

BOND AND DEVELOPMENT OF REINFORCEMENT



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ROBERTO LEON

Bond and Development of Reinforcement

A Tribute to Dr. Peter Gergely

Editor
Roberto Leon



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PREFACE

During the ACI Spring Convention 1997, ACI Committee 408, Bond and Development of Reinforcement, organized four sessions intending to assess the state-of-the-art in bond research, practical applications, and code developments. The sessions were co-sponsored by the Technical Group 2/5—Bond Models of the Comité Euro-Internationale du Béton (CEB).

The symposium drew a very large number of abstracts from its open call for papers. The final selection of the abstracts was based on both the technical merit and relevance of the topic. The majority of the presenters have provided written contributions to this volume, which is intended to serve as the third major conference on bond research in the past 15 years. The first major conference was organized in Scotland, UK, in 1982, and the second in Riga, Latvia, in 1992. Although these conferences are not formally connected, their intent has been to provide a forum for discussion of advanced mechanistic bond models, experimental research aimed both at model verification and development of design provisions, and practical design problems associated with bond and development of reinforcement.

The sessions held in Seattle, Washington, were organized into a symposium honoring Dr. Peter Gergely, a long-time member of Committee 408, who had recently passed away. This volume is dedicated to Dr. Peter Gergely, an outstanding engineer, educator and friend whose *joie de vivre* will not be forgotten by those who came into contact with him.

The editor would like to acknowledge the assistance of CEB 2/5 chairman, Dr. Rajels Tepfers of Chalmers University of Technology, Göteborg, Sweden, in organizing the symposium. Dr. Tepfers is retiring after a long and distinguished career in structural engineering that includes pioneering work in the area of bond and development of reinforcement.

Roberto Leon, Chairman, ACI Committee 408
School of Civil and Environmental Engineering
Georgia Institute of Technology
Atlanta, February 1998

A Tribute to Dr. Peter Gergely



The symposium held during the ACI meeting in Seattle in March 1997 and this Special Publication were organized to honor the contributions Dr. Peter Gergely made to the American Concrete Institute, especially his work in bond and anchorage, and his leadership of ACI Committee 408, Bond and Development of Reinforcement.

Peter Gergely was a professor of structural engineering at Cornell University until the time of his death on August 25th, 1995, after a long and courageous battle against cancer. He was born in Budapest, Hungary, on February 12th, 1936. He entered the Technical University of Budapest in 1954 but left in 1956 to join the freedom fighters in the Hungarian Revolution. He was one of the last resistance fighters to escape before Soviet troops invaded. He fled to Austria and then immigrated to Canada, where he received a bachelor's degree in civil engineering from McGill University in Montreal in 1960. He received a doctorate from the University of Illinois in 1963. Immediately afterwards, he joined the faculty of Cornell University.

Peter Gergely is survived by his wife Kinga, son Zoltan, and daughter and son-in-law, Illa and David Burbank. One of his greatest pleasures was to talk about his grandson, Istvan, who he was sure would carry on his love of chess.

Peter Gergely had an exceptional zest for life and took great pride in his Hungarian ancestry. He often spoke of his early school years in Budapest and about how lucky he was to have had wonderful, inspirational teachers, especially in mathematics and literature. These early experiences with great teachers inspired his teaching and his superb ability to motivate students to achieve their best. He was one of the most successful educators in the Cornell College of Engineering, and in 1995, he received a college award for his outstanding dedication to teaching.

Peter made major contributions to a wide variety of structural engineering problems. His research led to many important advances in understanding the mechanics of reinforced and prestressed concrete, and he worked to develop results that could be used to improve building codes. One of his early projects on cracking in concrete members resulted in an equation that has been in the ACI Building Code since 1971. He served on many ACI committees and was chairman of ACI Committee 408 from 1974 to 1980. Many of the provisions for development length and splices now in the Building Code were proposed and moved forward during his chairmanship.

In recognition of his outstanding contributions, Dr. Gergely received many ACI honors. He was elected a Fellow of the American Concrete Institute (1974); he received the ACI Delmar Bloem Distinguished Service Award (1981), was a co-recipient of the ACI Wason Award for Most Meritorious Paper (1993), and received the ACI Structural Research Award (1994). He was a co-recipient of the ASCE State of the Art of Civil Engineering Prize (1974) and the ASCE Raymond C. Reese Research prize (1976). Of all his honors, the one that meant the most to him was the Honorary Doctorate he received in 1992 from his alma mater, the Technical University of Budapest, given "for his outstanding international activities in advancing the development of his profession of mechanics and reinforced concrete."

His love of learning, joy in discovery, passion for music, chess, and all things Hungarian were evident to all who knew him. His dedication to teaching and to structural engineering will continue to impact his students and his colleagues. ACI will miss his unique talents and leadership abilities.

James O. Jirsa, Chairman
Civil Engineering Department
The University of Texas at Austin
Austin, January 1998

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Evaluation of Bond Performance in Reinforced Concrete Structures

by S. L. McCabe and S. J. Pantazopoulou

Synopsis: The increasing significance of performance-based criteria in modern structural design has motivated new considerations in bond design of conventional reinforcing steels, relating to more reliable assessment of both the demand and the supply sides of the anchorage/development design problem. Accurate identification of the required anchorage lengths needed to ensure strain compatibility, by proper consideration of the conditions affecting bond, is necessary to limit slippage of the steel relative to the concrete. While minimum development lengths calculated by designers imply that the bar is fully anchored, it is well established by experimental observation that in practice there is always some bar slip.

Recent research results from around the world provide the basis for improved understanding of the effects on bond performance of critical parameters such as confinement, spacing, and material properties. Much of this work has been empirical in nature and the applicability of empirical design expressions in calculations is limited. Nonlinear finite element calculations and other sophisticated analysis requires more information as to how the bond failure proceeds than simply an upper limit.

This paper will summarize the available information that exists both within North America through ACI and within the CEB as to the viable approaches and philosophies that can be applied to the bond problem. The range of application of the various techniques will be identified as will limitations and needs for more research.

Keywords: Anchorage; bond; bond models; design; detailing; development; finite element method; plasticity; reinforcement

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Steven L. McCabe is Associate Professor of Civil Engineering at the University of Kansas. His research interests include bond and development of reinforcement, earthquake engineering, finite element methods and damage mechanics, and the development of new reinforcing systems. He is an active member of CRSI, ASCE, CEB and ACI among others. Currently, McCabe is the chair of ACI Committee 439, Steel Reinforcement, a member of Building Code subcommittee 318-B, Development and Reinforcement, and a member of the CEB Bond Models Task Group.

Stavroula J. Pantazopoulou is an Associate Professor of Civil Engineering at the University of Toronto. She is active in ACI and CEB serving on ACI Committees 368, Earthquake Resisting Concrete Structural Elements and Systems; ACI 447, Finite Element Analysis of Reinforced Concrete Structures; ACI 408, Bond of Reinforcement, and the CEB Bond Models Task Group. Her research interests include bond and development of reinforcement, development design and analysis methods for earthquake resisting structural systems and the application of finite element in problems of continuum mechanics.

INTRODUCTION

In the traditional capacity design approach used in North America for reinforced concrete structures, designing for bond was considered to be a question of providing adequate strength along the bar anchorage for development of the bar yield force. Therefore, bond, as well as all other tension-failure controlled problems such as shear, were dealt with in design by establishing allowable stress limits for concrete, with stresses calculated from an admissible state of equilibrium without consideration of deformations. When needed, calculations of ductility capacity were performed after dimensioning and were associated with purely flexural response. Hence, of the two coordinates in the load-deformation response recorded in anchorage tests, the ACI code writing bodies have focused only on the load capacity side (with particular reference to yielding of reinforcement). The usual approach taken in the past was to synthesize a summary of experimental data for bond strength using empirical relations that subsequently formed the basis of the design requirements. These empirical relations attempt to describe the parametric dependence of bond strength on an array of important design variables. Although this process is fraught with debate, within ACI the main point of concern has really only been in selecting the form of the design expressions. The array of significant variables used, as well as the underlying philosophy of the approach, is generally accepted as a necessary accessory of the capacity design framework. Note that whereas deformation (slip) was universally acknowledged in the reported tests which formed the basis of current design requirements for bond, no systematic attempt had been made to process the available information in a manner analogous to the efforts placed on the side of strength.

The emergence of new performance based design approaches have refocused the problem of bond, since at the heart of these approaches lie considerations about deformation demand and supply, which inevitably bring forth questions about compatibility of deformations between concrete and the anchored bar throughout the various stages of response. For the design to meet certain predetermined performance requirements, it may no longer be adequate to simply assess the strength of a given anchorage without at the same time knowing the levels of deformation that would have to develop in order for the given strength to be realized, and whether these deformations can be realized in the structure without completely altering the assumed load path (redistribution). For this reason, code writing bodies which have already explicitly adopted performance-based design approaches, such as the CEB, require modeling of the complete load deformation envelope of the bond-slip re-

sponse of a given anchorage as reference for design purposes. Complete bond-slip relationships have been obtained either by empirical modeling of tests, or by analytical (mechanistic, numerical, etc.) methods. These approaches will be critically reviewed in the following sections with the objective to identify the salient features, range of applicability and common points between alternative methods. The scope of the review is limited to straight bar anchorages and will concentrate on monotonic loading, although many of the observations that can be made are common to both monotonic and cyclic loading.

REVIEW OF AVAILABLE MODELING APPROACHES

The array of important variables controlling the mechanics of bond both directly or indirectly appears from familiar experimental evidence to be quite extensive and has not been completely defined as of yet. The parametric complexity of the problem discourages formulation of straightforward mathematical descriptions. Even when restricting the scope to straight bar anchorages under monotonically increasing loads, the complexity of the problem persists: Position of the bar at casting, bar profile (rib geometry), transfer length, material properties, direct or incidental confinement of the surrounding concrete, and geometric boundary conditions all seem to not only have a profound influence on the shape and coordinates of the load deformation response, but also determine the mode of failure (splitting, pullout, or combination of the two modes).

Depending on the type of idealization and simplifying assumptions used in modeling the load deformation envelope for bond, published models can be classified as follows:

- (a) mechanistic vs. empirical models
- (b) global versus localized models
- (c) splitting vs. pullout models
- (d) numerical or hand-calculation models

CODE APPROACHES AND MODELS

The CEB design model and all ACI-related models are global, hand-calculation approaches: the influence of bond is evaluated by considering the free body equilibrium of a segment of the anchored reinforcing bar (Fig. 1). The critical section is at one end of the segment, at which point the applied bar stress is known. A distribution of bond stresses along the segment is postulated based on experimental results. The magnitude of bond stress is obtained from local bond stress - slip relationships that are often referred to as constitutive laws for bond. Note that this term is necessarily used loosely in this context, as bond is not an intrinsic physical property but the mechanical response of the bar - concrete assembly under certain specific boundary conditions. The force in the other end of the bar segment is obtained directly from equilibrium considerations. If the characteristic stress-strain relationships of steel have a positive definite tangent stiffness throughout the strain range, then it also is possible to obtain the deformations and the distribution of stresses along the anchored bars.

(1) Models Adopted by ACI Committees 318 and 408:

Various ACI committees (408, 318) have adopted several simplified versions of the above model over the years. In all cases the length of the segment considered involves the entire anchorage length, L_d , whereas the known stress input at the critical section is usually the yield or ultimate stress capacity of the bar (L_d corresponds to the length required for the stress in the bar to become zero at the unloaded end). The intensity of bond stresses u_b is taken as constant along the anchored length, set equal

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to an allowable maximum value multiple of the square root of f'_c in order to acknowledge the fact that bond failure is associated with tension failure of concrete in the principal directions. Note that by definition, bond stress is proportional to the rate of change of bar stress along the anchored length, therefore as a variable it has only local physical significance. Therefore the assumption of constant bond stress distribution is a simplifying idealization adopted for the sake of mathematical convenience. Experimental evidence suggests that whereas this assumption leads to acceptable results for normal weight concrete and uncoated bars, it cannot adequately model the behavior of epoxy coated bars, or of bars anchored in high strength concrete. (In these cases the bond stress distribution appears to be highly nonlinear with peak intensity at the loaded end of the bar.)

The apparent advantage of the ACI approaches is two fold. First, despite the overall empiricism inherent in the assumptions made, there is an underlying physical model, which explicitly establishes equilibrium in order to define the required L_d (in design) or in order to define allowable values for u_b (from experiments, where L_d is known). Second, the resulting expressions are simple enough for routine hand calculations. The obvious disadvantages of this approach are, first in the underlying assumptions of constant bond stress, which, as was mentioned in the proceeding, are unrealistic in the case of high strength concrete or epoxy coated steel. Secondly, the model provides no detailed information about the behavior of the bar under load; rather, the result is a binary answer as to the adequacy of the anchorage length for the development of the yield strength of the reinforcement. In a performance driven design framework it is necessary to predict structural behavior at specified load demands. Here it is not sufficient to know whether the anchorage length can or cannot develop the bar force capacity. For example, if the member is stressed up to the yield point or slightly beyond yield, then the bar slip relative to concrete will be limited and perhaps not important. In contrast, if the system is subjected to large inelastic deformations, then the demands on the bar and surrounding concrete will be such that significant bar slip may develop. This slippage will result in reduced apparent stiffness of the member and reduced energy dissipation capacity in the overall system. The assumption of perfect bond leads to higher estimates of the structural stiffness and to lower displacements than the actual values, the more so for systems undergoing cyclic load reversals. Sozen (1974) observed that the plastic rotation of a concrete joint may be doubled due to bond slip. Because this amount of increase in rotation affects significantly the story drift, it is important in design to limit bar slip to small values. In the ACI 408 report (1992), it is recommended that bond stresses be limited to about 80% of the ultimate values when designing for cyclic loads. The objective of this recommendation is to limit the demand on the bars – and consequently the intensity of bond stress – to values below the levels that would cause deterioration of the surrounding concrete.

