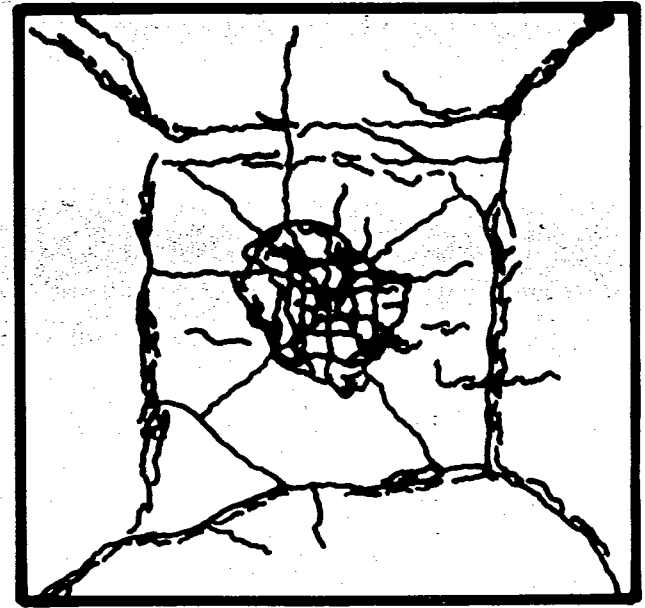
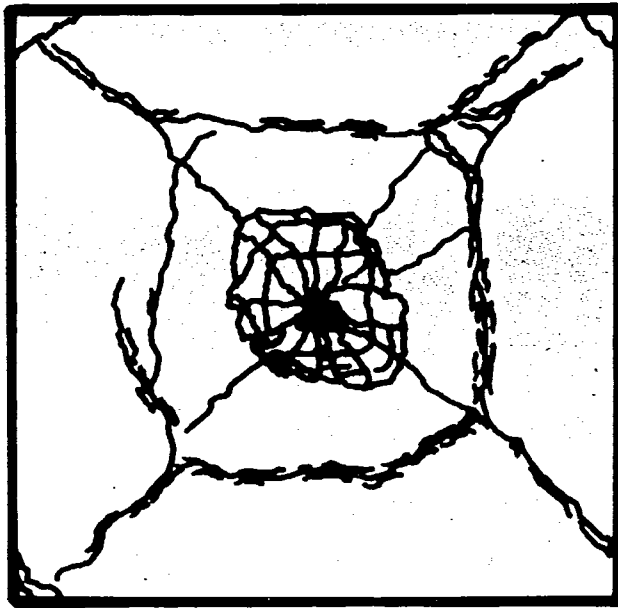
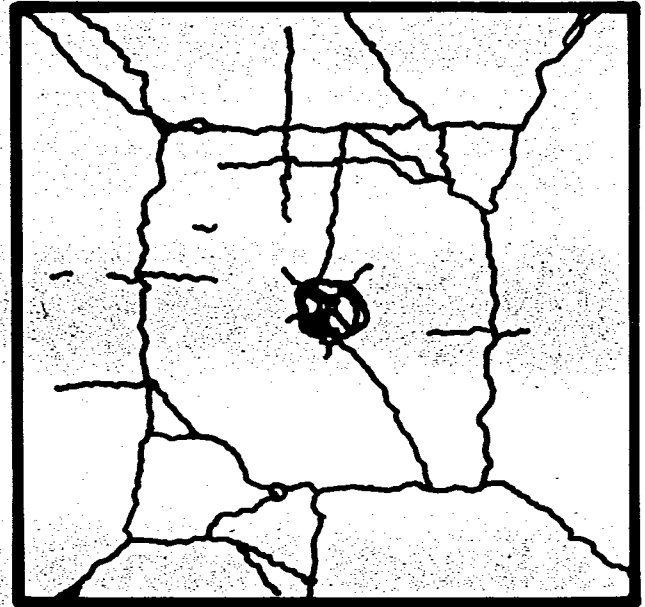
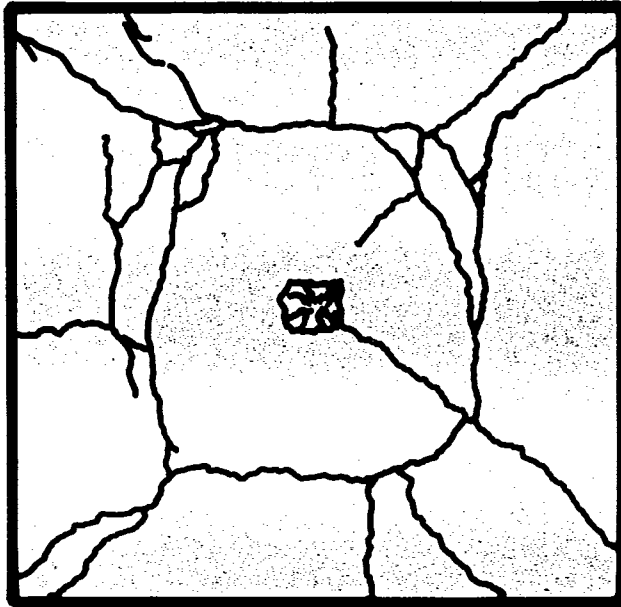


CRACKING IN PRESTRESSED CONCRETE STRUCTURES



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ERRATA FOR SP-113

The following corrections should be made to pages v and 157.

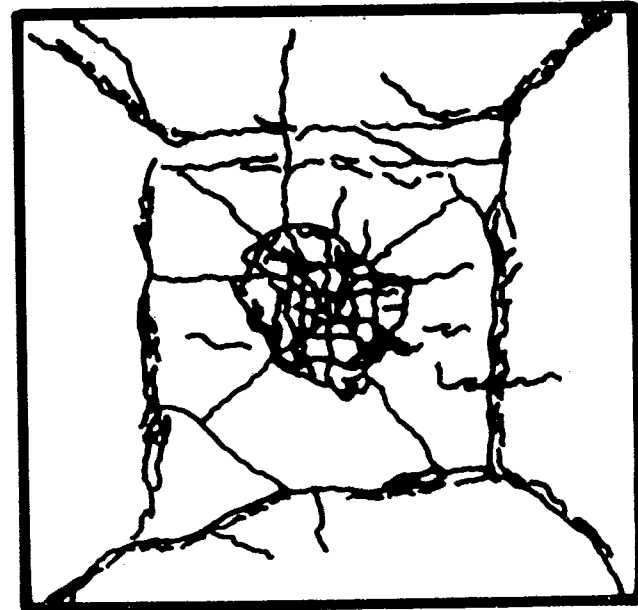
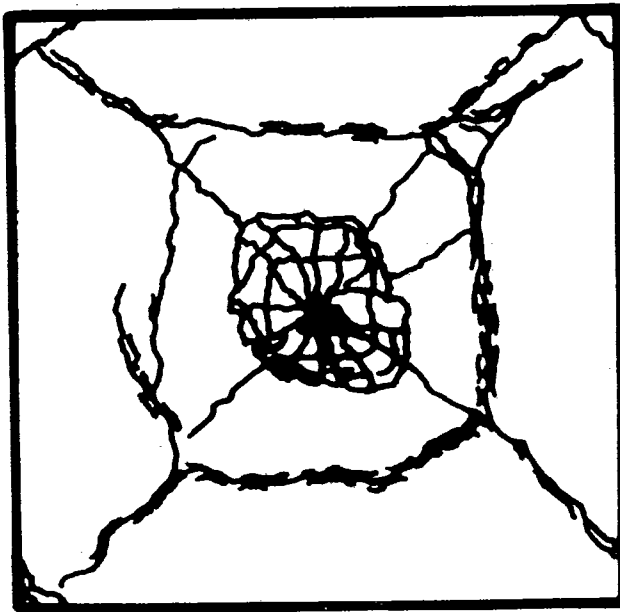
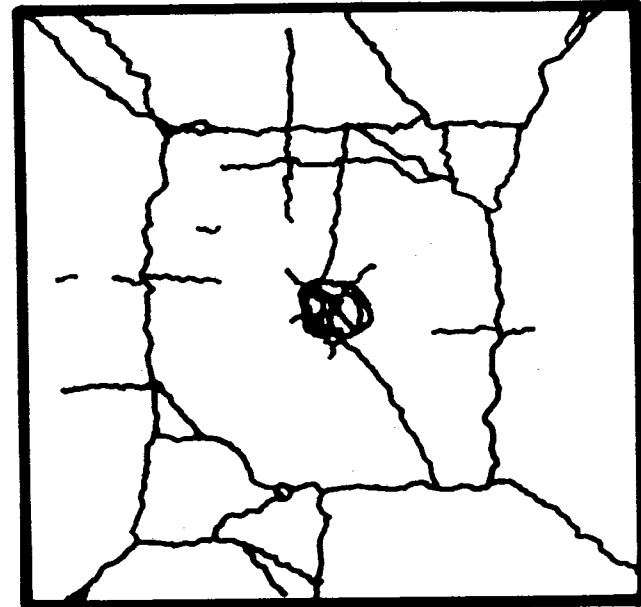
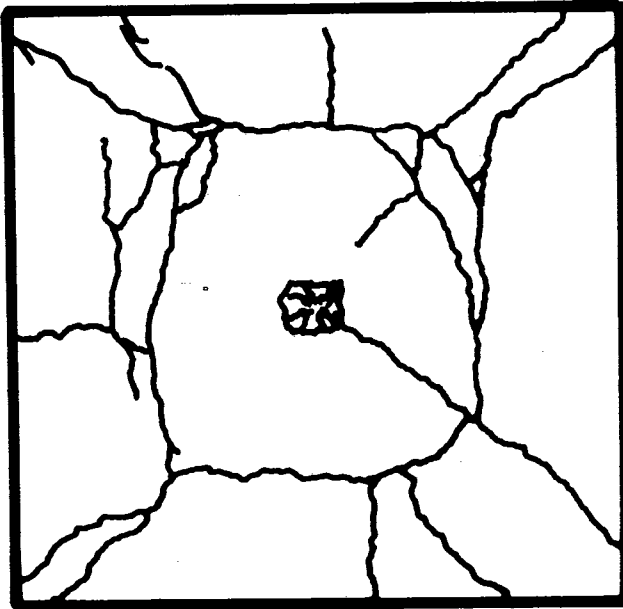
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Restraint Cracks and Their Mitigation in Unbonded Post-Tensioned Building Structures

by B.O. Aalami and F.G. Barth

CRACKING IN PRESTRESSED CONCRETE STRUCTURES



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DISCUSSION of individual papers in this symposium may be submitted in accordance with general requirements of the ACI Publication Policy to ACI headquarters at the address given below. Closing date for submission of discussion is October 1, 1989. All discussion approved by the Technical Activities Committee along with closing remarks by the authors will be published in the March-April 1990 issue of either ACI Structural Journal or ACI Materials Journal depending on the subject emphasis of the individual paper.

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Printed in the United States of America

Editorial production: Patricia Kost

LIBRARY OF CONGRESS CATALOG CARD NUMBER 88-83276

PREFACE

At the American Concrete Institute Fall Convention in November 1986, ACI Committee 224, Cracking, and ACI-ASCE Joint Committee 423, Prestressed Concrete, sponsored two technical sessions on "Cracking in Prestressed Concrete Structures." The sessions were organized and presentations selected by a committee consisting of Grant T. Halvorsen (Chairman, Committee 224), Randall W. Poston (Secretary, Committee 224), Antoine E. Naaman, and Ned H. Burns (members, Committee 423).

Gregory P. Chacos was chairman of Committee 423 when this venture was proposed; Robert N. Bruce, Jr., was chairman at the time of the sessions. Both enthusiastically supported the efforts of the organizing committee. The Baltimore sessions were chaired by Halvorsen and Burns.

This ACI special publication consists of seven papers, including six that were presented at the Baltimore sessions. The papers reflect the perspectives of both research and practice. Three papers deal with cracking in "partially prestressed concrete," two concern anchorage zone behavior, and two are about cracking in post-tensioned structures.

We wish to thank all authors and presenters for their contributions to the two technical sessions and this volume, the reviewers of the original papers for their helpful comments and suggestions, and the ACI staff for its assistance in organizing the sessions and preparing this special publication. Special thanks are also due to the members of both committees for support of this project.

Grant T. Halvorsen
Ned. H. Burns

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SP 113-1

Flexural Cracking Behavior of Partially Prestressed Pretensioned and Post-Tensioned Beams—State of the Art

By E.G. Nawy

Synopsis: More prestressed beams incorporating mild steel reinforcement are built today, with allowance of tension in the concrete in what is often termed as partial prestressing. Consequently, a study of their cracking behavior and control of the crack width and distribution are becoming more significant.

Available experimental data on the cracking behavior of prestressed beams is limited. Because of the importance of serviceability behavior of these elements, several experimental and analytical investigations have been undertaken and expressions proposed.

Simple mathematical expressions have been developed as a result of the work at Rutgers University corroborated by comparative analysis with other authors. The proposed expressions for evaluating crack widths in partially prestressed beams at working load and overload levels have been developed in terms of the controlling parameters, namely, the variation in the steel reinforcement and percentage of the prestressing tendons and the non-prestressed reinforcement.

The mathematical model is applied to tests on twenty simply supported pretensioned 9-ft. span beams and four 2-span continuous beams of effective 9-ft. span, and twenty-two simply supported post-tensioned beams of 7-ft. 6-in. span. The major controlling parameters were the variation in the steel reinforcement percentage of the prestressed tendons and the non-prestressed reinforcement. The effect of concrete cover was incorporated in the value of the concrete area in tension.

