2711	5808	3310	6224
SSR=	20720	80820	31320	1930	59120	344880	4700	140470
S.D.=	72	142	88	22	122	294	34	187
C.V.=	0.048	0.035	0.043	0.005	0.045	0.051	0.010	0.030

Fig. 3--Portion of the spreadsheet used for entering data from correlation testing program

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TRANSFORMED CORRELATION DATA								
NATURAL LOG OF PULLOUT LOAD								
1	7.856	8.661	7.970	8.763	8.279	8.802	8.300	8.850
2	7.537	8.718	7.881	8.949	8.217	9.069	8.317	9.009
3	7.661	8.762	7.911	8.811	8.198	8.907	8.179	9.085
4	7.595	8.562	7.921	8.592	8.212	8.903	8.338	9.108
5	7.745	8.755	7.907	8.683	8.096	8.927	8.495	9.103
6	7.680	8.438	7.881	8.559	8.223	8.779	8.464	8.596
7	7.680	8.804	7.931	9.020	8.347	8.994	8.454	9.007
8	7.637	8.774	7.892	8.852	8.127	9.083	8.418	9.038
9	7.789	8.729	7.996	8.947	8.218	8.986	8.482	9.115
10	7.816	8.829	7.935	8.900	8.282	8.968	8.526	9.091
11	7.530	8.776	7.826	8.835	8.252	8.919	8.472	8.863
n=	11	11	11	11	11	11	11	11
AVG=	7.684	8.710	7.914	8.810	8.223	8.940	8.404	8.988
SSR=	0.118	0.135	0.021	0.222	0.049	0.091	0.114	0.255
S.D.=	0.108	0.116	0.046	0.149	0.070	0.095	0.107	0.160
POOLED SD=	0.112							
NATURAL LOG OF CYLINDER STRENGTH								
1	7.384	8.316	7.654	8.429	7.903	8.711	8.100	8.754
2	7.346	8.309	7.563	8.435	7.955	8.611	8.118	8.706
3	7.310	8.327	7.673	8.437	7.840	8.618	8.093	8.759
4	7.265	8.373	7.611	8.425	7.886	8.719	8.112	8.760
5	7.286	8.279	7.645	8.434	7.937	8.671	8.100	8.700
n=	5	5	5	5	5	5	5	5
AVG=	7.318	8.321	7.629	8.432	7.904	8.666	8.105	8.736
SSR=	0.009	0.005	0.008	0.000	0.008	0.010	0.000	0.004
S.D.=	0.047	0.034	0.044	0.005	0.045	0.051	0.010	0.030
POOLED SD=	0.037							

Fig. 4--Portion of the spreadsheet used to calculate the natural logarithms of the test data shown in Fig. 3

LINEAR REGRESSION ANALYSIS

Equation: CYL(psi)= 0.606 PO(lb)** 1.021

N=	8	AVGX=	8.459	AVGY=	8.139	
SUMX=	67.672	SUMY=	65.111	SUMXY=	552.465	
SUMX*X=	574.091	SUMY*Y=	531.677	SUMX*Y=	552.465	*****
SXY=	1.688	SXX=	1.653	SYX=	1.743	ALPHA= 0.10
SLOPE=	1.021	INTCPT=	-0.501	SUM(SR)=	0.018	F(2,N-2)= 3.460
STERROR=	0.055	ERRSLOP=	0.042	ERRINT=	0.360	*****
CORRCOEF=	0.995					

X	Y	X*X	Y*Y	X*Y	Y-est	Y-low
7.684	7.318	59.047	53.558	56.235	7.347	7.247
7.914	7.629	62.628	58.205	60.376	7.582	7.503
8.223	7.904	67.616	62.477	64.996	7.898	7.840
8.404	8.105	70.627	65.686	68.112	8.083	8.032
8.710	8.321	75.860	69.238	72.474	8.395	8.337
8.810	8.432	77.617	71.100	74.287	8.497	8.433
8.940	8.666	79.917	75.099	77.471	8.630	8.556
8.988	8.736	80.779	76.314	78.515	8.679	8.601

DATA TO PLOT CORRELATION CURVE AND LOWER LIMIT

Ln(P)	Ln(C)	Ln(C) est	Ln(C) low	P	C	Cest	C low
				lbs	psi	psi	psi
7.684	7.318	7.347	7.247	2174	1508	1552	1404
7.914	7.629	7.582	7.503	2735	2057	1963	1813
8.223	7.904	7.898	7.840	3725	2709	2691	2541
8.404	8.105	8.083	8.032	4465	3310	3238	3076
8.710	8.321	8.395	8.337	6062	4109	4425	4176
8.810	8.432	8.497	8.433	6701	4592	4902	4597
8.940	8.666	8.630	8.556	7629	5802	5596	5197
8.988	8.736	8.679	8.601	8004	6222	5878	5437

Fig. 5--Portion of the spreadsheet used to perform regression analysis and provide values for plotting the correlation curve and its lower confidence limit.

