

PAPER NO. 1 An effort is made to bring into focus the knowns and unknowns relevant to the design of reinforced concrete columns. Both theoretical and practical concerns are covered in an effort to distinguish immediate problems which must be solved to make the reinforced column more economical and versatile.

The Reinforced Concrete Column in Perspective

By M. A. Sozen, B. B. Broms, I. Martin, and R. Diaz deCossio

■ BECAUSE OF ITS PROMINENT USE in ancient Egyptian and Greek temples, the column is the victim of understatement. It is most often associated with the image of the post and lintel: a vertical member of regular section subjected to axial compression. Actually the column in the modern reinforced concrete frame may be subjected to reversible axial forces, bending, torsion, and shear. Furthermore, the column may be nonprismatic and have an irregular cross section perforated by openings for conduits running along and across its axis. The relationship of such a column to the Attic shape is far less than that of a Daliesque beauty to a caryatid. Nevertheless, a careful study of most modern codes of practice will reveal that they deal primarily with the regular-shaped “post” with some recognition of bending moments.

Design considerations for vertical and horizontal members of a frame differ (stability of beams, for example, is seldom of consequence), but these differences are fewer and less important than has been assumed. Problems that concern the reinforced concrete column concern other members of the structure. This report attempts to bring to focus the knowns and unknowns in the host of technical information relevant to the design of reinforced concrete columns, and to distinguish the problems

which need urgent attention. Problems related to the steel reinforcement and to fatigue of concrete are not included.

The general problem is discussed at two levels: the response of the concrete and the response of the reinforced concrete unit. The discussion covers both theory and practice.

CONCRETE AS IDEA

Failure criterion

In view of experience with other less heterogeneous materials, the knowledge that we still lack an intelligible general explanation for the strength of concrete under different conditions of loading should not come as a surprise. Despite the overtones of alchemy, efforts toward the development of a criterion or set of consistent criteria to describe the failure of concrete are desirable from the practical as well as the scientific viewpoint.

Concrete has been observed to fail by (1) separation along a plane perpendicular to the maximum tensile stress or (2) sliding along a plane inclined to the axes of principal stress. Most of the failure theories are associated explicitly or implicitly with one of these modes.

Brandtzaeg's approach¹ is noteworthy in that it combines both modes of failure and provides a transition from one to the other. Reinius² has

ACI member **Mete A. Sozen**, professor of civil engineering at the University of Illinois, Urbana, is chairman of ACI Committee 104, Notation, and a member of ACI Committee 115 Research; ACI-ASCE Committee 421, Reinforced Concrete Slabs; ACI-ASCE Committee 428, Limit Design; and ACI-ASCE Committee 441, Reinforced Concrete Columns.

ACI member **Bengt B. Broms**, director, Royal Swedish Goethechnical Institute, Stockholm, serves on ACI-ASCE Committee 441, Reinforced Concrete Columns, and ACI Committee 543, Concrete Piles.

ACI member **Ignacio Martin**, vice-president, SACMAG International, Inc., San Juan, P. R., is a research lecturer at the University of Puerto Rico. He is a member of ACI-ASCE Committee 441, Reinforced Concrete Columns.

ACI member **Roger Diaz deCossio** is research professor and head of the graduate division, Institute of Engineering, National University of Mexico, Mexico City. He is a member of ACI Committee 115, Research; ACI Committee 318, Standard Building Code; ACI-ASCE Committee 426, Shear and Diagonal Tension; and ACI-ASCE Committee 441, Reinforced Concrete Columns.



developed an ingenious model primarily in relation to Mode 1, followed by similar models due to Baker,³ Roy,⁴ and Anson.⁵ The Coulomb-Mohr theory of failure is the most popular interpretation of Mode 2. The influence of the intermediate principal stress has been recognized through approaches adopted by Bresler⁶ and McHenry.⁷ Experiments by Richart,¹ Reinius,² Wastlund,⁸ Bellamy,⁹ Weigler,¹⁰ and Hilsdorf¹¹ have indicated finite influence of the intermediate principal stress, although the observed influence has varied from negligible^{1,11} to significant.¹⁰

Although some of the experimental observations are explained by all of the available theories and some observations by some of the theories, a theory to explain all bona fide observations of failure is still to come. Furthermore, the available theories are concerned primarily with compressive strength. The tensile strength of the concrete is more important than would be implied by its arbitrary elimination in calculations relating to flexure. Attempts such as that of Kaplan,¹² who made an effort to relate the tensile strength of concrete to Griffith's failure criterion,¹³ should be increased.

Load-deformation characteristics

In structural analysis, it is necessary but not sufficient to know the strength of a material. Its load-deformation characteristics must also be known. Considerable progress has been made in this area in recent years.¹⁴⁻¹⁶ One important point that has received little attention is the conversion from load-deformation to stress-strain. It is well known that, especially at advanced stages of loading, the average unit strain depends on the length and location of the measuring gage. It has also been shown¹⁷ that compressive stresses in an axially loaded short specimen are not uniform. Further study of the stress and strain distributions in short test cylinders or prisms is desirable in order to evaluate the significance of the standard tests.

The modulus of deformation E_c for concrete has been usually expressed as a function of the compressive strength only. La Rue,¹⁸ Baker,³ Hansen,¹⁹ and Hirsch²⁰ have related E_c to the stiffness of the hardened paste and the aggregate. This approach should be developed further and recognized directly in design recommendations.

The deviation of the stress-strain curve from linearity has been attributed to microcracks.^{19,21-23} Hsu²³ has observed that microcracks exist along the aggregate-mortar interfaces before loading and that the number and extent of these cracks increase with increasing load. At stresses equal to 70 to 90 percent of the maximum, cracks develop within the mortar matrix and form a continuous pattern at maximum stress. A better understanding of the relationship between microcracking and the shape of the stress-strain curve may lead to improvements in design, especially where sustained loads are concerned. Future studies should include the complete stress-strain relationship. The portion of the curve beyond peak stress

should receive particular attention. The effects of strain gradient, of the presence of reinforcement, and of autogenous healing^{24,25} on this portion of the curve need further study.

There has been considerable study of time-dependent effects on the stress-strain curve.^{15,19,26,27} Creep of the concrete is an important issue in design. Although the introduction of the concept of creep with and without moisture exchange¹⁹ is promising, there still does not exist a rational method for a quantitative prediction of creep.

CONCRETE AS A BUILDING MATERIAL

Strength

The actual strength of the concrete in the structure is a subject that appears to have eluded open discussion of any extent. The basic problem transcends control cylinder statistics. Even if the control cylinder strengths have no dispersion, the compressive strength of the concrete in the column may differ from that of the cylinder depending on casting and curing conditions. An obvious sample is the use of 85 percent of the cylinder strength for column strength. This reduction has been ascribed to an increase in the water-cement ratio in the upper portion of vertically cast columns by many investigators.^{28-31,49} On the other hand, Peterson's test results³² have indicated an increase in strength near the bottom of the vertically cast column rather than a decrease in strength near the top. Superimposed on this are the problems related to the effects of forming, curing, reinforcement, vibration, and "normal" dispersion.

Similar problems exist for tensile strength, compounded by the palpable effect of restrained shrinkage and the questions regarding the significance of the standard tests: the direct tension test, the modulus of rupture test, and the split-cylinder test.

The relationship between the actual strength of the concrete in the structure and the indication of the control specimen deserves immediate and concentrated attention. New developments in nondestructive testing methods may accelerate these studies.

Lateral restraint

For design, it has been found satisfactory to express the axial strength of confined concrete as the unconfined strength plus a constant times the uniform lateral pressure. This constant, used implicitly for the design of spirally reinforced columns in the ACI Building Code,³³ has been derived from experiments on a particular type of concrete.²⁸ Its validity for different types of concrete deserves critical study.³⁴

In the case of circular transverse reinforcement, a rational method is still lacking to determine the transverse reinforcement stress corresponding to maximum capacity of column.³⁵ Nor are reliable methods available for predicting the complete load-deformation curve of the confined concrete. There is even less information on the problem if the confinement is pro-

vided by noncircular transverse reinforcement.³⁶ Further information is also needed on the effect of confinement by adjoining concrete elements.³⁷

THE REINFORCED CONCRETE COLUMN AS IDEA

Axial load and bending

The reinforced concrete column section may be subjected to combinations of axial load, bending, shear, and torsion. Because of the classical definition of the column, most analytical work has been on combined axial load and bending.

If the critical properties of the materials involved are known or are assumed, the analysis of the response of the column section to axial loading and bending becomes simply a matter of the manipulation of the principles of statics and geometry. Nevertheless, because of the reluctance of the profession to move away from the realm of working stress design and to recognize bending in columns, the first comprehensive study of the problem was published in 1951.³⁸ The effects of length on an inelastic column were considered analytically by Broms,³⁹⁻⁴¹ Pfrang,⁴²⁻⁴⁴ and Mauch.⁴⁵ A significant recent development is the initiation of tests on long columns by Breen⁴⁶ in an effort to reconcile results of the analysis and the response of the column in a structural frame.

Recent progress in this area is rapid and promising, but it may be out of proportion to what is known about the loading conditions. Over and above the problem of predicting the possible loads on a structure, there is the problem of establishing what constraints are provided by other elements of the building, structural or nonstructural. Knowledge of what a column is asked to do should not be too far behind knowledge on what the column can do.

Shear and torsion

Shear and torsion may be dominant effects in columns, especially where lateral loads are involved. Nevertheless, they are often ignored. For example, although the report of ACI-ASCE Committee 426, Shear and Diagonal Tension, considers the influence of axial load on shear strength, applying the design recommendations given to a nonrectangular section requires more than routine interpretation.⁵⁰

When further work is done on shear and torsion, the fact that these phenomena are not limited to beams should be realized by the investigators.

THE REINFORCED CONCRETE COLUMN AS A BUILDING ELEMENT

There has been a rapid evolution in the types of reinforced concrete buildings. Consequently, there is urgent need for a critical reappraisal of certain practical considerations regarding construction and design. These are discussed in the following paragraphs.

Design limits

Limiting dimensions for columns—An 8 ft long reinforced concrete column with a 6 in. square section using 5000-psi concrete and four #4 intermediate grade steel reinforcing bars should develop an axial load capacity of 185,000 lb, a quantity which cannot be dismissed as insignificant. Nevertheless, this column cannot be used as part of a structure governed by the current ACI Building Code.³³ The development of pre-casting techniques and the attendant necessity of using minimum weight concrete structures demand a reappraisal of the limiting cross-sectional dimensions of reinforced concrete columns.

Limits on longitudinal reinforcement—The minimum limit of reinforcement in a column should be critically studied in view of the new knowledge developed on creep and shrinkage of the concrete and safety considerations of the column. The elimination of the maximum limit of reinforcement, with explicit warnings about its consequences, should be considered. In this connection, it should also be mentioned that the definitions of “composite column,” “pipe column,” and “combination column” may have no place in a modern building code. The design considerations for such columns could be approached at a more fundamental level.

ACI Committee 347, Formwork for Concrete, recommends⁴⁷ a tolerance of minus 0.25 in. and plus 0.5 in. for the cross-sectional dimensions of the column, and a tolerance of 0.25 inches in 10 ft of height, 0.375 in. for a height of 20 ft, and 0.75 in. for 40 ft or more. The impact of this provision on the safety of the column as designed by the ACI Building Code should be carefully studied. The same observation can be made with respect to the tolerances in placing the steel reinforcement.

Conduits and pipes embedded in concrete—In high-rise buildings it is architecturally convenient to place large-diameter conduits and piping within the column. The effect on column resistance of the eccentric positioning of the conduits or piping, taking into consideration the tolerance in their vertical placement and the effect of the horizontal piping connections, deserves careful study. The current limit on the column area which can be displaced by conduits without penalty should be related to the pertinent variables.

Reinforcement splices—The use of high strength concretes and more slender columns has contributed to reductions in the size of column sections. It is costly and virtually impossible to make conventional reinforcement splices in columns with small sections, especially if large-diameter high strength bars are used. This development, plus the fact that as more research is done on bond the required lap lengths get longer, creates a need for new types of splices such as welding or mechanical couplers.

Transverse reinforcement and concrete cover—The 1963 ACI Building Code has liberalized the requirements for lateral reinforcement of tied

columns. However, there appears to be room for further simplifications in these requirements, especially in the case of columns using high strength concrete and large amounts of reinforcement.

Architectural requirements and the desire to save space make it necessary to reduce the section of reinforced concrete columns to the point that in certain cases a concrete cover of 1.5 in. provides a hindrance in design. In the case of a column with an 8-in. side, this requirement leaves only 62.5 percent of the width of the column as the space available for placing reinforcement. Further work on the fire resistance of columns should provide an explicit relationship between concrete cover and time of resistance.

Bar spacing—The tendency to use high strength concrete in columns with high slenderness ratios, together with the different methods of depositing and vibrating concrete require an investigation of the limitations of minimum spacing for the bars of the columns. The investigation of the behavior of bundled reinforcement should be continued. Although the use of bundled reinforcement has been permitted by the ACI Building Code, investigations reported so far have been limited.⁴⁸

Construction

Tolerances—Eccentricity of loading is an important factor in columns with high slenderness ratios. The tolerances permitted in column construction affect the eccentricity of the load as well as the economy of the structure. The minimum eccentricity required by the building code should be related to the tolerance specified. A reinvestigation of permissible tolerances is in order.

Exposed columns—The use of exposed reinforced concrete columns in modern structures has raised new problems about the effects of dimensional stability of concrete at varying temperatures. The behavior of existing buildings with exposed columns should be carefully scrutinized and reconciled with theoretical considerations with the help of further tests under controlled conditions.

Shear walls

Concrete shear walls have been one of the successful solutions used by designers in high-rise buildings. The reinforced concrete shear wall resists vertical as well as lateral loads and is in effect a column.

The inherent properties of shear walls create problems in analysis and interpretation in the following areas:

1. Great differences between the dimensions of their sections
2. Stress concentrations at meeting points of orthogonal walls where one wall supports the other
3. Stress concentrations occurring at intersections with floor systems
4. Transmission of loads from walls to reinforced concrete columns
5. The effect of openings
6. Slenderness of the wall (*If* the requirements of Sec. 2202 of ACI 318-63 are applied, a reinforced concrete wall 402 ft high would require

minimum thickness of 22 in. However, buildings of that height have been constructed with walls only 11 in. thick.)

7. Limits on the amount of vertical and horizontal reinforcement.

CONCLUDING REMARKS

Any survey, either from the purely scientific or the practical viewpoint, of what is known about the reinforced concrete column would inevitably conclude that more needs to be known. To elevate that conclusion from a shibboleth to a serious call for action, recommendations must be given some order of priority. That requires a specific criterion and bitter pruning.

The writers have adopted the mundane criterion that research should makes a product cheaper or more versatile. Naturally, any and all endeavors in fundamental research may lead to a breakthrough and more than satisfy the criterion. However, the following topics have been selected for study from among those suggested on the assumption that further work along these lines promises reasonably quick and certain returns:

1. The relationship between the actual properties of the concrete in the structure and those in the control specimen
2. Determination of the actual forces acting on a column in a building
3. Reinforcement splices
4. Shear strength of columns with nonrectangular cross sections
5. Load-deformation characteristics of multistory shear walls
6. Limiting values on column size and amount of reinforcement
7. Construction tolerances.

Progress in the first two items would reduce to a minimum that part of the design process subject to judgment. The last two items are areas requiring immediate interaction among ACI committees concerned with structural analysis, materials, and construction.

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PAPER NO. 2 Compares ACI Building Code design procedure with Comité Européen du Béton (CEB) procedure and other more elaborate methods of predicting the behavior of tied and spirally reinforced columns. The factors of safety in the material specifications, in the load and capacity reduction factors, and in the design equations for short columns are analyzed separately. The ACI and CEB design equations for slender columns are compared with a theoretical method of analysis.