The ACI Design Equations for Bond: Over the recent years, ACI design expressions have progressively evolved from simple formulae to the more complex but also more accurate expressions of today. The process was primarily driven by an ever-expanding database of published experiments. Typically, best-fit lines have been used to describe the relationship between test parameters and performance. However the procedure was packaged, the resulting design expressions were intended to ensure (based on the experimental evidence) that the anchorage length was sufficiently long to fully develop the bar nominal yield force, while maintaining the bond stresses below allowable limits. The reason why converging to the design expressions of today has taken several iterations is that it has been shown over time, that the experimental values are sensitive to specimen shape and support conditions during testing. The perceived significance of other conditions, such as position at casting, has never been properly linked to physical properties of concrete such as porosity and composition. With such vital information missing from the database, and with the ongoing emergence of new qualities of concrete materials, the relative significance of

this parameter on the anchorage length is continually changing. An added layer of complication in the process of deriving design expressions from a collection of tests has been related to the mode of failure of the tests considered; note that whereas the most challenging aspect of bond design is to protect against splitting failures, until recently the majority of available tests in the literature had ended as pullout failures. Uniform specifications for designing and testing of bond specimens have been proposed only recently by the research community.

Early expressions for bond simply divided the yield force by the bond area surrounding the bar:

$$u_b = \frac{A_b f_s}{\pi d_b L_d} = \frac{d_b f_s}{4L_d} \quad (1)$$

In the ACI 318-56 Building Code, the limiting bond stress was taken as $0.10 f'_c$ for bottom deformed bars and $0.07 f'_c$ for top bars, with an upper limit on cylinder strength of 3500 psi. Thus an average bond stress of 350 psi (2.41 MPa) was assumed. Ferguson (1965) recommended the use of even lower bond stresses than permitted by ACI 318 in applications of low cover (i.e., a limit of $0.05 f'_c$ for smaller sized bars and $0.04 f'_c$ for No. 9, 10 and 11 bars). For Grade 60 (420 MPa) No. 8 bar embedded in 4000 psi (30 MPa) concrete, a development length of 30 inches (762 mm) would be required by this formula at a bar stress of 24 ksi (170 MPa) – the typical value employed in allowable stress design in 1956. Thus, here was the source of the 30 bar diameter rule of thumb used for many years in concrete design and construction in North America.

This method for computing development lengths was used until the early seventies when strength design replaced the allowable stress design. A primary study that drove the development of improved equations was the work of Orangun, Jirsa and Breen (1975, 1977). Design was based on the following empirical relationship:

$$u_b = \left[1.2 + \frac{3C}{d_b} + \frac{50d_b}{L_d} + \frac{A_{tr} f_{yt}}{500s d_b} \right] \sqrt{f'_c} \quad \text{U.S. Customary Units} \quad (2)$$

$$u_b = \left[0.1 + 0.25 \frac{C}{d_b} + 4.15 \frac{d_b}{L_d} + \frac{A_{tr} f_{yt}}{41.52s d_b} \right] \sqrt{f'_c} \quad \text{S.I. Units} \quad (3)$$

The above equation models bond strength as a linear function of bar diameter. The term $3C/d_b$ reflects the confining influence of cover and the negative effects of close bar spacing on bond. The square root power of f'_c is used in order to indicate that bond strength is a function of the tensile capacity of concrete and increases at a slower rate than cylinder compressive strength. Passive confinement is accounted for in a separate term that is proportional to the confining steel area. After simplification of terms, and by ignoring the confinement term, the above equation may be written in terms of the required development length:

$$L_d = \frac{0.04 A_b f_y}{\sqrt{f'_c}} \quad \text{U.S. Customary Units} \quad (4)$$

$$L_d = \frac{0.02A_b f_y}{\sqrt{f'_c}} \quad \text{S.I. Units} \quad (5)$$

Rearranging this equation provides a bond stress relationship of

$$u_b = \frac{8\sqrt{f'_c}}{d_b} \quad (6)$$

with an upper bound on bond stress of 625 psi (4.31 MPa) using ACI 318 limits.

Similar work by ACI Committee 408 has led to the following equation for the required development length of Grade 420 (60 ksi) reinforcement:

$$L_{db} = \frac{5500A_b}{\phi K \sqrt{f'_c}} \quad \text{U.S. Customary Units} \quad (7)$$

where K is the smaller of

$$(a) 0.5d_b + C_c + K_{tr} \text{ or}$$

$$(b) 0.5d_b + C_c + \left[\frac{\sum K_{tr}}{n} \right]; \text{ but no larger than } 3d_b$$

In the ACI 318-95 Building Code, the bond expressions have been reevaluated and restated in terms of bar diameter. The basic equation is,

$$L_d = \left(\frac{3d_b f_y}{40 \sqrt{f'_c}} \right) \left(\frac{\alpha \beta}{(c + K_{tr})} \right) \left(\frac{d_b}{d_b} \right) \quad \text{U.S. Customary Units} \quad (8)$$

For standard configuration, this equation reduces to

$$L_d = \left(\frac{d_b f_y}{20 \sqrt{f'_c}} \right) (\alpha \beta) \quad \text{U.S. Customary Units} \quad (9)$$

for No. 7 bars and larger with at least one bar diameter of cover, c , and minimum stirrups provided, K_{tr} . The coefficient in the denominator is 25 for smaller bars. Using an upper bound of 10,000 psi (69 MPa) for the compressive strength of concrete, the larger bar series has a required development length of 30 bar diameters. This corresponds to a bond stress of 500 psi (3.45 MPa).

From the preceding review it is evident that the ACI philosophy over the years has been consistent. The objective has been to limit bond stresses to values in the range of 500 - 600 psi (3.45 - 4.14 MPa), which are considered conservative based on experience from experimental studies. For different configurations in terms of bar spacing, cover or confining steel, adjustment factors are used to represent the influence of these parameters on the required development length. For example, the

K_{tr} term in the ACI 318-95 equation and the associated cover terms represent these factors. This approach was calibrated with research results and was originally developed by the ACI Committee 408 (1990).

In curve-fitting the available database of bond tests, the ACI 408 approach has been to include a ϕ factor in the analysis to account for localized uncertainties in the material properties and configuration. A ϕ value of 0.80 in the denominator is used to increase the required development length. On the other hand, the ACI 318 approach has been to avoid compounding the ϕ factors that appear in the bond expression and in flexure, shear or other applications. For this reason, the 318 expressions have a factor close to 1.0. It was mentioned in the introduction that an explicit relationship for bond stress vs. slip is not explicitly addressed in the ACI documents relating to bond. This is justified, since the overall approach of the ACI philosophy is to provide rules to preclude significant amounts of plasticity; in this range of response, detailed mechanistic models are not essential. Elaborate analytical models for bond have been developed by the research community, but are mostly used in the study of problems involving a significant degree of nonlinearity, and fall beyond the scope of the ACI framework of design.

(2) The CEB Approach in Modeling the Problem of Bond:

The CEB Model Code (1990) has taken a somewhat different approach to design: it is required that performance be checked within defined limit states. Two primary limit states are considered, the Service Limit State (SLS) and the Ultimate Limit State (ULS). The SLS represents a comparable state to that implied by the serviceability checks within the ACI 318 Building Code. Straightforward design rules are provided; the design objective is to ensure minimum levels of performance for crack control, deflections and other items.

The ULS represents a plastic limit state where the structure and its components are assumed to undergo significant inelastic deformations. Additional checks are needed to ensure the design satisfies pertinent performance and capacity requirements. The use of computer-based analysis, and in particular the Finite Element Method, is a hallmark of this approach.

A significant example of the differences between the SLS and ULS is that of moment redistribution. When considering the ULS, the Model Code allows up to 50% of the negative moment to be redistributed to the positive part of the moment diagram. Such levels of redistribution are not allowed by the ACI 318 code, where the maximum redistribution is 10%.

The moment redistribution provisions have an important corollary and that is the basic performance of the reinforcing steel. European reinforcing steels are generally comparable to ASTM A 706 steel. They are weldable, low-alloy steels of good quality and material traceability. The significant difference is the relative lack of strain hardening in these Euro-steels as compared to U.S. steels. For example, a Class A steel with a 500 MPa nominal yield has a required ratio of tensile strength to yield strength of 1.08. Within ASTM-A 706 it is required that the ratio of ultimate to yield strength be a minimum of 1.25. From a design perspective this means that a member forming a plastic hinge will have a relatively constant amount of moment capacity as the hinge is loaded and rotation occurs. In a similar situation but using U.S. steel the moment capacity will increase substantially as the steel hardens. The implication is that the increase in capacity will provide additional strength when needed under severe demands. In addition, considerations prevail relating to increased need for confinement as compared to the first onset of yielding as the internal loads are changing within the member. Therefore, the different amounts of moment redistribution permitted by the two codes complement the relative characteristics of

the reinforcing steels considered in the two cases.

In practical applications forces and moments within the structure may be computed either by nonlinear analysis, linear analysis, linear analysis with redistribution, or by plastic analysis. Each of these options has its own set of requirements and models that must be followed. It is not the intent of this paper to restate the entire Model Code; however it is evident that the Model Code is compatible with many alternative approaches of analysis and design.

Compared to the ACI approaches, another source of difference is the application of factors both to increase the level of loads on the structure for design (load factors) but also the reduction of material strengths by other factors. The design strength of reinforcing steel is defined as

$$f_{ydc} = \frac{f_{ytk}}{\gamma_s} \quad (10)$$

where the value of the partial factor γ_s is 1.15 for sustained loads. In the specific case of bond and development, similar comments can be made. The CEB requires that the designer check the bond stress to ensure that levels are not excessive.

CEB Design Rules: The CEB defines the basic length to transfer the yield force of a bar as

$$L_d = \frac{d_b f_{yd}}{4f_{bd}} \quad (11)$$

where d_b is the bar diameter in mm, and f_{yd} is the bar design yield force. Term f_{bd} represents the design bond stress of concrete, given by

$$f_{bd} = \eta_1 \eta_2 \eta_3 f_{td} \quad (12)$$

where η_1 is a geometry factor taken as 2.25 for ribbed bars, η_2 is an orientation factor for bond (a value of 1.0 is used in most cases). η_3 is a bar size factor set at 1.0 for 32 mm and smaller bars and taken as $(132 - d_b)/100$ for larger bars. The term f_{td} is the design tensile strength of concrete, defined as the characteristic tensile strength divided by 1.50.

The design anchorage length is determined as

$$L_{d,net} = \frac{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 L_{d,s,cal}}{A_{s,eff}} \geq L_{d,min} \quad (13)$$

In the above, factors α_i adjust the length based on:

- $i = 1$: Accounts for the effect of bar form, for straight bars = 1.0
- $i = 2$: Accounts for the presence of welded transverse bars = 0.7
- $i = 3$: Accounts for confinement provided by the concrete cover:

$$\text{For straight bars} = 0.7 \leq \left\{ \frac{1 - 0.15[c_d - d_b]}{d_b} \right\} \leq 1.0$$

i = 4: Accounts for confinement provided by traverse steel:

$$\left[1 = \frac{K (\Sigma A_{st} - 0.25 A_s)}{A_s} \right]$$

where K = 0.1 for bars in stirrup corners and 0.05 for bars away from stirrup corner

i = 5: Effect of pressure perpendicular to the splitting plane

For high bond bars, the product $\alpha_3 \alpha_4 \alpha_5 = 0.7$ as a limit. For a typical 25 mm diameter bar in a 30 MPa concrete, analogous to a No. 8 U.S. bar in a 4000 psi concrete, the required development length would be about 780 mm or 31 inches.

An attractive feature of the CEB approach is that it supplies the designer with a simplified bond model that may be used in practical applications. The model is based on research results and uses a nonlinear law to relate the applied bond stress, τ , to bar slippage, s . The model is illustrated in Fig. 2. The characteristic values s_1 , s_2 and s_3 are based on the state of confinement of the concrete and the quality of bond that is thought to be present.

The four segments of the relationship are defined as follows:

$$\tau = \tau_{\max} \left(\frac{s}{s_1} \right)^{0.4} \quad \text{for } 0 \leq s \leq s_1 \quad (14)$$

$$\tau = \tau_{\max} \quad \text{for } s_1 \leq s \leq s_2 \quad (15)$$

$$\tau = \tau_{\max} \frac{(\tau_{\max} - \tau_f)(s - s_2)}{(s_3 - s_2)} \quad \text{for } s_2 \leq s \leq s_3 \quad (16)$$

$$\tau = \tau_f \quad \text{for } s_3 \leq s \quad (17)$$

Terms for these equations are defined in Table 1. The parameter $\tau_{\max}$ represents the bond strength. Once the strength is attained, the bar slips at constant stress up to a value of slip, s_2 , of 0.6 mm for unconfined concrete and 3.0 mm for confined concrete. Beyond that point the bond resistance decays with increasing bar slip and it reaches a residual value of τ_f at a slip value s_3 of approximately 2.5 mm.

