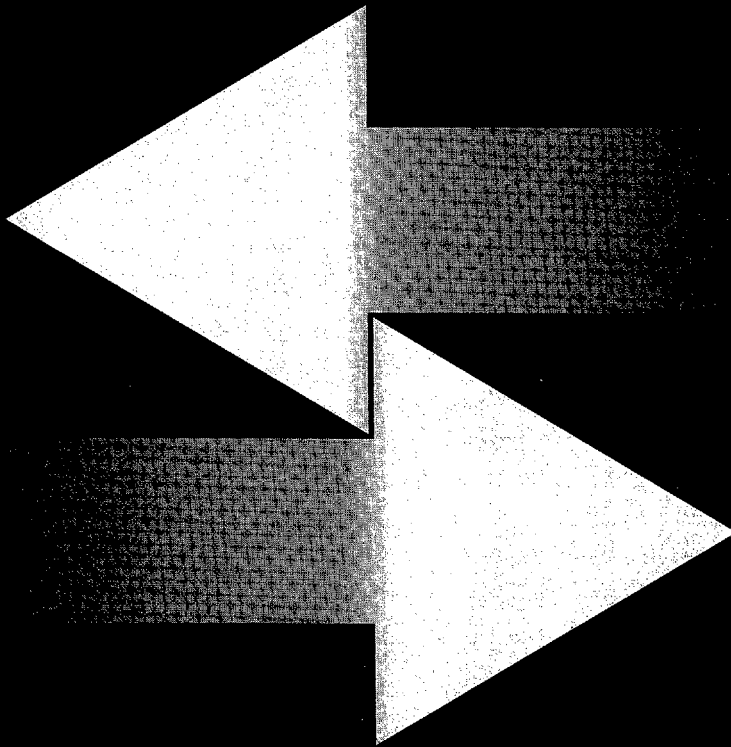


the Design of



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Editor

Two-Way Slabs



SP-183



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The Design of Two-Way Slabs

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PREFACE

At ACI's 1996 fall convention in New Orleans, La., ACI-ASCE Joint Committee 421, Design of Reinforced Slabs, sponsored two technical sessions. The focus of the morning session was "Design of Two-Way Slabs using Elastic Frame Analogies," and the afternoon session concentrated on "Design of Two-Way Slabs using Theorems of Plasticity." The sessions were moderated by Hershell Gill and Thomas C. Schaeffer.

This ACI Special Publication consists of 10 papers, all of which were presented at the sessions in New Orleans. The current ACI 318 Building Code specifically addresses two methods for the design of two-way slabs. These methods are the Equivalent Frame Method, and the Direct Design Method. However, the Building Code also "...permits a designer to base a design directly on fundamental principles of structural mechanics provided it can be demonstrated explicitly that all safety and serviceability criteria are satisfied." The papers contained in this volume should give the designer an overview of some of the different analysis and design techniques that are currently being used.

Committee 421 would like to thank all of the authors and presenters for their contributions to the two technical sessions and to this volume. We would also like to thank the reviewers of the original manuscripts, as well as ACI staff for their assistance.

T. C. Schaeffer
Editor

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Concept and Background of Elastic Frame Analogies for Two-Way Slab Systems

by S. Simmonds

Synopsis: The justification for using elastic frame analogies to determine design moments in two-way slab systems is discussed. A brief history of two-way reinforced concrete slab design leading to the current code procedures is presented. This history includes a description of the various elastic frame analogies that have existed in past codes, the reasons for changes and the research leading to improved frame analogies. This is followed by a critical review of the Equivalent Frame Method in the current code with suggestions for improving and simplifying provisions for elastic frame analogies in future codes.

Keywords: analysis; design; elastic frames; history; reinforced concrete slabs

Sidney H. Simmonds, currently Professor Emeritus, University of Alberta was for many years Secretary of the Canadian Concrete Code Committee A23.3. He also served on ACI Committees; 118 - Computers (Chairman 1979-83), 120 - History (Chairman 1991-95), 318F - s/c on Slabs, 334 - Shells, 340 - Handbook, 421 - Slabs, and was a founding member and first President of the Alberta ACI Chapter.

WHY AN ELASTIC FRAME ANALOGY?

Two-way slab systems are a common structural component in reinforced concrete construction. If asked how they design these slabs, many designers in North America would answer 'I use a computer program'. If pressed as to the methodology incorporated in the program they would likely respond 'elastic frame analogy'. Why an elastic frame analogy?

Traditionally, in reinforced concrete design, one uses a linear elastic theory to determine design parameters and then proportions members using an ultimate strength procedure. The justification for this apparent anomaly is that by designing for moments determined from elastic theory the amount of moment redistribution at service load conditions will be minimized thereby ensuring that serviceability requirements will generally be satisfied. Except for special cases such as deep beams or sudden changes in cross section where elastic theory is not applicable, this technique has served us well.

To apply a similar procedure to the design of two-way slab systems it is necessary to have a means of obtaining an elastic analysis. As early as 1811, Lagrange proposed an elastic theory for thin slabs which requires determining a function that will satisfy both a fourth-order differential equation and the boundary conditions. Solutions using this approach have been successful only for slabs with the simplest idealized boundary conditions, generally panels with non-deflecting boundaries. This method has been used to develop design procedures for slabs with beams between all supports. It was the need to provide a simple elastic analysis for column supported two-way slab systems that led to the concept of an elastic frame analogy.

Even today, although a number of ingenious techniques to obtain solutions for two-way systems have been proposed, for example Ang (1) and, more recently, numerical solutions based on finite element or finite difference techniques, none have proved practical for routine office use. Hence the continuing interest in elastic frame analogies.

WHAT IS AN ELASTIC FRAME ANALOGY?

The concept behind the use of elastic frame analogies is that satisfactory values for the design moments and shears in two-way slab systems can be obtained by considering a portion of the slab-column structure to form a design frame that can be analyzed as a plane frame.

The process consists of three parts:

- a) define the analogous plane frame including assigning member stiffness

- b) analyze frame with appropriate loading to obtain maximum frame moments, and
- c) distribute frame moments laterally across the corresponding critical sections of the slab.

Frame analogies can be used for both gravity and lateral loading on slab-column structures.

The basic approach for defining the geometry of the analogous elastic plane frames has remained essentially unchanged through various codes. The structure is considered to be made up of analogous or equivalent frames centered on the column lines taken longitudinally and transversely through the building, see Fig. 1. Each frame consists of a row of columns or supports and slab-beam strips bounded laterally by the centerline of the panel on each side of the centerline of the columns or supports. Frames adjacent and parallel to an edge are bounded by that edge and the centerline of the adjacent panel. Each frame may be analyzed in its entirety, or for vertical loading each floor or roof with attached columns may be analyzed separately.

Success in applying this analogy depends on the appropriate apportioning of stiffness to the members of the frame so that the elastic analysis of the two-dimensional frame will approximate that of the non-linear three-dimensional slab-beam-column system. This problem is made more complex by a fundamental assumption in the analysis of plane frames that does not apply to slab-column systems. In a typical plane frame analysis it is assumed that at a beam-column connection all members framing into that joint undergo the same rotation as shown in Fig. 2(a). For slabs supported by columns this assumption is valid only locally at the column. Portions of the slab laterally removed from the column will rotate lesser or greater amounts depending on the geometry and loading patterns as shown in Fig. 2(b). Furthermore, actual slab systems crack even under service loading, especially near the face of the column resulting in locally reduced stiffness. To account for the differences in behavior of the actual slab-column system and the idealized plane frame, it is necessary to modify the stiffness of the frame elements. Unfortunately, the modifications required to the member stiffness for lateral loading differ from those for gravity loading.

The definition of the analogous frame, the apportioning of stiffness and the rules for the lateral distribution of design moments across the slab have evolved through successive codes. To follow this evolution, it is helpful to review the history of the development of two-way slab construction.

'TWO-WAY SLABS' AND 'FLAT SLABS'

Since the 1971 ACI Code, the term 'two-way slab' refers to all slab systems reinforced for flexure in more than one direction with or without beams between supports. The term 'flat slab' is not used. Prior to 1971, the term 'two-way slab' referred only to those slabs with beams between supports along all sides of each panel and the term 'flat slab' referred to slabs without beams between supports but could have column capitals and/or drop panels. The need for the distinction in earlier codes was because of the different genesis of the two slab types and the resulting differences in design rules. The elastic frame method was initially

developed for two-way slabs without beams (flat slabs). In the remainder of this paper the term flat slab is used as defined above when discussing design rules prior to 1971.

EARLY HISTORY OF SLAB DESIGN

Reinforced concrete flat slabs were invented in the sense that they were not a logical extension of elastic theory or construction practice. Credit for this invention is generally given to C. A. P. Turner who constructed his first 'mushroom slab' (a reinforced concrete slab supported on columns with flared column capitals) for the five-story C. A. Bovey Building in Minneapolis in 1906. Lacking a rational analysis, the validity of his design was verified with a load test. So successful was this slab that almost immediately competitors were constructing slabs using various proprietary methods. Since there was no generally accepted procedure for analyzing such slabs, it is not surprising that the amount of reinforcement required varied considerably from design to design. A comparison of the amounts of reinforcement required in an interior panel by six different design procedures made by McMillan (2) in 1910 showed that some designs required four times as much steel as others.

In an attempt to reconcile these difference in designs, many of the slabs that were load tested had measurements of the strains in the reinforcement. Moments in the slab were computed from these steel strains using a straight line expression. These tests did not resolve the differences in design procedures.

In 1914, Nichols (3) examined the statics of a uniformly loaded interior panel of a slab without beams with square panels extending infinitely in both directions. In his original paper, he considered only a quarter of the panel but in the closure to his paper he considered as a free body the half panel designated as A, B, C, and D in Fig. 3. From symmetry no shears or twisting moments exist on faces B, C, and D but bending moments exist on all faces. He assumed that the shear forces on face A are uniformly distributed. Denoting the sum of the moments of all vertical forces about x-x as M_o , yields the simplified expression

$$M_o = \frac{WL}{8} \left(1 - \frac{2c}{3L}\right)^2 \quad \text{where } W \text{ is the total load on the panel.} \quad (1)$$

The difference between this expression and the exact expression is less than 1% for values of c/L smaller than 0.3.