Critical Review of the Design of Reinforced Concrete Columns

By I. Martin, J. G. MacGregor, E. O. Pfrang, and J. E. Breen

■ THE AMERICAN CONCRETE INSTITUTE BUILDING CODE requirements for reinforced concrete¹ were extensively revised between 1956 and 1963. In this revision, extensive ultimate strength design procedures were included and the column design sections were revised. During the same period a group of European countries organized the Comité Européen du Béton (CEB) and this institution has produced a series of recommendations on design procedures.² The CEB design recommendations on slender columns present a novel approach that departs appreciably from the traditional methods. This paper compares the present ACI Building Code design procedures with CEB recommendations and other more elaborate methods of predicting the behavior of tied and spirally reinforced concrete columns.

In a building code there are a number of sources of safety factors. In the 1963 ACI Code, for example, the actual safety factor for a column depends on such things as: (1) the definitions of f'_c and f_y , given in Sec. 504 and 1505; (2) the cross section design equations in Chapters 14 and 19; (3) the slender column equations in Sec. 915 and 916; and (4) the allowable

Ignacio Martin, vice-president, SACMAG International, Inc., San Juan, P. R., is a research lecturer at the University of Puerto Rico. He is an ACI member and serves on ACI-ASCE Committee 441, Reinforced Concrete Columns.

ACI member **J. G. MacGregor**, is associate professor of civil engineering, University of Alberta, Edmonton. He is a member of ACI-ASCE Committee 441, Reinforced Concrete Columns, and the Reinforced Concrete Research Council Task Committee 20 on Long Columns.

ACI member **E. O. Pfrang**, assistant professor of civil engineering, University of Delaware, Newark, is actively engaged in research into the behavior of inelastic columns. He is a member of ACI-ASCE Committee 441, Reinforced Concrete Columns, and ACI Committee 118, Use of Computers.

ACI member **John E. Breen** is assistant professor of civil engineering, University of Texas, Austin. He is a registered professional engineer in Texas and Missouri and is a member of ACI-ASCE Committee 441, Reinforced Concrete Columns.

working stresses in Chapter 10 or the load factors in Chapter 15. Each of these sources of safety will be discussed in this report. The safety factors inherent in material specifications, the load factors and design equations for short tied and spiral columns are each discussed independently. All this material is then combined to get an idea of the over-all factor of safety of short tied and spiral columns designed by various codes. The ACI and CEB design procedures for slender columns are then compared and discussed omitting any effects of either load factors or differing material strengths. Conclusions deal with strength of short columns and slender columns.

NOTATION

- A_{st} = total area of longitudinal steel
- b = width of compression face of flexural member or rectangular column
- d = distance from extreme compression fiber to centroid of tension reinforcement
- d' = distance from extreme compression fiber to centroid of compression reinforcement
- D = over-all diameter of a round column
- D_s = diameter of the circle through the centers of reinforcement arranged in a circular manner
- DL = design dead load
- E = initial modulus of elasticity
- e = eccentricity of axial load at end of member measured from plastic centroid of the section, calculated by conventional methods of frame analysis

- f_b = basic design strength to be assumed in the calculation
- f'_c = compressive strength of concrete determined by tests of standard 6x12-in. cylinders made and tested in accordance with ASTM specifications
- f'_{cr} = compressive strength of concrete specified by structural designer
- f^*_c = specified concrete strength to be used in design computations in the 1962 proposed revision of the ACI Building Code (not adopted)
- f_{ch} = the characteristic strength, given by $f_{ch} = f_m (1 - \nu)$
- f_m = the mean value of the strength, obtained from a series of tests on specimens
- f_y = yield strength of steel reinforcement
- f^*_y = reinforcement yield point to be used in the design computations in the 1962 proposed revision of the ACI Building Code (not adopted)
- h = actual unsupported length of member
- I = moment of inertia about the bending axis
- K = stiffness factor, EI/h
- K_f = coefficient to reduce the probability that the actual strength on the job could be less than the basic value
- K_L = coefficient to insure the same probability of occurrence or nonoccurrence of any greater load once during the life of the structure
- LL = design live load
- L_b = basic ultimate load
- L_{ch} = characteristic load
- m = $f_y/0.85f'_c$
- p_t = A_{st}/bt
- P_u = axial load capacity under combined axial load and bending
- r = radius of gyration of gross concrete area of the column section
- r' = the ratio of ΣK of columns to ΣK of floor members in a plane at one end of a column
- t = over-all depth of a rectangular section
- α = numerical constant that depends on the section of the column and quality of concrete and steel
- β = numerical constant that depends on the e/t ratio
- ν = coefficient of standard variation of the quality of the materials
- ρ = modified effective length coefficient in CEB buckling recommendations, defined in Appendix
- ϕ = capacity reduction factor

ASSUMPTIONS

Throughout this report it has been necessary to assume certain material properties and characteristics which are listed below.

Shape and behavior of column cross sections

For simplicity this study has been limited to tied columns with a uniform rectangular cross section and symmetrical reinforcement in the faces parallel to the bending axis, and to circular spirally reinforced columns with one ring of reinforcement. The longitudinal reinforcement in the spiral column was assumed to be distributed evenly in a single continuous ring for all steel percentages. No attempt was made to include the effect of the spiral on the strength of the core concrete since this effect does not increase the strength of eccentrically loaded or slender columns although spiral reinforcement does increase the ductility of short columns.

Properties of typical cross sections

Throughout this paper, comparisons are presented for one typical tied column and one typical spirally reinforced column cross section. For both tied and spiral columns this cross section has $\rho_m = 0.7$, a concrete strength of 3000 psi, a reinforcement yield point of 45,000 psi, a reinforcement ratio of 4 percent and a ratio of effective depth to over-all thickness, d/t , of 0.9 for tied columns and a ratio of diameter of spiral to column diameter D_s/D of 0.8 for spiral columns. A large number of other sections were compared in writing the report but to conserve space these comparisons have not been presented.

Properties of concrete

All interaction diagrams and other computations were based on the Hognestad stress-strain diagram for concrete in compression. The tensile strength of the concrete was ignored. The effects of sustained loads were approximated by multiplying all strain ordinates in the Hognestad stress-strain curve by two. Sustained load was assumed to have no effect on the strength ordinates of the curve. These assumptions were made somewhat arbitrarily to obtain an estimate of the effect of sustained loading in a simple, though approximate manner. The validity of these assumptions requires further theoretical and experimental verification. For the purposes of discussing the safety factors the concrete was assumed to have a coefficient of variation of 15 percent which would correspond to an average degree of control.

Properties of reinforcement

The reinforcement was assumed to have an elastic-plastic stress-strain curve in tension and compression. For the purposes of studying the effect of random variations in the reinforcement properties, the yield points were assumed to be normally distributed with a mean value of 1.05 times the design value and a standard deviation of 3 percent. This assumption is discussed in the section on material strengths.

Analysis of slender columns

The analysis of slender columns was based on the method outlined by Pfrang and Siess.³ In addition to the assumptions previously listed, the analysis assumed: (1) a linear distribution of strains; (2) linearly elastic end restraints which are affected by creep under sustained loads (assumed to remain elastic throughout the loading history of the column); and (3) the effects of different concrete stress-strain curves for loading and unloading are ignored.

A number of things affecting the load factors were not considered. These included, among others: (1) differences between in-situ strength and control specimen strength; (2) use of standard bar and section sizes rather than the exact amounts indicated by the design calculations (the effect of

this is beneficial in most cases); (3) differences in bar placing as compared to designed positions; (4) differences in dimensions as compared to designed dimensions; and (5) the effect on strength of splices, joints, and other practical details.

FACTORS OF SAFETY OF SHORT TIED AND SPIRAL COLUMNS

Safety factors in material specifications

In designing a column, the engineer arbitrarily chooses a concrete and steel strength for use in the various design equations. The actual strength of the materials in the member will probably differ from the assumed value to some extent. In this section, the relationship between the design strengths and the expected material strengths will be discussed briefly.

Concrete strength—Studies of concrete strength data indicate that the distribution of concrete strength tests is essentially “normal.” Based on a normal distribution, ACI Committee 214⁴ has defined the quality of control in terms of the coefficient of variation of the test results. Thus a coefficient of variation of less than 10 percent corresponds to excellent control, 15 percent corresponds to average control, and over 20 percent represents poor control. The studies reported in this paper are based on a coefficient of variation of 15 percent.

In the 1956 ACI Code the design strength for working stress design was defined in Sec. 304(c) and the design strength for ultimate strength design was outlined in Sec. A602(f) of the Code appendix. In the 1963 Code the strength requirements for ultimate strength design remain essentially the same as in the 1956 Code, but the strength requirements for working stress design have been simplified somewhat. These requirements are outlined in Sec. 504 of the 1963 Code.

The CEB Recommendations define a “characteristic concrete strength” equal to $(1 - \nu)$ times the mean strength, where ν is the coefficient of variation. This characteristic strength is approximately equal to the mean of the lower half of the test results. Otherwise, no attempt is made to specify a value of concrete strength for use in design.

The ratio of the required mean strength to the specified strength is summarized in Table 2-1 for the various standards of concrete control and various design specifications. This table indicates that the 1963 Code

TABLE 2-1 RATIO OF REQUIRED MEAN STRENGTH TO SPECIFIED STRENGTH, f_m/f'_c , FOR SEVERAL COEFFICIENTS OF VARIATION

Code	Coefficient of variation		
	10 percent	15 percent	20 percent
ACI working stress design (1956)	1.04	1.12	1.21
ACI working stress design (1963)	1.09	1.14	1.20
ACI ultimate strength design (1963)	1.15	1.24	1.35
CEB characteristic strength $(1 - \nu)$	1.11	1.18	1.25

requires a slightly higher strength concrete for working stress design than the 1956 Code and it also shows that the mean concrete strength required in ultimate strength design is about 9 percent higher (1.24 as compared to 1.14) than that required for working stress design.

Steel strength—Studies of steel yield point strengths indicate that the distribution of this strength is essentially log-normal. The current ASTM bar specifications do not allow any understrength tests without a retest. In a study of mill tests of reinforcing bars made by the ASCE committee on factors of safety,⁵ the strength distribution was log-normal with a ratio of mean strength to specified strength of 1.19 for 171 tests. The portion of this strength distribution curve representing strengths below the mean value could be crudely approximated by a normal distribution with a coefficient of variation of 3.4 percent of the mean. In assessing the variation of the steel strengths in a structure, it should be noted that the load is applied much more rapidly in mill tests than in the structure, with the result that for coupons from rolled structural sections, the mill test strength will be approximately 10 to 15 percent higher than the static yield strength.⁶ Tests of reinforcing bars under rapid strain rates suggest that the mill test strength of reinforcing bars would be about 10 percent higher than the static yield strength.⁷ For purposes of comparison it has been assumed that the steel has a mean static yield strength of 1.05 times the value used in design with a standard deviation of 3 percent.

Allowable stresses and load factors

Factor of safety of axially loaded columns designed by ACI working stress methods—By specifying different ratios between working stress and material strength for concrete and steel, the factor of safety for axially loaded columns in the ACI Code is a function of the steel percentage.⁸ Thus, when the steel percentage is low, the major portion of the load is carried by concrete and since concrete is more variable than steel, the factor of safety is higher than for a column with a high steel percentage.

In the 1963 Code the working stress equation for axially loaded tied columns was revised, resulting in a decrease in the factors of safety to 83 to 92 percent of the 1956 Code factors depending on steel percentage. The factors of safety for eccentrically loaded columns designed by working stress design are compared later in this paper.

Load factors in 1956 code appendix—The load factors adopted by ACI in the 1956 Code were taken from the report of the ACI-ASCE Committee 327, Ultimate Strength Design.⁹ Four equations were given for structures subjected to wind loads and two equations for structures which were not. The choice depended on the ratio of live to dead load. The load factors for structures not subjected to wind are plotted in Fig. 2-1. There were no member understrength factors on the material strength in the 1956 ACI Code.

For axially loaded columns an additional factor of safety is implied by the requirement that all spiral columns be designed for a minimum eccentricity of $0.05t$ and all tied columns be designed for a minimum eccentricity of $0.10t$. This tends to reduce the allowable load on tied columns below that on spiral columns. The minimum eccentricities also take account of the more ductile behavior of short axially loaded spiral columns. The two values of minimum eccentricity lead to axially loaded columns of approximately the same size for both WSD and USD.

USD load factors in the 1963 ACI code—In the 1963 ACI Code the load factors are divided into ϕ , an understrength factor on member strength, and an overload factor.

In the first draft of the Code,¹⁰ the design strengths were chosen assuming that only 1 test in 1000 would fall below the design value. The overload factors were derived on the assumption that the design loads should be

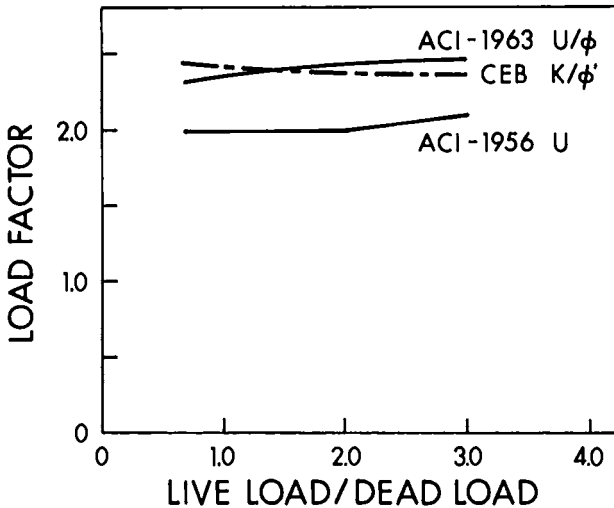


Fig. 2-1 Comparison of load factors

exceeded once in 100 times to once in 1000 times. Superimposed on this was a blanket 10 to 20 percent increase in design loads for spiral and tied columns, respectively.

In the final draft of the Code,¹ the probability of understrength material was increased from 1 in 1000 to approximately 1 in 100 and the probability of overloads was reduced. The understrength factors for concrete and steel and the blanket 10 and 20 percent design load increases for spiral and tied columns were combined to give a single understrength factor ϕ for each type of column. All of this was based on normal distributions of load and strength, which do not actually represent the true distributions, and certain gross simplifications regarding the superposition of loading and strength conditions.

The combined load factors and ϕ factors for tied columns are plotted in Fig. 2-1 for comparison with the effective CEB load factors. The load factors for the case of wind loading have not been compared.

Axially loaded columns are treated in the same way they were in the 1956 ACI Code. Thus axially loaded tied and spiral columns are assumed to have minimum eccentricities of $0.10t$ and $0.05t$ respectively.

European Concrete Committee load factors report—The load factors discussed in this report are summarized in a paper by Torroja.¹¹ These are a simplified set of expressions based on a more complicated statistical analysis. This set of safety factors is also divided into understrength factors and overload factors. A basic material strength f_b is defined as equal to the “characteristic strength” f_{ch} divided by a coefficient K_f to reduce the probability that the actual strength would be less than the basic value. The characteristic strength is $(1 - \nu)$ times the mean strength where ν is the coefficient of variation of the strength (Table 2-1). For concrete the constant K_f is 1.5 to 1.6, and for steel yield point it is 1.15 to 1.20. Thus the basic strengths are close to the values of f_c^* and f_y^* in the original draft of the ACI Code.¹⁰ When the average K_f factors are combined for columns, an effective value of ϕ' equal to 0.71 ± 0.04 is obtained.

