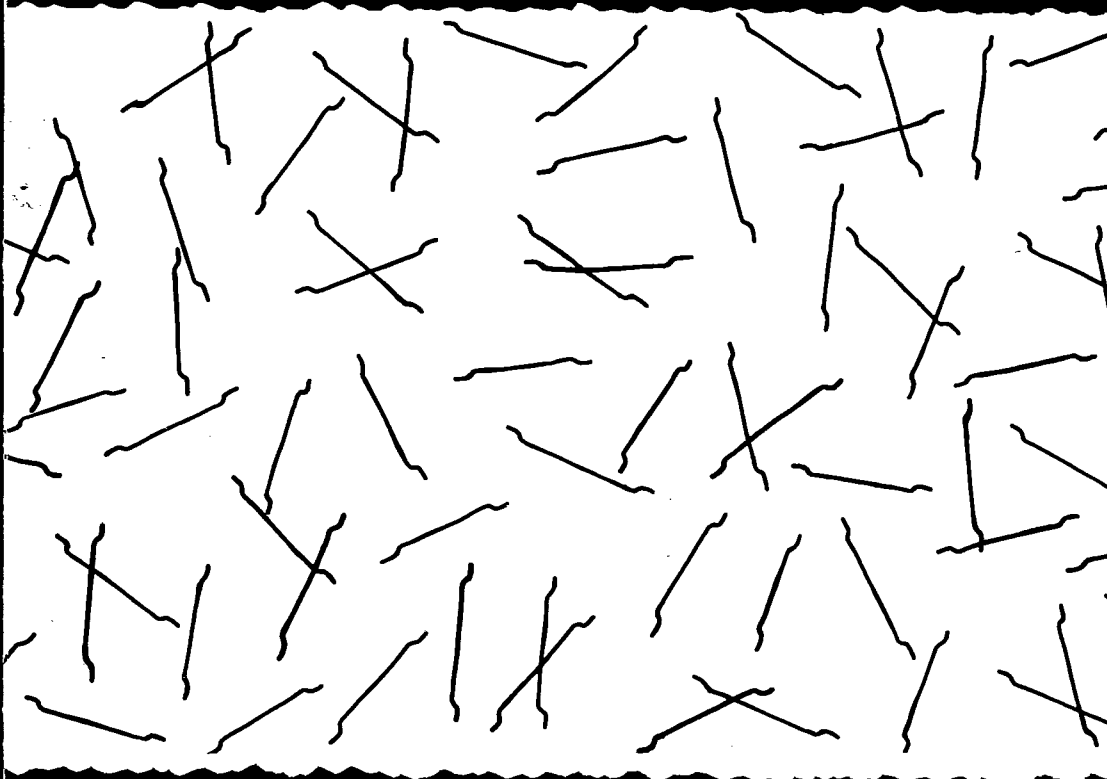


Fiber Reinforced Concrete

Properties and Applications

S.P. Shah
G.B. Batson
Editors



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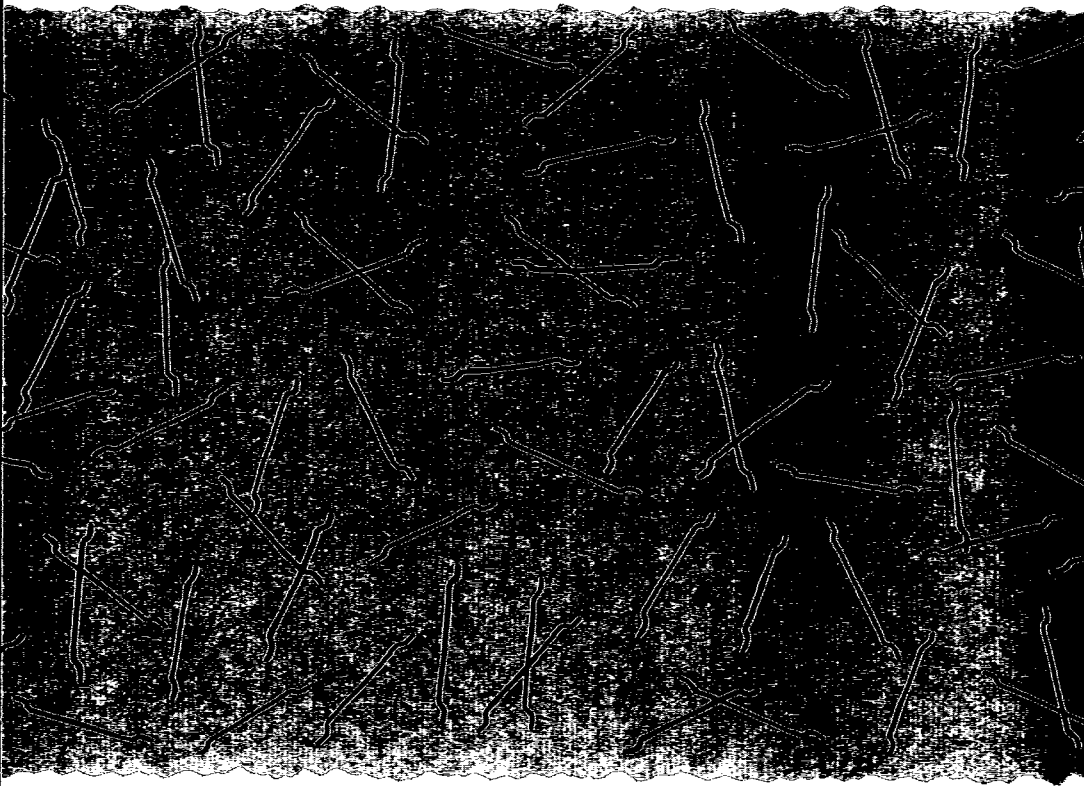
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PREFACE

Fiber reinforced concrete is used in a wide variety of applications. Concrete or mortar reinforced with short fibers is termed fiber reinforced concrete. The fibers can be made of steel, glass, or polymer; and can be added in a variety of ways. For example, they can be added to concrete in a ready mix truck and cast in a conventional manner for bulk applications such as highway pavements, slabs, and foundations. They can also be sprayed along with mortar slurry to form thin precast panels. New types of fibers, new methods of manufacturing precast products and fabricating cast-in-place members, and new applications for fiber reinforced concrete are currently being used throughout the world.

To gather, assess, and synthesize the current information on fiber reinforced concrete, ACI Committee 544, Fiber Reinforced Concrete, organized an international symposium. The original papers for this symposium were presented at the ACI Conventions in Baltimore, November 1986 and in San Antonio, March 1987. The papers were reviewed by members of the committee as well as members of the international community of experts on this subject. These original, reviewed, and revised papers constitute this ACI special volume.

The papers are divided into five groups: (1) Fracture and Mechanical Properties, (2) Polymer and Glass Fiber Reinforced Concrete, (3) Steel Fiber Reinforced Concrete and SIFCON (4) Pavements and Other Applications, and (5) Structural Behavior.

The editors gratefully acknowledge ACI Committee 544, reviewers, the authors, Mr. Reji John, doctoral candidate at Northwestern University and the staff of ACI who have made this volume a timely contribution to the state of the art of fiber reinforced concrete.

Surendra Shah, Coeditor and Chairman
ACI Committee 544
Gordon Batson, Coeditor

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Failure Mechanisms and Fracture of Fiber Reinforced Concrete

by V. Gopalaratnam and S. Shah

Synopsis: Several types of failure mechanisms and fracture of fiber reinforced concrete (FRC) composites are discussed. These include; multiple fracture of the matrix prior to composite fracture; catastrophic failure of the composite immediately following matrix cracking due to inadequate reinforcing; fiber pull-out following matrix cracking providing significant energy absorption with or without substantial strengthening of the matrix; and fracture of short fibers bridging the matrix crack without multiple fracture of the matrix. Aspects relating to the modelling of the two major causes for nonlinearities associated with fiber concrete composites, namely interfacial bond-slip, and matrix softening are also discussed. Analytical models available for predicting the tensile response of such composite are examined in light of the above mechanisms of failure.

Keywords: bonding; composite materials; cracking (fracturing); crack width and spacing; failure mechanisms; fiber reinforced concretes; mathematical models; pullout tests; strains; tensile strength

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INTRODUCTION

Potentially useful improvements in the mechanical behavior of tension-weak concrete (or mortar) matrices can be effected by the incorporation of fibers. The resulting interest in and research on fibrous concrete composites has led to an improved understanding of the mechanics of composite behavior under various simple modes of loading. Similar to the behavior of plain concrete, composite failure under most types of loading is initiated by tensile cracking of the matrix along planes where normal tensile strains exceed corresponding permissible values. This may be followed by multiple cracking of the matrix prior to composite fracture, if the fibers are sufficiently long (or continuous). However when short strong fibers (steel, polypropylene, glass etc.) are used to reinforce brittle matrices (concrete, mortar etc.), once the matrix has cracked, one of the following types of failure will occur:

(a) The composite fractures immediately after matrix cracking. This results from inadequate fiber content at the critical section or insufficient fiber lengths to transfer tensile stresses across the matrix crack.

(b) Although the maximum load on the composite is not significantly different from that of the matrix alone, the composite continues to carry decreasing loads after the peak. The post-cracking resistance is primarily provided by pulling out of fibers from the cracked surfaces. Although no significant increase in the composite tensile strength is observed, a considerable increase in composite fracture resistance (or toughness, computed as area under the stress-displacement curve) can be obtained.

(c) Even after matrix cracking, the composite continues to carry increasing tensile stresses; the peak stress and corresponding deformation are greater than those of matrix alone. During the inelastic range (between the matrix first crack stress and the composite peak stress) for type (c) behavior, progressive debonding and frictional slip at the fiber-matrix interface and some additional matrix cracking occur. It is clear that this mode of failure results in an improved performance of both the matrix and the fibers.

Analytical models used for the prediction of the mechanical behavior of fibrous concrete composites may be suitable for one or more of the above types of failure mechanisms. Based in part on the fundamental approach in their formulation, analytical models can be categorized as; models based on the theory of multiple fracture; composite models; strain relief models; fracture mechanics models; interface bond models and micromechanics models.