Equations are proposed relating the maximum weighted crack width to the increase in reinforcement stress and the concrete area in tension.

Keywords: beams (supports); cracking (fracturing); crack width and spacing; partial prestressing; post-tensioning; prestressed concrete; pretensioning; serviceability; tensile stress

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Edward G. Nawy, F.A.C.I., is Professor of Civil Engineering, Rutgers, The State University of New Jersey, holding the distinguished professor rank (Professor II). Active in ACI since 1949, Prof. Nawy is a former chairman and current member of ACI Committee 224, Cracking, and is currently a member of ACI Committee 340, Design Aids for ACI Building Codes, 435, Deflection of Concrete Building Structures; Joint ACI-ASCE Committee 421, Design of Reinforced Concrete Slabs; the Concrete Materials Research Council; Technical Activities Committee of PCI, and the Tall Buildings Council. His research interests are in reinforced and prestressed concrete, particularly in the areas of crack control and serviceability behavior. Prof. Nawy has published more than 100 papers and is the author of three major text-books: Reinforced Concrete - A Fundamental Approach, Simplified Reinforced Concrete, and Prestressed Concrete - A Fundamental Approach as well as numerous handbook chapters. He holds many honors and awards, including the ACI Chapter Activities Committee and Henry L. Kennedy Awards, was twice President of the N.J. ACI chapter, served two terms on the Rutgers University Board of Governors, was Honorary Vice President of the RILEM 30th Anniversary Congress, and is Honorary Professor of NIT, the Nanjing Institute of Technology. He is consultant to major organizations in the New York-New Jersey metropolitan area, and is a registered Professional Engineer in New Jersey, New York, and Pennsylvania.

INTRODUCTION

As more prestressed beams incorporating mild steel reinforcement are built today, with allowance of tensile strength in the concrete in a range of $0-12 \sqrt{f'_c}$, study of their cracking behavior and control of the crack width and distribution are becoming more significant.

Available experimental data on the cracking behavior of prestressed beams [1-12] is limited. Because of the importance of serviceability behavior of these elements, several experimental and analytical investigations have been undertaken and expressions proposed.

The expressions that are discussed in this paper are based on the concept advocated by J. Ferry Borges [6], Nawy-Potyondy [7,8], Holmberg [9,10], Nawy-Huang [12], Nawy-Chiang [21], and Nawy [24,25]. This approach is based on the reinforcement stress concept. It is more effective and more reliable than the flexural concrete tensile stress originally advocated in references [3] and [4], or the nominal concrete strain and

Cracking in Prestressed Concrete 3

curvature discussed in ref. [22], because it can accurately evaluate the strains and stresses in the steel reinforcement whereas such an evaluation of strain in the concrete at the beam or at the reinforcement level is very approximate at best.

Simple mathematical equations have been developed as a result of the work at Rutgers University [12,21] and corroborated by comparative analysis in the work of A.E. Naaman [19,23]. The proposed expressions for evaluating crack widths in partially prestressed beams at working load and overload levels have been developed in terms of the controlling parameters.

Tests were conducted on twenty simply supported pretensioned 9-ft. span beams and four 2-span continuous beams of effective 9-ft. span [12,21]. Tests were also conducted on twenty-two simply supported post-tensioned beams of 7-ft. 6-in. span. The major controlling parameters were the variation in the steel reinforcement percentage of the prestressed tendons and the non-prestressed reinforcement. The prestressing tendons were either 1/4" or 1/2" nominal diameter 7-wire strands having an ultimate strength of 250 ksi (1723.5 N/mm^2) for the 1/4" size and 270 ksi (1861.4 N/mm^2) for the 1/2" size. Mild steel reinforcement used was either #3, #4, or #5 deformed high-strength bars of yield strength varying between 60.9 ksi and 84.0 ksi ($419.8\text{--}579.1 \text{ N/mm}^2$). The effect of concrete cover was incorporated in the value of the concrete area in reinforcement stress and the concrete area in tension.

RESEARCH SIGNIFICANCE

Partial prestressing implies the allowance of some tensile stress in the concrete. The research reported permits control of the flexural cracks caused by the tensile stress through adequate proportioning of the bonded reinforcement percentage. As a consequence, serviceability of the prestressed beams can be maintained under normal and reasonable overload conditions.

TESTING PROGRAM

Materials

The concrete mix was designed for a nominal 28-day compressive strength of approximately 4000 psi (127.6 N/mm^2). Water/cement ratio varied between 5.4 and 7.1 gallons/sack of cement. The coarse aggregate used was crushed stone of 3/8 in. (9.35 mm) maximum size, while the fine aggregate was natural local sand. Slump varied between 5 and 7-1/2 in. (12.7 and 19.1 cm) as given in Table 1.

4 Navy

Uncoated, stress-relieved, 250 ksi 7-wire 1/4 in. (6.35 mm) strands were used for prestressing the pretensioned beams. In Series II of the post-tensioned beams, 1/2 in. (12.7 mm) diameter strands were used. The material had a unit elongation of 0.65 percent at 70.0 percent of the ultimate. The material satisfied ASTM A-416 specifications and had a typical stress-strain relationship as shown in Fig. 1.

Non-prestressed #3, #4, and #5 deformed bars were used as additional reinforcement at the tension side in all beams except B-1 to B-6. A typical stress-strain diagram for the deformed bars is shown in Fig. 2. Table 2 gives the details of the reinforcement used for the beams in this test program, and Fig. 3 gives typical cross-sections of the test beams.

Fabrication and Testing

Twenty single span and four continuous pretensioned beams were fabricated [1] and tested. The simply supported single span beams were as follows: 1) B-1 to B-6 were T-sections with pretensioned prestressed tendons only; 2) B-7 to B-18 were also T-sections reinforced with both pretensioned and prestressed tendons and non-prestressed mild steel; 3) B-19 and B-20 were I-sections with both prestressed and non-prestressed reinforcement; 4) B-21 through B-24 were I-sections continuous on two spans reinforced with both pretensioned prestressed tendons and mild reinforcement. Straight profile strand patterns were used in all beams. For continuous beams, strands were inserted into plastic tubes in the compression regions to achieve zero bond between concrete and strands. Before testing, those strands in the compression regions were cut through to eliminate the effectiveness of strands in these zones.

Twenty-two post-tensioned T-section beams were tested to failure. The test specimens were in two series and all of them had 7 ft. (2.13 m) spans. Series I: The prestressing tendons were 1/4 in. (6.35 mm) nominal diameter 7-wire strand [250 ksi (1740 N/mm^2)]; Series II: The prestressing tendons were 1/2 in. (12.7 mm) nominal diameter 7-wire strand [270 ksi (1950 N/mm^2)].

Pretensioned beams were 10 in. (25 cm) deep and post-tensioned beams were 11 in. (28 cm) deep as shown in Table 1. They were all overdesigned to resist diagonal tension. No. 3 deformed closed stirrups were used throughout the span.

All reinforcement was instrumented with electric resistive type strain gauges at critical locations. Readings were taken of the change in strain at all the necessary stages of prestressing and loading. In addition, demountable mechanical gauges were used to measure the variation of strain on the concrete faces of the beams.

Mechanical dial gauges with 2 in. (5 cm) travel and .001 in. (0.025 mm) accuracy were used to measure the change in deflection due to loading. Crack widths were measured with illuminated microscopes having a magnification of 25 times and a 0.05 mm accuracy. Crack spacings of all the developing cracks were also recorded as well as the crack penetration of the principal cracks.

For most of the beams, eight to nine increments of load were applied to failure. At the conclusion of each beam test, concrete control cylinders were tested for both compressive and tensile strengths.

FLEXURAL CRACKING TEST RESULTS

Maximum crack widths were measured at the reinforcement level and at the bottom tensile face of the concrete. The spacings of the cracks were measured on both faces of each beam at each loading stage. These spacings were summarized for each test specimen and the mean stabilized crack spacing was calculated. Crack spacing is considered stabilized when no additional cracks develop and the major cracks continue to increase in width with the increase in the applied load. Table 3 gives the mean stabilized crack spacing for load ratios of 50 to 70 percent of the ultimate load. Table 4 gives the measured crack width of the stabilized cracks at the reinforcement levels of the steel closest to outer fibers for the various stress levels.

Mathematical Model Formulation for Serviceability Evaluation

Crack Spacing - Primary cracks form in the region of maximum bending moment when the external load reaches the cracking load. As loading is increased, additional cracks will form and the number of cracks will be stabilized when the stress in the concrete no longer exceeds its tensile strength at further locations regardless of load increase. This condition is important as it essentially produces the absolute minimum crack spacing which can occur at high steel stresses, to be termed the stabilized minimum crack spacing. The maximum possible crack spacing under this stabilized condition is twice the minimum, to be termed the stabilized maximum crack spacing. Hence, the stabilized mean crack spacing a_{cs} is deduced as the mean value of the two extremes.

The total tensile force T transferred from the steel to the concrete over the stabilized mean crack spacing can be defined as

$$T = \gamma a_{cs} \mu \Sigma \sigma \dots \dots \dots (1)$$

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where

- γ = a factor reflecting the distribution of bond stress
 μ = maximum bond stress which is a function of $\sqrt{f'_c}$
 Σo = sum of reinforcing elements' circumferences

The resistance R of the concrete area in tension A_t can be defined as

$$R = A_t f'_t \quad \dots\dots\dots (2)$$

where f'_t = tensile splitting strength of the concrete. By equating Eqs. (1) and (2), the following expression for a_{cs} is obtained, where c is a constant to be developed from the tests:

$$a_{cs} = c \frac{A_t f'_t}{\Sigma o \sqrt{f'_c}} \quad \dots\dots\dots (3)$$

The concrete stretched area, namely the concrete area in tension A_t for both the evenly distributed and non-evenly distributed reinforcing elements, is illustrated in Fig. 4. With a mean value of $f'_t/\sqrt{f'_c} = 7.95$ in this investigation, a regression analysis of the test data resulted in the following expression for the mean stabilized crack spacing

$$a_{cs} = 1.20 A_t / \Sigma o \quad \dots\dots\dots (4)$$

Crack Width - If Δf_s is the net stress in the prestressed tendon or the magnitude of the tensile stress in the normal steel at any crack width load level in which the decompression load (decompression here means $f_c = 0$ at the level of the reinforcing steel) is taken as the reference point [3.5] then for the prestressed tendon

$$\Delta f_s = f_{nt} - f_d \quad \dots\dots\dots (5)$$

where

f_{nt} = stress in the prestressing steel at any load beyond the decompression load

f_d = stress in the prestressing steel corresponding to the decompression load

The unit strain $\epsilon_s = \Delta f_s / E_s$, and it is logical to disregard as insignificant the unit strains in the concrete due to the effects

of temperature, shrinkage and elastic shortening [4,5]. Hence, the maximum crack width can be defined as

$$w_{\max} = k a_{cs} \epsilon_s^\alpha \dots\dots\dots (6)$$

where k and α are constants to be established by tests, or

$$w_{\max} = k' a_{cs} (\Delta f_s)^\alpha \dots\dots\dots (6a)$$

where k' is a constant in terms of constant k .

Expressions for Pretensioned Beams

Eq. 6a is rewritten in terms of Δf_s so that analysis of the test data of all the simply supported test beams in Table 4a leads to the following expression at the reinforcement level:

$$w_{\max} = 1.4 \times 10^{-5} a_{cs} (\Delta f_s)^{1.31} \dots\dots\dots (7)$$

A 25 percent scatter band envelops all the data for the expression in Eq. 7 for $\Delta f_s = 20$ ksi to 80 ksi.

Linearizing Eq. 7 for easier use by the design engineer leads to the following simplified expression of the maximum crack width at the reinforcing steel level:

$$w_{\max} = 5.85 \times 10^{-5} \frac{A_t}{\Sigma o} (\Delta f_s) \dots\dots\dots (8a)$$

and a maximum crack width (in.) at the tensile face of the concrete:

$$w'_{\max} = 5.85 \times 10^{-5} R_i \frac{A_t}{\Sigma o} (\Delta f_s) \dots\dots\dots (8b)$$

where R_i is the distance ratio as defined in the notations.

A plot of the data and the best fit expression for Eq. 8a is given in Fig. 5 with a 40 percent spread, which is reasonable in view of the randomness of crack development and the linearization of the original expression of Eq. 7.

Expressions for Post-Tensioned Beams

The expression developed for the crack width in post-tensioned bonded beams which contain mild steel reinforcement is based on the test results given in Table 4b

$$w_{\max} = 6.51 \times 10^{-5} \frac{A_t}{\Sigma o} (\Delta f_s) \dots\dots\dots (9a)$$

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for the width reinforcement level closest to the tensile face, and

$$w'_{\max} = 6.51 \times 10^{-5} R_i \frac{A_t}{\Sigma o} (\Delta f_s) \dots\dots\dots (9b)$$

at the tensile face of the concrete lower fibers.

For non-bonded beams, the factor 6.51 in Eqs. 9a and 9b becomes 6.83. A regression plot of Eq. 9a is given in Figure 6. It shows a ± 40 percent scatter band, which is not unexpected in the randomness of flexural cracking behavior.

DISCUSSION OF TEST RESULTS

It is observed from this investigation that the initial flexural cracks started at a relatively low net steel stress Δf_s level between 3 and 8 ksi. These initial cracks formed in a rather random manner and with an irregular spacing. All major cracks usually developed at a net steel stress level of 25 to 30 ksi. At higher stresses the existing cracks widened and new cracks of narrow width usually formed between major cracks. Visible stabilized crack spacing condition was generally reached at 0.5 to 0.7 of the ultimate load.