The overload factors are defined as $L_b = K_L L_{ch}$ where K_L is a load factor and L_{ch} is the characteristic load equal to $(1 \pm \nu)$ times the mean load. The individual load factor depends on the quality of design and workmanship and risk of damage in the event of a failure. If the standard deviation is taken as 8 percent for dead loads and about 18 percent for live load, the load factor for average conditions of design and workmanship is $K_L = 1.78$ mean dead load + 1.94 mean live load. It should be noted that while the mean dead load is the value used in design in North America the mean live load will probably be less than the design live loads specified in our codes. If we assume the “characteristic loads” correspond approximately to the design live loads in our codes this load factor becomes $K_L = 1.78DL + 1.65LL$. If the effective value of ϕ' is included this becomes $K/\phi' = 2.50DL + 2.32LL$. This load factor is plotted in Fig. 2-1 for comparison with the ACI load factors and has been used in these comparisons.

Safety factors inherent in the design equations for short tied columns

The column design equations in building codes are, for simplicity, approximations of the true strength of an eccentrically loaded reinforced concrete column. In this section, the tied column design equations in the ACI working stress design (WSD) and ultimate strength design (USD) sections, and the design equations proposed in the CEB *Bulletin* No. 31¹² are compared to a more accurate estimate of the strength of a column cross section based on the Hognestad stress-block. The values obtained by the 1956 ACI WSD method have been plotted in Fig 2-2 and 2-6 but otherwise are not discussed. In order that the design equations themselves

might be compared, no load factors or understrength factors have been used and the same material strengths were used in both the theoretical and design strength computations. No attempt was made to consider biaxial bending or columns with bars in all four faces.

Fig. 2-2 shows a theoretical interaction diagram for what is considered to be a typical cross section, which has a $p_t m$ of 0.7. The properties of this cross section are listed in the section on assumptions. Also in Fig. 2-2 are design interaction curves for the several codes under consideration here. A study of this figure indicates the degree to which these codes approximate the theoretical strength for the particular section considered.

Fig. 2-3 to 2-5 show the relationship between the design equation strength and the theoretical strength for three design methods. In each figure there is a curve for the cross section with a $p_t m$ of 0.7. In addition to this cross section, approximately forty additional cases were studied covering a range of variation of d/t from 0.8 to 0.95, steel percentage from 1 to 8 percent, concrete strengths from 2500 to 5500 psi and steel yield points from 33,000 to 75,000 psi. These cases cover the practical range of variables for tied columns. The results of these additional cases are plotted as a shaded band. The high ratios of theoretical to design strength in Fig. 2-3 to 2-5 correspond in all cases to tension failures of lightly reinforced columns ($p_t = 0.01$) with large amounts of cover on the reinforcement ($d/t = 0.8$). In general, the strength ratios become much more uniform for $d/t \geq 0.85$ and $p_t \geq 0.02$.

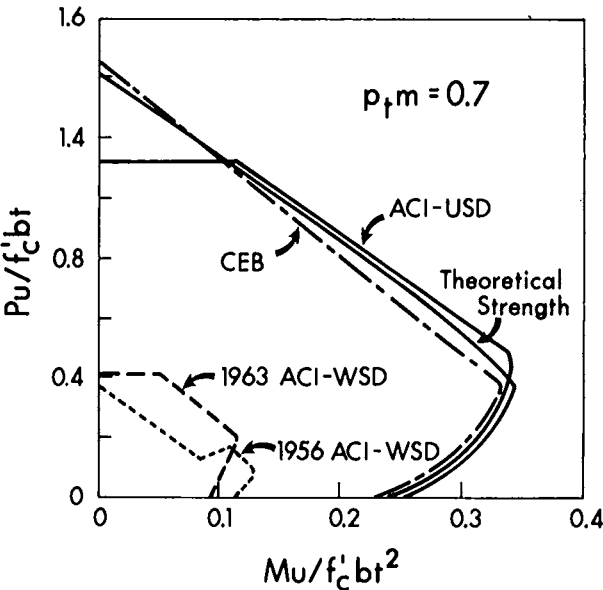


Fig. 2-2 Comparison of design equations for tied columns based on equal material strengths

ACI Code working stress design—The variation in factors of safety in the 1963 ACI Code WSD equations is illustrated in Fig. 2-3. The factor of safety appears reasonable in the compression failure range, ranging between 2.2 and 3.2. In the tension failure range it is reasonable for heavily reinforced columns but becomes erratic for lightly reinforced columns. The highest values correspond to lightly reinforced columns with thick cover.

Note that these factors of safety were computed assuming the eccentricity was kept constant as the load and moment were increased from the working load to ultimate. In the derivation of this design method the equations were chosen to produce a factor of safety of at least 2.5 for the following three loading conditions: (1) the eccentricity kept constant as the load is increased; (2) the moment kept constant as the load is increased; and (3) the load kept constant as the moment is increased to failure. For the third case the column with the largest factor of safety in Fig. 2-3 had a factor of safety of 3.07. As a result of the constant load method of defining the factor of safety, the 1963 Code equations for balanced eccentricity were set unrealistically low as may be seen in Fig. 2-2. This and the straight-line working stress design interaction diagram combine to give excessively high factors of safety in the tension failure region. If the correct

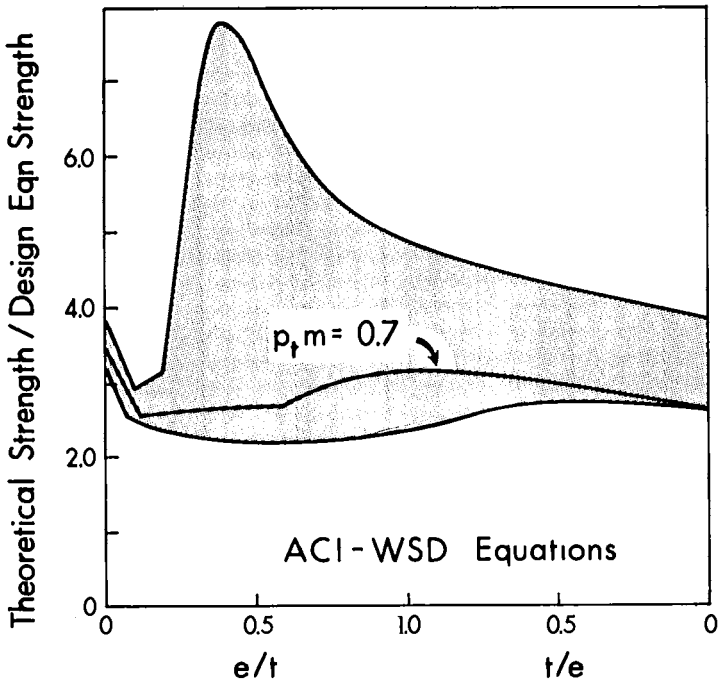


Fig. 2-3 Comparison of ACI working stress design equations for tied columns with theoretical strength based on equal material strengths

value of balanced eccentricity is used in this design method, the maximum constant eccentricity factor of safety is reduced by about 35 percent and the minimum constant load factor of safety to about 1.8.

ACI ultimate strength design equations—Fig. 2-4 and the interaction curve for the ACI USD case in Fig. 2-2 were plotted using Eq. (19-5) and (19-10) of the ACI Code, omitting the ϕ factor. Eq. (19-10) for compression failures tended to overestimate the column capacity while the tension failure equation agreed quite closely with the theoretical strengths for normal column dimensions. This agrees with the findings of ACI-ASCE Committee 327, Ultimate Strength Design⁹ as illustrated by Fig. 17 of their report.

The high ratios of theoretical to design strength correspond to columns with 1 percent steel and thick cover. The low strength ratios at low eccentricities correspond to 1 percent steel and very thin cover. The low factors of safety at high eccentricities correspond to 8 percent steel and thick cover.

For 2 to 6 percent steel and normal cover the ratios of theoretical to design strength were close to 1.0.

CEB design equations—Fig. 2-4 and the CEB interaction curve in Fig. 2-2 were plotted using Eq. (a) and (b) from CEB Appendix 1. These equations do not appear in the recently issued CEB Recommendations but were proposed in CEB *Bulletin* No. 31.¹² As shown in Fig. 2-2 and 2-5, the CEB equations agree fairly well with the Hognestad interaction

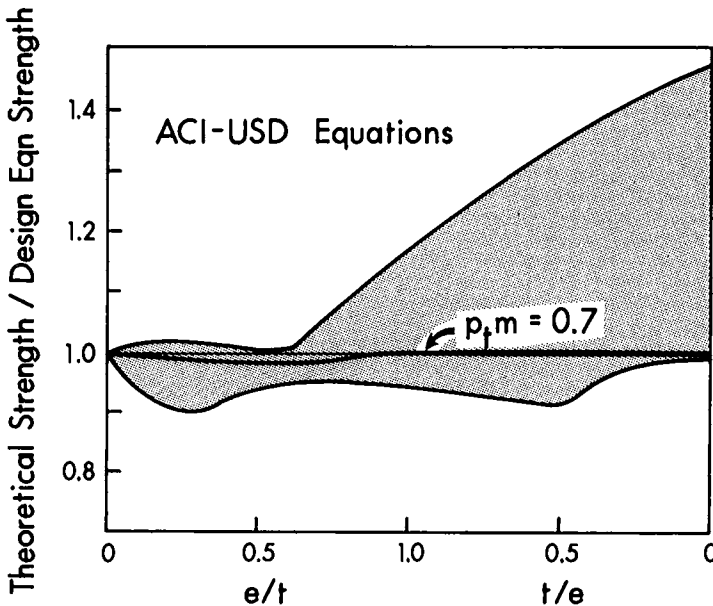


Fig. 2-4 Comparison of ACI ultimate strength design equations for tied columns with theoretical strength based on equal material strengths

diagram for normal column dimensions. The authors found the tension failure equation was much easier and less confusing to use than the corresponding ACI equation.

Comparison with test results—Both ACI and CEB design equations were checked against test results during their derivation. Mattock, Kriz, and Hognestad¹³ compare the ACI Code equations to 84 tied columns tested by Hognestad. The average ratio of test to calculated strength was 0.97 with a standard deviation of 0.059. No attempt was made in these computations to account for the effect of length. The CEB equations have similarly been compared to tests of 204 tied columns with a mean ratio of test to computed strength of 1.02 and a mean deviation of 6 percent.¹²

Over-all factor of safety for a short tied column

In previous parts of this report the factors of safety included in the material specifications, the load factors, and the design equations have been considered individually. This section combines all three of these sources of column safety factors to examine their over-all effect.

Interaction diagrams are presented in Fig. 2-6 to compare the service load capacity of a typical column to a band of theoretical strength curves. The material properties were chosen so that the mean concrete strength was 3000 psi and the mean yield point strength was 45,000 psi, giving a theoretical value of $p_m = 0.7$. The upper and lower limits on the band of theoretical strengths represent the strengths that one column in a

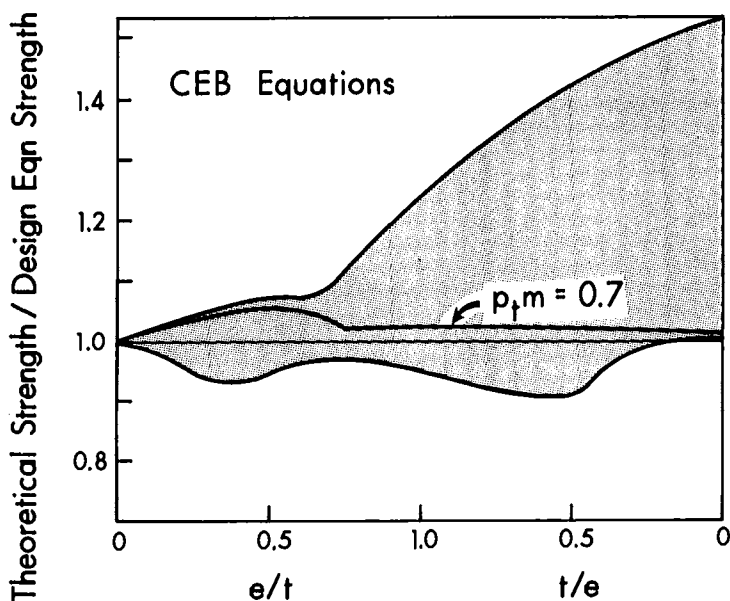


Fig. 2-5 Comparison of CEB design equations for tied columns to theoretical strength based on equal material strengths

hundred might reach. This was arbitrarily defined as occurring when the concrete and steel both had the high or low strength reached one time in ten.

The design strengths for steel and concrete and the load factors were evaluated in accordance with each design procedure prior to computing the service load capacities. The service load was assumed to be 60 percent dead load. Thus the design curves in Fig. 2-6 include all the factors of safety discussed in three previous sections of this report.

In the compression failure range there is good agreement between the CEB and ACI USD procedures and reasonable agreement between these procedures and the 1963 ACI WSD procedure. Both ultimate strength methods lead to a mean factor of safety of about 2.6 in this case.

This agreement between design procedures is not as good in the tension failure region. We have seen previously that there is close agreement between the Hognestad interaction diagram and the ultimate strength design equations in this region. However, when the various load factors and material understrength factors are introduced, complications arise. Thus, in the ACI USD method a transition is necessary from $\phi = 0.7$ for tied columns to $\phi = 0.9$ for beams, as shown by the vertical lower part of this interaction diagram. In the CEB procedure the steel and concrete strengths for design are discounted different amounts, changing the balance point and resulting in a rather sudden drop in factor of safety from about 2.6 for compression failures to about 2.2 to 2.3 for tension failures.

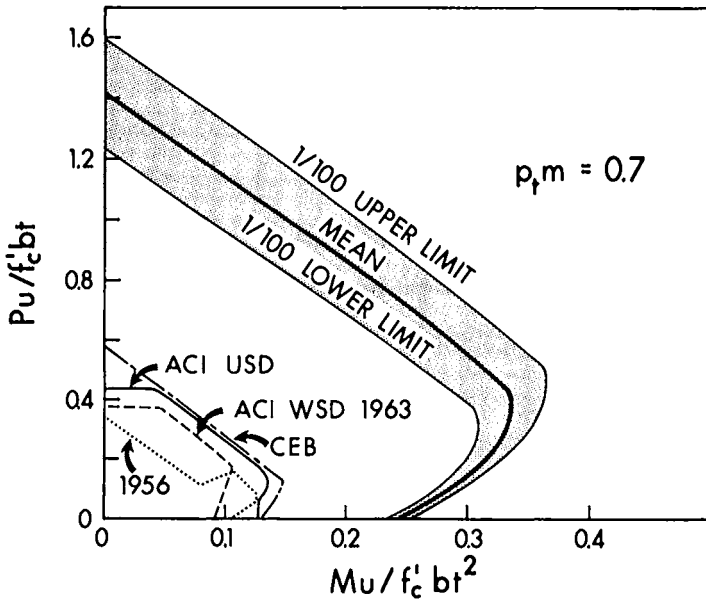


Fig. 2-6 Comparison of over-all factor of safety in design methods for tied columns

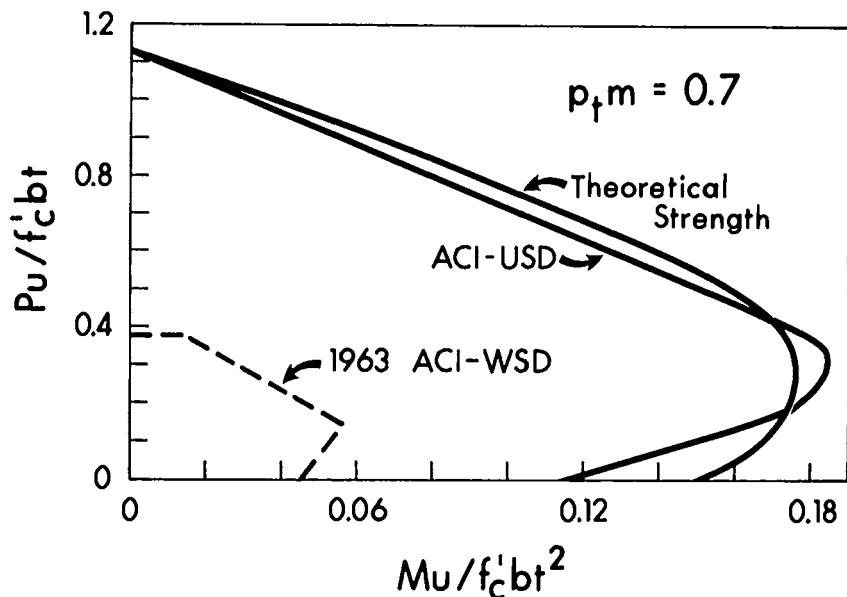


Fig. 2-7 Comparison of design equations for spiral columns based on equal material strengths

The now obsolete 1956 ACI WSD procedure is plotted on this curve for comparison (Fig. 2-6). In every case it is much more conservative than the 1963 WSD method in the compression zone. In the tension failure region it may be more conservative or less depending on the column properties, becoming more conservative for high steel percentages.

