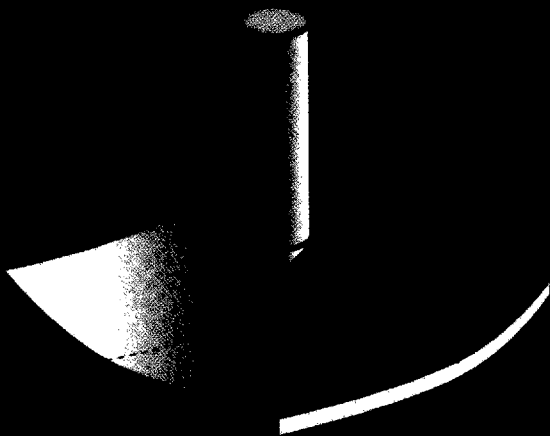


Recent Developments IN Deflection Evaluation OF Concrete



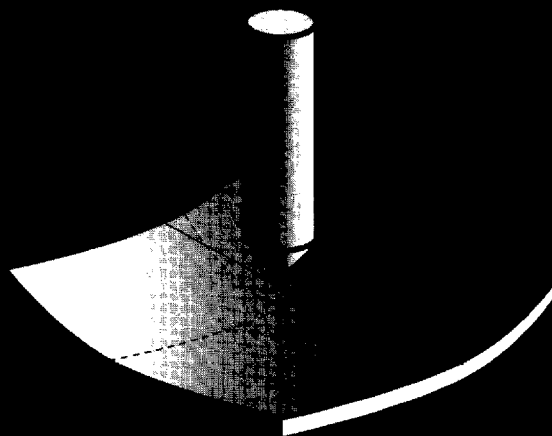
SP-161

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Edward G. Nawy

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Recent Developments IN Deflection Evaluation of Concrete



SP-161

Edward G. Nawy



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PREFACE

Design for serviceability and safety is central to the work of structural engineers, code-writing bodies and the users. The current era of high strength and high performance materials, exotic additives and limit states of design has necessitated better control of constructed facilities in their short and long-term performance as structural elements and systems.

The test is the outgrowth of a national symposium of the American Concrete Institute co-sponsored by ACI Committees 435 on Deflection and 421 on Two-Way Slabs, in honor of Dan E. Branson, a former chairman of ACI Committee 435 and major contributor in the area of deflection. The symposium concentrated on recent advances in deflection evaluation and covered in the 12 papers, topics ranging from interpretation of slab roof deflections to effect of openings on deflection and strength of floors, deflection analysis using finite element methods, and the identification and use of the developing fiber optic technology in evaluating strain and deformation in concrete structures. It came on the heels of the symposium inaugurating the recently developed comprehensive report of ACI Committee 435 entitled, "Control of Deflection in Concrete Structures," which covered all aspects of deflection reduction and control in reinforced and prestressed concrete beams and one-way and two-way slabs and plates.

All the papers presented in this publication were reviewed by recognized experts in accordance with the ACI review procedures. It is hoped that designers, constructors, and codifying bodies will be able to draw on the material presented in this volume in conjunction with the ACI 435 Committee Report, "Control of Deflection in Concrete Structures," in improving the long-term deflection behavior and performance of concrete constructed facilities.

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Dan E. Branson

This special publication is in honor of Dan E. Branson, a past chairman of the ACI Committee 435 on Deflection of Concrete Structures, 1962-68. Dr. Branson, Professor Emeritus, The University of Iowa, Iowa City, has contributed immensely to the engineering profession in the area of deflection and serviceability of concrete structures, both reinforced and prestressed. His technical contributions to the American Concrete Institute, both as a member for many years and as chairman of several committees and subcommittees, are well recognized. His $I_{\text{effective}}$ expression for evaluation of cracked section has been adopted and used in the ACI 318 Building Code and internationally for many years and continues to be used in the 318-95 Code. He received several awards from engineering societies for his accomplishments and this volume is a fitting tribute for Dr. Branson's contributions to the advancement of concrete.

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Deflection of a Concrete Beam and Slab Roof

by Russell S. Fling

Synopsis: An 8,000 sq.ft. (740 sq.m) portion of an industrial building was load tested and vertical movements measured to an accuracy of 0.0043 inches (0.11 mm). Measured deflections were compared to those estimated before testing and to revised estimates after testing. Using simplified procedures and judicious estimates of design parameters, computed deflection normally should be within 40 percent of actual average deflection and the coefficient of variation should be less than 50 percent. With a complete and accurate selection of design parameters, the accuracy and statistical variability can be improved to 15 and 40 percent respectively.

Keywords: Beams (supports); cracking (fracturing); creep properties; deflection; flexural strength; load tests (structural); prestressing; reinforced concrete; slabs; statistical analysis; stiffness; structural analysis; torsion; volume/surface ratio

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Russell S. Fling has been a practicing structural engineer for over 45 years. As a long-time member of ACI Committee 435 on Deflection, he has been chairman of several subcommittees that wrote reports on Allowable Deflections, and Deflection of Continuous Concrete Beams. Mr. Fling is a past President of the Institute.

INTRODUCTION

An 8,000 sq.ft. [740 sq.m] portion of a 500,000 sq.ft. [46,500 sq.m] industrial building built in 1942 and 1945 was load tested in June and July 1993 and measurements of vertical movements at ends and mid-point of each member (50 points), accurate to 0.0043 inches (0.11 mm), were taken at each of 40 load steps. Measured deflections were compared to those estimated before testing. After the load test, more careful estimates of deflection were computed. The purpose of the load test was to determine the safety of the structure because large shear cracks appeared in many beams and girders in zones where shear reinforcement was not provided. The structure did pass the load test. Details are given in another paper (1).

The building tested was one story, 30 ft [9m] high, 252 ft [77m] wide by 2,000 ft [610m] long with a reinforced concrete roof. Cast-in-place roof girders (24" by 60") [600 x 1800 mm] spanning 50 ft [15m] supported 40 ft [12m] beams (18" by 40") [555 x 1220 mm] spaced 16'-8" [5m] on centers. Six inch thick by 12 inch wide [150 x 300 mm], lightly prestressed, precast slabs rested on top of the beams. Deflections of four girders, 12 beams, and 12 slab panels were evaluated at each stage of loading. See the roof framing plan in Figure 1 which gives the general arrangement, member marks, and measuring points. Bays numbers indicate the loading sequence, i.e., Bay #1 was loaded first. The original design live loads were presumed to be 40 psf on the roof and 10 psf suspended below the roof.

Before load testing, estimates of deflections were made for the purpose of detecting incipient failure of members if this should occur. Later it was realized that the data accumulated in conduct of the test would be useful in evaluating the accuracy and variability of deflection calculated by the usual methods. Therefore, a second calculation of deflection was made using more accurate assumptions. There is an obvious risk that the assumptions after the test will be selected to fit the test results. However, assumptions listed here were reasonably foreseeable before the test. Both sets of calculations used the procedure outlined in the current ACI Code (2).

TEST PROCEDURES

The structure was loaded with water. Tanks similar to concrete formwork without form ties were built over each test member and lined with polyethylene film. Due to the physical dimensions of tank construction, tanks for beams and slabs were slightly shorter than the clear spans. Nevertheless, tanks were filled with the entire required uniformly distributed test loads with no allowance for the resulting increase in moment and shear in the members. In calculating required test weights of water, allowance was made for the weight of tanks.

Each 40 ft by 50 ft [12 x 15 m] bay was loaded in the sequence indicated on Figure 1 in five steps with deflection readings taken at the end of each step. First, full load was applied on beams and girders. Thereafter, one-quarter-load increments were applied on the slabs until the full load had been applied. A sixth reading was taken 24 hours later without the addition of more load. Twenty four hours after all bays were fully loaded, each bay was unloaded in one step, one day per bay, in the same sequence as they were loaded.

During the loading of bay #3, top bars in beam B12 delaminated, so the test was stopped, all bays unloaded, the beam repaired, then the test restarted and carried to completion. Loading of bays #1 and #2 are called Phase I and loading of bays #3 and #4 are called Phase II in this investigation.

To measure deflection, wires were hung from the ends and midspan of all members (midbay for slabs), connected to extension scales which in turn were connected to concrete blocks resting on the floor slab. Scales had an 80 lb. [36 kg] capacity graduated by 0.1 lb. [45g] increments and were set initially to read in the range of 30 to 50 lbs [13 to 22 kg]. Deflection of members reduced the scale reading. All scales (a total of 50) were read after application of each load increment, after the required waiting periods, and before loading started each day. Calibration indicated that a scale reading of 0.10 lb. [45g] equals an average movement of 0.0043 inches [0.11 mm] with a variability from one scale to another of up to 5 to 7 percent. The variability within one scale from pound to pound is unknown.

The quantity of water was measured by two meters as well as by measuring the depth of water in each tank. Temperature was almost constant at about 90° F [32° C] so that no adjustments were made for density of water at ambient temperature nor for the length of wires connected to the scales.

Raw data from scale readings at 50 points for 40 load steps were entered into a spreadsheet program and converted to reference elevations and measured deflections. The program also computed the expected deflection for each

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member at each step, ratios of measured to calculated deflection, and statistical analyses.

SLAB DEFLECTIONS

Assumptions used in calculating deflection before the load test (first calculation) and after the load test (second calculation) are summarized in Table 1 and discussed below, listed here in approximate decreasing order of effect on the result.