This investigation established that the maximum crack spacings after stabilization were close to twice the minimum possible spacings, having a mean value of 2.02 and a standard deviation of 0.29. The effect of the variation of percentages of the non-prestressed steel was significant both on the crack spacing and crack width. For beams with non-prestressed steel, the number of flexural cracks was almost twice as many as those for the case of no mild steel. These cracks were more evenly distributed, with considerably closer spacing and finer widths. This behavior can be attributed to the fact that the bond of the mild steel to the surrounding concrete played a pronounced role in crack control. A typical plot of the effect of the various steel percentages on the crack spacing at the various stress levels Δf_s is given in Fig. 7. It is seen from this plot that crack spacing stabilized at a net stress level of 36 to 40 ksi.

It is also observed that it is advantageous to locate the non-prestressed steel below the prestressed tendons. This is due to the fact that mild steel has larger diameters than the prestressed reinforcement, hence larger bond area of contact with the surrounding concrete. Also, by placing the mild steel close to the tensile concrete face, cracks will be more evenly distributed, hence crack spacing and consequently width will be smaller.

It should be noted that the areas of the bonded steel used in this research program were larger than the minimum required by the ACI 318-83 code. The code stipulates a minimum $A_s = 0.004 A$, where A is the area of the part of the concrete section between the flexural tension face and the center of gravity of the gross section considered. For the T-sections used in most of the test beams, $0.004 A \approx 0.07 \text{ in.}^2$ whereas the smallest area of bonded steel used as in beam B-1 was 0.108 in.^2 (Table 2a). The majority of the test beams had bonded longitudinal steel areas ranging between 0.144 in.^2 and 1.01 in.^2 with corresponding steel percentages $\rho = 0.17$ to 1.01 percent and a total steel percentage at the tensile side of $\rho = 1.24$ percent. Fig. 7 demonstrates that the crack spacing is better controlled using higher steel percentages than the minimum required by the ACI code. A range of not less than $\rho = 0.60$ to 0.70 percent for the bonded reinforcement is recommended to achieve a crack spacing of about 5 inches. As a result, acceptable crack widths can be achieved in partially prestressed beams.

The effect of spacing of the stirrups on the crack spacing was not pronounced. It was found that the final crack spacing and crack pattern did not necessarily follow the vertical shear reinforcement. Even though the first few cracks usually started at the stirrups, the vertical legs of the stirrups served only as initial weak areas of stress concentration. In most cases, the stabilized mean crack spacings were smaller than the spacings of stirrups.

Based on these discussions and the results of this investigation, it can be stated that the proposed expressions for crack control can be reasonably applied by the design engineer for maintaining the serviceability of pretensioned and post-tensioned partially prestressed beams under working load and overload conditions. Once the allowable crack width is established for the prevailing environmental conditions, the proper percentage of non-prestressed reinforcement can be determined to ensure serviceable behavior.

Discussion of Other Work

Based on the analysis of results of various investigators, Naaman produced the following modified expression for partially prestressed pretensioned members

$$(w_{\max})_{\text{in.}} = [42 + 5.58 \frac{A_t}{\Sigma o} (\Delta f_s)] \times 10^{-5} \dots\dots\dots (10)$$

This regression expression is very close to Eq. 8a by the author. If plotted against the experimental results of the various investigators it gives a best fit as shown in Fig. 8.

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The concrete cover has not been substantially varied in the reported Nawy's work since the magnitude of clear cover in prestressed beams is essentially standardized and is accounted for in the term A_t describing the concrete area in tension.

Additional tests being presently conducted by the author where the variation in the concrete cover is the major parameter have further verified that such variation does not affect the parameters of the expressions in Eqs. 8a and 9a for pretensioned and post-tensioned beams.

The study reported in Ref. [22] concentrating on the area of concrete in tension and the nominal strain in the concrete at the tensile face would not give a reliable prediction of the crack width. In particular it does not account for the actual stress in the steel reinforcement and depends on measurements of strain at the concrete surface which are difficult to reliably evaluate.

A comparison is made between the author's equations and the CEB-FIP equations using similar notations.

$$\text{Nawy: } w_{\max} = \xi \left(\frac{A_t}{\Sigma_o} \right) \Delta\sigma_p$$

$$\text{CEB-FIP: } w_{\max} = \left[K \frac{\phi}{\rho_t} + \alpha_o \right] \frac{\Delta\phi_p}{E_s} \left[1 - \beta \left(\frac{\Delta\sigma_{pr}}{\Delta\sigma_p} \right)^2 \right]$$

where

$$\Sigma_o \approx \lambda \left(\frac{\phi}{\rho_t} \right) \quad \text{with } \lambda \text{ being a multiplier}$$

K, β are variables,

and $\alpha_o, \left(\frac{\Delta\sigma_{pr}}{\Delta\sigma_p} \right)^2$ are terms in the CEB-FIP expression not of major significance and accounted for in the $\xi = 5.85$ for pretensioned beams and $\xi = 6.51$ for the post-tensioned beams in the author's expressions.

Fig. 9 shows a typical test set-up. Fig. 10 shows the cracking pattern at failure of one of the beams tested in this program, while Fig. 11 gives a close-up of the flexural cracks at failure of another beam at rupture in this investigation. One can recognize in these photographs the relatively even distribution of crack spacing and their extension to the compression flange at top.

CONCLUSIONS

1. The maximum crack width (in.) at the level of reinforcement closest to the tensile face can be predicted from the expression

$$w_{\max} = 5.85 \times 10^{-5} \frac{A_t}{\Sigma o} (\Delta f_s) \quad \text{for pretensioned beams,}$$

$$w_{\max} = 6.51 \times 10^{-5} \frac{A_t}{\Sigma o} \quad \text{for post-tensioned bonded beams}$$

$$\text{and } w_{\max} = 6.83 \times 10^{-5} \frac{A_t}{\Sigma o} \quad \text{for post-tensioned non-bonded beams.}$$

In all cases, the expression reflects inclusion in the analysis of the data from the beams which had no mild steel but only prestressed tendons.

The crack width at the outer tensile face of concrete is

$$w_{\max} = w_{\max} \times R_i$$

where R_i is the distance ratio.

2. Initial flexural cracks randomly form at irregular spacings at low net steel stresses Δf_s of 3 to 8 ksi. All major cracks usually develop at a net stress of 25 to 30 ksi.
3. Visible crack spacing stabilizes at 50 to 75 percent of the ultimate load.
4. The presence of non-prestressed steel in the prestressed members has a significant effect on crack control such that the cracks become more evenly distributed and the crack spacings and widths become smaller.
5. An increase in the percentage of bonded reinforcement effectively decreases the crack spacings and width in the partially prestressed beams. A total percentage of about 0.6 to 0.7 percent is needed to effect good crack control.
6. Long-term effects on the crack width increase in partially prestressed beams need to be further clarified. It should be expected that short-term crack widths can more than double with time under sustained and fatigue loadings.

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NOTATION

A_g	= gross area of concrete (in. ²)
A_s	= area of tension reinforcement (in. ²)
A_t	= the effective concrete area in uniform tension (in. ²)
a_{cs}	= stabilized mean crack spacing (in.)
a_{cs} mean	= mean crack spacing (in.)
a_{cs} max	= stabilized maximum crack spacing (in.)
a_{cs} min	= stabilized minimum crack spacing (in.)
E_c	= modulus of elasticity of concrete (psi)
E_s	= modulus of elasticity of steel (ksi)
e	= eccentricity of C.G. of steel with respect to the C.G. of the concrete area (in.)
ϵ_c	= compressive strain in concrete (in./in.)
ϵ_s	= strain in tension reinforcement
F	= prestressing force (lb., kip)
γ	= a factor reflecting the distribution of bond stress
f_c	= compressive stress in concrete (psi, ksi)
f'_c	= compressive strength of concrete
f_d	= stress in the prestressing steel corresponding to the decompression load
f_{nt}	= stress in the prestressing steel at any load level beyond the decompression load
f_r	= modulus of rupture of concrete (psi)
f'_t	= tensile splitting strength of concrete (psi)
f_u	= ultimate strength of reinforcement
f_y	= yield strength of reinforcement
Δf_s	= net stress in the prestressed tendon

h_1	= distance from the neutral axis to the centroid of the reinforcement (in.)
h_2	= distance from the neutral axis to the tension face
M_u	= ultimate resisting moment
Σo	= sum of perimeters of all effective bars crossing the section on the tension side (in.)
ρ	= percentage of tension reinforcement = A_s/bd where A_s is the total area of bonded tensile steel
ρ_t	= A_s/A_t
R	= resisting force of the tensile zone of the concrete
R_i	= h_2/h_1
T	= total tensile force transferred by bond from steel (lb., kip)
μ	= maximum bond stress (psi)
w	= crack width (in.)
w_{max}	= maximum crack width at steel level (in.)
Y_t	= distance from the centroidal axis of gross section to the extreme fiber intension

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TABLE 1--PROPERTIES OF CONCRETE IN BEAMS

Beam	Age At Test (Days)	Compressive Strength (psi) at test	Tensile Splitting Strength (psi)	Water/Cement Ratio (gals/sack)	Ave. Slump (in.)
1	2	3	4	5	6
<u>PRETENSIONED</u>					
B1-B4	28	4780	650	5.4	5
B5-B6	29	4100	545	6.2	6-1/2
B7-B8	31	4350	555	6.3	6-1/2
B9-B10	33	2915	470	7.1	9
B11-B12	35	4740	705	5.8	5
B13-B14	27	4200	650	6.4	7
B15-B16	29	4130	660	6.6	7
B17-B18	34	4925	695	5.5	7-1/2
B19-B20	34	3010	470	7.1	9
B21	37	4720	730	5.8	5
B22	36	4350	660	6.4	7
B23	39	4150	670	6.6	7
B24	33	4385	620	5.5	7-1/2
<u>POST-TENSIONED</u>					
B-1, B-2	28	4810	460	6.2	6.0
B-3, B-	31	4970	480	6.2	6.0
B-5, B-6	34	4755	450	6.2	6.0
B-7, B-8	32	2925	370	5.4	4.0
B-9	35	2850	350	5.4	4.0
B-10	29	3120	380	5.4	4.0
O-I-1, O-I-2	30	5190	450	5.4	3.0
O-II-1, O-II-2	29	5755	475	5.4	3.0
O-III-1, O-III-2	35	6155	500	5.4	3.0
O-IV-1, O-IV-2	33	6160	500	5.4	3.0
U-II-1, U-II-2	31	5415	450	4.8	2.5
U-IV-1, U-IV-2	28	5380	445	4.8	2.5

Note: 1 in. = 25.4 mm
 1 psi = 6.895 kPa
 1 gal. = $3.785 \times 10^{-3} \text{ m}^3$

TABLE 2a--GEOMETRICAL PROPERTIES OF PRETENSIONED BEAMS

Beam	Effective Depth* (in.)	Longitudinal Tensile Steel						Compress. Steel			Web Reinforcement	
		No. of 1/4" Tens- ions	No. of #3 Bars	No. of #4 Bars	No. of #5 Bars	A _s (sq. in.)	A _s (sq. in.)	A' _s (sq. in.)	A' _s (sq. in.)	A' _s (sq. in.)	Size of Bar	Spacing of Bar (in.)
1	2	3	4	5	6	7	8	9	10	11		
B-1	8.00	3	-	-	.108	0.17	.22	.34	#3	6		
B-2	7.75	4	-	-	.144	0.23	.22	.35	#3	6		
B-3	7.30	5	-	-	.180	0.31	.22	.38	#3	6		
B-4	7.00	6	-	-	.216	0.39	.22	.39	#3	6		
B-5	6.57	7	-	-	.252	0.48	.22	.42	#3	4		
B-6	6.25	8	-	-	.288	0.58	.22	.44	#3	4		
B-7	8.80	3	2	-	.328	0.47	.22	.31	#3	6		
B-8	8.80	3	2	-	.328	0.47	.22	.31	#3	6		
B-9	8.65	4	2	-	.364	0.53	.22	.32	#3	6		
B-10	8.65	4	2	-	.364	0.53	.22	.32	#3	6		
B-11	8.37	5	2	-	.400	0.60	.22	.33	#3	4		
B-12	8.37	5	2	-	.400	0.60	.22	.33	#3	4		
B-13	8.46	6	2	2	.616	0.91	.22	.33	#3	4		
B-14	8.46	6	2	2	.616	0.91	.22	.33	#3	4		
B-15	8.22	7	-	2	.652	0.99	.22	.33	#3	4		
B-16	8.22	7	-	2	.652	0.99	.22	.33	#3	4		
B-17	8.00	8	-	2	.688	1.08	.22	.34	#3	4		
B-18	8.00	8	-	2	.688	1.08	.22	.34	#3	4		
B-19	9.00	3	2	-	.328	0.61	.22	.41	#3	6		
B-20	9.00	4	-	2	.544	1.01	.22	.41	#3	4		
B-21	9.00	3	2	-	.328	0.61	.22	.41	#3	4		
B-22	9.00	3	2	-	.328	0.61	.22	.41	#3	4		
B-23	9.00	4	-	2	.544	1.01	.22	.41	#3	4		
B-24	9.00	4	-	2	.544	1.01	.22	.41	#3	4		

Note: 1 in. = 25.4 mm

*Total depth = 10 in. (254 mm); Web thickness = 3 in. (76 mm); Flange width = 8 in. (203 mm)