Safety factors inherent in the design equations for short circular spiral columns

In this section the spiral column design equations in the ACI Code WSD and USD sections are compared to a more accurate estimate of the strength of a spiral column cross section based on the Hognestad stress block. The comparison was limited to circular columns. In order that the design equations themselves might be compared, no load factors or under-strength factors have been used and the same material strengths were used in both the theoretical and design strength computations.

The design equation interaction diagrams are compared in Fig. 2-7 to a theoretical interaction diagram for the typical cross section with $p_t m = 0.7$ described in the section on assumptions. A study of this figure indicates the degree to which the ACI Code expressions approximate the theoretical strength for the particular section considered.

Fig. 2-8 and 2-9 each show the relationship between the design equation strength and the theoretical strength for a particular design method. On each figure a curve is plotted for the cross section with a $p_t m$ of 0.7. In addition to this cross section, nine others were studied with $p_t m$ ranging

from 0.1 to 2.0, D_s/D from 0.6 to 0.9, steel from 2 to 8 percent, concrete strengths from 3000 to 4000 psi, and steel yield points from 34,000 to 75,000 psi. The shaded area in Fig. 2-8 and 2-9 represents the range of the strength ratio for columns in this practical range of variables. Columns with 1 percent reinforcement fell above the shaded area in the tension failure region. The line plotted above the shaded area in Fig. 2-8 and 2-9 represents a column with 1 percent reinforcement, $D_s/D = 0.6$, and $\rho_m = 0.1$.

ACI working stress design—The variation of factors of safety in the 1963 ACI Code WSD equations are illustrated in Fig. 2-8. The factor of safety appears reasonable in the compression failure range, varying between 2.5 and 3.5. In the tension failure range the factor of safety is reasonable for moderately or heavily reinforced columns but becomes erratic for lightly reinforced columns. Again, the highest values correspond to lightly reinforced columns with thick cover. The factors of safety plotted in Fig. 2-8

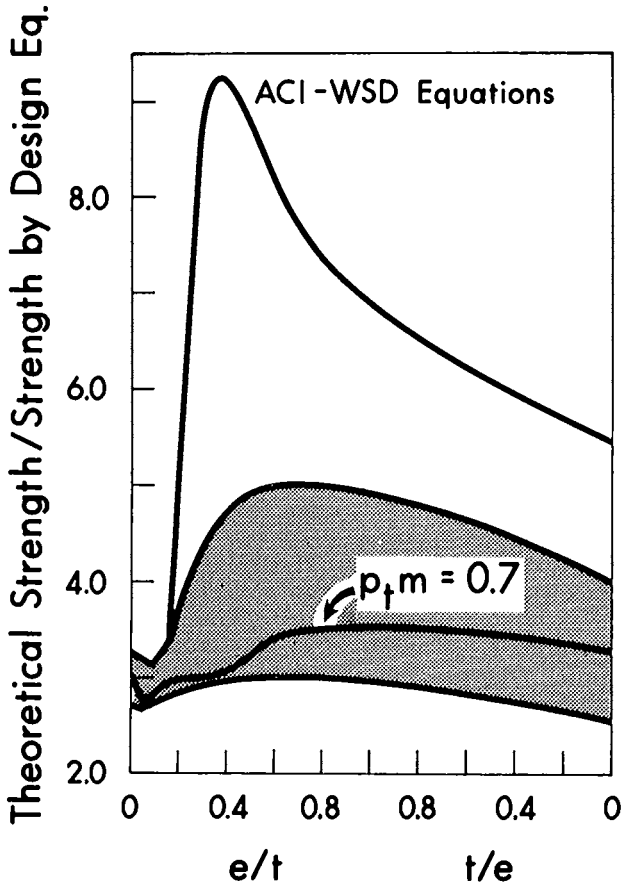


Fig. 2-8 Comparison of ACI working stress design equations for spiral columns with theoretical strength based on equal material strengths

are for the constant eccentricity loading case. The maximum factor of safety plotted on this figure corresponds to a factor of safety of 4.4 under constant load and increasing moment. If the correct value of balanced eccentricity is used in this design method, the maximum constant eccentricity factor of safety is reduced by about 35 percent and the minimum constant load factor of safety to about 2.0.

ACI ultimate strength design—Fig. 2-9 and the interaction curve for the ACI-USD case in Fig. 2-7 were plotted using Eq. (19-11) and (19-12) of the ACI Code, omitting the ϕ factor. The reinforcement yield requirements of Sec. 1505 of the Code were not included in the computations. Eq. (19-12) for compression failures tended to underestimate slightly the column capacity except for heavily reinforced columns with high yield point reinforcement (75,000 psi) and thick cover. The lowest values in Fig. 2-9 correspond to this case. For columns of normal proportions the tension failure equations tended to become excessively conservative as the pure bending case was approached.

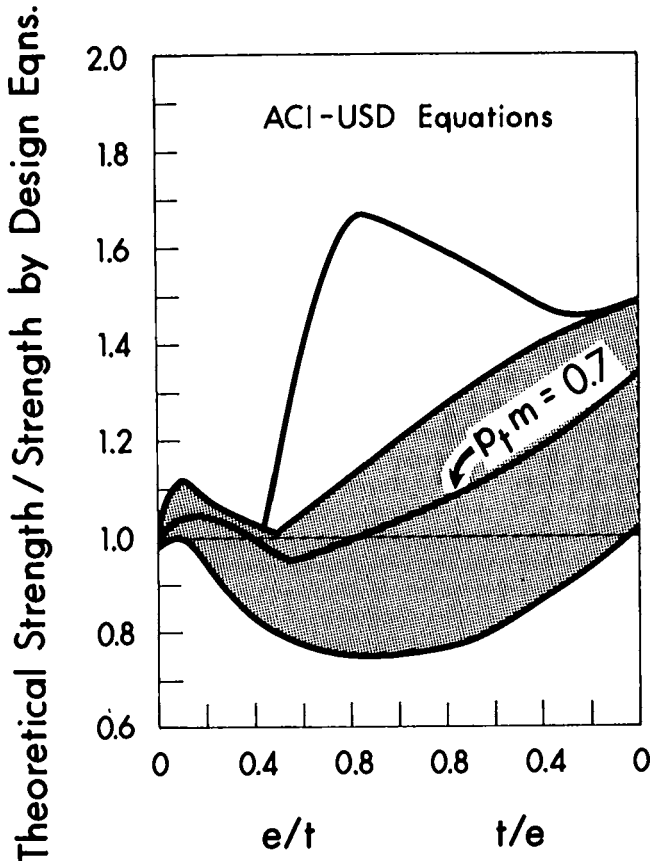


Fig. 2-9 Comparison of ACI ultimate strength design equations for spiral columns with theoretical strength based on equal material strengths

CEB design equations—The CEB Recommendations specifically limit spiral reinforcement to use in short axially loaded columns and for this reason this design method has not been considered.

Over-all factor of safety of a short spiral column

In this section the factors of safety included in material specifications, load factors and design equations are collected to examine their over-all effect. This is done by means of the interaction diagrams in Fig. 2-10 which compare the service load capacity of a typical spiral column with

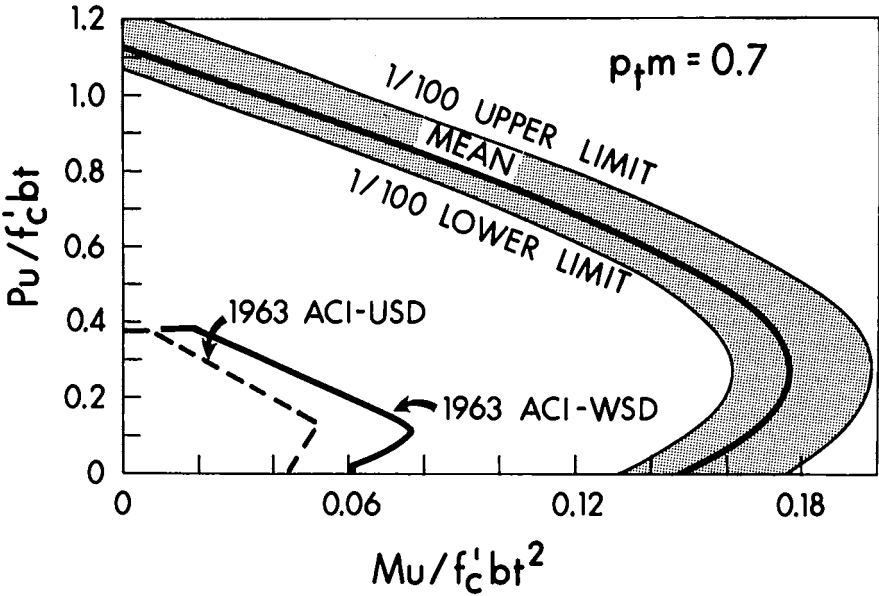


Fig. 2-10 Comparison of over-all factors of safety in design methods for spiral columns

$p_t m = 0.7$ and properties as listed in the section on short tied columns (p. 24) to a band of theoretical strength curves constructed in the manner described in that section.

In the compression failure range there is reasonable agreement between the two design procedures. The USD method leads to a mean over-all factor of safety of about 2.6 for this cross section compared to about 3.2 for the WSD equations.

In the tension failure region the comparison is not nearly as good with over-all factors of safety ranging from about 2.3 to 2.9 for the USD equations and from 3.2 to 3.8 for the WSD equations. The transition between $\phi = 0.75$ for spiral columns to $\phi = 0.9$ for beams is shown by the vertical lower part of the USD curve in Fig. 2-10.

SLENDER COLUMNS

Review of slender column design procedures

The behavior of the slender column as a stability problem has been studied by numerous investigators. Particularly noteworthy is the early work of Euler,¹⁴ Engesser,^{15,16} and Von Karman;¹⁷ and later Ros and Brunner,¹⁹ Westergaard and Osgood,²⁰ and Shanley.²¹ Behavior of slender reinforced concrete columns has been studied by Ernst, Hromadik, and Riveland,²² and Broms and Viest,²³⁻²⁵ among others.

Slender columns have been treated traditionally in building codes for reinforced concrete by means of a reduction factor that discounts the load-carrying capacity of the short column. This reduction factor is a function of the slenderness ratio of the column. The German and Russian building codes represent the reduction factor by means of a curve; the British building code, by means of two straight lines with different slopes; and the ACI Code, by means of one straight line.

The design procedures of applying a reduction factor to the load-carrying capacity of a short column have been based mainly on the experimental results of tests of concentrically loaded slender columns, taking into consideration only the end conditions (hinged or flat) during the test. The traditional equations were also applied to eccentrically loaded columns by inference since these columns were assumed to represent a rather unnecessary refinement in design practice.

In addition to the slenderness ratio traditionally considered in the long column expressions, these other factors that affect the strength of a slender reinforced concrete column can be mentioned: (1) the initial eccentricity of the loads; (2) the geometry of the column; (3) the geometry of all the other members of the structure; (4) the lateral displacement of the ends of the column; (5) the stress-strain curves of the concrete and steel in the column; (6) the elastic and plastic behavior of the column as part of a structure; and (7) the duration of loading.

The effects of some of these variables were studied by Von Karman¹⁷ who solved the stability problem of slender eccentrically loaded columns. He considered the eccentricity of the load and the stress-strain relationship for the materials and assumed a linear strain distribution across the section. The Von Karman theory has been followed by modern authors in trying to express the basic strength of slender columns. Some of these procedures are discussed in the next section.

In the 1963 Building Code,¹ ACI made substantial modifications to the design procedure applicable to slender columns, but retained the use of a long column reduction factor. The reduction factor is expressed as a linear function of the slenderness ratio and applies equally to load and moment. The main characteristics of the ACI design procedure are:

1. The initial eccentricity of the loads is taken into account in the expressions for the load capacity of a cross section but is not a part of the long column reduction factors.

2. The slenderness reduction factor is referred to the mechanical slenderness ratio h/r instead of the geometric slenderness ratio, h/l . This helps to take the shape of the cross section into account.

3. The relative rigidity of the column compared to the rigidity of the floor members is taken into account to determine the effective length of the column for columns free to sway.

4. The possibility of relative lateral displacement of the ends of the column is considered in the determination of the effective length of the column and in the reduction factor expressions.

5. Different reduction factors are specified for single and double curvature bending in the column.

6. The effects of sustained loads were included in the analyses used in deriving ACI Code equations. In addition, the Code requires the use of a reduced modulus of elasticity in any alternate analysis of the column strength.

The CEB, in its Recommended Practice,² has a different approach to the treatment of the slender column problem. The appropriate sections from the Recommendations are included as an appendix to this paper. The main characteristics of the CEB Recommendations for designing slender columns are:

1. The effect of slenderness is considered as a “complementary” or deflection moment to be added to the moment due to the initial eccentricity of the loads.

2. The initial eccentricity of the loads is taken into account in the equations for the complementary moment. The sum of the initial and complementary eccentricities are then used in the cross section design equations.

3. The complementary moment is expressed as a function of the geometric slenderness ratio. The constant ρ was derived assuming a rectangular column section.

4. The relative rigidity of the column as compared to that of the adjoining members as well as the possibility of relative lateral displacement of the ends of the column are very roughly taken into account by the term ρ in the equation for the complementary moment. This term is a function of the elastic effective lengths.

5. The effects of sustained loading, plastic deformation of the column, and special conditions such as vibrations may be included in the expression for the complementary moment.

Both ACI and CEB depart from the traditional design methods in that the relative rigidity of the slender column as compared to the rigidity of the adjoining members is considered, as well as the possibility of lateral displacement of the column. However, the approach of the design procedures of the two organizations is different; ACI considers the effect of slenderness with a reduction factor, while CEB considers this effect as a complementary moment to be added to the initial eccentricity moment.

Thus, the ACI procedure reduces the interaction diagram to scale, while the CEB procedure rotates the eccentricities of the interaction diagram.

Neither method accurately considers the influence of the stress-strain relationship of concrete and steel on the deflection of the columns, the influence of the lateral deflections on the ultimate capacity of the columns, the restraining effect of the structure on the ends of the column, and the plastic behavior of the column as part of a structure.

Parme¹⁸ has proposed a different approach to the design procedure for slender columns by applying a magnification factor to the moments as a function of the critical buckling load of the column. This procedure is presented in another paper in this symposium and will not be discussed here.

Determination of basic strength of slender columns

The strength of a slender column in a structure depends on the geometry of the column and the members of the structure, the lateral displacement of the column, the initial eccentricities, the stress-strain relationship of the concrete and steel, the plastic behavior of the column as part of the structure, and the effect of sustained loading.