MULTIPLE FRACTURE OF THE MATRIX

The law of mixtures approach has been used extensively to model the elastic behavior of many types of fibrous composites. Transformed elastic section for the analysis of conventional reinforced concrete is in fact based on the law of mixtures. Prior to cracking the effective composite modulus for a composite containing continuous aligned fibers can be expressed as

$$E_c = E_f V_f + E_m V_m \quad (1)$$

where E and V are the modulus of elasticity and volume content of the constituents identified through the subscripts: f for the fiber, m for the matrix and c for the composite. At loads beyond that causing the matrix to crack, relative displacement between steel and concrete is responsible for the crack-widths observed. As a result iso-strain assumptions of the law of mixture are violated. Tensile load transfer through bond stresses from the fiber to the matrix is achieved away from the crack. Due to further increases in load, additional matrix cracks will develop when the tensile strength of concrete is again exceeded. No more cracks can be formed when the distance between the existing cracks is not sufficient to transfer (by bond) the tensile force great enough to exceed the tensile strength of concrete (σ_{mu}). If uniform bond stress (τ) is assumed, then the minimum crack spacing can be calculated as

$$x_{min} = \frac{V_m \sigma_{mu} r}{2 V_f \tau} \quad (2)$$

where r is the radius of the fiber and σ_{mu} is the tensile strength of matrix. The maximum crack spacing is twice the minimum [1]. After multiple cracking has occurred, neglecting the tensile stress carrying capacity of the matrix, effective composite modulus may be obtained as

$$E_c = E_f V_f \quad (3)$$

The composite stress-strain characteristics is thus at this third stage of loading, proportional to that of the fiber. Composite failure occurs when the fibers fracture or yield. Aveston, Cooper and Kelly [1] have proposed a tensile stress-strain relationship similar to that described in Eqns. 1-3, for glass fiber reinforced concrete (GFRC). Assuming uniform distribution of bond stresses and the crack spacing given by Eq. 2, they have

proposed the following equation for computing the enhanced matrix cracking strain ϵ_{mu}

$$\epsilon_{mu} = \left[\frac{12 \tau \gamma_m E_f V_f^2}{E_c E_m^2 r V_m} \right]^{1/3} \quad (4)$$

where γ_m = surface energy of matrix fracture, and all other quantities are as defined earlier. This equation was obtained by considering the changes in energy that occur when a crack is formed across a tensile specimen. It was assumed that a crack will form when the sum of the work of fracture of the matrix, the energy absorbed due to friction at the fiber-matrix interface, and the increase in elastic energy in fibers is exceeded by the sum of the work done by the applied stresses and the elastic energy released by the matrix. Assumptions with regard to uniform bond, and ignoring the tensile capacity of the matrix, are not justified for such composites as discussed later. Additionally Eq. 4 is more sensitive to the fiber volume content than observed experimentally [2,3].

Aveston and Kelly [4] have further extended the model of Aveston et. al. [1] to include analysis of continuous fibers which are elastically bonded (non-uniform bond stress) to the matrix. The elastic solution of this problem is similar to solutions obtained by Cox [5] and Piggot [6] (which is discussed later). Also included in the modified model are considerations for the effects of random fiber orientation in 2-D and 3-D spaces. Expressions derived for crack spacing, however, still assume fibers long enough to be treated continuous. The difference in strains predicted by the much simpler model assuming frictional bond is not significantly different than that predicted by the modified model assuming elastic bond, to warrant the more cumbersome computations.

COMPOSITE MODELS

Cox [5] has analyzed the effect of orientation of the fibers on the strength and stiffness of paper (also a fibrous composite). For discontinuous fibers in a 2-D random distribution the composite modulus, E_c , has been determined to be $E_f V_f/3$ and the composite strength $\sigma_c V_f/3$. For a 3-D random distribution these values are $E_f V_f/8$ and $\sigma_f V_f/6$ respectively. In Cox's case, the length of the discontinuous fiber was assumed to be longer than the critical transfer length, ℓ_c , whereby composite failure is by fiber fracture rather than by fiber pull-out Fig. 1a.

Jayatilaka [7] has further discussed stress distributions likely if the fiber length is less than, equal to, or larger than the critical fiber length if a purely frictional shear transfer at the interface is assumed (constant τ). For the case in which

the fibers deform elastically and the matrix deforms plastically he has obtained an expression for the composite modulus, E_c , using an average fiber stress solution similar to Cox. It should be noted that the composite failure mechanism assumed by Jayatilaka is still by fracture of fibers.

Piggot [6] has extended the work by Cox, to develop fiber and interface stress solutions for fiber reinforced polymers and reinforced ceramics and cements when the interfacial bonding is elastic-frictional (constant τ) and elastic-nonlinear frictional respectively. The expressions for axial fiber stress and interfacial shear stress are more involved than those proposed by Cox, but as the nonlinear aspect of interfacial shear is incorporated in this model, composite tensile stress-strain behavior predicted exhibits nonlinearities observed in practice. The matrix and fibers are treated to be in a purely elastic state until the onset of interfacial bond degradation. Figs. 1b and c illustrate the fiber axial stress and interfacial shear stress distributions obtained by Piggot for the case of fiber reinforced polymers and fiber reinforced ceramics respectively. The phenomenon of slip introduced by Piggot will be useful in modelling the behavior of steel fiber reinforced cement composites, as described later.

Naaman, Moavenzadeh and McGarry [8] have analyzed FRC probabilistically for its tensile strength and post-cracking characteristics. Representing a tensile member as a chain link series they have obtained expressions for weakest link as

$$\left. \begin{aligned} \bar{\sigma}_{cc} &= \bar{\sigma}_{mu} (1 - V_f) + \alpha \bar{\tau} V_f \frac{\ell}{\phi} \\ \bar{\sigma}_{cu} &= \frac{\bar{\tau}}{\pi} V_f \frac{\ell}{\phi} \\ \bar{\gamma}_c &= \bar{\gamma}_{mu} (1 - V_f) + \bar{\sigma}_{cu} \frac{\ell}{12} \end{aligned} \right\} \quad (5)$$

where all "bar" quantities signify average quantities: $\bar{\sigma}_{cc}$ = composite cracking strength, $\bar{\sigma}_{mu}$ = matrix tensile strength, $\bar{\tau}$ = bond strength from single fiber pull-out test, $\bar{\sigma}_{cu}$ = composite post cracking strength, $\bar{\gamma}_c$ = composite surface energy, $\bar{\gamma}_{mu}$ = matrix surface energy, ℓ = fiber length and diameter, and α = empirical constant.

Depending upon the number of fibers pulling out from a unit area the composite cracking strength is obtained using an α value of 0.122 determined empirically. The model predicts a linear increase in both the cracking stress and post-cracking strength of FRC composites with an increase in the volume fraction and aspect ratio of the fibers. Strains or deformations at cracking and post-cracking strength levels cannot, however, be predicted using the model.

STRAIN RELIEF MODELS

The name for this category of models has been chosen because they essentially assume an elliptical zone around the crack to be partially relaxed. Outside the partially relaxed elliptical zone the material is assumed to be unaffected by the presence of the crack. Uniform uniaxial loading perpendicular to the plane of the crack and parallel to the direction of reinforcement is considered. Only strains parallel to the direction of loading are considered and shear interactions between the adjacent zones together with stress concentrations at the crack tips are neglected. For the unreinforced matrix, a linear change in strain is assumed parallel to the direction of loading from the crack face to the extremities of the elliptical zone. The size of the elliptical zone is chosen such that the strain energy released by the presence of the crack on the basis of the assumptions made, is the same as calculated by Griffith [9] for the classical case of crack in an infinite elastic material.

Korczynskyj, Harris and Morley [10] and Hannant, Hughes and Kelly [11] have extended the idea to study the influence of strong reinforcing fibers on the growth of cracks in brittle matrices. The strain-field in fibrous composites unlike in the unreinforced matrix, is modified by the presence of crack bridging fibers. The essential difference between Korczynskyj et.al. [10] and Hannant et. al. [11] is in the strain-field assumed within the elliptical zones. Fig. 2 shows the strain-field assumed by Hannant et. al. [11]. The fibers are assumed to be sufficiently longer than the crack length and are assumed to be uniformly distributed and aligned perpendicular to the crack (parallel to the loading direction). Stress is transferred between the fibers and the matrix via the fiber matrix interface which is assumed to be purely frictional (constant shear transfer). As a result of these assumptions the fiber and matrix strain distributions within a strip of the elliptical zone are as shown in Fig. 2.

Numerically the crack propagation problem is evaluated thus. The elliptical zones are divided into a number of strips, perpendicular to the crack. Only one quadrant of the ellipse is analyzed due to the two axes of symmetry. For a given crack length and farfield tensile strain the energy stored within these strips by the fibers and matrix are easily evaluated once the strain fields are established. The energy released, U_r , due to the presence of the crack can be evaluated by cumulating the contributions of the individual strips. Similarly the energy absorbed by friction, U_f , due to the difference in the matrix and fiber strains within ellipse L , can be computed. U_r and U_f are again evaluated for the same farfield tensile strain, for a small increment δc in the crack length, and thus the rate of energy released $\delta U_r / \delta c$ can be computed. If the rate of energy released is not greater than that absorbed, the farfield matrix strain is incremented and the procedure repeated until the condition of catastrophic crack growth is achieved. The catastrophic crack

growth condition in the modified matrix is given by

$$\frac{\delta U_r}{\delta c} > \frac{\delta U_f}{\delta c} + G_{cm} V_m \quad (6)$$

Where G_{cm} is the Griffith's critical strain energy release rate for the unreinforced matrix. The corresponding farfield matrix strain, ϵ_{my} , that just satisfies Eq. 6 is the "enhanced matrix failure strain" as a result of incorporation of fibers in the matrix. Constant bond stress and a traction free crack are assumed in the model. It is also very sensitive to the initial flaw size c assumed [2,3].

FRACTURE MECHANICS MODELS

Two broad classes of models can be identified among those developed using fracture mechanics concepts. The more fundamental class of models use concepts of classical linear elastic fracture mechanics (LEFM) or modified LEFM to solve the problem of crack initiation growth and stability under a farfield tensile loading. The others, do not attempt to predict the global composite behavior under tensile loading mode, but rather use its tensile post-peak stress-displacement relationship to model crack growth in the composite under other loading configurations. This class of model is popularly labelled as the fictitious crack model (FCM).

The mechanics of crack arrest in concrete reinforced with small diameter steel wires was studied by Romualdi and Batson [12]. Superimposing simplified solutions for a penny shaped crack in an infinite matrix subjected to farfield tension and the equivalent restraining effect of adjacent fibers (arranged in a square array and aligned in the loading direction) they obtained the effective stress intensity factor for cracks in fibrous composites. Implicit in their analysis is the assumption of a perfectly bonded fiber-matrix interface during the crack propagation process. Applying concepts of linear elastic fracture mechanics and adopting a critical stress intensity factor criterion, they have noted that the cracking stress (stress at which the crack propagates beyond the bounds of the adjacent fibers) is an inverse function of the square-root of fiber spacing. Romualdi and Mandel [13] have verified the fiber spacing concept based on splitting tension and flexural (3 point bending) tests on composites with randomly distributed short steel fibers. The applicability of LEFM using a single parameter fracture criterion for small crack lengths (corresponding to fiber spacing) has been questioned by Shah and Rangan [14] and Jenq and Shah [15].