Nichols concluded that for equilibrium this must be the sum of the bending moments on faces C and B plus the components about x-x on face A. While this analysis does not give the actual moment at any point or even across any section, it does provide a criterion against which proposed design moments could be evaluated. Since many of the designs that successfully passed load tests used moments that were significantly lower than this sum, his paper evoked a spirited discussion that was over five times the length of the original paper. While some of the discussions applauded his analysis others, including Turner, questioned even the validity of applying statics to two-way slabs.

Those who were opposed to Nichols' analysis referred to the results of slab tests. Values of the total moment obtained from steel strain readings for six slabs representative of the many slab tests reported in terms of the total panel static moment, M_o , were

Purdue test slab J	0.59 M_o
Purdue test slab S	0.74 M_o
Bell Street Warehouse	0.40 M_o
Western Newspaper Union	0.72 M_o
Sanitary Can Building	0.30 M_o
Shonk Building	0.38 M_o

This apparent disagreement between the requirements of equilibrium and the results of tests was a dilemma that was perplexing to engineers and code writers.*

In 1917, the Joint Committee on Concrete and Reinforced Concrete included principles of design for flat slabs in their Final Report (5). Influenced by Nichols' logic but unable to ignore the results of the load tests, they compromised by adopting the form of Nichols' expression but arbitrarily reduced the coefficient and hence the magnitude of the total panel moment by recommending the expression

$$M_o = 0.107WL \left(1 - \frac{2c}{3L}\right)^2 \quad (2)$$

However, the approved 1920 ACI Building Code (6) defied statics even more by further reducing the coefficient to yield

$$M_o = 0.09WL \left(1 - \frac{2c}{3L}\right)^2 \quad (3)$$

Although this expression gives total panel moments that are only 72% of the total panel moment required to satisfy equilibrium, it remained in the ACI Building Codes essentially unchanged until 1971. The only modification was in the 1956 Code (7) where the total moment was multiplied by a factor F (where $F = 1.15 - c/L$ but not less than 1.0) to increase slightly the moments for slabs supported by small columns.

Initially, the only procedure specified for the design of flat slabs was known as the Empirical Method. In this method each panel was divided into column and middle strips and the total moment given by Eqn. 3 was proportioned to the different critical sections using specified percentages. It is interesting to note that the 1920 code specified the distribution for only 80% of M_o leaving it up to the

* Later Westergaard and Slater (4) would show that the straight line method to compute moments from the measured steel strains greatly underestimated these moments as the effects of concrete tensile stiffening were not taken into account. Using statically determinate laboratory samples with similar reinforcement, they demonstrated that the sum of the actual moments corresponding to the measured steel strains were in close agreement with those predicted by Nichols' analysis. Unfortunately this information was not available to early code writers and, for many years, was ignored by others.

designer to distribute the remaining 20% "as required by the physical details and dimensions of the particular design employed". In the 1928 code and all following codes until 1971, designer discretion was removed and M_o from Eqn. 3 was distributed as follows:

	Negative Moment	Positive Moment
Column Strip	46	22
Middle strip	16	16

ORIGIN OF ELASTIC FRAME ANALOGIES

In 1929, a subcommittee of the reinforced concrete section of the Uniform Building Code, California edition, was established to investigate the idea of considering a flat slab and its supporting columns as a series of elastic frames. Although the report of this subcommittee was not published until 1938 by Dewell and Hammil (8), it did lead to the inclusion of an Elastic Frame Method as an alternative method for flat slab systems in the 1933 California edition of the Uniform Building Code. This method defined the frames as bounded by the centerlines of the panels as we do today. Columns were modeled as having hinges at the mid-height between the base of the capital and the top of the floor below. The column-slab joints were considered rigid.

Since the elastic frame analysis satisfied the equations of equilibrium, the design moments were considerably greater than those from the Empirical Method which accounted for only 72% of the static moment. This inconsistency was eliminated by arbitrarily reducing the negative moments obtained from the frame analysis by 40%. These positive and negative moments were distributed to column and middle strips in the same proportions specified for the Empirical Method.

An elastic frame method was first introduced into the ACI Code in 1941. The geometry for the equivalent frames followed closely that proposed by Dewell and Hammil except that the columns were assumed fixed at their far ends. The joints between columns and slabs were considered rigid. This rigidity was assumed to extend in the slabs a distance A , (where A was the distance from the center of the column, in the direction of the span, to the intersection of a 45 degree diagonal from the center of the column to the bottom of the flat slab or drop panel but not greater than one-eighth of the span) and in the column to the intersection of the sides of the column and the 45 degree line defining A . Negative moment was computed at a distance $0.073L + 0.57A$ from the column center-line, this distance being selected to result in total panel moments for interior square panels with uniform loading that agreed closely with those that would be obtained with the Empirical Method. Thus the effect was essentially the same as merely reducing the negative moments by 40 % as specified by the UBC, California Edition, but was not as transparent to or as easy for the designer.

The elastic frame method was modified in the 1956 Code so that the rigid joint extended in slabs from the center of the column to the edge of the column or capital and in the column from the top of slab to the bottom of the capital and the distance from the column center at which negative moments were computed was

simplified to the length A . With these changes the design moments remained essentially the same as before.

When the live load did not exceed three-quarters of the dead load, the design moments were assumed to occur with full live load on all spans. Otherwise, design moments were obtained with full live load on appropriate spans to give maximum values.

THE ILLINOIS SLAB STUDY

The situation in the early 50's was that flat slabs, even when using an elastic analysis with full span geometry and design loads, resulted in design moments that were substantially less than those required by considerations of equilibrium. On the other hand, two-way slabs (slabs with beams) were designed using coefficients that were in part developed from elastic plate theory and so satisfied equilibrium. As it was generally recognized that there is no essential difference in the behavior between slabs with or without beams, the differences in design procedures and differences in factors of safety needed to be addressed.

To resolve this situation, the Reinforced Concrete Research Council initiated a comprehensive study of slab systems at the University of Illinois, Urbana. This study, begun in September, 1956, involved both laboratory testing and analytical studies. A paper by Sozen and Siess (9) outlines the scope of this study and why it was commissioned.

Five tests of nine panel 1/4 scale slabs with and without beams were tested to failure. These tests confirmed that there was no fundamental difference in their behavior and the existing design rules led to smaller factors of safety for flat slabs. Using the Newmark plate analog to generate difference equations for beams and columns, elastic solutions were obtained for similar slabs for purposes of comparing with the test slabs and for extending the parameters.

The results of this study led to a unified approach to the design of two-way slabs with and without beams in the 1971 ACI Code.

1971 ACI CODE PROCEDURES

Two procedures, the Direct Design Method (DDM) and the Equivalent Frame Method (EFM) were introduced to replace the five methods for slab design in the 1963 code. Both methods are essentially frame methods.

The DDM originally considered a design strip similar to the elastic frame but without the interior columns. The total factored panel moment was computed for each span as:

$$M_o = 0.125 w l_2 l_n^2 \quad (4)$$

where w is the factored design load per unit area, l_2 is the span length perpendicular to the design strip and l_n is the clear span length. For interior panels 65% of M_o was assigned to the negative moment and 35% to the positive moment. For exterior spans, a modified frame analysis was performed by

computing a factor, α_{ec} , and distributing the moments in the exterior spans as functions of this factor as illustrated in Fig. 4. α_{ec} was defined as the ratio at a joint of the stiffness of the equivalent edge column (defined later for the EFM) to the stiffness of a slab-beam element based on gross concrete dimensions. Computing α_{ec} was extremely tedious and in the 1983 ACI Code this exterior column computation was replaced with a set of coefficients depending on the edge support. It now resembles the old Empirical Method except that the coefficient for M_o is such that it gives a value of total panel moment that is much closer to a Nichols' type analysis.

The EFM replaced the former elastic frame analysis. As part of the Illinois study Corley, Sozen and Siess (10) compared the moments calculated from the elastic frame analogy defined in the 1956 ACI Code with known elastic solutions. They concluded that, in general, the elastic frame analysis gave values of the positive moment that were too low and values of the negative moment at the column centerline that were too high. Generally the design negative moments after reducing to the critical section were either too high or too low for interior columns depending on the dimensions of the panel and column and too high for exterior columns. To overcome these difficulties required proposing new stiffnesses for the members of the elastic frame. These new stiffnesses incorporated in the EFM were presented by Corley and Jirsa (11).

The concept is to introduce torsional members between the columns and slab-beam elements. The reduction in column stiffness is achieved by defining an equivalent column formed by the actual column and attached torsional members as shown in Fig. 5. Torsional members are assumed to have a constant cross sections throughout their lengths consisting of a portion of slab having a width equal to that of the column or capital plus that portion of the transverse beam above or below the slab, if any. A stiffness K_t is computed from the expression

$$K_t = \sum \frac{9 E_c C}{l_2 (1 - \frac{c_2}{l_2})^3} \quad (5)$$

where the section parameter C is evaluated for the cross section by dividing it into separate rectangular parts and summing as follows

$$C = \sum (1 - 0.63 \frac{x}{y}) \frac{x^3 y}{3} \quad (6)$$

The stiffness of the equivalent column is defined as the sum of the flexibilities of the column and torsional member as

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \quad (7)$$

In computing the stiffness of the columns, K_c , the moment of inertia at any cross section outside the joint is based on the gross area of concrete taking into account any variation in section along the axis and at a joint is considered infinite from the top to the bottom of slab-beam. This is shown in Fig. 6.

Similarly, Fig. 7 shows the geometry for a typical slab-beam element where the moment of inertia is based on the concrete section outside the joint but from the center of the column to the face of the column or capital is assumed equal to the moment of inertia at the face of the column or capital divided by the quantity $(1-c_2/l_2)^2$.

Since the frame is defined using the centerline dimensions of the members, the negative moments at the column centerlines must be reduced to obtain the design moments at the critical section, defined generally at the face of the supports.

As with previous elastic frame analyses, when the specified live load was less than three-quarters of the specified dead load, design moments were obtained with full factored load on all spans. However, when the specified live load exceeds three-quarters of the specified dead load, the design moments are obtained with full factored dead load on all spans but only three-quarters of factored live load on appropriate spans to give maximum effects.

At the time the EFM was formulated, the only practical solution for elastic frame analysis was the moment distribution procedure, hence the need to determine fixed end moments, distribution factors and carry-over factors for the non-prismatic members resulting from the stiffness definitions. Although approximate values for these parameters for different geometric conditions have been tabulated to assist designers, for example Misis and Simmonds (12), the method is impractical for manual computation. However the concept has been incorporated successfully in computer programs written expressly for these definitions of stiffness and using a slope-deflection formulation.