TABLE 2b--GEOMETRICAL PROPERTIES OF POST-TENSIONED BEAMS

Beam	Effective Depth* (in.)	Longitudinal Tensile Steel										Web Reinforcement	
		No. of 1/4 in. Tendons	No. of D5 Wire	No. of #3 Bars	No. of #4 Bars	No. of #5 Bars	Chopped Wire (weight percent)	A _g (sq in.)	ρ percent	Size of D5 Wire (in.)	Spacing (in.)		
1	2	3	4	5	6	7	8	9	10	11	12	13	
B1, B2	7.25	4	-	-	-	-	-	-	0.1424	0.25	0.252	8	
B3, B4	8.61	4	-	-	2	-	-	-	0.3624	0.53	0.252	8	
B5, B6	8.91	4	-	-	-	2	-	-	0.5424	0.76	0.252	8	
B7, B8	7.25	4	-	-	-	-	1	1.01†	0.1424	1.01†	0.252	8	
B9	7.25	4	-	-	-	-	1.5	1.94†	0.1424	1.94†	0.252	8	
B10	7.25	4	-	-	-	-	2	2.75†	0.1424	2.75†	0.252	8	
O-I-1,	8.00	-	2	-	-	-	-	-	0.3060	0.48	0.252	6	
O-I-2	-	-	2	2	-	-	-	-	0.4060	0.60	0.252	6	
O-II-1	8.49	-	2	-	-	-	-	-	0.5260	0.74	0.252	6	
O-II-2	-	-	2	-	2	-	-	-	0.9260	1.24	0.252	6	
O-III-1	8.84	-	2	-	-	-	-	-	0.9260	1.24	0.252	6	
O-III-2	-	-	2	-	-	-	2	-	0.2530	0.37	0.252	6	
O-IV-1,	9.34	-	2	-	-	-	-	-	0.7730	1.03	0.252	6	
O-IV-2	-	-	1	2	-	-	-	-					
U-II-1	8.19	-	1	-	-	-	-	-					
U-III-2	-	-	1	-	-	-	2	-					
U-IV-1	9.41	-	1	-	-	-	-	-					
U-IV-2	-	-	-	-	-	-	-	-					

Note: 1 in. - 25.4 mm

*Total depth = 11 in. (279 mm); web thickness = 3 in. (76 mm); flange width = 8 in. (203 mm)

†By weight

TABLE 3a--MEASURED STABILIZED CRACK SPACINGS VERSUS
THEIR THEORETICAL VALUES -- PRETENSIONED BEAMS

Beam	2		3		4		5		6		7		8		9		10		11	
	Meas.		Theo.		acs		Meas.		Theo.		acs		Meas.		Theo.		acs		Corre-	
	Stab.min		Stab.min		acs		Stab.max.		Stab.max.		acs		Stab.		Stab.		acs		mean	
	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Crack	Spacing	Initial	M/M _u
	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}	a _{cs}
	min	min	min	min	min	min	max	max	max	max	max	max	max	max	max	max	max	max	mean	mean
	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)
B-1	5.00	4.08	4.08	4.08	1.23	7.25	8.15	8.15	8.15	8.15	0.89	0.89	6.12	6.33	6.33	6.33	0.97	0.97	0.65	0.65
B-2	4.13	3.93	3.93	3.93	1.05	7.63	7.86	7.86	7.86	7.86	0.97	0.97	5.90	5.34	5.34	5.34	1.11	1.11	0.65	0.65
B-3	4.31	3.73	3.73	3.73	1.16	6.63	7.46	7.46	7.46	7.46	0.89	0.89	5.60	5.13	5.13	5.13	1.09	1.09	0.67	0.67
B-4	3.13	3.26	3.26	3.26	0.96	6.38	6.52	6.52	6.52	6.52	0.98	0.98	4.89	4.75	4.75	4.75	1.03	1.03	0.67	0.67
B-5	2.38	2.77	2.77	2.77	0.86	5.00	5.53	5.53	5.53	5.53	0.90	0.90	4.15	4.39	4.39	4.39	0.95	0.95	0.70	0.70
B-6	2.50	2.41	2.41	2.41	1.04	5.13	4.81	4.81	4.81	4.81	1.07	1.07	3.61	3.92	3.92	3.92	0.92	0.92	0.66	0.66
B-7	2.13	2.13	2.13	2.13	1.00	3.94	4.26	4.26	4.26	4.26	0.92	0.92	3.20	2.97	2.97	2.97	1.08	1.08	0.75	0.75
B-8	1.84	1.98	1.98	1.98	0.93	4.06	3.96	3.96	3.96	3.96	1.03	1.03	2.97	2.97	2.97	2.97	1.00	1.00	0.75	0.75
B-9	2.13	1.90	1.90	1.90	1.12	3.75	3.80	3.80	3.80	3.80	0.97	0.97	2.85	2.72	2.72	2.72	1.05	1.05	0.73	0.73
B-10	2.50	1.84	1.84	1.84	1.36	5.00	3.68	3.68	3.68	3.68	1.36	1.36	2.76	2.72	2.72	2.72	1.02	1.02	0.76	0.76
B-11	1.75	1.63	1.63	1.63	1.07	3.13	3.26	3.26	3.26	3.26	0.96	0.96	2.44	2.67	2.67	2.67	0.91	0.91	0.66	0.66
B-12	1.75	1.65	1.65	1.65	1.06	3.38	3.30	3.30	3.30	3.30	1.02	1.02	2.47	2.67	2.67	2.67	0.93	0.93	0.66	0.66
B-13	1.44	1.58	1.58	1.58	0.91	3.56	3.16	3.16	3.16	3.16	1.13	1.13	2.37	2.21	2.21	2.21	1.07	1.07	0.68	0.68
B-14	1.38	1.51	1.51	1.51	0.91	3.63	3.03	3.03	3.03	3.03	1.20	1.20	2.27	2.21	2.21	2.21	1.03	1.03	0.71	0.71
B-15	1.25	1.39	1.39	1.39	0.90	3.13	2.77	2.77	2.77	2.77	1.13	1.13	2.08	2.46	2.46	2.46	0.84	0.84	0.75	0.75
B-16	1.75	1.50	1.50	1.50	1.17	3.00	3.00	3.00	3.00	3.00	1.00	1.00	2.25	2.46	2.46	2.46	0.91	0.91	0.76	0.76
B-17	1.63	1.50	1.50	1.50	1.08	3.25	3.00	3.00	3.00	3.00	1.08	1.08	2.25	2.29	2.29	2.29	0.98	0.98	0.70	0.70
B-18	1.50	1.37	1.37	1.37	1.09	3.06	2.75	2.75	2.75	2.75	1.11	1.11	2.06	2.29	2.29	2.29	0.90	0.90	0.71	0.71
B-19	1.88	2.09	2.09	2.09	0.90	4.25	4.18	4.18	4.18	4.18	1.02	1.02	3.13	3.52	3.52	3.52	0.89	0.89	0.72	0.72
B-20	1.69	1.85	1.85	1.85	0.91	3.75	3.70	3.70	3.70	3.70	1.03	1.03	2.78	2.57	2.57	2.57	1.08	1.08	0.70	0.70
Mean					1.04						1.03	1.03							0.99	0.99
S.D.					0.13						0.11	0.11							0.08	0.08

1 in. = 25.4 mm

TABLE 3b--MEASURED STABILIZED CRACK SPACINGS VERSUS
THEIR THEORETICAL VALUES -- POST-TENSIONED BEAMS

Beam	1		2		3		4		5		6		7		8		9		10		11	
	Meas.		Theo.		Meas.		Theo.		Meas.		Theo.		Meas.		Theo.		Meas.		Theo.		Corre-	
	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	Stab.min	Crack	acs	mean
	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Spacing	Initial
	acs	min	acs	min	acs	min	acs	min	acs	min	acs	min	acs	min	acs	min	acs	min	acs	min	acs	mean
	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	M/M _u
B-1	11.60	9.31	1.25	16.32	18.61	0.88	13.96	11.55	1.21	0.63												
B-2	10.08	9.03	1.12	17.00	18.05	0.94	13.54	11.55	1.17	0.57												
B-3	3.55	3.81	0.93	7.81	7.63	1.02	5.72	5.38	1.06	0.75												
B-4	3.62	4.12	0.88	8.50	8.24	1.03	6.18	5.38	1.15	0.82												
B-5	2.95	3.13	0.94	6.43	6.25	1.03	4.69	4.54	1.03	0.68												
B-6	3.02	3.33	0.91	7.05	6.67	1.06	5.00	4.54	1.10	0.71												
B-7	9.65	8.84	1.09	16.87	17.68	0.95	13.26	11.95	1.11	0.73												
B-8	11.06	9.42	1.17	17.20	18.84	0.91	14.13	11.95	1.18	0.67												
B-9	10.40	8.54	1.22	15.22	16.24	0.94	12.81	11.50	1.11	0.65												
B-10	11.30	9.27	1.22	16.50	18.53	0.89	13.90	11.85	1.17	0.70												
O-I-1	5.36	5.74	0.93	11.50	11.49	1.00	8.62	8.74	1.07	0.56												
O-I-2	5.83	6.21	0.94	12.68	12.41	1.02	9.31	8.74	1.07	0.61												
O-II-1	3.31	3.48	0.95	7.24	6.96	1.04	5.22	4.85	1.08	0.68												
O-II-2	3.22	3.39	0.95	7.10	6.77	1.05	5.08	4.85	1.05	0.73												
O-III-1	2.58	2.91	0.89	6.50	6.17	1.05	4.36	4.23	1.03	0.77												
O-III-2	2.47	2.68	0.92	5.75	5.36	1.07	4.02	4.23	0.95	0.64												
O-IV-1	2.05	2.20	0.93	4.90	4.40	1.11	3.30	3.27	1.01	0.71												
O-IV-2	1.95	2.05	0.95	4.25	4.11	1.03	3.08	3.27	0.94	0.59												
U-II-1	4.16	4.23	0.98	8.70	8.45	1.03	6.34	7.11	0.89	0.71												
U-II-2	4.25	4.31	0.99	9.14	8.63	1.06	6.47	7.11	0.91	0.78												
U-IV-1	2.75	2.50	1.10	5.50	5.00	1.10	3.75	4.03	0.93	0.45												
U-IV-2	2.61	2.41	1.08	5.05	4.81	1.05	3.61	4.03	0.90	0.37												
Mean:			1.015			1.012															1.05	
S.D.:			0.118			0.065															0.10	
1 in. = 25.4 mm																						

TABLE 4a--OBSERVED VERSUS THEORETICAL MAXIMUM CRACK WIDTH AT STEEL LEVEL OF PRETENSIONED BEAMS

Beam	Net Steel Stress Δf_s									
	30 ksi			40 ksi			60 ksi			
	$W_{obs.}$	$W_{theo.}$	$\frac{W_{obs.}}{W_{theo.}}$	$W_{obs.}$	$W_{theo.}$	$\frac{W_{obs.}}{W_{theo.}}$	$W_{obs.}$	$W_{theo.}$	$\frac{W_{obs.}}{W_{theo.}}$	
1	2	3	4	5	6	7	8	9	10	
B-1	.0089	.0089	1.00	.0122	.0118	1.03	.0198	.0177	1.12	
B-2	.0071	.0075	0.95	.0104	.0100	1.04	.0172	.0153	1.15	
B-3	.0062	.0072	0.86	.0090	.0096	0.94	.0149	.0144	1.03	
B-4	.0053	.0067	0.79	.0075	.0089	0.84	.0127	.0133	0.95	
B-5	.0047	.0065	0.72	.0067	.0087	0.77	.0108	.0130	0.83	
B-6	.0042	.0058	0.72	.0058	.0078	0.74	.0095	.0116	0.82	
B-7	.0040	.0041	0.98	.0055	.0055	1.00	.0095	.0082	1.16	
B-8	.0039	.0041	0.95	.0054	.0055	0.98	.0089	.0082	1.09	
B-9	.0037	.0035	1.06	.0050	.0047	1.06	.0081	.0070	1.16	
B-10	.0041	.0035	1.17	.0056	.0047	1.19	.0090	.0070	1.29	
B-11	.0036	.0043	0.84	.0050	.0058	0.86	.0080	.0087	0.92	
B-12	.0035	.0043	0.81	.0052	.0058	0.90	.0086	.0087	0.99	
B-13	.0030	.0034	0.88	.0045	.0046	0.98	.0077	.0068	1.13	
B-14	.0032	.0034	0.94	.0047	.0046	1.02	.0080	.0068	1.18	
B-15	.0025	.0040	0.63	.0042	.0053	0.79	.0077	.0080	0.96	
B-16	.0026	.0040	0.65	.0039	.0053	0.74	.0067	.0080	0.84	
B-17	.0027	.0037	0.73	.0042	.0049	0.86	.0074	.0074	1.00	
B-18	.0021	.0037	0.57	.0031	.0049	0.63	.0063	.0074	0.85	
B-19	.0041	.0049	0.84	.0057	.0065	0.88	.0090	.0097	0.93	
B-20	.0026	.0036	0.72	.0039	.0047	0.83	.0070	.0071	0.99	
Mean			0.84			0.90			1.02	
S.D.			0.15			0.14			0.14	
1 in. = 25.4 mm										