Kasperkiewicz [16] has proposed a novel approach to modeling

the tensile fracture of unreinforced and fiber reinforced cement composites. Matrix inhomogeneity is accounted for in his fracture model by making the surface energy a function of location, thus

$$\gamma_f(x) = \frac{\bar{\gamma} + \underline{\gamma}}{2} - \frac{\bar{\gamma} - \underline{\gamma}}{2} \cos \frac{2\pi x}{\lambda} \quad (7)$$

where $\bar{\gamma}$, $\underline{\gamma}$ are bounds of the variations of the local surface energy, and, λ , a material periodicity parameter to account for a representative volume in the inhomogeneous composite. A tensile specimen of matrix is considered to comprise a series of initial flaws of size c_0 . Using the Griffith criterion for stable crack growth in addition to assumptions regarding energy absorption in a cycle of unloading/reloading in concrete materials he has obtained solutions for relating the average tensile stress to crack size and the overall displacement of a finite sized tensile specimen. Softening behavior predictions for the matrix in tension have been accomplished qualitatively using these solutions. For the FRC specimens an additional term to account for energy absorbed during fiber pull-out has been incorporated to yield relationship between average tensile stress and corresponding displacements. The model, it should be noted, has only been used to qualitatively describe trends in softening behavior observed. It is quite sensitive to some of the assumed model parameters.

The fictitious crack models for fiber cement composites have been formulated, using approaches similar to the cohesive force model of Barenblatt [17] or the simpler plastic yielding model of Dugdale [18]. The major differences are eventually in the singularity assumptions at the crack tip, the criteria used for crack initiation and growth, and the stability of crack growth. A Fictitious Crack Model (FCM) has been proposed by Hillerborg [19] for fracture analysis of FRC, Fig. 3. The fracture zone (analogous to the plastic zone for metals) ahead of the real crack is assumed to act as a "fictitious crack" which has the ability to transfer stresses. Instead of conventional fracture parameters, the tensile stress-displacement curve of the composite is introduced to describe the behavior of the fracture zone. Elastic stress-strain relations are used to describe the behavior of zones outside the fracture zone. Incorporating the FCM in a finite element scheme, Hillerborg has shown that general trends observed from experiments on the flexural behavior of notched and unnotched FRC beams can be reproduced. Petersson [20] has used the FCM to study crack growth and development of fracture zones in plain and fiber reinforced concrete notched beams.

Wecharatana and Shah [21,22] have modelled the process zone ahead of the real crack in a similar fashion, with the closing pressure function determined experimentally. Unlike the Dugdale model, however, no assumptions have been made with regard to the singularity at the crack tip. An iterative procedure is set-up to determine the size of the process zone so that the predicted

Crack Tip Opening Displacement (CTOD) at a particular load matches the experimentally observed CTOD at the load level. The strain energy release rate has been modified in their formulation to account for slow crack growth as well as residual deformation (analogous to plastic deformations) on unloading. Resistance or R-curves (plot of strain energy release rate versus crack extension inclusive of the process zone) thus predicted for concrete and FRC compare well with experimentally obtained R-curves for different specimen sizes and geometries, namely: Double Cantilever Beam (DCB), Double Torsion (DT) and notched beam specimens. While simple to use in numerical computations the model does not incorporate conditions for crack stability.

Visalvanich and Naaman [23] have used the R-curve concept to model fracture of FRC. They use (i) an empirical stress-displacement relation for the stress transfer during fiber pull-out, ahead of an actual crack, and (ii) a straight line crack profile in this pseudo-plastic region as the basic data needed to generate the R-curve. Crack propagation is said to occur when the crack opening angle reaches a critical value (termed CCOA). The CCOA determined empirically, varies from 0.13° for plain mortar to 0.29° for some fibrous composites. The size of the pseudo-plastic zone varies from 6.35 cm - 226 cm. These are undefined functions of the composition of the composite (V_f and l/d of the fibers). The CCOA cannot as a result be prescribed a priori, which might restrict the use of the model.

Stang and Shah [24] have studied the fracture by fiber pull-out of composites made with continuous fibers aligned in the loading direction. The debonded zone is treated as an interfacial crack. Griffith type criterion is used for the growth of interfacial crack, yielding:

$$\frac{1}{2} \frac{\partial C}{\partial b} (P_{cr})^2 = 2\pi a (2 \gamma_i) \quad (8)$$

where P_{cr} = critical pull-out load for crack growth, γ_i = specific interfacial work of fracture, b = length of the debonded interface, a = fiber radius, and C is the compliance of the single fiber pull-out specimen. Thus knowing the maximum load one can calculate γ_i which is the material property characterizing the interface. The accuracy of the above equation was demonstrated by measuring changes in compliance resulting from artificially introduced debonded zones of different lengths [25]. In addition, methods to include frictional energy and calibrate the model parameter from the pull-out test was also discussed in [25]. This approach provides an alternative method of analyzing the results of the pull-out test in terms of fracture energy rather than bond strength (this method is discussed later).

Readily available asymptotic solutions have been used for the elastic compliance, C , of the fiber pull-out specimen

containing (i) rigid fibers $((E_f/E_c)(a/l)^2 \ln(2l/a) \gg 1$ and $l \gg a)$, and (ii) elastic fibers $((E_f/E_c)(a/l)^2 \ln(2l/a) \ll 1$ and $l \gg a)$, l is the fiber length and E_f and E_c are as defined earlier. The compliance of the single fiber pull-out problem has been built upon to yield the compliance of an idealized tensile specimen containing one major matrix crack, Fig. 4. Interfacial cracks are present at this stage up to a distance b on either side of the matrix crack along the continuous fibers aligned in the loading direction. The criterion for crack growth (Eq. 8) is once again applied to the idealized tensile specimen to yield the composite strength, σ_{cr}

$$\sigma_{cr} = 2 v_f \sqrt{\frac{E_c E_f}{E_c - v_f E_f} \frac{2 \gamma_i}{a}} \quad \text{(a) Elastic fiber case}$$

$$\sigma_{cr} = 2 v_f \left(\frac{l-b}{a} \right) \sqrt{\frac{E_m}{1 + v_m} \left(\ln \left(\frac{l-b}{a} \right) - 1 \right)^{-1} \frac{2 \gamma_i}{a}} \quad \text{(b) Rigid fiber case}$$

(9)

where E_c = composite elastic modulus given by the law of mixtures, v_m = Poisson's ratio of the matrix, and all other terms as defined earlier.

The solution Eq. 9 (b) shows that for stiffer and shorter fibers, the stiffness of the matrix and the length of the fibers become important in determining σ_{cr} and that crack propagation tends to be unstable soon after interfacial cracks are initiated (σ_{cr} maximum for $b = 0$). On the other hand Eq. 9 (a) is independent of fiber length. The possible model predicted solutions for the composite tensile stress-strain behavior is presented in Fig. 4 (b). It should be noted that the stress transferring capacity of cracked matrix is ignored in the model. Attempts to predict displacements or strains of the tensile specimen at σ_{cr} have also not been made.

More recently, Jenq and Shah [15] have proposed a fracture mechanics based model to predict the crack propagation resistance of fiber reinforced concrete. Fracture resistance in fibrous composites is separated into four regimes which include; subcritical crack growth in the matrix and the beginning of fiber bridging effect; post-critical crack growth in the matrix such that the net stress intensity factor due to the applied load and the fiber bridging closing stresses remain constant (steady-state crack growth); and the final stage where the resistance to crack separation is provided exclusively by the fibers. The model uses two parameters that describe the matrix fracture properties (K_{Ic} , modified critical stress intensity factor based on LEFM and the effective crack length, and CTOD the critical crack tip opening displacement), and a fiber pull-out stress versus slip (σ

- w) relationship as basic input information. It has been successfully used to predict the complete load displacement behavior of SFRC beams of several different sizes.

INTERFACIAL BOND AND ORIENTATION EFFECTS

Without the incorporation of the mechanics of bond failure, one cannot expect to realistically model the fracture behavior of fiber reinforced cement composites. Greczczuk [26] has analyzed fiber-matrix interfaces in metal matrix composites with a view to designing them for composite failure rather than interfacial failure. He has solved for the interfacial shear stress distribution during fiber pull-out, based on the size and shear modulus of the interface. Using this solution it is possible to evaluate the shear strength of the interface. The strength thus computed is always greater than the average bond strength computed assuming a constant shear at the interface.

Lawrence [27] has incorporated Cox's idea of relating the shear stress distribution to both the elastic properties of the matrix and the fiber into Greczczuk's formulation thus avoiding the use of interfacial width, which is difficult to establish for FRC. Lawrence has also extended the solution for the case of a partially debonded fiber configuration assuming constant frictional shear stress, τ_i , along the failed interface (where the shear stress exceeds the elastic strength of the interface.) A similar approach discussed later has been used by Gopalaratnam and Shah [3] for modelling stable interfacial debonding.

Laws, Lawrence and Nurse [28] using the analytical model developed by Lawrence [27] have studied the failure of glass fiber reinforced cement composites (GFRC). Extending the partially debonded pull-out solution to the analysis of multiple cracking of long aligned fiber composites they have predicted crack spacings in such GFRC specimens. Stress-strain curves of such composites have been predicted which compare satisfactorily with the experimentally observed behavior. Strength of random composites have also been modelled by them using an efficiency factor η describing the combined effects of the randomness in fiber length and orientation. Unlike in GFRC for short steel fiber reinforced concrete, orientation does not appreciably influence composite strength or the corresponding strain. The work of fracture for steel fiber composites, however, seems to be significantly affected by the randomness of fiber orientation.

Naaman and Shah [29] have concluded from experimental studies on pull-out tests of steel fibers from mortar matrix that; the peak pull-out load for fibers inclined with respect to the loading direction is almost as high as those for fibers parallel to the loading direction, and; the post-peak load carrying capacity for a fiber inclined to the loading direction is higher than that for a parallel fiber, with the highest value observed for a 45° orientation. Consequently, the work required to completely pull-out is higher for the inclined fibers.

Fig. 5 a-d describe the types of pull-out tests performed by them and also includes typical results from their study. The argument put forward by them for the increased work of fracture of inclined fibers is that inclined fibers undergo plastic deformations at the crack and this progressive plastic bending of successive sections of the fibers during pull-out contributes to the larger amount of work required. Also since normal force is applied to the fiber at the bend, increased friction stresses at the debonded interface also contribute to the increased work requirements.

Brandt [30] has conducted an analytic study with a view to optimize (energy absorption-wise) fiber orientation in cement based composites. He has derived expressions for the energy terms as functions of the fiber orientation (with respect to the loading direction) which include; the elastic strain energy stored in the interface; frictional fiber pull-out energy; plastic deformation energy due to fiber bending at the crack; energy due to compressive crushing of the matrix resulting from the fiber bearing stresses and energy due to additional friction at the bend. The contribution of the various energy terms obtained using realistic values for the properties of fibers, matrix and the interface indicate that frictional energy absorption is the most significant component, at the optimal orientation (about 45°). This substantiates experimental observations of Naaman and Shah (Fig. 5).

MICROMECHANICAL MODELS

Unlike many of the previously described approaches, micromechanical models derive their name from the fact that they systematically attempt to relate the local stresses in the matrix, the fiber and their interface, to the overall macroscopic composite behavior. This micro-approach enables the characterization of crack growth in the matrix, interfacial debonding and fiber pull-out, observed during the progressive failure of FRC.

