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Editor



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The primary goal of ACI Committee 118, Use of Computers, is to provide a forum for the presentation and dissemination of information pertaining to recent computer related developments which can be of practical use to the engineering community. Presentation is accomplished by sponsorship of technical sessions at the Annual ACI conventions and at committee meetings during ACI conventions. Dissemination is achieved primarily through ACI special publications, such as this one.

In reporting practical applications, many of the papers represent a technique or method which has been under investigation for years, and which has finally reached a degree of maturity which lends it to a practical application. As such, it should come as no surprise if depth coverage gave way to breadth of applications, and if theory is overshadowed by applications.

This volume contains the full length papers which were presented at the San Antonio ACI Convention in March 1987. Each author is gratefully acknowledged not only for contributing his paper, but also for reviewing another one.

Finally, special thanks should be extended to William Price for providing the leadership and guidance which were essential for the success of our committee.

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SP 106-1

Computer Analyses of Time-Dependent Behavior of Continuous Precast Prestressed Bridges

by J.D. Glikin, S.C. Larson, and R.G. Oesterle

Synopsis: Design and construction of bridges composed of simple-span, pretensioned girders made continuous for composite dead and live loads have become widespread. The design of these structures in the United States has been generally based on the procedure outlined in "Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders," published by the Portland Cement Association (PCA) in 1969. Although existing bridges designed by this procedure are generally performing well, it is believed that this method may not accurately predict the true behavior of these structures.

One of the major uncertainties in the design of these structures is prediction of positive and negative moments in the cast-in-place connections at the piers. This uncertainty is due to the different loading and construction stages, time-dependent effects, and details used to make the connections.

To resolve such uncertainties, Construction Technology Laboratories, Inc. has conducted an analytical study under the sponsorship of the National Cooperative Highway Research Project 12-29. The objective was to develop guidelines for more rational design of the continuity connections.

This paper summarizes results of an extensive parametric study to consider the effects of (1) construction sequence including simple span behavior before and continuous behavior after casting the deck and diaphragms, (2) time-dependent behavior including concrete creep and shrinkage, and steel relaxation, (3) live load applied at any stage of service life, (4) cracking resulting from both positive and negative moment including "tension stiffened" stress-strain relationships for reinforcement, and (5) closing of cracks when combined dead load plus time-dependent moments are reversed by application of live loads.

Keywords: bridges (structures); computer programs; continuity (structural); cracking (fracturing); creep properties; girders; loads (forces); moments; precast concrete; prestressed concrete; pretensioning; shrinkage; stress relaxation; structural analysis.



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INTRODUCTION

Background

Application of precast, prestressed girders to bridge construction started in the United States in the early 1950's. Use of pretensioned I-girders with cast-in-place concrete decks grew rapidly. Until the early 1960's, bridges built with pretensioned I-girders and cast-in-place concrete deck were designed as simply supported spans. However, longitudinal reinforcement placed in continuous deck slabs above the piers provided negative moment capacity. Therefore, these I-girders could be considered as partially continuous for negative moments at the piers. The degree of continuity depends on the time-dependent effects and the positive and negative moment connection details provided at the piers.

In a pretensioned member, prestress will usually cause the member to camber. If the member is simply supported, the ends of the member will tend to rotate, as shown in Fig. 1(a). When members are made continuous through the deck and pier diaphragms, the ends of the pretensioned girder are restrained from rotating. As a result, a positive restraint moment, as shown in Fig. 1(b) may occur at the pier. Positive moment also occurs at the piers when alternate spans have live loads. Reinforcement for positive moment connection is designed for

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the summation of positive moment due to time-dependent effects and live load application. Construction of the positive moment connection detail is generally expensive and time consuming.

In 1961, the Portland Cement Association (PCA) conducted an experimental research program on this type of bridge (1). The research program studied the influences of creep in the precast girders and differential shrinkage between the precast girders and the cast-in-place deck slab on continuity behavior after an extended period of time. As a result of these studies, procedures were developed for design of the positive moment connection and the negative moment reinforcement over supporting piers (2).

There are several uncertainties associated with the PCA procedures. Some of the uncertainty stems from the simplifying assumptions made in the PCA procedures. One assumption is that girder concrete and deck concrete have the same creep and shrinkage properties. This would not generally be the case, particularly if the sequence of construction results in significantly different ages between the girder, diaphragm, and deck concrete. Different concrete mixes and curing conditions for girder, diaphragm, and deck concretes also cause differences in creep and shrinkage properties. Also, for the PCA simplified analyses, the continuity connections are considered to have zero length and to be fully rigid. Full continuity is assumed in calculation of live-load positive and negative moments. The actual connections have finite lengths and rotational stiffnesses. The moment of inertia of the reinforced concrete section at the connection after cracking from either positive or negative moment will be significantly lower than the prestressed girder section. In addition, when positive restraint moment from time-dependent effects causes cracking in the diaphragm concrete, these cracks must close before the full section becomes effective for negative live load moment.

NCHRP Project 12-29

Since the PCA research was completed over 20 years ago, there have been significant advancements in understanding of material behavior and in methods of analysis. Therefore, CTL is conducting NCHRP Project 12-29 to investigate the behavior of precast, prestressed girders made continuous and to develop guidelines for determining design moments commensurate with the degree of continuity developed at the piers.

From a questionnaire survey carried out as a first task to Project 12-29, it was learned that there is significant variation in application of this type of bridge design by various state Departments of Transportation. This is particularly true with respect to the positive moment design of the continuity connection. Current practice includes:



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1. Designing and providing positive moment reinforcement or using standard details at the pier connection and considering negative moment continuity for reduction of live load positive moment near midspan.
2. Designing and providing positive moment reinforcement or using standard details but ignoring negative moment continuity for reduction of live load positive moment near midspan.
3. Providing no positive moment reinforcement but considering negative moment continuity for reduction of live load positive moment near midspan.
4. Providing no positive moment reinforcement and ignoring negative moment continuity for reduction of live load positive moment near midspan.

Another initial task of Project 12-29 was to develop and verify analytical procedures used to determine the degree of continuity and resulting moments. Descriptions of the computer program used in the project and its capabilities are given in the next section. Comparisons of analytical results and test observations for 1/2-scale bridges are given in Ref. 3.

The next major task of Project 12-29 was to carry out extensive computer analyses for a parametric study of the effects of various amounts of continuity reinforcement, different construction sequences, and variation in time-dependent material properties for concrete and prestressing steel in bridges with a range of girder type, span, and spacing. This paper summarizes the results of these computer analyses. The primary factor used to evaluate and compare results was the response to live load applied at various stages of service life. Implications of parametric study results on design procedures for this type of bridge are also discussed in the paper.

COMPUTER ANALYSES OF TIME-DEPENDENT RESPONSE

Program Description

Time-dependent deformations and restraint moments induced in multispan bridges built of prestressed girders made continuous were studied using a modified version of computer program PBEAM (4). This program, developed by C. Suttikan, is capable of analyzing composite prestressed concrete structures of any cross-sectional shape having one axis of symmetry. The program accounts for the effects of nonlinearity of stress-strain responses of materials and variations with time of strength, stiffness, creep, and shrinkage of concrete, and relaxation of steel. A step-by-step method is used in the time-dependent analysis with a tangent stiffness method implemented for solving nonlinear response.

Precast, prestressed bridge girders with composite cast-in-place decks are modeled using a discrete element method as developed by Hays and Matlock (5). Element deformations and forces are estimated by analyzing stress-strain relationships of a series of rectangular fibers distributed over the depth of the cross-section. It is assumed that strain in each fiber is constant at the centroidal axis of the fiber and strain distribution varies linearly through the depth of the section. For each time step, equilibrium at each element is maintained by determining the time dependent stress corresponding to the level of strain in each fiber. Stress multiplied by area is summed over all fibers and force equilibrium is checked. If necessary, the strain distribution is adjusted and the process is repeated until forces balance.

The A.C.I. Committee 209 (6) procedures to estimate the time variation of compressive strength, creep, and shrinkage of concrete and relaxation of prestressing steel are used in the PBEAM computer program. The rate of creep method and method of superposition are available in the program to account for the effect of concrete creep on the distribution of stresses in a section. The method of superposition was used for the studies conducted in this project.

The PBEAM computer program allows unlimited flexibility in analyzing various construction sequences and live load applications. The analysis accounts for construction sequence such that the simple span behavior of the girder before casting of deck and diaphragm, and continuous behavior thereafter are correctly modeled. Casting of deck and diaphragm can be done at any girder age and in any sequence. Live load can also be applied at any stage of service life and in any configuration. The program can therefore be used to investigate behavior under a wide variety of conditions likely to be encountered in actual use of this type of bridge.

The program's capabilities also allow a realistic analysis of the influence of diaphragm cracking on the behavior of this type of bridge. Depending on stress level and time-dependent material properties, the program accounts for cracking of girder and/or deck concrete under positive or negative moments. For each time step of the analysis, the program stores the stress-strain relationship in every fiber of each element. These stored conditions serve as the starting point for the behavior calculated for the succeeding time step. In this way, the program can follow both crack development and crack closing. This analytically models the situation shown experimentally in Ref. 1, in which cracks, which had opened at the bottom of the diaphragm under the influence of positive restraint moments, were closed by the negative moment induced upon application of live load. With initial application of live load, the diaphragm crack is open and girders behave essentially as simply-supported beams. With increasing live

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load and rotation at the diaphragm, the bottom crack closes and negative moment continuity becomes effective. The amount of rotation needed to close the crack is dependent on the creep and shrinkage properties of both the girder and deck concrete, the ages of the two concretes at the time of live load, the amount of restraint provided by the positive moment reinforcement in the diaphragm, and the girder type, span length and spacing. Therefore, the degree of negative moment continuity is dependent on all these parameters. With proper incremental application of live load, PBEAM correctly models the change in negative moment stiffness that accompanies closing of the diaphragm cracks, thereby providing an analytical tool to evaluate the effects of all these parameters.

A "tension stiffened" effective stress-strain relationship for top and bottom reinforcement at supports was also used in PBEAM in order to model behavior of reinforcing steel at cracked sections (7). These features allow PBEAM to accurately predict the changes in continuity at supports due to cracking of diaphragm and deck concrete as well as closing of cracks upon reversal of moments from application of live load.

Parametric Study

The program PBEAM was used to evaluate the effects of variations in several parameters on the behavior of bridges constructed of prestressed girders made continuous. The basic model used for the parametric study consisted of a bridge of four equal length spans, as shown in Fig. 2. Characteristics of the model which were varied included:

1. Girder type
2. Span length
3. Girder spacing
4. Positive moment continuity reinforcement
5. Time-dependent material properties
 - Concrete: Compressive strength
 - Creep coefficient
 - Shrinkage strain
 - Prestressing steel: Relaxation characteristics
6. Construction sequence
7. Girder age at application of live load

Values for these parameters were chosen to reflect the range of current practice as indicated by responses to the questionnaire. Following are brief discussions of the variables used in the parametric study.