In both the DDM and the EFM the last step is to distribute the design moments at the critical section across the width of the panel. In the 1971 Code, rules for this distribution to column and middle strips were given as part of the DDM. The definition of the column strip was defined with a width equal to half the

smaller of l_1 or l_2 centered on the column line instead of half the frame width as was used in previous codes. The middle strip was the remainder of the slab width. The distribution rules were specified for exterior negative, interior negative and positive moment critical sections and were functions of the panel aspect ratios.

LIMITS FOR APPLICATION OF ELASTIC FRAME ANALOGIES

Until the 1971 Code, the elastic frame method and all design specifications for flat slabs were explicitly limited to slabs with square or rectangular panels. All of the rules for assigning member stiffness and distributing design moments laterally across the slab both before and for the 1971 Code were developed by considering only square or nearly square panels.

Six limitations are listed before the DDM can be used. Three of them, namely, a minimum of three spans, limiting successive span lengths to one-third of the longer span and limiting the ratio of live to dead load are required for the DDM in order for the coefficients used to analyze the design strip to be valid. The remaining three limitations, namely, ratio of longer to shorter spans not greater than 2, column offset to maximum of 10% of span and limits to the effective beam stiffness ratio are required to ensure two-way behavior and the validity of the lateral distribution rules. As such these limitations must also apply for use of the EFM or any other elastic frame analogy.

While it may be argued that a frame can be defined for any irregular slab system and this frame analyzed for any arbitrary gravity loading including point loads, the use of elastic frame analyses as defined by the codes for other than rectangular panels should be viewed with caution. Certainly the lateral distribution of moments in irregular slabs may differ significantly from the rules given in the ACI code.

FUTURE IMPROVEMENTS TO ELASTIC FRAME ANALOGIES

While there are many areas in which elastic frame analogies may be improved, only two, simplifying member stiffness for gravity loading and specifying member stiffness for lateral loading are mentioned here.

It is acknowledged that the attached torsional member concept developed for the EFM generally gives solutions that are closer to elastic solutions than previous frame analyses. However, the method is unnecessarily complex and the

amount of computation involved for even simple design strips requires specially written computer programs. This might be justified if there were no alternatives or if slab moments had to be determined with great accuracy. Since the EFM is an elastic analysis, many designers interpret the code to permit varying the design moments by up to 20% providing the total static moment is satisfied thereby taking advantage of the moment redistribution in the slab. Also there are cases when the EFM does not work so well. For example, the attached torsional member concept breaks down when the span in the lateral direction is much greater than the span of the panel. This was recognized by Corley and Vanderbilt (13) who recommended that l_2 in Eqn. 5 be the smaller of l_1 or l_2 . A better designation for the EFM, since there are many ways of defining equivalent members, would be 'Elastic Frame with Attached Torsional Member Method'.

The primary purpose of the equivalent column for gravity load analysis is to reduce the effective stiffness of the column. The same result can be obtained by merely using the gross concrete section for the slab-beam and column to obtain the member stiffness but then multiplying the column stiffness by a column reduction factor, γ , less than 1. The resulting frame consisting of prismatic members can then be analyzed by any plane frame program based on the direct stiffness method or even by hand calculations. Values of γ presented in Table 1 derived by Mulenga and Simmonds (14) result in design moments that are in closer agreement with moments obtained using non-linear analyses at service loads than those given by the EFM. Such a method could be designated as 'Elastic Frame with Column Reduction Factor Method'. Other formulations for assigning stiffness to improve the agreement between the actual and equivalent frames and simplify the calculations may yet be proposed.

An elastic frame analogy can also be used to determine moments in slabs forming part of the lateral load resisting structure. The ACI code allows the results of a lateral load analysis to be algebraically combined with those from gravity load analysis but is silent on how member stiffness should be assigned. In this case it is the slab-beam element stiffness that must be reduced.

Traditionally, for lateral loading, the 'equivalent beam width' concept has been used wherein only a portion of the slab width is considered when determining the slab-beam moment of inertia. An excellent review of this method is given by Vanderbilt and Corley (13). Their paper also describes an 'equivalent beam method' which is similar to the equivalent column in the EFM except that the torsional members are in series with the slab-beam element instead of the column

element. Unfortunately, this procedure retains the complexities of the EFM and requires a specially written computer program to use. They also discuss the use of a reduction factor, β , to account for the effects of cracking where β is the ratio of effective moment of inertia to the gross moment of inertia and recommend a value of $\beta = 1/3$ for lateral load analysis. It is hoped that future ACI codes will include provisions to assist the engineer in assigning member stiffness for lateral loading of slab-column structures and that these provisions will not require specially written software.

CONCLUSIONS

Elastic frame analogies provide the only practical means for obtaining elastic analyses of column supported two-way slab systems with rectangular panels.

Future ACI Building Codes should simplify the current provisions for modifying stiffness for gravity loading. These simplifications should make stiffness a property of the member rather than the joint so that any plane frame analysis program based on the direct stiffness formulation can be used. Provisions for assigning member stiffness for lateral loading of slab-column structures so that they may analyzed as elastic plane frames should also be included.

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TABLE 1—VALUES OF COLUMN REDUCTION FACTOR, γ .

Stiffness Ratio	l_2/l_1		
	0.50	1.00	2.00
$\alpha \frac{l_2}{l_1} = 0.0$	0.30	0.30	0.90
$\alpha \frac{l_2}{l_1} \geq 1.0$	1.00	1.00	1.00

Note: Linear interpolation between values is suggested

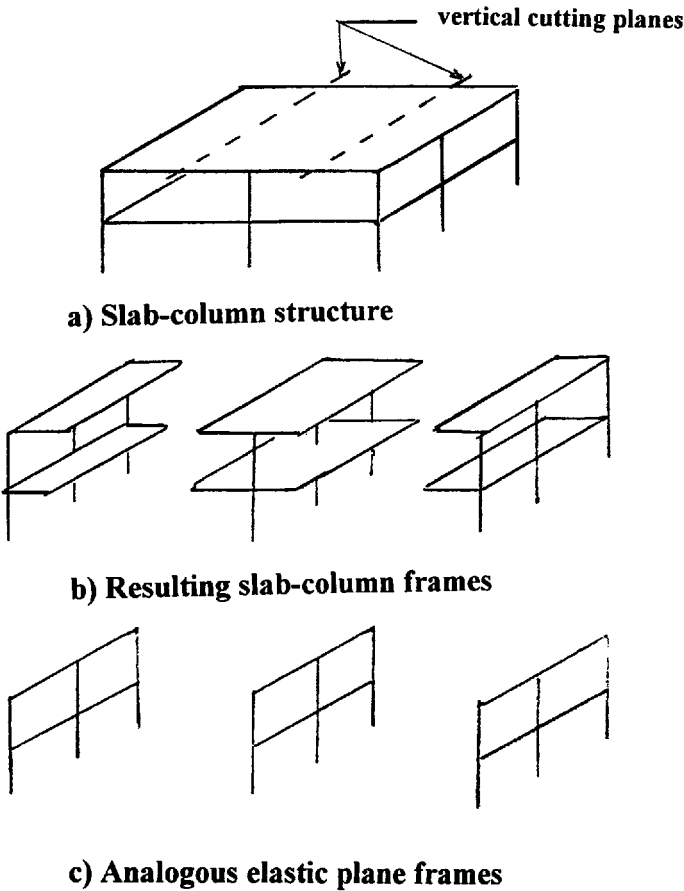
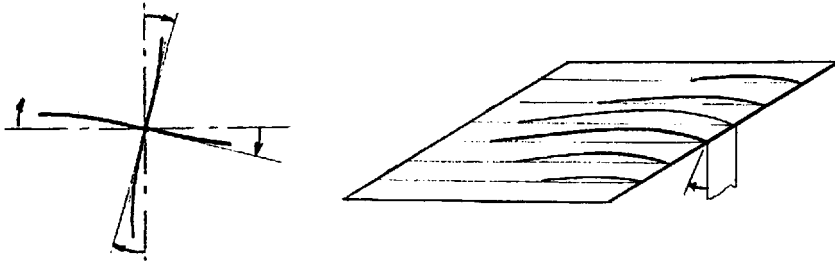
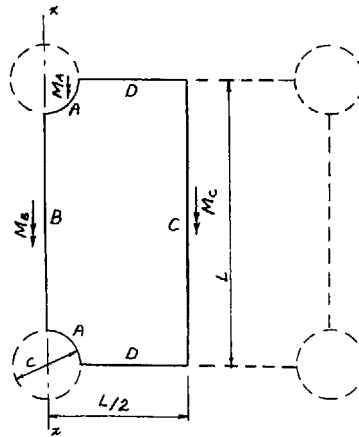


Fig. 1—Analogous frames (shown for one direction only).



(a) Joint rotation in 2-D frame (b) Joint rotation at slab-column connection

Fig. 2—Effects of rotation at different structural joints.



$$\text{Nichols' Analysis } M_o = 0.125WZ \left(1 - \frac{2c}{3L}\right)^3$$

Fig. 3—Nichols' analysis for flat slabs.

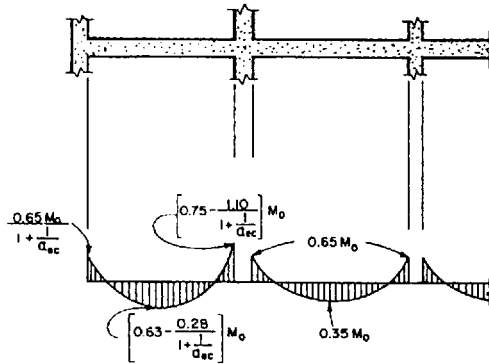


Fig. 4—Distribution of moments in exterior span, direct design method (1971-1983).

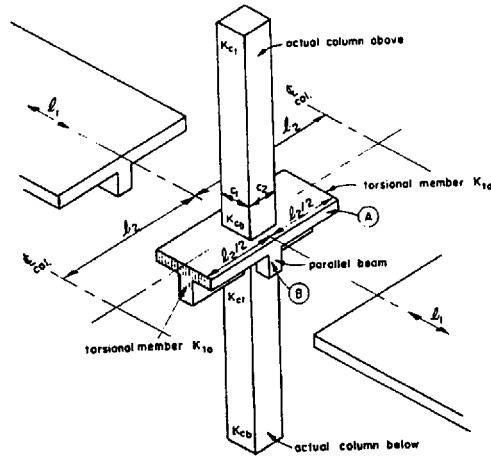


Fig. 5—Equivalent column—equivalent frame method.