Various procedures have been developed to determine the basic strength of a column following Von Karman's theory for eccentrically loaded inelastic columns. Ros and Brunner¹⁹ (1926) proposed a half sine wave deflected shape for the column. In 1928 Westergaard and Osgood²⁰ proposed a cosine wave deflected shape.

Ernst, Hromadik, and Riveland²² in 1953 applied the cosine wave proposed by Westergaard and Osgood to hinged reinforced concrete columns using a relationship between axial load, moment, and rotation determined from assumed stress-strain relationships for concrete and steel. They used Hognestad's stress block for concrete, with a 0.0038 strain at failure and an elasto-plastic stress-strain relationship for the steel.

In 1958 Broms and Viest²³⁻²⁵ applied the cosine wave proposed by Westergaard and Osgood to both hinged and restrained columns, by using Hognestad's stress block for concrete with a 0.0038 strain at failure and an elasto-plastic stress-strain relationship for the steel. Broms and Viest considered the restraining moments produced by the adjoining members as proportional to the end rotations of the column. Their recommendations formed the basis of the 1963 ACI Code.

Pfrang and Siess³ developed a method of determining the behavior of restrained slender columns by using Hognestad's stress block for concrete, with a 0.004 strain at failure, elasto-plastic stress-strain relationship for the steel, restraining moments produced by the adjoining members proportional to the end rotations of the column, and a numerical integration procedure to determine the lateral deflection of the column. This method of analysis was later modified by Breen and Ferguson²⁶ to analyze tests of columns in frames. The close correlation between measured and com-

puted loads and behavior represents a good check of this method of analysis.

For the purposes of this paper the Pfrang and Siess analysis has been used to compute the basic strength of the slender columns. The assumptions made in this analysis are listed in the early paragraphs of this report.

Comparison of slender column design equations

In this section of the paper, analytically derived interaction diagrams for slender columns will be compared to the corresponding design interaction curves. In each case the interaction diagrams for restrained columns are plotted in terms of the column loads and moments from a conventional first order frame analysis. Load factors, material strength factors, and the effects of the code definition of material strengths have not been included in deriving these curves.

Laterally restrained slender columns—Fig. 2-11 and 2-12 show the interaction diagrams according to the ACI Code and the CEB Recommendations when relative lateral displacement of the ends of the column is prevented. The curves are plotted for the assumed typical tied column section with $\rho_m = 0.7$ and slenderness ratios h/r of 40, 60, 80, and 100. Fig. 2-11 is plotted for single curvature bending and Fig. 2-12 for double curvature bending. Values for the CEB interaction diagrams were computed using an effective length factor ρ of 60, sustained loading factor ψ of 0.33, and a dead to total load ratio of 0.60.

For comparative purposes, these figures also show a band of analytical values based on the Pfrang and Siess analysis. These bands were obtained by considering extreme practical values of end eccentricity ratios, end restraint ratios r' varying from 1.0 to 10.0, and both short-term and sustained loading effects.

Fig. 2-13 and 2-14 show interaction diagrams for the typical spiral column section and slenderness ratios of 40, 60, 80, and 100. Design curves are presented for the ACI Code only, because the CEB does not allow the use of slender spiral columns (appendix). The analytical bands shown in these figures were based on the same assumptions as the analytical bands for the typical tied column section.

In the case of tied columns without lateral displacement and bent in single curvature the ACI Code is conservative when compared with CEB Recommendations and with the analytical band shown in Fig. 2-11. In some cases the CEB Recommendations are slightly on the unsafe side when compared with the analytical band.

In the case of tied columns without lateral displacement, and bent in double curvature (Fig. 2-12) the CEB Recommendations are conservative when compared with the ACI Code and the analytical band. In some cases the ACI Code is slightly on the unsafe side when compared with the analytical band.

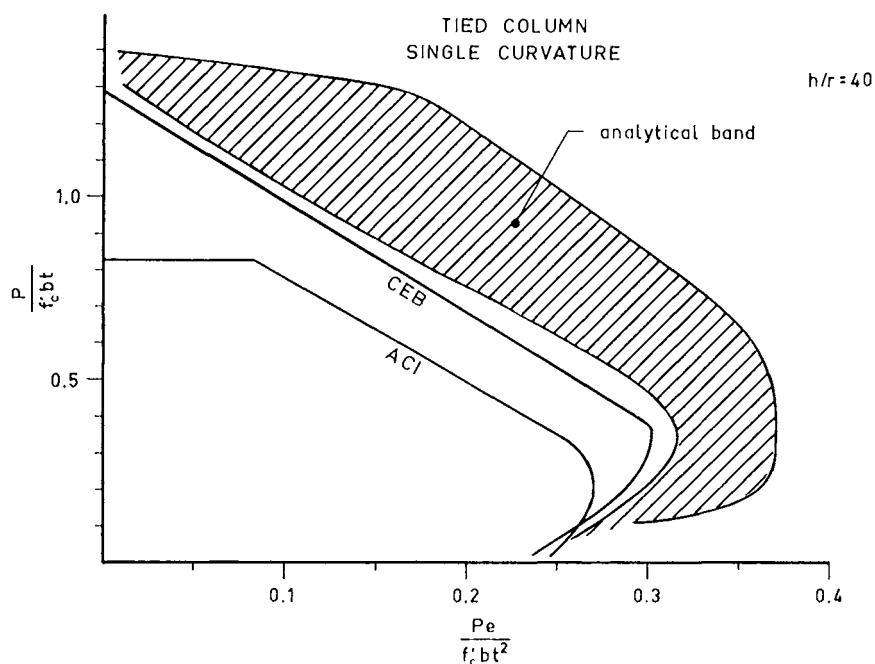
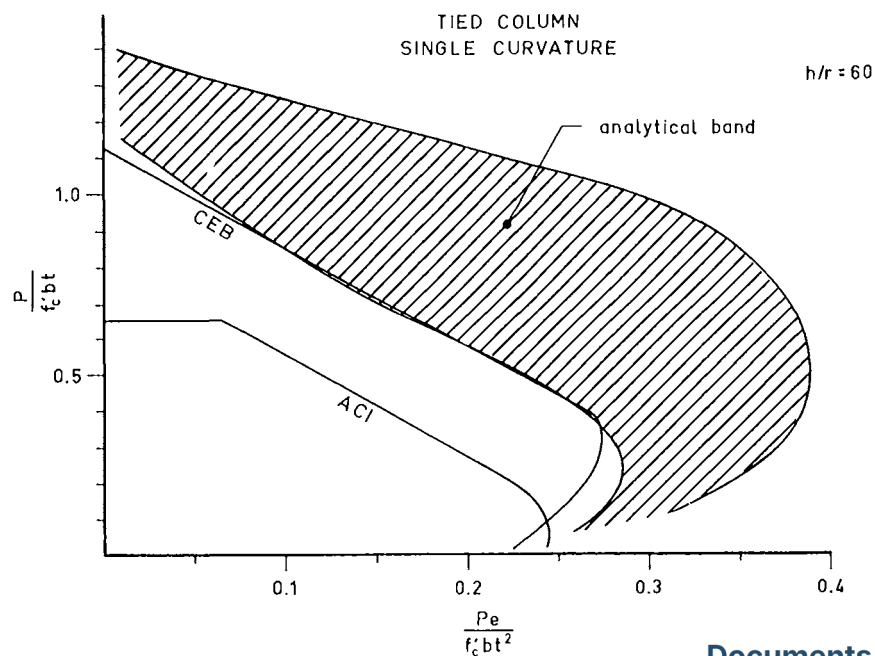


Fig. 2-11a (above) and 2-11b (below) Comparison of design equations for slender tied columns bent in single curvature with lateral deflection prevented



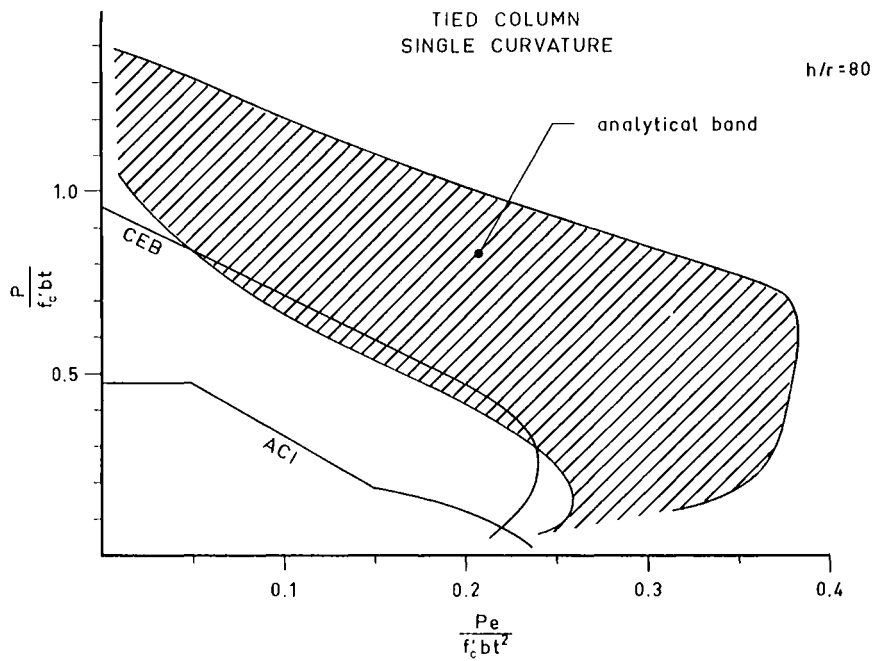
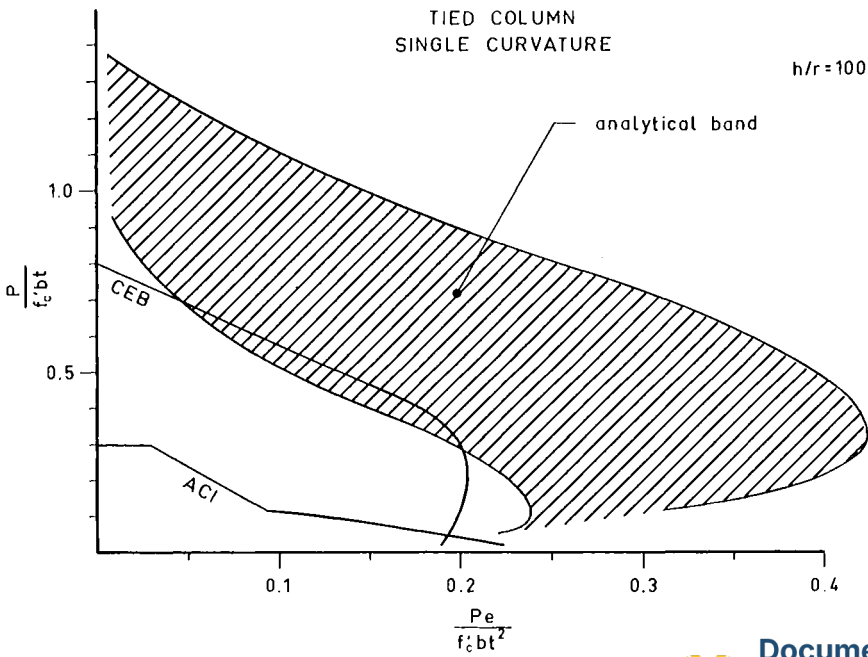


Fig. 2-11c (above) and 2-11d (below) Comparison of design equations for slender tied columns bent in single curvature with lateral deflection prevented



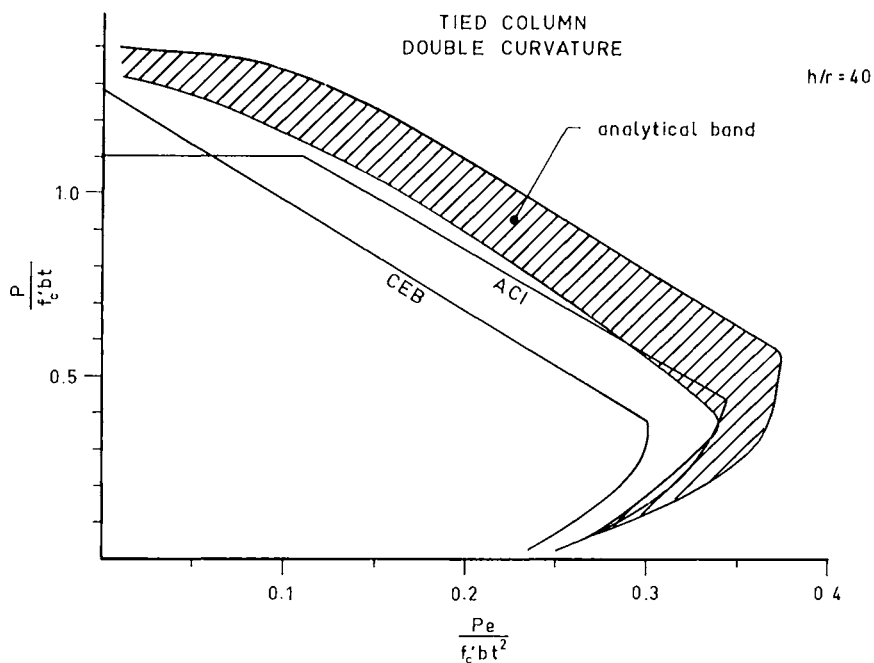
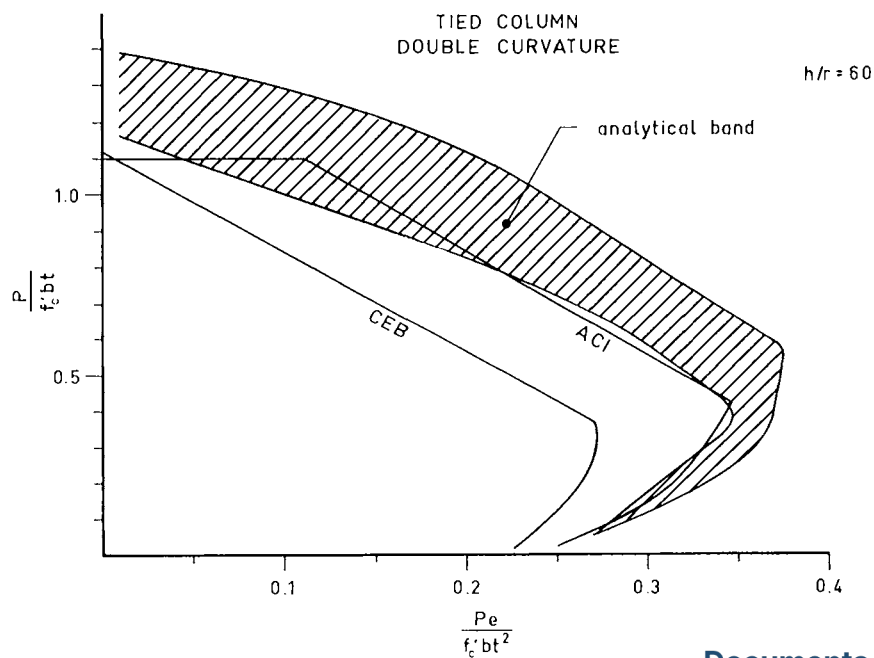


Fig. 2-12a (above) and 2-12b (below) Comparison of design equations for slender tied columns bent in double curvature with lateral deflection prevented