Four girder types were used in the parametric study, AASHTO Types IV and VI, Modified Bulb Tee BT72/6 (8), and a box section. From questionnaire responses, AASHTO standard sections were most commonly used. The AASHTO-IV and AASHTO-VI

sections were chosen to represent girders used for medium to long span bridges. The AASHTO-IV girder was used with span lengths of 70 and 100 ft (21 and 30 m) in the parametric study. The AASHTO-VI girder was used with span lengths of 100 and 130 ft (30 and 40 m) in the parametric study. These AASHTO standard girders have heavy cross-sections for their span length capabilities. Questionnaire responses indicated that a variety of lighter girder sections are in current use, particularly for longer spans. The BT72/6 section, from Ref. 8, was chosen to represent girders with a lighter, more efficient cross section. This girder was used with span lengths of 100 and 130 ft (30 and 40 m) in the parametric study. The girder spacing used for these three girders throughout the majority of parametric study analyses was 8 ft (2.4 m). Preliminary studies indicated that girder spacings varying between 4.5 and 8 ft (1.4 and 2.4 m) had a minor influence on bridge behavior. The box section which was used in the parametric study was a 48 in. (1200 mm) wide by 54 in. (1400 mm) deep section with side wall thicknesses of 5 in. (130 mm) and top and bottom wall thicknesses of 3.5 in. and 6.5 in. (90 and 170 mm), respectively. The box girder span length and girder spacing used in the parametric study were 100 and 11.7 ft (30 and 3.6 m), respectively. For AASHTO-IV, AASHTO-VI, and BT72/6 Sections, deck dimensions were 92 by 6.5 in (2340 by 170 mm). For the box girder section, deck dimensions were 140 by 9 in. (3560 by 230 mm).

One of the main priorities of the parametric study was to determine the effect on bridge behavior of varying amounts of positive moment reinforcement at supports. For each analyzed combination of parameters, positive moment reinforcement was set equal to one or more of 0.2, 3.6, or 7.2 sq in. (1.3, 23, or 46 sq cm). These values represent a range of reinforcement equivalent to one No. 4 bar up to twelve No. 7 bars. The 0.2 sq in. (1.3 sq cm) reinforcement is intended to represent an unreinforced section. A small amount of reinforcement had to be used in the PBEAM program to obtain a satisfactory numerical solution. Throughout the remainder of this paper, bridges analyzed with 3.6 or 7.2 sq in. (23 or 46 sq cm) of positive moment steel will be referred to as reinforced. Bridges analyzed with 0.2 sq in. (1.3 sq cm) will be referred to as unreinforced.

Preliminary PBEAM results indicated that variations in deck and girder concrete compressive strength had a minor effect on bridge behavior. Therefore, for the majority of parametric study analyses, girder and deck concrete compressive strengths were 6500 and 4000 psi (45 and 28 MPa), respectively. Two ultimate creep coefficients were used throughout the parametric study. A value of 3.25 represented the high end of the range and 1.625 was used for the low end value. Variation in ultimate shrinkage strain was found to be less influential than variation in ultimate creep coefficient. As a result, a value

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of 600 millionths was used for most PBEAM runs for the ultimate shrinkage strain for girder and deck concrete. For most parametric study runs, relaxation characteristics for stress-relieved strand were used. Preliminary PBEAM results indicated that the difference in bridge behavior between stress-relieved and low-relaxation strand was insignificant.

From responses to the questionnaire, it was learned that current bridge construction practices include a number of aspects. Girder age at the start of bridge construction varies from about 10 days up to about 300 days. The majority of respondents indicated that construction started at girder ages less than 90 days. The sequence of construction of deck and diaphragm fell into three general categories:

1. Casting the deck and diaphragms simultaneously at various girder ages.
2. Casting the deck approximately 7 to 10 days before the diaphragm at various girder ages.
3. Casting the diaphragm approximately 7 to 10 days before the deck at various girder ages.

For purposes of the parametric study, the majority of PBEAM runs were conducted assuming the deck and diaphragm were constructed simultaneously. The girder age at which continuity was established for these runs was either 17 or 67 days. Additional runs were conducted with girder age at continuity up to 320 days. Several runs were also done to investigate behavior when the deck was cast 7 days before the diaphragm and vice versa.

To evaluate the behavior of bridges under service conditions, bridge response was analyzed for the live load configuration shown in Fig. 2. The pattern of live load was chosen because it is symmetric about the central support and produces the maximum negative moment at the central support. The magnitude of the loads are based on the AASHTO HS20-44 lane load (9) with a girder spacing of 8 ft (2.4 m), two girders per lane of traffic, and an impact factor for span length equal to 130 ft (40 m). For purposes of comparison, the magnitude of load was not changed to account for different impact factors for 70 and 100 ft (21 and 30 m) spans. For bridges with girder age at continuity of 17 days, live load was applied at 650 days. For bridges with girder age at continuity of 67 days or greater, live loads were applied approximately 30 days after continuity was established. For several runs with continuity at 67 days, live loads were applied at 650 days.

CONTINUITY CONNECTION EFFECTS FOR POSITIVE MOMENT

The effects of providing varying amounts of positive moment connection steel on typical bridge behavior when positive support restraint moments develop are illustrated in Figs. 3 and 4. The results shown are from PBEAM analyses for AASHTO-VI girders with 130 ft (40 m) span lengths and ultimate creep coefficient of 3.25. The girder age at which continuity was established was 17 days and live load was applied at 650 days. The results shown in Figs. 3 and 4 and the discussion following are typical of all girder types, span lengths, and ultimate creep coefficient combinations analyzed in the parametric study for continuity age of 17 days and live load applied at 650 days. In these cases, a large proportion of girder prestress creep occurs after continuity is established, causing positive restraint moments to develop. Also, negative restraint moments caused by differential shrinkage between deck and girder concrete are lessened due to the fact that a relatively small amount of girder shrinkage has occurred before the deck is cast. The effects of varying amounts of positive moment reinforcement on support moments and midspan moments under these conditions are described below.

Moments at Supports

Time-dependent restraint moments at the central support of the four span bridge up to day 650 are shown in the left side of Fig. 3. The right side of Fig. 3 indicates moments upon incremental application of live load at day 650. The three curves on the graph are from PBEAM analyses with three quantities of positive moment reinforcement at supports, A_s . With positive moment reinforcement provided, A_s equal to 3.6 or 7.2 sq in. (23 or 46 sq cm), positive restraint moments accumulate over time. At an age of 650 days, restraint moment at the central support is equal to about 8000 kip-in (900 kN-m). With a small amount of positive reinforcement, A_s equal to 0.2 sq in. (1.3 sq cm), positive restraint moments are negligible. With a small amount of provided reinforcement, the girder end rotation causes cracks to develop in the bottom of the diaphragm. The rotational stiffness of the diaphragm is therefore reduced and restraints against girder end rotation decrease. When a substantial amount of reinforcement is provided, girder end rotation is more effectively restrained.

Figure 3 also shows response of central support moments to application of live load at 650 days. Live load was applied in three increments to reach 25, 50, and 100% of the load. It can be seen that the behavior of the reinforced and unreinforced sections differ. For analyses of bridges with reinforced diaphragms, application of live load causes immediate decrease of central support moment. At 100% live load, the moments are

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negative. In general, moments decrease linearly up to 100% of live load. For the analysis of the unreinforced diaphragm bridge, application of live load causes only a slight decrease in central support moment. This results from the fact that girder end rotation due to the applied live load is not restrained because of the low rotational stiffness of the cracked diaphragm. The positive moment crack at the bottom of the diaphragm must close prior to inducing negative moment at the continuity connection.

Mid-Span Moments

Time-dependent interior span midspan moments up to day 650 are shown in the left side of Fig. 4. The right side of Fig. 4 indicates midspan moments due to incremental application of live load at day 650. The three curves on the graph are from PBEAM analyses with three quantities of positive moment reinforcement at supports. With provided positive moment reinforcement at the diaphragms, midspan moments increase due to the positive restraint moments which develop at the supports. On the other hand, with unreinforced diaphragms, midspan moments remain essentially constant through day 650. Since the girder ends are virtually unrestrained, small restraint moments develop at supports, and midspan moments remain approximately equal to the composite section simple span dead load moment of 44500 kip-in. (5030 kN-m). The difference in midspan moments at 650 days between the unreinforced and reinforced diaphragm analyses is due to the difference in restraint moments at supports.

Upon application of live load at day 650, midspan moments increase in amounts dependent on the degree of continuity at supports. For analyses of bridges with reinforced diaphragms, support regions have relatively high rotational stiffnesses. As a result, continuous sections at supports resist some of the applied live load and the increase in midspan moment is reduced. For the analysis with negligible positive reinforcement, support regions have very small rotational stiffnesses because of diaphragm cracking. Therefore, essentially all of the applied live load is resisted by bending of the composite girder and midspan moments increase substantially until girder end rotation results in closing of diaphragm cracks.

An important feature of the data shown in Fig. 4 is that the resultant moment, defined as the final moment after application of live load, is the same regardless of the amount of positive moment reinforcement provided. The resultant moment is equal to 66000 kip-in (7460 kN-m) for the bridge analyzed in Fig. 4. The average change in midspan positive moment with application of live load for the reinforced diaphragm analyses is 11000 kip-in. (1240 kN-m). For the unreinforced diaphragm analysis, change in midspan moment with application of live

load is 20000 kip-in. (2260 kN-m). However, due to differences in time-dependent restraint moments, net positive moments at midspan resulting from the combination of dead load, time-dependent restraint and live load are virtually identical. This is a feature of behavior that was observed within all the analyses carried out for the various parameters used in this study.

Table 1 shows parametric study results for bridges with 100-ft (30-m) span lengths with various girder cross-sections and ultimate creep coefficients. Resultant moments at interior span supports and midspan are given. These results, which are typical of parametric study results for age of continuity of 17 days, indicate that the resultant midspan moment is practically identical with and without positive moment reinforcement provided at the pier connections. Since providing positive reinforcement connections at piers is a time consuming and expensive procedure, the economical effect of minimizing or not providing positive moment steel at the piers should be considered in the design procedure of multispan bridges consisting of precast, prestressed girders made continuous.

Results in Table 1 also show influences of girder type and creep coefficient on behavior of this type of bridge. The four girders used in the parametric study represent a range of cross-sectional areas in use for medium to long span bridges. Table 2 shows girder areas, composite section areas, and simple span composite dead load moments at midspan for the four sections. As shown in Table 1, sections with larger areas and as a result, larger dead loads, have larger resultant midspan moments. These range from about 30000 kip-in. (3390 kN-m) for BT72/6 to about 48000 kip-in. (5420 kN-m) for the box section. Higher ultimate creep coefficients result in more positive restraint moments at the supports. Also, upon application of live load, the girders with higher creep have more positive resultant moments at supports and midspan.