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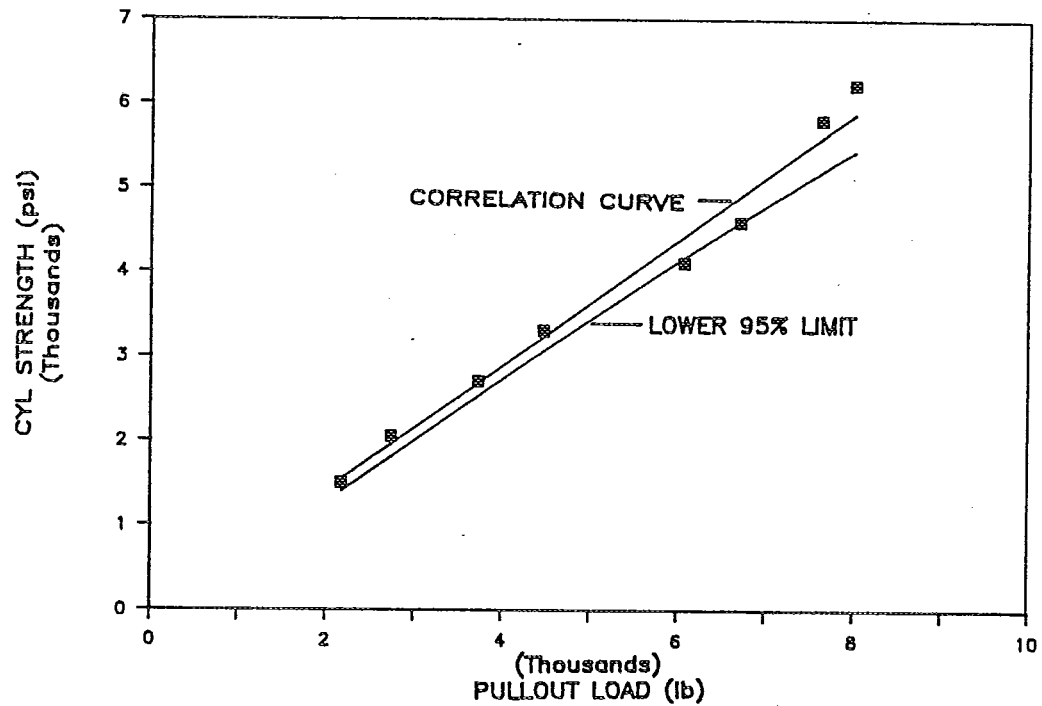


Fig. 6--Plot of data, the correlation curve, and the lower confidence limit

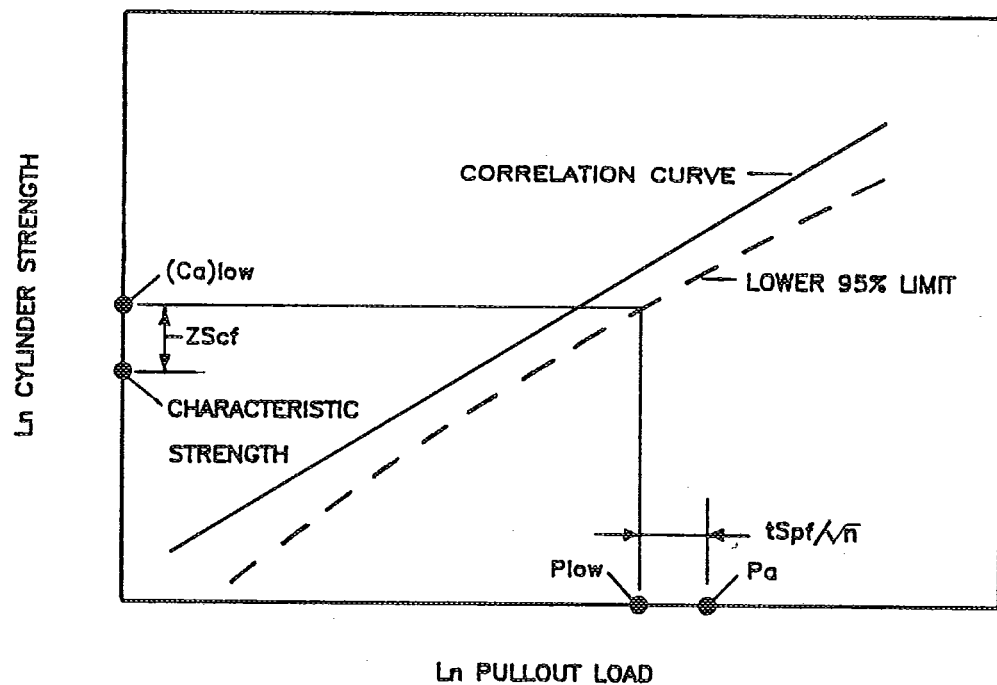


Fig. 7--Schematic illustration of method for estimating the in-place characteristic strength based on pullout tests

