

PAPER SP 18-1 The torsional actions and reactions which are generated in a structure are a function of its configuration and the distribution of the dead and applied live loads. This paper discusses aspects of the torsional problem with emphasis on over-all structural action, as opposed to the consideration of individual members only. Specific reference is made to spandrel beams, edge beams in shells, an orthotropic shell, certain grid systems and curved prestressed concrete bridges. The interrelationships between the applied live loads and torsion are discussed. The need for further research and the necessity to include provisions for torsion into engineering codes of practice is emphasized.

Aspects of Torsion in Concrete Structure Design

By K. G. TAMBERG

□ Torsion is becoming an increasingly important factor in structural design. Refinements in analysis, the greater use of novel structural forms by architects and the employment of ultimate strength rather than working stress design, have required explicit attention to torsion. Consideration of the combined effects of flexure, shear, and axial load is commonplace under present design methods, but torsion has been treated mostly informally and imprecisely. For example, many engineers plan structural frames in such a manner as to minimize the effects of torsion while others allow for torsion in a discretionary manner. It is, therefore, not surprising that some distress has been experienced.

A recent survey of structures conducted in the United States and Canada by the Portland Cement Association for ACI Committee 438 uncovered some classic cases of torsional distress. Distress occurred in connection with a variety of construction details, and the degree varied from minor to extensive cracking of a serious nature. The maximum cost of repair work encountered was over \$50,000 for one structure whose value is in excess of a million dollars. This does not include the legal fees incurred as a result of litigation, the inconvenience and cost of lost space during repairs, or the damage to the professional reputation of the structural engineer. The need to account properly for torsion is thus emphasized.

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Most discussions of combined loadings acting on structural members begin with the assumption that these forces are known. It is clear, however, that the whole structure must be analyzed as a unit in order to determine these forces. The interaction of the torsional and flexural stiffnesses inherent in a structure, and the restraints imposed by component parts upon one another must be considered. Torsion results from the monolithic character of cast-in-place concrete construction, and also occurs when precast elements are employed.

On the other hand, the torsional reactions present in a structure may prove beneficial rather than detrimental in certain cases. The extra strength of structures attributable to torsional stiffness may be best mobilized when applied loads act eccentrically. Design loadings presently in use do not always take advantage of this fact, which is particularly true for bridge construction. If, however, torsional stresses govern the design, a proper assessment of the eccentricity of the applied forces is, needless to say, imperative.

SPANDREL BEAMS

Fig. 1-1 shows cross-sections of spandrel beams which are frequently used in construction. Details a, b and c are suitable for use when precast slab or beam elements are employed, or when the member framing into the spandrel is cast later. Torsion is introduced into the spandrels by the eccentric action of the vertical forces V (details a and b), which are due to members supported as indicated in detail c. Details a and c are used at the

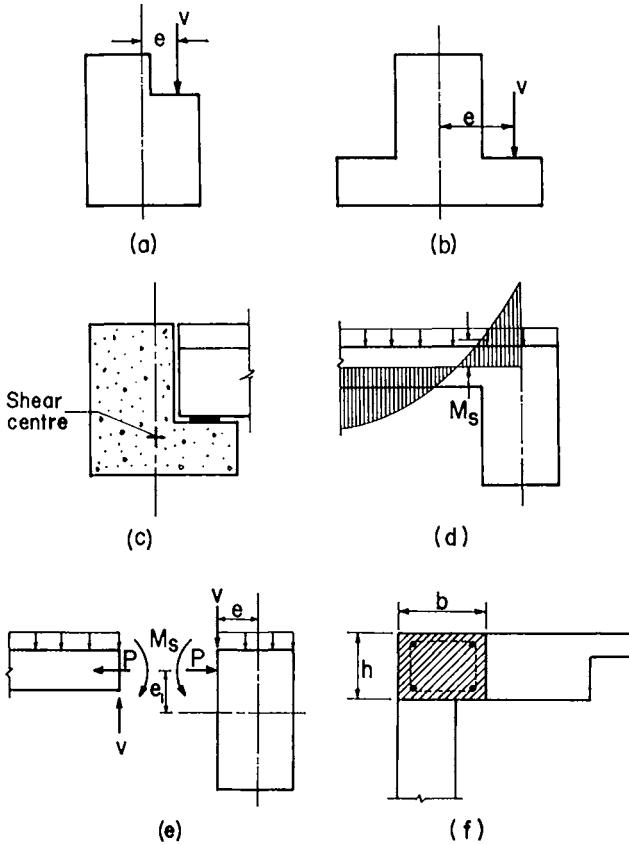


FIG. 1-1 SPANDREL BEAMS

discontinuous edges of floor systems. Detail b is generally used to support precast elements along an interior line of columns, in which case torsional moments may arise because of the presence of asymmetrical loading during erection. It may be said categorically that members under vertical load will be subjected to torsion in addition to shear and bending unless the line of action of the loads passes through the shear center of the cross-section employed. Design procedures should, of course, take this fact into account. It is noted that the shear center does not coincide with the centroid of the cross-section except for bisymmetrical or anti-symmetrical sections.

Cast-in-place spandrel beams which are integral with the floor slab present more complicated problems in design. Detail d shows the cross-section of a spandrel beam cast monolithic with the slab. The ordinate of the typical associated moment diagram,

per unit length of the spandrel, is M_S at the inside face of the beam. Detail e shows the resultant forces acting at the vertical interface of slab and beam, per unit length of spandrel. A strict evaluation of these forces requires that the whole structure be treated as a space frame; alternatively, some idea of the magnitudes of the forces involved may be obtained by use of approximate methods.¹⁻⁵ If the quantity $M_S + V_e - P_{e1}$ is known along the length of a spandrel, the resulting torsional moments acting at any cross-section may be evaluated. If concentrated wall loads are present, the torsional moments are modified accordingly.

Both creep and cracking will change the magnitude of the quantity $M_S + V_e - P_{e1}$ arrived at on the basis of an elastic analysis. The presence of creep will result in the reduction of the transverse moment M_S . The state of knowledge with regard to creep in reinforced concrete, however, does not allow a precise analytical evaluation of the resulting change. The presence of cracks due to torsion results in a pronounced reduction of the torsional stiffness of a spandrel. The significance of the consequent redistribution of moments in the structural frame should be investigated, and the advisability of permitting cracks to occur where architectural appearance is not a consideration must be established on the basis of serviceability criteria.

Shoobred and Holland,⁶ employing elastic analysis, have indicated that the influence of the force P in reducing the torsional moments applied to a spandrel is of minor consequence.

It has become a rather widespread architectural practice to use spandrel girders that are wide and shallow, as in detail f of Fig. 1-1, where the width of the spandrel beam is greater than that of the column. The consequent misalignment of the beam and column center lines not only complicates the calculation of torsional moments, but raises questions regarding both the transfer of torsional moments into the columns and the effective width b of the cross-section in torsion.

EDGE BEAMS OF SHELLS

In shells, the presence of edge beams affects the magnitude of the transverse moments, which, in turn, influence the transverse reinforcement and the thickness of the shell. Economy in shells requires the most efficient distribution of the applied forces. For example, the importance of edge beams for cylindrical shells is demonstrated in Fig. 1-2, which is due to Kemp.* The example considered, representative of a number of shells encountered in

*Dr. E. L. Kemp, West Virginia University, by private correspondence.

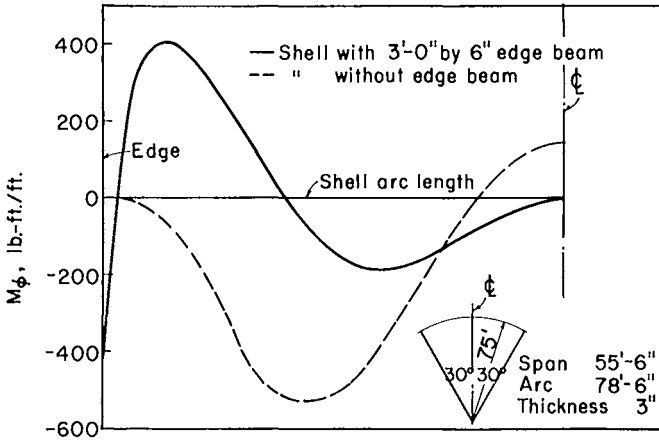


FIG. 1-2 TRANSVERSE BENDING MOMENTS FOR A BARREL SHELL

practice, is a 3-in. thick barrel shell of 55-ft span; the magnitude and distribution of transverse moments are compared for the case of no edge beams and for the case of edge beams 3 ft deep and 6 in. wide. Totally different moment distributions pertain for the two cases, and the absolute maximum moment is reduced by more than 20 percent when edge beams are provided.

Some edge beams thus may be subjected to torsional moments unless care is taken to reduce or eliminate the torsion by proper alignment of the shell reactions.

Fig. 1-3, for example, shows a typical cross-section through a tension ring of a circular dome which is assumed to be supported on a line of columns cast monolithically with the ring. The forces T_V and T_H , respectively, are the vertical and horizontal components of the force T , per unit length of tension ring, which the shell exerts on the ring. At the interface of ring and shell, there are moments M , per unit length of ring. Torsion in the ring is produced by the eccentric action of the forces T_H and T_V , moment, M , and the self-weight of the ring. If prestressing is introduced, the magnitude of the net torque is modified by the vertical and horizontal components P_V and P_H of a general prestressing force P , acting per unit length of edge beam. It follows that the net torsional moment introduced at

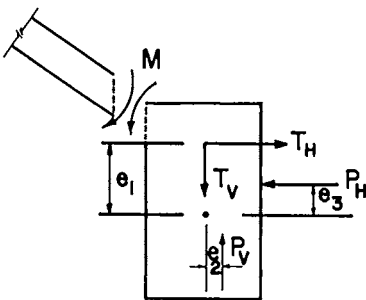


FIG. 1-3 TENSION RING OF SPHERICAL DOME

any section is then primarily a function of M , the forces T and P and their eccentricities, the distributed vertical load along the beam center line, the flexibility and spacing of columns and the diameter of the dome. The magnitude of these torsional moments may be effectively controlled by proper location and dimensioning of the edge support, and by the magnitude and location of the prestressing.

ORTHOTROPIC SHELLS

The torsional analysis of members with open, thin-walled cross-sections, such as cylindrical shell structures, is more complex in principle than the analysis of ordinary linear members, such as rectangular beams. Fig. 1-4 illustrates plain and monotropic cylindrical shells, both of which will deform excessively when subjected to twisting about the longitudinal axes because of low torsional stiffness. The addition of transverse ribs to the

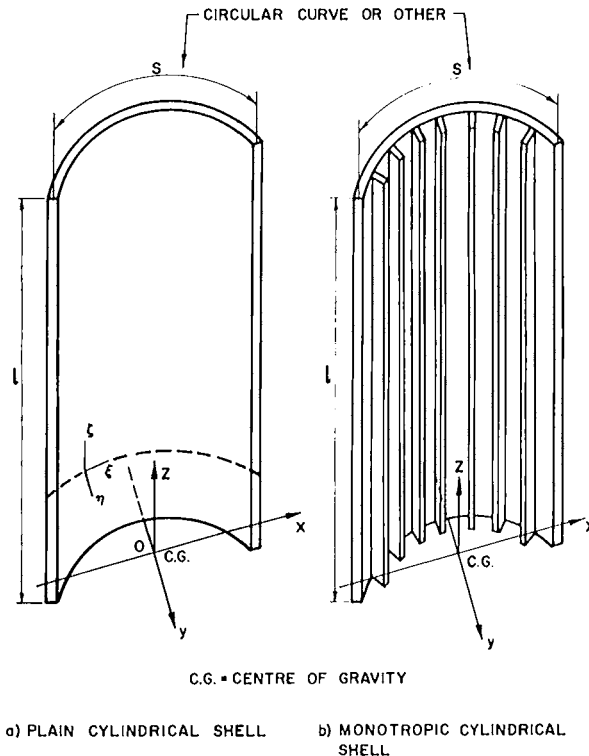


FIG. 1-4 CYLINDRICAL SHELLS WITH CARTESIAN COORDINATE SYSTEM THROUGH CENTER OF GRAVITY OF SECTION

monotropic shell converts it into an orthotropic shell and produces restraints which force warping of the cross-section to occur. This warping, which does not occur in the plain and monotropic shells, is responsible for substantially increased torsional stiffness and, moreover, makes it feasible to design entire structures of a shell-like shape. These structures may be subjected to twist by resultant transverse wind or earthquake forces which do not act through the flexural center.

A particularly interesting example involving both of these features is the Toronto City Hall (Fig. 1-5), selected to present a realistic notion of the stress and deflection characteristics of such a structure against the background of the main analytical expressions.⁷

The magnitudes of the stresses arising from torsion, bending and shear due to self-weight, superimposed live load and wind forces, are compared at the third-floor (podium roof) level (Fig. 1-6). A typical cross-section through the East Tower is shown in Fig. 1-7. The buttresses, walls, and columns are the longitudinal ribs, and the concrete floor slabs are the transverse ribs. The over-all structural action of the building must be considered when establishing the stress distributions involved. For design calculation purposes, the tower is treated as a cantilever, and may be considered fixed at the third-floor level (see Reference 7). The general theory of shells,^{8,9} modified for the special boundary conditions resulting from the presence of the diaphragms, must be used for rational determination of the internal stresses, strains, and deformations resulting from the action of external forces.

The following basic assumptions pertain:

- (1) Plane sections normal to the middle surface of the shell remain plane.
- (2) Stresses acting in a direction perpendicular to the generatrix of the shell may be neglected.
- (3) The middle surface of the shell is inextensible in the transverse direction.

The basic eighth-order differential equation which establishes the relationship between the external forces and the internal stresses and strains for a shell is:

$$\Omega \nabla \phi(z, s) + \frac{12}{h^3} \cdot \frac{\partial^4 \phi(z, s)}{\partial z^4} = P(z, s) \quad (1)$$

where

$$\Omega = \frac{\partial^2}{\partial s^2} \left(R \frac{\partial^2}{\partial s^2} \right) + \frac{\partial}{\partial s} \left(\frac{1}{R} \cdot \frac{\partial}{\partial s} \right)$$



FIG. 1-5 TORONTO CITY HALL

$\phi(z, s)$ = stress function

h = thickness of shell

z and s = coordinates of a point on the middle surface of the shell
(z is directed along the generatrix, s is directed along the contour of the shell)

$P(z, s)$ = a given load function expressed through the components P_z , P_s , P_n , of the vector of intensity of external forces acting on the shell

$R = R(s)$ = radius of the middle surface of the shell

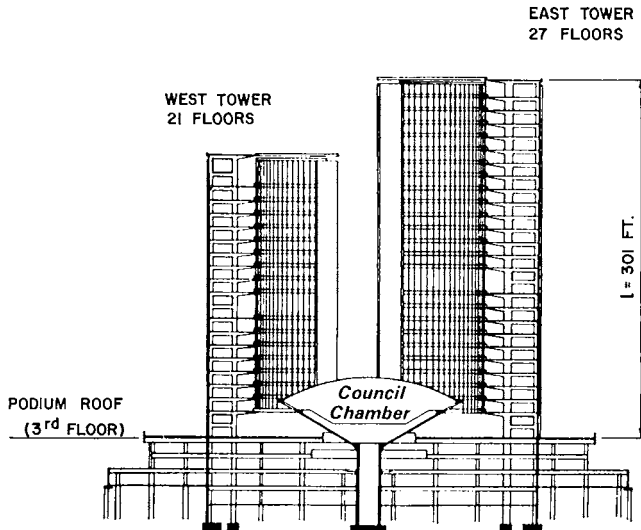


FIG. 1-6 EAST-WEST SECTION OF CITY HALL

The stress function $\phi(z, s)$ which satisfies the differential Eq. (1) and the boundary conditions provides the solution. If the stress function $\phi(z, s)$ is known, the longitudinal normal stresses $\sigma(z, s)$ and transverse bending moments $G(z, s)$ may be determined from the following formulae:

$$\sigma(z, s) = \frac{12}{h^3} \cdot \frac{\partial^2 \phi(z, s)}{\partial z^2} \quad (2)$$

$$G(z, s) = \Omega \phi(z, s) \quad (3)$$

Now, it has been found that Eq. (1) is equivalent to a system of two symmetrical differential equations:

$$\frac{\partial^2 [\sigma(z, s) h]}{\partial z^2} + \Omega G(z, s) = P(z, s) \quad (4)$$

$$\Omega \sigma(z, s) - \frac{12}{h^3} \cdot \frac{\partial^2 G(z, s)}{\partial z^2} = 0 \quad (5)$$

Eq. (4) may be derived directly by considering the equilibrium conditions of a shell element.