CONTINUITY CONNECTION EFFECTS FOR NEGATIVE MOMENT

Results of the analyses indicate that negative restraint moments can be developed in the diaphragms at the supports dependent primarily on the age of the girder when the diaphragm and deck are cast. The maximum resultant negative moment that can be developed depends on the differential age between the girder and deck, and on the age of the bridge when live load is applied. Figure 5 shows typical time-dependent restraint moments at the center support for a bridge with continuity made by casting the deck and diaphragm at a girder age of 67 days. The data shown in Fig. 5 are for AASHTO-IV girders with 100-ft (30-m) spans and ultimate creep coefficient of 1.625. This figure is typical of parametric study results for a late age of

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continuity of 67 days with deck and diaphragm cast simultaneously, regardless of girder type, ultimate creep coefficient, and amount of positive reinforcement at supports. High negative restraint moments develop initially due to differential shrinkage between the cast-in-place deck and the precast girder. At the age of 67 days, a large portion of girder concrete shrinkage has occurred. Therefore, the difference between deck shrinkage and remaining girder shrinkage is large and dominates restraint moment development immediately after continuity is established. As time progresses, the effect of prestress creep reduces the large initial negative restraint moments. Figure 6 shows resultant moments at the center support for live load applied at 100 days and 650 days. It can be seen that negative resultant support moments are higher for the earlier age of loading. At 100 days, resultant moment at the central support is -15,300 kip-in. (-1730 kN-m) and at 650 days, the resultant moment is -10,800 kip-in. (-1220 kN-m). The difference in resultant moments is due to the difference between support restraint moments at 100 and 650 days. There is a corresponding difference in resultant midspan moments for live load applied at the different ages.

Table 3 shows parametric study results for interior span support and midspan resultant moments for 100- and 130-ft (30- and 40-m) girder lengths for various girder sections and ultimate creep coefficients. It can be seen that different amounts of positive reinforcing steel at supports have a negligible effect on resultant support and midspan moments. Since the steel is in the compression zone of the section under negative moment, varying amounts of reinforcement have a small effect on bending stiffness. For a given girder type/span length combination, PBEAM analyses in Table 3 indicate that resultant moments are independent of ultimate creep coefficient for age of continuity of 67 days and live loading at 100 days. This is because early age restraint moments are dominated by the negative restraint moment contribution of differential shrinkage.

Table 4 shows the effects of increasing the girder age prior to casting the diaphragm and deck. These results are for analyses of AASHTO-VI girders of 130-ft (40-m) span with ultimate creep coefficient of 1.625. Each of these runs were done with positive moment reinforcing steel of 0.2 sq in. (1.3 sq cm). With increasing girder age at continuity, negative resultant moments at supports increase by about 50% for continuity at 320 days compared to 67 days. The corresponding decrease of midspan resultant moment is about 20%. There is a structural advantage gained for design of the prestressed girders for positive moment by delaying casting of the diaphragms and deck. However, construction of the bridge is then delayed. Also, the design moment for negative reinforcement in the deck over the supports is increased by delaying construction.

Data in Figs. 5 and 6 and Tables 3 and 4 are for girders with a relatively low creep coefficient. As indicated in Fig. 5, the restraint moment may remain negative for the life of the structure. The response will be as a fully continuous girder for application of live load. As shown in Fig. 7, with high creep values and/or smaller age of girder at time of casting the diaphragm and deck, the restraint moment can become positive at late ages. At this time, some degree of continuity will be lost.

EFFECTS OF CONSTRUCTION SEQUENCE

Restraint moments at supports of bridges constructed of simple span prestressed girders made continuous are highly dependent on the sequence and timing of construction. The primary factor is girder age at the time continuity is established by construction of the cast-in-place deck and diaphragm. However, the sequence of deck and diaphragm construction, either simultaneous or one before the other, also affects the development of restraint moments.

From responses to the questionnaire, actual construction sequences fell into three general categories: deck and diaphragms cast simultaneously, deck cast before diaphragms, and diaphragms cast before deck. In some cases, respondents indicated that portions of deck in positive moment regions were cast before the deck in negative moment regions and diaphragms. For purposes of the parametric study, the majority of PBEAM runs were conducted for deck and diaphragm cast simultaneously at girder ages of 17 or 67 days. Resultant moments from PBEAM analyses for deck cast before diaphragms and diaphragms cast before deck are compared to results for simultaneous casting in Table 5. The girder age at continuity is the age at which the diaphragm is cast. The analyses for the data in Table 5 were made for AASHTO-IV girders of 100-ft (30-m) span length with ultimate creep coefficient equal to 1.625. For all PBEAM runs, the deck is cast over the full length of the girder at the indicated time.

Results in Table 5 show that for the construction sequence of deck cast before diaphragm compared to simultaneous casting, resultant moments at the support and midspan are more positive for both 17- and 67-day continuity. This is shown by comparing Runs 1 and 2 with Runs 6 and 7 and by comparing Runs 3 and 4 with Runs 8 and 9. When the deck is cast before the diaphragm, the two components contributing to negative restraint moments are reduced. When the deck is cast, girder end rotation due to the additional dead load of the deck is unrestrained. More importantly, end rotation due to differential shrinkage between deck and girder is unrestrained until the diaphragm is cast.



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place. As a result, the negative restraint moment component is reduced, and the positive restraint moment contribution is relatively larger. In turn, resultant moments after application of live load are more positive as shown in Table 5.

When the diaphragm is cast before the deck, resultant moments are only slightly affected. For Run No. 10 with a girder age at continuity of 67 days and deck cast at 74 days, results for loading at 100 days are virtually identical to Run No. 3 with simultaneous deck and diaphragm casting. For loading at 650 days, however, the resultant positive moment at midspan is reduced slightly by casting the diaphragm 7 days before the deck, Run No. 11, as compared to simultaneous casting, Run No. 5. With the construction sequence of diaphragm before deck, the diaphragm has attained a higher strength at the time the deck is cast. Therefore, the stiffer diaphragm resists more of the negative restraint moment from differential shrinkage between the newly cast deck and the restrained girder. The end result is that restraint moments and resultant moments after application of live load are somewhat more negative. This sequence increases the probability of establishing full continuity and reducing resultant positive moments at midspan but only slightly.

SUMMARY AND CONCLUSIONS

Step-by-step computer analyses were used to study simple-span precast, prestressed bridge girders made continuous. Creep and shrinkage of concrete, relaxation of prestressing steel and aging of the materials were taken into account in the analyses. A parametric study was conducted using different creep and shrinkage properties, girder types, girder spacing, span lengths, and construction sequences.

The following conclusions summarize results of the parametric study.

1. With continuity established at an early girder age, positive restraint moments, and the degree of continuity with application of live load are highly dependent on the amount of positive reinforcement provided in the girder connections at the supports. Time-dependent positive restraint moment generally induces a crack in the bottom of the diaphragm concrete. With application of live load, the positive moment crack must close prior to inducing negative moment at the continuity connection. The presence of positive moment reinforcement helps to maintain closed cracks, thereby increasing live-load continuity.



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However, the positive restraint moment resulting from the presence of the reinforcement in the support connection increases the positive midspan resultant moment. The effect of the lack of full continuity caused by not providing positive reinforcement in diaphragm and allowing the bottom diaphragm crack to open, is virtually balanced by the increased positive restraint moment. Therefore, the resultant midspan moments, which include moments due to dead load, restraint moments due to creep and shrinkage, and live-load moments, are virtually independent of the area of positive reinforcement provided in the diaphragm connections at the supports. There is no structural advantage for providing this positive moment reinforcement.

2. Continuity performance is highly dependent on the age of the girder when the diaphragm and deck are cast. High negative restraint moments can occur at the support connections when continuity is established at late girder ages. Full continuity for live load can be established depending on the age of girder at time of casting the deck and diaphragm and on the creep coefficient for the girder concrete.
3. Sequence of deck and diaphragm construction affects the development of restraint moments. Casting of the deck prior to the diaphragm increases resultant positive moments at midspan. Casting of the diaphragm before the deck only slightly decreases resultant midspan positive moments. There is no major economic or structural advantage to sequencing the casting of the deck and diaphragm. Simultaneous casting is the simplest construction procedure.
4. The degree of continuity available in simple-span prestressed girders made "continuous" for live load by connections at the pier diaphragms can vary from zero continuity to full continuity. The degree of continuity is dependent primarily on the age of the girder at the time of casting the deck and diaphragm, and on the creep and shrinkage properties of the concrete. Full continuity can be established by delaying casting of the deck and diaphragm. However, delayed casting increases construction time and also increases negative moments required for design of reinforcement in the deck over the supports. There is a need for a simplified analytical tool to enable designers to account for the various parameters involved to find a good balance between construction timing and design moments. A simplified computer program approximating the PBEAM analytical results is being developed by CTL as part of NCHRP Project 12-29 to provide the designer with this analytical tool.

**Documents**

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TABLE 1--RESULTANT MOMENTS FROM PARAMETRIC STUDY — 100-FT (30-M) SPANS, GIRDER AGE AT CONTINUITY 17 DAYS, AGE AT LOADING 650 DAYS

Girder Type	Ultimate Creep Coefficient	Positive Reinforcement, in. ²	Resultant Moments, kip-in.		
			At Support B	At Mid-Span	At Support C
AASHTO-VI	3.25	7.2 0.2	2940 490	39200 40200	-4750 - 160
	1.625	7.2 0.2	-960 -400	36000 36600	-7150 -6500
BT72/6	3.25	7.2 0.2	-100 1100	31100 31900	-6750 -6310
	1.625	7.2 0.2	-3130 -2110	28600 29200	-8700 -8390
AASHTO-IV	3.25	7.2 0.2	6150 320	35800 36000	320 400
	1.625	7.2 0.2	3030 840	33300 32700	-1770 290
Box	3.25	7.2 0.2	720 -640	48800 47700	- 580 40

METRIC EQUIVALENTS: 1 in.² = 6.45 cm², 1 kip-in. = 0.113 kN-m

TABLE 2--GIRDER SECTIONS USED IN PARAMETRIC STUDY

Girder Type	Girder Cross Section Area, in. ²	Composite Section Area, in. ²	Midspan Composite Dead Load Moment,* kip-in.
AASHTO-IV	789	1387	21700
AASHTO-VI	1085	1683	26300
BT72/6	701	1299	20300
Box	920	2180	34100

*For simple span length = 100 ft (30 m)