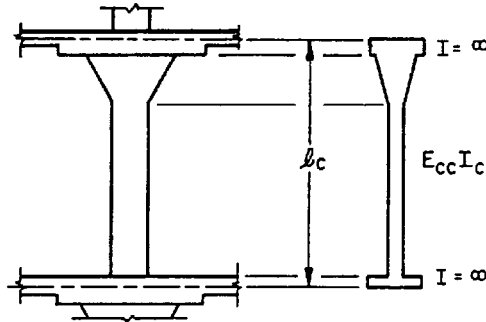


Fig. 6—Column geometry—equivalent frame method.

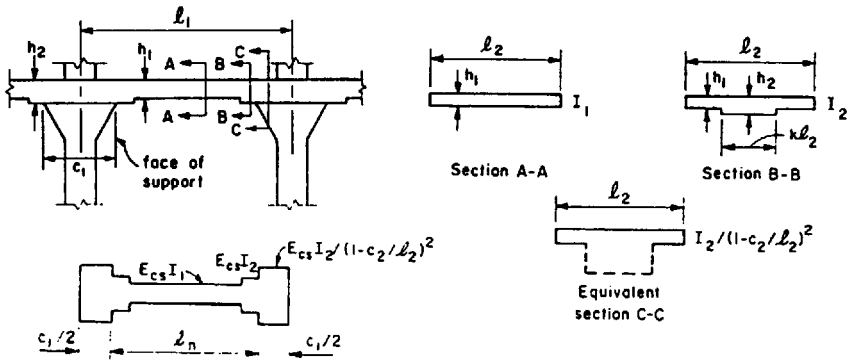


Fig. 7—Slab-beam geometry—equivalent frame method.

SP 183-2

Design for Severe Dynamic Loads

by S. Woodson and T. Krauthammer

Synopsis: Traditionally, U.S. Government agencies have developed and maintained manuals for the design of structures to resist severe dynamic loads, i.e. blast effects. However, such manuals have been primarily directed toward structures of a military nature, and relatively little attention has been given to the design of civilian buildings to resist blast effects. The lack of concern for the blast resistance of buildings is not surprising in that the threat has been minimal. Although some design guidance for blast resistance has been available to the general public, the primary users have been petro-chemical industries that are aware of potential accidental explosions related to their normal operations (i.e., chemical plants). Historically, general design guidance, such as that of the American Concrete Institute's Committee 318 (ACI, 1995) (1) has served the public well. However, two recent events, the World Trade Center and the Alfred P. Murrah explosions, have heightened awareness in the United States of the potential need to consider blast effects in the design of some buildings. The discussion presented herein summarizes existing blast-resistant design approaches and addresses issues that are critical to the development of buildings with improved resistance to severe dynamic loads. Emphasis is given to the design and behavior of reinforced concrete structures.

Keywords: design; dynamic analysis; reinforced concrete; structural steel

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INTRODUCTION

Manuals for Blast-Resistant Design

Numerous reports and publications on weapons effects and structural response to blasts are available in the open literature, but a single document for all aspects of blast-resistant design does not exist. Technical Manual (TM) 5-855-1 (Department of the Army, 1986) (6) serves as the Army's manual on protective construction, and TM 5-1300 (Department of the Army, the Navy, and the Air Force, 1990) (8) is the Tri-Service Manual for design against accidental explosions. The most widely used non-government blast-resistant design manual is published by the American Society of Civil Engineers (ASCE, 1985) (2) and is hereafter referred to as "ASCE Manual 42." A summary of the purpose of each manual, as well as some general guidance from each of these references, follows.

Tri-Service Manual (TM 5-1300), "Structures to Resist the Effects of Accidental Explosions"--Intended primarily for explosives safety applications, TM 5-1300 (Army designation) is the most widely used manual for structural design to resist blast effects. One reason for its widespread use by industry is that it is approved for public release with unlimited distribution. The manual states its purpose: "... to present methods of design for protective construction used in facilities for development, testing, production, storage, maintenance, modification, inspection, demilitarization, and disposal of explosive materials.

TM 5-1300 distinguishes between a “close-in” design range and a “far” design range for purposes of predicting the mode of response. Based upon the purpose of the structure and the design range, the allowable design response limits for the structural elements (primarily roof and wall slabs) are given in terms of support rotations. Such support rotations are computed simply by taking the arc tan of the quantity given by the predicted midspan deflection divided by one-half the clear span length, e.g. a three-hinge mechanism is assumed.

A unique requirement of TM 5-1300 is the use of lacing bars in certain conditions, particularly for very close-in explosions and for larger values of predicted support rotations. Figure 1 shows schematics of a structural element with lacing bars and various types of stirrups. Lacing bars are reinforcing bars that extend in the direction parallel to the principal reinforcement and are bent into a diagonal pattern, binding together the two mats of principal reinforcement. It is obvious that the cost of using lacing is considerably greater than that of using stirrups due to the more complicated fabrication and installation procedures.

Army Technical Manual 5-855-1--TM 5-855-1 is intended for use by engineers involved in designing hardened facilities to resist the effects of conventional weapons. The manual includes design criteria for protection against the effects of a penetrating weapon, a contact detonation, or the blast and fragmentation from a standoff detonation. TM 5-855-1 does not call for the use of lacing, but does require a minimum quantity of stirrups in all slabs subjected to blast. For beams, one-way slabs, and two-way slabs, the manual recommends a design ductility ratio (ratio of maximum response to yield response) of 5.0 to 10.0 for flexural design. The recommended response limits are only given in terms of ductility ratios, not support rotations. However, a more recent supplement to TM 5-855-1, Engineer Technical Letter (ETL) 1110-9-7 (Department of the Army, 1990) (7), provides response limit criteria based on support rotations.

ASCE Manual 42--The manual was prepared to provide guidance in the design of facilities intended to resist nuclear weapons effects. It attempts to extract concepts from other documents, such as the limited distribution design manual, “The Air Force Manual for Design and Analysis of Hardened Structures”; however, the intent is to provide a more general approach. ASCE Manual 42 presents conservative design ductility ratios for flexural response. Although the manual is an excellent source for general blast-resistant design concepts, it lacks specific guidelines on structural details, and it is out of print.

Design Issues

Distance/Loads--It is generally known that the greatest protection against blast effects is distance. The vehicle bomb has become a concern for buildings in the United States in regard to blast effects. Security and traffic route conditions that will prevent close access by an explosives-laden vehicle are significant protection measures. However, when distance cannot be reasonably guaranteed, specific design issues must be considered. Generally, the potential location for vehicle bombs is in a basement parking garage or in an exterior parking area. Structural protection against a detonation in a basement parking area is rarely feasible. Major structural components may be vulnerable to very close-in blast effects. In such cases, good structural design may only be successful at limiting catastrophic progressive collapse of the building. Additionally, the loading is worsened by partial confinement of the interior explosion. In contrast, external detonations are vented to the outdoor environment, and the potential distance to a major structural element is usually greater than for the basement parking condition.

The manuals listed above, as well as other available literature, provide a means to establish a reasonable approximation of the loading (peak pressure and duration) to be expected for a given explosive threat. The loading referred to is that applied to the exposed surfaces. As openings are created (e.g., windows, walls, and breached floor slabs), the blast loading on subsequent surfaces becomes very difficult to define, and research is continuing in this area. Failed components or sections of structural elements may be propelled into other elements, thereby complicating the loading process. Additionally, heavy failed sections of floor and wall slabs may simply fall by gravity, causing a "pancake" effect on lower floors. Thus, structural details are needed to help prevent progressive collapse.

Columns--A primary concern for buildings is the widespread use of columns. Columns (particularly exterior columns) are avoided in protective structures, and continuous walls are preferred. Columns are generally very stiff against lateral flexural response due to the relatively large quantities of longitudinal reinforcement. Also, compressive forces from gravity loads enhance the lateral strength at initial loading. The consequence is a structural element that will likely respond in a brittle mode, such as shear, unless specifically detailed with adequate confining reinforcement. Fortunately, columns are not generally vulnerable to "far-away" blast loadings, which tend to have lower peak pressures and engulf all sides of

the column. However, columns may be vulnerable to intense close-in blast loadings. In such cases, it is imperative that the total structure is capable of redistributing gravity loads when a column(s) has been destroyed.

Principal Reinforcement--Primary design issues include the placement (location) and continuity of principal reinforcement in slabs. In floor and roof slabs of protective structures, the principal reinforcement is usually equal in each face of the slab and is continuous throughout the span. If lengths prohibit continuous bars, then mechanical couplers or long splice lengths (prescribed in blast-resistant design manuals) are required. Additionally, all principal reinforcement is well anchored into the supporting elements. In contrast, floor slabs in buildings are primarily reinforced in tension zones and are vulnerable to "reversed" loading. Such reversed loading is very likely when shock waves from an explosion project upward and load floor slabs located at levels higher than the explosive source. When conventional slabs are loaded underneath by blast pressure, the reinforcement is generally in the "wrong" place. Consequently, the slabs are vulnerable to catastrophic failure and will probably come to rest in a pancake formation.

Membrane Behavior--As discussed in structural dynamics textbooks, such as Biggs (1964) (4), a fixed-supported beam or one-way slab element (Figure 2) under a slowly applied uniform load initially undergoes elastic deflection due to bending. As loading continues, plastic hinges first form at the supports and later at midspan. The load-deflection curve associated with the formation of the failure mechanism is referred to as a "resistance function." A common approach is to develop a single-degree-of-freedom model for dynamic analysis, using the resistance function to define the peak resistance (ultimate capacity) and response limit of the element. The ultimate capacity and response limit can be significantly enhanced by membrane forces. The ultimate capacity may be enhanced by compressive membrane forces, and the response limit may be enhanced by tensile membrane forces. Figure 3 presents a typical resistance function that includes membrane behavior.