Taya and Mura [31] have obtained solutions for the stiffness and strength aligned short fiber reinforced composites subjected to uniaxial tension containing penny shaped cracks at the fiber ends. The results obtained using a combination of Eshelby's equivalent inclusion method [32] and back stress analysis method is valid even for large fiber volume fractions. The Eshelby solution for the case of composites with no penny shaped cracks (only aligned fibers) shows that while the law of mixtures may be accurate at low fiber volume fractions (0-5%), it overestimates the stiffness by as much as 25% for composites containing 40% by volume, of fibers. The presence of penny shaped cracks at fiber ends it has been shown can reduce the stiffness by as much as 50%. ($V_f = 40\%$, $l/d = 10$ and $E_f/E_m = 10$). Composite strength has been computed using the Griffith^m criterion for the growth of the penny shaped crack. For short steel fiber reinforced concrete, however, failure by matrix cracking followed by interfacial cracking with eventual catastrophic pull-out of fibers

bridging the crack is the governing mechanism of fracture.

Takao, Chou and Taya [33] have solved the interesting problem of evaluating the effective longitudinal Young's modulus of random short fiber composites. The analysis is based again on the Eshelby's equivalent inclusion method and the average induced strain approach. The approach is unique in that it takes into account the interactions among fibers at different orientations. Using a purely elastic approach with no fiber debonding (or interfacial cracking) it has been demonstrated that fiber orientation distribution has a more significant effect on composite longitudinal Young's modulus than fiber volume fraction. However, their solution shows that for the reinforced concretes, orientation effects are not as dominant, a fact observed also from experimental modulus values for such composites.

Argon and Shack [34] have also used the Eshelby's equivalent inclusion analysis to solve Cox's problem of a short aligned fiber in an infinite matrix subjected to farfield tension. Unlike Cox [5] however, Argon and Shack have assumed that the fiber is capable of transferring stresses at its ends. Despite this difference in boundary conditions, the solution obtained for the distribution of shear stresses along the length of the interface are markedly similar with a rapid drop in these stresses within a short distance away from the fiber ends. Argon and Shack have shown that for fibers of short aspect ratios, this could lead to a total debonding of the fiber at the matrix first crack stress, thereby causing composite failure.

Although the Eshelby's inclusion type micromechanics models are powerful for the exact analysis of fibrous composite behavior in the elastic bonded stages of loading, they are not suited to solve the problem of growth of interfacial cracks and eventual pull-out of fibers.

Mandel [35], Mandel and Pack [36], Pack and Mandel [37] and Mandel, Pack and Tarazi [38] have investigated the problem of crack growth in the matrix, using the finite element method. The micromechanical numerical model includes triangular quarter point (square root singularity) crack tip element to represent the region of the matrix near the crack tip, isoparametric quadratic quadrilateral elements to model the fibers and the rest of the matrix material, and a zero thickness bond interface element. Since the failure mechanism is sequential, external load is applied in increments. After each increment of loading, the mesh and element stiffness are recomputed to reflect crack growth and associated changes in the constituent material behavior (fiber, matrix and the interface.) The failure criteria used for crack growth is based on LEFM. Pack and Mandel [37] have also included a three-dimensional analysis of the composite using a multiplane finite element analysis, where adjacent planes (treated as a lamina) are connected in the perpendicular direction by specially formulated connector elements. Phenomenologically, the multiplane model is reported to be capable of exhib-

iting the type of progressive failure observed in FRC. It will be interesting to compare model predicted results with those obtained experimentally after fiber delamination and pull-out features are also incorporated in the model [35].

Gopalaratnam and Shah [3] have proposed an micromechanical approach where the two primary aspects of nonlinearity associated with SFRC are incorporated, namely interfacial bond-slip and matrix softening characteristics [39].

To analyze the response of FRC (where fibers are short and discontinuous) to loads beyond that corresponding to matrix cracking, it is helpful to understand the mechanics of bond failure at the fiber-matrix interface. What happens to the fiber-matrix interface after matrix cracking can be better understood by examining the case of the single fiber pull-out test, Fig. 6. The calculated distribution of bond stress along the length of the fiber depends upon, among other things, the assumption made regarding the shear stress vs. shear strain relationship at the interface. For the small load P , it is reasonable to assume that the bond is elastic, (Fig. 6a). Under this condition, shear stress distribution can be calculated from the theory of elasticity using compatibility, equilibrium and boundary conditions. If it is assumed that the influence of the fiber is felt only within a concentric cylinder (the radius of the cylinder = half the fiber spacing), then using the method developed by Cox [5] Lawrence [27] and Laws, Lawrence and Nurse [28], Gopalaratnam and Shah [3] have shown that the elastic bond stress τ_e is given by:

$$\tau_e(x) = -\frac{\beta}{2\pi r} (C_1 \sinh \beta x + C_2 \cosh \beta x) \quad (10)$$

$$\text{where } \beta^2 = \frac{2\pi G_m}{\ln\left(\frac{1}{2}\sqrt{\pi/V_f}\right) A_f E_f} \quad \alpha = \frac{A_f E_f}{A_m E_m}$$

$$C_1 = -\alpha P \quad \text{and} \quad ,$$

$$C_2 = \frac{P \left(1 - \alpha \left(1 - \cosh \frac{\beta \ell}{2}\right)\right)}{\sinh \frac{\beta \ell}{2}}$$

and where G_m = shear modulus of the matrix, A_f and A_m are the cross-sectional areas of fiber and matrix and x is measured from the embedded end of the fiber (Fig. 6).

Note that the shear stress is maximum at $x = \ell/2$, that is at the cracked surface of the matrix. When the maximum shear stress exceeds the shear strength of the interface (τ_g), debonding of fiber is initiated. The debonded portion is assumed to have frictional resistance τ_i (Fig. 7).

Partially debonded case is shown in Fig. 6b. The fiber is assumed to be elastically bonded up to a distance $\frac{m\ell}{2}$ from the embedded end. The interface stress for $m\ell/2 \leq x \leq \ell/2$ is considered a constant and purely frictional. The relation between the pull-out load P and the debonded length $\frac{\ell}{2}(1-m)$ can be approximated as:

$$P = \left[\frac{2\pi r \tau_s}{(1-\alpha)} \tanh \frac{\beta m \ell}{2} + \frac{\pi r \ell \tau_i (1-m)}{(1-\alpha)} \right] \quad (11)$$

It can be seen that for a value of τ_i in the range $0 < \tau_i < \tau_s$, the maximum pull-out load ($dP/dm = 0$) is reached only after a stable debonding process (Fig 8). The average composite stress-strain curve will be nonlinear due to this stable debonding process prior to pull-out of the fiber. Typically in most experimental studies on the fiber-matrix bond behavior, the average interface strength $\bar{\tau}$ is computed from the maximum pull-out load assuming uniform shear stress distribution. This value, however, depends on the length of the fiber embedment, as well as on the radius of the fiber, and thus is not likely to be a material property. Gopalaratnam and Shah have shown that if a two parameter (τ_s, τ_i) model of interface behavior is assumed, then one can demonstrate that the calculated $\bar{\tau}$ will depend on the length and diameter of the fiber. The values of τ_s and τ_i were calibrated by them from the test results of Naaman and Shah [29] based on steel fibers of one diameter as shown in Fig. 9. From these values of τ_s and τ_i , the maximum pull-out load was theoretically calculated for different fiber diameters. The theoretical results are compared with test data [29] in Fig. 10. The theoretically predicted trend for various embedment lengths and diameters is shown in Fig. 11.

A calculation of the average stress-strain curve of a FRC composite can be obtained from the pull-out behavior described above, if it is assumed that the pull-out occurs at one critical (cracked) section. At that section, up to the start of matrix cracking, Eq. 1 can be applied, provided that it is assumed that the matrix, fiber and interface behave elastically. This point is taken to be the experimentally observed limit of proportionality (LOP). The matrix is assumed to crack when the tensile strength of the unreinforced matrix is exceeded. After the matrix has cracked, the composite response is obtained by superimposing the results for the fiber pull-out problem and the matrix softening problem (Fig. 12) [2,3]. The cracked (unreinforced) matrix continues to transfer a decreasing load, depending upon the average distance between the cracked surfaces (crack width). This type of a matrix softening behavior has been experimentally observed by Gopalaratnam and Shah [39]. The total crack width is the cumulative slip of the fiber from the matrix during

the pull-out (and can be calculated using the procedure outlined in [2,3]. Based on the above considerations, it is possible to calculate the average composite stress and average strain during stable debonding. The composite peak stress is assumed to be reached when the fiber-matrix interface subsequently starts to debond in an unstable manner. This procedure has led to a satisfactory prediction of the composite peak stress and strain for steel fiber reinforced concrete [2,3].

SUMMARY

In summary there are several valid approaches to the modeling of FRC. The type of model to be used is dictated by the anticipated mechanism of failure of the composite and the type of solutions desired (e.g. macroscopic composite behavior, local stress fields, resistance to crack growth, pre-peak behavior, behavior at the peak, or post-peak behavior etc.). In such instances it will be important that the user choose the best suited model judiciously and be aware of the assumptions and limitations of the approach used. The article provides some observations that will be useful towards that end.

ACKNOWLEDGEMENTS

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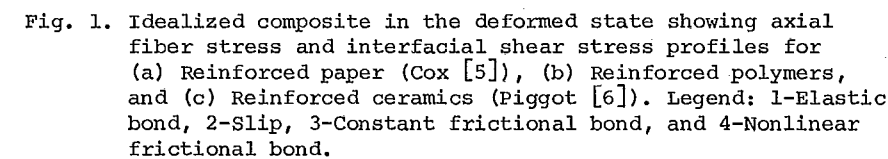
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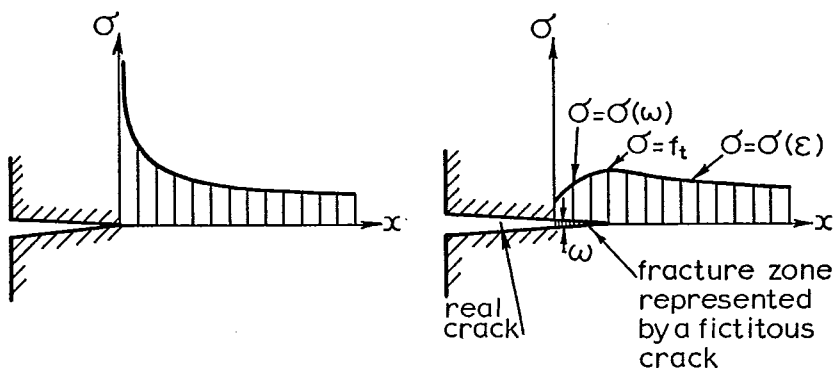
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The diagram illustrates the mechanical behavior of a fiber-matrix system. The left portion shows a cross-sectional view where a central fiber (dashed ellipse L_2) is embedded in a matrix (solid ellipse L_1). A crack (solid ellipse w) is present in the matrix. The vertical axis is labeled 'distance, X' and the horizontal axis is labeled 'strain, ϵ '. The fiber strain is denoted by ϵ_f , the matrix strain by ϵ_m , and the crack opening by $\omega/2$. The right portion shows a longitudinal view of the system under stress. The vertical axis is labeled 'A' and the horizontal axis is labeled '2c'. The stress in the matrix is denoted by $\sigma_{mu} = E_{mu} \epsilon_{mu}$. The vertical axis is also labeled 'x' and the horizontal axis is labeled 'y'.