ANALYTICAL BOND MODELS

The heart of the issue in bond modeling is the bar slippage that occurs while the system is being loaded. If compatibility of strain between bar and concrete is maintained, then no relative movement occurs (i.e., no bar slip). This behavior is not found in actual members where there is always some bar slip as the reinforcement is loaded. Moreover, the assumption of perfect bond can be unconservative in analysis. Some shear failures are in fact bond failures that cause a reduction of shear capacity as a secondary effect (McCabe 1997). From a practical viewpoint, it is of interest to determine, for a given reinforcing arrangement, the amount of bond-slip likely to develop and its effect on performance. Note that if the slippage is too great then stiffness is reduced causing large member displacements. This behavior has been well-documented in experimental studies of monotonic and cyclically loaded structures and members. In the case of monotonic loading, local bond-slip relationships are difficult to measure experimentally. Results are generally reported as force at the loaded end vs. slip at either the loaded or unloaded end. Darwin et al. (1992) reported that the amount of actual slip measured under load was significant, even at low bar stress levels. Adhesion of the bar to concrete is seen to be destroyed nearly immediately. Following loss of adhesion, the amount of slip increases with increasing bar force in a nonlinear manner leading to complete loss of resistance to slippage at about 0.002 inches (0.05 mm) for a 25 mm diameter bar. The behavior noted is specimen and configuration dependent with changes in confinement, stirrups and cover, and bar size having large effect on the amount of slippage and the slope of the curve. Thus, the actual bar slip vs. bar stress relationship is complex and one that is quite difficult to characterize experimentally in a manner that can be readily used in design. Rather the information is of more interest to researchers.

A traditional approach taken in modeling the behavior is to use experimental data and represent the experimental trends by rules. These empirical approaches are very practical and may be the only workable model for large scale computations where complete structural systems must be modeled. However, more robust analytical models of the bar-concrete interface behavior have been proposed; these are derived from basic principles and to a large extent they avoid the limitations implicit in empirical models. Studies have included three primary approaches (ACI 408 1992): (1) solve for bond performance based on an assumed set of equilibrium and compatibility equations; (2) assume a function to model bond-stress distribution along the bar; and (3) use non-associated plasticity theory or fracture mechanics to model the interface between the steel and concrete where the bond develops.

Models that fall in categories (1) and (2) represent the background of the existing code methods and as such, have been implicitly described in the preceding sections. Alternative modeling approaches that attempt to mathematically define the mechanics of the bond are reviewed below:

(a) Mechanics-Based Models for Bond--Recently, a number of experimental and analytical studies have been presented, seeking a more fundamental definition of the local bond slip relationships (Gambrova et al. 1989, Giuriani et al. 1991, Cox and Herrmann 1992, Rosati and Schumm 1992, Rostasy et al. 1987). Giuriani et al. (1991) considered the development of an axially loaded bar, confined by transverse stirrups and anchored in concrete which has already developed splitting cracks. Provided that some reinforcement is crossing the splitting plane, bond action is still possible past the development of splitting. Based on the work of Gambrova et al. (1989), the local bond stress-slip relationship is expressed as a function of the crack opening, w :

$$u = u(w, s) = u_{m,w} \left[1 - \exp \left[- \left(\beta_1 + \left(\frac{\beta_2 w}{d_b} \right) \right) \left(\left(\frac{s}{d_b} \right) - \left(\frac{\gamma_2 w}{d_b} \right) \right) \right] \right] \quad (18)$$

where,

$$u_{m,w} = u_{m,0} \left[1 - \gamma_1 \left(\frac{w}{d_b} \right) \right]$$

and $u_{m,w}$ the maximum bond stress that can be reached for a given crack opening w ; $u_{m,0}$ the maximum bond stress that can be reached if there is no splitting crack ($w = 0$); γ_1 , γ_2 , β_1 and β_2 are coefficients determined from experimental data. For a given crack opening w , the stress developing in the transverse reinforcement crossing the crack is estimated as follows:

$$\sigma_t = E_s \sqrt{\frac{\alpha_2}{\left(\frac{\alpha d_t}{d_b} \right)^2} \left(\frac{w}{d_b} \right)^2 + \frac{\alpha_1}{\frac{\alpha d_t}{d_b}} \left(\frac{w}{d_b} \right) + \alpha_0} \quad (19)$$

where the coefficients α_2 , α_1 , and α_0 are related to the assumed anchorage conditions of the transverse reinforcement (Giuriani et al. 1991). Additional confining action, such as that provided by the residual tensile strength of cracked concrete was also accounted for in the model, and was expressed as a function of the crack opening and the maximum aggregate size, d_a :

$$\sigma_{rc} = \frac{\sigma_0}{\left[k \frac{w}{d_a} + 1 \right]} \quad (20)$$

where σ_0 the radial confining stress at $w = 0$ and k an empirical constant. The local bond stress is also related to the total radial force produced by the anchored reinforcement for a given splitting crack opening:

$$u = u_0 \left(\frac{1}{1 + k_1 \frac{w}{d_b}} \right) + u_1 \left(\frac{1}{1 + k_2 \frac{w}{d_b}} \right) (\Omega \sigma_{st} + B \sigma_{rc}) \quad (21)$$

where W is the total cross section of stirrup legs crossing the splitting plane divided by the cross section of the principal bar A_b , and B is the total area of concrete in the splitting plane divided by A_b . The nonlinear system of equations presented here is solved numerically to establish the relationship between bond stress, crack width, slip, and transverse confinement. The pitfall of the model is in the many empirical constants required for the definition of the various stress components; these were obtained from a series of specially designed experiments by Gambarova et al. (1989). The merit of the model is in that it takes a fundamental approach to the mechanics of bond, and therefore provides a crisp mathematical relationship between the important design variables.

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(b) A Plasticity Model for Bond--Cox and Herrmann (1992) used an alternative analytical approach in order to derive local bond stress - slip relationships from consideration of the mechanics of anchored bars. Key to this formulation is that bond stress and slip are fully coupled with radial confinement stress and dilation. As before, the objective was to produce a bar scale model that would be able to reproduce rib phenomena, while at the same time be usable in member scale computations. In this study, a two degree of freedom non-associated plasticity model for bond was formulated based on the experimental results of Malvar (1991) and Gambarova et al. (1989). Plasticity was chosen as the framework for studying bond because of the existence of a contact interface between bar and concrete, with Coulomb-like frictional behavior.

The model relates local slip and radial dilation to bond stress and radial confinement stress in a finite thickness region around the bar- so called process zone. The generalized stress and strain vectors are defined as,

$$\mathbf{Q}^T = [\tau, \sigma]; \mathbf{q}^T = \frac{1}{d_b} [\delta_t, \delta_n] \quad (22)$$

where τ and σ are the shear and normal stresses at the interface, d_b the bar diameter, and δ_t and δ_n represent the slip and the radial dilation, respectively (i.e., these are relative displacement quantities between the inner and the outer surface of the interface zone). Using experimental data, the following elastic moduli were derived, relating the stress vector $\mathbf{Q}$ with the vector of relative displacements $\mathbf{q}$:

$$\mathbf{D}^e = \mathbf{E}_c \begin{bmatrix} 0.1 & -0.012 \text{sgn}(\delta_t) H(-\delta_t - \varepsilon) \\ -0.012 \text{sgn}(\delta_t) H(-\delta_t - \varepsilon) & 0.04 \end{bmatrix} \quad (23)$$

In the above, E_c is the elastic modulus of concrete, H is the heaviside step function, and ε denotes *numerical* zero. The physical interpretation of the terms in the property matrix is as follows: The diagonal terms are obtained either from experimental measurements or from calibration studies of anchorage tests. The off-diagonal terms are small, reflecting that prior to the occurrence of plastic deformation, negligible radial stress will arise if shear deformation is imposed at the interface (similar reasoning for the other entry). As used, the heaviside function will set the off-diagonal terms to zero in the case of no-slip (because in that case the problem reduces to one of radial contraction, with no resultant shear force along the bar axis). The sgn function adjusts the direction of the stress vector to the direction of applied slip, since the stress state considered is a contact problem.

The strength of the interface is regulated by a yield surface:

$$\left[\frac{\tau}{f_t} \right] = C(d) \left(W_e(d) \left[1 - e^{-\alpha \left(\frac{-\sigma}{f_t} + \rho(d) \right)} \right] + M(1 - W_e(d)) \left[-\frac{\sigma}{f_t} \right]^\alpha \text{sgn} \left(-\frac{\sigma}{f_t} + \rho(d) \right) \right) \quad (24)$$

and a non-associated flow rule:

$$[\dot{\mathbf{q}}^p]^T = \langle \lambda \rangle [\text{sgn}(\tau), g] \quad (25)$$

In these equations, f_t represents the tensile strength of concrete, $C(d)$ and $\rho(d)$ are the isotropic and kinematic hardening functions respectively, g is the rate of

plastic dilation with respect to plastic slip, and λ is the loading index. Parameters C , W_c , and p are functions of a *damage measure* d , defined as the ratio $(\delta p/s_r) < 1$, where s_r the rib spacing (the upper limit 1 was enforced based on the results of Eligehausen et al. (1983) which show that bond is completely failed and only the frictional resistance remains when slip is equal to rib spacing). The selected flow rule has the following parametric behavior: For small values of the normal stress σ , failure is dominated by longitudinal and transverse cracking and the rib is able to slide along the interface surface causing small amount of crushing and large dilation. For large values of the radial confining pressure failure is dominated by local crushing and by shearing of the concrete between ribs, and in this case, the amount of dilation is limited. The merit of this formulation is that it only requires two input property variables for concrete (i.e., tensile strength and elastic modulus), and two geometric properties of the reinforcement (diameter and rib spacing). The model was used successfully to correlate several independent experimental studies.

(c) Detailed Modeling of the Bar Profile - Fracture Mechanics Approach

Ideally, many of the limitations and influences on the observed bond behavior which are caused by the experimental setup could be avoided or quantified if a detailed mathematical model of an anchored bar embedded in concrete could be used to supplement the experimental studies. A complete model accounts for the exact geometry of the bar surface (such as spacing and height of the ribs), the constitutive properties of concrete under triaxial states of stress, and represents accurately the boundary conditions of both concrete and steel in all directions. This is possible only with three dimensional finite element idealizations of the anchored length of the embedded bar. In such analyses, local bond - slip relationships are no longer input properties of the model, but rather, they emerge from the computations as a result of prescribed geometry, material properties, and global boundary conditions. Although the computational demands of the finite element models of the type described in this section render them impractical for large scale modeling of complete structural systems, yet, results from analysis of anchored bar examples could be particularly useful in deriving characteristic bond stress - slip relationships for global models, as was done in the past with experimental results.

Several 3-D finite element models have been published in recent years, including work by Ingraffea et al. (1983), Ozbolt and Eligehausen (1992), Pijaudier-Cabot et al. (1991), Mainz et al. (1992), Rots (1989), Eligehausen (1989), Brown et al. (1993). The most difficult aspect in the development of these models is in the definition of constitutive relations for the stress-strain response of plain concrete. To reduce the amount of computational effort required for a solution, most published finite element models consider the nonlinearity in the material behavior only for a narrow zone of elements which includes the contact region between reinforcement and concrete. Stress concentrations, which arise at the interface of the ribs with the interlocking concrete, cause localized damage and irreversible energy absorption. This region is referred to in the literature as interface zone, fictitious interface (Pijaudier-Cabot et al. (1991), Rots (1989), Eligehausen (1989)), or process zone (Ingraffea et al. (1983), Malvar (1991), Cox and Herrmann (1992)) and is modeled by nonlinear elements with special constitutive properties. It is common practice to model concrete as linear elastic outside the interface zone.

Because the process of stress transfer from rib to concrete invariably occurs by cracking and crushing of the surrounding concrete, it has been argued that modeling bond warrants a fracture mechanics approach (Rots (1989), and Eligehausen (1989), Mainz et al. (1992), Brown et al. (1993)). Formulation of the models in this framework classifies the deformation mechanisms that contribute to bond-slip into four types, i.e., elastic deformations, secondary cracking (cone shaped), longitudinal splitting cracking (radial), and crushing in front of the ribs (Ingraffea et al. (1983), Rots (1989)). Of those, it is commonly assumed that secondary and longitudinal

cracking are the most important contributions. These are closely related to the occurrence of fracture (cracks). In numerical analysis, they can be accounted for by (1) lumping all contributions to the fictitious interface, or (2) by modeling the individual contributions in detail (in a narrow layer of interface elements which is treated as a continuum with all the failure mechanisms that go with this assumption, i.e., crushing, softening, cracking, etc.). The latter approach is computationally very intensive and hence not appropriate for practical applications. Research by Ozbolt and Eligehausen using a micro plane model offers the promise of solving this type of problem with a fracture mechanics theory but in a more efficient manner since the actual characteristics of the bars do not need to be modeled (Ozbolt and Eligehausen, 1992).