As discussed by Park and Gamble (1980) (18), the ultimate flexural capacity is enhanced by compressive membrane forces in slabs whose edges are restrained against lateral (outward) movement. As the slab deflects, changes in geometry cause the slab's edges to tend to move outward and to react against stiff boundary elements. The membrane forces enhance the flexural strength of the slab sections at the yield lines. Research has shown that compressive membrane forces can increase the ultimate capacity of

both one-way and two-way slabs, with values up to 17 times the yield-line (pure flexure) resistance as shown by Roberts (1969) (19). An ultimate capacity of 1.5 to 2 times the yield-line value is more common. Actually, past design manuals have not fully utilized compressive-membrane theory in defining resistance functions, but criteria currently under development will closely follow the theory presented by Park and Gamble (1980) (18). Designers should be cautious in relying on compressive membrane behavior in buildings, but may find confidence in applying the theory to slab systems that include stiff beams. In any case, knowledge that compressive-membrane forces exist will provide a "silent" safety factor.

Although compressive-membrane behavior may be limited, tensile-membrane behavior can be a significant factor in limiting catastrophic failure and progressive collapse. The tensile-membrane region defined in Figure 3 represents a region where the load resistance increases as the deflection increases. As discussed by Park and Gamble (1980) (18), after the ultimate load resistance has been reached, the load resistance decreases until membrane forces in the central region of the slab change from compression to tension. In pure tensile-membrane behavior, cracks penetrate the whole thickness, and yielding of the steel spreads throughout the central region of the slab. The load is carried mainly by reinforcing bars acting as a tensile net or membrane; thus, the resistance increases as the steel is strained until rupture occurs. The increase in load resistance that accompanies this action is often called "reserve capacity."

Reserve capacity is important in the design of protective structures since moderate-to-severe damage is often acceptable if collapse is avoided. It is possible for a slab's peak reserve capacity to equal or be greater than the ultimate capacity. Tensile membrane behavior can only occur if the principal reinforcement extends across the entire length of the slab (or is properly spliced) and is well anchored into supporting members or adjacent elements. Additionally, stirrups placed throughout the length of the slab will help to bind the top and bottom layers of principal steel, thus enhancing the integrity of the concrete element during the large deflections associated with tensile membrane behavior. Also, consideration should be given to the strength of supporting members for determining the extent of potential reserve capacity.

Numerical Application of Membrane Effects

Although the information on membrane effects in structural concrete slabs has been known for more than three decades (Park and Gamble 1980, Wood 1971) (18, 21), its application for both numerical analysis and/or design has not yet matured. In blast-resistant structural behavior, until recently, the elastic and elasto-plastic models that have been in use since the early 1960s, as discussed by Biggs (1964) (4), were the most frequently used approaches. Those approaches, however, considerably underestimated the load-carrying capacity of slabs, as previously discussed herein. This underestimation of capacity, although acceptable for design, was not acceptable for performing detailed analysis of slab-type structures subjected to severe dynamic loads. Here, it should be noted that blast loads have durations that are about 1/1,000 earthquake durations, and under such conditions the inertia effects could dominate the behavior. In statically loaded slabs, the capacity underestimation was a hidden safety factor. Under severe dynamic conditions, however, the load-response relationship is much more complicated, and one needs to know how the loads are resisted to determine the structural safety.

The model presented by Park and Gamble (1980) (18) included an expression for estimating the peak load-carrying capacity, $w_{\max}$, of one-way reinforced concrete slabs in the compression membrane domain. It was noted, based on test data obtained by various researchers (Park and Gamble 1980) (18), that this capacity was associated with central deflections to slab thickness ratios (δ/h) between 0.1 and 0.89. Woodson (September 1994) (22) reported δ/h ratios near 0.3 for one-way slabs with length-to-effective depth (L/d) ratios of 10. However, for deep one-way slabs (L/d of 3 and 5), Woodson (November 1994) (23) reported that the δ/h ratios varied between approximately 0.03 and 0.07. Obviously, the peak load capacity in deep slabs is reached at much smaller δ/h values, as compared with those in more slender slabs. Park and Gamble (1980) (18) recommended that for slabs with $L/h = 20$, the peak capacity $w_{\max}$ can be estimated at $\delta/h=0.5$. For slabs with lower L/h ratios, the δ/h values are expected to be lower (i.e., the peak capacity will be reached earlier). Furthermore, the peak load will be reached at lower δ/h values in strips (i.e., one-way action), as confirmed by Woodson (September, November 1994) (22, 23). The transition into the tensile membrane domain was noted to occur in the range $1 \leq \delta/h \leq 2$, and it corresponded in load to the yield-line capacity of the slab. Beyond that transition, the resistance is governed by the tensile strength of the steel, and such a model was also discussed by Park and Gamble (1980) (18).

Employing these models for both the compression and tension membranes enables one to describe a complete load-deflection relationship for structural concrete slabs subjected to uniformly distributed loads, and for several typical support conditions.

Compressive membrane action can develop if sufficient resistance exists at the supports to both outward motion and rotation. Similarly, preventing inward motion at the supports is essential for the development of tensile membrane effects. Park and Gamble (1980) (18) discussed the requirements of support restraint to enable compressive membrane action. They showed that compressive membrane action can be achieved when the lateral support stiffness has the value $S = 2E_c / (L/h)$, in which L/h is the span-to-slab thickness ratio and E_c is the slab's modulus of elasticity. The ratio S/E_c defines the relative support stiffness, as compared with the slab's modulus of elasticity. Although significant compressive membrane enhancement can be achieved even for low lateral support stiffness values, these support conditions must be adequately considered to ensure compressive membrane behavior.

Krauthammer (March 1984) (9) showed that considering membrane effects (both in compression and tension) improved considerably the ability to explain some observations in slabs tested under explosive loads. These preliminary results motivated further attention to detailed structural models that included both improved membrane effects and direct shear resistance mechanisms (Krauthammer et al. April 1986, Krauthammer August 1986) (11, 12). This improved model was a modified version of that presented by Park and Gamble (1980) (18) to be used for two-way slabs, and it included the effects of externally-applied in-plane forces. In these later studies, such load-deflection relationships were adopted for dynamic structural analysis based on a single-degree-of-freedom (SDOF) approach, as discussed next.

The relationship between load and structural response has to be emphasized, and the following brief discussion is aimed at illustrating the potential difficulties in the dynamic domain. Limiting the illustration to the linear response regime, the relationship between load and response under a static load for an SDOF mass-spring-damper system (i.e., the equation of static equilibrium) is given as:

$$F = Kx \quad (1)$$

in which, F is the magnitude of the applied load, K is the structural stiffness, and x is the corresponding deflection. The static external load is resisted only by the spring, and the mass and damper do not contribute to resisting the load.

In the dynamic domain, the corresponding equation of equilibrium is given by:

$$F(t) = M\ddot{x} + C\dot{x} + Kx \quad (2)$$

in which F , K , and x were defined above (here, however, the force is time dependent). M is the mass and C is the damping coefficient, respectively; and $\ddot{x}$ and $\dot{x}$ are the acceleration and velocity, respectively.

Mass and stiffness parameters for the structural system under consideration are selected based on the type of problem. These include the load source, type of structure, and general conditions for load application to the structure (i.e., localized load on a small part of the structure, distributed load over a large part of the structure, etc.). Obviously, the expected behavioral domain (linear-elastic, elastic-perfectly plastic, nonlinear, etc.) will affect this relationship. The general approach for selecting such parameters has been discussed in detail by Biggs (1964) (4), and most design manuals contain similar procedures. Although neither Biggs (1964) 94) nor the design manuals provide information on the treatment of fully nonlinear systems by SDOF simulations, such approaches were presented in the literature, cited above, and they can be employed for analysis and design. The structural element is represented by an equivalent SDOF system, which is a modified version of Eq. (2):

$$F(t)/M(x) = \ddot{x} + 2\xi\omega'(x)\dot{x} + [R(x)/M(x)]x \quad (3)$$

in which x , $\dot{x}$, and $\ddot{x}$ are the displacement, velocity, and acceleration, respectively, of the point on the structural element that is being modeled. $M(x)$ represents the equivalent mass of the system, $R(x)$ is the resistance function, and $\omega'(x)$ is the damped natural circular frequency. M , R , and ω' are numerical functions that depend on the state of deformation of the system. ξ is the damping ratio of the system, given as a ratio of the critical damping (i.e., $\xi = C/C_{cr}$). $F(x)$ is the equivalent time-dependent loading function. The nonlinear load-deflection relationship serves as the skeleton resistance curve, based on which the dynamic resistance function and the dissipation of energy are evaluated. An equivalent mass of the SDOF

system is derived based on the deflected shape of the structural member. This approach bears similarities to previous methods, as described by Biggs (1964) (4) and as shown in design manuals, while the derivation of nonlinear resistance functions for various structural behavior mechanisms and other parameters was discussed by Krauthammer et al. (1990) (14). The effect of structural damping is usually small (ξ is typically between 2% and 5%; however, this should be assessed based on the specific case under consideration). Therefore, the damping contribution, $2\xi\omega(x)\dot{x}$ may be ignored for many cases by setting $\xi=0$ in Eq. (3).

The differences between the static and dynamic cases arise from the effects of inertia ($M\ddot{x}$) and damping ($C\dot{x}$) that do not participate in the static response. Although the effect of structural damping is small, the inertia effect could be significant. Inertia may dominate the response whenever loading durations are much shorter than structural response times. Furthermore, unlike the static case where the magnitudes of force and stiffness determine directly the corresponding deflection, in the dynamic domain the response (i.e., deflection, velocity, and acceleration) is obtained by solving the differential equation (2 or 3). Such solutions are usually obtained by employing a numerical approach, as discussed in Biggs (1964) and Clough and Penzien (1993) (4, 5). The system response will depend not only on the magnitude of the force, but also on the relationship between the dynamic characteristics of the force and the frequency characteristics of the structure. These are defined by the ratio K/M and the effect of damping. A detailed discussion of these issues is presented in Biggs (1964) (4), Clough and Penzien (1993) (5), and in Chapter 7 of Manual 42 (ASCE 1985) (2). The various design manuals (such as TM 5-855-1, TM 5-1300, and Manual 42) contain dynamic response charts and tables that are based on SDOF considerations, and these can be used for design.

The differences between the approaches discussed by Biggs (1964) (4) and the various design manuals, cited above, and the modified approach proposed by Krauthammer (1984, 1986) (10, 12) and Krauthammer et al. (April 1986) (11) include the use of a different structural resistance function. Instead of employing the classical concept for a resistance function (elastic, elastic-plastic, or plastic), the term $R(x)$ was represented by the modified load-displacement relationship that included membrane effects, as described above. That relationship included membrane effects (compression and tension for one- or two-way slabs), and the influence of externally induced in-plane forces, shear effects, and strain rate effects on

the materials. Another modification extended the application of this approach to cases where the loads were localized rather than uniformly distributed.