Eq. (5) may be derived directly by considering the compatibility of deformation of the middle surface of the shell; namely, from the condition that

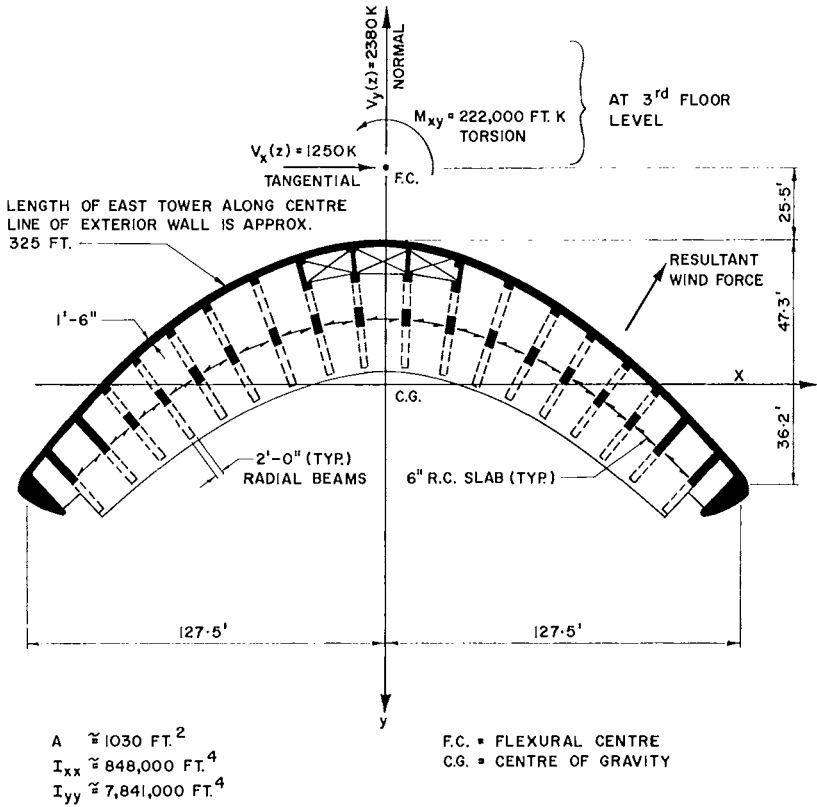


FIG. 1-7 CROSS-SECTION THROUGH EAST TOWER

$$\Omega \epsilon(z, s) + \frac{\partial^2 \chi}{\partial z^2} = 0 \quad (6)$$

and from the well-known equations of elasticity:

$$N(z, s) = E h \epsilon(z, s); \quad G(z, s) = -\frac{E h^3}{12} \chi(z, s) \quad (7)$$

where

$N(z, s)$ = normal force

E = modulus of elasticity

$\epsilon(z, s)$ = longitudinal tensile strain

χ = transverse bending strain

The important feature to note is that Eq. (1) can be derived by way of Eq. (4) and (5) as well as by first principles.

A fourth basic assumption is introduced; namely, that, because of the presence of rigid reinforced concrete floor slabs, the contour of the shell remains unchanged (i.e., $\chi = 0$). For this condition, Eq. (5), (6) and (7) yield

$$\Omega \sigma(z, s) = \frac{\partial^2}{\partial s^2} \left(R \frac{\partial^2 \sigma(z, s)}{\partial s^2} \right) + \frac{\partial}{\partial s} \left(\frac{1}{R} \cdot \frac{\partial \sigma(z, s)}{\partial s} \right) \quad (8)$$

Eq. (8) essentially states that the longitudinal normal stresses, being a function of the contour coordinate, s , z , follow the law of sectorial areas and that warping of the originally plane cross-sections of the shell occurs.

Taking into consideration the fundamental interrelationships of stress, strain, and geometry from the theory of elasticity, Eq. (8) may be solved without any difficulties. Its solution is:

$$\sigma(z, s) = F_1(z) + F_2(z) y(s) + F_3(z) x(s) + F_4(z) w(s) \quad (9)$$

where

$F_1(z); F_2(z); F_3(z); F_4(z)$ = functions of integration

x, y = Cartesian coordinates of the middle surface of the shell (Fig. 1-4)

$w(s) = \int_s r \, ds$ = sectorial coordinate representing twice the area enclosed between the initial and moving radius vectors and by the shell contour (Fig. 1-8)

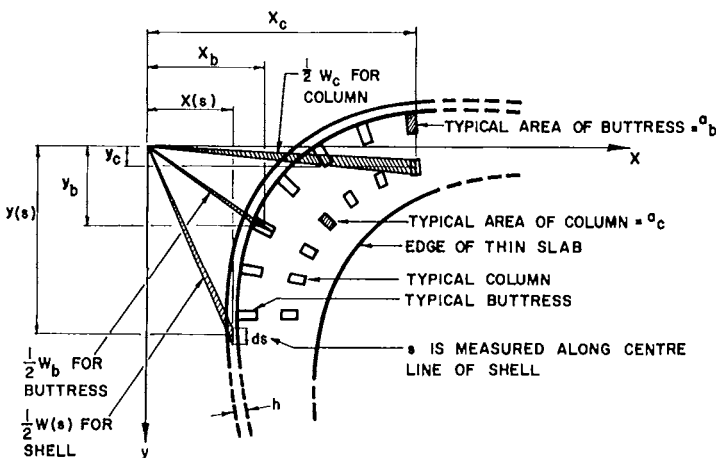


FIG. 1-8 LAYOUT FOR CALCULATING THE LOCATION OF FLEXURAL CENTER OF AN ORTHOTROPIC SHELL

Exact geometric interpretations may be given to functions $F_1(z)$, $F_2(z)$, $F_3(z)$ and $F_4(z)$, transforming Eq. (9) as follows:

$$\sigma(z, s) = \frac{P(z, s)}{A} \pm \frac{M_x(z)}{I_x} y(s) \pm \frac{M_y(z)}{I_y} x(s) \pm \frac{M_B(z)}{I_W} w(s) \quad (10)$$

where

$P(z, s)$ = total normal force parallel to z -axis

A = cross-sectional area of shell

$M_x(z)$ = bending moment with respect to x -axis

I_x = moment of inertia with respect to x -axis

$M_y(z)$ = bending moment with respect to y -axis

I_y = moment of inertia with respect to y -axis

$M_B(z)$ = bimoment

I_W = sectorial moment of inertia

The first three right-hand terms of Eq. (10) are clearly those normally pertaining to short columns subjected to biaxial bending. They are easily evaluated, provided the normal force, the mutually perpendicular bending moments, and section properties of the shell are known. The fourth term, however, is the torsional component which makes the shell problem different from the short column, and it warrants special attention. It is constructed to be of the same form as the second and third terms, and is a generalized computational device permitting convenient calculation of the torsional stress component. I_W is a new geometrical characteristic called the sectorial moment of inertia and is determined by the following formula:

$$I_W = \int_s w^2(s) h ds \quad (11)$$

where $w(s)$ has been defined previously as twice the shaded area shown in Fig. 1-8. Similarly, $M_B(z)$, called the bimoment, is a new concept and a generalized force. This bimoment, due to the external torsional moment applied to the shell, is an internally balanced force arising from the warping of the cross-section in the z -direction. The bimoment $M_B(z)$ is not a moment; nor is I_W a moment of inertia in the usual sense, but together with $w(s)$ they form a dimensionally correct expression for stress.

In order to calculate $M_B(z)$, it is necessary to express the bimoment with respect to the angle of rotation, $\theta(z)$, related to the flexural center:

$$M_B(z) = -EI_W \theta''(z) \quad (12)$$

However, before a value for $\theta''(z)$ may be obtained, θ must be evaluated from the following fundamental equation, given in Reference 9:

$$EI_W \theta^{iv}(z) - GI_p \theta''(z) + m(z) = 0 \quad (13)$$

where

EI_W = sectorial rigidity

GI_p = torsional rigidity

$m(z)$ = external torsional moment per unit length of the shell in the longitudinal, z -direction

Eq. (13), with corresponding boundary conditions, determines an angle of rotation, $\theta(z)$, as a function of the coordinate z . Its solution is:

$$\theta(z) = C_1 + C_2 z + C_3 \sinh \alpha z + C_4 \cosh \alpha z + \bar{\theta}(z) \quad (14)$$

where

C_1, C_2, C_3, C_4 = functions of integration

$\bar{\theta}(z)$ = particular solution of the differential equation

$$\alpha = \sqrt{\frac{GI_p}{EI_W}}$$

The required derivatives of Eq. (14) may be readily obtained and evaluated.

The angle of rotation, $\theta(z)$, is related to the flexural center, and coordinates x and y may be evaluated according to the following formulae:

$$x = \frac{\int_S y(s) h w(s) ds}{\int_S y^2(s) h ds} \quad (15)$$

$$y = \frac{\int_S x(s) h w(s) ds}{\int_S x^2(s) h ds} \quad (16)$$

Eq. (15) and (16) may be simply derived from the condition that no torsion is generated by forces passing through the flexural center. This means that the location of the center can be determined as a point of intersection of the lines of action of two mutually perpendicular forces for two independent flexural deformations in the planes of two principal axes.

Referring to Fig. 1-8, Eq. (11), (15) and (16) are modified as follows:

$$I_W = \int_S w^2(s) h ds + \sum_n w_b^2 a_b + \sum_n w_c^2 a_c \quad (17)$$

$$x = \frac{\int_S y(s) h w(s) ds + \sum_n y_b w_b a_b + \sum_n y_c w_c a_c}{\int_S y^2(s) h ds + \sum_n y_b^2 a_b + \sum_n y_c^2 a_c} \quad (18)$$

$$y = \frac{\int_S x(s) h w(s) ds + \sum_n x_b w_b a_b + \sum_n x_c w_c a_c}{\int_S x^2(s) h ds + \sum_n x_b^2 a_b + \sum_n x_c^2 a_c} \quad (19)$$

In Eq. (17) to (19), inclusive, n refers to the number of buttresses or columns.

In addition to $\sigma(z, s)$, it is also important to evaluate the tangential (shear) stresses, $\tau(z, s)$. An equation (Reference 8) may be written as follows:

$$\tau(z, s) = \frac{E}{h} [\zeta''(z) h ds + \xi'''(z) S_y(s) + \eta'''(z) S_x(s) + \theta'''(z) S_w(s)] \quad (20)$$

where

$$S_y(s) = \int_S x(s) h ds \rightarrow \text{for Fig. 8: } \int_S (s) h ds + \sum_n x_b a_b + \sum_n x_c a_c$$

$$S_x(s) = \int_S y(s) h ds \rightarrow \text{for Fig. 8: } \int_S y(s) h ds + \sum_n y_b a_b + \sum_n y_c a_c$$

$$S_w(s) = \int_S w(s) h ds \rightarrow \text{for Fig. 8: } \int_S w(s) h ds + \sum_n w_b a_b + \sum_n w_c a_c$$

ξ, η, ζ = deformations of the shell along x, y, z directions at any point of the shell (Fig. 1-4)

The solution to Eq. (20) may be presented as follows:

$$\tau(z, s) = F_1'(z) + F_2'(z) S_y(s) + F_3'(z) S_x(s) + F_4'(z) S_w(s) \quad (21)$$

where

$$F_1'(z); F_2'(z); F_3'(z); F_4'(z) = \text{functions of integration}$$

Exact geometric interpretations may be given to functions $F_1'(z)$, $F_2'(z)$, $F_3'(z)$, and $F_4'(z)$, as follows:

$$\tau(z, s) = \frac{M_K(z)}{I_P} h + \frac{V_x(z)}{I_y h} S_y(s) + \frac{V_y(z)}{I_x h} S_x(s) + \frac{M_W(z)}{I_W h} S_w(s) \quad (22)$$

where

$M_K(z)$ = torsional moment

I_P = torsional (polar) moment of inertia

$V_x(z)$ = shear force in xoz plane along x-axis

$V_y(z)$ = shear force in yoz plane along y-axis

$M_W(z)$ = torsional warping moment

As before, the fourth term is the novel generalized stress due to orthotropic shell action.

The moments $M_K(z)$ and $M_W(z)$ may be expressed as functions of the angle of rotation θ :

$$M_K(z) = GI_P \theta'(z) \quad (23)$$

$$M_W(z) = M_B'(z) = -EI_W \theta'''(z) \quad (24)$$

Similar to the arguments presented in connection with Eq. (10), it follows that Eq. (13) must be solved before $\tau(z, s)$ values may be established.

Finally, it is pointed out that the deflection of the shell at a particular point may be calculated by adding vectorially the deflections resulting from external loading in the x and y directions and torsion. The deflection resulting from torsion is obtained by multiplying the appropriate $\theta(z)$ value with the distance from the flexural center to the point in question (refer to Fig. 1-9).

Design data relating to the East Tower of the City Hall in Toronto are presented next. The information was made available by Dr. P. T. Mikluchin of P. T. Mikluchin and Associates, Ltd., Consulting Engineers, Toronto, Ontario. This company acted as a special consultant to Viljo Revell and John B. Parkin Associates, Toronto, and was responsible for the complete analysis of the shell structures.

For the East Tower:

$$\begin{aligned} A &\approx 1030 \text{ ft}^2 \\ I_x &\approx 848,000 \text{ ft}^4 \\ I_y &\approx 7,841,000 \text{ ft}^4 \\ I_W &\approx 885,000,000 \text{ ft}^6 \end{aligned}$$

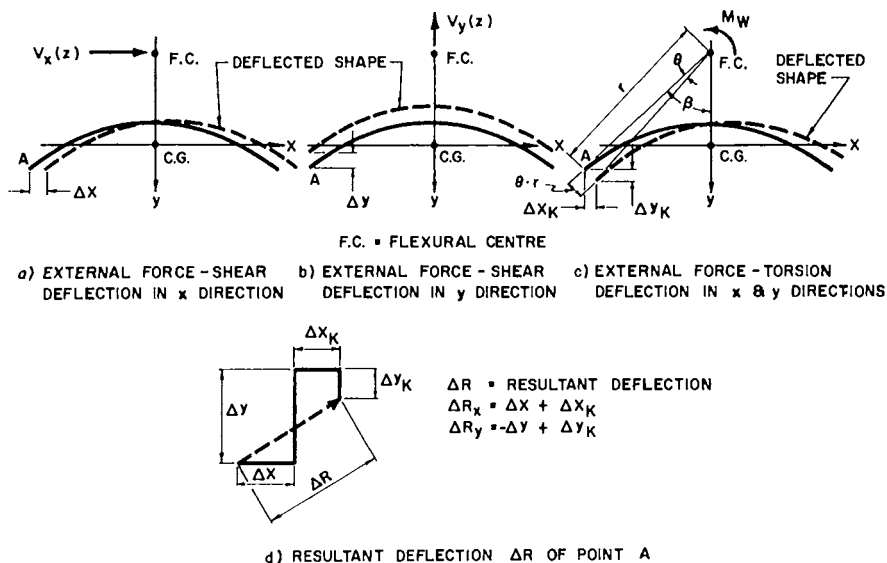


FIG. 1-9 DEFLECTION MODE OF SHELL

$$I_p \approx 1700 \text{ ft}^4$$

$$h \approx 1.5 \text{ ft}$$

$w(s)$ = sectorial coordinate*

$S_y(s)$ = area moment with respect to y-axis*

$S_x(s)$ = area moment with respect to x-axis*

$S_w(s)$ = sectorial area moment*

Further, in Eq. (10), the quantity $P(z, s)$ is the dead load of the tower, including the reduced permissible live load acting on the concrete floor slabs, above the level under consideration. At the third-floor level $P(z, s) = 96,000$ kips. The term $M_x(z)$ consists of the moment which is induced by the self-weight of the floor slab, radial beams, and the live load supported by the slab, and the moment induced by the normal component of the wind force which is parallel to the y-axis and acts through the flexural center. At the third-floor level, the contribution of the dead-live load component is 160,000 ft-k, whereas the moment arising from the wind force is approximately 340,000 ft-k. Study makes evident that the dead load moment must occur because the origin of the reference

*These geometrical properties were evaluated for each location where stresses were calculated, but are not listed in this paper.

system is located at the center of gravity of the cross-sectional area of the shell, which includes the buttress wall and column areas, whereas the resultant gravity force is eccentric therefrom because it includes the weight of the floor slabs and beams.