1. For the first calculation, prestressing was not considered. For the second calculation, the cracking moment was increased by the amount of the prestressing moment, using an assumed initial stress of 20,000 psi [140 MPa] with a consequent reduction in the amount of cracking and increase in stiffness. While low levels of prestress are commonly ignored in strength calculations because creep in concrete reduces or even eliminates the initial prestress in the steel, it seems evident that creep does not reduce the cracking moment.
2. In the first calculation, the deflection under test load was calculated by taking the difference between the initial deflection under dead load only using an assumed concrete strength of 3,500 psi [24 MPa] when the slabs were removed from the forms and the final deflection computed for dead and test loads with strength of 9240 psi [64 MPa]. For the second set of calculations, deflection was calculated for each step using only the test load and the stiffness at the end of each load step. The first load step was over 20 lbs per sq.ft. [960 Pa] and probably exceeded the maximum live load experienced by the structure at any time before the load test. During the load test, as the slabs became increasingly more cracked and less stiff, the measured deflection increased correspondingly. Of course, evaluation of the stiffness at each load step had to consider the pre-test load and loading history.
3. In the second calculation, the effective depth of the reinforcement was increased to 5-1/16 inches [13 mm] which still allows for a 3/4 inch [19 mm] concrete cover on the bottom. This change increased both the prestressing moment and the cracked moment of inertia.
4. A concrete strength of 9,240 psi [64 MPa], the same as assumed for the cast-in-place concrete, was used in the first calculation. Concrete strength was reduced to 6,500 psi [45 MPa] in the second calculation because the lower value is more consistent with the probable current strength. In any case, this factor had a small effect on the computed deflection.

5. In the second calculation, the test load used was reduced 4 percent because this represents the weight of test equipment applied before deflection measurements were started. On the other hand, moments were increased 5.5 percent because the full test load was applied to less than the full length of the span. The net effect of these two adjustments was small.

For both the first and second calculation, a modulus of rupture of $7.5(f'_c)^{0.5}$ was used, and a span length equal to the clear span with no restraining moments was assumed (simple span).

BEAM and GIRDER DEFLECTIONS

Assumptions used in calculating deflection before the load test (first calculation) and after the load test (second calculation) are summarized in Table 2 and discussed below, listed here in approximate order of effect on the result.

1. For simplicity, no column stiffness was assumed in the first calculation. In the second calculation, a pseudo-column supporting the ends of each member was used. The pseudo-column included the stiffness of the real column as well as all columns in the same line and parallel, unloaded beams (for the beams) and parallel, unloaded girders (for the girders). The increase in stiffness over the real column was 24 percent for the girders G3-G5, 42 percent for girders G4-G6, and an average of 690 percent for beams. Moments were computed only for G3-G5 and proportioned for G4-G6.

Inclusion of parallel flexural members and adjacent columns is justified by the torsional resistance of the massive members, especially the girders. It was evaluated by assuming that the pseudo-column stiffness K_{pc} is the sum of the stiffnesses of the adjacent real members (columns, beams, or girders) reduced by the ratio of [rotation due to flexural stiffness of the real member] to [total rotation due to flexural stiffness of the real member plus rotation due to twist of the girder or beam] as shown in the following expression and Figure 2, assuming a constant modulus of elasticity, E_c , of the concrete.

$$K_{pc} = \sum K_b * \frac{1/K_b}{1/K_b + 1/K_t} = \sum \frac{1}{1/K_b + 1/K_t}$$

where K = unit rotation = flexural stiffness
 $= 4I/L$

K_{pc} = K of the pseudo-column

K_b = K of the column, beam or girder resisting the torsional moment

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- K_t = the unit rotation due to twist of the torsional member
 $= 0.1X^3Y/L_t$
 I = gross concrete moment of inertia
 L = center to center span length
 L_t = length of torsional member from point where torsional moment is applied to where it is resisted
 X = shortest cross-sectional dimension of the torsional member
 Y = longest cross-sectional dimension of the torsional member

To reduce the volume of computations, an average value for the pseudo-column stiffness was used to calculate moments for all beams. The range in individual pseudo-column stiffnesses, compared to the average, was from +45 percent for beams nearest the center of the building to -19 percent for beams nearest the edge. The variation in computed moments was much smaller than these values.

Beam B4 framing directly into columns deflected 87 percent of the four adjacent beams while beam B8 deflected 101 percent of the four adjacent beams. The average of 94 percent is within the range of variation of beam deflections but might indicate that beams framing directly into columns deflect less than those that do not.

In some cases, the torsional moment induced into the girders probably exceeds their nominal torsional strength as computed by Section 11.6.1 of ACI 318 Code (2) and increased by a factor of four as suggested by the corresponding Section in the Commentary. However, using uncracked torsional stiffness was simpler. Using torsional stiffness increased the accuracy of calculated deflections in the subject structure. Without this refinement, the similarity in deflection behavior between adjacent beams would have been difficult to explain.

2. Past experience indicates that members with a large volume/surface ratio have a higher flexural strength than given in the ACI Code Section 9.5.2.3, resulting in increased cracking moment and stiffness, and reduction in deflection. In this project, beams have a V/S ratio of 6.2 inches [160 mm] and girders have a V/S ratio of 8.6 inches [220 mm] compared to V/S ratios of 3 to 4 inches [75 to 100 mm] for typical concrete structural members. The same value was used for both before-test and after-test calculations.
3. For simplicity in the first calculation, assuming that the beams and girders had been previously loaded to the load-test level, the beam stiffness at each load stage was assumed to be proportional to the

stiffness at the final load stage. After the load test, it was obvious that the members had never been loaded more than the current code design live load of 25 psf [1200 Pa] and certainly not to the project design live load of 50 psf [2400 Pa]. The load-deflection curves in Figures 3 through 9 clearly indicate an increasing rate of deflection with an increase in load. Therefore, an attempt was made to calculate deflection based on the flexural stiffness determined by the amount of cracking produced by the load at that stage. Even with the help of a computer this proved to be a massive undertaking. Hence, the stiffness at each load stage was assumed to be proportional to the stiffness at each load stage for beams B2 and B3 which were believed to be representative. Adjustment were made for members receiving only one-half the test load such as beams B1 and B5. This assumption is reasonably accurate for some members but introduces error for others.

Stiffness at the haunch end of girders is much larger than at midspan so that the stiffnesses at the ends and the midspan were averaged for the girders in accordance with Eq. 2.15 of the ACI 435 Report (3). The average stiffness was 9 to 28 percent more than midspan stiffness only, depending on span and loading conditions. The stiffnesses at ends and midspan for beams was similar, so the stiffness at midspan only was used in the calculations.

4. In the second calculation, the haunch was used in computing moments as well as stiffnesses, as mentioned above. As a result, maximum positive moments increased about one percent. No attempt was made to evaluate the effect on deflections of the distribution of maximum, or near maximum, moments along the length of the span. The shape of the moment curve could be significant.
5. A series of impact hammer tests before the load test indicated a compressive strength of 9240 psi [64 Mpa] in the beams and girders. The average compressive strength of nine cored cylinders taken after the load test was 2900 psi [20 MPa] and this value was increased to 3410 psi [24 Mpa] by dividing 2900 by 85 percent as suggested by ACI318 Commentary Section R5.6.3.4 (4).
6. For the first calculation, the full test load in the tributary area was applied to the full span. For the second calculation, since construction of the tanks prevented loading beams near their ends, only water load (not tank weight) was applied to a portion of the span corresponding to the actual conditions. The net affect of these two modifications was very small (about one percent). Also, the concentrated load applied by the beams to the girders was adjusted to account for the shear transfer in the beams due to unbalanced moments.

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The actual cracked beam stiffness was not used for moment distribution at each load stage because of the computational effort. Instead, the customary assumption of moment distribution based on the stiffness of uncracked concrete cross-section was used. Preliminary analysis indicated cracked stiffness may affect distribution of moments as much as five percent or more.

RESULTS

The maximum calculated and measured deflection of members is summarized in Table 3 and shown on Figures 3 through 9. In these figures, loading occurred in steps 8 through 19 for Phase I, steps 20 through 31 for Phase II, and unloading occurred in steps 32 through 38. In all cases, the before-test calculated deflection is shown.

In all cases, the values in Table 3 are for the load giving the maximum deflection. Thus, when both interior and exterior spans are loaded, the actual deflection would be the combination of the downward and upward deflections.

Creep Deflection

In the 16 to 24 hours after each bay was loaded, during which there was no additional load, most members continued to deflect. The average increase in deflection for beams was 4.4 percent and for slabs was 6.4 percent. Because simple-span slabs are not affected by loading of adjacent members, their creep deflection for longer periods could also be computed. Their average creep per day for periods of 1.5 to 5 days was 5.6 percent. On average, deflection of girders did not increase over the first 24 hours.

Column Shortening

Column shortening was measured as a matter of interest, but it had no effect on beam, girder or slab deflections. Data were quite erratic, perhaps because beams along lines (A) and (51) may have gained support from walls under them and because gages rested on the floor over column footings. No attempt was made to estimate footing settlement caused by the test loads.

Ratio of Measured to Computed Deflections

The ratio of measured to computed deflections was calculated for each member and each load increment during the loading stages. The average ratio of measured to calculated deflection is given in Table 4. Due to the large quantity of deflection readings it was possible to evaluate the data by statistical means using Eq. 5 of ACI 214 (5) and the results are also given in Table 4. A statistical analysis was performed for Phases I and II separately because there appeared to be an unexplained, slight difference in deflection

behavior between them. For unexplained reasons, deflection of slab S10 was consistently about two-thirds that of the other slabs. Therefore, all other slabs were analyzed without slab S10 and the results are also shown in Table.