TABLE 4b---OBSERVED VERSUS THEORETICAL MAXIMUM CRACK
WIDTH AT STEEL LEVEL OF POST-TENSIONED BEAMS

Beam	Net Steel Stress Δf_s									
	30 ksi			40 ksi			60 ksi			
	Wobs.	Wtheo.	$\frac{Wobs.}{Wtheo.}$	Wobs.	Wtheo.	$\frac{Wobs.}{Wtheo.}$	Wobs.	Wtheo.	$\frac{Wobs.}{Wtheo.}$	
1	2	3	4	5	6	7	8	9	10	
B-3	0.0055	0.0064	0.859	0.0086	0.0085	1.012	0.0136	0.0128	1.063	
B-4	0.0058	0.0064	0.906	0.0094	0.0085	1.106	0.0144	0.0128	1.125	
B-5	0.0054	0.0056	0.964	0.0085	0.0075	1.133	0.0138	0.0112	1.232	
B-6	0.0053	0.0056	0.946	0.0080	0.0075	1.067	0.0128	0.0112	1.143	
O-II-1	0.0054	0.0062	0.871	0.0080	0.0083	0.964	0.0134	0.0124	1.081	
O-II-2	0.0049	0.0062	0.790	0.0074	0.0083	0.892	0.0124	0.0124	1.000	
O-III-1	0.0044	0.0053	0.830	0.0070	0.0071	0.986	0.0110	0.0107	1.028	
O-III-2	0.0048	0.0053	0.906	0.0073	0.0071	1.028	0.0118	0.0107	1.103	
O-IV-1	0.0035	0.0041	0.854	0.0056	0.0055	0.982	0.0088	0.0083	1.060	
O-IV-2	0.0037	0.0041	0.902	0.0059	0.0055	1.073	0.0096	0.0083	1.157	
U-II-1	0.0072	0.0093	0.774	0.0111	0.0124	0.895	0.0178	0.0187	0.952	
U-II-2	0.0065	0.0093	0.699	0.0107	0.0124	0.863	0.0168	0.0187	0.898	
U-IV-1	0.0038	0.0053	0.717	0.0061	0.0071	0.859	0.0098	0.0107	0.916	
U-IV-2	0.0043	0.0053	0.811	0.0067	0.0071	0.944	0.0104	0.0107	0.972	
Mean:			0.99			0.98			1.05	
S.D.:			0.09			0.12			0.10	
1 in. = 25.4 mm										

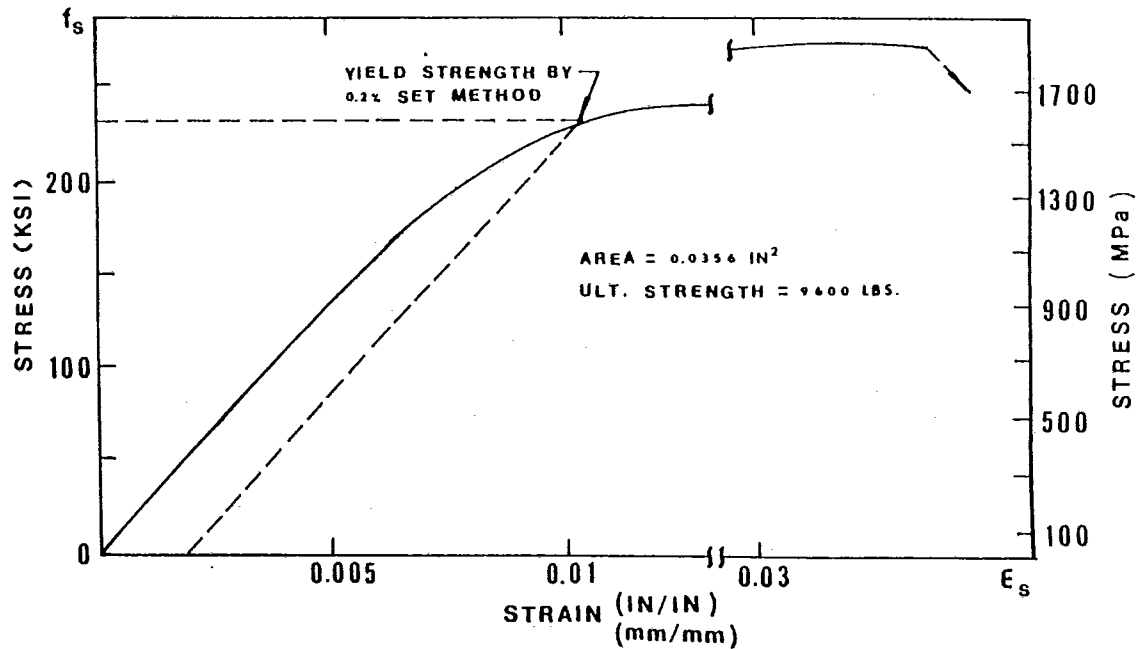


Fig. 1--Typical stress-strain relationship of prestressing steel

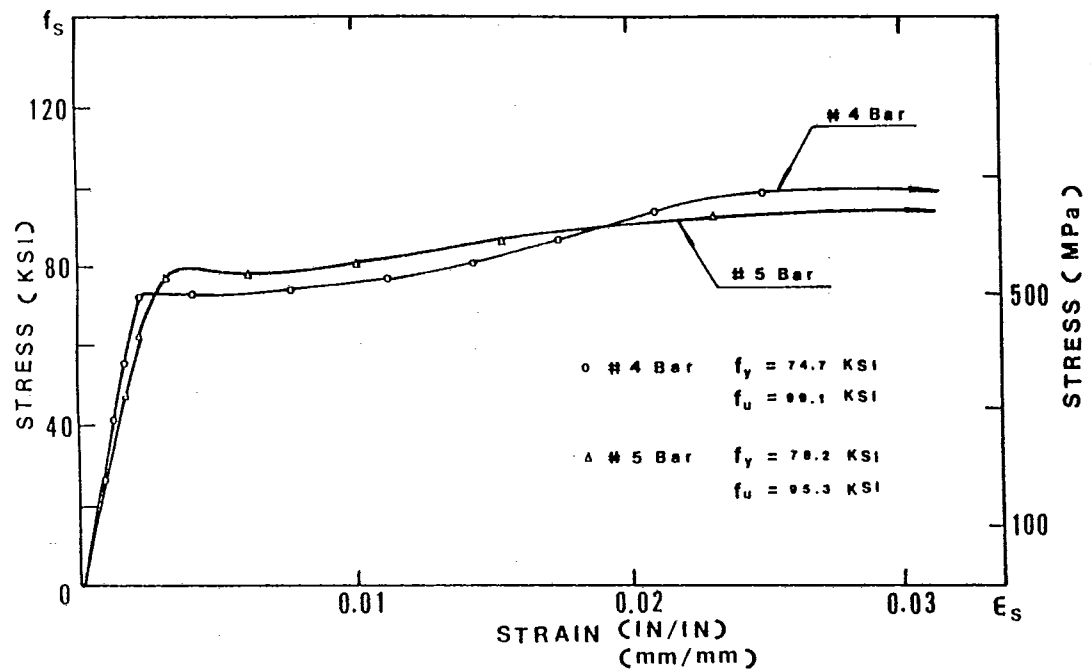
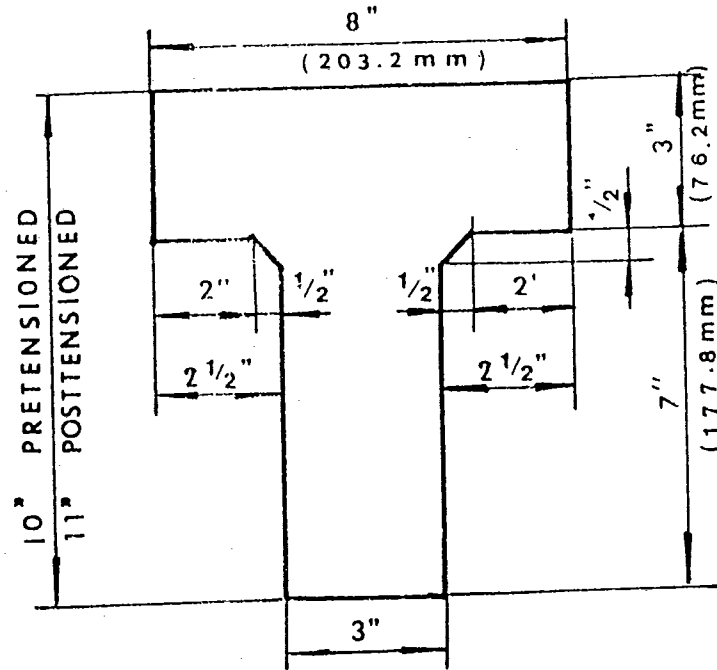
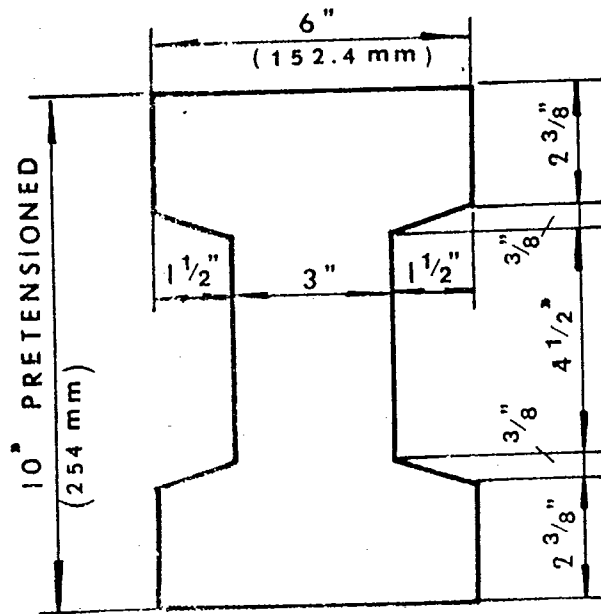