Through such analytical formulations, bond strength may be studied as a function of rib height, rib shape, concrete cover, lead length, embedded length, and transverse reinforcement. While the effort in researching models to predict bond performance has been significant, the ACI 318 Building Code and other design codes used in the US have not yet considered such analysis options and instead rely on the experimentally based anchorage equations to determine adequacy of embedment. No official endorsement of any particular approach in modeling bond and bar slippage is provided.

INFLUENCE OF BOND ON STRUCTURAL PERFORMANCE

Determining the performance of reinforced concrete structures under load is of great practical importance, particularly when considering dynamic load conditions such as strong ground motion or explosion. From a design perspective it is desirable that the structure survives such extreme load conditions with no loss of strength although it may undergo significant displacements and rotations at members and connections. To achieve this objective the designer needs pertinent computational and modeling tools that will enable determining the characteristics of structural response during loading.

Consider for example the familiar practice of moment redistribution. It can be an attractive design alternative to move a portion of the negative moments from support locations and to relocate this demand into the positive moment portion of the span. During response to lateral load, flexural members seek moment capacity at the negative moment locations (supports), due to large rotational demands at these points. If insufficient capacity is available, a plastic hinge will form at the corresponding member end, whereas the residual load not accounted for is carried by positive moments. It was mentioned earlier that at present, ACI practice limits this redistribution to about 10%, while CEB allows designers to redistribute up to 50% of the total negative moment demand. However, the designer is required to verify that the system can indeed support this level of demand along with the resulting rotations of the hinge.

Observations on Bond Modeling

From the foregoing discussion it is evident that both ACI and CEB have provided a comprehensive set of simple rules that govern the design of reinforced concrete structural systems. These rules only require hand calculations and define the required anchorage lengths for development of the ultimate bar force. The objective of this requirement is to ensure that given the detailing and the section geometry, the engagement of the bar deformations in concrete are sufficient to anchor the bar when it develops its full force capacity.

In determining the anchorage status the basic question is stated in the form of empirical equations developed over the years from experimental studies of bond.

Whereas the equations are simple, the behavior certainly is not; however, the equations give the sense that the reinforcing bars truly are anchored in every sense of the word (i.e., there is no movement of the bar as it supports load). Though the concept of a perfectly bonded bar in concrete is convenient in design, it is not representative of the true behavior of the bar under load. As the load increases bond-slip occurs, which may become large enough to push the bar deformations (ribs) into the surrounding material resulting in dilation of concrete and a resulting tensile stress field. If the slippage continues, then at some point fracture initiates as a result of stress concentration near the first loaded rib. The fracture energy is released instantaneously (brittle response) whereas the fracture surface grows through the thickness and along the bar. The splitting crack develops along a minimum energy path in concrete, from the bar surface to the nearest free surface of concrete (i.e., through the cover).

Thus, the checks that are made by the designer using either the ACI or CEB code equations are an effort to preclude large enough bond slippage to cause a splitting fracture of the cover over the reinforcement. The code equations are an attempt to deter this slippage by providing a long enough bar embedment, or large enough confinement through cover or transverse steel.

Experience has shown that these equations lead to reasonable designs and that research and design expertise has provided a framework that generally provides a correct assessment of the state of stress and slippage of the bars being developed. However, several issues are becoming increasingly relevant. One is that much of the data that were used in deriving these equations were obtained from specimens of low to moderate strength concrete whereas high strength concrete is more widely used today. Bond in high strength concrete has been shown to be quite different in that less redistribution of load occurs in that case from one deformation (rib) to another. The result is less ductile bond performance and a greater tendency to "unzip" the bar from the concrete as the bond failure begins. An other issue is that many new reinforcing systems are becoming important such as welded wire, high strength bars, headed bars; all these products behave differently from the classical deformed bars (350-520 MPa yield strength) that were used in the past for much of the supporting experimental work. Today we are essentially beginning to use a new set of materials that behave differently than those in use even as recently as 10 years ago. Because of the nature of the design problem with such new materials, accurate determination of bond behavior becomes even more important than before, since a localized bond failure would be more likely to result in a large scale problem in a member, connection or in the structure as a whole.

It also has become apparent that the existing equations are truly only applicable when designing new structures under conventional loads. The checks provided by the code equations may not be sufficient in assessing existing structures that have been significantly modified for some new use or subjected to an unforeseen overload such as from blast or earthquake. In a case such as this there are no simple design alternatives. One option is to apply the Finite Element Method (FEM) or other computer-based structural models in order to assess how the system behaves under load. This type of analysis can provide essential information as to how the system moves and redistributes load. A key ingredient is modeling of the nonlinearity that occurs as yielding of the steel and cracking of the concrete occurs. The application of nonlinear FEM has been the source of ongoing research and additional work is needed to provide straightforward, practical tools for use by designers. However, it is indeed becoming more and more apparent that nonlinear FEM may be the best practical alternative when evaluating structures that have been overloaded, or loaded in a manner unforeseen in the original design.

The accuracy of the overall analysis lies in modeling of the phenomena at the steel-to-concrete interface. The assumption of perfect bond, in essence the lack of a

bond model, is an easy way to model the structure but produces unconservative results; in that case the steel-concrete system is assumed to work at a perfect efficiency transmitting the strains of the steel directly into the concrete. A major problem is lack of consensus of how exactly to model the bond. The general view has been that any individual bond model is capable of providing good results so long as the model has been calibrated against experimental data. Thus, given a set of data the model may be tuned to match these values. While this approach may be an acceptable step in process of developing new modeling techniques, it is not proper for design; here efficient methods for determining conservative estimates of anchorage capacities are required.

Therefore, the assessment of structural performance using nonlinear FEM will depend in part on removing much of the guesswork that exists at the present time in modeling of bond. This need is one that can be filled by organizations like ACI and CEB. There is a clear need for guidance and for clear concise modeling rules for bond. This need will only become greater as more and more structural engineers turn to nonlinear analysis methods to determine the adequacy of members and structures under load. At the present time, designers may choose between utilizing the present CEB bond model which is over a decade old and developing case-specific models consistent with their own interpretation of the literature. Those using the ACI guidelines for bond will find no information as to bond modeling in the Building Code and will find only passing reference to the concept in the various bond reports of ACI Committee 408.

Oversupply of Bond Resistance or How Much Bond is Too Much?

The possibility of oversupply of bond resistance has become recently an important consideration of performance based design. In the past, it was generally assumed that there never can be too much bond and that improvements in bond through changes in the bar geometry or other factors, will only improve the performance of the structure. However, this is not supported by either theory or experiment: note that if bond is improved beyond the ability of the member to sustain it, the mode of failure will change from pullout to splitting. In such circumstances there is too much pullout resistance for the member as detailed. To remedy this situation additional transverse reinforcement would be necessary in order to confine the bars and thereby increase the splitting bond capacity. This may mean additional cost that must be weighed against the gains in performance. It may indeed be the case that the additional bond is not needed and that the structural performance is acceptable if the bar slips, provided that the slippage is controlled and accounted for in the design.

With regards to the performance of plastic hinges, it has been suggested that increases in bond may lead to decreases in plastic hinge rotation (referred to in the literature as rotational capacity). Load redistribution in structures subjected to severe excitations occurs because of ductile performance of those hinges. Thus, any loss of ductility resulting from increases in bond may be counterproductive and may lead to premature failure. This topic is in need of systematic investigation, because of the many types of reinforcing steels available. Existing data is not sufficient to determine whether with North American and European steels there is a potential upper limit to the bond that is advisable with all steels or if a limit exists for only certain types of steels, if at all.

CONCLUSIONS

The conclusion is that we are moving into a new era of practice. The designer is faced with using many new tools for the design and construction of structures. These tools include new materials such as high strength concrete and new types of reinforcing systems, both of which may be outside the bounds of the data used to

develop the existing bond relationships. Moreover, new analysis tools principally include computer-based methods. While linear elastic models used to determine behavior under normal loads will continue to represent the vast majority of applications, it will become more and more commonplace for practicing engineers to rely on nonlinear methods for assessing structural behavior. A key ingredient will be the accurate modeling of all aspects of structural response, including the very real phenomena of bond slip.

It has been shown in a number of studies that the assumption of perfect bond in reinforced concrete members subjected to large demands is not an accurate assumption. Moreover, it may indeed be unconservative as well. Thus, the analysis of reinforced concrete systems under large demands requires the type of model provided within the CEB framework to accurately determine system response. It is incumbent on the standards and code writing organizations to provide a unified approach for bond modeling in such applications to ensure accuracy and safety in the analysis.

The question can be asked whether any form of bond modeling in large scale analysis is practical for design. At this time, the practicality is limited to those situations where detailed determination of post yield behavior is needed. Also, the practicality is limited by the analysis platform that the model is analyzed on as well as the software. Certainly, using analytical bond models in structural design is not a practical alternative for typical applications. It still is a primary element in the determination of displacements and drift. However through ongoing research, it is likely that the application of bond models to performance determination will become a reasonable alternative for designers even for typical structures. This change is due in part to research and in part to improvements in computing hardware. The future will no doubt include design and analysis software that will enable designers to predict – accurately – bond stresses and bar-slip in an easy and straightforward manner.

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TABLE 1—CEB BOND MODEL PARAMETER DEFINITIONS (MODEL CODE 1990)

	Column 2	Column 3	Column 4	Column 5
	Unconfined concrete*		Confined concrete†	
	Good bond conditions	All other bond conditions	Good bond conditions	All other bond conditions
s_1	0.6 mm	0.6 mm	1.0 mm	1.0 mm
s_2	0.6 mm	0.6 mm	3.0 mm	3.0 mm
s_3	1.0 mm	2.5 mm	Clear rib spacing	Clear rib spacing
α	0.4	0.4	0.4	0.4
τ_{max}	$2.0\sqrt{f_{ck}}$	$1.0\sqrt{f_{ck}}$	$2.5\sqrt{f_{ck}}$	$1.25\sqrt{f_{ck}}$
τ_f	$0.15\tau_{max}$	$0.15\tau_{max}$	$0.40\tau_{max}$	$0.40\tau_{max}$

*Failure by splitting of the concrete.

†Failure by shearing of the concrete between the ribs.

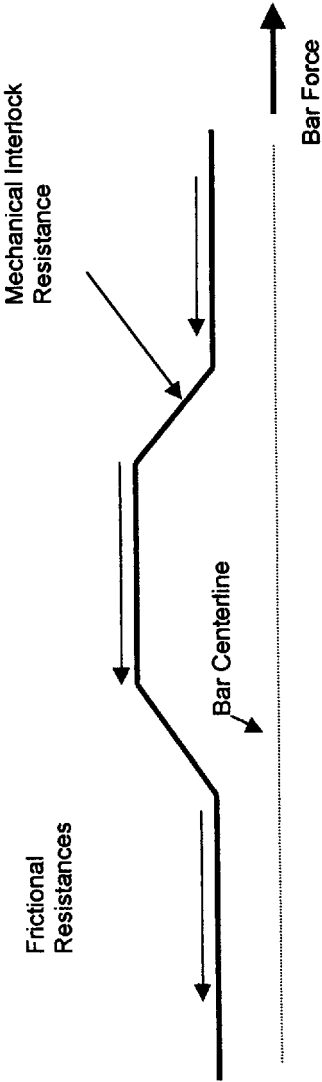


Fig. 1—Applied load versus resisting forces on reinforcing bar deformations

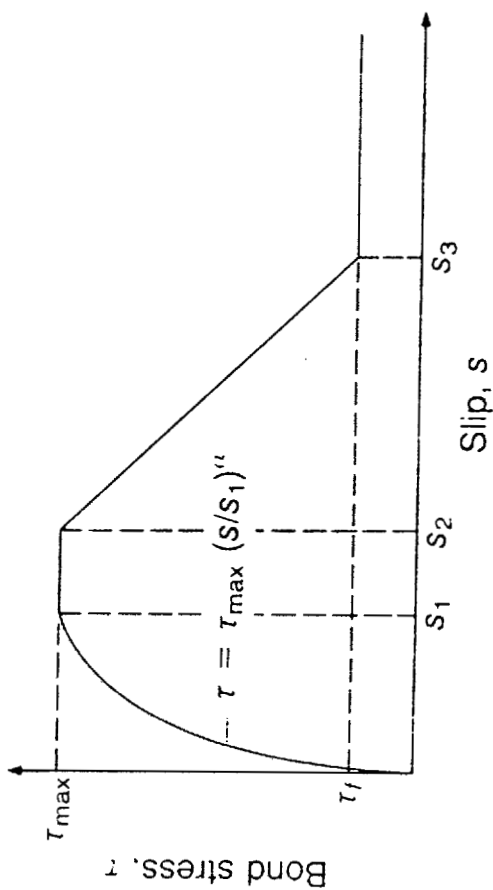


Fig. 2—CEB bond model from model code (1990)

SP 180-2

Bond and Splitting: A Vexing Question

by P. G. Gambarova, G. P. Rosati and C. E. Schumm

Synopsis: Splitting does always occur in some way prior to bond failure, in the form of either partial splitting (quite often undetected) or full splitting, the latter being the subject of several recent papers, owing to the importance of cover splitting in R/C elements. Starting from the test results on fully-split specimens (like those by the authors on special specimens having a fabricated crack) it is possible to formulate suitable bond stress-confinement stress relationships. These models can be introduced into the limit-analysis models developed lately for the description of partial splitting up to the onset of full splitting and bar pull-out in short anchorages. In this way, a linkage between the bar-concrete pressure (studied here through a limit-analysis elastic-cohesive model) and the bond stress is established, in order to evaluate the ultimate bond capacity and to investigate the transition from a splitting-type failure to a pull-out failure. At the same time, such important topics as concrete tensile strength and fracture energy, crack cohesion and localization, concrete cover and bar diameter, fiber content and external pressure can be incorporated into the model. A set of diagrams showing the bond capacity and crack number/opening/penetration versus concrete cover is presented, and the design implications of both the theoretical and experimental results are discussed.