Because of the unorthodox shape of the Toronto City Hall, the wind loads acting on the structure were obtained from wind tunnel tests carried out on a model at the Institute of Aerophysics, University of Toronto.¹⁰ These loads may be replaced by a resultant acting eccentrically with respect to the flexural center as shown in Fig. 1-4. In turn, this resultant force may be decomposed into normal, $V_y(z)$, and tangential, $V_x(z)$, forces and a torsional moment, $M_{xy}(z)$, acting at the flexural center, for computational purposes (see Fig. 1-7). Thus $M_x(z)$ is established. $M_y(z)$ arises entirely from the wind force and is approximately 196,000 ft-k at the third-floor level. The bimoment, $M_B(z)$, arises from the torsional reaction of the shell, induced by wind, and is equal to approximately 31,700,000 k-ft².

At the third-floor level, Eq. (10) thus becomes:

$$\sigma(z, s) \cong -93 + 0.59y(s) + 0.025x(s) + 0.036w(s) \quad (25)$$

If $y(s)$ and $x(s)$ values in ft and $w(s)$ in ft² are substituted into Eq. (25), the computed stress $\sigma(z, s)$ is obtained in kips per sq ft.

In Eq. (22), at the third-floor level, $M_K = 0$, the tangential shear force $V_x(z) = 1250$ kips, the normal shear force $V_y(z) = 2380$ kips and $M_W(z) = 222,000$ ft-k, all of these forces arising from wind action. Eq. (22) thus becomes:

$$\tau(z, s) \cong 10^{-4} [1.06S_y(s) + 18.7S_x(s) + 1.67S_w(s)] \quad (26)$$

If $S_y(s)$ and $S_x(s)$ values in ft³ and $S_w(s)$ in ft⁴ are substituted into Eq. (26), the computed stress $\tau(z, s)$ is obtained in kips per sq ft.

The numerical coefficients in Eq. (25) and (26) change when the stresses at a different floor level are computed.

The variations of the $M_B(z)$, $M_K(z)$ and $M_W(z)$ values with the height of the East Tower, measured from the third-floor level, are presented in Fig. 1-10. It is significant to note that the torsional warping moment, $M_W(z)$, is almost solely responsible for the shear stress arising from torsion. The rate of change of the bimoment increases when proceeding from the top of the tower downward.

Stress diagrams corresponding to the solutions of Eq. (25) and (26) are presented in Fig. 1-11 to 1-14, inclusive. The sign convention used for shear stresses is indicated in Fig. 1-15.

Fig. 1-11 illustrates the variation of the individual stress components when moving along the shell. The magnitude of the normal stresses corresponding to gravity forces and to normal, tangential plus torsional wind forces are plotted, as appropriate, along the center line of the curved shell, or the center line of the buttresses, walls, and columns as indicated. For clarity, the symmetrical gravity forces are shown for the left half only. The symmetrical normal stresses acting on the columns and walls due to the normal component (Fig. 1-7) of the wind force are shown on the right half only, whereas those for the shell are shown in their entirety. The stresses acting on the buttresses and arising from the normal wind force component are not shown but are of magnitude similar to those acting on the curved shell. The normal stresses due to the tangential component (Fig. 1-7) of the wind force are small in magnitude and are shown only along the center line of the shell and the three end walls on the right-hand side. The normal stresses due to the torsional effect of the wind force are plotted for the shell all along its center line, for the columns and walls on the right only and for the end wall on the left. It is to be noticed that these normal stresses are of opposite sign on the right- and left-hand sides of the diagram.

Eq. (25) was used in order to derive the stresses presented in Fig. 1-11, except that the gravity part of the second right-hand term should be lumped with the first term in order to permit a ready calculation of the component curves. This means that the first right-hand side term of Eq. (25) becomes $[-93 + 0.19y(s)]$ and the second term becomes $0.40y(s)$, the other terms remaining unchanged.

Fig. 1-12 shows the total normal force at each location, as compiled from Fig. 1-11, with the stresses from the torsional effect of the wind force superimposed. It is most important to note that the latter stresses account for a substantial portion of the total stress. At the maximum stress of 1040 psi, torsion accounts for 27 percent of the total. If at the same location only wind forces are considered, then torsion accounts for 78 percent of the total stress. It is thus clear that when the contributions of the wind force to the normal stresses are evaluated, for towers similar to the one under consideration, it is of utmost importance to understand the torsional action of the structure clearly.

Fig. 1-13 presents shear stresses which arise from the normal, tangential, and torsional components of the wind force (Fig. 1-7).

The sum total of these components, with the torsional part superimposed, is shown in Fig. 1-14. It is seen that, at the maximum stress of 181 psi, torsion accounts for 67 percent of the total shear stress.

The shear stresses presented in Fig. 1-13 and 1-14 may be calculated by using Eq. (26), provided that the $S_y(s)$, $S_x(s)$ and $S_w(s)$ values are known. As indicated earlier, these values are not shown in this paper, but Eq. (26) has been presented in order to allow the reader to picture more readily the process of solution.

The deflection mode of the shell is indicated diagrammatically in Fig. 1-9. The East Tower is considered fixed at the third-floor level, and the deflections 301 ft above this level, at the top of the building, may be computed. The Δx component of Fig. 1-9a arises from the tangential wind force component. The tower acts as a cantilever about the y-y axis, and $\Delta x = 0.009$ in. Corresponding to Fig. 1-9b, the tower acts as a cantilever about the x-x axis, with $\Delta y = 0.140$ in. In order to calculate the deflection component of Fig. 1-9c, the angle θ must be known. Corresponding to Eq. (14), successive values for the East Tower are: $\theta = 0$, 1.49×10^{-4} , 5.49×10^{-4} , 8.94×10^{-4} , 14.2×10^{-4} for elevations above the third floor of 0, 0.25 ℓ , 0.5 ℓ , 0.75 ℓ and ℓ , respectively, (where $\ell = 301$ ft). This means that $\Delta x_K = \theta r \cos \beta = 14.2 \times 10^{-4} \times 2059 \times \cos 48.4^\circ = 1.94$ in. and $\Delta y_K = 14.2 \times 10^{-4} \times 2059 \times \sin 48.4^\circ = 2.19$ in. at the edge of the shell at the top of the tower, resulting in total deflections in the x and y directions of $0.009 + 1.94 = 1.95$ in. and $0.14 + 2.19 = 2.33$ in., respectively. It is clear that torsional action is responsible virtually for the whole deflection.

Finally, it is pointed out that the normal stresses, shear stresses, and deflections applicable to the East Tower of the City Hall and referred to in this paper are somewhat high because excessively large design wind pressures were used. Later discussion with the Institute of Aerophysics of the University of Toronto indicated that the design wind pressures originally established and used could have been lower. The maximum stresses and deflections should be multiplied by approximately 0.68 in order to give more realistic values.

From the above discussion it follows that it is important to understand the torsional behavior of members and structures, and to establish the applied forces with reasonable accuracy. In the case of Toronto City Hall, the provision of the buttresses, walls and columns as part of the external concrete shell resulted in a torsionally stiff structure. This may be considered analogous to providing reinforcing steel in a concrete beam in order to increase its resistance to torsional or combined loading. The orthotropic action of the East Tower limits deflections to small values.

Fig. 1-10 through 1-15 appear on pp. 26 through 31. Text continues on p. 32.

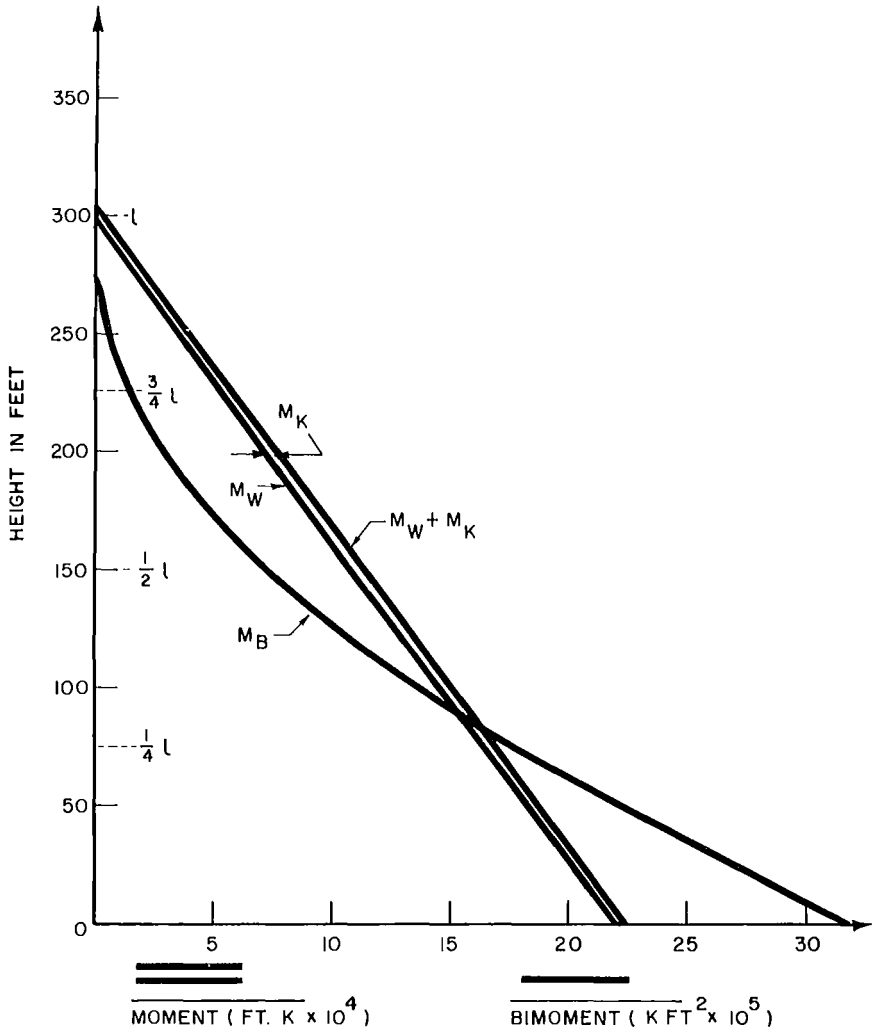


FIG. 1-10 HEIGHT OF SHELL VERSUS MOMENT
 $[M_B(z), M_K(z) \text{ and } M_W(z)]$ PLOTS

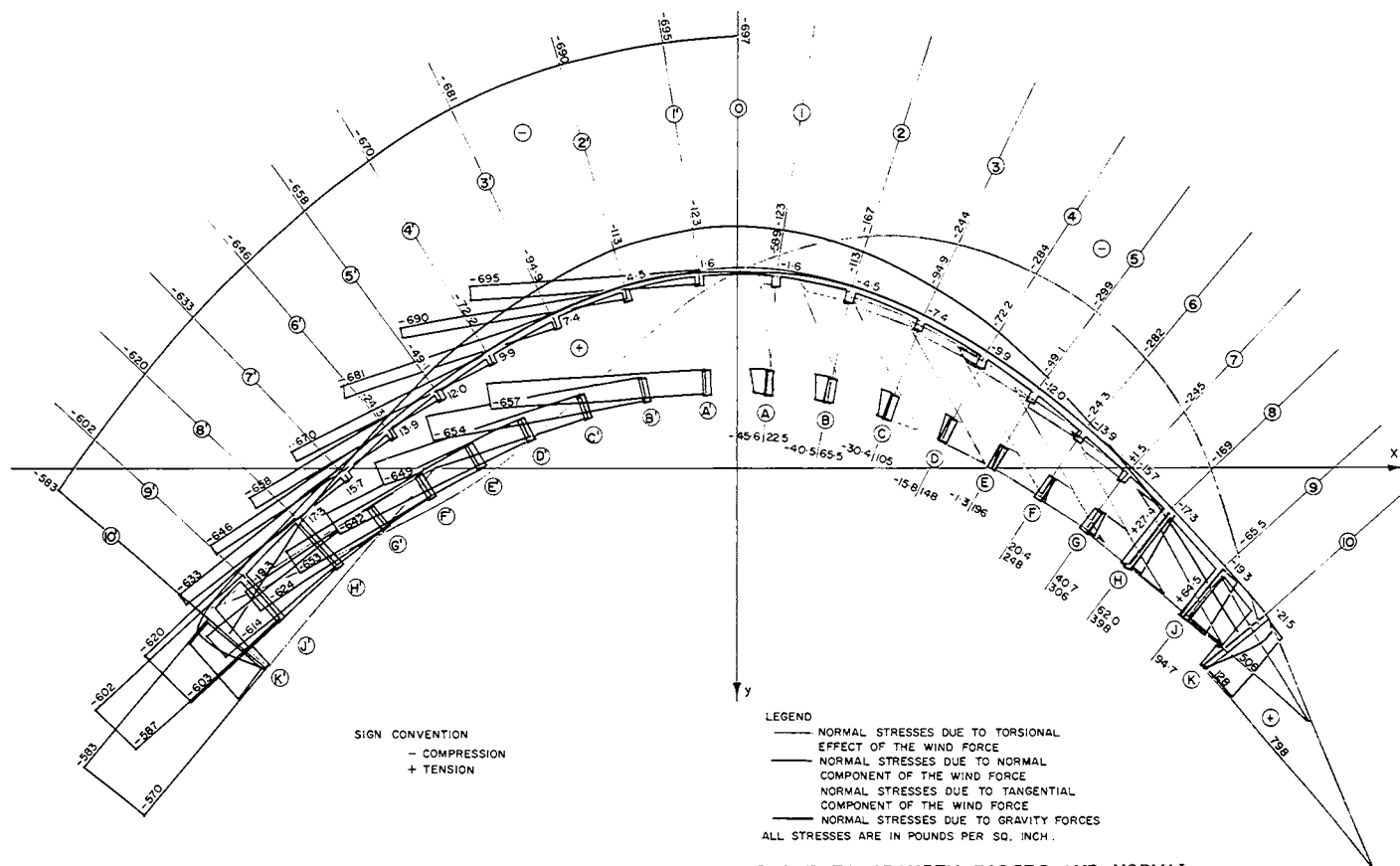


FIG. 1-11 $\sigma(z,s)$ - STRESS COMPONENTS DUE TO GRAVITY FORCES AND NORMAL, TANGENTIAL, TORSIONAL WIND FORCES (THIRD-FLOOR LEVEL)