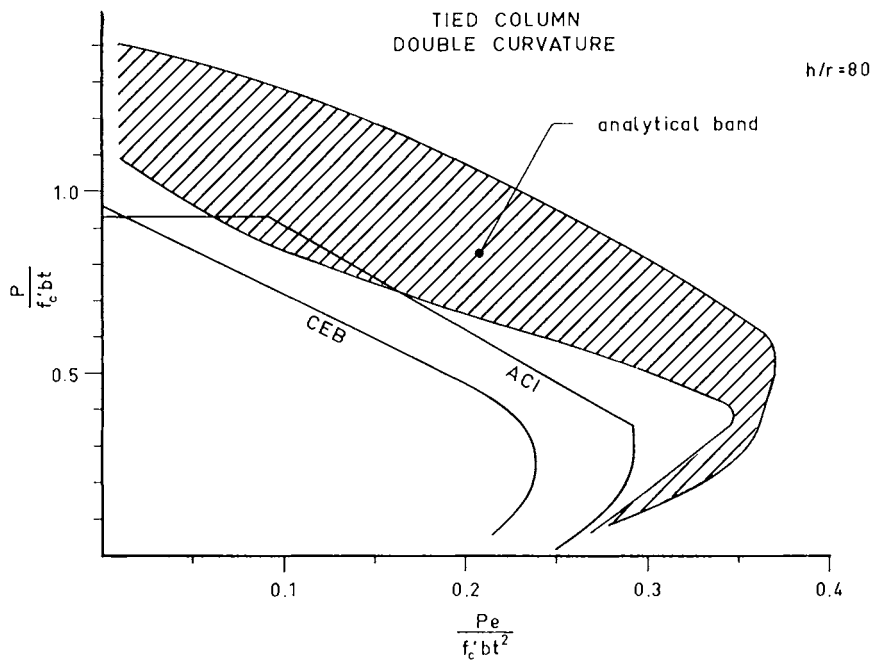
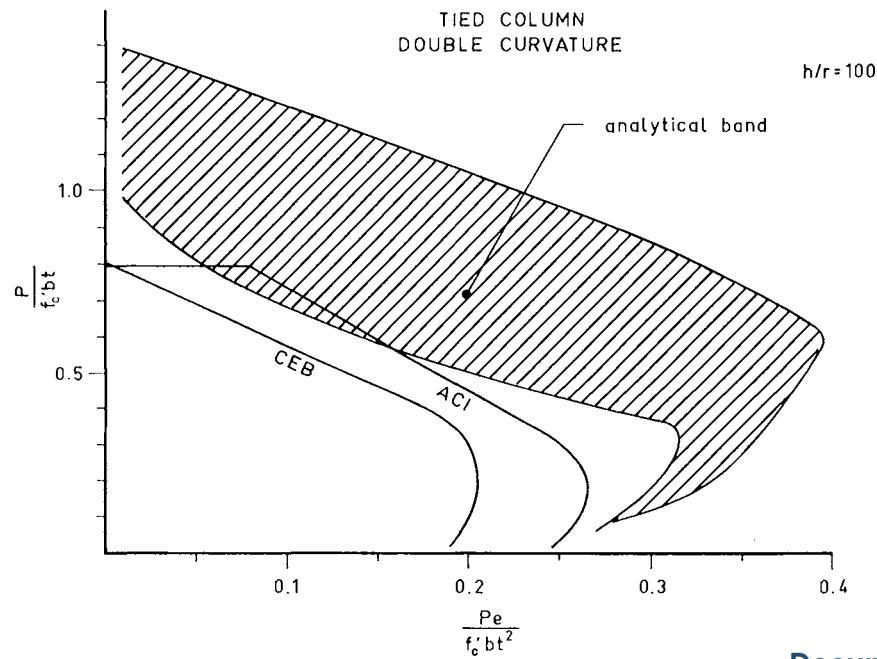


Fig. 2-12c (above) and 2-12d (below) Comparison of design equations for slender tied columns bent in double curvature with lateral deflection prevented



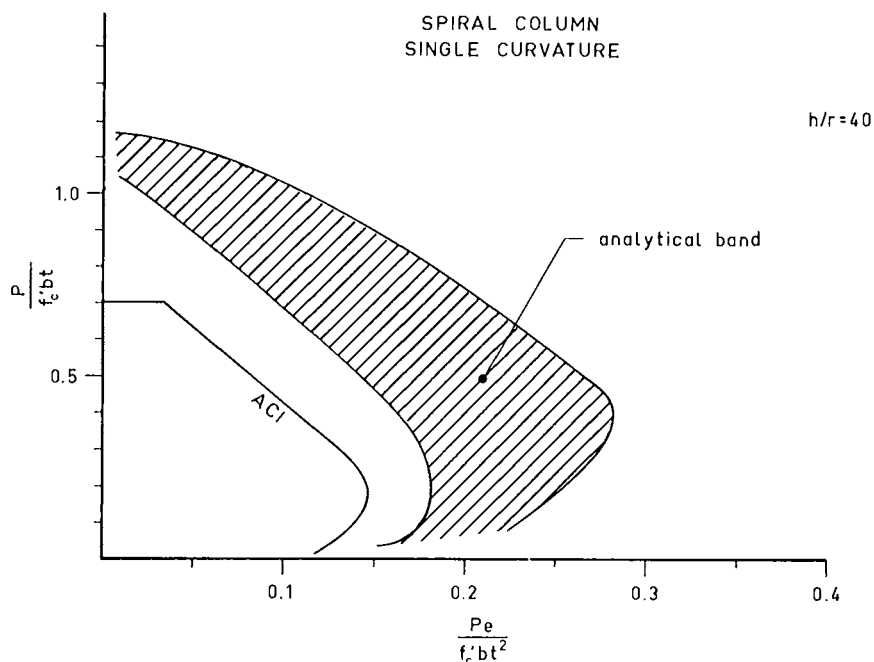
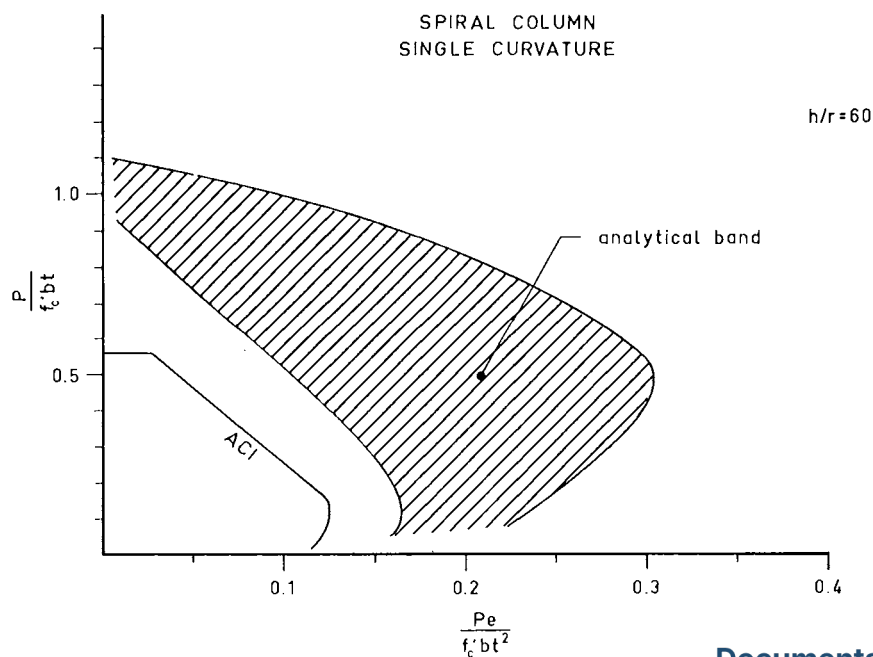


Fig. 2-13a (above) and 2-13b (below) Comparison of design equations for slender spiral columns bent in single curvature with lateral deflection prevented



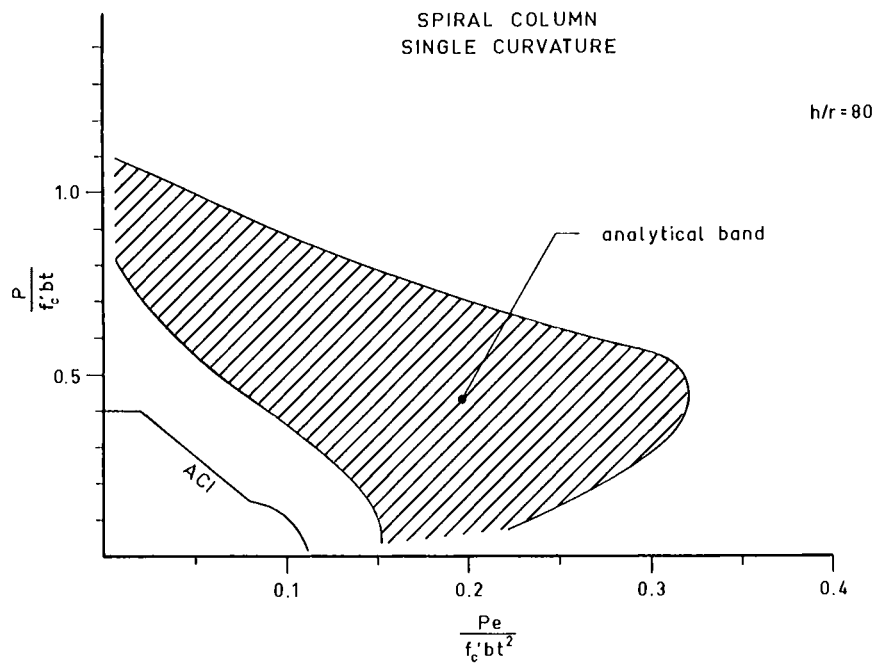
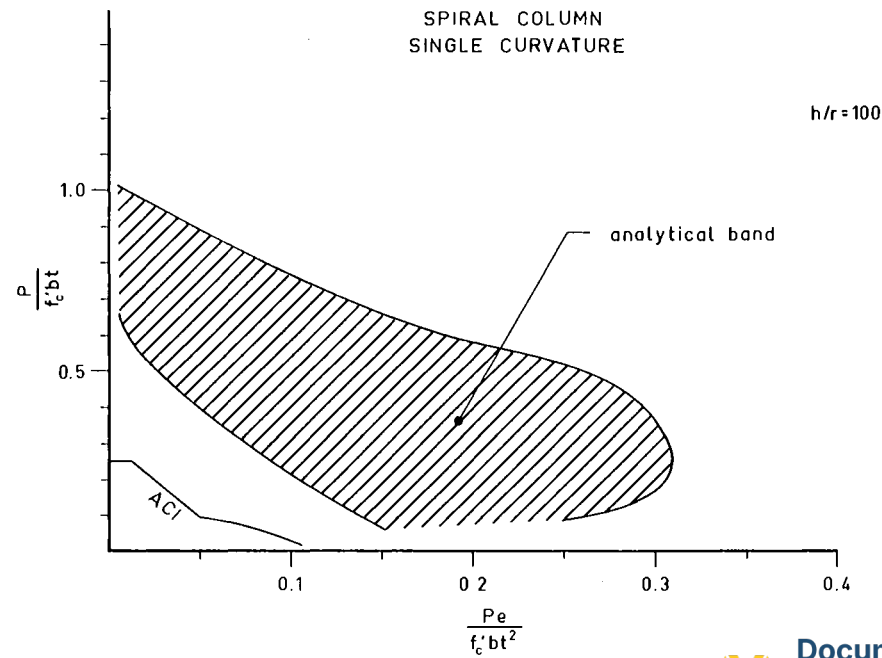


Fig. 2-13c (above) and 2-13d (below) Comparison of design equations for slender spira columns bent in single curvature with lateral deflection prevented



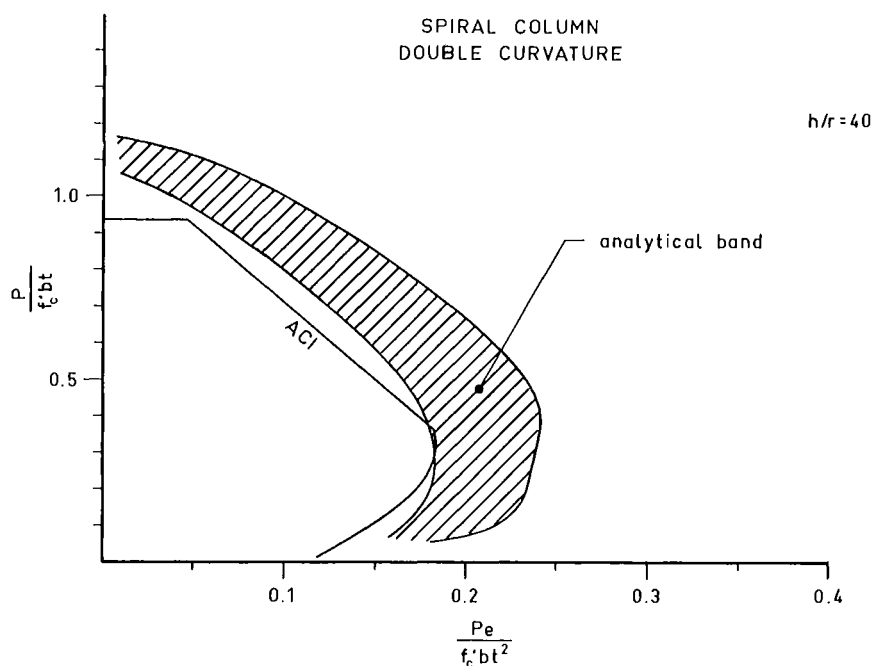
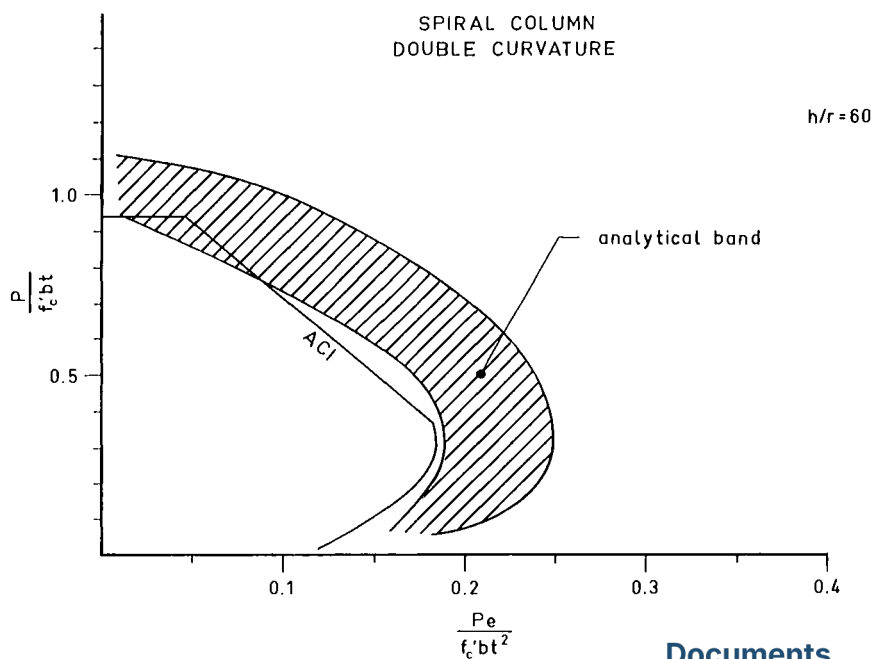


Fig. 2-14a (above) and 2-14b (below) Comparison of design equations for slender spiral columns bent in double curvature with lateral deflection prevented