Keywords: Bond (concrete to reinforcement); bond strength; reinforced concrete structures; strength; splitting (cracks); test

Pietro G. Gambarova has been a professor since 1981 at Milan University of Technology, where his scientific interests are concentrated on reinforced concrete mechanics, bond, high-performance concretes and high-temperature effects on cementitious composites; he is also active in the Task Group TC 2/5 (Bond Models) of CEB Commission 2 (Material and Behaviour Modelling). Prof. Gambarova is a graduate of Milan University of Technology and was a visiting scholar at Northwestern University (Evanston, IL) in 1978 and 1982.

Gianpaolo Rosati has been an assistant professor since 1992 at Milan University of Technology, where he received both his MS and PhD degrees, the latter with a thesis on tension stiffening and crack cohesion; his main field of research includes bond, concrete fracture mechanics and very high-strength concretes. Dr. Rosati is a member of CEB Task Group TG 2/5 (Bond Models).

Cristiano E. Schumm studied structural engineering at Milan University of Technology (MS and PhD degrees), where he became an assistant professor in 1993; his main field of research includes punching in FRC slabs, limit-analysis models for bond, and concrete fracture mechanics. Dr. Schumm is a member of CEB TG 2/7 (Concrete Tensile Strength in R/C Design).

INTRODUCTION AND NATURE OF PROBLEM

Concrete splitting and concrete-steel bond are so deeply intertwined that bond failure is always accompanied by a more or less extended splitting. The latter makes bar pull-out easier [1], but goes often undetected because the radial cracks remain limited to the concrete layer closest to the bar (Fig. 1).

In spite of its local nature, splitting affects the structural behavior from the point of view of safety (structural strength and stiffness diminish owing to splitting) and durability. Furthermore, splitting is destined to become even more important as high-strength concrete (HSC, more brittle and prone to cracking) enters the scene and is increasingly used in a variety of structures, not to speak of epoxy-coated and non-metallic rebars, which can develop a larger wedging action against the cover, owing to the lack of chemical adhesion (in both cases) and to the high Poisson ratio (in the latter case).

Though through-splitting (often resisted by transverse reinforcement or external confinement) is the most severe situation, partial splitting is definitely more subtle and difficult to study, since many aspects of concrete mechanics come into play, such as concrete tensile strength and multiaxial behavior, crack cohesion, concrete-bar friction, crack penetration (or extension) and opening, and crack stability in the radial direction [2-9].

In this paper concrete splitting and bar pull-out are treated from the viewpoints of both experimental investigation and mathematical modelling. Since both shear and compressive stresses develop at the bar-concrete interface, the tests are instrumental in formulating a suitable bond stress-confinement stress relationship, where the crack opening or the confinement are a predetermined parameter. The test specimens are generally presplit or cast with a fabricated crack (Fig. 2) [10-12].

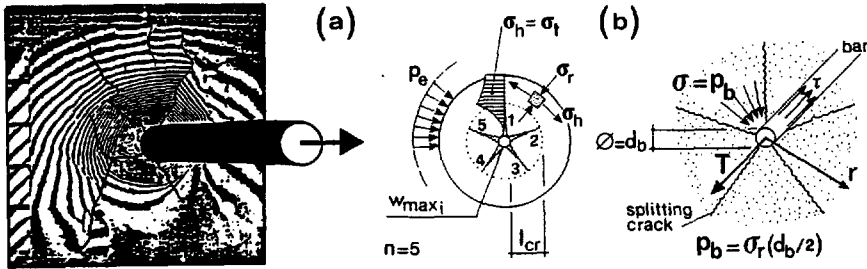


Fig. 1—Bond and splitting in R/C: (a) experimental evidence (moiré holography, Baranek, IBBC-TNO, Delft, 1980), and (b) mathematical modelling (limit-analysis models [2-4, 6-8, 19])

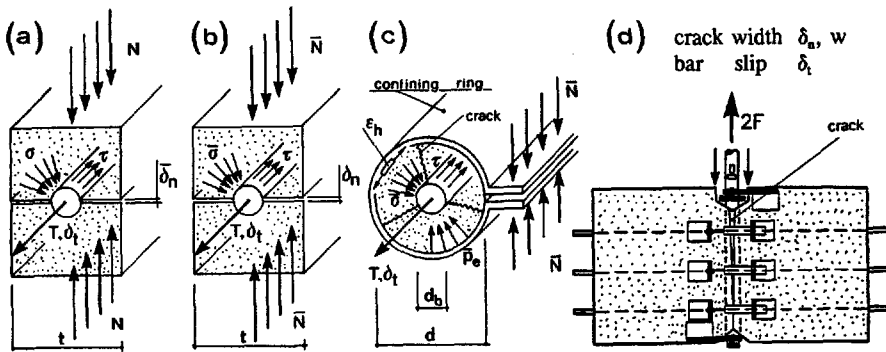


Fig. 2—Bond-confinement interaction: different test philosophies; short anchorages: (a,b) single fabricated cracked plane (crack number $n=2$), and (c) multiple splitting; (a) constant crack width, and (b,c) constant confinement; (d) long anchorage

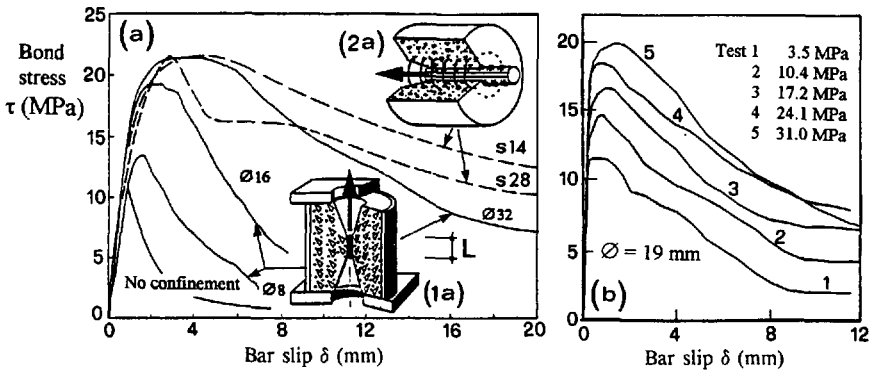


Fig. 3—Bond response ($f_c = 38\text{--}40$ MPa; $L/\varnothing = 3.50\text{--}3.75$): (a) passive confinement [7] (1.a pure pull-out tests: severe confinement, sleeve thickness 10 mm; 2.a splitting tests: moderate-severe confinement, spiral pitch 14,28 mm, $\varnothing_s = 6$ mm), and (b) active confinement [10] (the different values refer to the nominal pressure acting on the bar).

The formulation of the bond-confinement relationship is a first, necessary step in modelling the failure of a short anchorage. It is treated first in the paper, as required also by the limit-analysis models still under development [4,6-8] which do not address explicitly the evaluation of the bond stress and focus their attention on bar-concrete pressure (i.e., confinement stress on the bar owing to the slip-induced wedging action).

In the second part of the paper, on the basis of a few fundamental assumptions regarding the concrete (considered as a two- or three-phase material, partial splitting included, Fig. 1b), the cohesion of the radial cracks, the static admissibility at crack tip and the kinematic admissibility of the crack pattern (splitting cracks), the following quantities are calculated by maximizing bar-concrete pressure with respect to crack penetration: (a) the maximum pressure attainable at the bar-concrete interface, p_{max} ; (b) the "critical" crack penetration, s_{cr} or l_{cr} (beyond which cracking becomes unstable in the radial direction); and (c) the number and opening of the radial cracks, n and δ_n (or w). As a result, once the opening of the splitting cracks and the pressure along the bar are known, the bond stress-confinement stress relationship allows for evaluating the maximum bond stress (= bond strength or capacity, τ_u), for any bar and concrete diameter, and for any applied external pressure.

Since the experimental bond-confinement relationships are governed by crack opening in low-confinement conditions (large crack width), and by the crushing and shearing-off of the concrete along the bars, under severe confinement (small crack width), the limit-analysis models can describe both "splitting failures" and "pull-out failures", once the above mentioned relationships are introduced. This last issue is relevant not only to a better understanding of the two fundamental failure modes of bond, but also to the detailing in R/C design, since - for instance - overreinforcing an anchorage region in the transverse plane may be even detrimental to bond, owing to such side-effects as bar congestion.

The limit-analysis models [6,8,12] are also instrumental in describing crack localization in the radial direction, which is an important and still open issue. As a matter of fact, bond efficiency depends on the width of the splitting cracks, but the width is in turn a function of the crack number! Moreover, the knowledge of the crack number is badly needed also as an input data for F.E. modelling of tension stiffening and anchorages; previously adopted idealizations have included a predefined number of splitting cracks [4,5,7], smeared cracks without crack cohesion [16] and a "process zone", within the framework of elastoplasticity [17].

Beside partial splitting and bond capacity, there is full splitting, when bond capacity is guaranteed only by the transverse confinement, either active or passive. Through cracks may be caused by the applied loads (generally in the ultimate load situation) or by other factors, such as shrinkage and exposure to high temperatures (which may occur also in the working load situation). However, only by knowing the bond stress/confinement stress relationship is it possible to infer the ultimate capacity of a short anchorage from either the width of the through cracks, or the maximum available confinement (which is strictly related to the amount of transverse reinforcement [15]).

TEST RESULTS ON BOND AND SPLITTING

Bond stress-confinement stress laws for cover splitting can be formulated if suitable test results are available. Four basic parameters are at issue: pull-out force, confinement action, bar slip and crack width (i.e. the width of the splitting crack/cracks). Since test results have to be clear and possibly unaffected by the structural setting, short anchorages should be preferred, to ensure a well-defined bond behavior, with a uniform distribution of both the static and kinematic parameters.

In the last fifteen years, different approaches have been adopted, all based on planar or axisymmetric specimens, generally reinforced with a single bar.

In the planar specimens tested in Milan in the 1980s and early 1990s (see Refs. in [12]), a pull-out force was applied to a single, short bar embedded in a concrete thick plate (Figs. 2a and 4), whose formwork had special separators, to create a fabricated splitting crack. The 4 basic parameters were measured directly; in most cases the crack width was kept constant by adjusting the confinement throughout the test, and only in a few cases the confinement was kept constant; in all tests the confinement was controlled by means of special mechanical devices.

Similar tests were carried out by Modena et al., on either ordinary or epoxy-coated bars subjected to constant confinement (Fig. 2b, [11]).

With regard to axisymmetric specimens, both active and passive confinements were adopted. In the former case (Malvar [10], Fig. 3b) the cylindrical specimens were enclosed with a hydraulically-tightened ring (confining ring, Fig. 2c), radial precracking was forced at the beginning of each test, and the radial dilation was measured; the crack number was not prefixed (2 to 4 cracks). In the latter case (Noghabai [7]), both outer steel sleeves and inner steel spirals were used (Fig. 3a), and both normal-strength and high-strength concretes were studied. In Noghabai's tests the specimens were initially unsplit, and the splitting cracks formed during the test (crack number: 2-5).

Going back to planar specimens, those tested by Plizzari et al. [15] are unique since they allow studying the formation and propagation of splitting along the reinforcement of a long anchorage (one or two bars). The confinement was provided by 3 two-leg stirrups (Fig. 2d), and the moiré technique was used for an early detection of the cracks.

Summing up, all tests have their strong and weak points: axisymmetric tests with passive confinement are realistic and instrumental in checking the soundness of mathematical models; the same holds for long anchorages. On the other hand, only by testing precracked bidimensional specimens (as in Figs. 2a, b,d) can the four basic parameters (crack opening, bar slip, bond stress and confinement) be controlled separately, in order to investigate their interaction. Of course, more complex tests with a more realistic bar arrangement are needed too [13] for the validation of F.E.codes.

Finally, when considering the different test series, comparing their results is always a difficult task, since critical assumptions are required. For instance, in the case of Noghabai's splitting tests (passive confinement, Figs. 3a and 8), the spirals were assumed to be at the onset of yielding, and the

extension of the radial cracks was put equal to 1.5 times the bar diameter, while a linear distribution was adopted for the cohesive stresses.

BEHAVIORAL LAWS

The test results at constant crack opening [12] (Figs. 5,6) show that the mechanical characteristics of bond are unfavorably affected not only by crack opening, but also by the diameter of the bars. Small-diameter bars have an edge over medium- and large-diameter bars, with reference to bond strength, initial bond stiffness, confinement-slip coupling and initial friction coefficient at the bar-concrete interface. Nevertheless, the test results are far from exhaustive, since - for instance - no attention has been devoted so far to the effects of bar diameter under a given and constant confinement (active confinement). On the other hand, in the case of passive confinement (Fig. 3a, steel sleeve [7], severe confinement and negligible splitting), the scanty experimental data show a favorable effect of bar diameter.