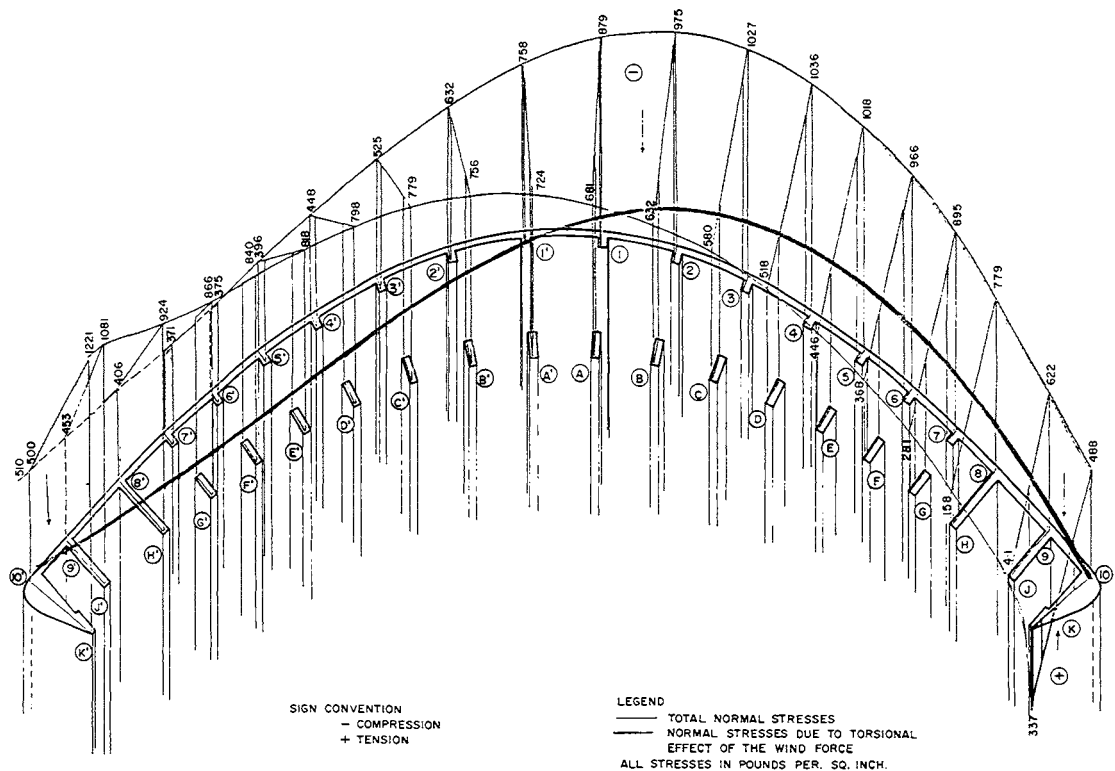


FIG. 1-12 TOTAL NORMAL STRESS $\sigma(z,s)$ - GRAVITY PLUS WIND EFFECTS (THIRD-FLOOR LEVEL)

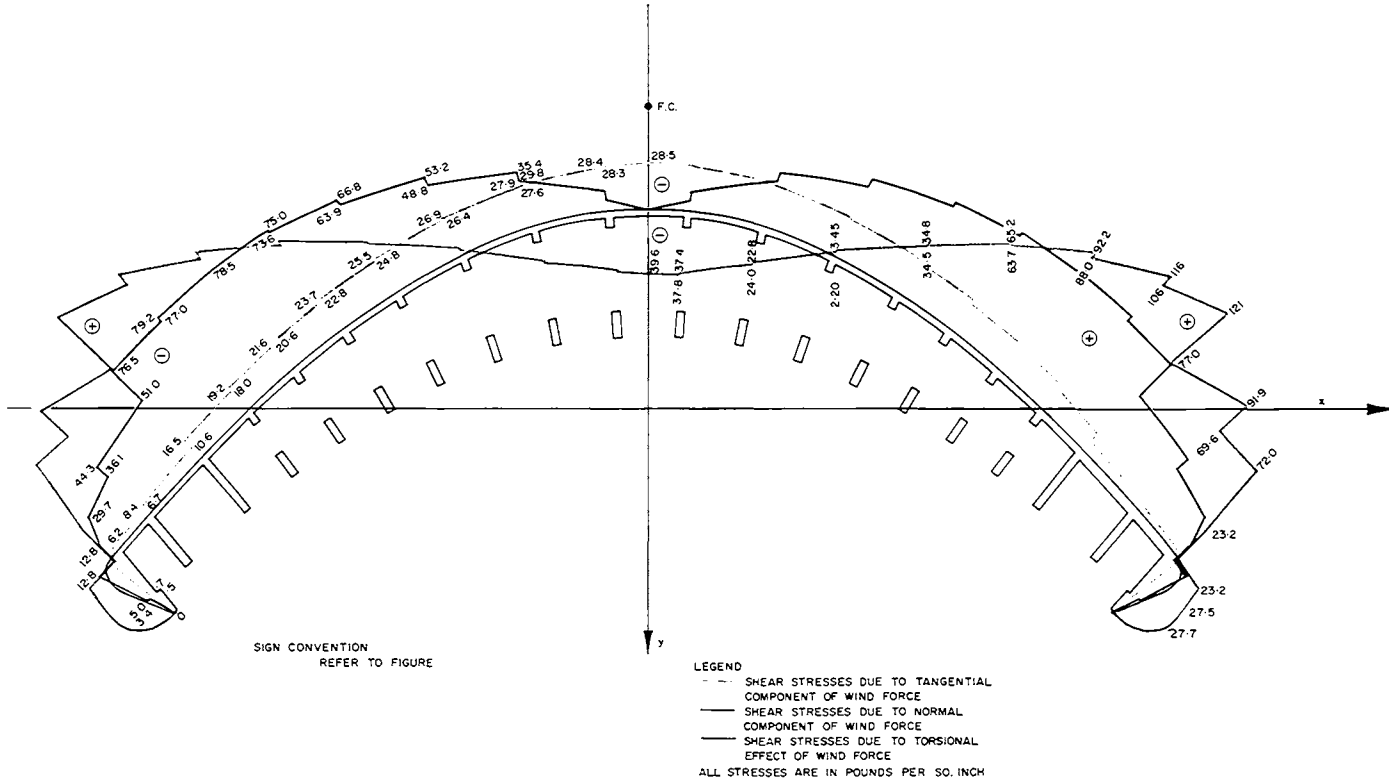
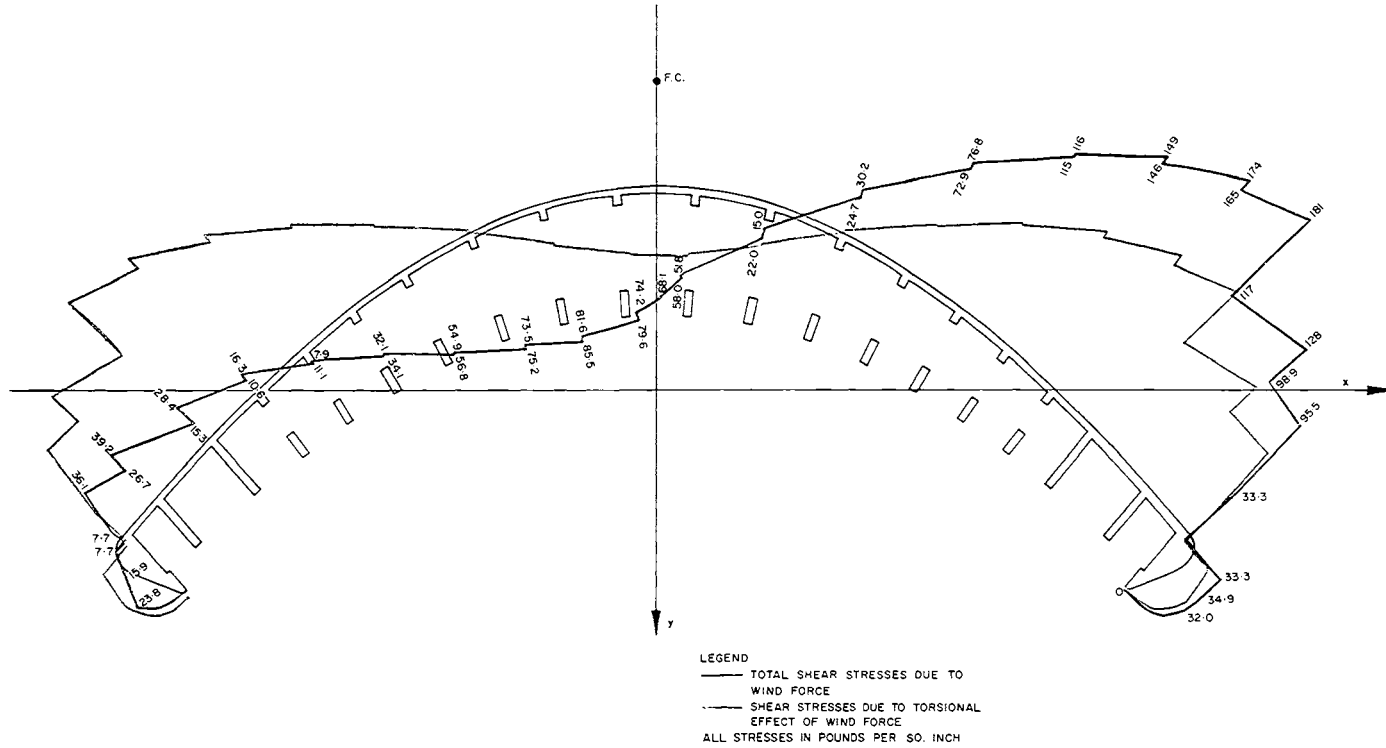


FIG. 1-13 $\tau(z,s)$ - STRESS COMPONENTS DUE TO NORMAL, TANGENTIAL AND TORSIONAL WIND FORCES (THIRD-FLOOR LEVEL)

FIG. 1-14 TOTAL SHEAR STRESS $\tau(z,s)$ ARISING FROM WIND EFFECTS

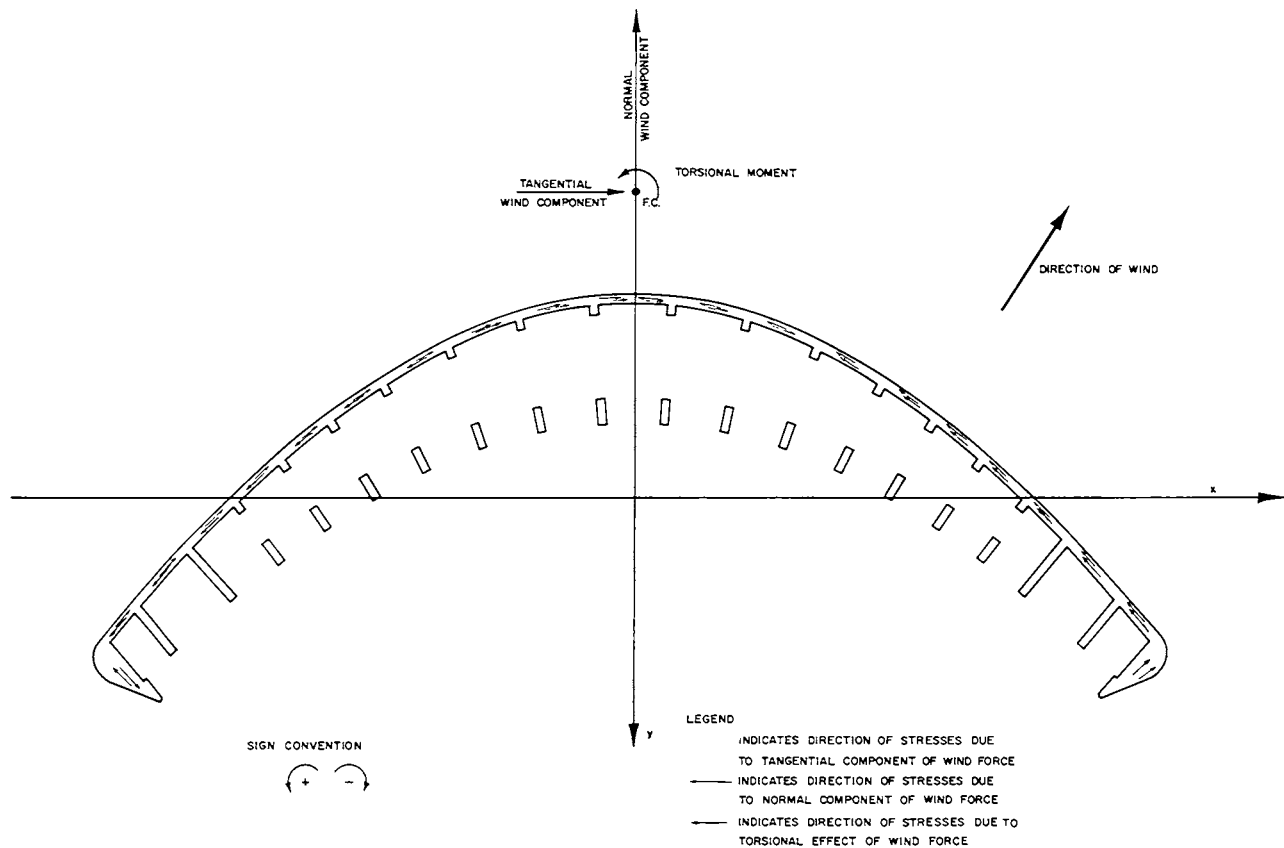


FIG. 1-15 SIGN CONVENTION USED FOR SHEAR STRESS

GRIDS

Another case in which torsional reaction may play an important part in assisting to support externally applied loads occurs for plate and grid frameworks. The magnitudes of the governing shears, moments, and deflections are affected by the torsional stiffness.

It is the purpose of this section to:

- (1) Illustrate, by means of an example, a rectangular grid supported along all sides and subjected to a uniformly distributed load, the importance of making proper allowance for torsion, and the effects which arise from various approximations used;
- (2) Emphasize that the employment of Bach's method may result in large errors when calculating torsional stiffnesses;
- (3) Discuss in elementary terms the importance of the interrelationship of the torsional and flexural stiffnesses in right multi-girder bridge decks, with specific reference to five-girder decks;
- (4) Emphasize that the electronic computer may be employed successfully in the analysis of bridge decks, considering the latter in terms of equivalent grids.

It is also pointed out that a number of methods and procedures of analysis are at the disposal of the engineer and may be used to investigate the efficiency of structures in distributing applied loads.¹¹⁻²⁴ These methods may be broadly classed into three main groups:

- (1) The structure is divided into longitudinal and transverse members of particular flexural and torsional stiffnesses, and deflection compatibility equations are derived;
- (2) The longitudinal or primary members are considered to be interconnected by some form of cross-connection which represents the behavior of the transverse members, and equations are derived accordingly;
- (3) The structure is analyzed in terms of anisotropic or orthotropic plate theory.

Clearly, with such wide choice, experience and judgment are required in selecting the most suitable method of analysis for the solution of a particular problem. The accuracy of the results obtained depends on how closely the actual structure is in agreement with the assumptions inherent in the employed method of analysis.

A rectangular concrete grid structure 48 x 60 ft in plan, composed of beams and integral slab, and simply supported along all four sides, is shown in Fig. 1-16. The self-weight of the grid is 137 psf, which corresponds to a loading expected to occur on the floors of stock rooms in libraries.²⁵ The total load is 287 psf.