INPLACE STRENGTH PREDICTION				

INPLACE				
PO LOAD	LN (PO)			
(lbs)				
1	2830	7.948	STAND. DEVIATIONS OF TRANSFORMED DATA	
2	2770	7.927	SD(P1)=	0.112 SD(Pf)= 0.208
3	3460	8.149	SD(C1)=	0.037
4	2670	7.890	INPLACE ESTIMATES	
5	2640	7.879	SD(CF)=	0.069
6	3890	8.266	Ln(Ca)=	7.668
7	3680	8.211	Ca=	2140 psi
8	2150	7.673	Ln(P)low=	7.907
9	2410	7.787	Ln(C)low=	7.496
10	3840	8.253	Clow=	1800 psi
-----		Ln(CS)=	7.408	
N=	10	10	*****	
AVG=	3034	7.998	ALPHA= 0.1	
SD=	628	0.208	*CHR STR=	1650 psi
CV=	0.207		*****	
			* t(N-1)=	1.383

Fig. 8--Portion of the spreadsheet for entering in-place test results and computing the in-place characteristic cylinder strength

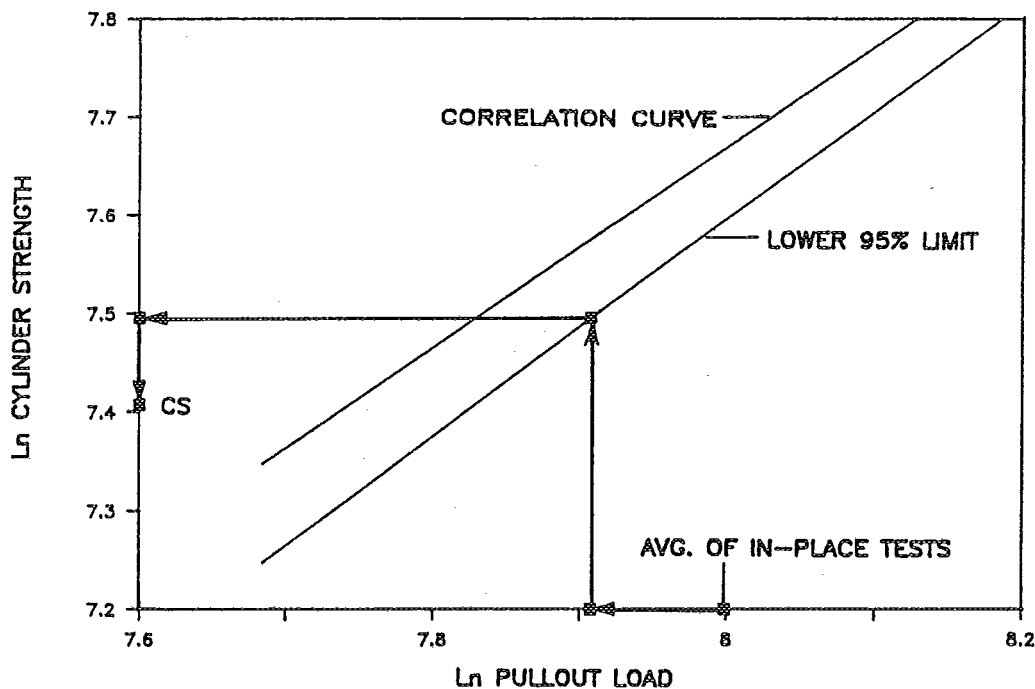


Fig. 9--Illustration of how characteristic strength in Fig. 8 was obtained

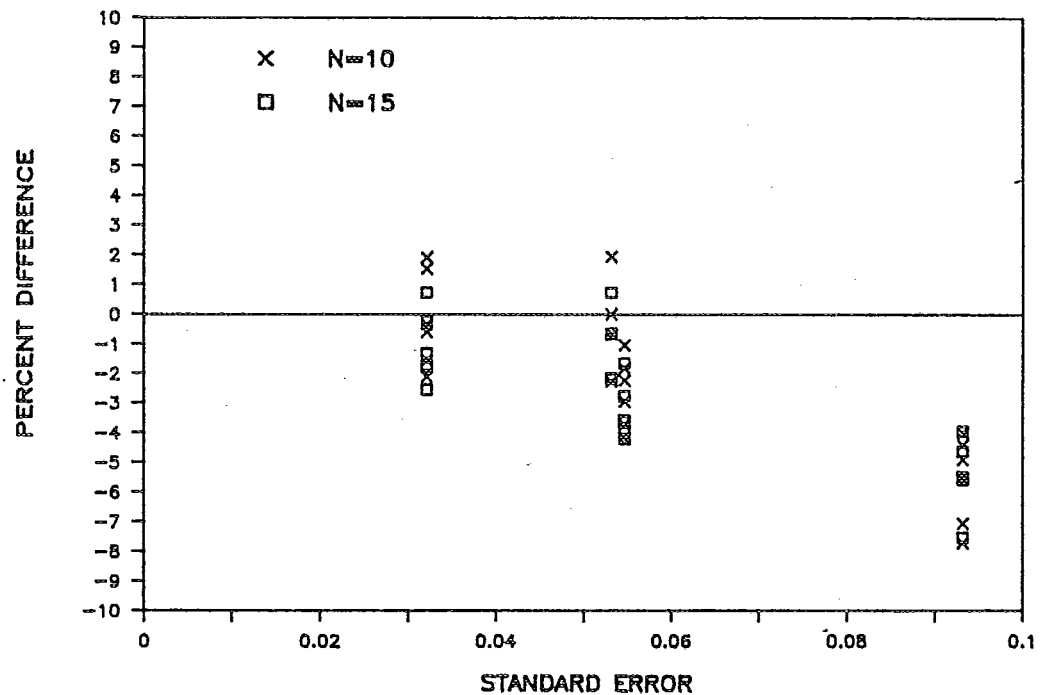


Fig. 10--Effect of the standard error of the correlation relationship on the difference between characteristic strengths computed by the proposed and the rigorous methods

SP 101-2

Analysis of Maturity/Pullout Testing Data

by R. Dilly, V. Beizai, and W. Vogt

Synopsis: Pullout testing provides a measurement of in-place strength of concrete and maturity can be used to estimate strength development. Test statistics are useful tools for interpreting the significance of the data. Maturity may correlate to pullout strength by regression analysis. The simplest correlation would be linear using the logarithm of maturity. Linear relationships may be compared using test statistics to determine if slopes and/or intercepts are significantly different. Statistical analysis techniques are used to evaluate pullout strength and maturity data. Some limitations associated with these techniques are discussed. A programmable calculator was utilized in developing the programs to illustrate the application of these analysis techniques.

Keywords: age-strength relation; coefficient of variation; computer programs; concretes; evaluation; nondestructive tests; pullout tests; regression analysis; standard deviation; statistical analysis

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INTRODUCTION