Statistical analysis for the first calculation used data for all steps including both loading and unloading, upward and downward, except for loading steps where calculated or measured deflection was 0.008 inches [0.2 mm] or less. This value was twice the reading accuracy of the gages. Thus the error introduced by the reading accuracy level ranged from 1 to 2 percent for readings at maximum deflection to 22 percent (± 0.002 in./0.009 in.) for readings at the minimum reading evaluated. This error was in addition to error introduced by inaccuracies in the scales themselves, as noted above.

For the second calculation, only downward deflections in the loading cycle were included. Upward deflections and deflections on unloading or 24 hours after full load was applied were not included. Limiting the data in this manner may have improved the accuracy for the second cycle.

Slabs

For slabs, the calculated deflection was quite close to the measured average deflection and the variability was very low. See Figure 3 for slab load-deflection curves. Possible reasons the computed slab deflections were more accurate than the beam computed deflections include the following.

- The simple span almost totally eliminated the uncertainty of moments in the slabs, whereas the beams were continuous with subjectively indeterminate moments.
- The slabs were keyed together so that the measured deflection represented not just the one slab to which the equipment was attached but the average of a large number of slab units.
- Concrete in the precast plant was probably of more uniform quality and strength than that produced in the field.

Beams and Girders

The accuracy of calculation of beam and girder deflections was improved by more careful selection of assumptions. An important factor limiting the accuracy of deflection calculations was that the beams and girders deflected unpredictably. For example, interior beams B6 and B7 deflected more than exterior beams B2 and B3, contrary to normal expectations. This behavior might be explained by one of the following construction or service life conditions.

- Some columns could have settled and caused higher moment in the beams and girders than those calculated. Indeed, there was indication that some columns in the area had settled but the data were too incomplete to make a careful analysis of moment redistribution.
- Decentering procedures of the contractor or other construction aberration may have caused higher moments in certain beams and girders than those calculated or caused them to be more thoroughly cracked. At this late date, it would be difficult if not impossible to determine what procedures were used and how they might have affected the structure.
- Some bays could have been overloaded many years earlier, cracking these beams, and resulting in a lower stiffness. However, it seems unlikely that only certain bays would have been overloaded and not adjacent bays or that beams and girders were overloaded but not the slabs.

See Figures 4, 5, 6, and 7 for beam load-deflection curves and Figures 8 and 9 for girder load-deflection curves.

CONCLUSIONS and RECOMMENDATIONS

For reliably accurate deflection calculations, it is essential that an accurate estimate of the distribution of moments in a member be determined. Moments must be computed using actual member stiffnesses. A complete and careful selection of design parameters will improve the accuracy and statistical variability of deflection computations. The possibility of framework action caused by torsional stiffness of supporting flexural members should be considered. With these precautions, an engineer should be able to compute deflections within 15 percent of the actual average value, with a coefficient of variability of 40 percent or less. In this investigation, after the load test, calculated deflection was within 8 percent except for beams and girders in Phase II which had unexplained, irrational deflection behavior, and the coefficient of variation was always under 40 percent.

Availability of accurate design parameters such as loading history, concrete compressive and flexural strength and modulus of elasticity, etc. and inability to measure deflection accurately will prevent reaching this level of accuracy in many, or even most cases.

Using simplified procedures and judicious estimates of design parameters, computed deflection normally should be within 40 percent of actual average deflection and the coefficient of variation should be less than 50 percent unless unknown factors affect deflection dramatically. In this investigation, before the test, calculated deflection was within 40 percent for beams and girders and 52 percent for slabs even though important factors were not considered such as column stiffnesses and prestressing in the slabs. Had

these two factors been considered, in all cases the calculated deflection would have been well within 40 percent of the actual measured deflection. Likewise, the coefficient of variation for all members would have been well within 50 percent if these two factors had been considered.

Further Research

Further research is needed in the following areas.

- Expected creep in the period immediately following the loading of a structure, that is, within 30 days.
- The relationship between the volume/surface ratio and the modulus of rupture and whether the shape of a member affects the modulus, and whether sealed surfaces should be considered in computing the V/S ratio.
- Rules and procedures to be applied to torsion in the analysis of a framework when determining moments for deflection computations.
- The residual variability of deflections when all presently known procedures are used correctly and all required data is accurately known.
- Variability to be expected when normal simplifying assumptions are used.

The preceding discussion and recommendations probably apply to deflection of members loaded for periods of up to one month although none were loaded more than five days in this investigation. None of the beams in this investigation were Tee-beams. While the general discussion and recommendations are probably applicable, the stiffness of Tee-beams is not well understood (6) and may lead to greater variation than reported here.

ACKNOWLEDGMENTS

Timothy G. McCrate and Carlton W. Doncaster of AMG, Inc. Consulting Engineers, were responsible for assistance in planning the load test, preparing all contract documents and coordinating all field operations. The Owner, who wishes to remain anonymous, financed the load test and urged us to conduct it in strict accordance with all test recommendations. Without his encouragement and support, this investigation would not have been possible.

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TABLE 1 — SUMMARY OF ASSUMPTIONS FOR CALCULATING SLAB DEFLECTION

Parameter	First Calculation Before Test	Second Calculation After Test
1. Prestress	Not considered	20,000 psi [140 MPa]
2. Slab Stiffness	I_e at final load stage	I_e at each load stage
3. Effective depth, d	4-3/4 in. [12 mm]	5-1/16 in [13 mm]
4. Concrete Strength f'_c	9240 psi [64 MPa]	6500 psi [45 MPa]
5. Load	Full test load on full span	Water load only on partial span

TABLE 2 — SUMMARY OF ASSUMPTIONS FOR CALCULATING BEAM AND GIRDER DEFLECTION

Parameter	First Calculation Before Test	Second Calculation After Test
1. Column Stiffness	none	Pseudo-column using I_g of all columns and parallel beams or girders
2. Concrete flexural Strength, $(f_r/(f'_c)^{0.5})$	10	10
3. Beam stiffness	I_e at final load stage I at midspan	I_e at each load stage Average I at midspan and ends of girders
4. Consider girder haunch?	No	Yes
5. Concrete Strength f'_c	9240 psi [64 MPa]	3410 psi [24 MPa]
6. Load	Full test load Tributary area	Water load on partial span. Concentrated load on girders increased for unbalanced moment in beams

TABLE 3 — MAXIMUM DEFLECTION OF MEMBERS (in.) (A)

Member (b)	Quan. of members (c)	CALCULATED		MEASURED	
		First calc. (d)	Second calc. (d)	average	step (e)
Precast slabs	12	0.352	0.183	0.165	12, 18, 24, 30
Beams, Interior, down	6	0.075	0.081	0.115	12, 24
B6, 7, 11, 12 up		-0.028	-0.014	-0.014	32, 36
Exterior, down	6	0.104	0.112	0.167	32, 36
B2, 3, 9, 10 up		-0.028	-0.016	-0.025	12, 24
Girders, Interior, down	2	0.095	0.190	0.116	18
G3 up		-0.030	-0.086	-0.067	34
G5	2	0.204	0.231	0.243	34
Exterior, down up		-0.030	-0.051	-0.069	18
Column shortening	9	-0.084	-0.084	-0.040	

(a) Deflection under load conditions giving the maximum value.

(b) Listed members received load causing maximum deflection.

(c) Quantity of members for which deflection was analyzed.

(d) In some cases, assumed parameters were averaged, see text.

(e) Steps in the test procedure at which maximum deflection occurred.

(f) One inch = 25.4 mm.

TABLE 4 — STATISTICAL ANALYSIS OF DEFLECTION RATIOS — FIRST DEFLECTION CALCULATION (BEFORE TEST)

Phase I	Girders	Beams	Slabs
n = Number of tests	55	119	72
X = Average value	1.779	1.214	0.495
s = Standard deviation	0.773	0.635	0.064
V = Coefficient of variation	41%	52%	13%

Phase I and II	Girders	Beams	Slabs
n = Number of tests	75	204	168
X = Average value	1.320	1.389	0.483
s = Standard deviation	0.780	0.785	0.080
V = Coefficient of variation	59%	57%	17%

SECOND DEFLECTION CALCULATION (AFTER TEST)

Phase I	Girders	Beams	Slabs
n = Number of tests	40	39	48
X = Average value	1.075	0.943	1.003
s = Standard deviation	0.249	0.370	0.067
V = Coefficient of variation	23%	39%	7%

Phase II	Girders	Beams	Slabs	Slabs*
n = Number of tests	39	17	48	37
X = Average value	0.847	1.188	0.900	0.984
s = Standard deviation	0.312	0.368	0.191	0.124
V = Coefficient of variation	37%	31%	21%	13%

* Without slab #S10

Phase I and II	Girders	Beams	Slabs	Slabs*
n = Number of tests	79	56	96	85
X = Average value	0.961	1.012	0.952	0.995
s = Standard deviation	0.308	0.400	0.152	0.096
V = Coefficient of variation	32%	40%	16%	10%