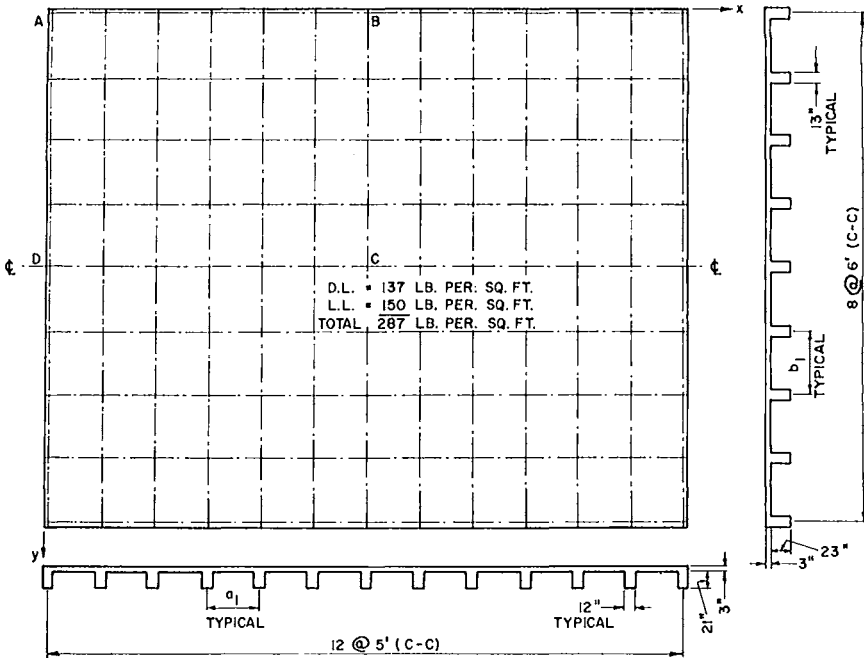


FIG. 1-16 GRID SYSTEM SIMPLY SUPPORTED ALONG ALL FOUR EDGES

This structure has been previously analyzed by means of the orthotropic plate analogy,^{26, 27} which makes use of the basic differential equation:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \quad (27)$$

where

w = deflection

x, y = Cartesian coordinates

q = intensity of load

D_x = bending stiffness of plate in x direction

D_y = bending stiffness of plate in y direction

$2H = \nu D_x + \nu D_y + 2(D_{xy} + D_{yx})$

ν = Poisson's ratio

D_{xy} = torsional rigidity

$$D_x = \frac{E_x I_x}{b_1}; D_y = \frac{E_y I_y}{a_1}; 2H = \frac{C_1}{b_1} + \frac{C_2}{a_1}$$

$E_x = E_y = E$ generally = modulus of elasticity

I_x = moment of inertia of each beam parallel to x-axis

b_1 = distance center-to-center of beams parallel to y-axis
($b_1 = 6$ ft in Fig. 1-16)

I_y = moment of inertia of each beam parallel to y-axis

a_1 = distance center-to-center of beams parallel to x-axis
($a_1 = 5$ ft in Fig. 1-16)

C_1 = torsional rigidity of each beam parallel to x-axis

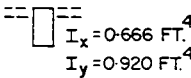
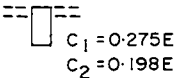
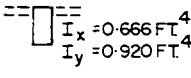
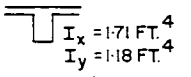
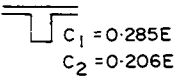
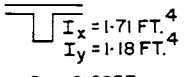
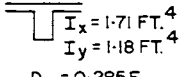
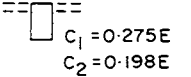
C_2 = torsional rigidity of each beam parallel to y-axis

For the case under consideration, w in Eq. (27) may be expressed in terms of a double trigonometric series. After determination of the particular coefficients of this series and the deflection function, the moment and shear expressions may be found by successive differentiation. Maximum values obtained from this analysis, corresponding to five distinctly different assumptions, are summarized in Fig. 1-17. The dotted portions of the cross-sections were neglected.

Case 3 of Fig. 1-17, which includes all elements of the structure, may be considered closest to reality. The reliability of the results obtained for the remaining four cases may then be checked by comparison to case 3, leading to the following conclusions:

- (1) Negligibly small errors result when the contribution to torsional stiffness arising from the 3-in. thick concrete slab is neglected. Compare cases 5 and 3. This means that the structure may also be analyzed as an open grid framework.
- (2) The torsional moments of cases 1, 3 and 5 at the corners of the grid are large, and torsional distress may occur. This illustrates the importance of properly proportioning edge supports, and the importance of accounting for torsional effects in the analysis.
- (3) When neglecting torsion, the resulting increases in deflection and governing design moments are a maximum, approximately 28 percent, for the open grid framework. Compare cases 1 and 2.
- (4) In contrast to its negligible contribution in torsion, the effect of the concrete slab in flexure is extremely significant. Compare cases 2 and 4.

In connection with the above, it is pointed out that the elastic torsional properties of cross-sections may be calculated by well-known formulas,²⁸ by valid approximations, or by finite differences.²⁹ If approximations are introduced, the designer must assure himself that he understands their significance. For example, an approximation frequently used in calculating the

CASE	CHARACTERISTICS		w (IN.)	M _x (FT.K)	M _y (FT.K)	M _{xy} (FT.K)	M _{yx} (FT.K)	Q _x (KIPS)	Q _y (KIPS)
	BENDING ($I = \frac{bd^3}{12}$)	TORSION ($C = \beta b^3 d G$)							
1	 $I_x = 0.666 \text{ FT.}^4$ $I_y = 0.920 \text{ FT.}^4$ $D_x = 0.153E$ $D_y = 0.135E$	 $C_1 = 0.275E$ $C_2 = 0.198E$ $2H = 0.852E$	2.80	254	292	94.6	-68.5	18.7	23.2
2	 $I_x = 0.666 \text{ FT.}^4$ $I_y = 0.920 \text{ FT.}^4$ $D_x = 0.153E$ $D_y = 0.135E$	$C_1 = 0$ $C_2 = 0$ $2H = 0$	3.57	325	371	0	0	16.9	24.2
3	 $I_x = 1.71 \text{ FT.}^4$ $I_y = 1.18 \text{ FT.}^4$ $D_x = 0.285E$ $D_y = 0.236E$	 $C_1 = 0.285E$ $C_2 = 0.206E$ $2H = 0.888E$	1.72	289	311	60.5	-44.0	18.7	23.1
4	 $I_x = 1.71 \text{ FT.}^4$ $I_y = 1.18 \text{ FT.}^4$ $D_x = 0.285E$ $D_y = 0.236E$	$C_1 = 0$ $C_2 = 0$ $2H = 0$	1.99	336	361	0	0	17.6	23.7
5	 $I_x = 1.71 \text{ FT.}^4$ $I_y = 1.18 \text{ FT.}^4$ $D_x = 0.285E$ $D_y = 0.236E$	 $C_1 = 0.275E$ $C_2 = 0.198E$ $2H = 0.852E$	1.73	291	313	58.8	-42.6	18.7	23.2

NOTE:

w IS THE DEFLECTION AT CENTRE 'C' OF GRID

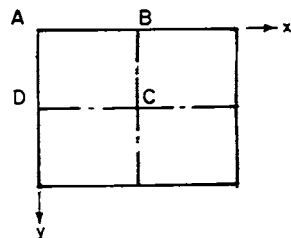
M_x IS THE BENDING MOMENT AT 'C' IN BEAM PARALLEL TO x - AXISM_y IS THE BENDING MOMENT AT 'C' IN BEAM PARALLEL TO y - AXISM_{xy} IS THE TORSIONAL MOMENT AT 'A' IN EDGE BEAM ALONG x - AXISM_{yx} IS THE TORSIONAL MOMENT AT 'A' IN EDGE BEAM ALONG y - AXISQ_x IS THE SHEAR FORCE AT 'D'Q_y IS THE SHEAR FORCE AT 'B'

FIG. 1-17 MAXIMUM VALUES OF DEFLECTION, MOMENT AND SHEAR FOR GRID OF FIG. 1-16 (5 CASES)

torsional stiffness of sections is to employ Bach's method,³⁰ which divides the section into component rectangles and sums the stiffnesses of the individual components. The magnitude of the error introduced depends on the shape of the cross-section. To illustrate the order of magnitude of the errors in Bach's approximation, which may be expected to occur in certain cases, Fig. 1-18 is

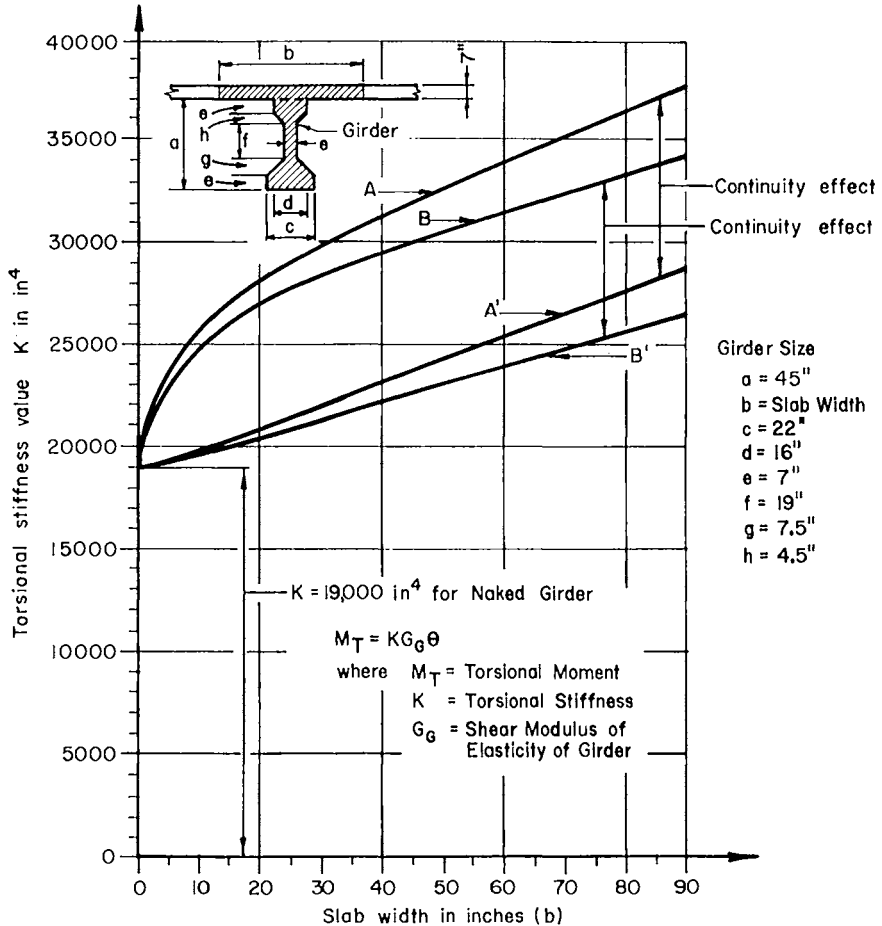


FIG. 1-18 TORSIONAL STIFFNESS VALUES K FOR
AASHO GIRDER III PLUS SLAB

presented. This figure originates from Reference 29 and shows torsional stiffness values, K , as a function of slab width, corresponding to the conditions listed for Type III prestressed concrete AASHTO girders. The effects arising from slab width, continuity, and two types of concrete are clearly illustrated. The slab contributes a great deal more to the torsional stiffness than is the case in Fig. 1-17. When Bach's method is used, the continuity effect at the slab-beam junction is lost and, in addition, the torsional stiffness is underestimated by a substantial margin for the naked girder.

Torsional stiffness assumes significance in bridge construction also. It is important to achieve a distribution of the live load for economical design of multi-girder decks. The effectiveness of such a deck in distributing loads is a function of the ratio of the longitudinal to the transverse flexural stiffness of the deck and of the torsional stiffness of the longitudinal girders. The transverse flexural stiffness may be varied by the introduction of diaphragms, whereas the torsional stiffness of the girders is dependent on the type of cross-section selected. For rational design of bridge decks, it is thus necessary to understand the interaction of the torsional and flexural stiffnesses in order to arrive at optimum designs.

The following discussion limits itself to the consideration of simply supported, rectangular, five-girder bridge decks with identical and uniformly spaced girders only, but provides the designer with a qualitative as well as quantitative means of visualizing the effects exerted by torsional and flexural stiffnesses on the distribution of loads in a large number of bridge decks occurring in practice.

Corresponding to the characteristic flexural stiffness ratio, $H = (E_G I_G / E_S I_S L)$, and the torsional stiffness K_G of a longitudinal girder,

where

E_G = modulus of elasticity of longitudinal girder material

I_G = moment of inertia of one longitudinal girder

E_S = modulus of elasticity of transverse medium (slab alone or slab plus diaphragm in case of composite section)

I_S = moment of inertia per unit length of transverse medium (slab alone or slab plus diaphragm in case of composite section)

L = bridge span

the following extreme conditions, embracing all possible cases of the five-girder bridges under consideration, may be visualized:

Case 1a $\rightarrow H = \infty, K_G = 0$

Case 1b $\rightarrow H = 0, K_G = 0$

Case 2a $\rightarrow H = \infty, K_G = \infty$

Case 2b $\rightarrow H = 0, K_G = \infty$

The smaller H is, the larger is the relative transverse stiffness; $H = 0$ thus denotes infinite transverse stiffness.

An understanding of the interrelationships between torsional and flexural stiffnesses affecting the distribution of applied loads within the grid system may be best obtained through consideration of the applicable influence lines. Extreme cases 1b and 2b are the more interesting and important for the present discussion.

The transverse distribution of a concentrated deck load to the various girders, in the case of 1b, is easily obtained by employing a redefined form of the well-known flexural formula $\sigma = (P/A \pm Mc/I)$. In accordance with Fig. 1-19, for example, the portion of the load P distributed to girder A, when this load is at A, is $(P/5 + P \times 2S \times 2S/10S^2) = 0.60P$. Similarly, when the load P is at D, the portion of P carried by girder A is $(P/5 - P \times S \times 2S/10S^2) = 0$. Other points on the influence lines, which are straight, may be obtained in a similar manner. With two points known, the influence lines for case 1b may be drawn.

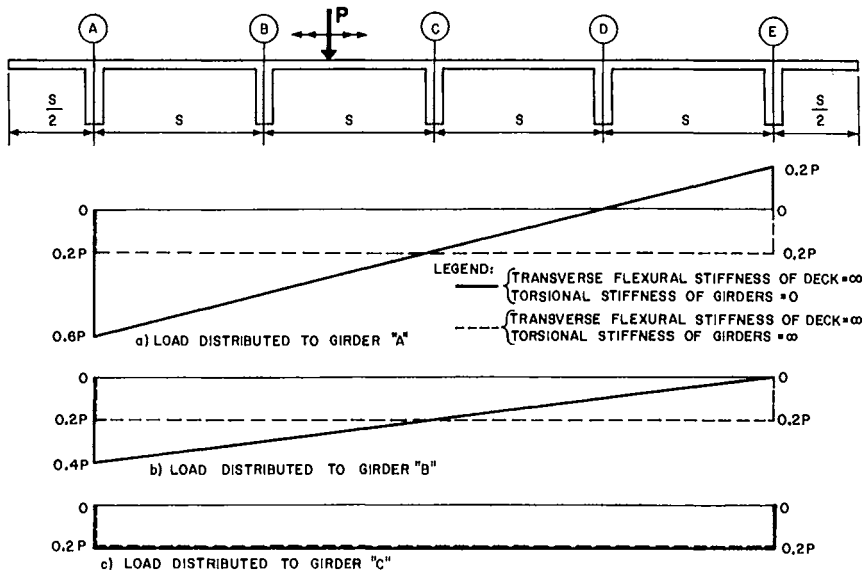


FIG. 1-19 FRACTION OF MOVING LOAD "P" DISTRIBUTED TO THE GIRDERS OF A 5-GIRDER BRIDGE, SHOWN AS A FUNCTION OF POSITION FOR TWO EXTREME CONDITIONS

It is observed that for zero torsional stiffness, in Fig. 1-19, the load on the outer girders A and B increases linearly with the eccentricity of P, whereas the presence of infinite torsional stiffness (case 2b) would have the effect of producing an equal distribution of the load among the girders. The load distribution to girder C is constant for conditions 1b and 2b, irrespective of the position of P. Girder A, with the load at A, must carry three times as much load when the torsional stiffness of the girders is zero than when infinite torsional stiffness prevails. If a uniformly distributed load of intensity P acts over a distance 2S between A and C, the load acting on the same girder at zero torsional stiffness is two times the value at infinite torsional stiffness. Therefore, the resulting girder loads for any combination of discrete loads P between A and C will vary between the ratios 3.0 and 2.0. The corresponding limits for girder B are 2.0 and 1.5, respectively. It is clear then that torsional stiffness assumes importance in helping to distribute eccentrically applied loads only when flexurally stiff diaphragms are present.