METRIC EQUIVALENTS: 1 in.² = 6.45 cm², 1 kip-in. = 0.113 kN-m

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TABLE 3--RESULTANT MOMENTS FROM PARAMETRIC STUDY — GIRDER AGE AT CONTINUITY 67 DAYS, AGE AT LOADING 100 DAYS

Girder Type	Span Length, ft	Ultimate Creep Coefficient	Positive Reinforcement, in. ²	Resultant Moments, kip-in.		
				At Support B	At Mid-Span	At Support C
AASHTO-IV	100	3.25	7.2 0.2	-16,100	19,800	-15,500
				-16,000	20,000	-15,300
		1.625	7.2 0.2	-16,100	19,800	-15,500
				-16,000	20,000	-15,300
BT72/6	100	3.25	7.2 0.2	-16,800	18,500	-15,200
				-15,100	19,300	-15,200
		1.625	7.2 0.2	-16,500	18,700	-15,000
				-14,900	19,500	-15,100
Box	100	3.25	0.2	-25,800	19,700	-27,200
AASHTO-VI	130	3.25	0.2	-18,700	46,500	-20,000
				-18,700	46,700	-20,000
		1.625	7.2 0.2	-19,000	46,300	-20,200
				-19,000	46,300	-20,200
BT72/6	130	3.25	0.2	-15,200	39,600	-18,500
				-15,200	39,600	-18,500
		1.625	7.2 0.2	-16,400	38,500	-19,400
				-15,300	39,000	-19,000

METRIC EQUIVALENTS: 1 ft = 0.305 m, 1 in.² = 6.45 cm²,
1 kip-in. = 0.113 kN-m

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TABLE 4--RESULTANT MOMENTS FOR VARIOUS GIRDER AGES AT ESTABLISHMENT OF CONTINUITY

Girder Age at Continuity, Days	Girder Age at Loading, Days	Resultant Moments, kip-in.		
		At Support B	At Mid-Span	At Support C
67	100	-19,000	46,300	-20,200
97	130	-21,200	42,900	-24,700
190	230	-25,300	40,200	-26,100
320	350	-29,900	36,000	-29,700

METRIC EQUIVALENT: 1 kip-in. = 0.113 kN-m

TABLE 5--RESULTANT MOMENTS FOR VARIOUS CONSTRUCTION SEQUENCES --
AASHTO-IV, 100-FT (30-M) SPANS, ULTIMATE CREEP COEFFICIENT = 1.625

Construction Sequence	Run No.	Girder Age at Casting, Days	Girder Age at Continuity, Days	Girder Age at Loading, Days	Positive Reinforcement, in. ²	Resultant Moments, kip-in.		
						At Support B	At Mid-Span	At Support C
Deck and Diaphragm Simultaneous	1	17	17	650	7.2	1230	33300	-5900
	2	17	17	650	0.2	250	32700	290
	3	67	67	100	7.2	-16100	19800	-15500
	4	67	67	100	0.2	-16000	20000	-15300
	5	67	67	650	0.2	-11700	24400	-10800
Deck Before Diaphragm	6	10	17	650	7.2	5320	37600	-3400
	7	10	17	650	0.2	390	36000	490
	8	67	74	100	7.2	-14200	22700	-11600
	9	67	74	100	0.2	-13900	23000	-11300
Diaphragm Before Deck	10	74	67	100	7.2	-16300	19800	-15400
	11	74	67	650	0.2	-13200	23200	-11700

METRIC EQUIVALENTS: 1 in.² = 6.45 cm², 1 kip-in. = 0.113 kN-m

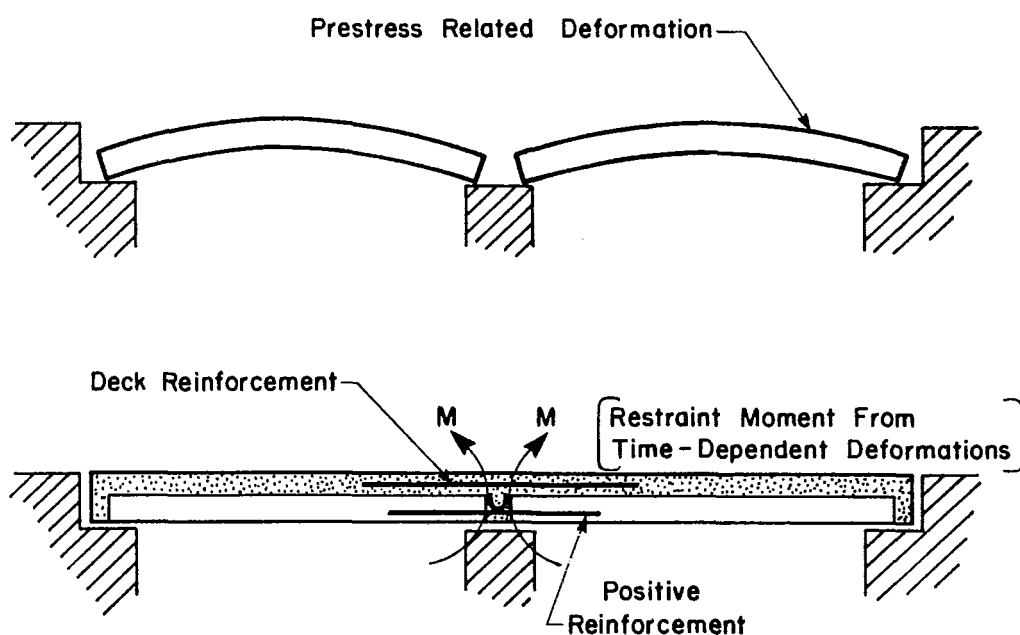


Fig. 1--Two-span bridge with precast, prestressed girders made continuous

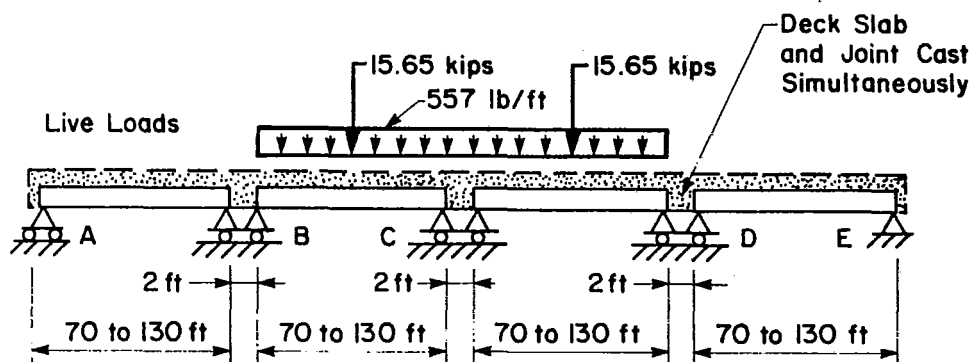


Fig. 2--Bridge with four spans of various girder types

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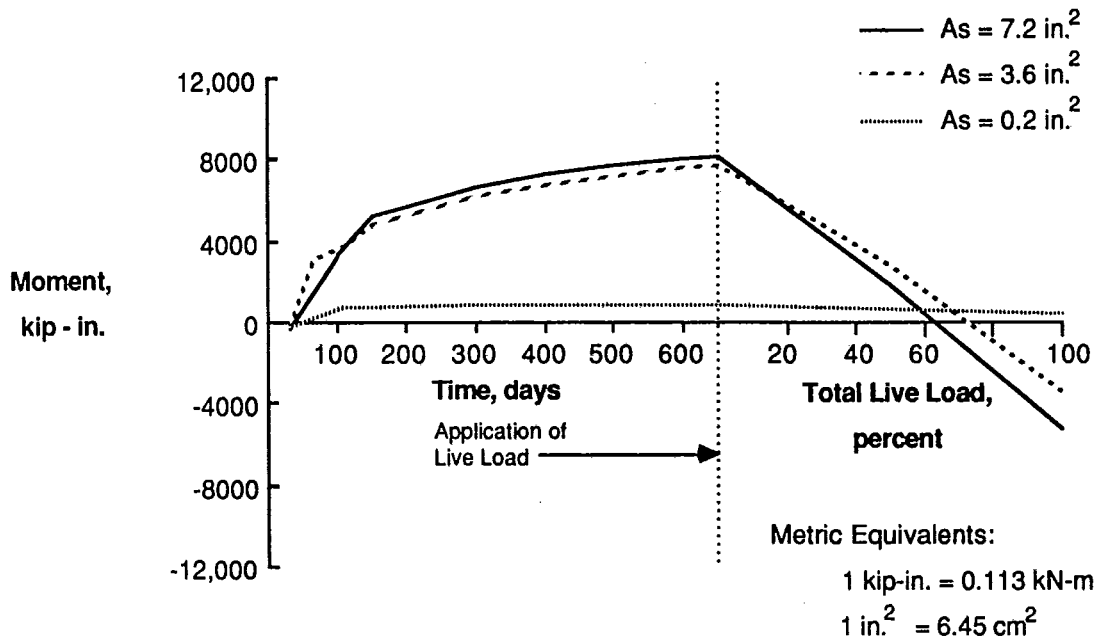


Fig. 3--Central support moments for continuity at girder age of 17 days, loading at 650 days

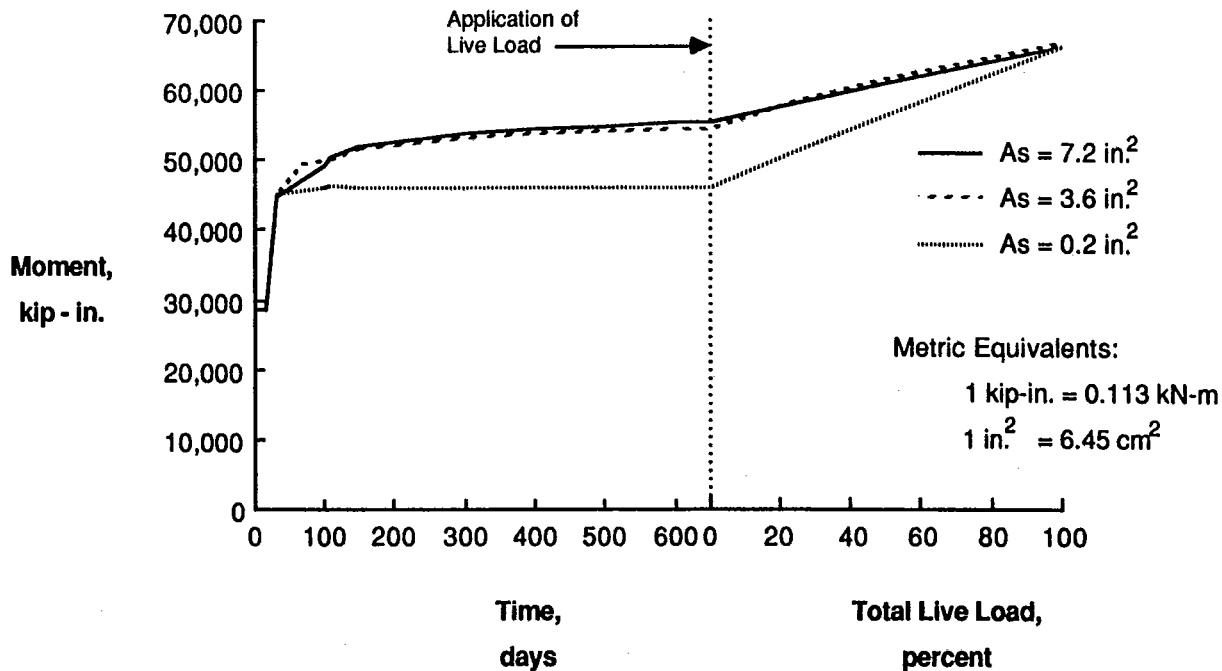


Fig. 4--Interior span midspan moments for continuity at girder age of 17 days, loading at 650 days