* Without slab #S10

Note that average values and standard deviations are ratios, not dimensions

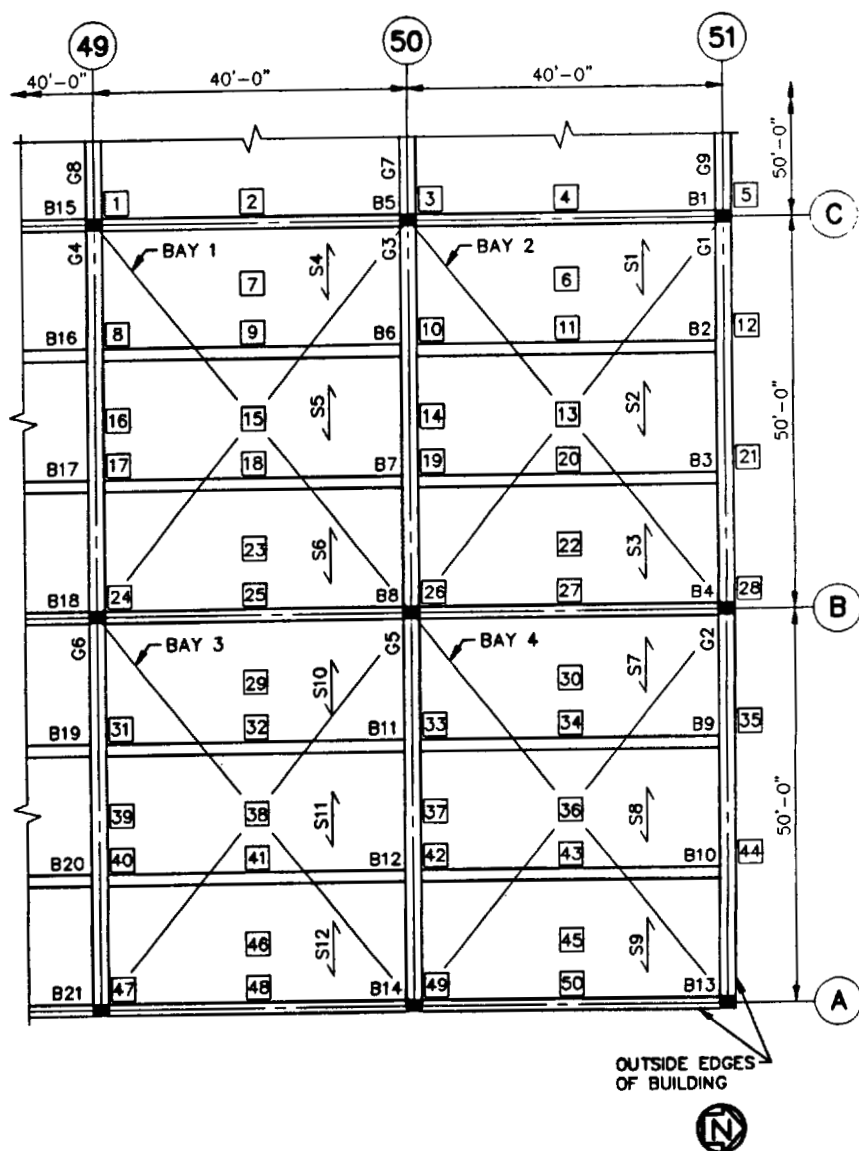


Fig. 1—Roof plan of the test area

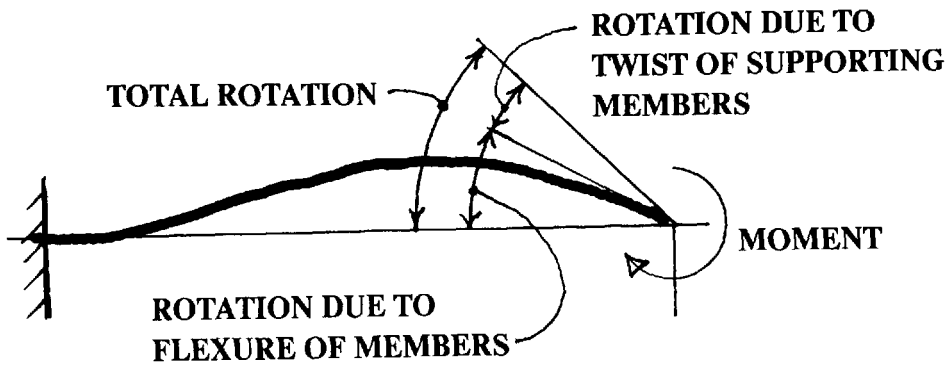


Fig. 2—Rotation of a joint due to combined flexural and torsional stiffnesses

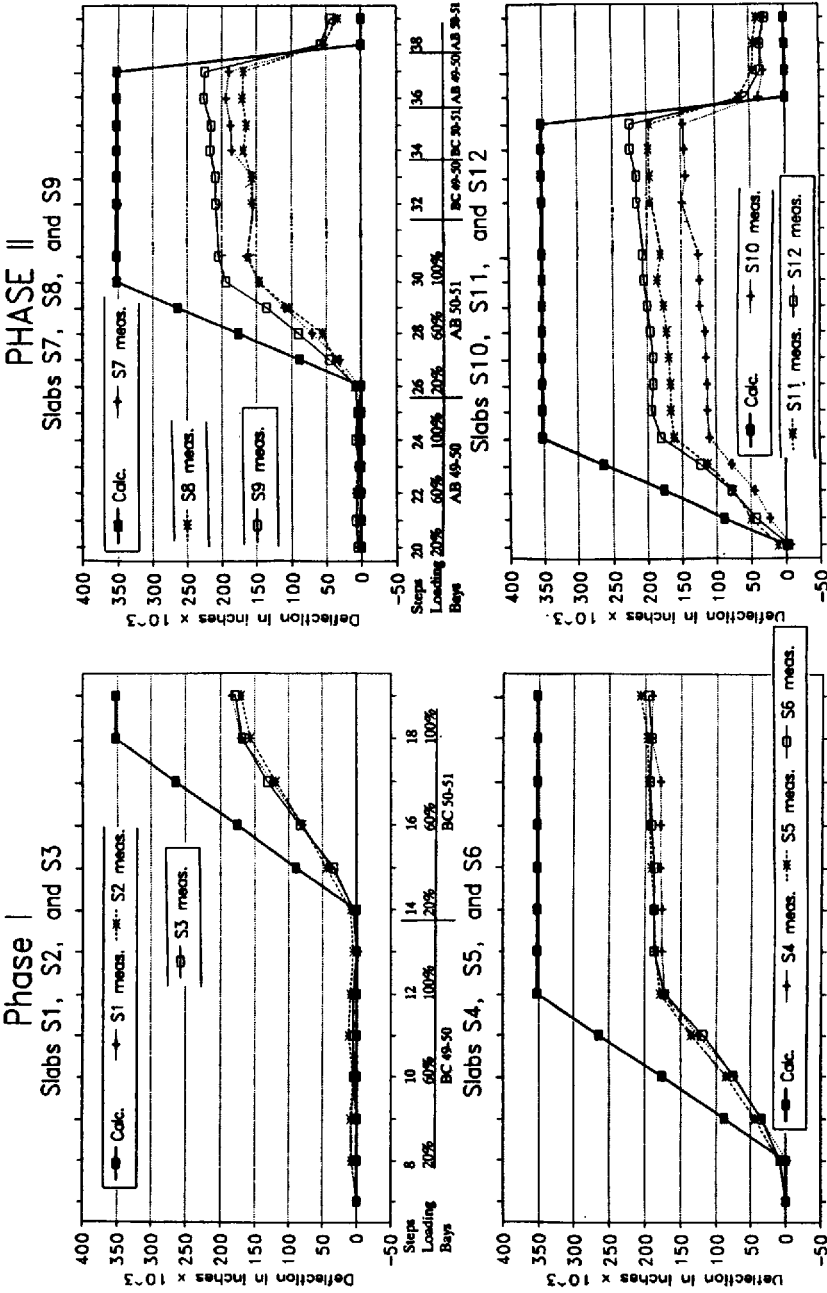


Fig. 3—Load-deflection curves of slabs

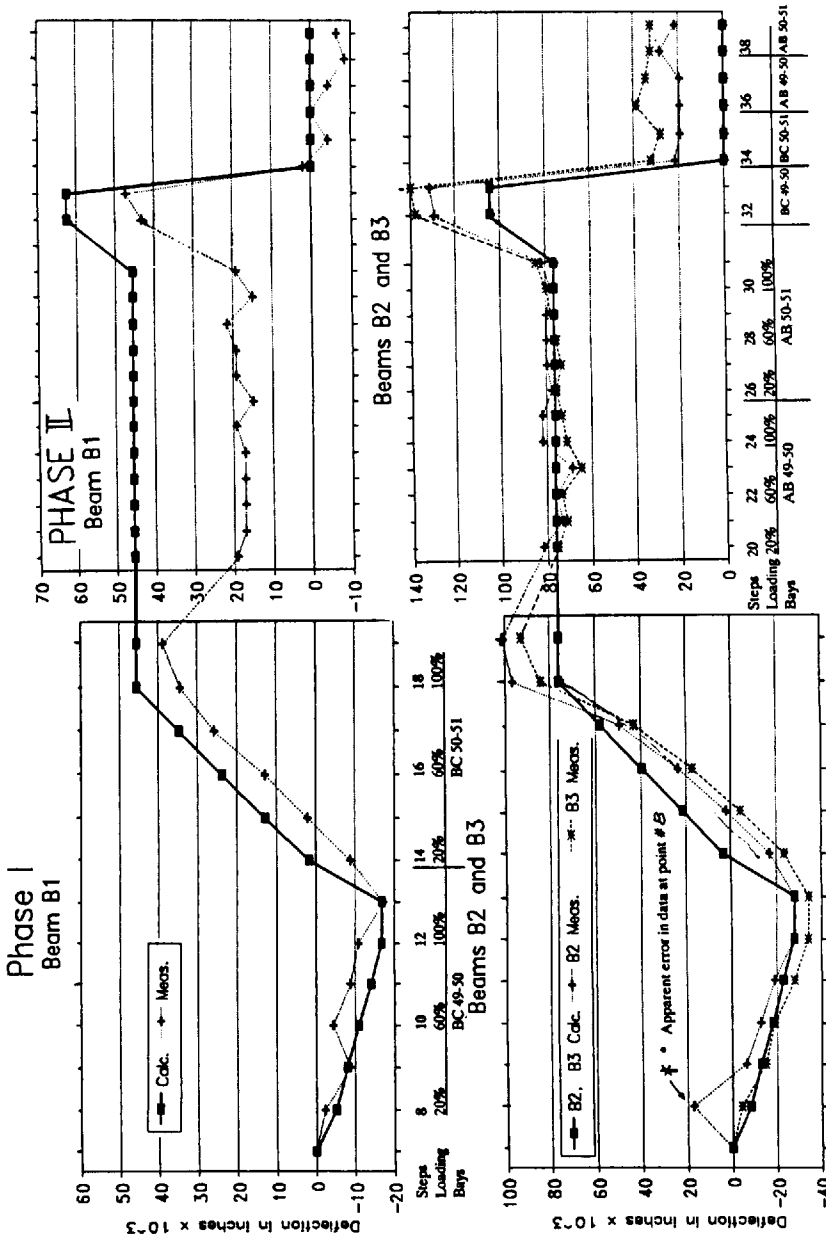


Fig. 4—Load deflection curves of beams B1, B2, and B3

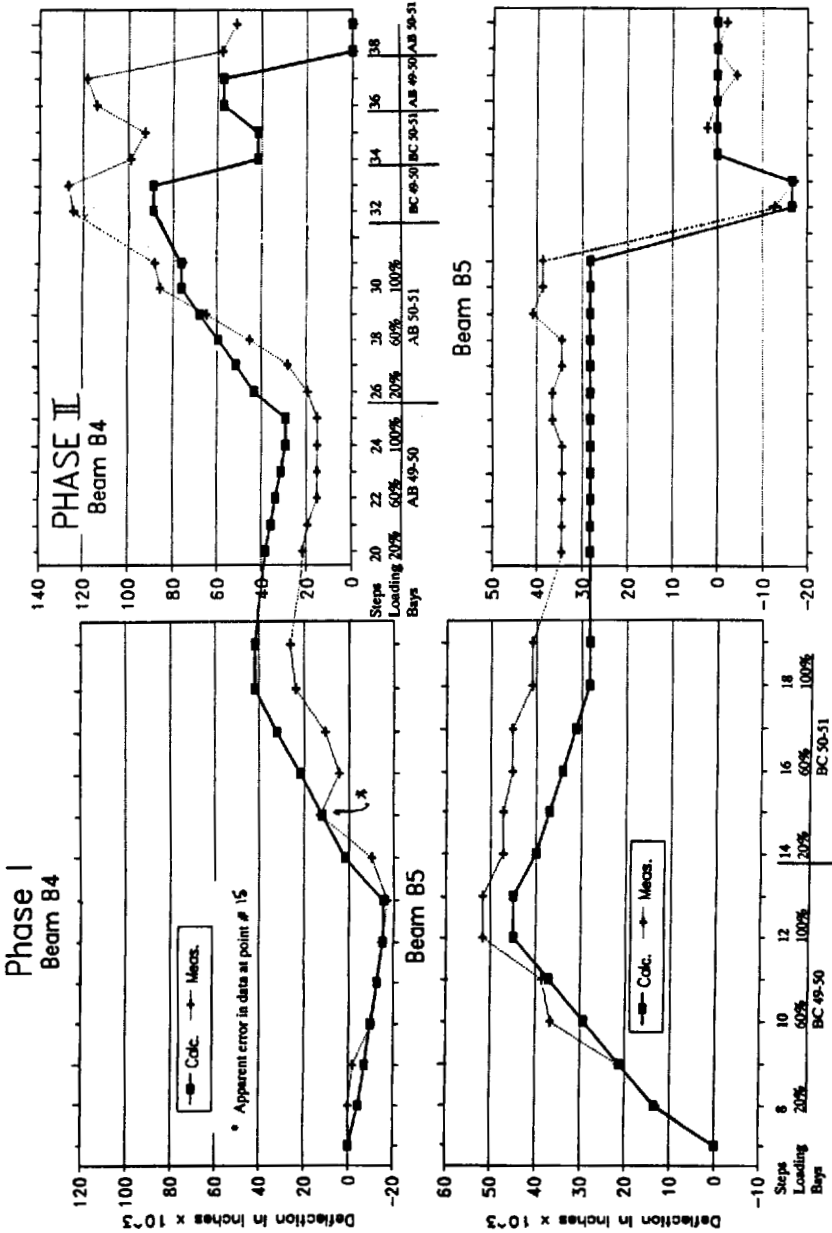