It may be concluded also that the importance of the contribution rendered by torsion to load distribution is dependent on the live load design criteria employed. It is suggested that the present trend toward heavy and special type vehicles makes the continued use of the widely accepted AASHO design loadings questionable insofar as they do not distinguish between ordinary and special-permit truck traffic. On the other hand, the division of truck traffic into ordinary and special-permit truck traffic, representing the former by uniformly distributed loads and the latter by an extra heavy vehicle, would seem to take the maximum advantage of the torsional stiffness inherent in a bridge deck.³¹ Therefore, the preceding analysis may well form the basis for a reappraisal of present highway loading standards, leading possibly to improved economy in design and greater understanding of bridge behavior.

Influence ordinates at intermediate values of torsional stiffness may be conveniently evaluated from the extreme values in Fig. 1-19 by using the interpolation formula derived by Hendry and Jaeger:²¹

$$\rho_{\beta} = \rho_0 + (\rho_{\infty} - \rho_0) \sqrt{\frac{\beta \sqrt{\alpha}}{3 + \beta \sqrt{\alpha}}} = \rho_0 + (\rho_{\infty} - \rho_0) \Gamma \quad (28)$$

where

ρ_{β} = distribution coefficient or influence value at torsional stiffness parameter β

ρ_0 = distribution coefficient or influence value when the torsional stiffness equals zero

ρ_{∞} = distribution coefficient or influence value when the torsional stiffness equals infinity

with the qualities α and β redefined as follows:

$$\alpha = 0.124 \frac{1}{H} \left(\frac{L}{S} \right)^3$$

$$\beta = 4.94 \frac{K_G G_G}{I_G E_G} \times H \times \frac{S}{L}$$

K_G = torsional stiffness of one longitudinal girder

G_G = shear modulus of elasticity of longitudinal girders

S = spacing of longitudinal girders

The larger the coefficient r becomes, the closer the infinitely stiff condition is realized. Fig. 1-20 presents values of r with respect to β for values of $\alpha = 0.1, 1.0, 10$ and 100 , corresponding directly with values of $H = 1.24(L/S)^3$, $0.124(L/S)^3$, $0.0124(L/S)^3$ and $0.00124(L/S)^3$, respectively. The values for each of these expressions for H with respect to L/S are given in Fig. 1-21, along with superimposed graphs representing actual bridge designs.

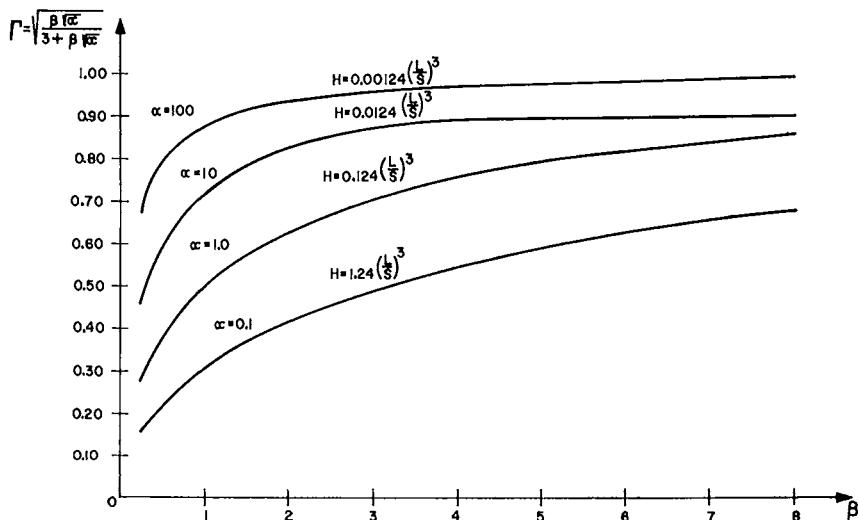


FIG. 1-20 PLOT OF r VERSUS β FOR FOUR FUNCTIONAL VALUES OF H

It may be concluded from Fig. 1-20 that, in order to take advantage of torsional stiffness, the characteristics of bridge decks preferably should be such that the α and β values are larger than, say, 1.0. Fig. 1-21 indicates that a large number of bridges in

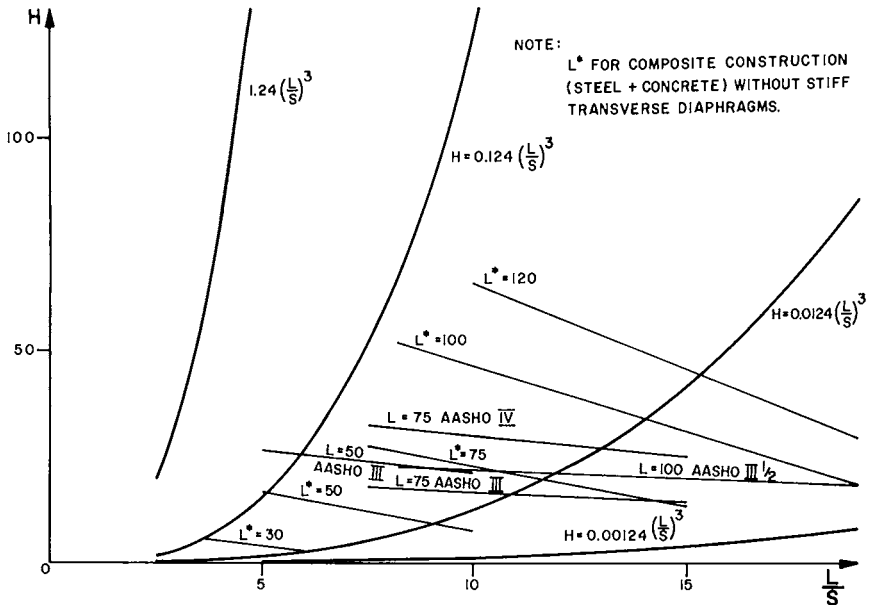
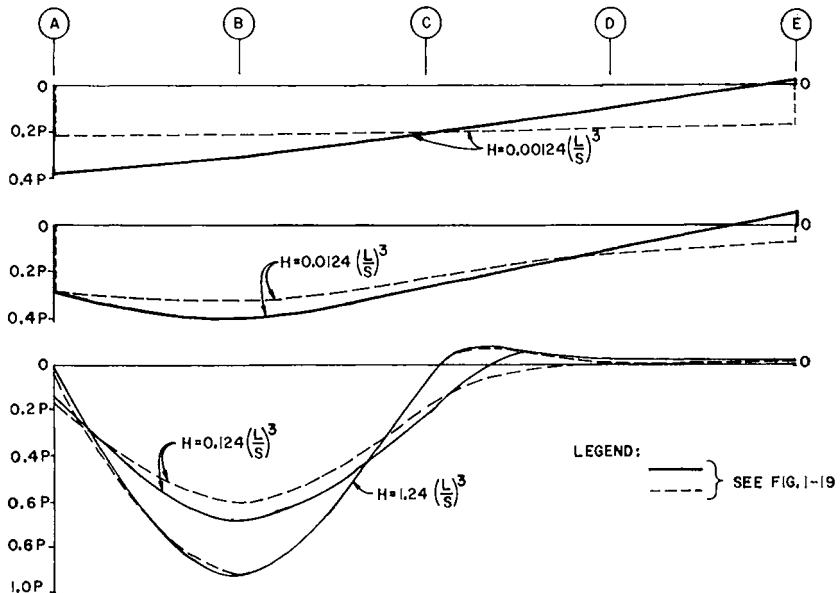
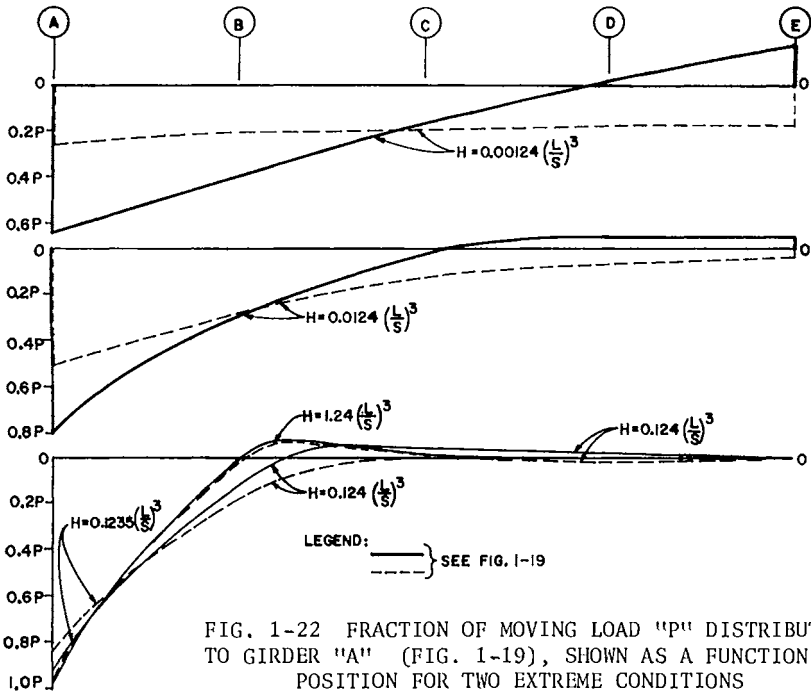


FIG. 1-21 PLOT OF H VERSUS L/S FOR FOUR FUNCTIONAL VALUES OF H

the highway system have α -values larger than the specified minimum. The bridges represented in Fig. 1-21 are of composite design. The decks consist of steel and prestressed concrete AASHO girders, generally with 7-in. thick concrete slabs. The H-values applicable to concrete girder decks also fall within the range covered by these bridges. It should be noted, however, that decks with steel girders have practically zero torsional stiffness, that the β -values for AASHO girder decks are smaller than 1.0, and that decks with concrete girders frequently have β -values in excess of 1.5. The Interim Specification of the 1961 AASHO Code²³ specifies distribution coefficients of $S/6.0$ and $S/5.5$, respectively, for decks consisting of concrete T-beams and for concrete slabs acting together with steel beams or prestressed concrete girders. These coefficients apply to bridges designed for two or more traffic lanes, $S \leq 10$ and 14 ft, respectively. Even more advantageous load distribution, namely $S/7.0$, is permitted for concrete box girder bridges because of their large torsional stiffness.

For the same selected H-values of Fig. 1-20 and 1-21, approximate load influence lines for girders A, B and C for zero and infinite torsional stiffness are given in Fig. 1-22 to 1-24. Each influence line applies to a large number of bridges since H is a function of $(L/S)^3$, and L/S may be freely specified.

Fig. 1-22 shows load limits which can be expected to act on girder A, and indicates that the influence ordinates may be either



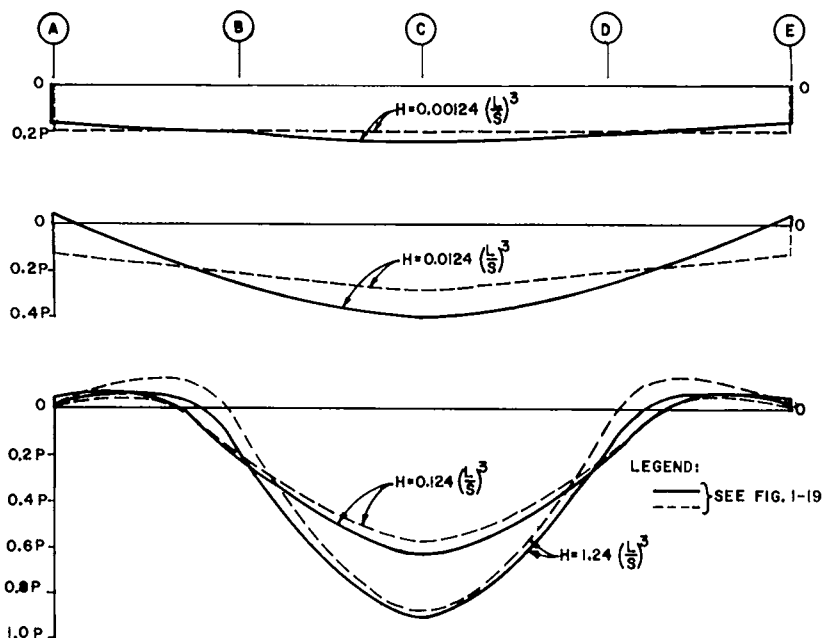


FIG. 1-24 FRACTION OF MOVING LOAD "P" DISTRIBUTED TO GIRDER "C" (FIG. 1-19), SHOWN AS A FUNCTION OF POSITION FOR TWO EXTREME CONDITIONS

positive or negative quantities. Since the location of design loads such as H20S16 on bridge decks is specified in the AASHTO Code, the reader may readily use Fig. 1-22 to determine how such loads are distributed to the girders. To determine the number of lanes which must be loaded in order to produce governing moment, account must be taken of the characteristic flexural stiffness ratio, H , and the torsional stiffness of girders. For example, for high values of H , one or two loaded design lanes closest to girder A may produce the governing moment, but this moment may arise also from all design lanes loaded for $H \leq 0.0124(L/S)^3$, depending on the value of the torsion coefficient β . Similar arguments may be put forth in connection with Fig. 1-23 and 1-24.

Spot checking has indicated that the H20S16 live load moments in some five-girder decks with AASHTO girders (Fig. 1-21), in spite of the low values of β , are about 10 percent smaller than the moments calculated by neglecting the torsional stiffness of the longitudinals. This observation is confirmed by Reference 32. It appears that an over-all saving of 5 percent or more of the total cost of a bridge may be realized if, because of the reduction in live load moment, the number of girders may be reduced from five to four. This indicates that the taking into account of torsional

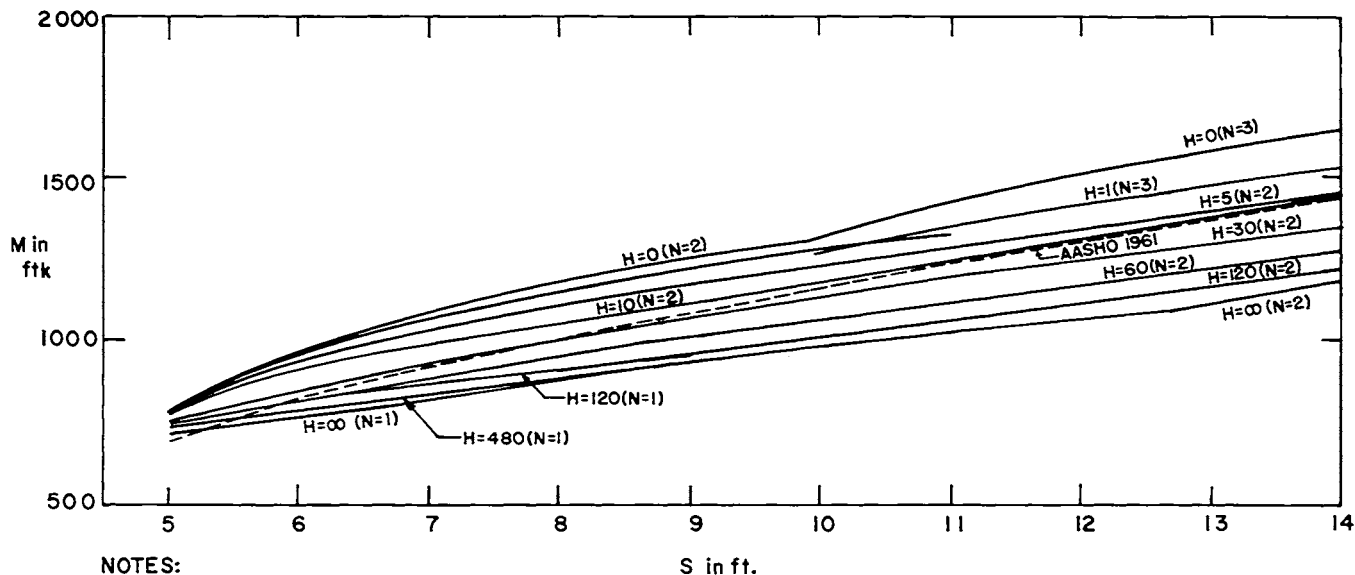
influences when using precast bridge girders may result in significant savings, even when no diaphragms are used. Moreover, the permissible gross vehicle weight of a single vehicle traveling down the middle of a deck employing AASHTO girders may frequently be increased by 20 percent if torsional stiffness is taken into account.