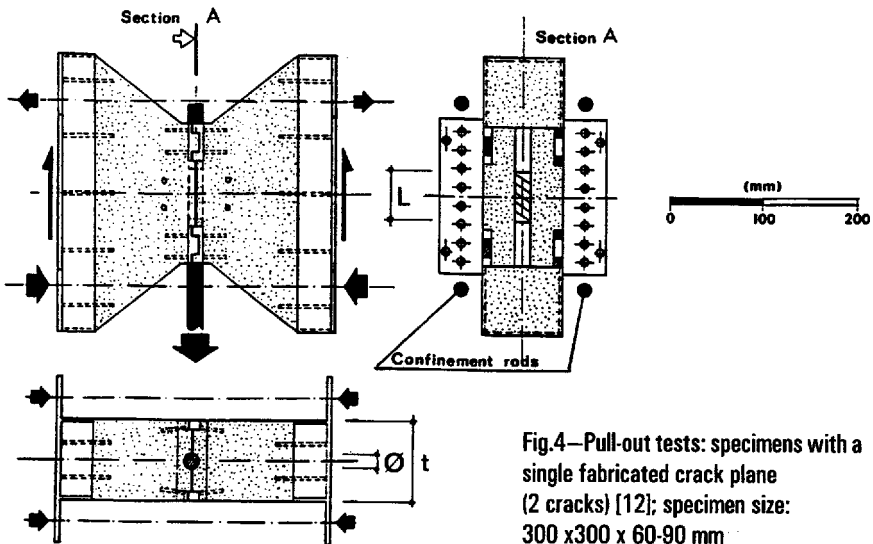


Fig.4—Pull-out tests: specimens with a single fabricated crack plane (2 cracks) [12]; specimen size: 300 x 300 x 60-90 mm

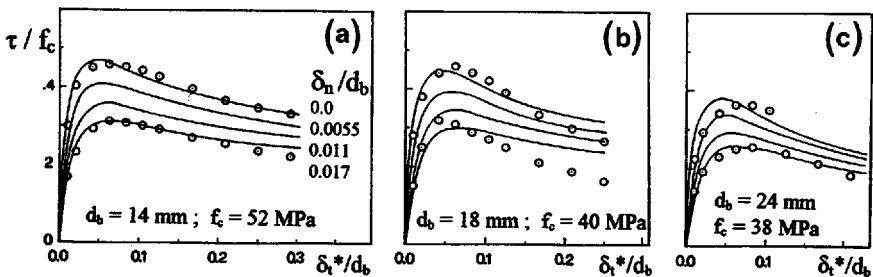


Fig. 5—Pull-out tests at constant crack width [12]: bond stress-bar slip curves (○ test results and interpolating curves, Eq. 4); $\bar{\delta} = 14, 18, 24$ mm; $L/\bar{\delta} = 3.0, 2.9, 2.4$; $t/\bar{\delta} = 4.3, 5.0, 3.8$

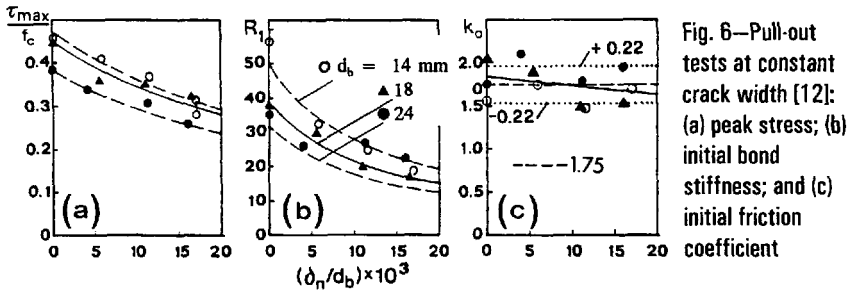


Fig. 6—Pull-out tests at constant crack width [12]: (a) peak stress; (b) initial bond stiffness; and (c) initial friction coefficient

Bond strength and stiffness

Owing to the more uniform contact between a small-diameter bar and the concrete (which is quite close to a mortar in the transition zone from the bar to the undisturbed concrete), both the bond strength and the initial bond stiffness are definitely higher for small-diameter bars (tests at constant crack width, Fig. 6). The following relationships fit the test results shown in Fig. 5 quite well:

$$\frac{\tau_{max}}{f_c} = k_1 \left[\frac{a_1}{W+a_2} - a_3 \right] ; R_1 = \frac{\partial(\tau/f_c)}{\partial(\delta_i^*/d_b)} = \frac{0.5 (1.2 + 0.8 \bar{D}^2)}{a_4 + a_5 W} \quad (1,2)$$

where: $k_1 = 0.907 + 0.5 D [1 - 0.81 D]$, $D = d_b/\bar{d}_b$ with $\bar{d}_b = 18$ mm,

$$W = \delta_n/d_b, a_1 = 30 \times 10^{-3}, a_2 = 50 \times 10^{-3}, a_3 = 150 \times 10^{-3},$$

$$\bar{D} = 1/D = \bar{d}_b/d_b, a_4 = 2.61 \times 10^{-2}, a_5 = 200 \times 10^{-2},$$

with δ_n = crack opening (or width, Fig. 2),

$\delta_i^* = \delta_i - \delta_i^0$ = (total bar slip) - (initial free slip),

$\delta_i^0/d_b = 0.57(\delta_n/d_b)$: δ_i^0 is the rigid slip ensuing from crack opening (see Refs. in [12]).

On the whole, the initial bond stiffness (Fig. 6b) seems to be more sensitive to both crack opening and bar diameter than bond strength (Fig. 6a).

The initial friction coefficient k_o is a slowly-decreasing function of crack opening, as shown by the linear regression in Fig. 6c (solid line). However, the dispersion of the test results is so large that k_o may be considered as independent of crack opening (1.75 is the mean value and ± 0.22 is the standard deviation).

The linear regression on the test results [12] leads to the following relationship:

$$k_o = 1.25 + 0.48 D \quad (3)$$

As for δ_n (opening or width of a splitting crack), assuming a uniform distribution in the radial direction (Fig. 2) is a simplification, which disregards the elastic deformation of the embedment.

Bond-slip relationship

The test results on small-, medium- and large-diameter bars [12] make it possible to formulate a bond-stress/bar-slip relationship, taking into account Eqs.1 and 2 for bond strength and stiffness:

$$\frac{\tau}{f_c} = \frac{R_1(W) \Delta}{1 + a_6 \Delta^m} \quad (4)$$

where: $\Delta = \delta_i^*/d_b$, $a_6 = \frac{R_1^m (m-1)^{m-1}}{m^m (\tau_{\max}/f_c)^m}$,

$$m = 1.36 + 0.015 d_b (1 - D).$$

In spite of its apparent complexity, Eq. 4 is easy to handle, once $R_1(W)$, $\tau_{\max}/f_c$ and m have been evaluated, starting from the bar diameter and the crack opening. Of course, in Eq. 4 the dimensionless crack opening W is a parameter and not a current variable. Consequently, the use of Eq. 4 as a relationship between the bond stress and the crack opening requires some care. In Fig. 5, Eq. 4 is plotted for 3 bar diameters and 4 values of the dimensionless crack opening: on the whole the fitting of test results is satisfactory.

Bond-confinement interaction

The test results (see Refs. in [12]) show that bond-confinement interaction is characterized by closed envelopes which have the following properties:

- little, but not negligible, initial cohesion (in Fig. 7 for $\sigma=0$), mostly independent of crack opening (τ_o , Eq. 5);
- frictional behavior ensuing from bar wedging action and concrete restrained dilatancy; the frictional behavior is represented by the mostly linear branch following the initial cohesion; its slope is very close to the friction coeff. k_o ;
- extended damage in the concrete surrounding the bar, with little wedging action and concrete dilatancy, leading eventually to the shearing-off of the concrete keys between the steel lugs; such behavior is represented by the flattening-out of the envelope;
- greater bond strength $\tau_{\max}$ for smaller bars, as already pointed out.

The above-mentioned behaviors (initial cohesion, dry friction and extended damage) suggest a trilinear approximation of the bond-confinement envelopes or a polynomial formulation:

$$\tau/f_c = \tau_o/f_c + k_i \cdot (\sigma/f_c) - k_u \cdot (\sigma/f_c)^2 \quad (5)$$

with $\tau_o/f_c = 0.042$, $k_i = 1.45$, $k_u = 1.26 + 11.10 (\delta_n/d_b) + 2190 (\delta_n/d_b)^2$. The polynomial approximation should be stopped when the bond stress reaches its maximum.

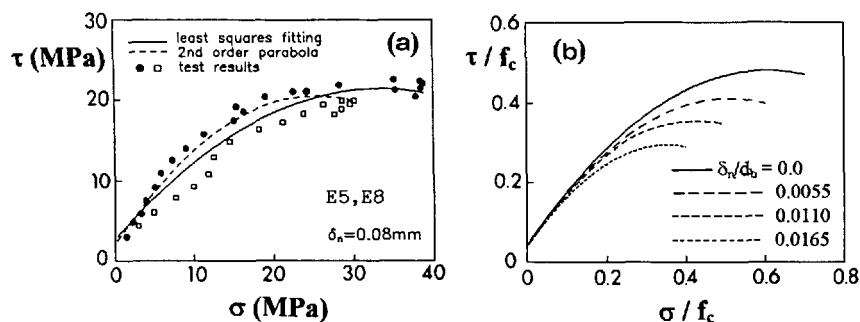


Fig. 7—Bond stress-confinement stress envelopes: (a) test results and fitting (small-diameter bars, $\varnothing = 14$ mm), and (b) proposed law (Eq. 5)

BOND STRENGTH VERSUS CONFINEMENT

In Fig. 8 several test results [7,10-12], at constant confinement (variable crack opening), at constant crack opening (variable confinement) and with passive confinement are represented, in terms of bond strength and confinement stress.

As already pointed out in previous papers (see for instance [11]), for small and medium confinement levels ($\sigma/f_c < 0.25$) there is a definitely linear relationship between the bond capacity and the confinement stress, after concrete splitting.

The linear regression on the test results regarding small-, medium- and large-diameter bars ($d_b = 14$ -24 mm) leads to Eq. 6.

At higher confinement levels ($\sigma/f_c = 0.25$ -0.80) the dispersion of the test results is considerable, but the results show clearly that the bond strength does not keep up with the confinement level. Such an occurrence is confirmed also by the results at constant crack width, which are quite close to those at constant confinement. In other words, it does not matter whether the bond peak-stress τ_{max} is reached during a loading process at constant confinement ($\sigma = \text{constant}$) or at variable confinement, since τ_{max} is mostly related to the actual confinement stress. This is nothing new, since the test results quoted in [12] show that the higher the confinement level, the less the path-dependency of bond.

The linear regression on the test results regarding medium-diameter bars ($d_b = 19$ mm, Fig. 8) leads to Eq. 7:

$$\tau_{max}/f_c = 0.075 + 1.20 \sigma/f_c \quad (\sigma/f_c \leq 0.25) \quad (6)$$

$$\tau_{max}/f_c = 0.293 + 0.247 \sigma/f_c \quad (\sigma/f_c > 0.25) \quad (7)$$

Eqs. 6 and 7 (full and dot-dashed straight lines in Fig. 8) represent two different behaviors regarding the failure of a bar after concrete splitting.

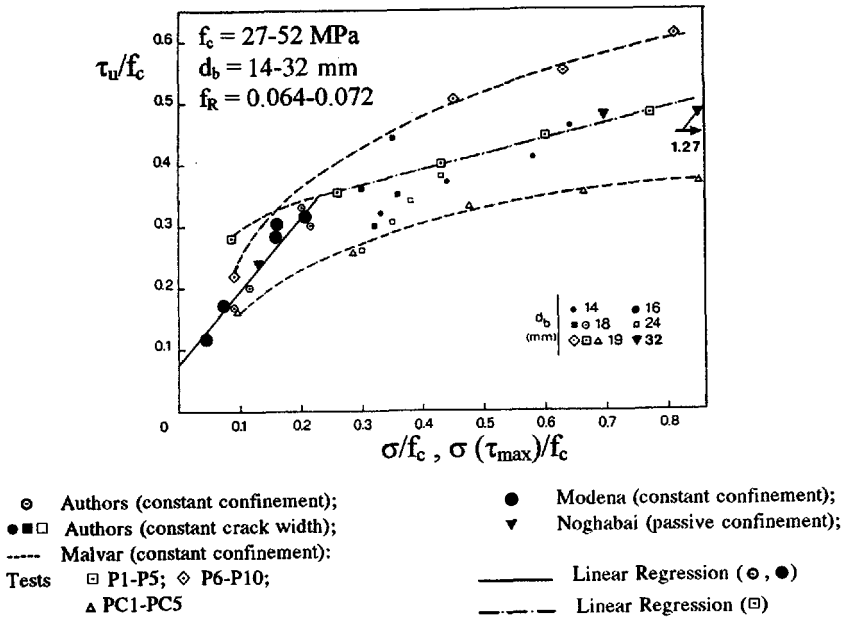


Fig. 8—Ultimate bond capacity as a function of the confinement level; as a rule, the normal stress refers to the peak of the bond stress-bar slip curve [7,12]

At low and medium confinement levels ($\sigma/f_c < 0.20-0.25$) the bond-slip response is characterized by the steady opening of the splitting cracks and by the ensuing increase of the radial and circumferential strains [10,11]; thus, the bond strength is greatly affected by the confinement level, which tends to keep the cracks closed.