Fig. 2--Typical stress-strain relationship of non-prestressing steel

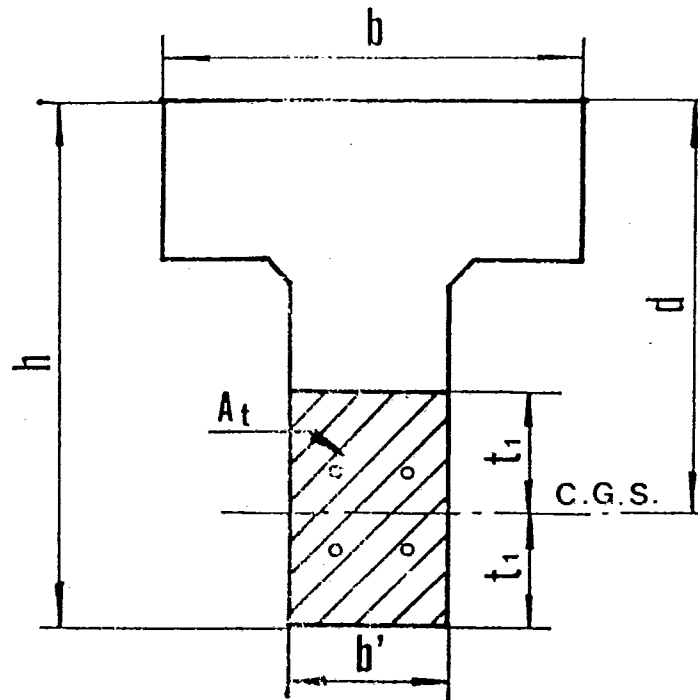


[a] T - SECTION

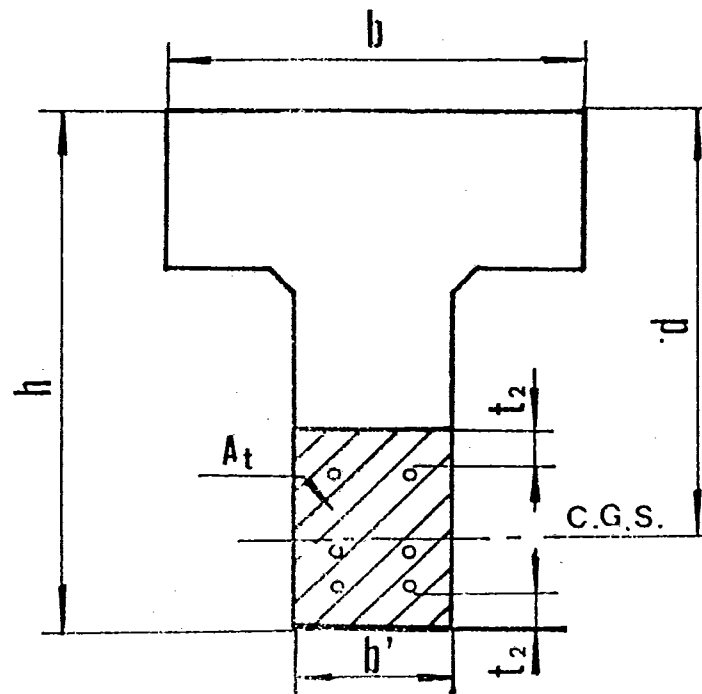


[b] I - SECTION

Fig. 3--Geometrical beam cross section dimensions



(a) FOR EVEN DISTRIBUTION OF REINFORCEMENT
IN CONCRETE



(b) FOR NON-EVEN DISTRIBUTION OF REINFORCEMENT
IN CONCRETE

Fig. 4--Effective concrete area in tension

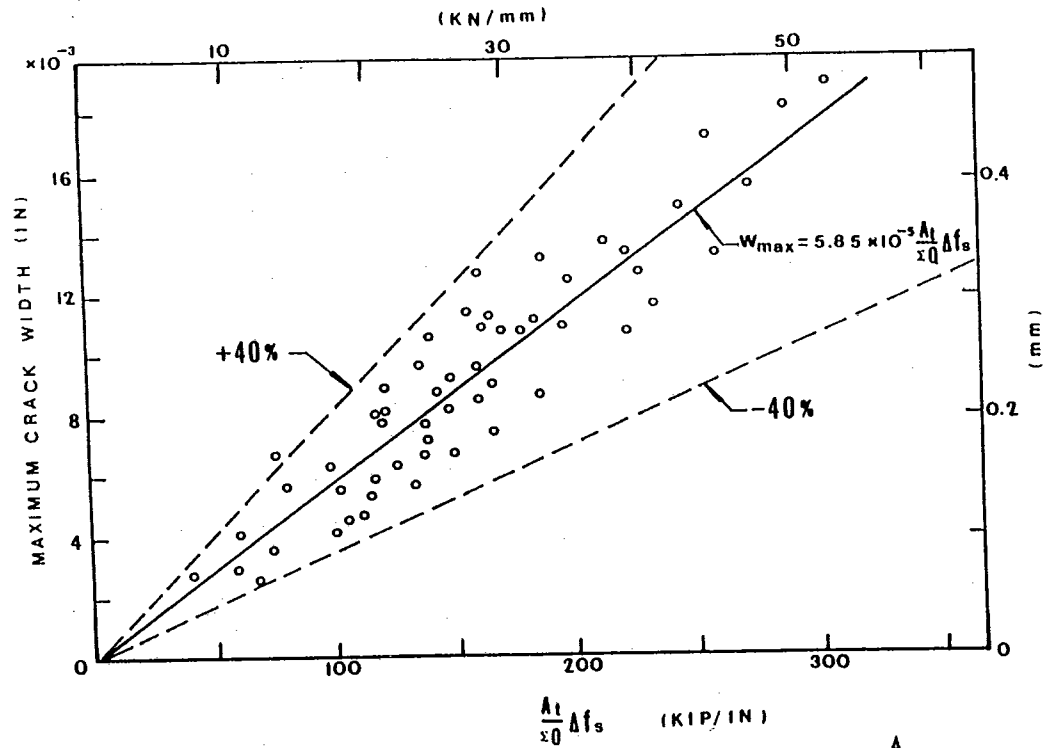


Fig. 5--Linearized maximum crack width versus $(\frac{A_t}{\Sigma o}) \Delta f_s$ in pretensioned beams

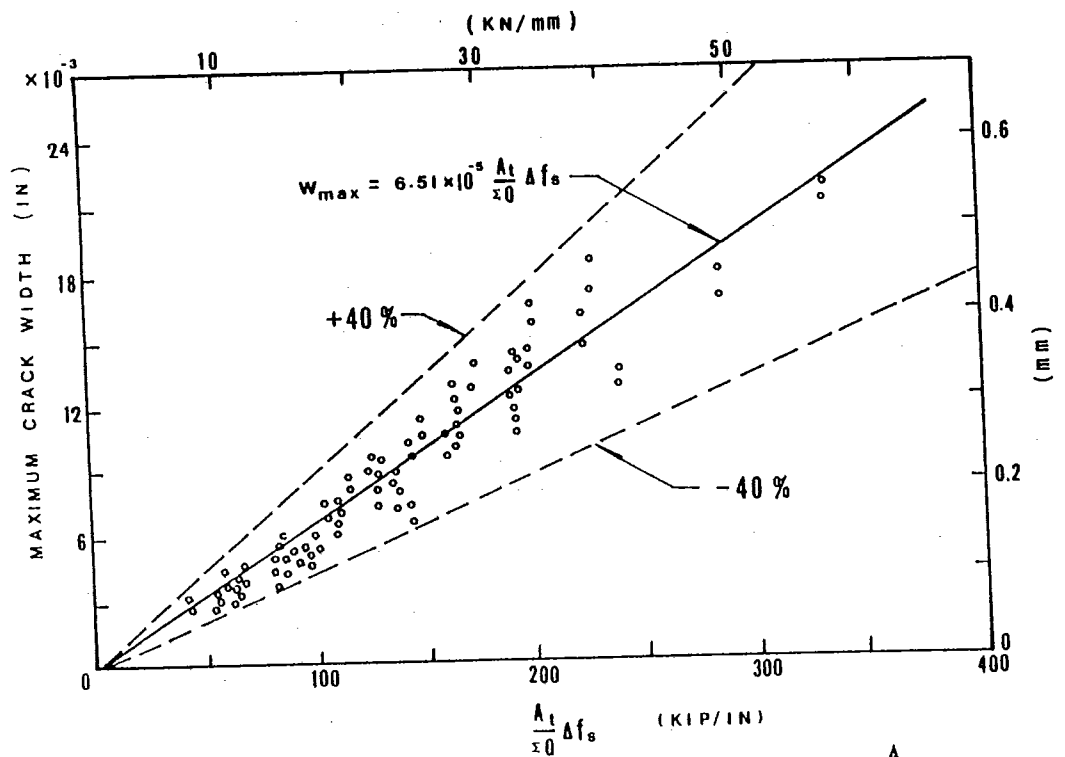


Fig. 6--Linearized maximum crack width versus $(\frac{A_t}{\Sigma o}) \Delta f_s$ in post-tensioned beams

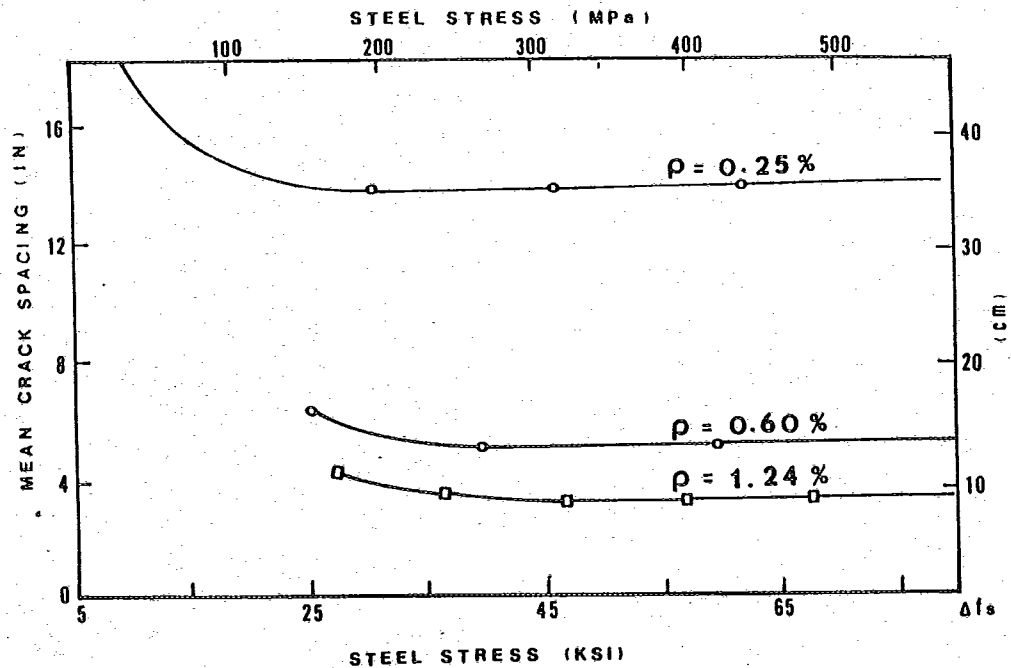


Fig. 7--Effect of steel percentage on mean crack spacing

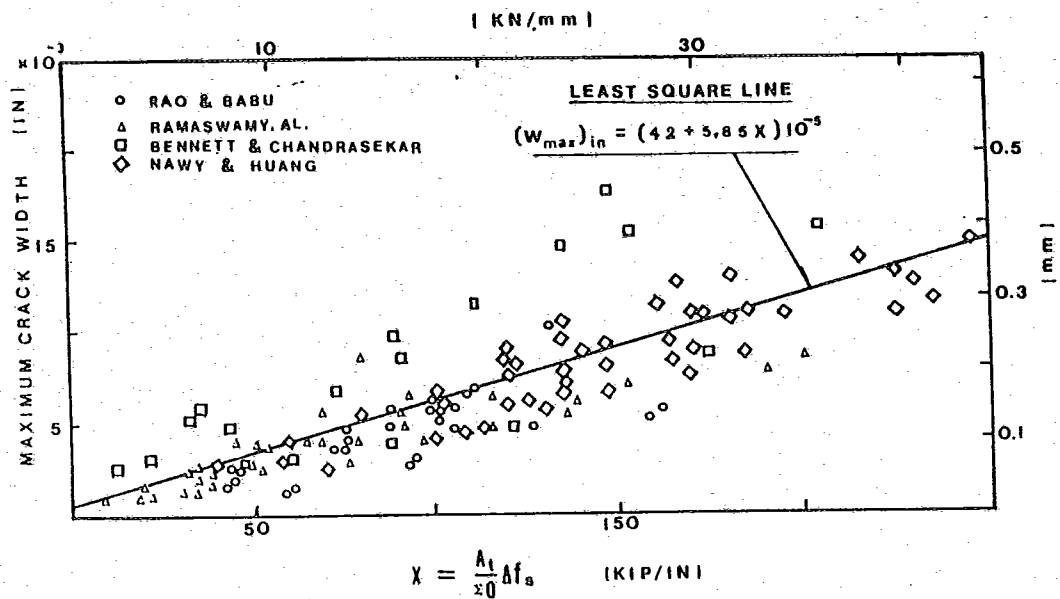


Fig. 8--Reinforcement stress versus crack width (best fit data of several investigators)

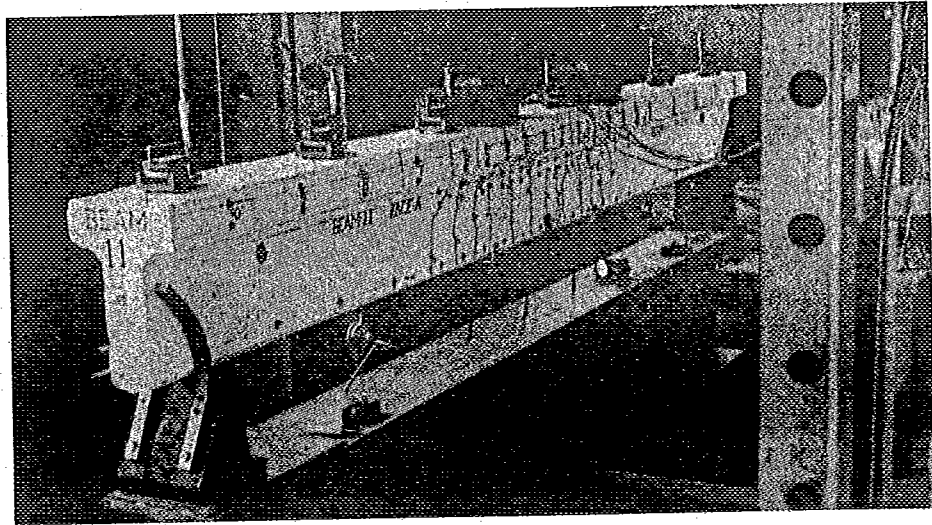


Fig. 9--Typical test setup of a prestressed T-beam

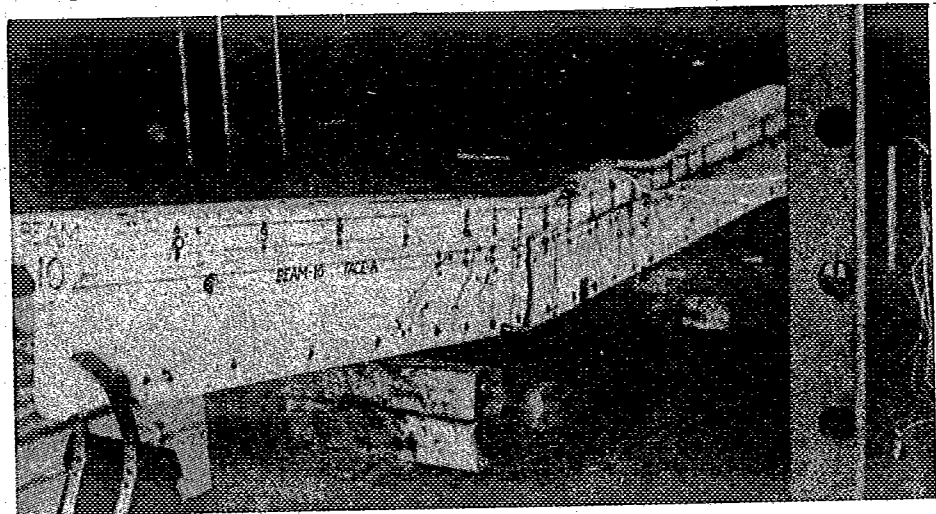


Fig. 10--Cracking pattern at failure of a pretensioned beam

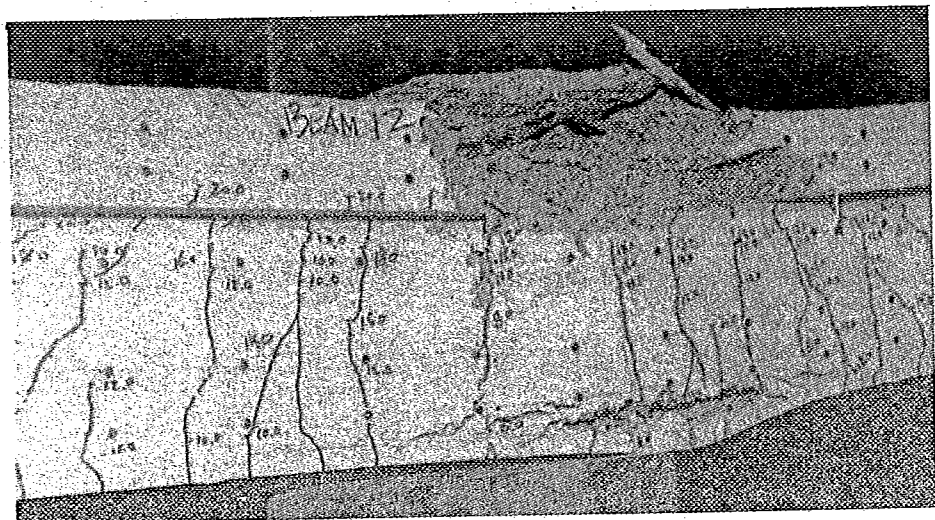


Fig. 11--Closeup of flexural cracks at failure of a prestressed beam

SP 113-2

Cracking in Partially Prestressed Beams Under Static and Fatigue Loading

By M.H. Harajli and A.E. Naaman

Synopsis: This study was conducted to evaluate cracking in partially prestressed concrete beams under both static and cyclic fatigue loading. It comprises two parts: an experimental investigation, and analytical modeling of crack width increase under cyclic fatigue loading. In the experimental part, twelve different sets of beams were tested in four point bending. Each set comprised two identical beams. One beam was tested to failure under monotonically increasing load, while the second beam was tested in fatigue at a constant load range simulating full live load. In the analytical part of the study, a model for computing the increase in crack width under cyclic fatigue loading is developed. The model accounts for the effect of change in steel stress due to cyclic creep of concrete in compression, the increase in slip due to bond redistribution, concrete shrinkage, and cyclic creep of concrete in tension. Results predicted by the model were found to be in good agreement with the experimental results observed in this study as well as results reported elsewhere.