Fig. 5—Load-deflection curves of beams B4 and B5

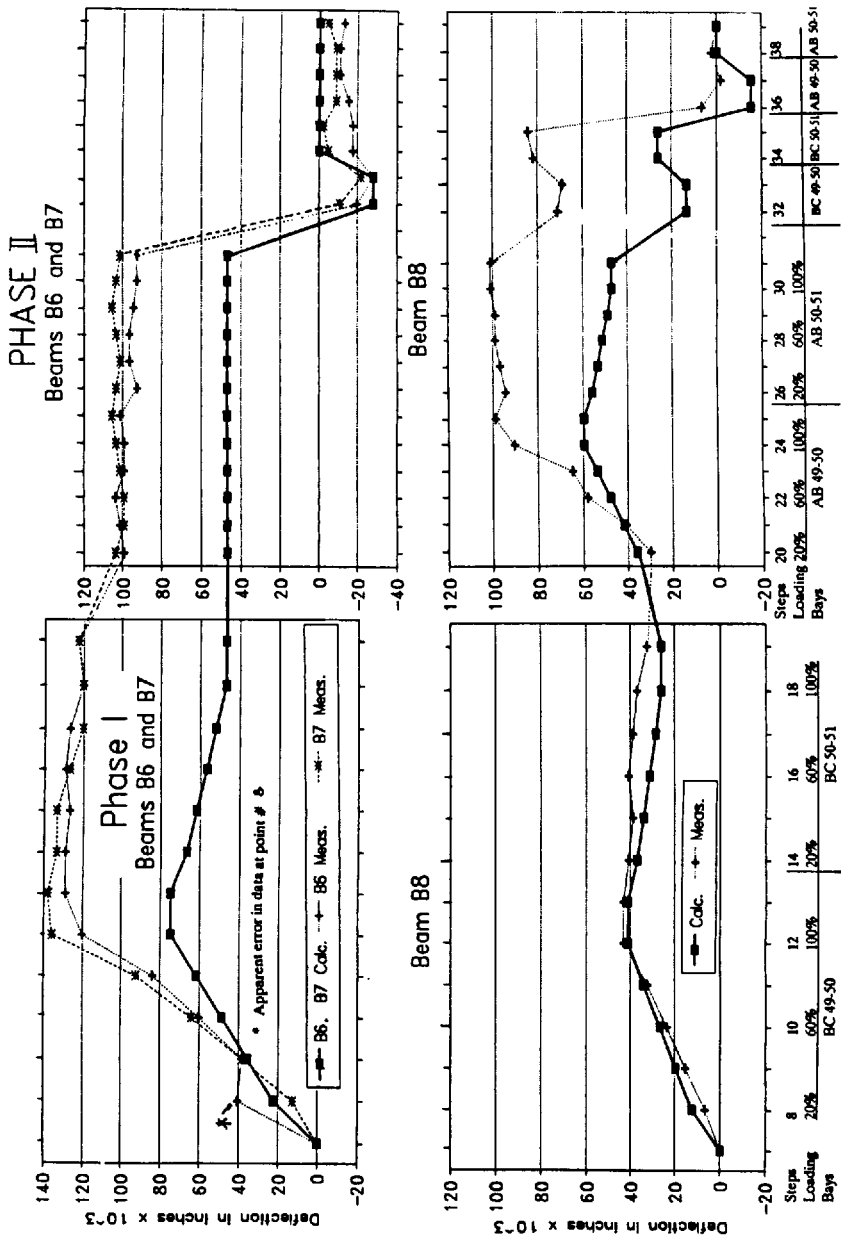


Fig. 6—Load-deflection curves of beams B6, B7, and B8

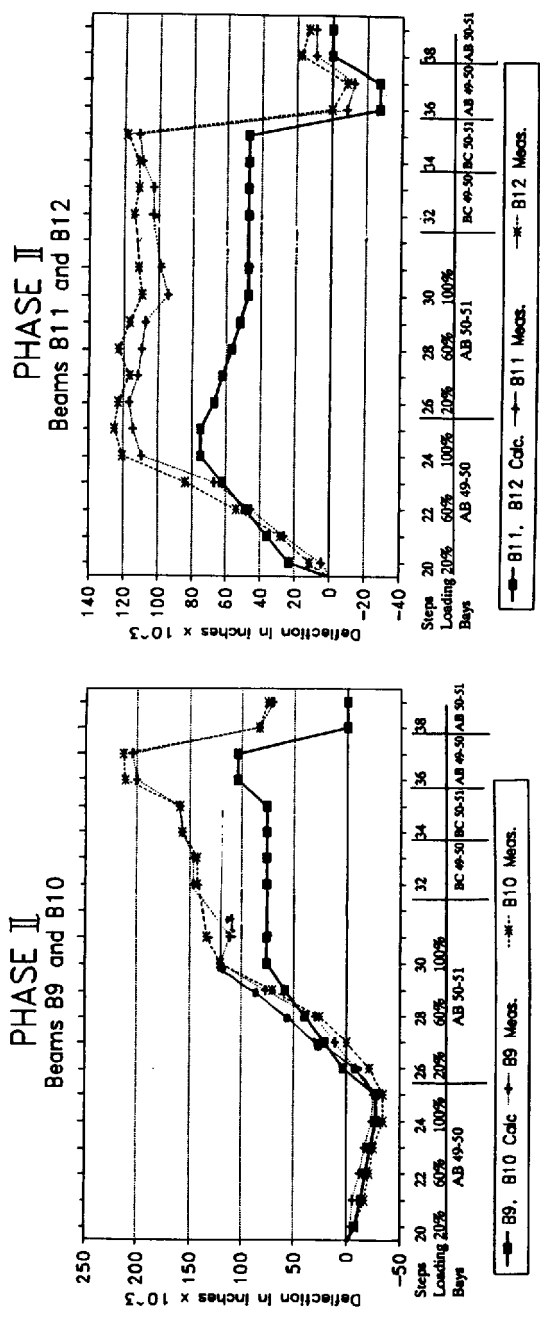


Fig. 7—Load-deflection curves of beams B9, B10, B11, and B12

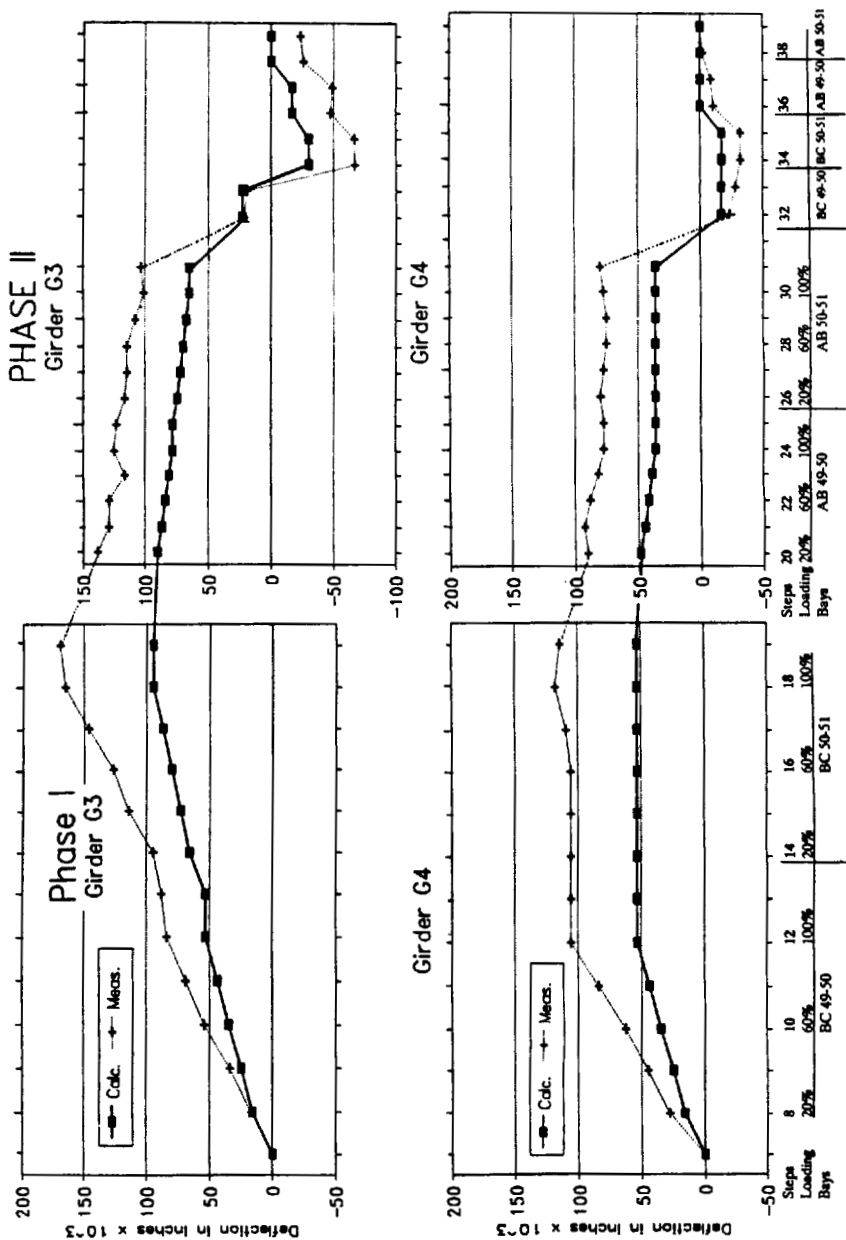


Fig. 8—Load-deflection curves of girders G3 and G4

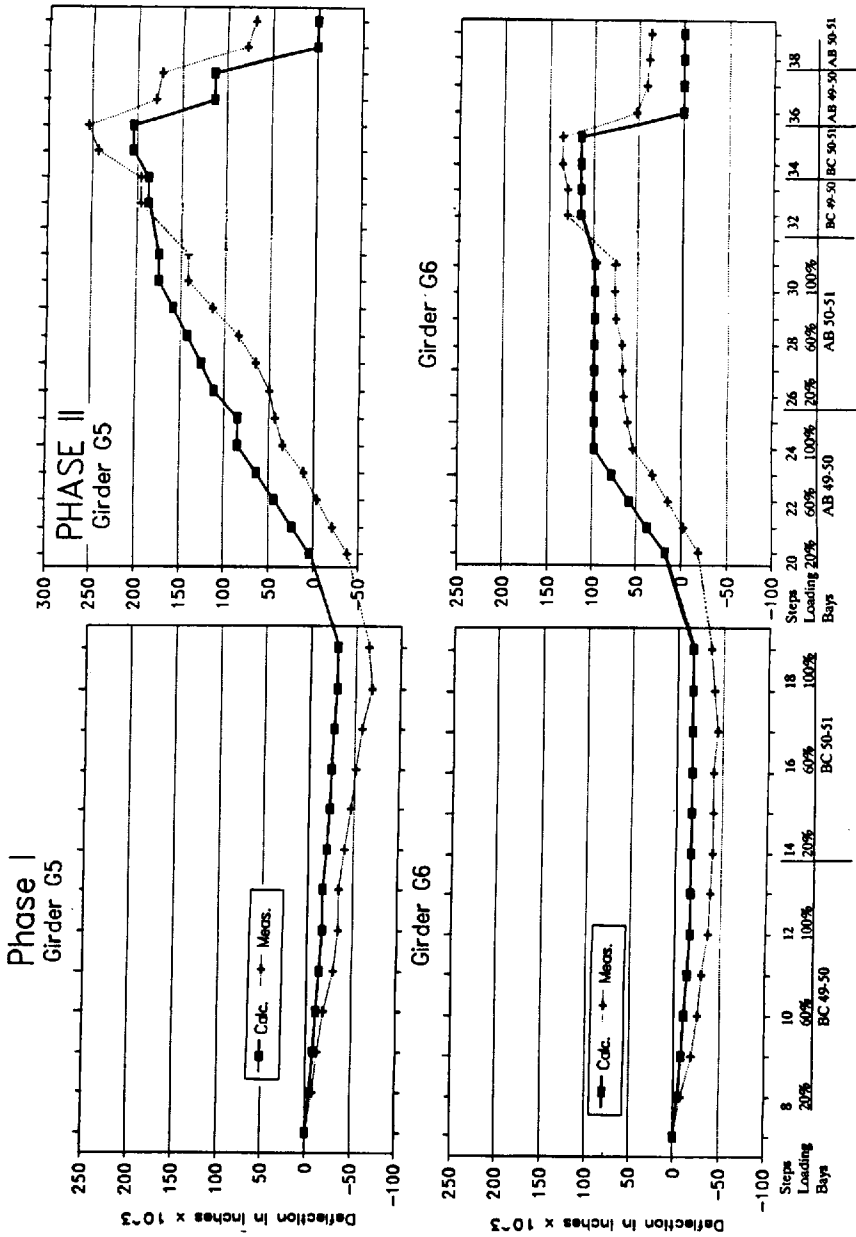


Fig. 9—Load-deflection curves of girders G5 and G6

SP 161-2

Deflection and Minimum Thickness Requirements in the CEB and ACI Codes

by H. Solanki

Synopsis: This paper presents American and European design provisions for the control of deflections. Both minimum and computed versus allowable deflections of CEB-FIP Model Code(MC 1990) and ACI Code(ACI 318-89) are considered. The methods in both Codes for predicting deflections, and applying span/depth-thickness ratio are evaluated in design examples. Based on the simplified method for computing the long term deflection, no significant difference is found between the Codes.