The pullout test provides a method of determining in-place strength of concrete. The maturity method can be used to estimate concrete strength development. These methods may be combined and used when construction activities require monitoring of strength development. Statistical analysis can be used to interpret test data acquired with these methods and to determine if significant differences occur between test ages, test locations, or curing regime.

BACKGROUND

The maturity concept is reported to have been presented by Saul(1) in 1951 as an empirical approach for predicting concrete strength development based upon time-temperature histories(2).

The time-temperature curing history of a concrete mixture can be used to calculate maturity by:

$$M = \Sigma(T - T_0) \Delta t \dots (1)$$

where M is the maturity of the concrete, T is the average concrete temperature where the change occurs

at an approximately constant rate for an increment of time, Δt ; and T_0 is the datum temperature at which or below concrete ceases to gain strength with time.

This concept postulates that strengths will be approximately equal at the same maturity. However, the results of an experimental investigation conducted by Carino, Lew, and Volz(3) confirmed that different early age temperatures can affect the strength-maturity relationship, as recognized earlier by McIntosh(4) and Klieger(5).

The maturity concept has been investigated by a number of other researchers. Strength prediction equations using the logarithm of maturity for accelerated curing methods were developed by Ramakrishnan and Dietz(6) using a linear form of an equation originally suggested by Plowman(7). Nonlinear regression analysis has been used by researchers to develop compressive strength relationships with maturity. An experimental investigation by Naik(8) has shown that a predetermined strength/maturity relationship for a concrete mixture can be used with about the same degree of accuracy as a core strength. ASTM C 918 is a standard method for estimating later-age strengths using early-age compression test values with maturity calculations(9). A datum temperature is not used in this method. ACI 306 has recognized the use of the maturity concept(10).