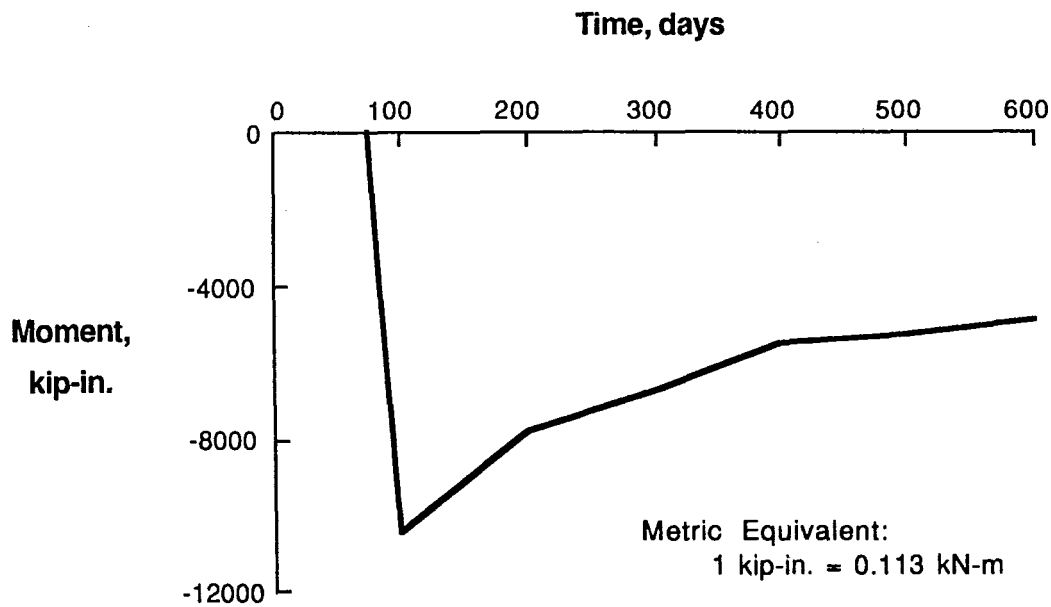


Fig. 5--Central support restraint moments for continuity at girder age of 67 days

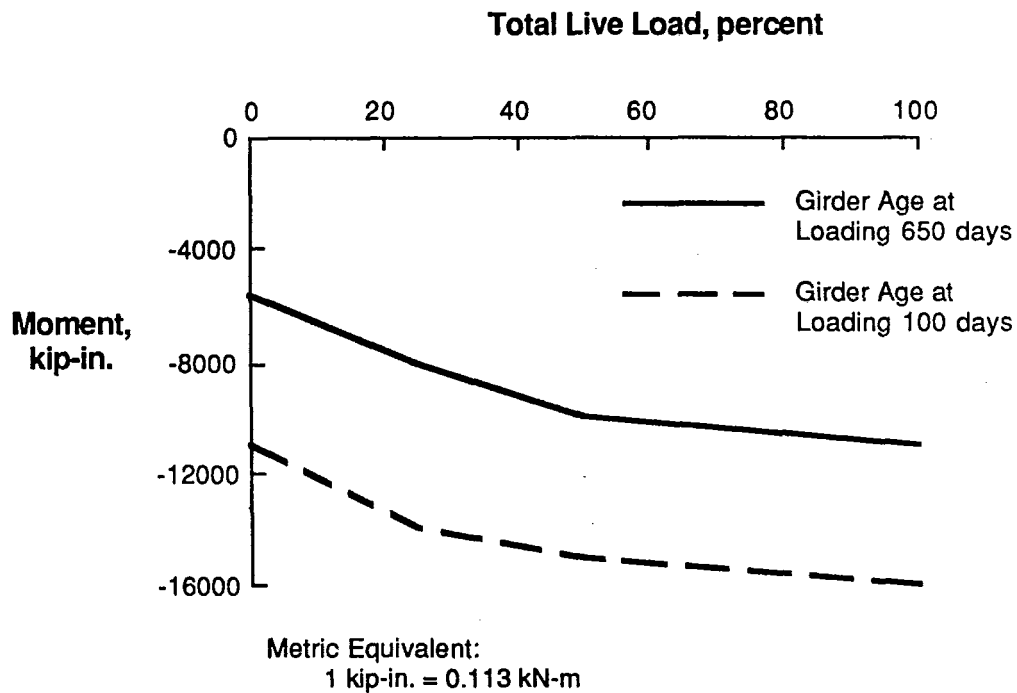


Fig. 6--Central support resultant moments for continuity at girder age of 67 days, loading at 100 and 650 days

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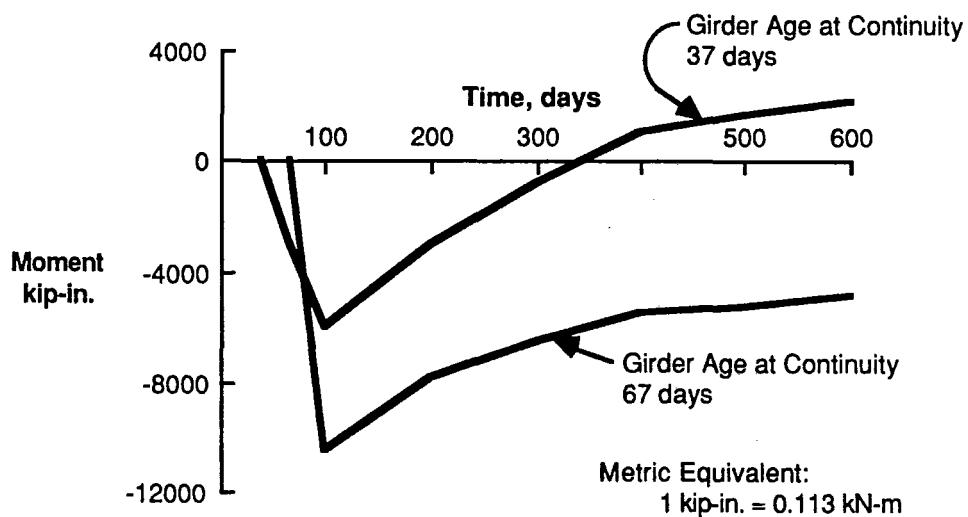


Fig. 7--Central support restraint moments for continuity at girder age of 37 and 67 days

SP 106-2

Substructure Modeling of Two-Way Slab Systems

by M. Hoit, E. Fagundo, and T. Hamilton

Synopsis: A substructuring technique to analyze equivalent frames in accordance with ACI 318-83 is presented. A more consistent implementation of the Equivalent Frame Method is formulated. The width of the column strip and the percentage of torsional stiffness required are studied. A program that will analyze a building system subjected to gravity as well as lateral loads is developed.

Keywords: beam-column frame; computer programs; connections; framing systems; lateral pressure; loads (forces); moments; stiffness; structural analysis; two-way slabs

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INTRODUCTION

It could be said that adequate modeling of a two-way slab concrete building system can only be achieved with a sophisticated three dimensional finite element computer program with plate-bending and frame elements. Where the forces recovered from the analysis are used for selecting member sizes and reinforcement and the displacements indicate the serviceability of the structure. Unfortunately, this type of analysis is rarely cost effective for the small consulting office when considering a standard concrete building system. As a result, some type of analytical simplification of the building system is required. This is usually accomplished by subdividing the structure into a system of two-dimensional frames. In the current American Concrete Institutes' "Building Code Requirements for Concrete Structures" (ACI 318-83) this procedure is described as the Equivalent Frame Method (1).

The objective of this paper is to describe the development and implementation of a substructuring technique to analyze equivalent frames in accordance with ACI 318-83. Program TWSLAB will analyze a building system subjected to gravity as well as lateral loads. Lateral load analysis requires the use of the displacement compatibility method, which connects each floor with a relatively stiff truss bar thereby ensuring equal displacements at each floor under lateral loads.

TWSLAB was developed from an existing structural analysis program STAN (2), which utilizes the Direct Stiffness or Finite Element Method for three dimensional truss and frame elements as well as membrane elements. Conversion of STAN to TWSLAB was

accomplished by developing subroutines to perform the following tasks:

- (1) Read the geometric configuration of the concrete two-way slab system including:
 - Flat plate with drop panels and column capitals.
 - Two way slab with longitudinal and transverse beams.
 - Any combination of the above.
- (2) Create the element stiffness for the slab-beam and column from their respective geometric configurations in accordance with ACI 318-83.
- (3) Recover the forces, moments and displacements necessary to check design and serviceability.

This three dimensional version is used for all the studies performed on the modified Equivalent Frame Method. Further simplification to the program can be made consistent with the recommendations found in the conclusions of this paper to create a production version.

DERIVATION OF THE SLAB-BEAM SUBSTRUCTURE

An orthogonal equivalent frame system consists of two types of elements, the slab-beam element and the column element. In TWSLAB it is assumed that the systems are orthogonal frame systems, no transformation from local to global coordinates is required. All substructure stiffness matrices are formed in a global coordinate system with slab-beams and columns in horizontal and vertical positions respectively. Both exterior frames as well as interior frames can be considered.

Torsional Member

In two-way slab systems, the major difficulty lies in modeling the beam-column connection. The efficiency of the moment transfer varies with the width of the column. For example, in Figure 1a the efficiency of the moment transfer between the wall and the slab is 100 percent. However, the efficiency of the beam-column connection in Figure 1b approaches zero as c_2 approaches zero. It is this discrepancy in moment

transfer which led to the development of a torsional member for "softening" the analytical connection between the slab-beam element and the column element. Vanderbilt and Corley (4) present three approximate methods for modeling the connection stiffness. They are:

- (1) Equivalent beam width model
- (2) Transverse torsional member model
- (3) Stub-beam model

Vanderbilt and Corley indicate that the most generally applicable model is (2) because it was calibrated using two flat plates, two flat slabs and two two-way slabs. The transverse torsional member model is incorporated in ACI 318-83 for equivalent frame analysis. It is this torsional member which differentiates the equivalent frame models in use today.