Keywords: beams (supports); cracking (fracturing);
crack width and spacing; cyclic loads; fatigue
(materials); partial prestressing; prestressed concrete

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Muhammed H. Harajli is Assistant Professor of Civil Engineering at the American University of Beirut. He obtained his Ph.D. degree from the University of Michigan in 1985. His research interests include prestressed concrete and the dynamic response of concrete structures. The present study is part of his Ph.D. thesis research.

ACI Fellow Antoine E. Naaman is Professor of Civil Engineering at the University of Michigan. He is active on several technical committees of ACI, PCI, and ASCE. His research interests include prestressed concrete and fiber reinforced cement based composites.

INTRODUCTION

It is generally agreed that serviceability criteria in prestressed and partially prestressed concrete structures primarily control their design. The most commonly critical criteria are related to allowable stresses, fatigue, deflection, corrosion, and cracking. Partially prestressed beams are designed on the basis that cracking is allowed under service loads. Wide cracks may be safe from a strength design point of view but are often unacceptable for aesthetic reasons and because they may provide an easier path to the steel for corrosive agents.

Numerous models are available in the technical literature to predict crack widths in partially prestressed beams subjected to static monotonic loads. Cracks widths are known to keep increasing under sustained loads (1). Repetitive loads accelerate the creep process of concrete (2,3,16,30) and may lead to significant bond deterioration between concrete and the reinforcement. These factors tend to contribute significantly to the increase in crack width (7,8). They should be considered in design when applying a crack width criterion. However, little information exists on the prediction of growth in crack widths under sustained and cyclic loads simulating respectively sustained dead loads and repetitive live loads.

The main objective of this study was to investigate experimentally and model analytically the increase in crack widths in partially prestressed beams subjected to static and cyclic fatigue loading. The experimental part is briefly summarized in the next section. The analytical model developed here takes into consideration the following factors that influence the growth in crack width when cyclic loads are considered: 1) cyclic creep of concrete in compression, 2) bond stress redistribution between the reinforcement and the surrounding concrete, and 3) shrinkage and tensile creep of concrete in the tensile zone between two successive cracks.

EXPERIMENTAL PROGRAM

A detailed description of the experimental program and a comprehensive set of data and results can be found in References 14-15. Twelve different sets of beams designed for 12 different sets of reinforcing parameters (three levels of reinforcing index and, for each, four levels of partial prestressing including full reinforcing and full prestressing) were tested. All beams were rectangular in cross section (4.5x9 in (114x228 mm)), simply supported on a 9 ft (2.75 m) span and loaded in

four point bending. Each set of beams comprised two identical beams; one was tested under static monotonic loading up to failure, and the other under cyclic fatigue loading ranging from 40% to 60% of the failure load observed in the monotonic test. The cyclic load level was selected to simulate full live load in a real size beam. Cyclic loading was carried out up to five million cycles or failure whichever occurred first. The cyclic tests were interrupted at selected cycling intervals and measurements of crack widths, deflections, and strains in the reinforcement were taken for a complete loading-unloading cycle. Similar measurements were also taken for the beams tested in monotonic loading.

Crack widths during the static and cyclic tests were measured at the location of a crack former at midspan by two linear voltage differential transducers (LVDTs) attached to the beam with a special fixture. The two LVDTs were placed level with the centroid of the non-prestressed tension reinforcement at 1 in (25 mm) from the bottom tension fiber.

ANALYTICAL MODEL FOR CYCLIC SLIP AND STRESSES

The main approach adopted in developing a general time and cyclic dependent model to compute crack widths is based on : 1) equilibrium between steel stresses, bond stresses, and concrete tensile stresses in a concrete tensile prism joining two successive cracks (Fig. 1), and 2) a known local bond stress-slip relationship for the reinforcement.

Assumptions

1. The segment of beam considered is assumed subjected to a constant bending moment only.
2. The ascending portion of the bond stress-slip relation is assumed linear up to a maximum value $\tau_{\max}$. A similar assumption was adopted in References 9,13, and 29. It leads to the following relation:

$$\tau_x = kS_x \leq \tau_{\max} \quad (1)$$

in which τ_x and S_x are the local bond stress and local slip respectively at a distance x , and k is the stiffness of the bond stress-slip relation.

3. The bond stress distribution varies linearly in magnitude from zero at the primary crack to the peak bond stress $\tau_{\max}$ at a distance L_T from the center of the cracked region (Fig. 1). This will occur only if the steel stress is large enough to cause $\tau_{\max}$ to develop between two primary cracks. The location L_T of the peak bond stress can be determined analytically as shown later in Eq. 21. This assumption of linear bond stress distribution in the plastic region has the advantage of bypassing the need to know the descending portion of the local bond stress-slip relationship (which is not known to date).

4. During redistribution of bond stresses under cyclic loads the location of the maximum bond stress remains stationary at a distance L_T from the center of the

prism.

Local Bond Stress-Slip Relationship Under Static Loading

The local bond stress-slip relation under monotonic load adopted in this study was taken from an experimental investigation by Eligehausen et al. (11). For a 30 MPa (4.35 ksi) concrete strength, the following ascending portion of the local bond stress-slip relation was derived:

$$\tau_x = 13.5 S_x^{0.4} \leq 5 \text{ MPa} \quad (2a)$$

in which τ_x is in MPa and S_x is in mm. For the US system Eq. 2a becomes:

$$\tau_x = 7.14 S_x^{0.4} \leq 0.72 \text{ ksi} \quad (2b)$$

in which τ_x is in ksi and S_x is in inches. It was also observed that while the bond stress increases, the slip decreases, and both could be related to f'_c , the compressive strength of concrete. Using the experimental results described in (11), the following idealized equations for estimating k (taken at 50 % $\tau_{\max}$) and $\tau_{\max}$ for deformed bars in unconfined concrete were derived (Fig. 2):

$$k = 145 f'_c \quad (\text{kip/in}^3) \quad (3)$$

$$\tau_{\max} = 0.72 \sqrt{\frac{f'_c}{4.35}} \quad (\text{ksi}) \quad (4)$$

in which f'_c is in ksi.

An analysis of the experimental results of Gylltoft (13) led to a value of k for prestressing strands equal to 650 kip/in³. This value is consistent with the range of values (665 to 850 k/in³ (181 to 231 N/mm³)) recommended by Edward and Picard (10). The value of $\tau_{\max}$ adopted in this study was taken equal to 0.56 ksi (3.86 MPa) for a 6 ksi (41.4 MPa) concrete compressive strength using the test results of Stocker and Sozen (28). Equation (3) and (4) for reinforcing bars and the preceeding results for prestressing strands were used in the verifying the predictions of the model developed here.

Cyclic Bond Stress-Slip Relationship

Rehm and Eligehausen (25) observed in their experiments that the increase in slip under cyclic bond stress in the linear range of the bond stress-slip relation, is similar to the cyclic creep behavior of concrete in compression. The increase in slip is due to: 1) a time dependent slip component (creep in shear), and 2) cyclic dependent slip component caused by bond damage due to cyclic loading. The following expression was derived from their experimental results:

$$S_{x,N} = S_x N^{0.107} \quad (5)$$

in which $S_{x,N}$ is the local slip at any number of loading cycles N . For the range

of variable tested, $S_{x,N}$ was found to be independent of the stress range in the cycle, the bar diameter, and the concrete compressive strength. It was moreover observed that the increase in slip under cyclic bond stress is similar to that under sustained bond stress with a time lag. Using the experimental data given in (25), and taking into consideration the frequency of loading (1 Hertz), the following expression was derived in this study to describe the variation of local bond slip with time and number of loading cycles:

$$S_{x,N} = S_x(t^{0.107} + 0.58 N^{0.107}) \quad (6)$$

in which t represents time in hours ($t > 1$) and N the number of cycles ($N > 0$).

Equation 6 can be used to derive an expression for the apparent stiffness of the bond stress-slip relation at any time t and cycle N , that is (Fig. 3):

$$k_N = k / (t^{0.107} + 0.58 N^{0.107}) \quad (7)$$

Therefore Eq. 1 can be rewritten as:

$$\tau_{x,N} = k_N S_{x,N} \quad (8)$$

Time and Cyclic Dependent Stresses

Consider a segment of beam bound by two primary cracks and subjected to pure bending (Fig.1). The variation of slip $S_{x,N}$ of reinforcement with respect to x at any time t and cycle N can be expressed as follows:

$$\frac{dS_{x,N}}{dx} = \epsilon_{sx,N} - \epsilon_{cx,N} + \gamma_N \epsilon_{SU} \quad (9)$$

in which $\epsilon_{sx,N}$ and $\epsilon_{cx,N}$ are the steel strain and the concrete tensile strain at a distance x from the center of the segment, γ_N is a time dependent shrinkage factor, and ϵ_{SU} is the ultimate free shrinkage strain. All strains are assumed positive. Differentiating Eq. 9, using Eq.8, and writing that equilibrium between steel stress, bond stress, and concrete stress is satisfied lead to the following equation:

For $0 \leq x \leq L_T$

$$\frac{d^2 S_{x,N}}{dx^2} = K_{1,N}^2 S_{x,N} \quad (10)$$

in which:

$$K_{1,N} = \sqrt{k_N \Psi m_N} \quad (11)$$

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$$m_N = \frac{1}{A_s E_s} + \frac{1}{A_b E_{ct,N}} \quad (12)$$

$$A_b = \frac{A_t}{n_b} \quad (13)$$

in which A_s and Ψ are the area and perimeter of a reinforcing bar or strand respectively, E_s is the modulus of elasticity of the tension steel, $E_{ct,N}$ is the apparent modulus of elasticity of concrete in tension (which accommodates the effect of tensile creep of concrete), A_t is the effective area of concrete tension prism surrounding the principal reinforcement, and n_b is the equivalent number of bars (total area of reinforcement divided by the area of the largest diameter bar). The definition of A_b and n_b is in accordance with section 10.6 of the ACI Building Code Requirements for Reinforced Concrete and its commentary (ACI 318-83). The solution of Eq. 10 leads to the following equation:

For $0 \leq x \leq L_T$

$$S_{x,N} = A_N \cosh(K_{1,N} x) + B_N \sinh(K_{1,N} x) \quad (14)$$

in which A_N and B_N are constant to be determined from the following boundary conditions: 1) the slip at the center of the segment is equal to zero (due to symmetry), and 2) the tensile stress in the concrete is equal to zero at the crack while the steel stress is equal f_{so} as determined from linear elastic cracked section analysis at first cycle of loading. For prestressing tendons, f_{so} represents the increase in stress above the effective prestress. Using these boundary conditions leads to:

$$A_N = 0 \quad (15)$$

$$B_N = K_{2,N} = \frac{\left(\frac{f_{sT,N}}{E_s} - \frac{f_{cT,N}}{E_{ct,N}} + \gamma_N \epsilon_{su} \right)}{K_{1,N} \cosh(K_{1,N} L_T)} \quad (16)$$

in which $f_{sT,N}$ and $f_{cT,N}$ are the steel stress and the tension stress in the concrete at a distance L_T from the center of the cracked region; $f_{sT,N}$ and $f_{cT,N}$ can be calculated from the assumed bond stress distribution in the plastic region as:

$$f_{sT,N} = f_{so} - \frac{1}{2} \tau_{T,N} \Psi L / A_s \quad (17)$$

$$f_{cT,N} = 1/2 \tau_{T,N} \Psi L_A / A_b \quad (18)$$

$$L_A = L - L_T \quad (19)$$

where $\tau_{T,N}$ is the peak bond stress at time t and cycle N at a distance L_T from the center of the cracked region and is expressed as follows:

$$\tau_{T,N} = \frac{k_N \left(\frac{f_{so}}{E_s} + \gamma_N \epsilon_{su} \right) \text{Tanh}(K_{1,N} L_T)}{K_{1,N} + \frac{1}{2} \Psi L_A k_N m_N \text{Tanh}(K_{1,N} L_T)} \quad (20)$$

The distance L_T from the center of the cracked region can be calculated by trial and error using the following transcendental equation ($0 < L_T < L$):

$$\frac{k}{K_{1,N} \tau_{\max}} \left\{ \frac{f_{so}}{E_s} - \frac{1}{2} \tau_{\max} \Psi m (L - L_T) \right\} \text{Tanh}(K_1 L_T) = 1 \quad (21)$$

where K_1 and m are the same as $K_{1,N}$ and m_N evaluated at first cycle of loading. If for $L_T = L$, the left hand side of Eq. (21) is less than 1, then $\tau_{\max}$ did not develop at first cycle and the bond stress distribution is elastic everywhere inside the cracked region.