Keywords: Beams (supports); concretes; creep properties; deflection; flat concrete plates; flat concrete slabs; modulus of elasticity; moments of inertia; reinforced concrete; reinforcing steels; shrinkage; span-depth ratio; standards; structural analysis; structural design; thickness

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INTRODUCTION

Design provision for deflection and minimum thickness of concrete elements are available in various codes, but the approaches differ considerably. This paper compares the provisions of the Comité Européen du Béton Model Code CEB, 1990(1) and the American Concrete Institute (ACI 318-89)(2) as they relate to requirements for deflection and minimum thickness of concrete elements.

The criteria outlined for the deflection and minimum thickness of concrete elements are based on pure bending and deflection occurring at service load level under static conditions. They do not apply for loads with strong dynamic characteristics such as those due to earthquake, transient wind, and machinery operation.

CODE PROVISIONS

CEB - FIP Code:

The calculation of deformation is based on curvatures i.e. moment curvature relationship.

(a) Moment Curvature Relationship:

When more precise data such as properties of the materials, ambient conditions, loading and re-loading conditions, etc are not available, the mean curvature, instantaneous or long term, in any section of the element can be defined as follows (fig. 1).

$$\frac{1}{r} = \frac{1}{r_1} \quad \text{for State I (1a)}$$

$$\frac{1}{r} = \frac{1}{r_2} - \left[\frac{1}{r_{2r}} - \frac{1}{r_{1r}} \right] \beta_b \begin{bmatrix} M_r \\ M \end{bmatrix} \quad \text{for State II (1b)}$$

$$\frac{1}{r} = \frac{1}{r_y} - \left[\frac{1}{r_{2r}} - \frac{1}{r_{1r}} \right] \beta_b \begin{bmatrix} M_r \\ M_y \end{bmatrix} + \left[\frac{M - M_y}{2K_{III}} \right] \quad \text{for } M > M_r \text{ (1c)}$$

where,

$$K_{III} = M_u - M_y / (1/r_u - 1/r_y)$$

M = moment acting in the section;

M_y = yielding moment

M_u = ultimate moment

M_r = cracking moment; $M_r = W_1 \cdot f_{ct}$

W_1 = section modulus in State I (including the reinforcement)

$$1/r = (\epsilon_{s,m} - \epsilon_{c,m})/d$$

$\epsilon_{s,m}$ = the mean tensile strain in the steel

$\epsilon_{c,m}$ = the mean compressive strain in the concrete at the extreme fibre

$1/r_y$ = curvature corresponding to M_y

$1/r_u$ = curvature corresponding to M_u

$f_{ck} = 0.7f_{ctk}$ if local deformations are to be considered

= f_{ctm} if the effects of an overall deflection are to be considered

f_{ck} = characteristic strength

f_{ctk} = characteristic tensile strength

f_{ctm} = mean tensile strength

d = effective depth

Table 1 shows the values of f_{ck} , f_{ctk} , f_{ctm} .

$1/r_1, 1/r_{1r}$ = curvature in State I (uncracked section) corresponding to the action of M and M_r respectively

$1/r_2, 1/r_{2r}$ = curvature in State II (cracked section) naked corresponding to the action of M and M_r respectively

$\beta_b \geq \beta_1 \cdot \beta_2$

β_1 = coefficient characterizing the bond quality of the reinforcing bars

= 1.0 for high bond bar

= 0.5 for smooth bar

β_2 = coefficient representing the influence duration of application or of repetition of loading

= 0.8 at first loading

= 0.5 for long-term loading or for a large number of load cycles

At time t the mean curvature ($1/r$) can be obtained by the sum of the initial (instantaneous) curvature $1/r_0$ and the increment of the curvature ($\Delta(1/r)$) due to time dependent effects (creep and shrinkage of concrete, relaxation of prestressing steel):

$$\frac{1}{r} = \frac{1}{r_0} + \left[\Delta \frac{1}{r} \right]$$

$1/r_0$ = initial (instantaneous) curvature

$(\Delta(1/r))$ = curvature due to the time dependent effects

$1/r$ = mean curvature at time t

The mean curvature at time t_2 due to permanent loads(g) (introduced at time $t_1 < t_2$) and instantaneous reloading/unloading(q) is given by the following relation (Fig. 1 and Fig.2)

$$\frac{1}{r(g+q)} = \frac{1}{r(g)} + \frac{1}{r_0(g+q)} - \frac{1}{r_0(g)} \quad (2)$$

$1/r(g+q)$ = curvature at time t_2 due to g and q .

$1/r(g)$ = curvature at time t_2 due to g .

$1/r_0(g+q)$ = instantaneous curvature at time t_2 due to g and q .

$1/r_0(g)$ = instantaneous curvature at time t_2 due to g .

The deflection can be calculated from Eqs. 1a, 1b, 1c or 2 by applying appropriate procedures, such as the principle of virtual work or double integration.

(b) Simplified Method

For building members, long term deflections excluding the effect of shrinkage can be evaluated by the following relations based on a bilinear relationship between load and deflection:

$$a = (1 + \Phi) a_c \quad \text{for } M_d < M_r \quad (3a)$$

$$a = (h/d)^3 \eta (1 - 20 \rho_{cm}) a_c \quad \text{for } M_d > M_r \quad (3b)$$

with cracking moment $M_r = W_1 f_{ct}$

where,

a_c = the elastic deflection calculated with the rigidity $E_c I_c$ of the gross section (neglecting the reinforcement)

M_d = the bending moment at mid span of a beam or a slab, or at the fixed end of a cantilever under frequent actions,

h = nominal size of member

d = effective depth of section

ρ_{tm} = the geometrical mean percentage of tensile reinforcement

$$\rho_{tm} = \rho_a (l_a/l) + \rho (l_o/l) + \rho_b (l_b/l)$$

ρ_a, ρ_b = percentage of tensile/compressive reinforcement at the left and the right support respectively.

ρ = percentage of tensile reinforcement at the M_{max} section
(An estimation of the length l_a, l_b is generally sufficient see Fig. 3)

ρ_{tm} = the geometrical mean percentage of compressive reinforcement

Φ = creep coefficient

η = a correction factor (see Table 2), which includes the effects of cracking and creep.

$$= \Phi_{RH} \beta(f_{cm}) \beta(t_o)$$

with

$$\Phi_{RH} = (1 + (1 - RH/RH_0)/0.46(h/h_c)^3)$$

$$\beta(f_{cm}) = 5.3/(f_{cm} / f_{cmo})^3$$

$$\beta(t_o) = 1/(0.1 + (t_o / t_1)^{0.2})$$

where,

$$h = 2 A_c / u$$

f_{cm} = mean compressive strength of concrete at the age of 28 days(MPa)

$$= f_{ck} + \Delta f$$

where $\Delta f = 8 \text{ MPa}$ and f_{ck} is the characteristic compressive strength

$$f_{cmo} = 10 \text{ MPa,}$$

RH = relative humidity of the ambient environment (%)

$$RH_0 = 100\%$$

h = notional size of member (mm), where A_c is the cross section and u is the perimeter of the member in contact with the atmosphere

$$h_0 = 100 \text{ mm}$$

$$t_1 = 1 \text{ day}$$

$$t_o = \text{age of concrete at loading (days)}$$

(C) Span- Depth Ratio

A check on the deflection may not be necessary where the span/depth ratio of reinforced concrete elements do not exceed a certain value.

For simple building elements calculating the deflection is not required if the span/depth ratio l/d , are respected as follows

$$l/d \leq 1 / \sqrt[3]{\delta \eta (l/a)_{lim}} \quad (4a)$$

$$\delta = a_c h^3 / l^4$$

$$(l/a)_{lim} = \text{span / deflection limiting value} \\ = 300$$

a_c = the elastic deflection calculated with rigidity $E_c I_c$ of the homogeneous sections (neglecting the reinforcement)

The above formula (Eq. 4a) is primarily developed for $A_s/(bd) \leq 0.6\%$ (slabs).

The Deflection Limits for CEB Code are:

1. The deflection may cause damage to partitions, to member attached to, or in contact with the member considered, and to fixtures or finishes, if the calculated deflection occurring after construction of the elements could suffer damage is excessive. A limit of deflection is span/500.
2. The appearance and general utility of the structure may be impaired when calculated deflection of a beam, slab or cantilever subjected to quasi-permanent loads exceeds span/250.

The deflection is assessed relative to the supports.

The limiting deflections given above are derived from ISO 4356.

For reinforced concrete flexural elements, without axial force, the following formula may be applied

$$l/d < \lambda$$

$$= \lambda_o k_T k_f (400/f_{yk}) \quad (4b)$$

where λ_o = constant from Table 3

k_T = 1.0 for flanged section with flange to web width lower than 3.0

= 0.8 for ratio higher than 3.0

k_f = $7/l \leq 1$, with l in meter,

l = span in meter

f_{yk} = yield stress of the reinforcing steel in MPa.

ACI Code

Two methods are given in the ACI Code for controlling deflections of one way and two way flexural systems.

The deflection may be controlled by means of minimum thickness or directly by limiting computed deflections.

(a) Minimum Thickness

The deflections of beams and one way slabs (non-prestressed) supporting loads commonly found in buildings are normally satisfactory when the minimum thickness of the Code are met or exceeded (Tables 4).

For slabs with beams spanning between the supports on all sides, the minimum thickness shall be as follows:

For α_m greater than 0.2 but not greater than 2.0, the thickness shall not be less than

$$h = \frac{l_n (0.8 + f_y/200,000)}{36 + 5\beta[\alpha_m - 0.2]} \quad (5a)$$

but not less than 5 in.