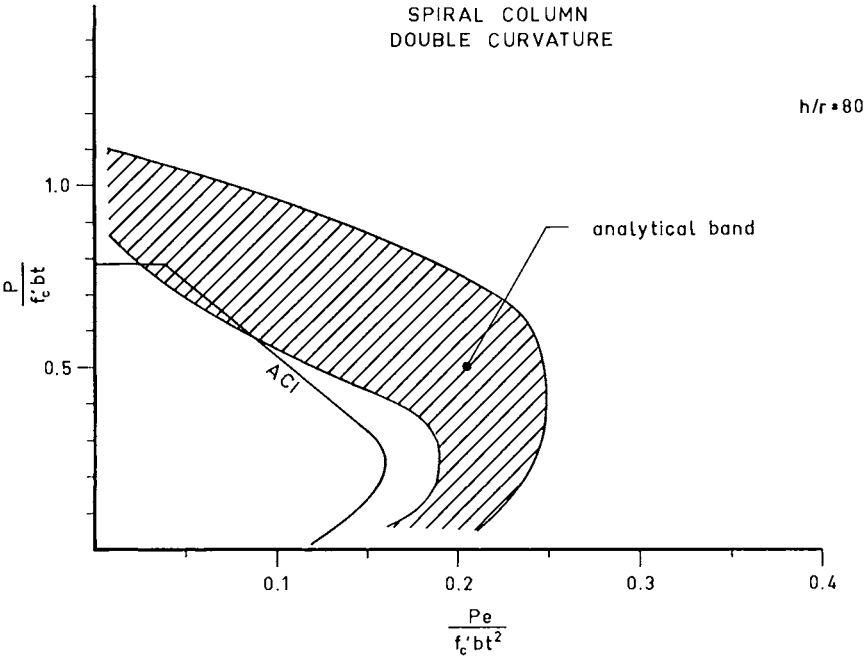
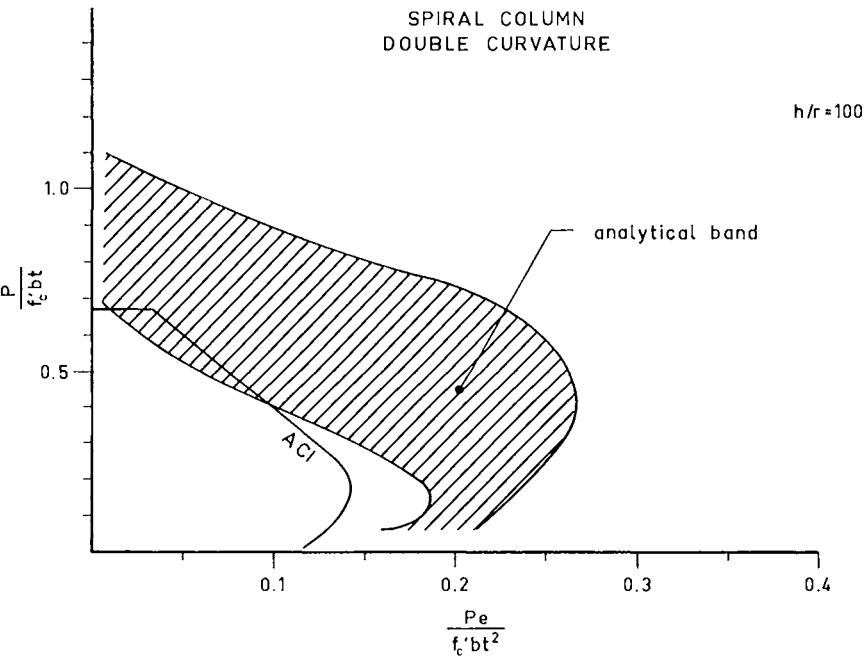


Fig. 2-14c (above) and 2-14d (below) Comparison of design equations for slender spiral columns bent in double curvature with lateral deflection prevented



The ACI Code is conservative compared to the analytical band for spiral columns bent in single curvature without lateral displacement and it may be slightly unsafe for certain extreme cases of spiral columns bent in double curvature.

Columns free to sway—If there is no lateral stiffening in a given story of a structure, that story may fail by translational or sidesway instability. The load at which this type of instability will occur is strongly affected by the loading conditions and the geometry of the structure. For example, the critical load of a symmetrically loaded symmetrical structure will be affected very little by the initial moments in the columns, while the critical load of an antisymmetrically loaded structure will be strongly affected by the magnitude of the initial column moments. In addition, the translation of a column or structure greatly increases the end moments in the restraining beams, and the moment at which these beams hinge becomes a major limitation on the sidesway strength.

The structure considered in this paper was an axially loaded column restrained against rotation at both ends by elastic beams which did not yield during the life of the structure. The lower end of the column was held against translation while the beam and column at the upper end were free to translate laterally. Moments were induced by applying a lateral load to the top of the column to displace it laterally. This structure was chosen because it represents a very severe case of sidesway. The model chosen is similar to one column from the frames tested by Breen and Ferguson²⁷ and good agreement was found when the computer program was used to analyze these tests.

Fig. 2-15 compares the design interaction curves for a typical tied column with $p_m = 0.7$ to computed interaction curves for h/r values of 40, 60, 80, and 100. Fig. 2-16 is a similar plot for the assumed typical spiral column. In each part of these figures, curves are presented for $r' = 1.0$ and a higher value of r' which was felt to be an extreme for a structure free to sway. In these figures the upper ACI, CEB, and theoretical curves were based on the same value of r' , as were the lower three curves. The CEB curves in Fig. 2-15 are based on $\rho = 52/\sqrt{1 + 0.75 r'}$ (see appendix).

In deriving Fig. 2-15 and 2-16 the restraining beams were assumed to be elastic throughout the loading range. The effect of yielding in the restraining beams is illustrated in Fig. 2-17 which is a reproduction of the theoretical curves for $h/r = 40$ from Fig. 2-15 with additional curves (dashed lines) which represent frames in which the restraining beams have a total end rotation of 0.006 radians at the start of yielding. This end rotation corresponds to a yield strain of 0.0015, a neutral axis depth $k = 0.375$, restraining beams fixed at the far end with a length of $l/d = 10$.

It can be seen from Fig. 2-17 that if the restraining beams yield in the manner assumed above, the column strength is very strongly affected by

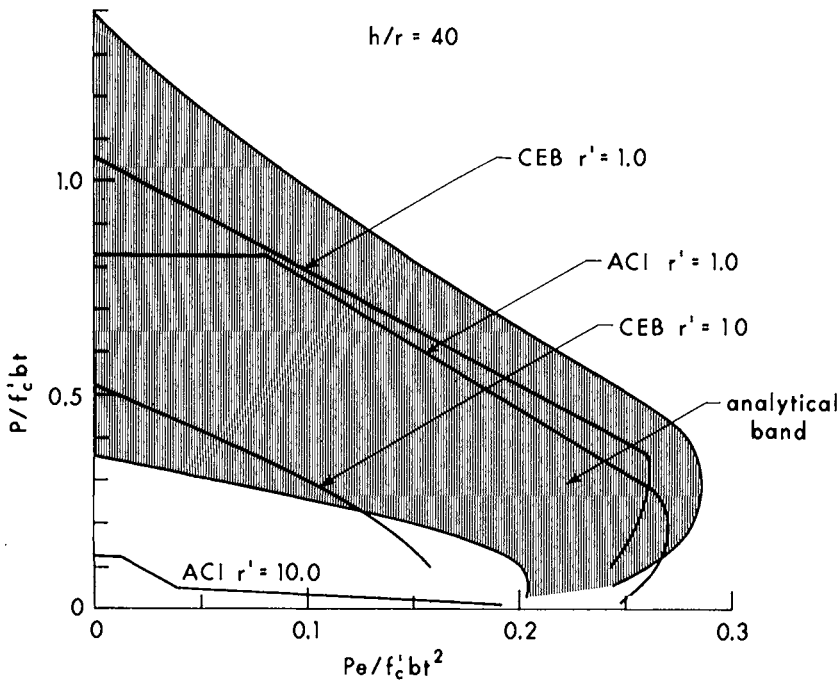
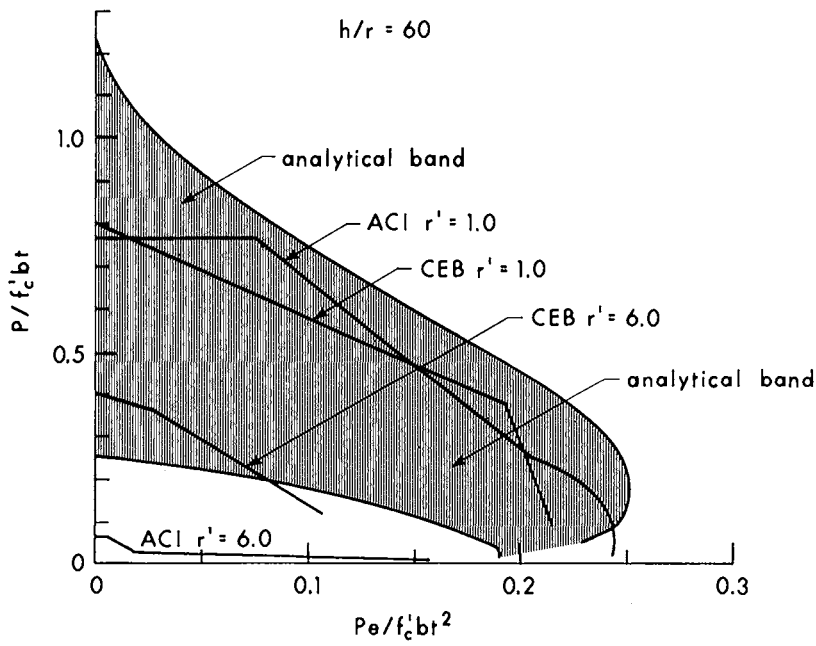


Fig. 2-15a (above) and 2-15b (below) Comparison of design equations for slender tied columns subjected to sidesway instability



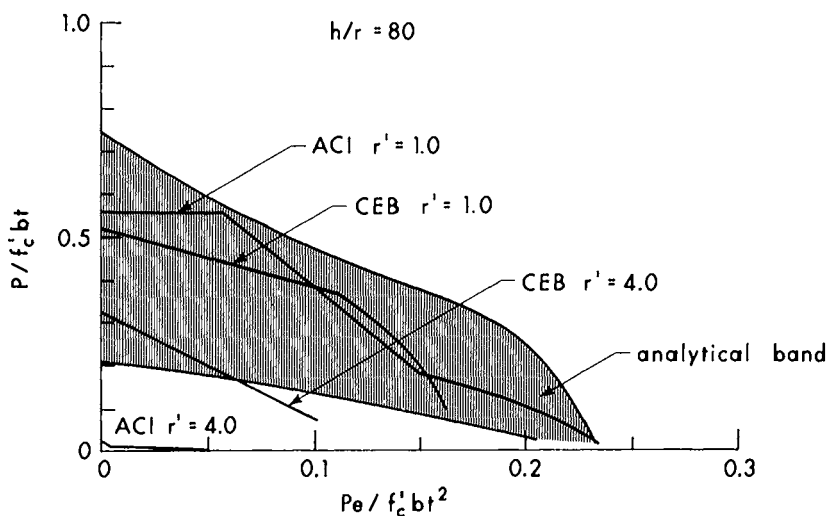
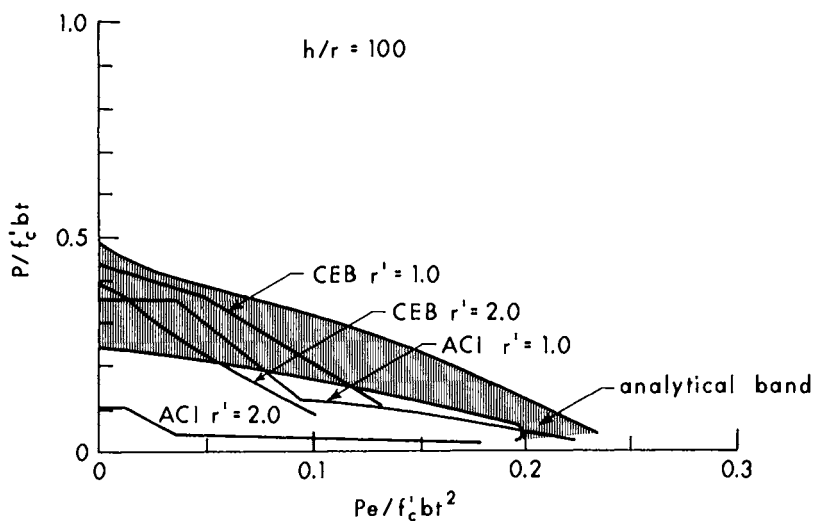


Fig. 2-15c (above) and 2-15d (below) Comparison of design equations for slender tied columns subjected to sidesway instability



the yield moment capacity of the beams. This is especially true for large eccentricities.

For tied and spiral columns subjected to the type of sidesway assumed in this analysis and restrained by elastic beams the ACI design methods range from slightly conservative for $r' = 1.0$, to very conservative for high values of r' . The CEB design method for tied columns is slightly conservative for $r' = 1.0$ and becomes only moderately unconservative for high values of r' . If the restraining beams yield as assumed in plotting Fig. 2-17,

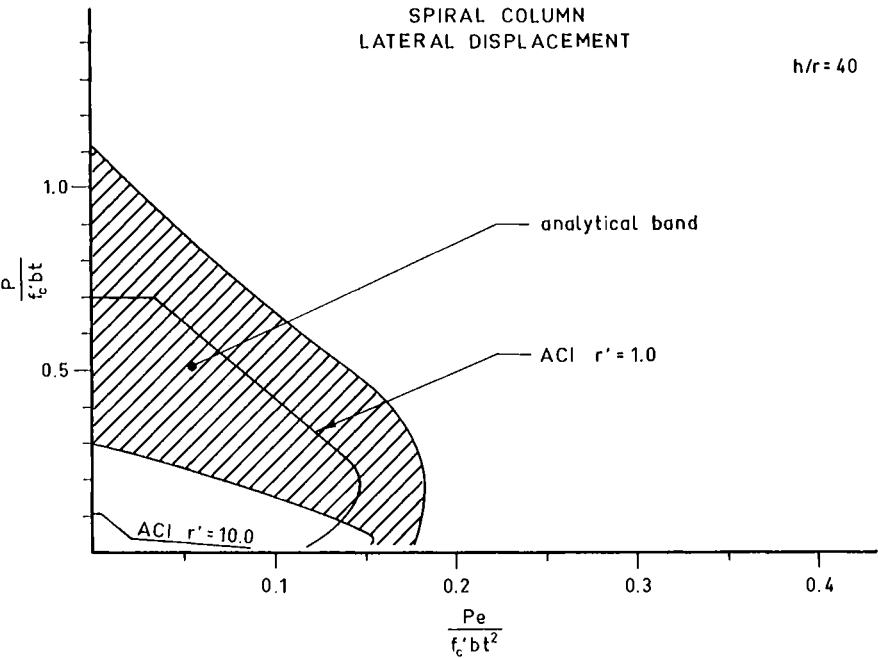
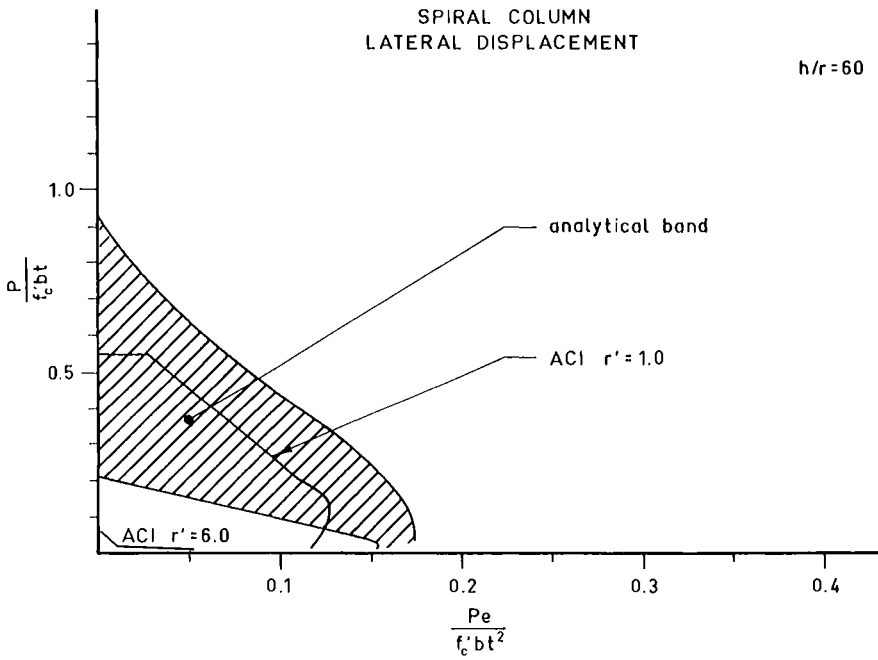