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(a) Stress solution-elasticity (b) Fictitious crack model

Fig. 3. Schematic of the Fictitious Crack Model used to represent the crack tip behavior in FRC (Hillerborg [19]).

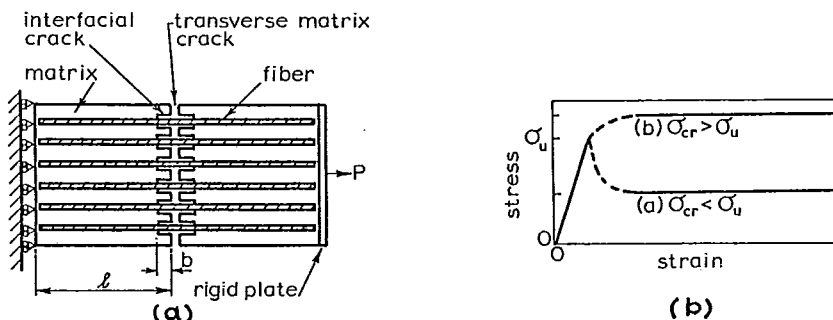


Fig. 4. (a) Idealized tensile specimen considered by Stang and Shah [24], (b) Possible model predicted solution for the tensile behavior of FRC.

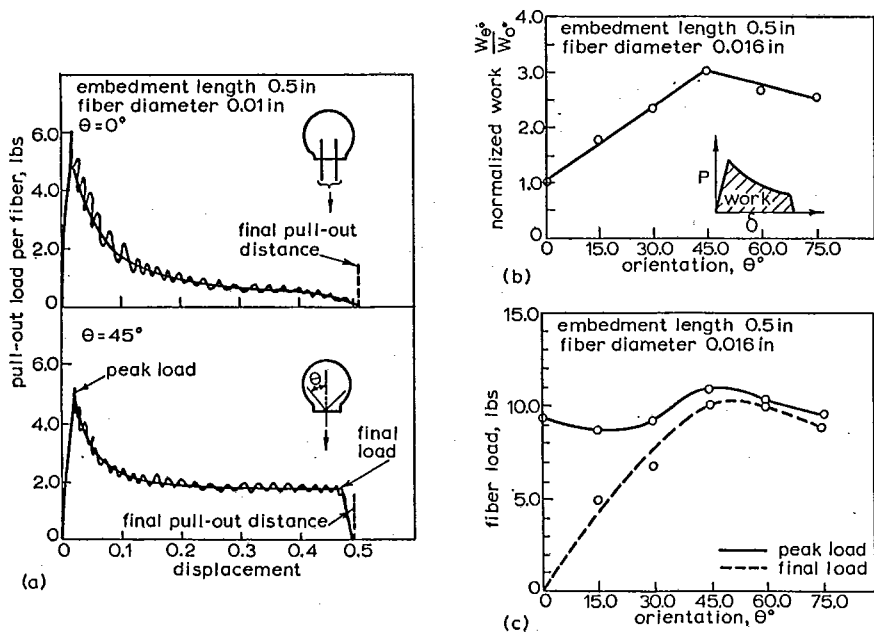


Fig. 5. Experimental results from pull-out tests conducted by Naaman and Shah [29]. Smooth steel fibers of circular cross-section are pulled out from mortar matrix briquettes. (a) Typical load-displacement behavior for aligned fiber and inclined fiber pull-out, (b) Pull-out work requirement, and (c) Peak and final load carrying capacities as functions of fiber orientation.

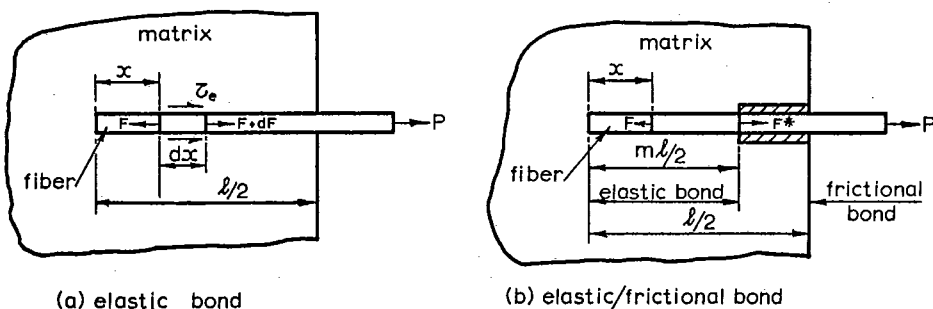


Fig. 6. Schematic of the fiber pull-out problem solved showing definitions of the model parameters. (a) Elastic stress transfer, and (b) Elastic-frictional stress transfer.

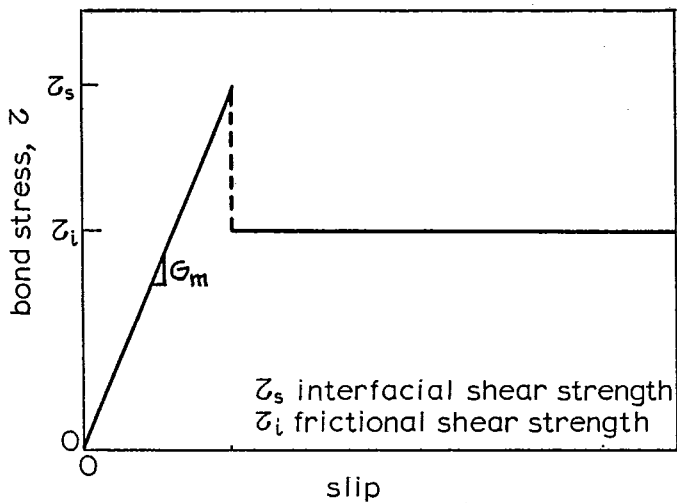


Fig. 7. Schematic of the interface property assumed. A purely elastic behavior until the shear strength of the interface τ_s , is reached, is followed by a constant frictional bond of strength τ_i .

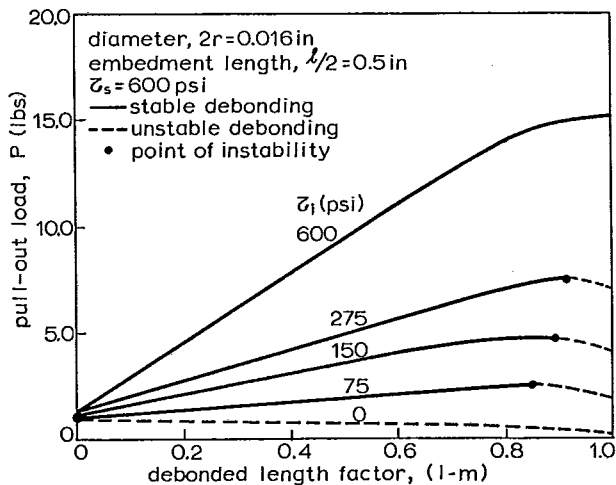


Fig. 8. Model predicted pull-out load versus the debonded length factor $(l-m)$, showing stable and unstable debonding regimes depending upon the ratio τ_s / τ_i assumed.

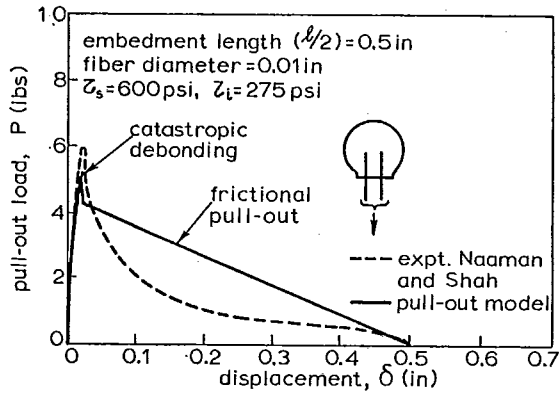


Fig. 9. Comparison of the model predicted pull-out load versus displacement behavior with experimental results reported by Naaman and Shah [29].

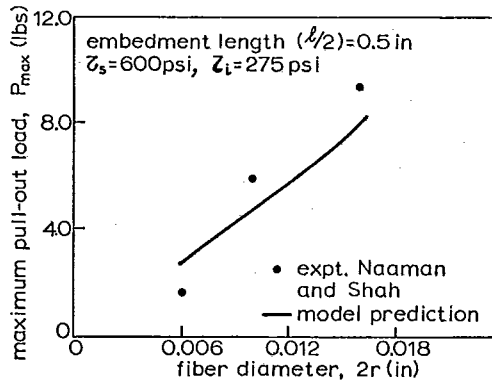


Fig. 10. Experimental [29] and model predicted variation of the maximum pull-out load with respect to the fiber diameter.

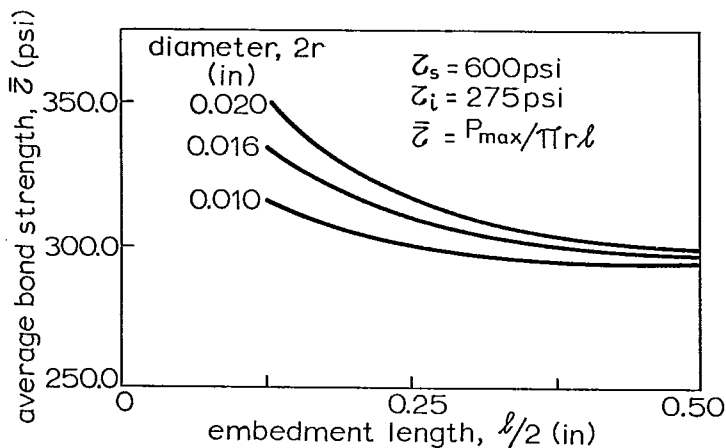


Fig. 11. Parametric study of the model predictions. Average interfacial bond strength for different embedment lengths and fiber diameters is shown.

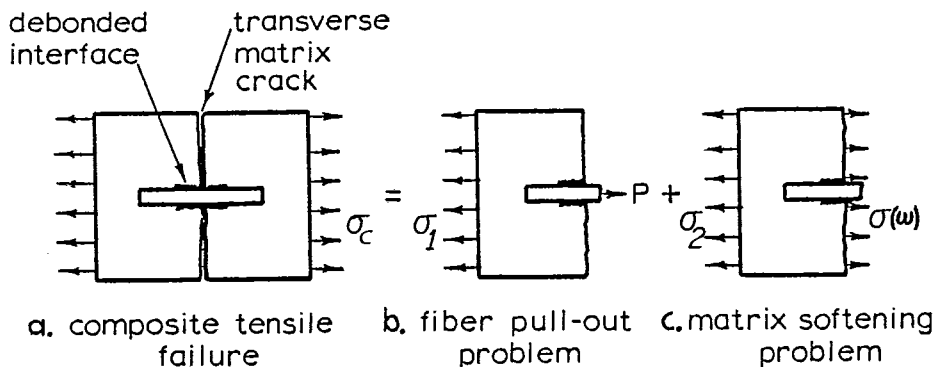


Fig. 12. Coupled superposition of solutions for the fiber pull-out and the matrix softening problem yields the stress-crack-width behavior of the idealized composite. For purposes of clarity only one aligned fiber is shown bridging the transverse matrix crack.