When more advanced numerical approaches are considered important (e.g., finite-element or finite-difference methods), one needs to ensure that the approach can represent membrane effects correctly. Certain types of elements may include tension membrane effects, but may lack the ability to exhibit compression membrane behavior. Others may introduce “numerical locking,” which is unrelated to the physical phenomenon of compression membrane behavior. Obviously, analysts and designers must understand the characteristics of the particular numerical tool they intend to use to ensure that the results would be interpreted correctly. Since a detailed discussion on such topics is beyond the scope of this paper, the reader can find further information in the literature (for example, Bathe 1996) (3).

Application for Analysis and Design--The design of slabs in typical construction is often governed by serviceability requirements rather than by strength considerations. Therefore, the consideration of full compression and tension membrane enhancements may not be feasible. Since the peak compressive membrane capacity is expected to be associated with a central deflection of about $0.5h$, it is quite possible that such conditions would meet ACI deflection control requirements (ACI 1995) (1). Under certain conditions, considering some parts of membrane contribution could be advantageous. Furthermore, for special cases, the consideration of both compression and tension membrane effects could be justified. These issues need to be discussed further to illustrate such points of view.

Park and Gamble (1980) (7) showed that by considering compression membrane effects, designers could reduce the amount of steel in the slab to less than that required by the yield-line theory. However, additional steel had to be added to the supports to ensure sufficient restraining of the slab. Nevertheless, it was expected that more steel would be saved by considering compression membrane effects than that added to the supporting beams. However, designers would need to consider loading patterns on the floor slab to prevent overstressing the tie reinforcements in the supporting beams. In summary, one can include consideration of compressive membrane effects, but the detailing of steel in the support regions must be carefully examined to ensure that the enhanced slab capacity will not jeopardize the support integrity.

The consideration of membrane effects in special structures could be much more attractive. In fortifications, one must rely on every possible contribution to structural resistance for surviving severe loading environments. Often, under such conditions, survivability rather than serviceability may govern the required performance criteria. Obviously, a slab that can resist higher loads, even if it is associated with large deflections, could be very useful. Test data discussed by Krauthammer et al. (April 1986) (13) and Woodson (September 1994) (22) can be used to show such advantages. However, very careful attention must be given to shear reinforcement and to the support design to ensure that the slab can reach the corresponding resistance and deformation levels. Another type of structure that could benefit from membrane effect considerations would be a culvert. Again, based on operational needs for such structures, one may limit the design to only compressive membrane enhancement if deflections must be controlled. Otherwise, both compressive and tensile membrane effects could be considered. Besides the information presented by Park and Gamble (1980) (18), two recent examples of how compressive membrane enhancement could be utilized in design may illustrate this approach further.

Krauthammer et al. (1986) (13) adopted the approach in Park and Gamble (1980) (18) for the redesign of reinforced concrete culverts. A one-barrel box culvert 40 ft long with a 12-ft by 12-ft opening was considered. Slab thicknesses were 11.5 in. for the roof, 12 in. for the floor, and 8 in. for the walls. The findings confirmed the expectations in Park and Gamble (1980) (18) as follows: First, in an existing slab, even small support stiffness can provide significant compressive membrane enhancements. When the ratio S/E_c was varied between 0.005 and 0.16 for the roof slab, the compressive membrane enhancements varied between 1.1 and 1.47. Second, when the slab was redesigned (the same steel was redistributed to accommodate compressive membrane action), its compressive membrane enhancement varied between 1.2 and 1.65. Third, a large S/E_c ratio would not ensure a much higher compressive membrane enhancement. When S/E_c was set to 50 in the original slab, the compressive membrane enhancement increased from 1.47 to 1.5. The slab was then checked for meeting serviceability requirements. It was found that cracking would be controlled by the given amount of steel, and the δ/h ratio varied between 0.5 and 0.2 for the given range of S/E_c ratios. For the roof slab, these ratios corresponded to deflections between 0.57 in. and 2.3 in. For a span of 144 in., these deflections correspond to deflection-to-span ratios (δ/L) in the

range between 0.004 and 0.016, which may be quite acceptable according to ACI deflection control limits (ACI 1995) (1).

Meamarian et al. (November-December 1994) (15) continued the development of this approach by considering improved material and failure models and implemented them into an analysis and design computer code. In that approach, the general method presented by Park and Gamble (1980) (18) was used to derive the ultimate load, deflections, and sectional forces. Then, the Modified Compression Field Theory (Vecchio and Collins, March through April 1986) (20) was used to compute the internal stresses, strains, crack angles, and total deflections. The proposed approach was used to simulate the behavior of 10 slabs for which test data were available. The average ratios of computed results to test data for axial force, moment, load, and deflection were 1.07, 1.07, 1.04 and 1.0, respectively. However, the standard deviations for these comparisons were in the range between 0.13 and 0.38, showing that this approach will require additional attention before it could be used in support of design activities. Interestingly, for these slabs the compression membrane enhancement was estimated in the range between 1.7 and 2.9. Furthermore, this enhancement was important for thinner slabs with moderate amounts of steel ($0.007 \leq \rho \leq 0.012$), but it did not change for slabs with lower reinforcement ratios (e.g., $\rho = 0.005$).

Connections and Support Conditions--It has been shown that the effect of the supports' lateral restraint is very significant on the development of membrane enhancement in slabs. Although small magnitudes of support stiffness can introduce some membrane enhancement, one needs well-designed supports to enable the higher ranges of membrane enhancement. Attention to connection details is, therefore, an important factor that can ensure membrane enhancements. Reinforced concrete structures cannot develop their full capacity if their reinforcement details are inadequately designed. In connections, for example, the joint size is often limited by the size of the elements that frame it. This restriction, along with poor reinforcement detailing, may create connections without sufficient capacity to develop the required strength of adjoining elements. Knee-joints are often the most difficult to design when continuity between adjoining members is required. The work by Nilsson (1973) (16) provided extensive insight into the behavior of joints under static loads, and on the relationships between performance and internal details. He showed that even slight changes in a connection detail had significant effects on the strength and behavior of the joint. Park and Paulay (1975) proposed the diagonal strut and the truss mechanisms to describe a joint's internal

resistance to the applied loads. The diagonal compression strut is obtained from the resultant of the vertical and horizontal compression stresses and shear stresses. When yielding in the flexural reinforcement occurs, the shear forces in the adjoining members are transferred to the joint core through the concrete compression zones in the beams and columns. The truss mechanism is formed with uniformly distributed diagonal compression, tensile stresses in the vertical and horizontal reinforcement, and the bond stresses acting along the beam and column exterior bars. These two mechanisms can transmit shearing forces from one face of a joint to the other, and their contributions were assumed to be additive.

The design approach for reinforced concrete connections by Park and Paulay (1975) was adopted in TM 5-1300 (Department of the Army 1990) (9). Results from recent investigations of a blast containment structure by Krauthammer and Ku (accepted for publication) (14) and Otani and Krauthammer (accepted for publication) (17) can be used to illustrate the importance of structural concrete detailing in connection regions. The findings showed that the location of the diagonal bar across the inner corner of the connection affected the joint's strength. The relationships between the maximum stress and the location of the diagonal bar, and/or its cross-sectional area, could be examined to produce design recommendations that can ensure a desired level of performance. It was observed that the radial reinforcing bars across the connection (in the direction normal to the diagonal bar) affected the tensile stress in the diagonal bar at the inner corner of the connection. Furthermore, the diagonal compressive strut to resist the applied loads could be mobilized effectively by proper combination of radial and diagonal bars.

The strengthening of the joint regions by the diagonal bars caused the formation of plastic hinge regions in the walls near the ends of the diagonal bars. The relocation of the largest rotations from the support faces to the plastic hinge regions showed a shift of maximum moment and shear along the slabs. Examination of the stresses in the flexural bars along the slabs' planes of symmetry revealed that yielding and the maximum stresses in the interior flexural and tension bars were in the plastic hinge regions. Besides the damage to the flexural reinforcement, maximum shear stresses in the concrete and large tensile shock wave stresses also occurred in the plastic hinge regions. These stress patterns in both concrete and steel were in addition to the presence of direct in-plane tension in the slab (due to the expansion of the walls and roof caused by the interior explosion). They showed that excessive damage could occur at these regions if not properly designed. Current design procedures (in TM 5-1300) do not address the

issue of plastic hinge regions, nor do they anticipate the location of maximum negative moment (negative yield-line location) to occur near the ends of the diagonal bars. Furthermore, the computation of structural capacity, in current design procedures, is based on the assumption of yield-line formation at the supports. Clearly, the shift in hinge location must be included to derive a more realistic structural capacity.

Using support rotations as a parameter that can indicate the extent of damage is customary, as defined in TM 5-1300. For the containment structure under consideration, it was required that such rotations would not exceed 2 degrees (Otani and Krauthammer, accepted for publication) (17). A support rotation (sometimes defined as the global rotation) is estimated by the slabs' peak mid-span deflection divided by half the span length. The localized rotations computed in the hinge regions exceeded 12 degrees, and the global rotations exceeded 2 degrees. In this structure, the sidewall/roof connection, which had the largest local and global rotations, also had the largest diagonal and longitudinal stresses. This observation confirmed that the magnitudes of the local and global rotations provide a good indication of damage to the slab. Furthermore, these findings emphasize the need to address both local and global rotations in design approaches.

Conclusions

Research findings and design criteria developed for protective structures are available to assist engineers in the design of buildings to resist severe dynamic loads, particularly blast loads. Presently, no requirements for consideration of blast effects have been incorporated into general building design codes. It is clear that construction costs will be affected when the structural resistance to blast effects is improved. Protective construction has led to practical and more economical design concepts and details; however, significant refinements of such approaches are required before they can be adopted in civilian construction.