The advantages to be gained by providing a large degree of torsional stiffness in five-girder bridge decks may also be assessed by referring to Reference 33, which discusses torsionally weak systems. Fig. 1-25 to 1-27, taken from that source, present maximum live load (H20S16) moments for zero torsion which occur at center span of girders A, B and C (Fig. 1-19) of a simply supported 100-ft span deck as a function of H , of the girder spacing, S , and of the number of lanes loaded, N . In addition to the notes of Fig. 1-25, the following specifications apply:

- (1) When calculating the maximum live load moment for girder A, design vehicles within their 10-ft lanes were located within 11-ft design lanes adjacent to girder A, with one wheel load always placed at girder A.
- (2) Similarly, for girder B, 11-ft design lanes were used. No wheel load was located on the outside of the exterior girders when calculating maximum moments.
- (3) For girder C, the design vehicles were located within 11-ft lanes on either side of girder C.

The choice of using an 11-ft lane throughout $S = 5$ to 14 ft was made arbitrarily and produces rather severe conditions of loading. Fig. 1-25 implies that the moments at $H = 0$ are of a high value in comparison with the values yielded by the AASHTO line; moments increase with a decrease in the H -values. In Fig. 1-26, the $H = 0$ line yields considerably higher moment values than in Fig. 1-27. The maximum moments in girders B and C decrease with a decrease in H .

As discussed earlier, the provision of torsionally stiff girders tends to even out the magnitudes of the moments supported by individual members when reasonably stiff diaphragms are provided. In the extreme, for the 100-ft bridge under consideration, each girder would resist a maximum moment of 305 ft-k for each lane of H20S16 live load, not including impact. For two, three and four lanes loaded, the corresponding figures are $2 \times 305 = 610$, $3 \times 305 \times 0.90 = 824$ and $4 \times 305 \times 0.75 = 915$ ft-k, respectively, including the statistical reduction factors. Specifically, for $S = 9$ ft, $H = 0$, this implies that the maximum moments in girders A (Fig. 1-25) and B (Fig. 1-26) would change from 1260 to 824 ft-k and 1010 to 824 ft-k, respectively, corresponding to reductions of 34.6 and 18.4 percent. The moment in girder C (Fig. 1-27) would increase from 610 ft-k to 824 ft-k. The governing design moment for the deck is 824 ft-k plus an appropriate increase for impact.



NOTES:

1. Effects from statistical load factors as defined by AASHTO (sec. 1-2-9) are included.
2. Impact effects are not taken into account.
3. Graphs are based on minimum lane width = 11'-0".
4. N = Number of lanes loaded.
5. --- For girders B and C see sec. 1-3-1 (B1) of 1961 AASHTO code - $\frac{S}{5.5}$.
For girder A see sec. 1-3-1 (B2a) of 1961 AASHTO code.

FIG. 1-25 MAXIMUM DESIGN LIVE LOAD MOMENTS (H20-S16) FOR GIRDER "A" (FIG. 1-19),
OF TORSIONALLY WEAK 5-GIRDER BRIDGE--L = 100 FT 0 IN.

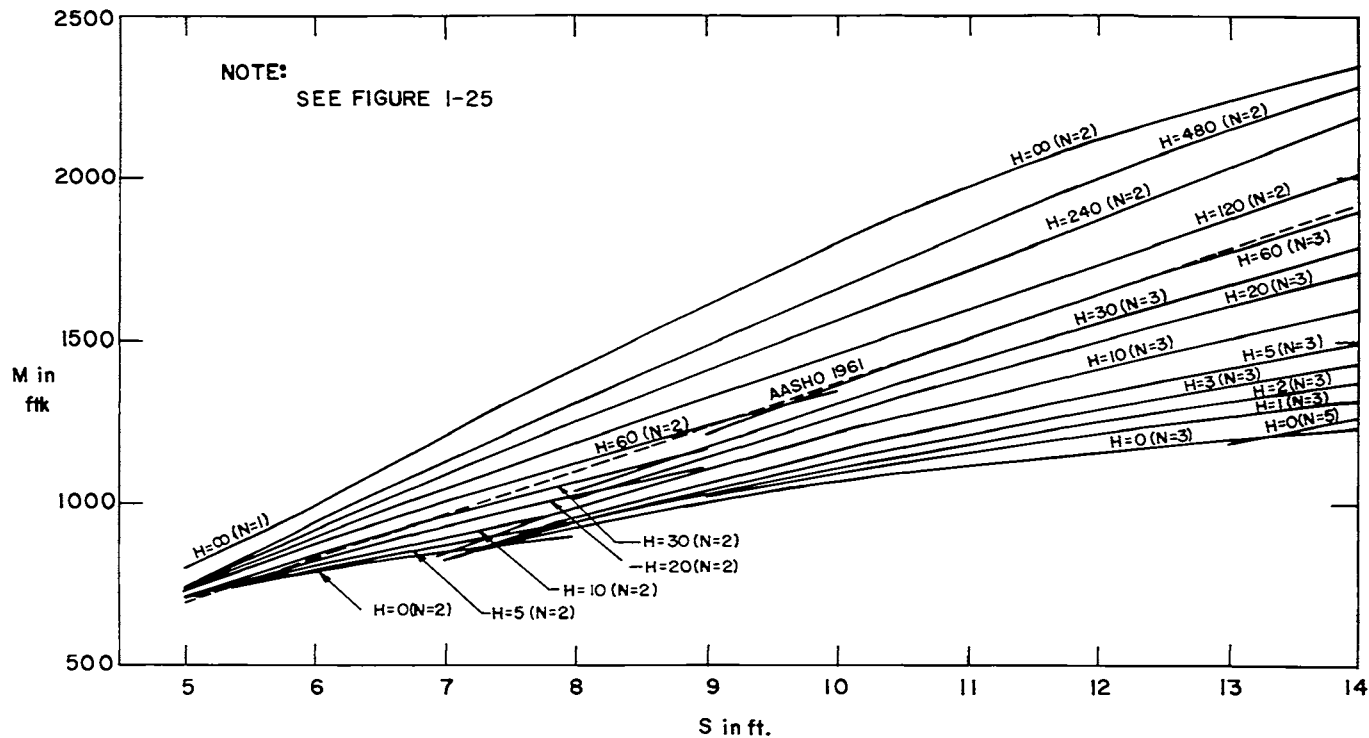


FIG. 1-26 MAXIMUM DESIGN LIVE LOAD MOMENTS (H20-S16) FOR GIRDER "B" (FIG. 1-19),
OF TORSIONALLY WEAK 5-GIRDER BRIDGE-- $L = 100$ FT 0 IN.

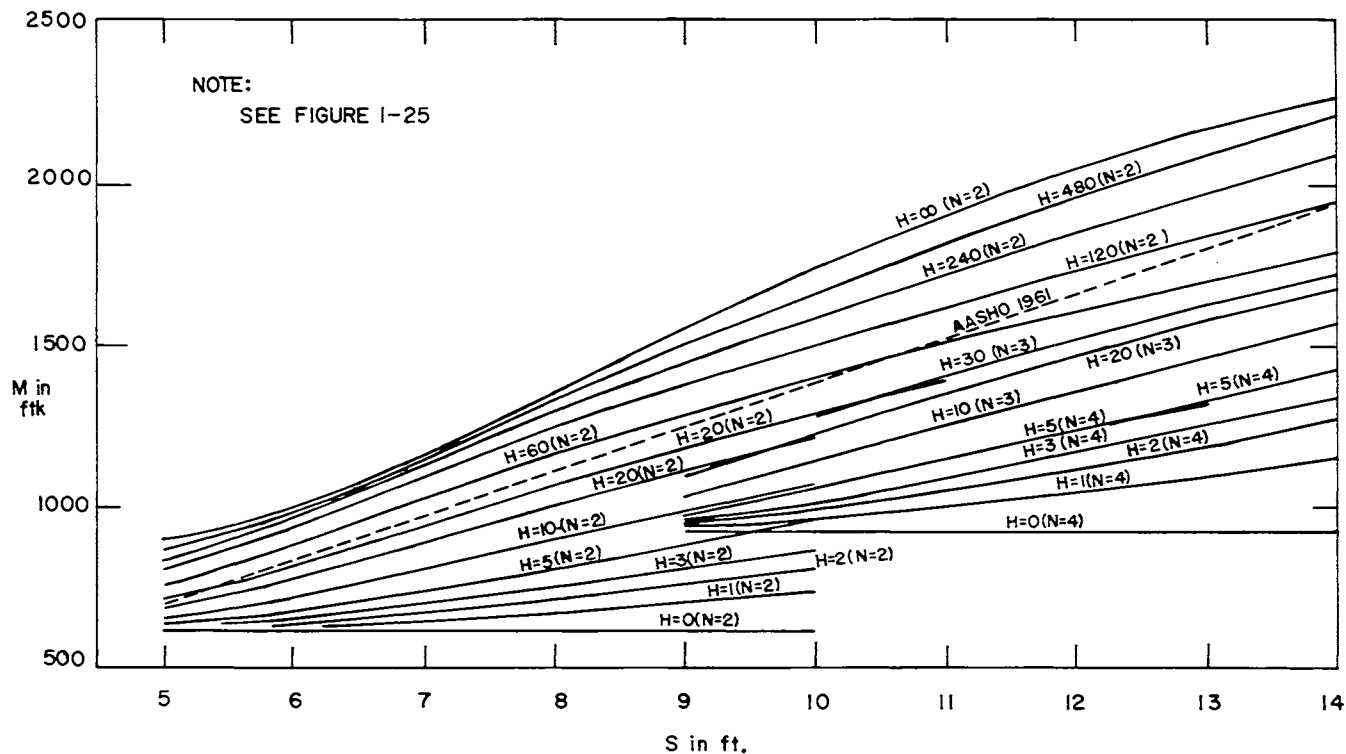


FIG. 1-27 MAXIMUM DESIGN LIVE LOAD MOMENTS (H20-S16) FOR GIRDER "C" (FIG. 1-19), OF TORSIONALLY WEAK 5-GIRDER BRIDGE--L = 100 FT 0 IN.

According to the AASHO lines of Fig. 1-25 to 1-27, the maximum moment for girders B and C is 1230 ft-k, and the equivalent figure for girder A is 1090 ft-k. The governing AASHO design moment, therefore, is about 50 percent larger than the design moment, 824 ft-k.

In design it is, of course, impossible to achieve infinite flexural and torsional stiffness. Practical limitations are set by economy and the shape of cross-sections employed. In Reference 33, the following formula, which is applicable to torsionally weak bridge decks and defines the longitudinal to transverse flexural stiffness ratio, H' , including extra diaphragms, has been derived:

$$H' = \frac{1}{\frac{1}{H} + rn} \quad (29)$$

where

$$r = \frac{E_D I_D}{E_G I_G}$$

E_D = modulus of elasticity of diaphragm

I_D = moment of inertia

n = number of additional diaphragms ($n \geq 3$)

The formula was derived assuming the same shape of deflection profile for the concrete slab and continuous diaphragm. The value H' is nothing but a new H -value which reflects the influence arising from the added transversals. It is thus possible, on the basis of Eq. (29), to investigate the suitability of providing diaphragms and to establish the resulting increases or reductions in girder moments.

Since the AASHO lines in Fig. 1-25, 1-26 and 1-27 refer to bridge decks "without" diaphragms, it is evident that little is to be gained by providing diaphragms in the lower regions of S . The concrete slab dominates the load distribution. At $S = 9$ ft, however, more significant reductions in moment may be expected to occur as diaphragm action assumes dominance over slab action. For $H = 50$, $r = 0.20$ and $n = 3$, $H' = 1.61$. This means that the moment in girder A is increased from 1090 to about 1220 ft-k. In girder B the moment is decreased from 1230 to about 1020 ft-k, with the corresponding figures for girder C being 1230 and 730 ft-k, respectively. From this specific example of a torsionally weak system, it may thus be concluded that diaphragms may be incorporated which closely approximate the infinitely stiff flexural condition, providing continuity prevails.

Now consider the effect of torsion. Let $L = 100$ ft, $S = 9$ ft and $H' = 1.61$, in which case the upper curve of Fig. 1-20 applies. Consequently, if $\beta > 1$, the infinitely stiff torsional condition is approximated. It was pointed out earlier that concrete girder decks frequently have β -values in excess of 1.5; large β -values apply to prestressed concrete box girders also. Thus, the 100-ft bridge may be framed in a manner that the governing live load design moment is about 70 percent of the governing AASHTO moment (1230 ft-k).

The provision of torsionally stiff longitudinals and flexurally stiff transversals enhances the overload capacity of a bridge deck enormously. In the extreme case ($H = 0$, $K_G = \infty$), a 100-percent increase in live load on the bridge means only a 20-percent increase for each member of a five-girder bridge, with the same girders, equally spaced.

The stress distribution in multi-girder decks may be improved further by decreasing the spacing between the outside and first interior girders. This configuration produces a more favorable stress distribution in the slab spanning between the girders. The Guyon-Massonet theory, modifications thereof,²² and the Hendry and Jaeger theory,²¹ which are in common use, are not directly applicable to such cases. These cases are, however, easily solved by employing the electronic computer.³⁴

The electronic computer may be employed with particular advantage in case of a complex layout of one deck grid system such as the Sheppard Avenue Canadian Pacific Railway crossing at Agincourt, Ontario.^{35,36} The irregularly shaped three-span, continuous deck of this railroad bridge consists of hollow post-tensioned boxes, and was replaced by an equivalent grid system for purposes of analysis. The torsional stiffness of the resulting grid could not be neglected as this would have given an incorrect picture of the stress distribution.

The foregoing discussions pertaining to load distribution have centered around simply-supported five-girder bridge decks with equally-spaced and stiff members. Similar reasoning applies to continuous bridge decks and to decks with more or fewer girders. It is noted, however, that the ability to carry overloads is not as pronounced, for example, for three- as for five-girder bridges. Nevertheless, the use of bridge decks with a low number of girders has become increasingly popular, particularly in prestressed concrete construction.

The double box section, with the boxes being separated by a stretch of slab, and the single box section are both commonplace in Europe.

ELEVATED ROADWAYS

The most characteristic feature of modern urban-area highways is the large number of geometrically complex multi-level interchanges and grade separation structures necessary for smooth and safe traffic flow. Such structures frequently take the form of long curved bridges with a minimum of space available for piers. A typical example is the Spadina Interchange (Fig. 1-28) at Toronto, Ontario.