Fig. 2-16a (above) and 2-16b (below) Comparison of design equations for slender spiral columns subjected to sidesway instability



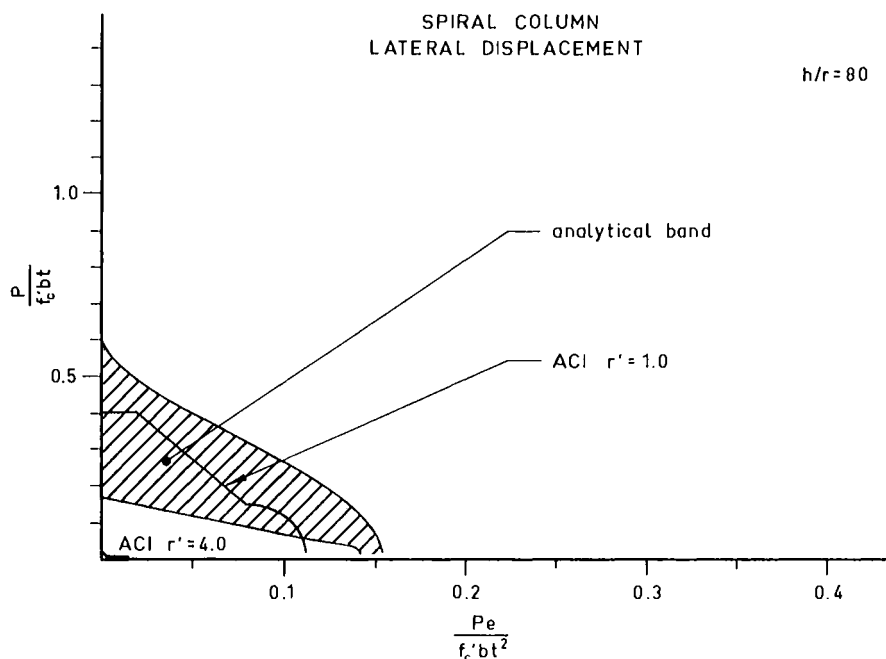
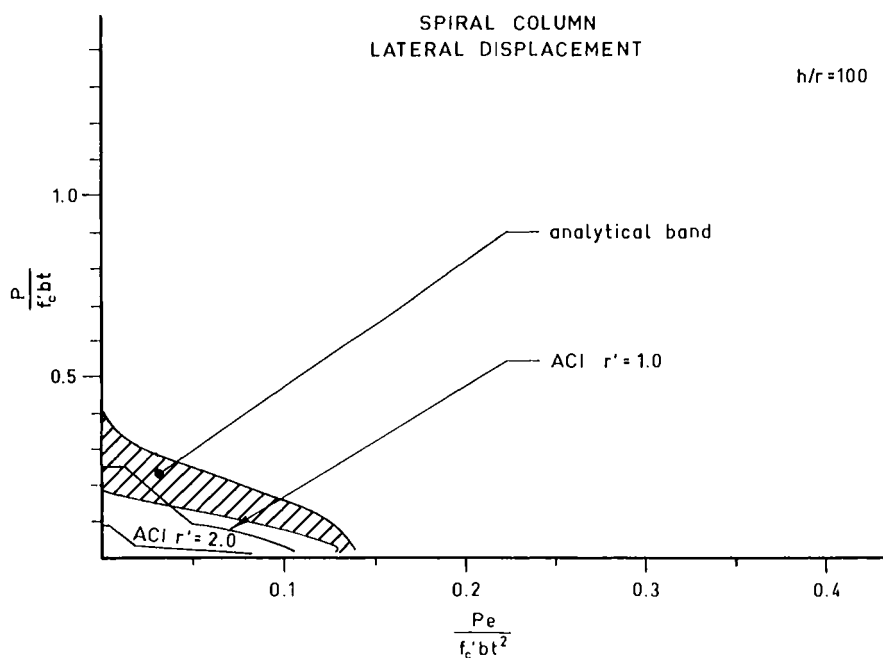


Fig. 2-16c (above) and 2-16d (below) Comparison of design equations for slender spiral columns subjected to sidesway instability



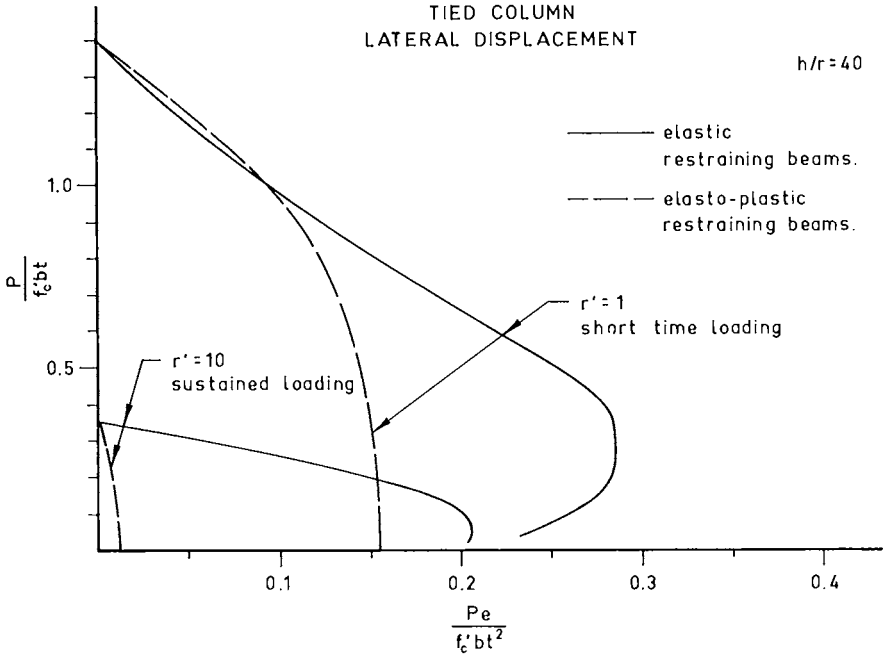


Fig. 2-17 Effect of yielding restraints on sidesway buckling strength

the ACI and CEB methods both become excessively unconservative for large eccentricities.

For other loading cases the sidesway effect may be less critical although it still will be strongly affected by the yield strength of the beams. To obtain a correct idea of the sidesway strength of a structure, the entire structure must be analyzed.

General comments on long column design procedures

For slender columns there is often a marked disagreement between the behavior predicted by the design procedures and the theoretical behavior from a column analysis. Thus the CEB and the analytical interaction diagrams may predict tension failures for cases that the ACI design equations would predict as compression failures. In this regard the CEB design procedures tend to be in better agreement with the isolated test results available than the ACI interaction diagrams, because in general slenderness increases the possibilities of ultimate tension failures.

Neither ACI nor CEB design procedures predict the position of the failure section. This is perhaps most severe for the case of double curvature without sidesway since the CEB equations were derived strictly on the basis of single curvature deflection.

The economics of column construction do not require accuracy in the design of columns, but the increasing demand for slender columns for

aesthetic and functional reasons requires the use of more refined methods of design. However, both ACI and CEB design procedures for slender columns are far from the refinement used in the design of flexural members.

An accurate design procedure would require the consideration of the geometry of the members of the structure of which the slender column is a part and the consideration of stress-strain relationship of concrete and steel to determine the deflected structure and the final loads, moments, and shears to which the column is subjected. It would also require the consideration of the plastic behavior of the column as part of the structure under sustained loading, and it should predict the position of the failure section as well as the type of failure. In the present stage, ACI and CEB design procedures are only crude approximations of this accurate design procedure.

CONCLUSIONS

Strength of a short tied column

1. There is little difference between the results of ACI and CEB design methods for computing the ultimate strength of short tied columns. Both methods lead to essentially the same design for e/t larger than 0.1 t , the ACI minimum. The CEB equations appear to be slightly easier to use in design.
2. For low steel ratio p and for small depth to thickness ratios d/t , all the design procedures studied tended to become conservative in some part of the tension failure range. In general this was not true for d/t greater than 0.85 and p greater than 0.02.
3. The use of reduced material strengths in the CEB design method where the concrete strength is reduced more than the steel strength leads to a lower factor of safety in the tension failure region. The ACI ϕ factors, however, lead to an abrupt transition from $\phi = 0.7$ for tied columns to $\phi = 0.9$ for beams.
4. For compression failures or for average and large values of steel percentage, the 1963 ACI working stress design method leads to more uniform factors of safety than the 1956 ACI Code. For low steel percentages the factor of safety of the 1963 ACI working stress design procedure varies erratically in the tension failure range. The 1963 procedure for computing the balanced eccentricity always yields a value smaller than the true balanced eccentricity.
5. All methods for the ultimate strength or working stress design of columns are inadequate for columns with reinforcement in four faces or for columns subjected to biaxial bending.
6. The factors of safety of tied short columns designed by both the ACI and CEB design procedures seem adequate, considering the present state

of the knowledge of reinforced concrete columns, and construction practice.

Strength of a short spiral column

1. For compression failures, both the ACI working stress and ultimate strength design procedures lead to an acceptable design except in the case of high reinforcement yield points and thick cover when the ultimate strength equations tend to be unconservative. The lowest over-all factor of safety was 1.78 for this case compared to a low of 2.32 for the more typical $p_m = 0.7$ case.

2. Both design methods for spiral columns lead to conservative design in some parts of the tension failure region when compared to strength predictions based on the Hognestad stress block.

3. The factors of safety for short spiral columns designed by both of the ACI design procedures seem adequate for columns with reinforcement yield points not in excess of 60,000 psi.

Slender columns

1. Slender columns designed by the ACI and the CEB procedures show larger variations than do short columns.

2. For slender tied columns without lateral displacement and bent in single curvature, the ACI Code is conservative compared to strength predictions based on the Pfrang and Siess analysis. The CEB Recommendations are slightly on the unsafe side in some cases. Note that the load factors and material strength factors have not been considered in these comparisons.

3. For slender tied columns without lateral displacement, bent in double curvature, the CEB Recommendations are conservative compared to the analysis. The ACI Code is slightly unsafe in some cases.

4. For slender spiral columns without lateral displacement, the ACI Code is conservative for columns bent in single curvature and slightly unsafe in some cases for columns bent in double curvature. CEB Recommendations do not allow the use of slender spiral columns.

5. For slender columns with lateral displacement the instability of the structure is greatly affected by the loading conditions and the geometry of the structure. In some cases, both the ACI Code and the CEB Recommendations may yield results which differ significantly from the results obtained by analytical procedures.

6. CEB design procedure better approximates the type of failure of the slender column in single curvature than the ACI design procedure, because the moment acting in the column is amplified and thus better simulates the actual behavior of the slender column.

7. Both the ACI and CEB design procedures fail to determine the behavior of the slender column, as to the position of the failure section and the plastic behavior of the column as part of a structural frame.

General

The comparisons of design methods made in any study of this sort are only as good as the underlying assumptions. In the future, more complete experimental results and more precise analytical methods may change certain aspects of these comparisons. Similarly, the limiting variables chosen for the theoretical computations may need revision as future practice changes. At the present, however, the writers are satisfied that the assumptions made and the ranges chosen for the variables are representative of current practice.

ACKNOWLEDGMENTS

The writers wish to acknowledge the helpful comments of Boris Bresler and other members of ACI-ASCE Committee 441, Reinforced Concrete Columns. The electronic computer analyses presented in this report were carried out jointly by SACMAG International, Ltd. and the Universities of Delaware, Texas, and Alberta. The National Science Foundation, the Reinforced Concrete Research Council, and the National Research Council of Canada, respectively, helped finance the computer solutions at these three universities.

APPENDIX

CEB DESIGN EQUATIONS

Cross section strength

Although there are no equations for combined axial load and bending in the CEB Recommendations,² a set of such equations was proposed in CEB *Bulletin* No. 31.¹² The comparisons in this paper were all based on these equations which are given below.

1. Tied column section controlled by compression:

$$P_u = 0.85 f'_c b t \frac{(1 + p m)}{(1 + \alpha e/t)}$$

where

$$\alpha = 4 \left(\frac{1 + p m}{1 + 2 p m \frac{d - d'}{t}} \right)$$

2. Tied column section controlled by tension:

$$P_u = 0.76 f'_c b t \left[-\beta + \sqrt{\beta^2 + 1.1 p m \frac{d - d'}{t}} \right]$$

where $\beta = e/t - 0.5$.

The CEB does not give equations for spiral columns and under the heading, "CR4.132: Members Reinforced by Binding," they state:²

"the binding is efficient only for short columns and slight eccentricities. For long columns reinforced by binding, the influence of the binding on the buckling is very small and there is often premature failure of the column by bursting."

Slender columns

The CEB Recommendations² treat slender columns in the following manner:

R4.141: Axially Loaded Columns

For analyzing slender members with regard to failure, an additional moment M_c^* is introduced which will be added to the loadings calculated by the first-order theory

$$M_c^* = \frac{N^* (h_t + e_o) E_b}{3,300 \sigma_E}$$

where

N = characteristic value of the normal force, assumed calculated according to first-order theory, multiplied by the load increment coefficient γ_s

h_t = total geometric depth of the section measured parallel to the bending plane

e_o = eccentricity of the normal force in relation to the center of gravity of the concrete section alone, calculated according to first-order theory, if there is a transverse bending moment

E_b = longitudinal strain modulus of the concrete, kg/cm²

σ_E = Euler stress calculated on the elastic theory with the value chosen for the E_s modulus, taking into account the effective configuration of the system concerned

γ_s = load factor usually taken as 1.4 (defined in CEB R2.321 and CR2.321).

CR4.141: Axially Loaded Columns

For columns with constant inertia and a rectangular cross section, the expression of the additional moment M_c^* given in the Recommendations, may be written

$$M_c^* = N^* (h_t + e_o) \left(\frac{l}{\rho h_t} \right)^2$$

where l is the free length of the column and ρ is the numerical coefficient taken equal to:

26, if the member is free at one end and fixed at the other

52, if the member is articulated (hinged) at both ends

52, if the member may be considered as fixed at both ends, the ends being capable of moving in relation to each other in a direction perpendicular to the longitudinal axis of the member and situated in the principal plane for which buckling is investigated

75, if the member is articulated (hinged) at one end and fixed at the other.

For multistory buildings, where the continuity of the columns and their sections is assured, ρ shall be taken equal to:

75 if the ends of the column are fixed in the foundation slab or connected to floor beams with an inertia at least similar in the direction in question and running through it from one side to the other

52/ $\sqrt{1 + 0.75r'^{\dagger}}$ if the ends of the column are free to sway, where

$$r' = \sum \frac{EI_c}{L_c} \div \sum \frac{EI_b}{L_b}$$

60 in all other cases.

[†]The sidesway instability case is not included in the CEB comments. The value used here has been suggested in a private communication from Dr. Aas-Jakobsen, reporter for CEB Commission 8, "Buckling," dated Nov. 1, 1965.