Malhotra(11) and Richards(12) undertook many of the initial experimental investigations of pullout testing conducted in North America. The pullout test is a method of determining the in-place strength of concrete. A standard method is provided by ASTM C 900(13). Bickley has reported on the use of pullout testing to evaluate in-place strength during construction(14). Vogt, Beizai, and Dilly(15) reported the results of pullout testing conducted on a project in which ACI 214(16) statistical methods were used to report the data, although these methods were developed exclusively for compressive strength evaluation.

The pullout strength of concrete is determined by measuring the force required to extract an embedded insert from hardened concrete and dividing by the surface area generated from fracturing the concrete. Relationships have been developed between pullout strength and concrete maturity to evaluate in-place concrete strength. Maturity-pullout relationships have been reported by Parsons and Naik(17), and Dilly and Ledbetter(18).

APPLICATION OF STATISTICS FOR INTERPRETATION OF TEST RESULTS

Single variable tests can be used to estimate if significant differences exist between different test locations or test ages. Multivariable methods, such as linear regression analysis, can be used to estimate if significant differences exist between different curing regimes.

Two HP, Hewlett-Packard 41C/41CV programs were developed for statistical analysis of test results. Each program utilizes the calculator's statistical functions. The first program designated "ST" provides an analysis of two test sample means, while the second program, "LT" performs a least square regression multiple variable analysis of slopes and intercepts of two linear relationships. "ST" includes a test statistics for significant differences between sample means, and "LT" includes test statistics for significant differences between the slopes and the intercepts of least square regression relationships. Reference 19 was used to develop program "ST", and references 20 and 21 were used to develop program "LT". Flowcharts, user's manuals, and program listings appear in the Appendix.

Two examples are discussed, illustrating the use of these programs. Table 1 is a summary of test statistics used in "ST", and Table 2 is a summary of test statistics used in "LT". The data in the examples was the result of an experimental investigation reported by references 18 and 22. Both of these refer to the same experimental investigation in which a hydraulic centerpull ram pullout testing apparatus was used to estimate the in-place strength of a 6.0 ft (1.8 m) tall concrete column. Sets of inserts were placed at the base and top of the column at a vertical spacing of 5.0 ft (1.5 m). When inserts were extracted, the resulting approximate conic frustums had heights of 1.8 in (30 mm) and base diameters of 2.76 in (70 mm) and 1.8 in (30 mm). The apex angle for the assumed conic frustum is 67.4 degrees, resulting in a surface area of 8.79 in² (56.6 cm²).

The pullout strengths which appear in the first example were obtained by dividing the force on the ram at failure by the assumed conic frustum area. The average pullout strengths which appear in the second example were obtained by averaging three pullout strengths determined in the same manner.

Example using program "ST"

At an age of 28 days, the following pullout strengths resulted from tests conducted on the column:

fp-Individual pullout strength, psi	
<u>Top of column</u>	<u>Base of column</u>
376	681
376	704
400	634

Note: 1 psi = 6897 Pa

Program "ST" performs an analysis of this limited data. The input and output shown in Figure 1 indicate that for set 1 (top of column), the average is 384 psi, the standard deviation is 13.9 psi, and the coefficient of variation is 3.6%. For set 2 (base of column), the average is 673 psi, the standard deviation is 36 psi, and the coefficient of variation is 5.3%. "F" calculated is 6.63, and "F" tabulated is 19.00 at the 5% significance level. "F" calculated is less than "F" tabulated, so the calculated "t" test statistics value of 13.08 applies. For this example, the two "t" test statistics values are the same due to the equal number of observations from each set. At a significance level of 5%, "t" tabulated is 2.13, thus a significant difference exists between top and bottom average pullout strength. Refer to reference 19 for details on these statistical testing concepts.

Example using program "LT"

At ages 3, 7, 14, and 28 days, the average pullout strengths shown below resulted from tests conducted on a column. The values of logarithm of maturity shown on each of the following test ages was calculated using Equation (1), where 14°F (-10°C) was used as the datum temperature, T_o .

Test age days	Log of maturity °F-hrs	fp-average of three pullout strength tests	
		<u>Top of column</u>	<u>Base of column</u>
3	3.8104	224	490
7	4.1671	290	548
14	4.4476	361	575
28	4.7249	384	673

Note: $\text{Log of maturity}(\text{°F-hrs}) = 0.2553 + \text{Log of maturity}(\text{°C-hrs})$

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Program "LT" provides an analysis for this limited data. The linear regression expression corresponding to the output would have the general form:

$$fp = a + b \times \text{Log of maturity} \dots\dots(2)$$

where "a" is the intercept and "b" is the slope.