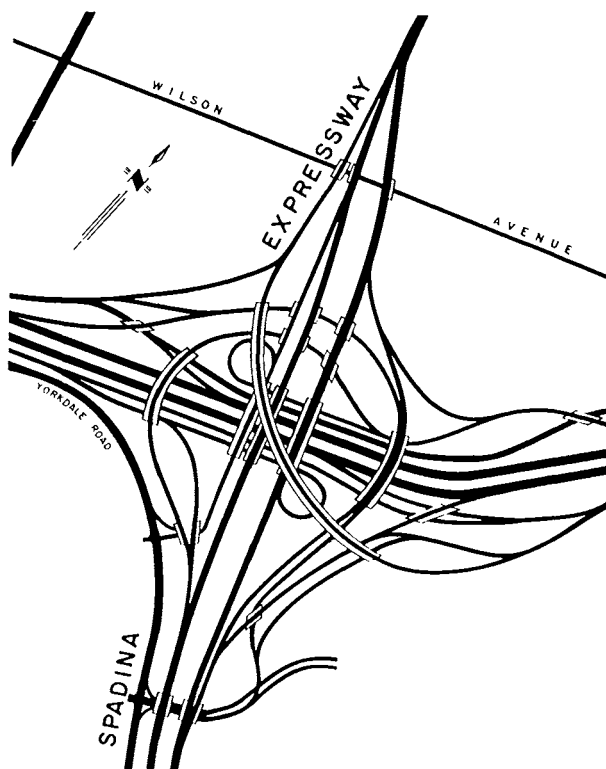


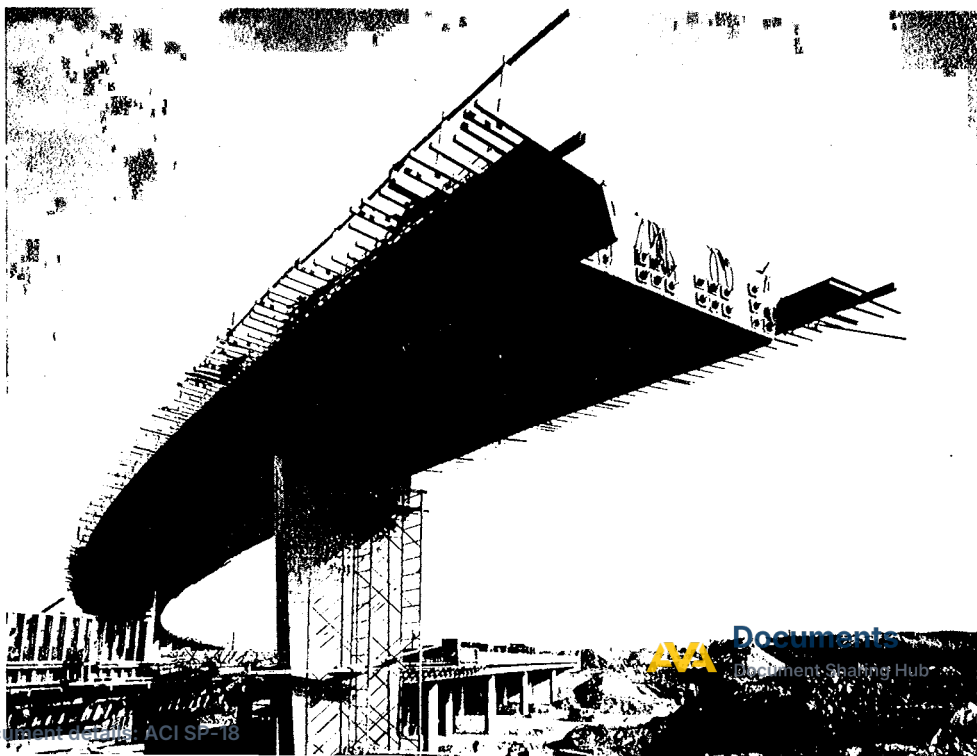
FIG. 1-28 SPADINA INTERCHANGE AT TORONTO

It is the purpose of this part of the paper to describe briefly the significance of torsional forces acting on certain prestressed concrete superstructures, on curved alignments, which are supported on a minimum number of piers. The size, number, and position of piers and the construction depth of the superstructure are the significant factors determining the suitability of a structure to the geometrical requirements of an interchange. These factors are, in turn, related to the structural configuration employed.

If piers are constructed monolithically with the deck, there is a transfer of forces from the deck to the piers, which does little to reduce the forces in the deck and increases the strength requirements for the piers. If, as is the present trend, it is required that single columns be used along the center line of the structure, such a transfer of forces would require excessively large pier shafts. It has been found advantageous to prevent the transfer of longitudinal forces to piers by the use of sliding or elastomeric bearings. A recent development has been the use of bearings which permit the deck not only to move horizontally and to deflect under longitudinal bending independent of the pier, but also to twist under the action of torsion. The only longitudinal forces transmitted to the piers then are the small horizontal forces caused by friction in the bearings. In some cases even friction can be eliminated by providing a bearing at both top and bottom of the pier (Fig. 1-29). The forces in the piers thus are axial compression only, permitting minimum pier size.

Torsion which is not resisted by a pier is transmitted along the bridge deck to a point at which it can be conveniently resisted, either by a more massive pier or a row of columns, preferably situated either at a less congested part of the interchange or at an abutment. The magnitude of the torsional moments arising at the restraints depends upon the distance between adjacent restraints.

FIG. 1-29 ELEVATED ROADWAYS UNDER CONSTRUCTION
AT TORONTO



For decks which are on a curved alignment, the torsion induced by permanent loads may be reduced by offsetting piers and by varying the prestressing force across the width of the bridge. The latter may be accomplished by changing the spacing between prestressing cables.

The cross-section of the superstructure is of primary importance. Adequate torsional stiffness and strength, as well as flexural, and a minimal depth are generally required. Fig. 1-30 shows some basic forms used for bridge cross-sections. Detail a shows the torsionally least stable arrangement; detail g, the rectangular solid slab, provides the maximum torsional stiffness for a given depth. The solid slab also has the highest flexural strength, but is inefficient for long spans because of the large dead load involved.

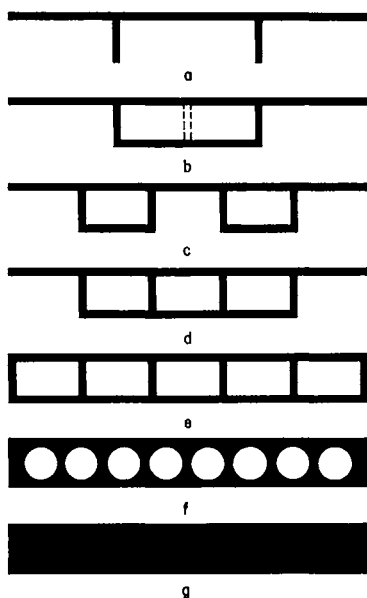


FIG. 1-30 BASIC FORMS USED FOR BRIDGE CROSS-SECTIONS

Elastic torsional stiffness is of interest for rotation control, but the magnitude and location of torsional stresses resulting from the applied torque are the primary considerations. Torsional shears of significant magnitude must be combined with other stresses in order to calculate the resulting principal stresses governing the selection of a cross-section. Sophisticated variations of basic cross-sectional forms, used for some of the post-tensioned concrete bridges in the Province of Ontario, are shown in Fig. 1-31. References 37 to 44 are relevant to the design of elevated roadways.

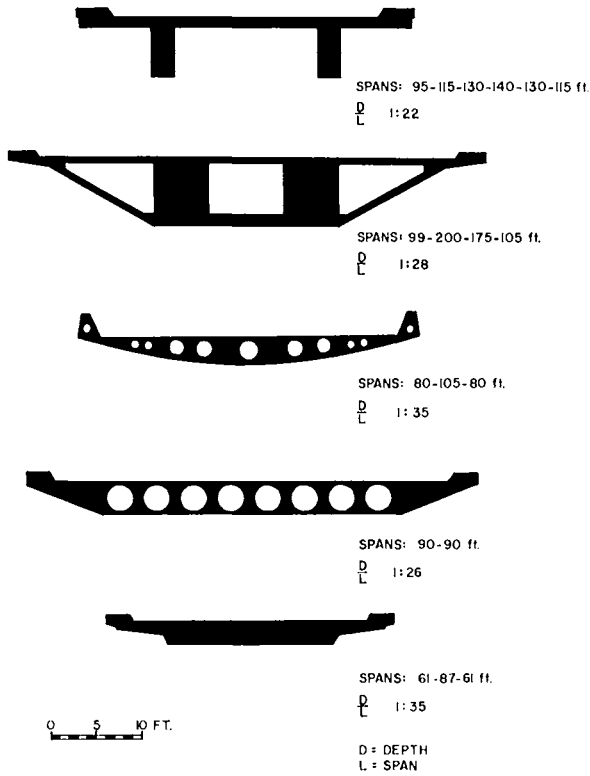


FIG. 1-31 CROSS-SECTIONS USED IN ONTARIO--CONTINUOUS POST-TENSIONED PRESTRESSED CONCRETE DECKS

Strain energy analysis is well suited to space structures such as curved bridges. For relatively large numbers of unknowns, matrix formulation and computer solution are advantageous. The total strain energy is expressed as:

$$U = \int \frac{M^2 dx}{2EI} + \int \frac{T^2 dx}{2GJ} \quad (30)$$

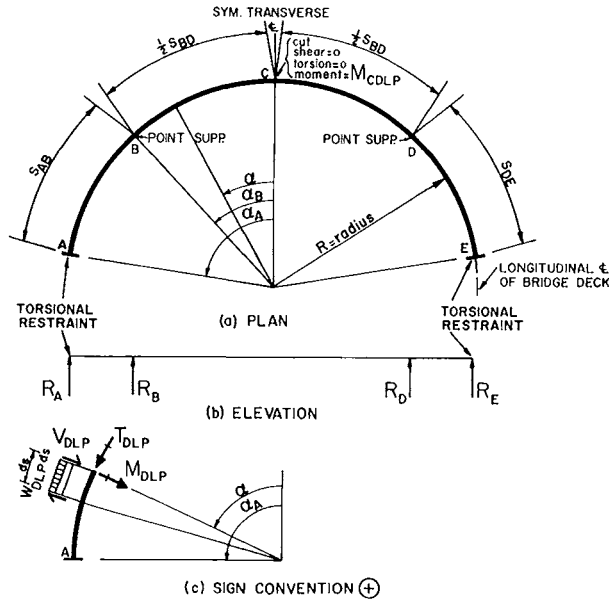
where M and T are the bending moment and torque, respectively, in terms of selected forces M_n , T_n and V_n .⁴⁵ Integer n refers to location. Shear energy is generally negligible. Deformations consistent with the selected redundant forces are defined by: $\partial U / \partial M_n = 0$ (flexural rotation), $\partial U / \partial T_n = 0$ (twist) and $\partial U / \partial V_n$ (vertical deflection), which yield a set of linear deformation equations in terms of selected redundant forces.

A simplified method of investigating the effects of torsion in symmetrical curved bridge decks, developed at the Department of

Highways of Ontario, is believed to be of considerable practical interest. The following analytical expressions are derived for symmetrical, three-span continuous structures as shown in Fig. 1-32, and are later applied to the analysis of an actual structure.

Fig. 1-32 shows a structure with torsional restraints at abutments A and E; otherwise free rotation may take place. Point supports are provided at locations B and D, situated symmetrically with respect to the transverse center line. A fictitious cut is introduced at C as indicated to render the structure statically determinate for analysis purposes. The following stipulations apply:

- (1) For uniformly distributed loads and for curves up to approximately 30 deg, which is rarely exceeded in highway bridges, the resulting effect of torsion on the longitudinal bending moments may be neglected. This assumption permits the



FOR $\alpha \leq \alpha_B$

$$T_{DLP} = (w_{DLP} R^2 + t_{DLP} R + M_{CDLP} \sin \alpha - w_{DLP} R^2 \alpha = T_{DLP}(\alpha)$$

FOR $\alpha > \alpha_B$

$$T_{DLP} = T_{DLP}(\alpha) + R [R_{BMDLP} + R_{BWDLP} + R_{BIDL P} R_{BTP}] [-\cos(\alpha - \alpha_B)] + T_{BTP} \cos(\alpha - \alpha_B)$$

FOR $0 \leq \alpha \leq \alpha_A$

$$R_{BDLP} = \frac{1}{\sin(\alpha_A - \alpha_B)} \{ (1 - \cos \alpha_A)(w_{DLP} R + t_{DLP}) - \frac{M_{CDLP}}{R} \cos \alpha_A \} + \frac{T_{BTP}}{R}$$

$$R_{ADLP} = \alpha_A R w_{DLP} - R_{BDLP}$$

FIG. 1-32 SYMMETRIC THREE-SPAN PRESTRESSED CONCRETE BRIDGE--INDUCED TORSION

curved deck to be developed into a straight alignment for the purpose of calculating the longitudinal moments.⁴⁶

- (2) Longitudinal bending has a significant effect on the magnitude of torsional moments.
- (3) Maximum torsional moments are the result of loads applied symmetrically with respect to the transverse center line (Fig. 1-32a).

It follows that, for symmetrically applied loads about the transverse center line, only one redundant force, namely, the longitudinal moment, acts at the cut at location C. The shear and torsional forces are equal to zero because of symmetry of loading.

Deferring for the moment a discussion of the effects created by live loading, the torsional moments due to initial and superimposed dead loads and the prestressing force may be expressed in the following terms:

- (1) Longitudinal moment M_{CDLP} at C.
- (2) Load w_{DLP} per unit length of deck, distributed along the longitudinal center line (Fig. 1-32a)
- (3) Torque t_{DLP} per unit length of deck, distributed along the longitudinal center line.
- (4) Torques T_{TP} which arise from cable forces at the supports of the bridge deck when the prestressing force is varied across the width of the deck.

The subscripts refer to location (C), dead load (DL), prestressing force (P) and torque at supports due to prestressing force (TP). In turn, the subscripts to be introduced for the initial and superimposed dead loads are IDL and SDL, respectively. Note that $DL = IDL + SDL$.

Now,

$$M_{CDLP} = M_{CIDL} + M_{CSDL} + M_{CP} \quad (31)$$

and, referring to Fig. 1-33,

$$w_{DLP} = w_{IDL} + w_{SDL} + w_P \quad (32)$$

The component w_P is due to the prestressing force, whose effect may be represented in terms of a uniformly distributed load by using the equivalent load method.⁴⁷ Similarly then,

$$t_{DLP} = t_{IDL} + t_{SDL} + t_P \quad (33)$$

The torque t_{IDL} is the resultant of the initial dead load times the eccentricity, Δ_{IDL} , measured from the longitudinal center line of the section, or

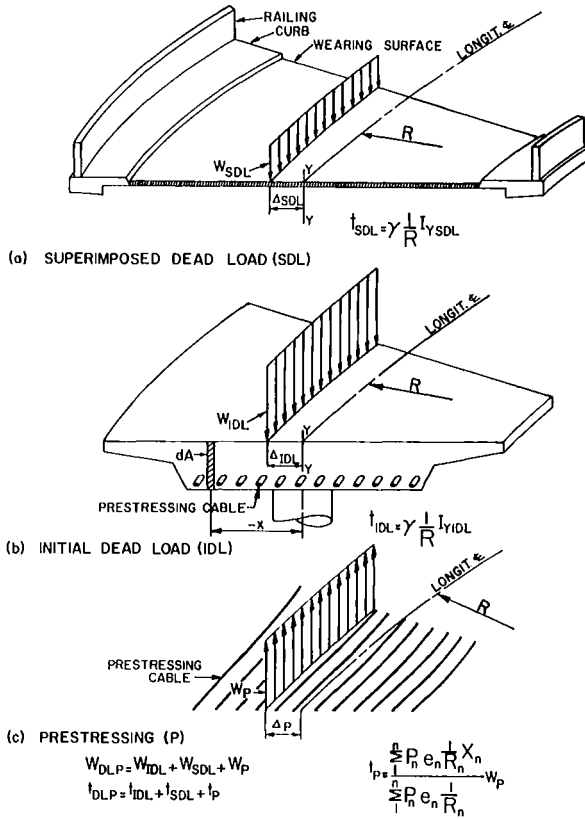


FIG. 1-33 UNIFORMLY DISTRIBUTED LOAD AND TORQUE DUE TO INITIAL AND SUPERIMPOSED DEAD LOAD AND PRESTRESSING

$$t_{IDL} = \gamma \frac{1}{R} \int_A (R + x) x dA = \gamma \frac{1}{R} I_{YIDL} \quad (34)$$

where

R = radius

x = distance of area dA from the longitudinal center line (Fig. 1-33b)

I_{YIDL} = moment of inertia about Y-Y axis

γ = density of material (0.150 kcf for concrete)

The torque $t_{SDL} = W_{SDL} \times \Delta_{SDL}$, or alternately:

$$t_{SDL} = \gamma \frac{1}{R} I_{YSDL} \quad (35)$$