On the other hand, at high confinement levels ($\sigma/f_c > 0.25$) the bond-slip response is characterised by very limited radial strains in the concrete surrounding the bar [10], which implies a limited crack opening; as a result, the bond strength is little affected by the confinement level.

In the former case the failure of bond is due to a combination of crack opening and concrete crushing underneath the lugs of the bar, while in the latter case a veritable pull-out occurs, with partial or negligible splitting and with the lugs crushing and furrowing the concrete surrounding the bar, or shearing off the concrete keys between the lugs.

The value $\sigma/f_c = 0.25$ (which is the threshold of "true" pull-out) agrees quite well with the value $\sigma/f_c = 0.30$ quoted by Nagatomo and Kaku [18].

At the close of this part of the paper, it is worth noting that the term "confinement" is used here to indicate the normal stress at the bar-concrete interface (σ , $\bar{\sigma}$ or p_b in Figs. 1b and 2a,b,c). This stress has nothing to do with the external pressure p_e (if any, Fig. 1b), which - of course - tends to increase the confinement stress.

LIMIT-ANALYSIS MODELS

Elastic-Cohesive Model

Several limit analysis models [4,6,7,8] have recently been developed (Fig. 9) to predict the capacity of a short anchorage subjected to pull-out and to describe the pattern of the splitting cracks and the hoop action in the concrete cover. These models are based on the elastic-cracked axial-symmetric model introduced by Tepfers in the Seventies [2].

Tepfers' model, though simple and based on clear and sound assumptions, does not encompass a few important aspects of the splitting failure, such as (a) crack cohesion (which is related to concrete softening in tension), (b) crack number, (c) the bidimensional nature of the state of stress in the transition region between the cracked core and the solid "ring" (tension + compression), and (d) the favorable effects of external pressure (as occurs in proximity to a direct support).

Tepfers (1979) [2]	Elastic-cracked model (no cohesion) ECR		
Reinhardt van der Veen (1990) [4]	Elastic-cohesive model (n=1) ECH	$\sigma_w = f_{ct} \left[1 - \left(\frac{w}{w_c} \right)^k \right]$	
Rosati Schumm (1992) [6]	Elastic-cohesive model (n var.) ECH	$\sigma_w = f_{ct} \frac{D_a}{w_c} \frac{w_c - w}{D_a + kw}$	
Noghabai (1995) [7]	Elastic-cohesive model (n=2) ECH	$\sigma_w = f_{ct} \left(1 - \frac{w}{w_c} \right)$	
Rosati Schumm (1995) See Refs. in [19]	Elastic-plastic- cohesive model (n var.) EPC	$\sigma_w = f_{ct} \frac{D_a}{w_c} \frac{w_c - w}{D_a + kw}$ Proc. 2nd Int. Workshop on HPFR Cement Composites, Ann Arbor (Mi, USA), June 1995, pp.349-381.	

Fig. 9—Limit analysis models to predict confining capacity and crack pattern of concrete cover

Several models have been developed lately to introduce crack cohesion (Fig.9), which in turn requires the introduction of crack kinematics and number, since the crack width is the ruling parameter of any cohesive law. As a result, a statically-admissible approach (as in Tepfers' model) is no longer sufficient, since a suitable assumption on crack kinematics must be introduced.

All of the models which have been developed since 1990 (Reinhardt and van der Veen [4], Noghabai [7] and Rosati-Schumm [6,8]) are based on the assumption of partial cracking in the cover and perfect elasticity in both the outer ring and the concrete keys between the splitting cracks. They are also based, in the case of no external pressure, on a uni-axial failure criterion, with ultimate tensile strain ϵ_c equal to f_{ct}/E_c .

Different stress displacement laws have been adopted for crack cohesion (exponential [4]; non linear polynomial [4]; linear [7]; hyperbolic or linear [6,8]).

The law adopted here for concrete softening [20] contains some parameters related to concrete fracture, such as concrete tensile strength f_{ct} and critical crack opening w_c :

$$\sigma_w(w) = f_{ct} \frac{D_a}{w_c} \frac{w_c - w}{D_a + \kappa w} \quad (8)$$

where σ_w is the cohesive tensile stress (i.e. hoop stress along a splitting crack), w and w_c are the crack opening and its critical value ($\sigma_w(w=w_c)=0$), D_a is the maximum aggregate size and κ is a shape parameter related to fracture energy G_f :

$$G_f = \int_0^{w_c} \sigma_w(w) dw = \frac{f_{ct} D_a}{\kappa} \left(\left(1 + \frac{D_a}{\kappa w_c} \right) \ln \left(1 + \frac{\kappa w_c}{D_a} \right) - 1 \right) \quad (9)$$

As for crack kinematics, different assumptions may be introduced for the circumferential elongation in the cracked core, leading to many relationships for the ratio α_r between the elongation of any circumferential fiber ($R_b \leq r < R_c$) and the elongation at the interface between the cracked core and the outer elastic ring ($r=R_c$) (Fig.10):

$$\alpha_r(r, R_c, n) = \frac{r}{R_c} \left(\frac{\sigma_w(r, R_c, n) / E_c}{\epsilon_c} + \frac{n w(r, R_c, n)}{2 \pi r \epsilon_c} \right) = \alpha_{r1} (\alpha_{r2} + \alpha_{r3}) \quad (10)$$

In particular, the following assumptions may be adopted, according to the actual bar-to-concrete interface behavior:

- constant circumferential elongation, with each fiber subjected to the same stress

- which develops at the tip of the splitting cohesive cracks ($\epsilon_t(r)=f_{ct}/E_c$) (the equilibrium in the hoop direction is not satisfied inside each concrete key) [4];
- constant circumferential elongation, with each fiber subjected to the same stress which develops along the splitting cohesive cracks ($\sigma_r(r)=\sigma_w(r)$) (the equilibrium in the hoop direction is satisfied) [6-8];
 - variable circumferential elongation, i.e. according to the hoop-strain distribution of an "equivalent" elastic case, or to a hyperbolic distribution.

Depending on the kinematic assumption, different crack-width and cohesive stress distributions are obtained (Fig. 10) and, by imposing the equilibrium along any radial, partially-split section, the bar-concrete pressure can be formulated as follows:

$$p_b(R_c, n) = \left(f_{ct} \frac{R_c}{R_b} \right) \frac{R_m^2 - R_c^2}{R_m^2 + R_c^2} + \frac{1}{R_b} \int_{R_b}^{R_c} \sigma_w(r) dr = e(R_c) + c(R_c, n) \quad (11)$$

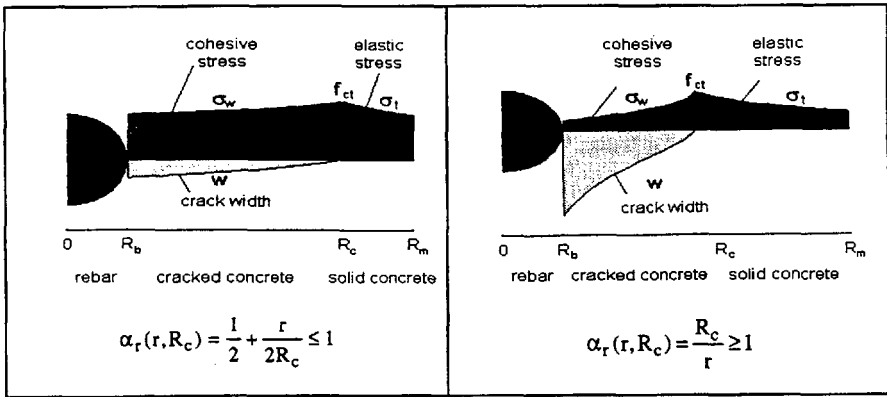


Fig. 10—Crack-width (w), and cohesive (σ_w) and elastic (σ_t) stress distributions for different kinematic conditions α

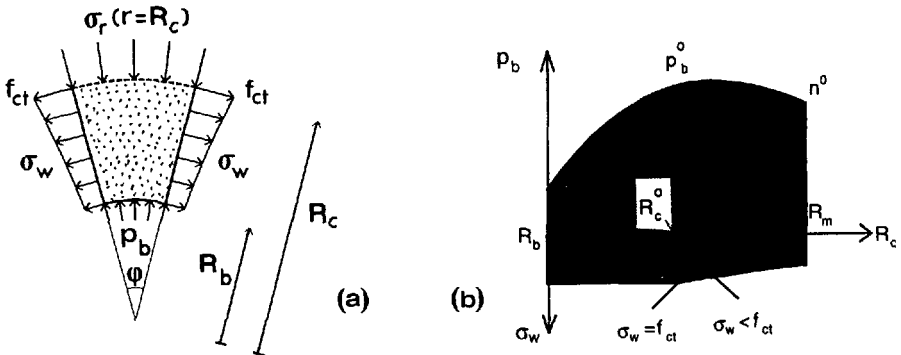


Fig. 11—Sketch of a concrete key between two radial cracks (a); plots of hte confining pressure P_b and crack-tip cohesive stress σ_w as a function of crack penetration P_c (b)

In [6,8] the crack number is not given a-priori, but is one of the unknowns of the problem. By definition, it must be the number n^0 (Fig. 11) that satisfies both static consistency (positive cohesive stresses smaller than f_{ct} or equal to f_{ct} at the crack tip) and kinematic consistency (positive crack opening):

$$n^0(R_c) = \frac{2\pi f_{ct}}{E_c w_c} \left(1 + \frac{\kappa w_c}{D_a} \right) R_c \quad (12)$$

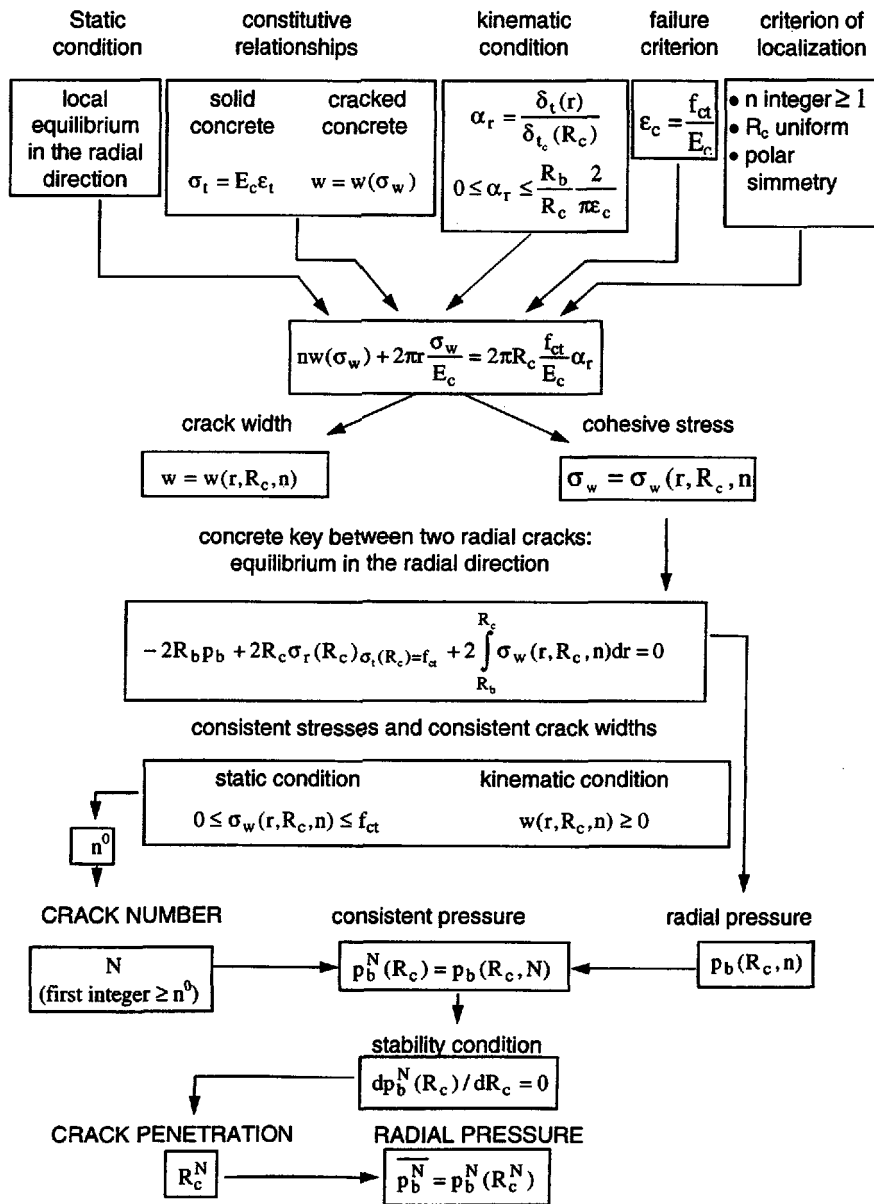


Fig. 12—Flow-chart of the ECH model by Rosati and Schumm, see refs. in [19]

According to experimental evidence, it seems better to adopt the first integer number N of radial cracks bigger than $n^0(R_c)$, so that the consistent radial pressure becomes:

$$p_b^N(R_c) = p_b(R_c, N) \quad (13)$$

By maximizing the pressure with respect to crack extension ($dp_b^N(R_c)/dR_c = 0$) and by giving the integer value N to the crack number n , the effective crack penetration R_c^N and the effective confining capacity $\bar{p}_b^N$ of concrete cover are found [6,8].