The input and output shown in Figure 2 indicate that for set 1 (top of column), the intercept is -470 psi, the slope is 183, the standard error of the estimate squared is 212, and the coefficient of determination is 0.97. For set 2 (base of column), the intercept is -238 psi, the slope is 189, the standard error of the estimate squared is 573, and the coefficient of determination is 0.93. "F" calculated is 2.71 and "F" tabulated is 19.00 at the 5% significance level. "F" calculated is less than "F" tabulated, so the test statistics for slope and intercept apply. The "t" calculated values are 0.14 for the slope, and 1.30 for the intercept. At a significance level of 5%, which is generally used, "t" tabulated is 2.13 suggesting that there is no significant difference between the slopes or the intercepts, and the data may be pooled. If this pooling was done, the accuracy of prediction would be expected to decrease. If the sample sizes had been larger, significant differences may have been more apparent for the intercepts.

DISCUSSION AND CONCLUSIONS

A requirement for the statistics incorporated in these programs is for the data to be acquired randomly. Pullout testing of in-place concrete using preplaced inserts may not represent a random sample due to factors such as installation constraints. Even though this requirement may not be satisfied for single variable analysis, two-sample tests of means may be used to distinguish whether significant differences are apparent between concrete strength at different locations. Correct interpretation of in-place strength differences can influence project safety and the construction schedule. These differences may be attributed to many factors, and to be properly evaluated, the influence of top-to-bottom effects may have to be considered.

Linear regression analysis test statistics for comparing slopes and intercepts requires assumptions in addition to random sampling. These additional

assumptions are based upon conditional distributions of the dependent variable, Y , on the regression line. For pullout strength vs. logarithm of maturity relationship, these other assumptions as well as the assumption of randomly obtained observations, may not be satisfied. With an accurate temperature recorder for maturity data collection, an additional assumption that X values are known without error may be nearly satisfied. Even though all assumptions may not be met, the logarithm of maturity may still be used in reducing errors of predicting required pullout strength and in determining whether significant differences are apparent between slopes and/or intercepts due to variations in concrete mixture and curing conditions.

The results of the logarithm of maturity relationships established in the second example were not significantly different. However, the resulting general trends established with limited observations indicate that top and bottom differences are apparent and perhaps the vertical locations of embedded inserts should be considered in order to reduce variations of f_c about the best fit line established with logarithm of maturity.

Although not specifically stated in this paper, larger sample sizes improve estimates of the mean pullout strength and pullout strength vs. logarithm of maturity relationships. Significant differences may not be detected unless larger sample sizes are used.

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REFERENCES

1. Saul, A.G.A., "Principles Underlying the Steam Curing of Concrete at Atmospheric Pressure," Magazine of Concrete Research (London), V. 2, No. 6, March 1951, pp. 127-140.
2. Carino, N. J., and Lew, H. S., "Temperature Effects on Strength-Maturity Relations of Mortar," ACI Journal, Proceedings, V. 80, No. 3, May-June 1983, pp. 177-182.
3. Carino, N. J., Lew, H. S., and Volz, C. K., "Early Age Temperature Effects on Concrete Strength Prediction by the Maturity Method," ACI Journal, Proceedings, V. 80, No. 2, March-April 1983, pp. 93-101.
4. McIntosh, J. D., "The Effect of Low-Temperature Curing on the Compressive Strength of Concrete," Proceedings, RILEM Symposium on Winter Concreting (Copenhagen, 1956), Danish National Institute for Building Research, Copenhagen, 1956, 17 pp.
5. Klieger, Paul, "Effect of Mixing and Curing Temperature on Concrete Strength," ACI Journal, Proceedings, V. 54, No. 12, June 1958, pp. 1063-1081.
6. Ramakrishnan, V., and Dietz, J., "Accelerated Methods of Estimating the Strength of Concrete," Transportation Research Record, No. 558, 1975, pp. 29-44.
7. Plowman, J. M., "Maturity and Strength of Concrete," Magazine of Concrete Research, Vol. 8, No. 22, March 1956, pp. 13-32.
8. Naik, T.R., "Concrete Strength Prediction by the Maturity Method," ASCE Journal of Engineering Mechanics Division, EM3, June 1980.
9. "Standard Method for Developing Early Age Compression Test Values and Projecting Later Age Strengths," (ASTM C918-80), 1985 Annual Book of ASTM Standards, Section 4, Volume 04.02, ASTM, Philadelphia, pp. 586-591.
10. ACI Committee 306, "Cold Weather Concreting," (ACI 306R-78), American Concrete Institute, Detroit, 1978, pp. 16-20.