The Fracture Characteristics of FRC Materials in Shear

by B. Barr

Synopsis: The paper reviews seven test specimen geometries which have been used to determine the shear performance of fibre-reinforced concrete (FRC) materials. All the geometries are modified standard quality control test specimens - modified cubes, beams or cylinders. The performance of FRC materials can be characterized by two fracture parameters - fracture toughness (which gives the resistance to cracking) and toughness index (which quantifies the post-first-crack toughness).

The shear strength results are similar for the various test geometries used in the study. The shear strength of steel FRC mixes is shown to be independent of fibre content whereas the shear strength of polypropylene FRC decreases with increasing fibre content and the shear strength of glass fibre reinforced concrete increases with increasing fibre content. The post-cracking toughness increases uniformly with increasing fibre content over the range of fibre contents studied. This increase in toughness is observed for all three fibres - steel, polypropylene and glass fibre.

Keywords: cracking (fracturing); fiber reinforced concretes; glass fibers; metal fibers; polypropylene fibers; shear strength; tests

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INTRODUCTION

In fracture mechanics, there are three basic modes of crack extension as shown in Fig. 1. The three types of crack extension are as follows :

- (a) Mode I or opening mode: the crack surface displacements are normal to the crack plane
- (b) Mode II or edge sliding mode: the crack displacements are in the crack plane and normal to the crack border
- (c) Mode III or tearing mode: the crack surface displacements are in the crack plane and parallel to the crack border

Most of the early fracture tests developed for concrete concentrated on the Mode I type of failure. However, in recent years shear failure has received greater attention. Fibrous composites are generally weak in shear and hence Mode II and Mode III fracture of composites is an important area for future research. Shear failure is also important in many concrete structures including deep beams, prestressed concrete members and corbels.

The development of FRC materials in the 1970s occurred at the same time as fracture mechanics was being used increasingly in the study of the fracture characteristics of more ductile materials. Figure 2 shows a typical load-deflection response from a fracture test on a notched specimen. The fracture toughness of a material in any fracture Mode may be determined from the initial linear response where Linear Elastic Fracture Mechanics (LEFM) conditions are assumed. The validity of the application of fracture mechanics theory in concrete technology will not be discussed here - a review of the application of fracture mechanics to concrete has been given by Swamy(1) and a large number of papers in this field have been published in recent years.

It is well established that FRC materials are not particularly stronger in compression than plain concrete. The main advantages of FRC over plain concrete include :

- (a) improved toughness or apparent "ductility" and
- (b) improved impact resistance

Although impact tests can be used to show the enhanced properties of FRC materials, they have not been widely used until recently since they are expensive and give a high degree of variability in their results (2,3). Therefore much research effort has been given to the development of various test methods for evaluating the toughness of FRC materials. The enhanced toughness of FRC materials is reflected in many ways including improved cracking resistance, improve post-first-crack strength and its ability to absorb large amounts of energy during fracture.

A large number of test methods have been proposed in recent years for measuring the performance of FRC materials. In 1978, ACI Committee 544(4) reported on proposed "Measurements of Properties of Fiber Reinforced Concrete". New test methods were suggested for toughness-energy absorption, impact strength and workability as these are the three properties which required detailed study. Although the slump test is a fast and economical measure of workability for plain concrete it is not an acceptable method for measuring the relative workability of FRC mixes. ACI Committee 544 recommended the use of either the Vebe test or the Inverted Cone test for determining the workabilities of FRC mixes.

ACI Committee 544 proposed evaluating toughness from the load-deflection graphs obtained in modulus of rupture tests. The toughness index proposed was defined as the ratio of the area under the load-deflection curve out to 0.075in (1.9mm) to the area under the load-deflection curve of the fibrous beam up to the point of first crack. The ACI report recommended the use of a dimensionless toughness index since it eliminates the need for conversion between units. The ACI definition of toughness index gives results which are dependent on specimen size whereas dimensionless indices are expected to be independent of specimen size. The main advantage of the ACI toughness index is that it is readily obtained from a standard specimen and thus it does not require a new test geometry or new testing equipment. Furthermore, the index readily discriminates between high and low-energy-absorbing fibres.

If a new test is to be readily accepted by industry it must be simple, straightforward and result in similar results irrespective of the laboratory and the operator. It should be noted that similar results can only be ensured if similar machines are used, e.g. similar stiffness characteristics or control of deflection rates. In concrete technology, it is most desirable to have tests based on existing test specimen geometries i.e. cubes, beams or cylinders. In the work reported here on shear test specimens, all specimens are modified cubes, beams or cylinders and are thus ideal specimens for commercial development. All the test specimens may be described as fracture specimens since they use notched specimens and may be used to evaluate fracture toughness results.

FRACTURE MEASUREMENTS

The fracture toughness in any fracture Mode may be determined from the load at cracking (maximum load on the linear portion of the load-deflection curve, see Fig. 3), provided that certain conditions, discussed later, are satisfied. It is proposed that the fracture characteristics in Mode I and Mode II fracture can be represented by the two following properties :

- (a) fracture toughness (K_{IC} or K_{IIc}). Where the expression for K_{IIc} has not been determined, the Mode II fracture toughness may be replaced by the shear strength in Mode II.
- (b) toughness index (Mode I or Mode II - see Figs. 4,5 and 6)

Fracture toughness or shear strength gives the resistance of a material to cracking while the toughness index gives an indication of the post-cracking performance of the material. The toughness index gives the relative importance of area B in Figs. 5 and 6.

The main difficulties associated with the application of fracture mechanics theory in concrete technology are the effects of slow crack growth that may occur during testing and the large size of the test specimens required to ensure valid fracture toughness results. The influence of slow crack growth under Mode I loading conditions has received considerable attention. Swamy(5) studied steel, glass and polypropylene FRC beams in a very stiff testing machine and concluded that slow crack growth cannot be neglected in any fracture toughness test. A similar conclusion was reached by Brown(6). It is proposed here that the importance of slow crack growth varies with the relative stiffness of the test specimen and the testing arrangement and, in particular, with the strain energy stored in the test specimen prior to crack initiation. Mode II shear test specimens have a large amount of strain energy stored in them prior to fracture. The specimens fail with unstable crack propagation accompanied with a considerable amount of noise i.e. a typical brittle type of fracture and hence slow crack growth is not considered to be a major factor with shear strength testing.

The second potential difficulty associated with the application of fracture mechanics in concrete technology is the minimum size requirement for the test samples. The test specimens should be sufficiently large to ensure that the plastic zone which may develop at the crack tip is not large in comparison with the crack length, specimen thickness, uncracked ligament etc. This potential problem is enhanced with FRC materials where the aggregate size and/or fibre length may be relatively large compared with the uncracked ligament length. This effect is best investigated by varying the overall size of the test specimens and introducing various notch depths into the specimens. The influence of test specimen size (for Mode I specimens) has been investigated at Cardiff and the results confirm the work of Mindess(7). The fracture toughness values increase with increasing specimen size - but only for large test specimens (300x300x100mm specimens or greater). For smaller specimens the fracture toughness values are approximately constant and independent of notch depths used (8). This variation is contrary to that expected from fracture mechanics theory and requires further detailed study.

Most of the shear tests carried out to date and reported here have concentrated on the determination of the shear strength rather than Mode II fracture toughness. The inplane shear strength, τ , is determined from the load at cracking as follows :

$$\tau = \frac{\text{Cracking load}}{\text{Area of cracked surface between roots of notches}}$$

The next stage in the development of the Mode II fracture specimens reported here will be a detailed finite element analysis

to obtain suitable expressions for the Mode II stress intensity factor K_{II} .

In view of the ability of FRC materials to absorb large amounts of energy under various service conditions, considerable research work has been made in the last decade to arrive at an acceptable method of evaluating the enhanced toughness. The proposals of ACI Committee 544 for measuring toughness (4) have not been readily accepted. Johnston(9) introduced two definitions of toughness index as shown in Fig. 4. The first definition is given by the ratio of total area at a given multiple of first-crack deflection to the area up to the point of first-crack. The second definition is given by the ratio of only the total post-first-crack area to the area up to the point of first-crack. The first definition gives a minimum toughness of unity and is referred to as the base-unity system whereas the second definition gives a minimum toughness of zero and is referred to as the base-zero system. Johnston (10) has also investigated the precision of flexural strength and toughness parameters for steel FRC and concluded that the use of multiples of first-crack deflection reduce the variation in toughness index results compared with the results obtained using the ACI method.

The toughness results reported by Johnston in 1982 were all obtained from modulus of rupture specimens. He concluded (9) that "there is a need for a definition of toughness that accommodates changes in specimen cross section, span and mode of loading and takes account of changes in matrix properties that influence deflection. It should not be based on arbitrarily chosen levels of load or deflection... Finally, the variability of any toughness parameter complying with these requirements should be reasonably low". The above requirements have been satisfied to a large extent by the toughness index proposed in 1982 by Barr et al (11). The definition of the toughness index is given in Fig. 5 and it differs from the ACI definition in four ways :

- (a) There is no restriction on the type of test specimen used
- (b) A prescribed deflection is not used
- (c) The definition gives an upper (unity) and a lower-bound value (0.25) to the results
- (d) The definition may be used for various fracture Modes

An alternative definition of toughness index has been proposed recently (12) and is illustrated in Fig. 6. This new definition is similar to the base-zero definition proposed by Johnston whereas the definition given by Fig. 5 is similar to the base-unity definition proposed by Johnston.

The toughness indices defined by Figs. 5 and 6 are based on the load-deflection curves obtained out to twice the deflection at first-crack. Toughness index results have been obtained for various fibre reinforcement using the range of shear test specimens reported here. This type of toughness definition allows the toughness index to be measured for both Mode I and Mode II type of loading. Further details on toughness index measurements are given

in (12). These toughness definitions may also be used for GRC material (13).

TEST GEOMETRIES

A review of some of the shear test geometries proposed in recent years has been given by Liu et al (14) but only some of these geometries are applicable in concrete technology. A comparative study of the three shear test geometries shown in Fig. 7 has been given elsewhere (15). The three geometries are modified beams, cylinders and cubes i.e. the standard quality control test specimens. The modified beams and cylinders are similar, being double notched specimens loaded longitudinally to give a shear stress distribution along the length joining the roots of the two notches. In all cases the notches were cut at approximately 21 days. The stress distributions between the two notch roots are shown in Fig. 7(b). Although tensile stresses are created along this line, the tensile stresses are approximately an order of magnitude less than the corresponding shear stresses. The beam and cylinder specimens gave good results for shear strength but unfortunately considerable difficulty was encountered in producing shear failures in the geometry shown in Fig. 7(d) and this geometry is not recommended for further study. Most of the failures observed were Mode I type of failures rather than Mode II type of failures in the case of the cube specimens.