The ACI 318-83 code and commentary imply the use of the model shown in Figure 2 through the description of its components:

"The components of an equivalent frame shall comprise three parts: 1) The horizontal slab strip with any beams spanning in the direction of the frame 2) Columns or vertical supporting members, extending above and below the slab and 3) elements of the structure that provide moment transfer between the horizontal and vertical members."

Several methods were made available for simplification of the space frame into a plane frame. The first approach to simplification was the equivalent column method developed by Corley et al. (5) in 1961. This method divides the torsional element stiffness in proportion to the relative stiffnesses of the columns above and below. Each torsional stiffness, in the form of a spring, connects the appropriate column to the slab-beam (Figure 3a). This method was developed for gravity loads assuming a single story with the far ends of the columns fixed.

Vanderbilt (6) investigated the use of the equivalent column method and found poor performance under lateral loads. He proposed a new method known as the equivalent beam method (Figure 3b). The equivalent beam method divides the torsional element and allocates the partial stiffness to each beam on each side of the column in proportion to their relative stiffness.

The final step in this process was to develop an analytical system to model a complete concrete building equivalent frame system. Vanderbilt proposed a method for extending the analysis of one story segments to the full height of the building, allowing a full elastic analysis for the effect of lateral loads. Vanderbilt (6) developed the computer program EFRAME which utilized either the effective beam width model or the transverse torsional member model (with the option of the equivalent beam or equivalent column method). EFRAME was tested against the experimental results of a flat plate mortar model. In a later publication Vanderbilt and Corley (4) show the same test and recommend the use of the equivalent column method only for gravity load analysis and both the equivalent beam and equivalent column method for lateral loads with the equivalent beam giving slightly better (conservative) results for drift.

As an alternate to the equivalent column and equivalent beam method, Wang and Salmon (7) suggest the use of a model as shown in Figure 4. The stiffness of the column strip is placed in ABC, the stiffness of the two half middle strips in A'A'A' and the torsional stiffness in A'A, B'B and C'C. The model to used in program TWSLAB is similar to the one used by Wang and Salmon. The rational behind the choice is twofold. First is to create an element which can be used confidently for both lateral and gravity load analysis. Second is to create an element which more closely approximates the actual three-dimensional behavior of the two-way slab system.

Substructure Modeling of the Slab

The slab-beam element is modeled through the use of analytical substructuring and is shown in Figure 5a. It consists of 20 degrees of freedom and 8 elements. Elements 1 through 5 are standard frame elements with a rotational, shear and axial degree of freedom at each node. Elements 6 and 7 are torsional elements and will only transfer torsional forces. Element 8 is a bending member which has only rotational degrees of freedom at each end.

Inspection of ACI 318-83 shows that the moment of inertia changes a maximum of 5 times between the centerline of the adjacent columns as the result of the combination of beams, capitals and panels. This variation in stiffness is accounted for by placing five different elements representing the slab-beam between

the column centerlines. Each subelement is composed of the standard six degrees of freedom frame element. These subelements are connected rigidly at each change in moment of inertia and will readily transfer moment, axial and shear forces.

The 20 x 20 substructure stiffness and loads are condensed to the 8 degrees of freedom shown in Figure 5b. This condensed 8 x 8 substructure is considered to be the slab-beam element and is consistent with the direct stiffness method. It should be noted that the slab-beam element is interconnected to columns and other slab-beams at degrees of freedom (dof) 3, 4 and 5 and 6, 7, and 8. Testing has been completed to determine if dof 1 and 2 require coupling between adjacent slab elements or if the dof can be condensed prior to analysis in two dimensions.

A major assumption in developing TWSLAB is that the column strip stiffness is connected directly to the columns and the middle strip stiffness is connected to the columns through the torsional element. TWSLAB divides the column and middle strip before calculating the moment of inertia and the exact dimensions are used for calculating the moment of inertia. The percentage of the equivalent frame width which is to be used in calculating the moment of inertia for the column strip is denoted as γ . The remaining width is assumed to be the middle strip. γ is the fraction of the torsional stiffness, as defined in the Equivalent Frame Method, that is considered to be effective.

The moments of inertia for elements 1, 2 and 3 are calculated using the full section of the column strip including beams and panels. The moments of inertia for elements 4 and 5 are based on the moments of inertia for 2 and 3 respectively. ACI 318-83 requires an increase in stiffness of the slab-beam element to account for the finite size of the joint. For example, the formula required to compute the moment of inertia for element 4 (I_4) is:

$$I_4 = I_2 / (1 - c_2 / l_2)^2$$

Where:

c_2 = Column width

l_2 = Equivalent frame width

Because the only axial dof for the slab-beam substructure is located in the column strip, the full equivalent frame cross sectional area is utilized for the axial dof in each of the column strip subelement stiffness matrices. The rotational stiffness coefficients are added into the middle strip dof 13 and



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14. However, since there is no transfer of shear across the torsional elements, the shear stiffness coefficients for the middle strip must be added directly to dof 16 and 19.

In addition to the shear stiffness terms, the fixed end reactions for the middle strip are added into dof 16 and 19, while the fixed end moments are added into dof 13 and 14. This technique is based on the assumption that the torsional elements are rigid in bending and directly transfer the reaction from the middle strip to the column strip.

The uniform pressure load is reduced to a line load on the column strip and middle strip by the ratio of the relative stiffnesses between the column strip and the middle strip. The fixed-end forces calculated from these line loads are calculated and added into the appropriate nodes in the subelement load matrix.

The torsional elements connect the middle strip to the column strip and are only capable of transferring moment between the middle and column strip. The torsional element was originally derived assuming that the entire stiffness for the slab-beam is located at the end of the torsional element (ACI CODE). The stiffness for this element is calculated in accordance with the empirical formulas developed in ACI 318-83. It should be emphasized that the torsional stiffness is an empirical equation derived from actual test data and based on the assumption that all of the slab-beam stiffness is placed at the end of the torsional element and not on the column. Before actual use is made of this program, further studies are required to adjust or verify the stiffness of the torsional element as calculated.

DERIVATION OF COLUMN SUBSTRUCTURE

The derivation of the column element stiffness requires less analytical interpretation than the slab-beam element due to its inherent ability to be analyzed as a two-dimensional frame element. Since the column element requires no analytical simplification for two dimensional frame analysis, its form remains the same between various equivalent frame methodologies. ACI 318-83 defines the column stiffness based on the length of the column from mid-depth of slab above to mid-depth of slab below. The code goes on to say that the column moment of inertia is computed on the basis of cross section taking into account the increased stiffness of the column capital. The capital stiffness should be



taken to vary linearly from the base of the capital to the bottom of the slab-beam. Finally, ACI 318-83 suggests that the column stiffness be assumed infinite through the depth of the slab above and below. An excerpt from ACI 318-83 clearly depicts the intent of the code (Figure 7).

As with the slab-beam element, the column element is divided into a substructure of elements with a constant or linear varying moment of inertia. Inspection of Figure 7 indicates a maximum of 3 possible elements for the column substructure assuming the infinite stiffness is accounted for by the use of rigid-end offsets. As a result, only two elements are required.

The elements are assembled into a 9×9 global stiffness matrix for the substructure corresponding to the dof shown in Figure 8a. This global matrix is then condensed to the 6 dof shown in Figure 8b. The 6×6 condensed matrix is transformed to account for the rigid-end offsets and used in the direct stiffness method. It is assumed that no element forces are applied to the column elements. All lateral forces are applied at the nodes. Therefore, no element load matrix is required for the column element.

Column Capital Stiffness

The moment of inertia of subelement 2 (Figure 8a) depends on the presence of a column capital. If there is a capital present, the variance in the moment of inertia must be accounted for. Assuming the depth and width of a typical capital vary linearly across the length would result in an exponentially variable moment of inertia. ACI 318-83 permits the assumption that the moment of inertia varies linearly along the length of the column capital.

The stiffness coefficients for the column capital are derived here using the Hardy-Cross column analogy method (8). The stiffness coefficients for the standard frame element are analogous to the stiffness and carry-over factors (COF) used in the moment distribution method (MDM). For non-prismatic sections $k_{22} \neq k_{33}$ and $\text{COF} \neq 1/2$. Therefore, the Hardy-Cross moment distribution factors can be derived using the column analogy method and substituted into the 3×3 stiffness matrix for transformation to the 6×6 matrix. The coefficients are derived here in general form to produce an equation for k_{22} , $k_{23} = k_{32}$ and k_{33} . The equations derived are a function of the moment of

inertia at the bottom and top of the column capital.

$$S_b = \frac{2}{1+} X \left| \begin{array}{c} 1 \\ - \end{array} \right| + \frac{2(1+2)}{(1+4+)} \left| \begin{array}{c} 2 \\ - \end{array} \right|$$

$$S_{bt} = \frac{6(1+)}{1+4+}$$

$$S_t = \frac{2}{1+} X \left| \begin{array}{c} -1 \\ - \end{array} \right| + \frac{2(2+)}{(1+4+)} \left| \begin{array}{c} 2 \\ - \end{array} \right|$$

The axial stiffness coefficient is derived in McGuire and Gallagher (3) using the force equilibrium equation $d(A_y)/dy = 0$ and the strain-displacement equation $e_y = y dv/dy$ to form:

$$S_a = \frac{1 - A_t/A_b}{\ln(A_t/A_b)}$$

To account for the infinite stiffness in the area of the slab-beam, the matrix method of rigid end offsets is used to transform the 6 x 6 stiffness matrix.

The force recovery subroutine for the column element recovers the moments and the axial and shear forces. The results are at the intersection of the slab-beam and column, as would be required in design, rather than the geometric nodal points.

VERIFICATION AND TESTING OF TWSLAB

TWSLAB was tested against other computer programs which are designed to perform equivalent frame analysis and against results from tests on physical models. In addition, a number of analyses were run using TWSLAB while varying the values of t and t . These tests are used to correlate the appropriate t value for t and also to determine the coupling requirements of the middle strip elements for the slab-beam elements.

A flat plate system is analyzed using TWSLAB and compared to the results from analyses using the models described by Wang and Salmon (7) and the commercial program ADOSS(9). Reference 7 incorporates the equivalent beam (EB) method, equivalent column (EC) method, and the ACI implied model or space frame with



torsional elements (SF). An analysis from each method in reference 7 will be compared to TWSLAB. ADOSS utilizes the equivalent frame method as prescribed by ACI 318-83 (similar to the SF method of reference 7). As an additional comparison the design moments are calculated for the test structures in accordance with the direct design method of ACI 318-83.