After calculating the constants $B_N = K_{2,N}$, it is possible to calculate the time and cyclic dependent slip and, correspondingly, the bond stress, steel stress and concrete tensile stress distribution at any time t and cycle N . These were derived and are given in Appendix I, Equations Ia, Ib, Ic and Id respectively.

Evaluation and Validation

Typical bond stress, steel stress, and concrete tensile stress distribution between two primary cracks predicted using Eq. Ia to Ie are shown in Figs. 4 and 5 for two possible cases of bond stress. Fig. 4 covers the case where $\tau < \tau_{\max}$ while Fig. 5 covers the case where $\tau = \tau_{\max}$. As shown in the figures the stress f_{so} in the steel at the primary cracks at first cycle was assumed constant for this comparison. It can be observed from these figures that as the number of loading cycles increases the bond stress decreases at a given section due to bond redistribution, and the corresponding stress in the steel increases. Thus the steel stress profile changes progressively from the case of bonded reinforcement to that of an equivalent unbonded reinforcement. This in turn leads to an increase in the

slip of the reinforcement and contributes significantly to the increase in crack width at the primary cracks.

To check the adequacy of the analytical predictions, the steel stress distribution using Eq. 1c is compared with the experimental results obtained by Bresler and Bertero (8) for three different steel stresses f_{s0} at the primary cracks. As shown in Fig. 6, predicted and experimental results are in good agreement suggesting that the model is reasonably valid.

PREDICTION OF CYCLIC DEPENDENT CRACK WIDTHS

In the preceding section, the change with number of loading cycles in bond stress, steel stress, and concrete tensile stress between primary cracks, was evaluated assuming a constant stress in the steel f_{s0} at the primary crack. In effect, for constant loading, the stress at the primary crack changes with the number of loading cycles due to the cyclic creep of concrete in compression and in tension. Hence f_{s0} must be determined from a cracked section analysis using equilibrium and compatibility. Once f_{s0} is calculated at the primary crack, the stress distribution between two primary cracks can be determined and the crack width calculated. The approach followed in this research to account for the cyclic creep of concrete is described next.

Cyclic Creep of Concrete In Tension

The effect of tensile creep of concrete in tension between two primary cracks was taken into consideration by using the apparent modulus of elasticity of concrete in tension $E_{ct,N}$ calculated as follows:

$$E_{ct,N} = f_{ctmax} / \epsilon_{ct,N} \quad (22)$$

in which f_{ctmax} and $\epsilon_{ct,N}$ are respectively the concrete tension stress calculated for the first cycle of loading and total tension strain (including cyclic creep strain) at the center of the cracked region. By assuming that the cyclic creep of concrete in tension is similar to that in compression (6), the simple expression proposed by Balaguru (5) was used to calculate $\epsilon_{ct,N} (x 10^6)$ as follows:

$$\epsilon_{ct,N} = \epsilon_{cti} + 129 \sigma_{tm} t^{1/3} + 17.8 \sigma_{tm} \Delta \sigma_{tc} N^{1/3} \quad (23)$$

where:

$$\epsilon_{cti} = 10^6 f_{ctmax} / E_{cti} \quad (24)$$

$$\sigma_{tm} = (f_{ctmax} + f_{ctmin}) / (2f_r) \quad (25)$$

$$\Delta \sigma_{tc} = (f_{ctmax} - f_{ctmin}) / f_r \quad (26)$$

in which E_{cti} is the tension modulus of elasticity of concrete (assumed equal to that in compression); f_{ctmin} is the minimum tensile stress at the center of the cracked region corresponding to the minimum steel stress level in the cycle (computed similarly to f_{ctmax}); f_r is the modulus of rupture of concrete and t is the time in hours. Expressions derived in this study to compute f_{ctmax} , σ_{tm} and $\Delta\sigma_{tc}$ are given in Appendix I, Eqs. Ie, If, and Ig respectively.

Cyclic Creep of Concrete In Compression

Cyclic creep of concrete in compression causes stress redistribution in the concrete section at the primary cracks and leads to an increase in steel stress thus in crack width. The effect of cyclic creep was taken into consideration by using the concept of apparent modulus of elasticity of concrete. The stress in the steel is calculated at any time t and cycle N using linear elastic cracked section analysis using the general equations derived by Naaman and Siriaksorn (22). In this analysis, the real modulus of elasticity of concrete is replaced by an apparent cycle dependent modulus given by:

$$E_{c,N} = f_{max} / \epsilon_{c,N} \quad (27)$$

in which f_{max} is the maximum concrete compressive stress in the uppermost fiber calculated at first cycle of loading and $\epsilon_{c,N}$ is the total concrete compressive strain including cyclic creep strain at the same fiber; $\epsilon_{c,N}$ was calculated using the expression suggested by Holmen (16) and is given by Eq. Ii of Appendix I. The number of cycles to failure N_f used in Eq. Ii is determined from the following relationship recommended by Rilem Committee 36-RDL (26):

$$\sigma_{max} = 1 - \beta_o(1 - R) \log N_f \quad (28)$$

where β_o = material constant (varies between 0.064 and 0.080) and :

$$R = \sigma_{min} / \sigma_{max} \quad (29)$$

$$\sigma_{max} = f_{max} / f_c \quad (30)$$

$$\sigma_{min} = f_{min} / f_c \quad (31)$$

f_{min} is the minimum stress in the uppermost fiber at the primary crack. An average value for β_o equal 0.072 was adopted in this study.

Prediction Equations For Crack Width

For a given crack spacing a_{cs} ($a_{cs} = 2L$) the crack width at first cycle and at

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any number of loading cycles N , can be calculated from the slip at the primary crack and the increase in steel stress due to cyclic creep of concrete in compression as follows (Fig.7):

$$W = 2S_o\beta \quad (32)$$

$$W_N = 2S_{o,N}\beta_N + (f_{so,N} - f_{so})a_{cs}\beta_N/E_s \quad (33)$$

in which W and W_N are the crack widths at the first and N_{th} loading cycles respectively; β and β_N are distance ratios (see Fig 5) evaluated at first cycle and cycle N respectively; S_o and $S_{o,N}$ are the slip of the reinforcement at the primary crack at first cycle and at cycle N respectively; and $f_{so,N}$ is the steel stress at the primary crack at cycle N . The slip S_o and $S_{o,N}$ can be evaluated from Eq.1a of Appendix I as follows:

$$S_o = K_2 \sinh(K_1 L_T) + f_{so} L_A / E_s - 1/6 \tau_{max} \Psi m L_A^2 \quad (34)$$

$$S_{o,N} = K_{2,N} \sinh(K_{1,N} L_T) + f_{so,N} L_A / E_s - 1/6 \tau_{T,N} m_N L_A^2 + \gamma_N \epsilon_{SU} L_A \quad (35)$$

where K_2 is the same as $K_{2,N}$ evaluated at first cycle of loading.

The ratio of the crack width at cycle N to that at first cycle can be reasonably approximated using the following expression:

$$\frac{W_N}{W} = \frac{S_{o,N}}{S_o} \frac{f_{so,N}}{f_{so}} \frac{\beta_N}{\beta} \quad (36)$$

If the maximum bond stress is not developed in the first cycle (bond stress is elastic everywhere inside the tension prism), the following expression for computing $S_{o,N}/S_o$ can be derived from Eqs. 6 and 1a:

$$\frac{S_{o,N}}{S_o} = \mu_N S_N \sqrt{\frac{m}{m_N} (t^{0.107} + 0.58 N^{0.107})} \quad (37)$$

where

$$\mu_N = \frac{\text{Tanh}(L\sqrt{k_N\Psi m_N})}{\text{Tanh}(L\sqrt{k\Psi m})} \quad (38)$$

$$S_N = 1 + \frac{\gamma_N \epsilon_{SU} E_s}{f_{so}} \quad (39)$$

In case a combination of reinforcing bars and prestressing strands is present (partially prestressed beams), the increase in crack width for a particular crack spacing, is governed by the type of reinforcement that results in the lowest $S_{o,N}/S_o$ values. It was observed in the analytical part of this study that the tension reinforcement with the larger bar diameter or higher tensile stress leads to a lower slip ratio and therefore controls the increase in crack width under cyclic loading. This implies that in partially prestressed beams the previous analysis is applicable to that single reinforcing bar or prestressing strand with the largest diameter and the largest stress f_{so} obtained from linear elastic cracked section analysis at first cycle.

To illustrate the computation procedure and the sequence of steps to be followed in computing crack width under cyclic loading an example is developed in Appendix II.

COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

The data obtained from the experimental program described earlier were compared with the analytical predictions derived in this study. Typical comparisons of results are shown in Figs. 8 and 9 for the monotonically and cyclically loaded beams respectively. It can be generally observed that the theoretical predictions are in very good agreement with the experimental results. Although numerical accuracy is not excellent in all cases, the trends predicted are accurate throughout. A good correlation also was obtained when the analytical predictions were compared to experimental data reported by Lovegrange et al.(18) as shown in Fig.10. The experimental beam B1 shown in Fig.10 was cycled with a load range producing maximum and minimum steel stresses at first cycle of 43 ksi (295 MPa) and 7.0 ksi (48 MPa) respectively.

CONCLUSIONS

Following is a summary of conclusions derived from the experimental and analytical results obtained in this study.

1. The analytical model developed in this study for partially prestressed beams realistically predicts the growth in crack width under cyclic fatigue loading and helps identify as well as quantify the mechanisms of this growth.
2. The increase in crack width under cyclic loading was shown theoretically to be primarily due to the increase in steel stresses resulting from cyclic creep of concrete in compression and to bond redistribution in the cracked region between two primary cracks. Experimental observations supported the

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analytical predictions with reasonable accuracy.

3. The increase in crack widths in partially prestressed beams under cyclic fatigue loading is greatly influenced by the cyclic dependent changes and stress conditions that occur in the reinforcing bars.
4. The presence of reinforcing bars in partially prestressed beams is beneficial for the control of cracking under static and cyclic fatigue loading. They help reduce the increase in steel stress due to cyclic creep of concrete, minimize the influence of bond redistribution, and reduce the growth in crack width. Compared to precracked fully prestressed beams subjected to cyclic fatigue loading, partially prestressed beams show smaller crack spacings and smaller growth in crack width during cycling.
5. Experimental observations indicate that for approximately equal partial prestressing ratios (PPR) and for equal load ranges expressed in percent of ultimate load, the reinforcing index (ω) appears to have little influence on the rate of crack width growth with the number of loading cycles.

ACKNOWLEDGEMENT

This research was supported by the National Science Foundation under Grants No. CEE 82-11563 and CEE 84-02194 respectively to the University of Illinois at Chicago and to the University of Michigan with Dr. Michael P. Gaus and Dr. Gifford Albright as program directors respectively. The authors are very grateful for that support. Any opinions, findings, and conclusions expressed in this paper are those of the authors and do not necessarily reflect the views of the sponsor.

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APPENDIX I. - SOLUTION EQUATIONS

For $0 \leq x \leq L_T$

$$S_{x,N} = K_{2,N} \sinh(K_{1,N} x)$$

For $L_T \leq x \leq L$

$$S_{x,N} = K_{2,N} \sinh(K_{1,N} L_T) + \frac{f_{so}}{E_s} (x - L_T) + \frac{\tau_{T,N} \psi m_N}{L_A} \{(L - x)^3 - L_A^3\} + \gamma_N \epsilon_{su} (x - L_T) \quad (Ia)$$

For $0 \leq x \leq L_T$

$$\tau_{x,N} = K_{3,N} \sinh(K_{1,N} x)$$

For $L_T \leq x \leq L$

$$\tau_{x,N} = \tau_{T,N} \left\{ 1 - \frac{(x - L_T)}{L_A} \right\} \quad (Ib)$$

$$K_{3,N} = k_N K_{2,N}$$

For $0 \leq x \leq L_T$

$$f_{sx,N} = f_{sT,N} - \frac{\psi K_{3,N}}{A_s K_{1,N}} \{ \cosh(K_{1,N} L_T) - \cosh(K_{1,N} x) \} \quad (Ic)$$

For $L_T \leq x \leq L$

$$f_{sx,N} = f_{so} - \frac{\tau_{T,N}}{2A_s L_A} (L - x)^2$$

For $0 \leq x \leq L_T$

$$f_{cx,N} = \frac{\tau_{T,N} L_A}{2A_b} + \frac{\psi K_{3,N}}{A_b K_{1,N}} \{ \cosh(K_{1,N} L_T) - \cosh(K_{1,N} x) \} \quad (Id)$$

For $L_T \leq x \leq L$

$$f_{cx,N} = \frac{\tau_{T,N} \psi}{2A_b L_A} (L - x)^2$$