11. Malhotra, V. M., Testing Hardened Concrete: Non-Destructive Methods, ACI Monograph No. 9, Iowa State University Press, 1976.
12. Richards, O., "Pullout Strength of Concrete," Reproducibility and Accuracy of Mechanical Tests, STP-626, ASTM, 1979, pp. 32-40.
13. "Standard Test Method for Pullout Strength of Hardened Concrete," 1984 Annual Book of ASTM Standards, Section 4, Volume 04.02, ASTM, Philadelphia, pp. 582-585.
14. Bickley, J. A., "The Variability of Pullout Tests and In-Place Concrete Strength," Concrete International, Vol. 4, No. 4, April 1982, pp. 44-51.
15. Vogt, W. L., Beizai, V., and Dilly, R. L., "In Situ Pullout Strength of Concrete with Inserts Embedded by "Finger Placing," In Situ/Nondestructive Testing of Concrete, ACI Publication SP-82, American Concrete Institute, Detroit, 1984, pp. 161-175.
16. ACI Committee 214, "Recommended Practice for Evaluation of Strength Test Results of Concrete," (ACI 214-77), ACI Manual of Concrete Practice 1983, Part 1, American Concrete Institute, Detroit.
17. Parsons, T. J., and Naik, T. R., "Early Age Concrete Strength Determination by Pullout Testing and Maturity," In Situ/Nondestructive Testing of Concrete, ACI Publication SP-82, American Concrete Institute, Detroit, 1984, pp. 177-199.
18. Dilly, R. L., and Ledbetter, W. B., "Concrete Strength Based on Maturity and Pullout," Journal of Structural Engineering, ASCE, Vol. 110, No. 2, February 1984, pp. 354-369.
19. Hawkins, Clark A. and Jean E. Weber, Statistical Analysis: Applications to Business and Economics. New York: Harper & Row, 1980, pp. 234-239, 250-252.
20. Summers, George W., Peters, William S., and Charles P. Armstrong, Basic Statistics in Business and Economics, California: Wadsworth Publishing, 1981, pp. 281-318.

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21. Kleinbaum, David G. and Lawrence L. Kupper, Applied Regression Analysis and Other Multivariable Methods, Massachusetts: Duxbury Press, 1980, pp. 95-106.
22. Dilly, R. L., "The Use of the Maturity Concept in Evaluating Development of Concrete Pullout Strength," Thesis for a Master of Science in Civil Engineering, Texas A & M University.

TABLE 1--SINGLE VARIABLE ANALYSES (TESTS CONCERNING MEANS)

Function	Standard symbol	Display symbol	Expression
Average of X values	$\bar{X}$	XM^*	$\frac{\sum X}{n}$
Standard deviation of X values	S_x	SDM^*	$\sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}}$
Coefficient of variation	V	$VM^*, \%$	$\frac{S_x}{\bar{X}} \times 100$
Sample size	n	NM^*	-
F test to determine the equality of variances	F	F	$\frac{S_x^2 \text{ largest}}{S_x^2 \text{ smallest}}$
Tabulated F values <u>If $F < F_T^{***}$</u>	$F_{m1, m2, r.t.}^{**}$	F_T	Obtain from F Distribution tables
Student t Distribution	t	T	$\frac{ \bar{X}_1 - \bar{X}_2 }{\sqrt{\frac{(n_1-1)S_{x1}^2 + (n_2-1)S_{x2}^2}{n_1+n_2-2} \times (\frac{1}{n_1} + \frac{1}{n_2})}}$
Degrees of freedom <u>If $F > F_T^{***}$</u>	d.f.	D.F.	n_1+n_2-2
Student t Distribution	t	T	$\frac{ \bar{X}_1 - \bar{X}_2 }{\sqrt{\frac{S_{x1}^2}{n_1} + \frac{S_{x2}^2}{n_2}}}$
Degrees of freedom (approximate)	d.f. approx.	D.F. APRX	$\frac{\left[\left(\frac{S_{x1}^2}{n_1} \right) + \left(\frac{S_{x2}^2}{n_2} \right) \right]^2}{\left(\frac{S_{x1}^2}{n_1} \right)^2 \left(\frac{1}{n_1-1} \right) + \left(\frac{S_{x2}^2}{n_2} \right)^2 \left(\frac{1}{n_2-1} \right)} - 2$