The equivalent frame used in testing is three bays long and has two levels. The top floor system is modeled as a rigid system with lateral displacement allowed. Each column is assumed to be fully fixed against rotation and displacement. Lateral loads are applied as concentrated loads at the junction of the floors and the left most columns. A typical dead and live gravity load is applied to each span of the floor system.

The objective of this parametric study is to:

- (1) Determine α and t for this flat plate configuration, which correlate with acceptable analytical models.
- (2) Determine if coupling of slab-beam is required for this flat plate configuration.

Results

Selected moments recovered from each analysis are used in a comparison of TWSLAB with reference 7. This comparison is consolidated into a representative graph shown in Figure 9. Each family of curves represents the variation of the negative moment at the indicated node in relationship to the variation of α and t . Also plotted are the resulting moments from the analysis performed with Reference 7, ADOSS, and DDM.

An additional set of moments are plotted for an $\alpha = 0.001$. This is provided to show that the ACI space frame (as provided by Reference 7) can be modeled exactly by TWSLAB. This is logical since an analysis using TWSLAB with $t = 1.0$ and a small α would place the equivalent frame stiffness in the middle strip at the end of the torsional elements.

It should be noted that the DDM and ADOSS give moments at the face of the column, while Reference 7 and TWSLAB give moments at the centerline of the columns. In order to make the moments consistent, the DDM and ADOSS moments are converted to the centerline of the column.

Comparison of TWSLAB with Canadian model

One important consideration in concrete frame structures is the lateral drift under lateral loading conditions such as wind or earthquake forces. Vanderbilt and Corley compare the results of an analysis on their EFRAME program to the actual drift of the model under static lateral loads. Eight flat plate structures were built and tested as a part of a study by the National Research Council of Canada (10). Of the eight models constructed, only one series of results for the lateral loading of these multistory unbraced structures are available. Vanderbilt uses this test as a comparison for the program EFRAME presented in (4,6). The results (drift or lateral displacement) from the lateral loading test of the Canadian structure are compared to the results of an analysis using TWSLAB with the parameters chosen from the parametric study of $\alpha = 0.20$ and $t = 0.50$.

The equivalent frame system in the E-W direction is chosen for the analysis. The model is divided into 3 equivalent frames each composed of 4-21' bays. The three equivalent frames are connected at each floor level with stiff truss bars to enforce displacement compatibility during loading. A load of 100 pounds is applied to floor level 6 for load case number one and to floor level 8 for load case number two. Vanderbilt finds that with an infinitely stiff torsional element and a reduced slab-beam stiffness of 22% of the original stiffness, the results from EFRAME correlate well with the test results. This indicates that the flat plate analytical system requires a large amount of "softening" in order to accurately estimate the lateral drift. Therefore, the parameters α and t are chosen as 0.2 and 0.15 respectively. A small α and t will "soften" the structure by reducing the torsional stiffness and the stiffness located between the columns.

The test results and the results from the TWSLAB analysis are presented in Figure 10 and 11. Both lateral load cases from TWSLAB correlate well with the Canadian models test results with a maximum difference of 22 percent.

DISCUSSION AND CONCLUSION

Two tests were performed on TWSLAB in order to determine the range of α and t required, as well as the coupling requirements for the slab-beam. The flat plate configuration was tested and compared with other analytical modeling techniques for both gravity and lateral loads. This same configuration was also tested using a microconcrete model in order to determine TWSLAB's analytical ability with predicting lateral drift on building systems.

Analytical Comparison

The parametric study of TWSLAB including ADOSS, Reference 7 and DDM show good correlation for both gravity and lateral loads. As can be seen in Figure 9 the alternate methods of analysis correlate with low α and t for both the interior and exterior supports. It is noted however that the uncoupled version of TWSLAB for gravity loads has a tendency to overestimate the moment at the exterior support, and underestimate the moment at the interior support.

This is based on comparison with the alternate methods of analysis shown in the graphs. Comparing the moment for $\alpha = 0.2$ and $t = 0.4$, at the interior support, to the moment t from ADOSS gives a difference of approximately 100%. When the ends of the middle strips are coupled the moment in the middle strip can be transferred directly to the adjacent middle strip. However, if the middle strips are uncoupled, the ends are free to rotate and can only transfer moment through the "soft" torsional element to the columns. Since, in general, the middle strip has more stiffness than the torsional element, the moment that would have been transferred across the coupled node is instead transferred through the middle strip to the exterior nodes. Considering the good correlation between the coupled version and the other analytical models, it is recommended that the slab-beam be coupled for gravity load analysis.

Examination of the lateral load moments between the coupled and uncoupled slab-beam shows a different trend. The plots for both interior and exterior moments, comparing uncoupled and coupled slab-beams, are essentially the same. The uncoupled exterior and interior moments tend to be greater and less than the coupled moments, respectively. However, the largest difference is approximately 10%, which indicates that

both uncoupled and coupled slab-beams will give good results. Since coupling is required for gravity load it is recommended that lateral loads be analyzed in a similar fashion.

The next consideration in the parametric study are the values of α and t . These parameters can be fixed based on comparison with the results from the geometric configuration, although it is likely that the parameters chosen here will not be valid for all geometric configurations. This provides impetus for further study and testing of TWSLAB.

Considering only gravity loading there are a range of α and t parameters which satisfy the comparison with the alternate analysis techniques. Inspection of figures in Appendix A show that an α of 0.20 and an t of 0.40 satisfy the range of moments produced by the alternate techniques. This is true for both the interior and exterior moments. The lateral load cases show a similar correlation with the same α and t .

Canadian Model

From the discussion above it would seem that the values chosen for α and t would be applicable to prediction of drift for tall concrete structures. However, the problem associated with lateral loading and drift is the reduction of stiffness in the system due to cracking. As shown in Figure 10 and 11, a lateral load analysis on a Canadian microconcrete structure with TWSLAB using an α of 0.2 and an t of 0.15 still underestimates the actual drift in the microconcrete structure by 22%. This contradiction indicates that different parameters are required for an analysis for drift, compared to an analysis for forces and moments used in the design.

There are several factors which influence the lateral drift in a concrete building system:

- (1) The reduced stiffness at the column-slab interface and in the members due to cracking from gravity loads
- (2) Added moments on the columns due to P-delta effects
- (3) Material nonlinearity

The overall effect of these factors is a reduction of the system stiffness. As a result an elastic analysis will usually underestimate the drift. Since these factors are not directly accounted for in TWSLAB, the stiffness must be reduced arbitrarily as was done in the TWSLAB analysis of the Canadian building. Similarly, Vanderbilt and Corley (4) selected a range for the stiffness reduction factor based on parametric studies of various flat plate structures. This factor was correlated with the static lateral drift criteria of $h/500$, where h = structure height.

In closing, it is appropriate to note that TWSLAB requires further study and development in order to be useful as a design tool for concrete structures. Additional capabilities are required as well, such as the ability to handle geometric and material non-linearity, and recover positive moments and midspan deflections. The inherent problem with adding these capabilities is that it takes the program out of the range of personal computer use. Therefore, further additions must be tempered with the practical application of the program to personal computers.

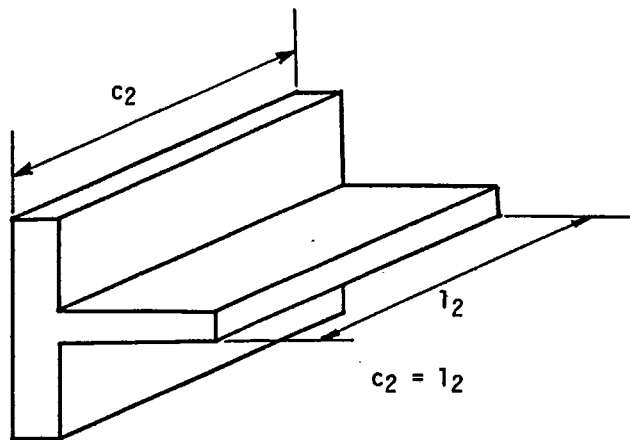
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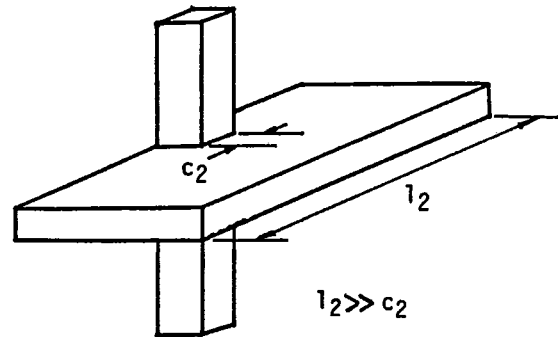
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- 8) Wang, C., Indeterminate Structural Analysis, McGraw-Hill, New York, 1983.
- 9) Portland Cement Association, "PCA ADOSS," Version 1.02a, July 1985.
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(a) Wall-Slab Connection