It is worth noting that the effects of external pressure have also been introduced [8]: bond capacity increases, but not beyond an upper limit, which is related to the compressive strength of the concrete in the radial direction; the crack number, critical extension and maximum width decrease because of the external pressure.

To evaluate bond strength on the basis of the bar-concrete pressure p_b obtained with the above-formulated model, a bond stress - radial pressure relationship (Eq. 5) is needed. This relationship depends on the crack width at the bar-to-concrete interface and it has been calibrated with reference to a single splitting plane, characterized by two polar symmetric radial cracks ($N=2$). For this reason, an equivalent crack width δ_n has to be evaluated for every radial crack pattern ($N \geq 2$). For the sake of simplicity [12], its value can be calculated as being equal to:

$$\delta_n(R_c, n) = \frac{nw(R_b, R_c, n)}{2} \quad (14)$$

A comparison of the proposed bond model with experimental results by Malvar [10], obtained for different and predetermined values of the external pressure p_e , is shown in Fig. 14a: of course the burt-concrete pressure p_b could not be measured during the test, and is one of the unknowns of the proposed model.

Similarly, Fig. 14b shows the comparison of the present model with Orangun's formula:

$$\tau_b^o = \sqrt{f_c} \left(k_1 + k_2 \frac{C}{D_b} + k_3 \frac{D_b}{l_s} \right) \quad (15)$$

where no cohesive contribution is accounted for (f_c = compressive strength, C = concrete cover, l_s = lap of the splice). In particular, the solid line corresponds to Eq. 15, the dotted and dashed lines correspond to normal concrete ($G_f = 0.08$ N/mm and $w_c = 0.15$ mm) and to fiber-reinforced concrete ($G_f = 5$ N/mm and $w_c = 3$ mm) respectively.

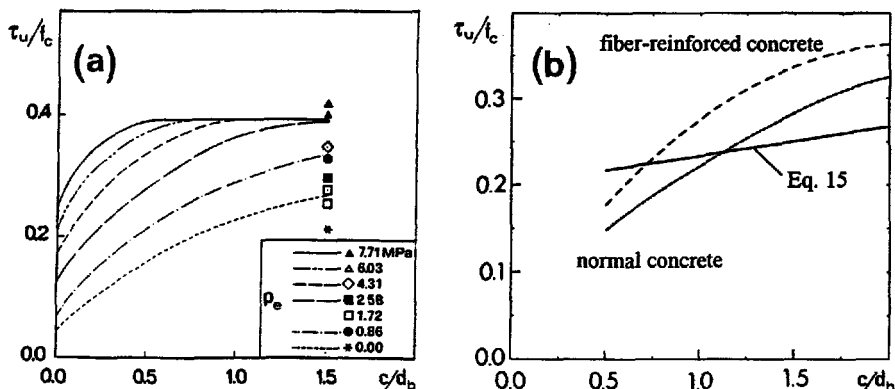


Fig. 13—Comparison of proposed bond model with Malvar's test results [10] (a), and Orangun, Jirsa and Breen's equation (ACI JOURNAL, V.74, No. 3, 1977) (b)

Elastic-plastic-cohesive model

The fundamental aspects of the elastic-plastic-cohesive model developed from the elastic-cohesive model presented in [6,8] (Fig. 14a) are as follows:

1. The concrete mass surrounding the bar consists of a three-phase material with an inner radially-cracked and cohesive core, an intermediate plastic ring and an outer elastic ring; the intermediate ring is a recent development, since the hoop action in [6,8] was developed only by the elastic ring and by the cohesion of the radial cracks in the original model.
2. The hoop action in the elastic and plastic rings (with the contribution of external pressure p_e , if any [8]) prevents the pull-out of the bar as long as the penetration of the radial cracks R_c is less than or equal to a "critical" value which maximizes bar-concrete pressure p_b and, consequently, pull-out strength. Beyond this critical value, the equilibrium of the bar is no longer possible, since the radial cracks become unstable and spread across the whole cover.
3. The solid parts of the radially-cracked core behave elastically and are subject to uniaxial compression in the radial direction ($\nu=0$).
4. The radial cracks have some residual strength in tension (crack cohesion [20]) and can transfer tensile stresses, as shown by the softening branch of the stress-displacement curve (Eq. 8, Fig. 14c). In the case of fiber-reinforced concrete, both f_{ct} and w_c depend on fiber content.
5. The radial cracks stabilize if, and when, the hoop stress at their apex is equal to concrete strength in uniaxial tension (uniaxial criterion) or in tension-compression (biaxial criterion); tension-compression interaction is detrimental

to both tensile and compressive strength, and should be introduced (biaxial criterion) in the case of high external pressure [8]).

6. Crack width in the concrete core and bar-concrete pressure can be formulated in terms of a few geometric and mechanical parameters by imposing (a) the equilibrium condition along any radially-cracked section, and (b) the kinematic condition in the cracked core, which governs the shape of the radial cracks, i.e. the dependence of the width on the radial coordinate r .
7. In the uncracked region, the hoop strains are assumed to have an elastic-type distribution according to an elastic constitutive law, energetically equivalent to the actual elastic-plastic law. However, the hoop stresses are introduced according to the actual elastic-plastic behavior of the concrete (Fig. 14c), and as such they appear in the equilibrium equation.
8. The critical crack extension, representing the maximum exploitation of bond, is obtained by maximizing bar-concrete pressure p_b with respect to crack extension, for any given crack number.
9. The crack number is defined as the number corresponding to the smallest value of the pressure maximums, in fulfillment of both static and kinematic consistency. Smaller or greater crack numbers, albeit statically and kinematically consistent, prevent the maximum exploitation of bond in the former case, or may cause unstable situations in the latter case.

By applying this model, the behavior of a bar embedded in plain concrete (Fig. 14c, curve 1) or in high-volume fiber-reinforced concrete (Fig. 14c, curves 2 and 3) can be compared. In Fig. 14c, the stress-strain/stress-displacement curves do not refer to any specific normal or fiber-reinforced concrete, but are three possible and realistic, albeit arbitrary, constitutive laws. The three curves have the same initial slope (fiber content does not significantly alter the elastic behavior), slightly different peaks (fibers add some strength in tension), different plateaus (High-Performance Fiber-Reinforced Cement Composites have a remarkable pseudo-plastic behavior, depending on fiber content) and slightly different softening branches, representing crack localization. The extension of the plastic plateau $\Delta\epsilon$ is 0, 7 ϵ_1 , 15 ϵ_1 respectively (ϵ_1 = peak strain in plain concrete = f_{ct}/E_c).

Fig. 14b shows how rapidly bond behavior tends to approximate to fully plastic behavior, because of the contribution of the fibers (fiber content should be in excess of 4-5% by volume).

Fig. 15 shows the variation of crack width (at the bar-concrete interface, Fig. 15a), crack number and critical penetration (Fig. 15b) as a function of concrete cover.

In summary, high fiber-contents (which turn the behavior of concrete under tension from elastic-brittle to elastic-plastic-softening) guarantee much better bond characteristics for the same concrete cover, as well as thinner and less extended splitting cracks, and - in the end - greater strength and durability.

CONCLUDING REMARKS

Concrete splitting is something that designers, builders and researchers have to learn to live with, even more since the new materials entering into the scene,

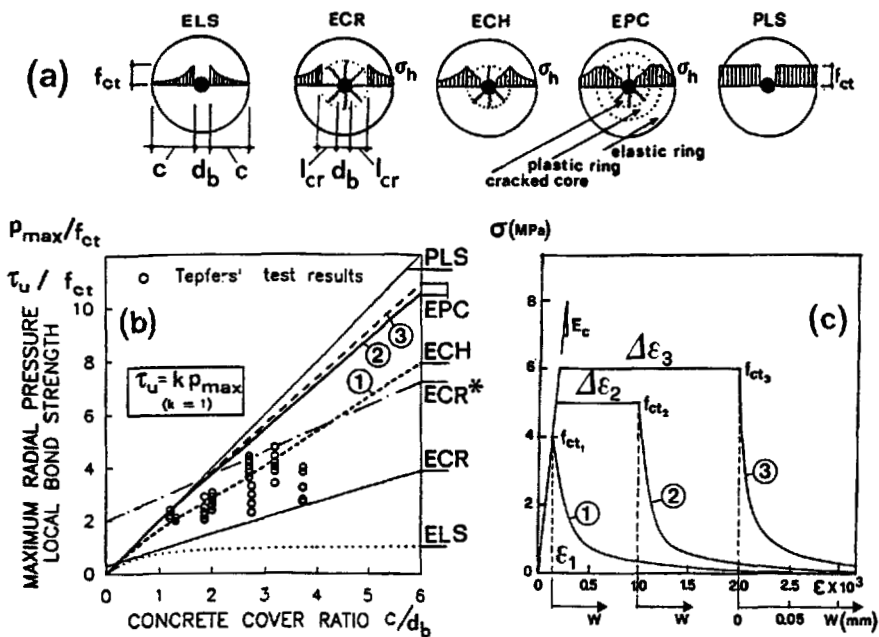


Fig. 14—(a) 2-D Limit Analysis Models for cover splitting: ELS=ELaStic, ECR=Elastic-CRacked, ECH=Elastic-CoHesive, EPC=Elastic-Plastic-CoHesive, and PLS=PLaStic, see also Figure 9; (b) bar-concrete pressure and bond strength versus concrete cover (ECR* = Elastic-CRacked with bond cohesion [3]); (c) typical stress/strain and stress/crack-opening laws for plain concrete (1) and fiber-reinforced concretes (2), (3) ($v_f > 5$ percent by volume = high-volume fiber content)

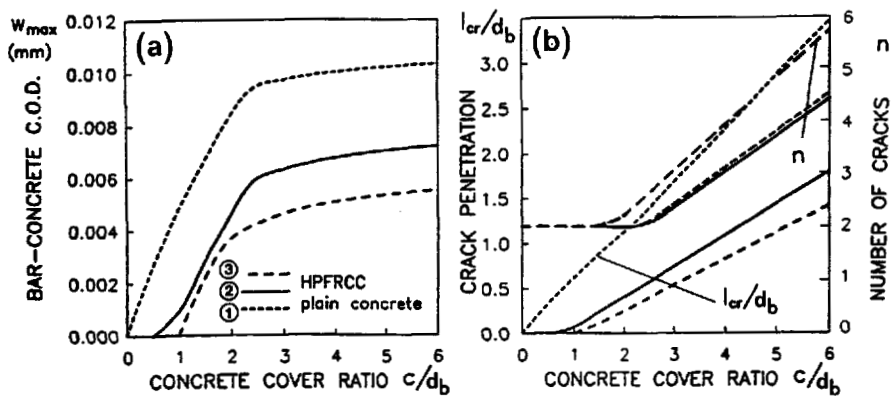


Fig. 15—Plots of maximum crack-width (a), and crack number/crack penetration (b) versus concrete cover

such as high-strength concretes and non-metallic bars, favor cover splitting, owing to their lack of toughness in the former case (silica-fume concretes) and to their reduced chemical adhesion and enhanced wedging action in the latter case. Other factors too play a relevant role, like bar rusting, concrete shrinkage and concrete exposure to extreme environmental conditions (low and high temperatures). However, the mixed effects of the environment-related factors need still to be thoroughly investigated.

Owing to the number of parameters affecting cover splitting, further efforts are needed to work out the interface equations (bond stress-confinement stress laws) valid for each specific situation, as well as the constitutive laws of the concrete, since splitting is not a pure interface problem.

Once the above equations have been formulated, the limit-analysis models come into play. In spite of their 2-D description of the 3-D bond problem, these models are capable of shedding light on each specific parameter, and allow a comparison between different materials (concrete and reinforcement). In this sense, 2-D models are instrumental in the calibration of the code provisions regarding high-strength and fiber-reinforced concretes, and special bars, with reference to bar cover and interspace, splice length and anchorage capacity.

Beside the parameters related to the concrete embedment, the issue of the confinement needs special attention, and new or improved formulations for both active and passive confinement (transverse reinforcement) are necessary. With regard to this point, the 2-D Limit-Analysis models are promising because their favorable effects as crack arrestors can be investigated in a relatively easy way.

Finally, the crack number should be carefully considered too, since more cracks (generally thinner and less extended) enhance both bond strength and durability, as shown by the elastic-plastic-cohesive model introduced in this paper (Fiber-Reinforced Concrete).

It is true that in many cases the crack number is dictated by the geometry of the cover and by the arrangement of the reinforcement, but it is equally true that in well-embedded bars (isotropic cover) the crack number should be dictated by the static and kinematic consistency of the crack pattern, and by the maximum exploitation of bond (maximum interface pressure = "minimum" crack number). However, further effort is needed to identify the actual solution ("true" crack number), in terms of stable equilibrium, but to do that an energy-based approach should be adopted.

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