* M reflects the set number

** m1 is the degrees of freedom for the numerator, (n_1-1)m2 is the degrees of freedom for the denominator, (n_2-1)

r.t. is the right tail of the distribution

***See text for details

TABLE 2--LEAST-SQUARES ANALYSES

Function	Standard symbol	Display symbol	Expression
Slope	b	BM*	$\frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$
y-intercept	a	AM*	$\bar{Y} - b\bar{X}$
Variance of X values	S_x^2	SQ SDXM*	$\frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$
Square of the standard error of estimate	$S_{y/x}^2$	SQ SDY/XM*	$\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}$
Coefficient of determination	r^2	SQ R	$\frac{a\sum Y + b\sum XY - n\bar{Y}^2}{\sum Y^2 - n\bar{Y}^2}$
t test for parallelism	t_s	TB	$\frac{ b_1 - b_2 }{\sqrt{S_{P,y/x}^{**} \left(\frac{1}{(n_1-1)S_{x1}^2} + \frac{1}{(n_2-1)S_{x2}^2} \right)}}$
t test for common intercept	t_i	TA	$\frac{ a_1 - a_2 }{\sqrt{S_{P,y/x}^2 \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{\bar{X}_1^2}{(n_1-1)S_{x1}^2} + \frac{\bar{X}_2^2}{(n_2-1)S_{x2}^2} \right)}}$
Degrees of freedom	d.f.	D.F.	$n_1 + n_2 - 4$
z test for parallelism	z_s	ZB	$\frac{ b_1 - b_2 }{\sqrt{\frac{S_{y/x1}^2}{(n_1-1)S_{x1}^2} + \frac{S_{y/x2}^2}{(n_2-1)S_{x2}^2}}}$
z test for common intercept	z_i	ZA	$\frac{ a_1 - a_2 }{\sqrt{S_{y/x1}^2 \left(\frac{1}{n_1} + \frac{\bar{X}_1^2}{(n_1-1)S_{x1}^2} \right) + S_{y/x2}^2 \left(\frac{1}{n_2} + \frac{\bar{X}_2^2}{(n_2-1)S_{x2}^2} \right)}}$

* M reflects the set number

$$** S_{P,y/x}^2 = \frac{(n_1-2)(S_{y/x1}^2) + (n_2-2)(S_{y/x2}^2)}{n_1 + n_2 - 4}$$

NOTE: Tabulated F values are obtained as explained in Table 1, except:

$m_1 = n_1 - 2$, and

$m_2 = n_2 - 2$.

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ENTER SET1	XEQ "ST"	ENTER SET2	RUN	RUN
376.0000	$\Sigma+$	681.0000	$\Sigma+$	F=6.6302
376.0000	$\Sigma+$	704.0000	$\Sigma+$	RUN
400.0000	$\Sigma+$	634.0000	$\Sigma+$	IF F<FT
RUN	RUN	RUN	RUN	T=13.0779
X1=384.0000	RUN	X2=673.0000	RUN	RUN
SD1=3.8564	RUN	SD2=35.6791	RUN	D.F.=4.0000
Y1=3.6084	RUN	V2=5.3015	RUN	IF F>FT
RUN	RUN	RUN	RUN	T=13.0779
N1=3.0000	RUN	N2=3.0000	RUN	RUN
				D.F.APRX=0.5899
				RUN
				ENTER SET1

Fig. 1--Input and output for program "ST"

ENTER SET1	XEQ "LT"	ENTER SET2	RUN	RUN
224.0000	ENTER↑	490.0000	ENTER↑	F=2.7096
3.8104	$\Sigma+$	3.8104	$\Sigma+$	IF F<FT
290.0000	ENTER↑	548.0000	ENTER↑	RUN
4.1671	$\Sigma+$	4.1671	$\Sigma+$	TB=0.1422
361.0000	ENTER↑	575.0000	ENTER↑	RUN
4.4476	$\Sigma+$	4.4476	$\Sigma+$	TA=1.3022
384.0000	ENTER↑	673.0000	ENTER↑	RUN
4.7249	$\Sigma+$	4.7249	$\Sigma+$	D.F.=4.0000
RUN	RUN	RUN	RUN	RUN
N1=4.0000	RUN	N2=4.0000	RUN	ENTER SET1
A1=-469.5005	RUN	A2=-237.9504	RUN	
B1=182.9156	RUN	B2=188.7931	RUN	
X1=4.2875	RUN	X2=4.2875	RUN	
Y1=314.7500	RUN	Y2=571.5000	RUN	
SQ SDX1=0.1530	RUN	SQ SDX2=0.1530	RUN	
SQ SDY/X1=211.5455	RUN	SQ SDY/X2=573.1985	RUN	
SQ R=0.9732	RUN	SQ R=0.9345	RUN	

Fig. 2--Input and output for program "LT"