(b) Column-Slab Connection

Fig. 1--Efficiency of moment transfer between beams and columns

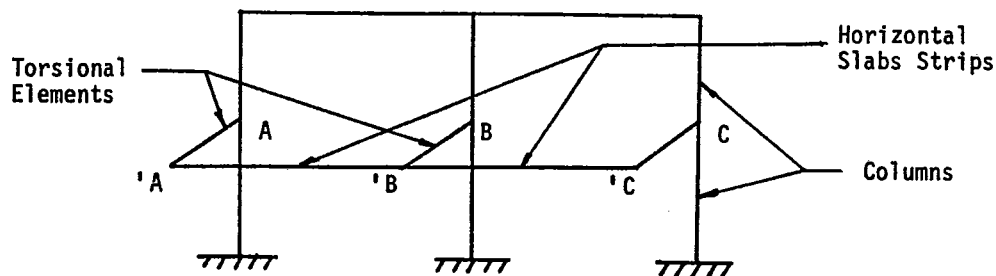


Fig. 2--Interpretation of ACI model

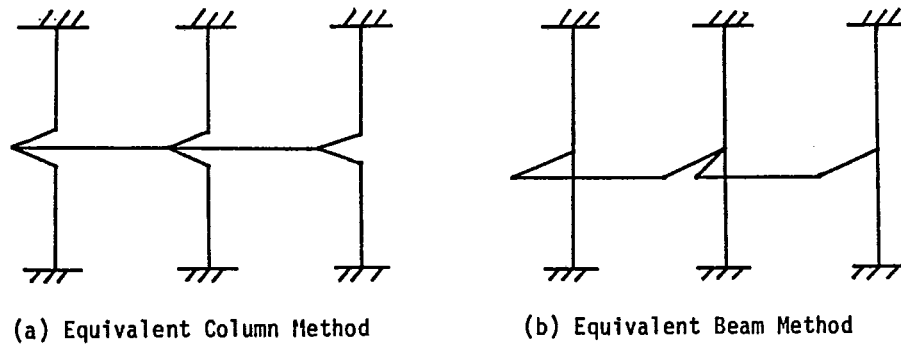


Fig. 3--Methods of modeling the torsional element

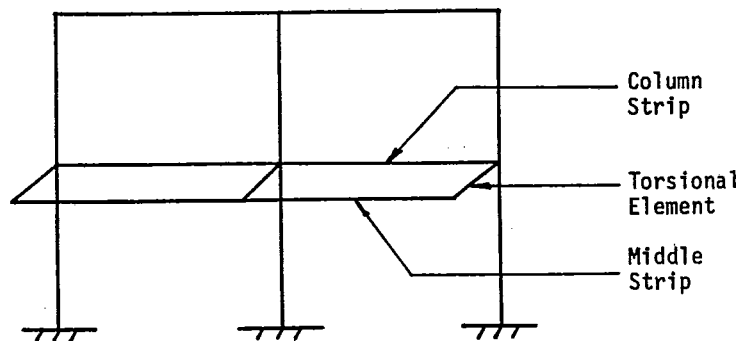
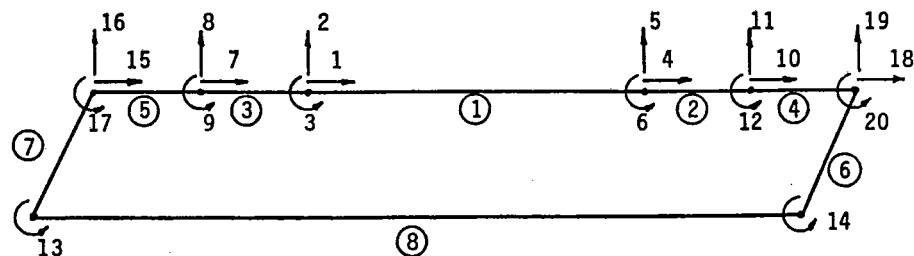
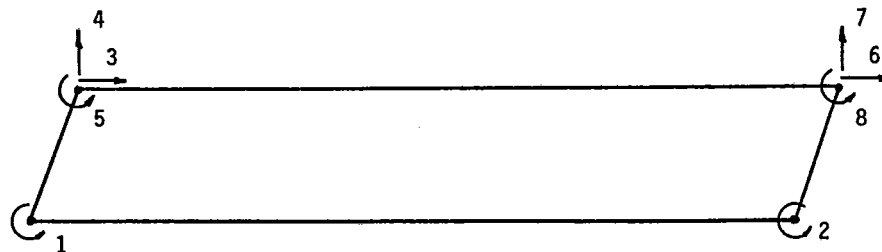


Fig. 4--Wang and Salmon model



(a) Full Substructure Element

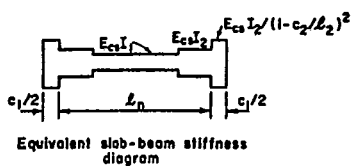
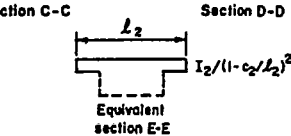
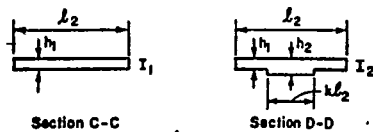
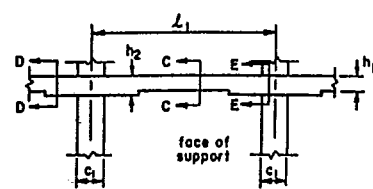


(b) Condensed Substructure Element

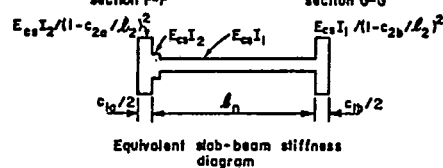
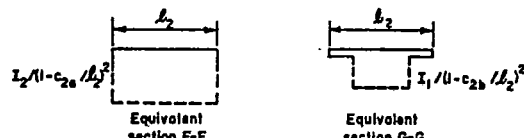
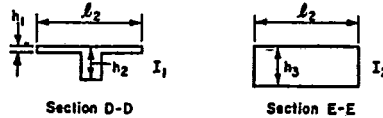
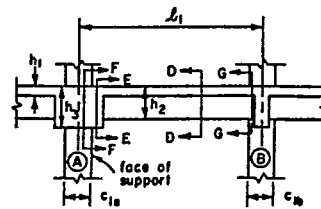
○ Indicates Subelement #

Fig. 5--Slab-beam element degrees of freedom

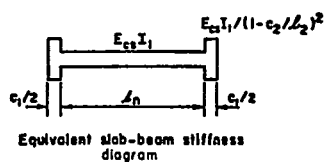
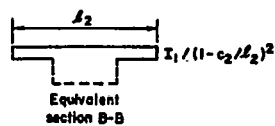
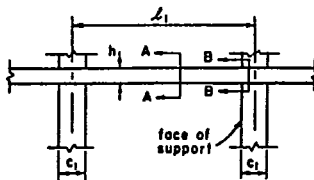
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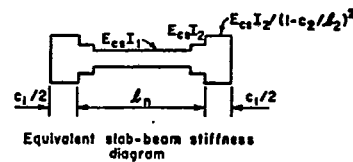
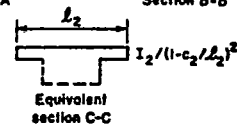
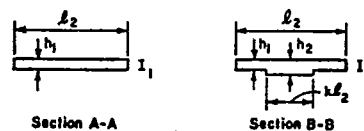
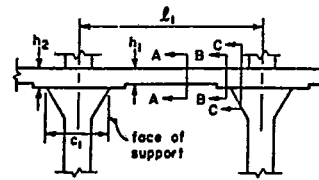
(b) Slab system with drop panels



(d) Slab system with beams



(a) Slab system without beams



(c) Slab system with column capitals

Fig. 6--Sections for calculating slab-beam stiffness

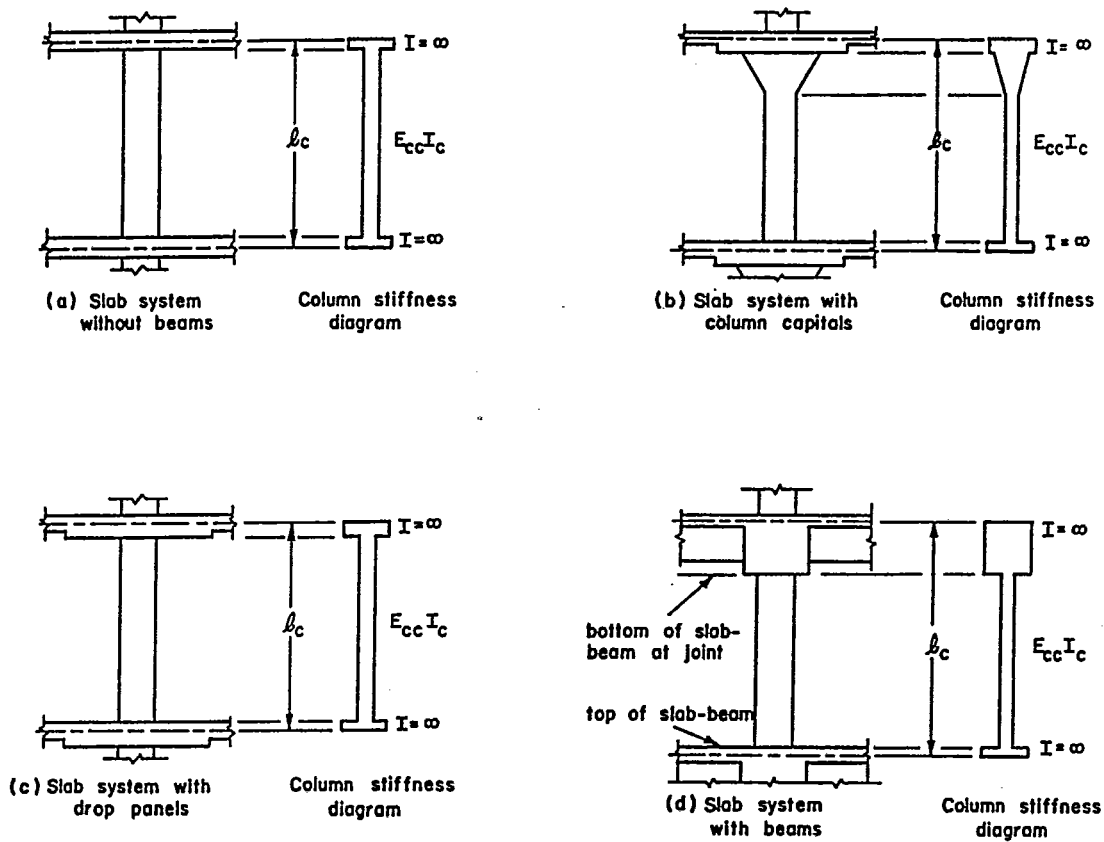
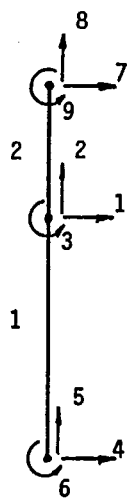
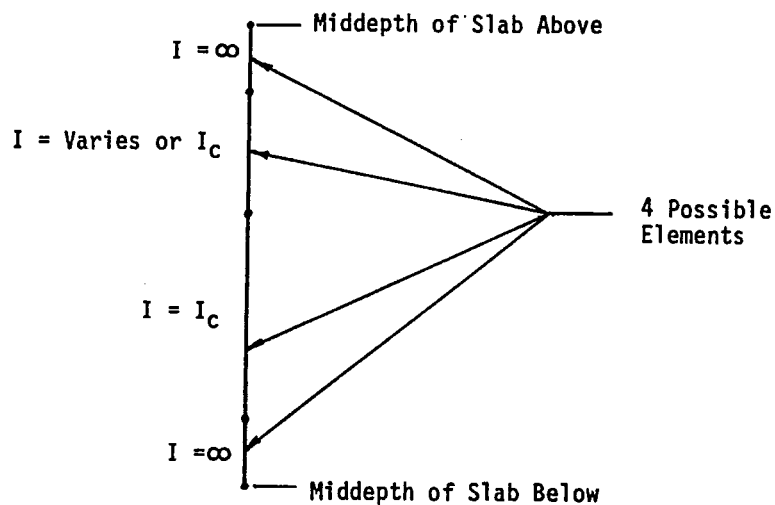
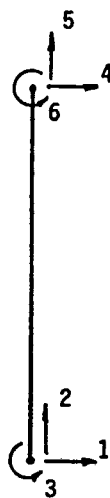


Fig. 7--Sections for calculating column stiffness

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(a) Substructure DOF's



(b) Condensed Substructure DOF's

Fig. 8--Condensation of column substructure