Following the study on the three geometries shown in Fig. 7, the main objective of the next stage of the work was to produce more compact shear test specimen geometries. Under certain circumstances it would be difficult to obtain either the short-beam shear specimen (Fig. 7(a)) or the cylindrical shear specimen (Fig. 7(c)). In the UK it is more likely that cubes would be available rather than beams and that concrete cores would be more readily available than cylinders. Furthermore, it is probable that many cores would not extend to 200mm in length and their diameter may be less than 100mm. The first compact shear test specimen geometry is illustrated in Fig. 8(a). The geometry is similar to the short-beam specimen. In this case the overall length of the specimen has been reduced from 200mm to 100mm i.e. a cube has been substituted for the short-beam specimen. Two equal and opposite 50mm deep notches were introduced as shown. In this study two slot separation distances, a , were used - 25mm and 35mm. The 25mm slot separation distance corresponds geometrically to the 50mm slot separation distance in the short-beam specimen (both having $a/H = 0.25$). Since the cube faces were flat and parallel, this test geometry was much easier to load than the corresponding short-beam test specimen.

The second compact shear test specimen geometry developed in this study is illustrated in Fig. 8(b). The cylinder has a diameter of 100mm and is also 100mm long. Thus its overall dimensions are similar to that of the cube shown in Fig. 8(a). The cylinder was subjected to a line load in the same way as loading is carried out in the standard indirect tension test for concrete. As in the case

of the cube, the load was applied along a smooth cast surface and was thus much easier to apply than in the case of the cylindrical specimen shown in Fig. 7(c). In the case of the cylindrical specimens shown in Fig. 8(d) three values of a were studied - 25mm, 30mm and 35mm.

Although all the tests reported here are for 100mm diameter specimens 100mm long, there is no need for the length to be maintained at 100mm. (Work is currently under way in the same laboratory on the use of this test geometry to study the shear strength of rocks. A number of rock samples are only 60mm to 80mm - due to the difficulties in producing sufficiently long cored samples). Furthermore, this type of test can be applied to the two broken parts obtained from the test illustrated in Fig. 7(c). In this case the available length of specimen will be approximately 75-80mm.

A number of other shear test specimens have also been proposed for fibrous cement composites. Oakley and Unsworth (16) described three test methods to determine the shear strength of GRC and a comparative study of four test specimen geometries suitable for evaluating the in-plane shear behaviour of fibrous composites has been given by Herakovitch et al (17). The shear test geometry shown in Fig. 8(c) is currently being developed at University College, Cardiff to study the shear strength of GRC materials (18). This geometry has already been used successfully by Bazant (19) to determine the shear strength of FRC beam specimens.

Work has recently started on the circumferentially notched cylindrical shear test specimen shown in Fig. 8(d). In countries where cylinders rather than cubes are the traditional quality control specimens, this geometry has obvious advantages. Furthermore, this geometry is ideal for use with concrete cores. (Cylindrical test specimen geometries may also be used for rock cores and the geometry shown in Fig. 8(d) is a potential geometry for a number of civil engineering materials). The potential compactness of this geometry is currently under investigation. One of the main advantages of this geometry is that the notch position and depth may be accurately determined during specimen preparation.

EXPERIMENTAL DETAILS

A nominal Grade 50 (i.e. 50N/mm² compressive strength) standard mix has been used throughout the study of the test geometries shown in Figs. 7 and 8. The mix proportions (by weight) were 1:1.8:2.8 of cement:fine aggregate:coarse aggregate. The cement was Ordinary Portland Cement, the fine aggregate was Zone 3 sea dredged sand and the coarse aggregate was 10mm single size crushed limestone. The water/cement ratio was kept constant at 0.50 and the mix gave adequate workability for plain concrete and all the fibre reinforced mixes used in the studies. In some of the more recent work both 5mm and 20mm coarse aggregate have been used.

Various fibres have been added to the above mix. The main fibre additions include :

- (a) polypropylene fibre of 12000 denier (700/kg) cut into 50mm single size strands
- (b) straight steel fibre 40mm long and 0.4mm diameter
- (c) crimped steel fibre of two types - 30mm long and 0.3mm diameter and also 40mm long and 0.3mm diameter
- (d) Duoform fibre 40mm long and 0.4mm diameter

All the mixing was carried out by a pan mixer. The constituents, excluding the fibres, were initially mixed. The fibres were then added in small amounts and the mix turned over after each amount was added onto the surface. This procedure was repeated until all the fibres had been added to the mix and ensured that "balling" of the fibres was kept to a minimum. Compaction was carried out by means of a vibrating table and the specimens cured under water at about 20°C for 28 days before testing. Notches were cut into the specimens at 21 days.

Most of the tests were carried out by means of a 1251 Model (4.5T capacity) Instron machine at room temperature (20°C). Some of the tests had to be carried out in an Avery-Denison hydraulic machine (60T capacity) due to the high loads required. In both machines the tests were carried out under deflection controlled conditions. The rate of loading was 0.5mm/min and the load-deflection graphs were produced autographically by the machines. Typical load-deflection graphs for various fibre reinforced concrete materials are shown in Fig. 3. The graphs show a linear response up to a point where the concrete matrix fails. Thereafter the load reduces to a level of residual strength which is provided by the steel or the polypropylene fibres. In the case of GRC the load continues to increase after the point of first-crack due to the high toughness of the material.

The GRC specimens were obtained from two sources. In the first instance specimens were prepared from commercially available sheets. In the latter stages the specimens were prepared in the laboratory using chopped strand glass fibre. A water-cement ratio of 0.6 was used for the cement matrix. Some slurry was initially poured into a mould and a layer of chopped strand worked into the slurry by means of a roller. This process was repeated until the required number of glass layers had been added.

RESULTS

The best results (using the test geometries shown in Fig. 7) were obtained from the short-beam and notched cylindrical test geometries (15). In both cases the more consistent results were obtained for notch separation distances of 40-50mm with the best overall precision being given by the double-notched cylindrical specimen. However, during the experimental study it became clear that a great deal of care had to be taken to align the cylindrical specimens (so that the roots of the two notches were along the line passing through the points of application of the loads) to ensure

the correct fracture mode. In a number of the initial tests a Mode I type of failure occurred. The greatest difficulty in the development of the compact-cube shear specimen (Fig. 7(d)) was in devising suitable methods of load application to the test specimen. Due to this and other difficulties only a limited amount of results were obtained for the cube specimen.

Typical results for shear strength of polypropylene FRC and steel FRC are presented in Figs. 9 and 10 respectively. Both Fig. 9 and Fig. 10 suggest that the shear strength given by the short beam tests are less than those given by the cylindrical specimens. However, the difference is probably due more to the method of compaction than to variation due to the geometry used. In the case of the cylinders, compaction took place in the same direction as the direction of loading whereas for the short beam specimens compaction took place at right angles to the direction of loading. Furthermore, in the case of the cylinders and the beams, compaction results in the fibres tending to concentrate into the horizontal plane with a corresponding slight reduction in the vertical plane as compaction takes place. Thus the effective fibre density tends to increase in the horizontal plane and reduce in the vertical plane. This effect is more likely to occur in the case of the steel FRC mixes.

In the case of cylinders, the failure plane joining the roots of the two notches passes through a zone of increased effective fibre density and the converse is true in the case of the beam specimens. This effect is shown clearly with the results for steel FRC, illustrated in Fig. 10. A further complication in the comparison of the cylinder and beam specimens is the fact that the cylinder results were obtained partly from 100mm diameter specimens and partly from 150mm diameter specimens. A size effect was observed in the results for the steel FRC mixes for the higher concentrations of fibre with higher shear strength values being achieved from the larger 150mm cylinders. The shear strength should be independent of the specimen geometry - however, the only method of testing this proposition would be by testing cores taken from large blocks of FRC mixes. Although Figs. 9 and 10 suggest that the shear strength of polypropylene FRC decreases with increasing fibre content whereas the steel FRC shear strength remains constant (or increases depending on the geometry used for testing) this conclusion should be tested by further work on concrete cores.

A general comparison of Mode I and Mode II fracture results has been given recently by Liu et al (14). Both fracture toughness and toughness index results for polypropylene and steel FRC were reported. The K_{IIc} results were shown to be independent of fibre content for both types of fibre. This result is in contrast to K_{Ic} results for steel FRC which increases with increasing fibre content. The definition of toughness index given in Fig. 5 (for Mode I loading condition) can also be used to define a Mode II toughness index. The loading was continued in the study of steel FRC until the load-deflection graph extended to twice the deflection at first

crack. Hence the Mode II toughness index was evaluated using the same ratio of areas as given in Fig. 5. The toughness index of steel FRC increases with increasing fibre content but only at half the rate which is achieved under crack opening conditions. These results are summarized in Fig. 11.

In developing the more compact shear specimens shown in Fig. 8(a) and Fig. 8(b), it was important to determine if the results for these two geometries were independent of the notch separation distance, a . Thus the possible effect of varying the distance ' a ' was an important part of the study. The results show that the shear strength is independent of ' a ' in the range of 25-35mm. The shear strength results obtained from these geometries together with the corresponding results obtained from short-beam and cylinder geometries for the polypropylene and steel FRC mixes are shown in Fig. 12 and Fig. 13 respectively.

Figure 13 shows that the shear strength of steel FRC is relatively unaffected by the fibre concentration for a fibre content in the range 0-2% by weight. In contrast, Fig. 12 suggests that the shear strength is reduced by the presence of the polypropylene fibre. This is due to the fibres creating areas of weakness (at the points where the shear cracks pass through the fibres) in the shear plane. However, once the shear cracks have formed, the fibres then contribute to the post-cracking toughness of the matrix.

In the case of both polypropylene and steel FRC there is good agreement for the results obtained using the compact specimens and the double-notched cylinders. In both cases the results obtained from the short-beam test specimen are lower than the other three values. This is probably due to the method of sample preparation, particularly in the case of the steel FRC mixes, where the density of the effective fibres in the area where shear takes place is reduced during compaction of the short-beam specimens. Furthermore, it is more difficult to load the short-beam specimens, which have been cut from 500mm long beams, with the same precision as that which may be achieved with the other three specimens. As indicated earlier, the compact cube and disc specimens are such that the applied loads are in contact with smooth parallel faces which are produced during the casting of the standard specimens.

The double-notched beam shear test specimen shown in Fig. 8(c) has been used by Bazant (19) to determine the shear performance of concrete beams. This geometry has been used by the author to study the shear strength performance of GRC which is reported in detail elsewhere, (18). The enhanced shear strength and toughness of GRC is illustrated in Fig. 14. Work is currently under way on the development of the test specimen geometry shown in Fig. 8(d).