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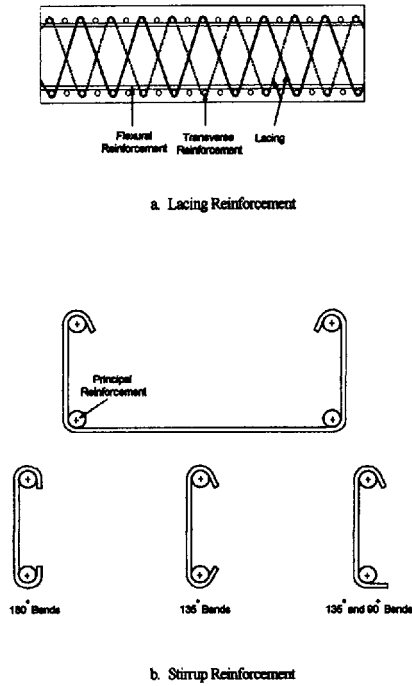


Fig. 1—Shear reinforcement.

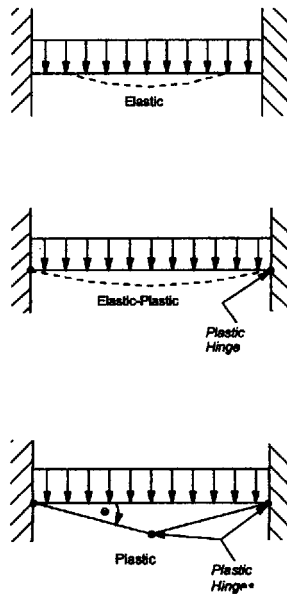


Fig. 2—Element response.

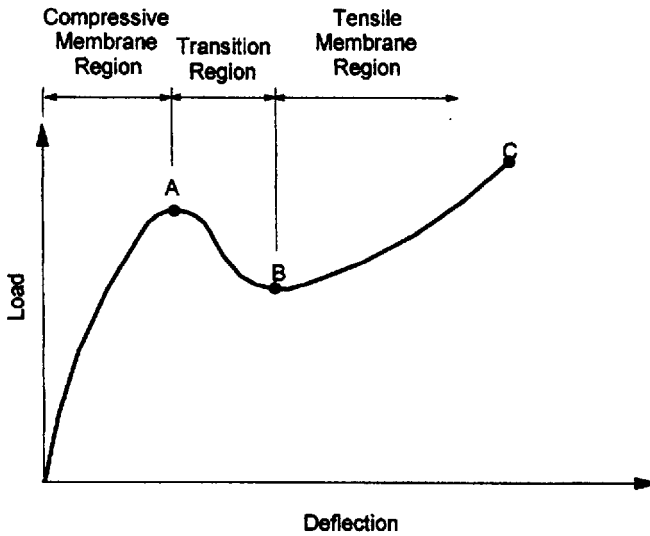


Fig. 3—Typical resistance function.

SP 183-3

Design for Punching Shear in Concrete

by S. Megally and A. Ghali

Synopsis:

Design of connections of columns to flat slabs to ensure safety against punching failure is presented. The connections transfer shearing forces and moments between the columns and slabs. The objective is to cover the design procedure in most practical situations including: interior, edge and corner columns, prestressed and nonprestressed slabs, slabs with openings and slabs with shear reinforcement. The ACI318-95 code requirements are adhered to where applicable. The designs are demonstrated by numerical examples. Design of shear reinforcement in raft slabs, footings and walls subjected to concentrated horizontal forces is also discussed.

Keywords: columns (supports); connections; ductility; footings; prestressed concrete; punching shear; reinforced concrete; seismic design; shear strength; slabs; structural design

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INTRODUCTION

One of the most common floor systems is the flat plate. It provides architectural flexibility, more clear space, less building height, easier formwork and consequently shorter construction time. A serious problem that can arise in flat plates is brittle punching failure due to transfer of shearing forces and unbalanced moments between slabs and columns. Cracking due to shear occurs on inclined surfaces within the slab thickness. Thus up to or close to failure, the cracks are not visible on the slab surfaces. Occasionally, drop panels are used; this is an increase in the slab thickness over a small rectangular area above the supporting columns. Often more economical and more architecturally acceptable solution is to increase the strength and the ductility by use of shear reinforcement.

Requirements of ACI318-95 (1) building code for design of slabs against punching shear are reviewed. Design steps, following the code when applicable are presented. Prestressed and nonprestressed slabs and slabs with openings are covered. The design steps are demonstrated by numerical examples for interior, edge and corner columns. Reference is made to an available computer program which can be used for design. Design and arrangement of punching shear reinforcement in raft slabs, footings and walls is also discussed.

TYPES OF SHEAR REINFORCEMENT

The most commonly used type of shear reinforcement is vertical shear

reinforcing bars in the form of closed stirrups or shear studs (2) (Figs. 1 and 2). Vertical reinforcing bars crossing the surfaces of the crack (Fig. 3) will prevent their widening, provided that no slip occurs. A small amount of slip commonly occurs in the bends at the top and bottom of the vertical branch of a stirrup. The tensile force in the vertical branches of stirrups can reach yield only in non-shallow members. In slabs, the vertical branch of a stirrup is relatively short; thus small slip causes large strain reduction, which does not allow the full yield strength of the stirrup to be developed. For this reason the Canadian Standard (3) allows use of stirrups only when the slab thickness is ≥ 12 in. (300 mm). The ACI318R (1) code commentary requires that stirrups be closed and be provided with heavy horizontal bars lodged in the stirrup corners to assist in anchorage (Fig. 1a).

The shear studs detailed in Fig. 4 have anchor heads sufficiently large to develop the yield strength of the stud stem. Crushing of the concrete below the heads does not take place and no measurable slip occurs. This has been confirmed experimentally (4).

To be most effective, the shear reinforcing bars must be as tall as possible by being anchored close to the slab surfaces (see the left-hand side of Fig. 3). A short shear reinforcing bar, as shown on the right-hand side of Fig. 3, is not effective because the inclined cracks can pass above and below the bar without intersection. Figure 5 is a vertical section perpendicular to a stud strip showing the position of the Stud Shear Reinforcement (SSR) and the flexural reinforcement in the slab thickness. For most effectiveness of the SSR, its overall height should be as long as possible. Thus the overall length of the studs should not be less than the thickness of the slab minus the specified covers at top and bottom; a tolerance value, e.g. one half bar diameter of flexural reinforcement may be allowed.

Figure 6 represents load-deflection graphs for three slabs of thickness 5.9 in. The slabs differ only in the absence (5) or presence of conventional stirrups (6) or studs (5) as shear reinforcement. It can be seen that the beneficial effect of conventional stirrups in such a thin slab is negligible. This represents partial justification of the Canadian Standard (3) provision that stirrups shall not be used in thin slabs.

ACI318-95 CODE REQUIREMENTS

Nonprestressed slabs

ACI318-95 (1) requires that at a critical section at $d/2$ from column face (Fig. 7):

$$v_u \leq \phi v_n \quad (1)$$

where v_n is the nominal shear stress (Eqs. 4-6); ϕ is the strength reduction factor ($\phi = 0.85$); v_u is maximum shear stress caused by the transfer of a factored shearing force V_u and moments M_{ux} and M_{uy} between the slab and column and acting at critical section centroid,

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{vx} M_{ux}}{J_x} y + \frac{\gamma_{vy} M_{uy}}{J_y} x \quad (2)$$

where b_o is length of perimeter of shear critical section; d is the distance from extreme compression fiber to centroid of longitudinal tension reinforcement; the subscripts x and y refer to centroidal axes in directions of both spans; (x, y) are coordinates of the point at which v_u is maximum and J is a property of critical section "analogous to polar moment of inertia". Figure 7 indicates the positive directions of x and y axes and the force V_u and moments M_{ux} and M_{uy} ; in this figure and others in this paper the arrows represent the directions of force and moments exerted by the column on the slab. γ_{vx} and γ_{vy} are fractions of the moments transferred by eccentricity of shear about the x and y axes, respectively:

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{b_1 / b_2}} \quad (3)$$

ACI318-95 (1) defines b_1 and b_2 , respectively, as widths of shear critical section measured in direction of the span for which moment is determined and perpendicular to it.

In absence of shear reinforcement, the code requires that the nominal shear stress of nonprestressed slabs be the smallest of (using lb and in. units):

$$v_n = v_c = 2 + \frac{4}{\beta_c} \sqrt{f'_c} \quad (4)$$

$$v_n = v_c = \frac{\alpha_s d}{b_o} + 2 \sqrt{f'_c} \quad (5)$$

$$v_n = v_c = 4 \sqrt{f'_c} \quad (6)$$

where v_c is the nominal shear stress provided by concrete; β_c is ratio of long side to short side of column; f'_c is specified concrete compressive strength; $\alpha_s = 40$ for interior columns; $\alpha_s = 30$ for edge columns and $\alpha_s = 20$ for corner columns.

When $v_u > \phi v_n$; slab thickness must be increased or shear reinforcement provided. When slab thickness is increased in the column vicinity, by means of

column capitals or drop panels, Eq. 1 must be satisfied at a critical section outside the thickened area of the slab. Although the stud shear reinforcement are shown (6) to be more effective than conventional stirrups because of the superiority of anchorage, ACI318-95 (1) code does not distinguish between the two types of shear reinforcement, and expresses the nominal shear stress in presence of shear reinforcement as:

$$v_n = v_c + v_s \leq 6\sqrt{f'_c} \quad (7)$$

$$v_c = 2\sqrt{f'_c} \quad (8)$$

$$v_s = \frac{A_v f_{yv}}{b_o s} \quad (9)$$

where v_s is nominal shear stress provided by shear reinforcement; A_v is area of shear reinforcement within a distance s ; f_{yv} is specified yield strength of shear reinforcement and s is spacing of shear reinforcement. The upper limit for s is 0.5 d . Shear reinforcement must be extended for a sufficient distance until the critical section outside the shear-reinforced zone (Fig. 8) satisfies Eq. 1 with

$$v_n = v_c = 2\sqrt{f'_c}$$

Other provisions for prestressed slabs and slabs with openings in the column vicinity will be discussed in following sections.

Prestressed Slabs

For prestressed slabs with no shear reinforcement, ACI318-95 (1) replaces Eqs. 4 to 6 by:

$$v_n = v_c = \beta_p \sqrt{f'_c} + 0.3 f_{pc} + V_p / b_o d \quad (10)$$

where V_p is the vertical component of all effective prestress forces crossing the critical section; f_{pc} is average value of f_{pc} in two vertical slab sections in perpendicular directions, with f_{pc} being the compressive stress at section centroid after allowance for all prestress losses; β_p is the smaller of 3.5 and $[(\alpha_s d/b_o) + 1.5]$. Eq. 10 can replace Eqs. 4 to 6 only if the following conditions are satisfied: (a) no portion of the cross section of the column shall be closer than 4 times the slab thickness to a discontinuous edge; (b) f'_c shall not be taken greater than 5000 psi and (c) f_{pc} in each direction shall not be less than 125 psi nor be taken greater than 500 psi.