For α_m greater than 2.0 the thickness shall not be less than

$$h = \frac{l_n (0.8 + f_y/200,000)}{36 + 9(\beta)} \quad (5b)$$

but not less than 3.5 in.

α_m = average value of the ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerline of adjacent panel on each side of beam.

β = ratio of clear spans in long to short direction of two-way slabs.

All discontinuous edges, an edge beam shall be provided with a stiffness ratio α not less than 0.80 or the minimum thickness required by Eqs. 5a or 5b shall be increased by at least 10 percent in the panel with discontinuous edge.

Also the minimum thickness must meet the following values.

- | | |
|--|---------|
| 1. Slabs without beams or drop panels
(also for $\alpha_m < 2.0$) | 5.0 in. |
| 2. Slabs without beams but with drop panels | 4.0 in. |
| 3. Slabs with beams on all four edges with
a value of $\alpha_m \geq 2.0$ | 3.5 in. |

(b) Maximum Permissible Computed Deflections

The initial or short term deflection of a beam is calculated by integrating the curvature along the length of the beam. For an elastic beam the curvature, Φ , is calculated as $\Phi = M/EI$, where EI is the flexural stiffness of the cross section. In the ACI Code, the flexural stiffness is based on the effects of cracking and reinforcement.

Unless the stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity E_c for concrete and with the effective moment of inertia as follows but not greater than I_g (2,3).

$$I_e = \left\{ \frac{M_{cr}}{M_D} \right\}^3 I_g + \left[1 - \left\{ \frac{M_{cr}}{M_D} \right\}^3 \right] I_{cr} < I_g \quad (6)$$

where

$$M_{cr} = f_r I_g / y_t$$

$$f_r = 7.5 \sqrt{f'_c} \text{ for normal weight concrete}$$

$$M_D = \text{maximum service load moment (unfactored moment)}$$

y_t = distance from centroid to extreme tension fiber

f'_c = cylinder compressive strength

I_g = moment of inertia of the homogeneous section (neglecting the reinforcement)

E_c = modulus of elasticity of concrete

I_{cr} = moment of inertia of cracked transformed section

(c) Long-Term Deflections

Additional long term deflection resulting from creep and shrinkage of flexural members (normal weight or light weight concrete) shall be determined by multiplying the immediate deflection caused by the sustained load considering by the factor

$$\lambda = \frac{\xi}{1 + 50 \rho'} \quad (7)$$

ρ' = compression reinforcement ratio at mid span for simple and continuous spans, and at support for cantilever

ξ = time dependent factor for sustained loads

5 years or more	ξ	=	2.0
12 months		=	1.4
6 months		=	1.2
3 months		=	1.0

Deflection computed in accordance with the ACI Code shall not exceed the limit of span/240.

DISCUSSIONS AND CONCLUSIONS

DISCUSSIONS

1. The CEB Code limits calculated deflection to span/500 if the the construction of the elements could suffer excessive damage. ACI Code limits deflection under similar conditions to span/480.

2. ACI Code limits the deflection to span/240 for roof or floor construction supporting or attached to non-structural elements not like to be damaged by large deflection. CEB Code limits this type of deflection to span/250.
3. Both Codes provide a simplified method for the long-term deflection computations but the approaches are different. CEB Code proposes a simpler version than ACI Code because
 - (a) The ACI Code considers the long term effect by using the long term multiplier, λ , and applies it to the short term/initial deflection. The total long term deflection is computed by adding the two deflection values or adding 1.0 to the multiplier, λ . The CEB Code computes the long term deflection by the multiplication of various variables to the elastic deflection,
 - (b) The ACI Code presumes the maximum long term deflection is three times the short term/initial deflection for member without compression reinforcement while the CEB Code does not set any limits, and
 - (c) For continuous structures, the ACI Code permits using I_e at mid-span for prismatic members. For non prismatic members it permits averaging I_e at mid span at end span sections. The CEB Code computes compression/tension reinforcement ratios and based on the net total ratio it computes the multiplier η .
4. The CEB Code considers the span/effective depth ratio to compute the minimum thickness of the structural elements. ACI Code uses the span/thickness ratio for computation of minimum thickness of the structural elements. The ACI Code approach is somewhat simpler than CEB Code because it computes directly the minimum thickness.
5. ACI Code limits span to thickness ratio for non-prestressed beams and one way slabs in Table 4 (ACI Code Tables 9.5(a)). The limits of span to thickness ratio for two-way slabs in Table 5 (ACI Code Table 9.5(c)) considers the reinforcement yield strength. The ACI Code does not distinguish between concrete highly stressed and concrete lightly stressed. The CEB Code provides two basic limits of span to effective depth ratio categorized by concrete highly stressed and concrete lightly stressed (Table 3). These values must be multiplied by various variables (Eqs. 4a and 4b).

6. When ACI Code eqs. 6 and 7 and CEB Code eqs. 4a and 4b are used to compute the long term deflection of a flat plate, the deflection values do not differ significantly (example 1). In case of the long term deflection of a beam, the ACI Code values differ significantly, when they are compared with the CEB Code value. The CEB Code result is found greater than the ACI Code results (example 2). Based on the simplified method for computing the long- term deflection insignificant difference is found between the two Codes (see Tables 6 and 7).
7. When span to effective depth or thickness ratio is considered for a flat plate, the ACI and CEB Codes give similar results (example 1). In case of a beam, the ACI Code value falls between the CEB Code values (example 2).

CONCLUSIONS

1. Both Codes provide a simplified version for computing the deflection and the minimum thickness of the reinforced concrete elements, but the approaches differ significantly.
2. The maximum allowable deflection values slightly differ in both Codes. The CEB Code allows span/500 and span/250, while the ACI Code allows span/480 and span/240 respectively, probably due to rounding values in English versus SI units.
3. In the ACI Code, tension stiffening is accounted for interpolation between gross and cracked moments of inertia of the cross section using an empirical interpolation developed by Branson (3).
4. In the CEB Code, strain in cracked sections can be determined by two analyses. 1) Ignoring cracking (State I) and 2) assuming that the concrete cannot carry any stress in tension (State II). The value of strain can be determined in the two stages and the values for actual conditions can be obtained by interpolation between the two analyses.
5. Both Codes provide a general guidance on deflection limits to the designer but the designer should choose the limits which he or she believes to be appropriate taking into account the nature of the structure and the intended use.
6. In the ACI Code, a simple equation similar to CEB Eq. 4b should be considered. This would help to the designer to include several variables such as longer span, higher steel ratio, etc. The equation could be further modified to take into account the concrete strength greater than 4000 psi (27.8 MPa.).

ACKNOWLEDGMENTS

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TABLE 1 — TENSILE STRENGTH FOR VARIOUS CONCRETE GRADES (MPa) (CEB-FIP MC CODE 1990 TABLE 2.1.2)

Concrete Grades	C12	C20	C30	C40	C50	C60	C70	C80
f_{ck}	12	20	30	40	50	60	70	80
f_{ctm}	1.6	2.2	2.9	3.5	4.1	4.6	5.1	5.6
$f_{ctk,min}$	1.1	1.5	2.0	2.4	2.8	3.1	3.5	3.8
$f_{ctk,max}$	2.1	2.9	3.8	4.7	5.4	6.1	6.8	7.4

In CEB Code, the term "tensile strength" refers to the axial tensile strength f_{ct} determined in accordance with RILEM CPC7.

TABLE 2 — CORRELATION FACTOR η FOR ESTIMATION OF DEFLECTIONS (CEB-FIP MC CODE 1990 TABLE 7)

ρ_{tm} [%]	0.15	0.20	0.30	0.50	0.75	1.00	1.50
η	10	8	6	4	3	2.5	2

The mean percentage ρ_{tm} of tensile reinforcement should be based on the bending moment diagram (Fig.3).

TABLE 3 — VALUES OF λ_o FOR RC MEMBERS WITHOUT AXIAL COMPRESSION (CEB-FIP MC CODE 1990 TABLE 7.5.2)

Structural System	Concrete highly stressed	Concrete lightly stressed
1. Simply supported beams, one or two way spanning simply supported slabs.	18	25
2. End span of a series of continuous spans two-way spanning slab continuous over one long side.	23	32
3. Interior span of beam or one way or two way spanning slab.	25	35
4. Slab supported on columns without beams (Flat slab), verification effectuated on the on the longer span.	21*	30*
5. Cantilever	7	10

*These value should be verified.

Note: Members where the concrete is lightly stressed are those where $\rho < 0.5\%$ ($\rho = A_s/bd$). It may normally be assumed that slabs are lightly stressed.

If reinforcement ratio is known, (λ_o)-values between the "highly stressed" and "lightly stressed" values in Table (CEB Table 7.5.2) may be obtained by interpolation assuming the "lightly stressed" values to correspond to $\rho = 0.5\%$ and the "highly stressed" to $\rho = 1.5\%$.

TABLE 4 — MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED* [ACI CODE TABLE 9.5(a)]

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Canti-lever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

*Span length ℓ is in inches.

Values given shall be used directly for members with normal weight concrete ($w_c = 145$ pcf) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:

(a) For structural lightweight concrete having unit weights in the range 90-120 lb per cu ft, the values shall be multiplied by $(1.65 - 0.005 w_c)$ but not less than 1.09, where w_c is the unit weight in lb per cu ft.

(b) For f_r other than 60,000 psi, the values shall be multiplied by $(0.4 f_r/100,000)$.