Th results from these studies show that the shear strength is independent of the test specimen geometry and specimen dimensions. However, only a limited range of test specimen sizes

has been investigated in the work reported here and a detailed study of the effect of specimen size on the shear characteristics of FRC mixes has recently been initiated. The shear strength, in the case of polypropylene FRC and steel FRC, is relatively independent of the fibre content used in the range 0-1% fibre content by volume. In the case of GRC materials the shear strength increases with increasing fibre content. Some typical shear strength results are given in Table 1.

A detailed account of the toughness index results in shear is beyond the scope of this paper. Figures 15 and 16 illustrate the toughness index results with various fibre contents for the polypropylene and steel FRC mixes respectively. These two Figures show the results obtained from the two compact specimens illustrated in Fig. 8. Furthermore, these two Figures show results for two types of coarse aggregate - 10mm and 5mm aggregate size.

The toughness index results are in close agreement and the increasing toughness with increasing fibre content is similar for the two maximum sizes of coarse aggregate used and also for the two types of fibres used. Thus, although the shear strength of the polypropylene and steel FRC mixes appear to be independent of the fibre content (or show a small decrease in value with increasing fibre content in the case of polypropylene FRC) the addition of both types of fibres enhance the post-cracking toughness performance of the mixes. The increasing toughness in shear with increasing fibre content is as expected and less than that which occurs in a crack opening situation. The range of typical toughness index values in shear are given in Table 2.

CONCLUSIONS

Seven test specimen geometries used to evaluate the shear strength and post-cracking toughness of FRC are reported. All geometries were modified standard quality control test specimens - modified cubes, beams or cylinders. Most of the specimens were double-notched specimens loaded so that shear stresses were created in the zone between the roots of the two notches. The cube specimen and disc shear specimen are compact test specimen geometries. The cylindrical and disc specimens may be prepared either from standard cylinders or from concrete cores. Results are presented for both polypropylene and steel fibre reinforced concrete mixes and also GRC materials.

The performance of FRC materials may be characterized by two fracture parameters - fracture toughness and toughness index. The fracture toughness gives the resistance to cracking and the toughness index quantifies the post-first-crack toughness of the material.

The results for shear strength were similar for the various test geometries used in the study. The shear strength of steel FRC mixes was independent of fibre content whereas the shear strength of polypropylene FRC mixes shows a small decrease with increasing

content. The decrease in the shear strength of polypropylene FRC was probably due to the weakening of the matrix by the presence of the fibrillated fibres cutting across the shear failure plane. On the other hand, the shear strength of GRC increases with increasing fibre content. The results suggest that the various test geometries (with the exception of that shown in Fig. 7(d) give shear strength results which are independent of the test geometry.

The results show that a great deal of care must be exercised in the interpretation of the shear strength values. Both compaction (which results in a variation of the effective fibre density) and size effects (observed between 100mm and 150mm diameter cylinders) have an effect on the shear strength results. The results should be independent of geometry and size effects and hence further work is required in this area of study. These effects should be studied by casting large cubes of various FRC mixes and sampling a range of cores taken from the cubes.

The post-cracking toughness increased uniformly with increasing fibre content over the range of fibre contents studied. This increase in toughness index was observed for all three fibres-steel, polypropylene and glass fibre. The results show the importance of determining both the cracking strength and the post-cracking toughness of FRC materials. The shear strength alone does not show the enhanced toughness of FRC materials subjected to shear loading.

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TABLE 1--TYPICAL SHEAR STRENGTH RESULTS

Geometry	Type of Fibre	Fibre Content (% wt.)	Shear Strength (N/mm ²)
Fig. 7(a)	Polypropylene	0.1	6.42
		0.2	6.06
		0.3	5.35
Fig. 7(c)	Steel	1.0	7.6
		2.0	8.9
		3.0	9.6
		4.0	8.5
Fig. 8(a)	Polypropylene	0.1	8.04
		0.2	7.38
		0.3	8.36
		0.4	8.90
Fig. 8(b)	Polypropylene	0.1	8.03
		0.2	7.80
		0.3	8.08
		0.4	7.00
Fig. 8(c)	GRC	2.0	6.92
		4.0	11.98
		8.0	13.10
		12.0	11.81
Fig. 8(d)	Polypropylene	0.3	7.49
		0.5	6.85
	Steel	2.0	8.05
		4.0	7.81

TABLE 2--TYPICAL VALUES FOR TOUGHNESS INDICES

Type of Fibre	Fibre Content	Toughness Index	
		Mode I	Mode II
Polypropylene	0.1	0.35	
	0.2	0.45	
	0.3	0.55	0.36
	0.4	0.62	0.40
Steel	1.0	0.49	0.40
	2.0	0.60	0.46
	3.0	0.73	0.47
	4.0	0.84	0.54
GRC	2.0	0.87	0.76
	4.0	0.87	0.80
	6.0		
	8.0	0.87	0.85
	10.0		
	12.0	0.90	0.81

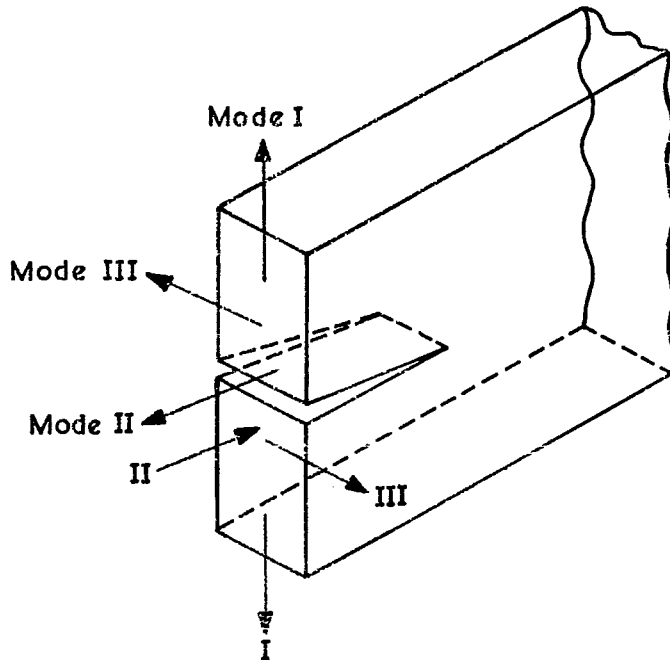


Fig. 1--Three modes of crack extension

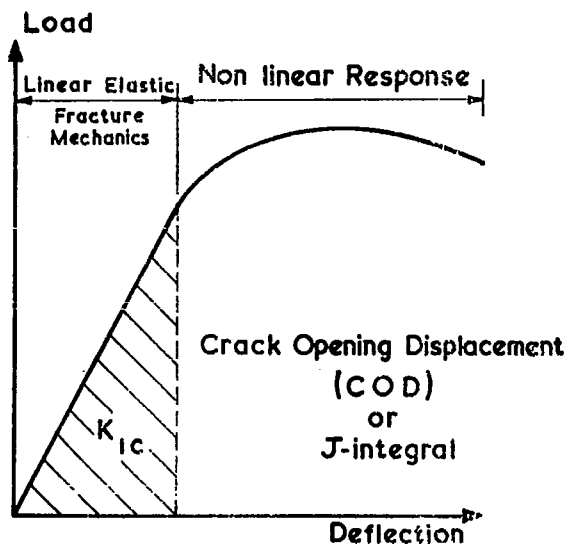


Fig. 2--Typical load-deflection response from fracture test of FRC material

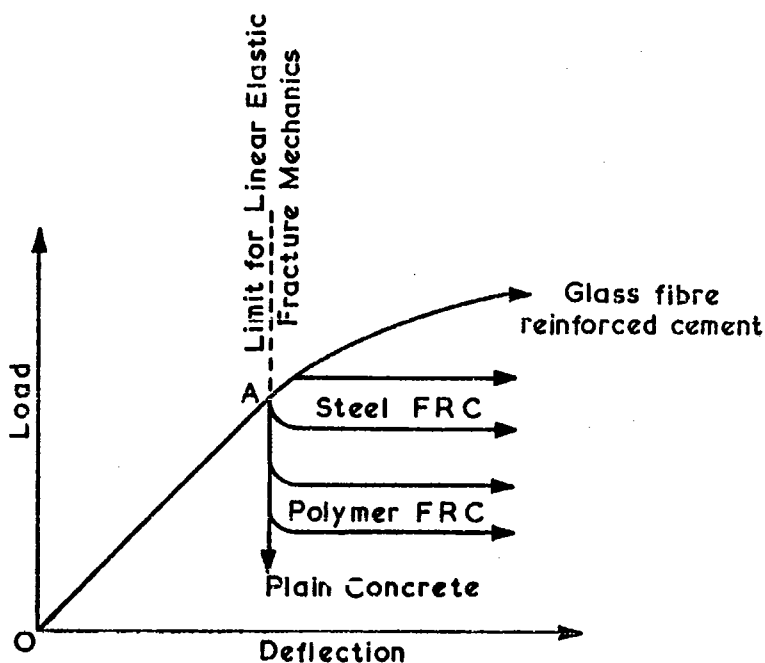
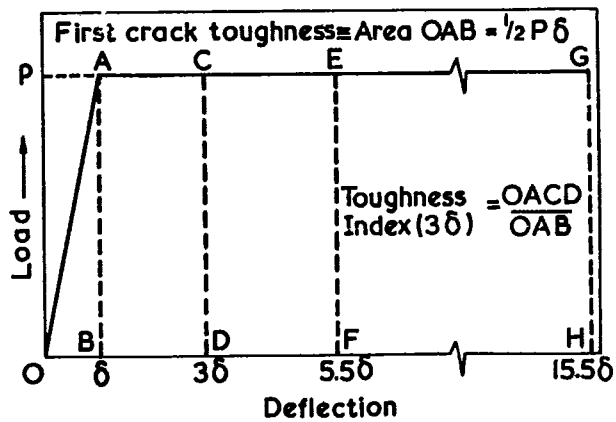
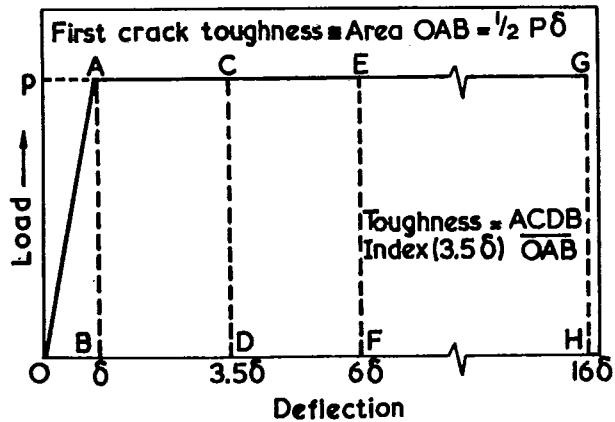


Fig. 3--Sketches of typical load-deflection curves for three types of fiber used in study



(a) Base-unity system



(b) Base-zero system

Fig. 4--Toughness Index definition proposed by Johnston