In thin slabs, it is difficult to control the slope of tendon profile at the point it crosses a critical section. Thus for practical consideration, the last term in Eq. 10 may be neglected, or V_p reduced to account for the inaccuracy that can occur in the execution of the tendon profile.

Within the shear-reinforced zone, v_n is to be calculated using the same equations as for nonprestressed slabs. Section 11.5.4.1 of ACI318-95 allows, for prestressed members, spacing of shear reinforcement, s to reach $3h/4$ but not to exceed 24 in., where h is overall thickness of member. It is considered here that this limit is excessive in slabs and it is recommended that the spacing should not exceed $3d/4$. This is because the difference between d and h is more important in slabs than in beams and cracks could bypass the shear reinforcement as discussed earlier in conjunction with Fig. 3.

Slabs With Openings

ACI318-95 (1) requires that effect of openings on punching shear resistance of a slab-column connection must be considered when openings are located at a distance less than 10 times the slab thickness from the column, or when openings are located within the column strip. The effect of openings is taken into account by considering part of shear critical section to be ineffective. The ineffective part is that part of the critical section perimeter that is enclosed by straight lines projecting from the column centroid and tangent to the boundaries of the openings (see Example 4).

Reduction of the Coefficient γ_v

Section 13.5.3.3, introduced for the first time in ACI318-95 (1), allows, in design of slab-column connections without shear reinforcements, the use of values of γ_v smaller than what is given by Eq. 3. This option is allowed only when the value of V_u is relatively small and when other conditions are satisfied. Discussions (7) of the new section, when it was proposed, indicated that this can lead to unsafe design. The present authors are of the same opinion; thus this option is not recommended here.

Design for One-Way Shear

Although this paper focuses on two-way (punching) shear design of slabs, one-

way shear resistance should also be checked. Figure 9a shows the one-way shear critical sections for interior and edge slab-column connections, according to the ACI 318-95 building code (1). The critical section extends in a plane across the entire slab width and is located at a distance d from the column face. Figure 9b shows the one-way shear critical section, according to the Canadian Standard (3), for corner slab-column connections in which case the critical section is a straight line of minimum length located not farther than $d/2$ from the column corner. If slab cantilevers extend beyond the face of the corner column, the length of the critical section may be extended into the cantilevered slab portion by a length not exceeding d .

The one-way shear check should satisfy the inequality:

$$v_u = \frac{V_u}{b_o d} \leq 2\phi \sqrt{f'_c} \quad (11)$$

where b_o is the length of the critical section.

For the corner column in Fig. 9b, the angle ψ corresponding to the minimum b_o should satisfy:

$$\tan \psi = \frac{\sqrt{(c_2 + 0.5d \cos \psi)}}{\sqrt{(c_1 + 0.5d \sin \psi)}} \quad (12)$$

This equation can be used to solve for ψ by trial. In the first trial $\psi = 45^\circ$, one additional trial will commonly be sufficient.

It should be mentioned that the one-way shear critical section shown in Fig. 9b is not specified by the ACI 318-95 code (1). However, it is suggested here to use the section shown in Fig. 9b for one-way shear design of corner slab-column connections.

HIGHER ALLOWABLE VALUES FOR THE NOMINAL SHEAR STRESS AND SPACING OF SHEAR REINFORCEMENT

Shear reinforcement is not effective unless it is well anchored. ACI 318R-95 code commentary (1) emphasizes the difficulty of anchorage of stirrups in slabs thinner than 10 in.; for such slabs, the commentary requires that stirrups should only be used if they are closed and enclose a longitudinal bar at each corner. The Canadian Standard CSA-A23.3-94 (3) does not allow stirrups in slabs thinner than 12 in. (300 mm).

Because of the superiority of anchorage of the stud shear reinforcement, justified by tests (5,8-12), ACI421.1R-92 (2) suggests the following deviations from ACI318 Code when shear studs are used: (a) the nominal shear stress, v_n resisted by concrete and shear reinforcement (Eq. 7) can be as high as $8\sqrt{f'_c}$, instead of $6\sqrt{f'_c}$. This enables use of thinner slabs; (b) the upper limits for s_o and s can be based on the value of v_n at the critical section at $d/2$ from column face:

$$s_o \leq 0.5d \text{ and } s \leq 0.75d \text{ when } v_n / \phi \leq 6\sqrt{f'_c} \quad (13.1)$$

$$s_o \leq 0.35d \text{ and } s \leq 0.5d \text{ when } v_n / \phi > 6\sqrt{f'_c} \quad (13.2)$$

where s_o is the distance between first peripheral line of studs and column face.

ACI421.1R-92 (2) considers a vertical branch of a stirrup to be less effective than a stud in controlling shear cracks because the stud stem is straight over its full length while the ends of the stirrup branch are curved, and the mechanical anchors at the stud ends ensure that the yield strength is available at all sections of the stem; which is not the case with a vertical branch of a stirrup.

For the same reasons the Canadian standard CSA-A23.3-94 (3) allows, in presence of shear studs; a value of v_c one-and-half times the allowable value when stirrups are employed. The same approach is adopted in the remainder of the paper; thus Eq. 8 is replaced by $v_c = 3\sqrt{f'_c}$.

The value of v_c , the nominal shear stress provided by concrete in presence of shear reinforcement, represents the resistance of concrete after occurrence of shear cracks. Experiments indicate that studs are more efficient than conventional stirrups in controlling the width of inclined cracks. Thus, a higher value of v_c should be expected. The value $v_c = 3\sqrt{f'_c}$ is verified to be safe by experiments (10).

SHEAR REINFORCEMENT ARRANGEMENTS

Figures 10 and 11 show typical arrangements of stud shear reinforcement at rectangular and circular columns, respectively. Each group of studs on a line perpendicular to column face are welded to a steel strip, Fig. 2. ACI421.1R-92 (2) recommends that, in direction parallel to column face, the maximum distance, g between the steel strips be less than $2d$. This limitation is to ensure that the studs confine the concrete and prevent widening of shear cracks over the perimeter of the critical section.

When stirrups are used, they should be placed in rows parallel to the column face

(Fig. 1). In direction parallel to the column faces, the distance g between stirrup legs (Fig. 1a) should satisfy the requirement $g \leq 2d$; or because stirrups are less effective than shear studs, a more restrictive limit should apply.

THE PARAMETER J

The vertical shear stress v_u calculated by Eq. 2 has vertical resultant component $= V_u$; but has resultant moment components slightly smaller than $\gamma_{vx} M_{ux}$ and $\gamma_{vy} M_{uy}$. Replacing J_x and J_y by the moments of inertia I_x and I_y , respectively gives linearly varying stress distribution, whose resultants are exactly equal to V_u , $\gamma_{vx} M_{ux}$ and $\gamma_{vy} M_{uy}$. Thus Eq. 2 becomes:

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{vx} M_{ux}}{I_x} y + \frac{\gamma_{vy} M_{uy}}{I_y} x \quad (14)$$

This equation satisfies the linear distribution required in Section 11.12.6.2 of ACI318-95 (1). The equation satisfies equilibrium only when x and y are centroidal principal axes (e.g. Figs. 7a, b and 8a, b).

The symbols J_x and J_y , defined by the code (1) as section properties "analogous to polar moment of inertia", have no known meaning in mechanics. For this reason, calculation of the two parameters is ambiguous when the critical section is irregular (e.g. slabs with openings and critical sections outside shear-reinforced zone). Ambiguity of the symbols J_x and J_y is avoided by replacing them by the well-known second moments of area I_x and I_y .

THE COEFFICIENT γ

The empirical Eq. 3 adopted by ACI318-95 is shown by numerous experiments to be satisfactory for interior columns in which the critical section, at $d/2$ from column faces, has the shape of the perimeter of a closed rectangle. At the same location, the critical section for edge and corner columns has three or two sides, respectively (Fig. 7b and c). Outside the shear-reinforced zone, the critical section follows the perimeter of a closed or open polygon, whose sides are not all parallel to a column face (Fig. 8). Problems arise (13) when the empirical Eq. 3 is employed for corner and edge columns.

Elgabry and Ghali (14) showed by numerous finite element analyses that Eq. 3 does not apply for all cases and for all critical sections. ACI 318-95 code Eq. 3 proved to be satisfactory for interior columns only, but for other cases the finite element results showed that the same equation cannot be applied. Elgabry and

Ghali (14) supplemented the ACI code equation to give a set of equations applicable for all cases (Fig. 12):

At interior columns:

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\ell_y / \ell_x}} \quad (15)$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\ell_x / \ell_y}} \quad (16)$$

At edge columns:

$$\gamma_{vx} = \text{same as Eq. 15} \quad (17)$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{(\ell_x / \ell_y) - 0.2}} \quad \text{when } \frac{\ell_x}{\ell_y} < 0.2, \gamma_{vy} = 0 \quad (18)$$

At corner columns:

$$\gamma_{vx} = 0.4 \quad (19)$$

$$\gamma_{vy} = \text{same as Eq. 18} \quad (20)$$

where ℓ_x and ℓ_y are projections of the critical section on principal axes x and y respectively.

The safety of design using the above equations has been verified using published experimental results (14).

DESIGN PROCEDURE

The data required for design of slab-column connections are: d , c_1 , c_2 , V_u , M_{uxO} and M_{uyO} and f'_c (Fig. 7). It is required to determine whether d is sufficient for safety against punching without the use of shear reinforcement and if not, design the necessary shear reinforcement. The steps of design are:

Step 1: Replace V_u , M_{uxO} and M_{uyO} at column centroid, O by the statical equivalents V_u , M_{ux} and M_{uy} at the centroid of the critical section at $d/2$ from column face (Fig. 7):

$$M_{ux} = M_{uxO} + V_u y_O ; M_{uy} = M_{uyO} + V_u x_O \quad (21)$$

where x_O and y_O are coordinates of the column centroid. Appropriate signs for the force and moments must be used; the positive sign convention is indicated in Fig. 7.

Step 2: Calculate v_u by Eq. 14 which is repeated here: