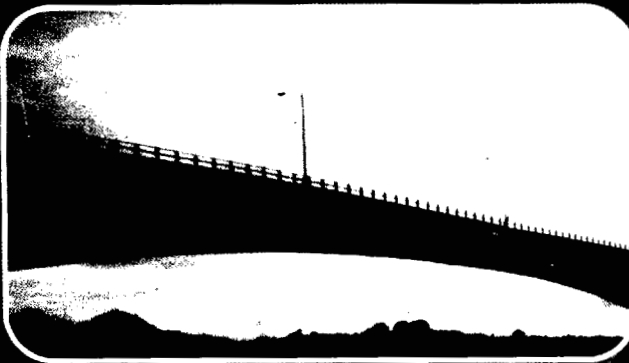


# The Adam Neville Symposium: Creep and Shrinkage – Structural Design Effects



SP-194

**Creep-induced excessive  
sag of a bridge in  
Palau, Carolina Islands**  
Photos by Adam Neville

Editor: Akthem Al-Manaseer



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# Adam Neville Symposium: Creep and Shrinkage— Structural Design Effects

*Editor*

Akthem Al-Manaseer



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## PREFACE

### ABOUT ADAM NEVILLE

This book is dedicated to Adam Neville and contains the papers presented at the Adam Neville Symposium held at the ACI Convention in Atlanta in November 1997. This begs the question: why a symposium on creep honoring Adam Neville? The answer goes a long way back, but then, Neville goes a long way back too.

After World War II service in Iraq, Persia, Palestine, and the Italian Campaign (where he was decorated for bravery) Neville enrolled at Queen Mary College, University of London, where he graduated with first class honors (corresponding to *summa cum laude*) in engineering in 1950. This was the period when prestressed concrete in England was in its infancy. During his final year, Neville undertook an experimental research project on prestress losses in beams. This project involved losses due to the creep of concrete and gave Neville a taste for that little researched topic.

Although Neville won a scholarship to work toward a Ph.D., he decided to move into the world of real engineering. He firmly holds the view that academic engineers who lack practical experience of design and construction, run the risk of doing excellent research to answer questions that are not posed by practitioners.

Nevertheless, creep got under his skin, and Neville worked on his own on research dealing with the creep of concrete and obtained a M.S. and a Ph.D. from the University of London. Later as a faculty member, he continued his research and wrote numerous papers on factors influencing creep as well as its structural effects. This combination reflects Neville's firm belief that a study of material behavior should be linked to the study of structural behavior, and that a dichotomy between materials specialists and structural specialists is unlikely to serve the engineering profession well. Further degrees awarded for his research were a Doctor of Science in Engineering from the University of London and a Doctor of Science from the University of Leeds, England.

Neville's interest in creep of concrete has a fairly wide-ranging effect. While Head of the Department of Civil Engineering at the University of Leeds, he involved a number of colleagues in research on creep, which continues to this day. While at the University of Calgary, Canada, Neville and his colleagues coauthored joint papers and books. One of the earliest books on creep of concrete, published in 1970, was titled *Creep of Concrete: Plain, Reinforced, and Prestressed*, by A. M. Neville. Four chapters of its 622 pages were written in collaboration with W. H. Dilger. In 1983 Neville coauthored a book with W. H. Dilger, and J. J. Brooks titled *Creep of Plain and Structural Concrete*.

Neville's research covers a broad field of concrete topics. His book titled *Properties of Concrete*, first published in 1963, with the final edition published in 1995, is lightheartedly referred to as the concrete bible. Translated into 13 languages, this book has sold more than half a million copies worldwide.

Neville's research has not been limited to the properties of concrete. In the 1960s and 1970s he wrote a number of papers dealing with shear in reinforced and prestressed concrete. In addition, Neville has coauthored several other successful books including *Basic Statistical Methods for Engineers and Scientists* with J. B. Kennedy and *Structural Analysis: A Unified Classical and Matrix Approach* with A. Ghali.

Over the past 50 years, Neville has written eight books and over 220 research and technical papers. In the last four years, he has written 20 papers published by ACI and RILEM.

It is arguable that his output is not large for 50 years of work. This would be true, had he done nothing but “concreting;” however, Neville spent, or wasted, nine years being Principal (President) of the University of Dundee, Scotland, and four years being Foundation Dean of Engineering and Foundation Dean of Graduate Studies as part of establishing the University of Calgary, Canada. All this is not the full explanation; he has been a skier from the age of five, with his latest season in 2000. Whether this was his swan song will not be known until the next snow falls. If you add all the hours spent on the ski slopes, they would suffice to write another book or two.

By the way, his skiing was not entirely divorced from the University. At Calgary, he insisted on graduate students accompanying him on “faculty ski days.” Some of them, from hot countries, were apprehensive about exposure to snow, but felt it wise to humor the Dean!

During the last 12 years, Neville has been working full-time as a consultant and expert in problems and disputes concerning concrete structures of all kinds including highways, nuclear power stations, suspension bridges, and dams.

Adam Neville has received wide recognition: three honorary doctorates, several honorary fellowships, the title of commander of the order of the British Empire for his contribution to science and technology, numerous prizes and awards, and honorary membership of ACI.

It is my pleasure to be the editor of this book dedicated to Adam Neville.

**Akthem Al-Manaseer**  
**Editor**

## FOREWORD

I am delighted to be honored by the publication of the *Adam Neville Symposium: Creep and Shrinkage—Structural Design Effects* as a Special Publication of the American Concrete Institute. This Special Publication No. 194 reminds me that I wrote one of the very early Special Publications: Monograph No. 6, titled *Hardened Concrete: Physical and Mechanical Aspects*, now of course well forgotten.

I deeply appreciate the eponymous symposium and this book on creep, especially as I have a long connection with the creep of concrete and with ACI Committee 209 on Creep, which I was chairman of in the early 1960s. Later, I organized and edited two volumes of a Bibliography of Creep and Shrinkage published by ACI. Even at that time, there were several hundreds of papers on those topics.

The output of papers on creep since then has been prodigious, and much useful information has been obtained. The understanding of the phenomena involved has greatly improved, but we have not yet reached the stage of establishing an unchallenged mechanism of creep, either basic or drying. A large volume of information on the parameters involved and on factors influencing creep of concrete has been built up. This volume is the latest contribution, but I am sure not the last.

The stumbling block to progress has been the attempts to produce a single expression to predict creep and another single expression to predict shrinkage. These attempts have not proved easy; indeed, in my opinion, they are not practicable. What has happened instead is a split into groups of researchers, each group believing that it possesses the “truth” about the creep and shrinkage expressions. And yet, it seems that none of the expressions has a universal validity as tested by the available data bank. I should add that some of the data are far from recent and they refer to concrete less complex in composition and in the range of materials than is the case today, especially in creep-sensitive structures.

In my view, such a search for a single expression capable of predicting creep at all ages as a function of a number of parameters is unrealistic. Let me explain the rationale of this assertion.

We all have a great interest in being able to predict the compressive strength of concrete, say at 28 days, as a function of various parameters so that we could say: with previously untried materials to be used to make concrete, we can achieve a desired compressive strength and a particular workability by using certain mixture proportions. The fact is that we cannot do so and we must, to a greater or lesser extent, resort to experimental procedures and testing. We have lived with this situation quite comfortably and we continue to do so.

Admittedly, the necessary strength tests take, at most, 28 days. With respect to creep, however, the necessary long-term testing is often not practical. This is why we search for an expression to fit all combinations of intrinsic and extrinsic factors influencing creep. Over a broad range of possible mixture ingredients and proportions, as well as exposure conditions, every expression that has been proposed has wide confidence limits; to know that the “true” value of creep has a 95% probability of being within 25% or even more, is not very useful. I believe that we must learn to live with this situation, at least for quite a long time to come.

My question is this: Is this situation so bad? The observation is not a criticism of those who attempt to develop the creep and shrinkage expressions, but rather a critique of their unduly focused efforts.

I have to confess that I am somewhat bemused by these continuing efforts, each on an individual track, but the tracks are not convergent. Are these efforts a search for

the holy grail? Are they a determined effort to help the designers? Or are they, at least some of them, an ego trip to show oneself right?

There is a story that bears upon the search for a single creep prediction expression. Imagine a room painted black, with people in it all dressed in black, and a black cat in it. They are trying to catch the cat; that is philosophy. Imagine the same black room, people, and cat. Every now and again, someone calls out "I've got it!"; that is religion. Again, imagine a black room, with people dressed in black, and every now and again someone calls out "I've got it!", but there is *no* cat. That is the search for the creep expression.

Much effort has gone into a "reconciliation" of the opposing views. This is not new. Approximately 20 years ago, together with the late Bob Philileo, I was a "conciliator" in an attempt to find common ground between the then propounded expressions. We failed abysmally.

Today, we are not much further. The difference is that in the present-day situation it is not a failure but a realistic acknowledgment of the inherently intractable material, concrete, as far as creep and, to a large extent, also shrinkage, are concerned. After all, we are dealing with a composite material consisting both of manufactured and of natural materials, partly processed in a semi-industrial manner.

So, we should accept that this is the reality of "concrete life." For most concrete purposes, this is a tolerable situation. For complex, sensitive, and special structures, some experimental work is necessary. Because these structures are rarely built in a great hurry, an input is practicable.

Now let me briefly turn to the papers in this volume. As a contribution to knowledge, they are highly valuable and we should be grateful to their authors for writing them. As this is "my" symposium volume, I am especially grateful.

It is not practicable to comment on all the papers in the present volume; on the other hand, it would be invidious to select only some for comment. There are, however, three authors I should like to mention: John Gardner, because some 40 years ago he was my undergraduate student; Jeff Brooks, because he was my Ph.D. student; and Walter Dilger, because he was my post doctorate fellow. They all now are very senior academics.

Overall, this volume contains 12 papers written by authors from seven countries: Belgium, Canada, France, Italy, Japan, the United Kingdom, and the United States. The topics include: factors in creep and shrinkage, such as admixtures and various cementitious materials, temperature, and humidity; special consideration of high-performance concrete; kinetics approach to creep and shrinkage; creep and shrinkage models; testing and field verification of laboratory tests; and also, very importantly, structural effects of creep and shrinkage in reinforced and prestressed concrete, their analysis, and design procedures.

We should also acknowledge the contribution of ACI Committee 209 over many years to the study of creep and shrinkage. A committee functions well only if led by an able chairman. For the last few years, this function has been performed by Akthem Al-Manaseer. I have an almost grand-parental relation with him as I was the Ph.D. supervisor of his post-doctoral supervisor. It is fitting to recognize Akthem's hard work both inside the Committee and in the ensuing preparation of this volume, the Adam Neville Symposium. For this we should all, and especially I, thank him.

**Adam Neville**

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# Creep and Shrinkage Prediction Model for Analysis and Design of Concrete Structures: Model B3

by Z. P. Bažant and S. Baweja

## Synopsis:

The present paper presents in chapter 1 a model for the characterization of concrete creep and shrinkage in design of concrete structures (Model B3), which is simpler, agrees better with the experimental data and is better theoretically justified than the previous models. The model complies with the general guidelines recently formulated by RILEM TC-107. Justifications of various aspects of the model and diverse refinements are given in Chapter 2, and many simple explanations are appended in the commentary at the end of Chapter 1 (these parts are not to be read by those who merely apply the model). The prediction model B3 is calibrated by a computerized databank comprising practically all the relevant test data obtained in various laboratories throughout the world. The coefficients of variation of the deviations of the model from the data are distinctly smaller than those of the latest CEB model (1990), and much smaller than those for the previous model in ACI 209 (which was developed in the mid-1960s). The model is simpler than the previous models (BP and BP-KX) developed at Northwestern University, yet it has comparable accuracy and is more rational. The effect of concrete composition and design strength on the model parameters is the main source of error of the model. A method to reduce this error by updating one or two model parameters on the basis of short-time creep tests is given. The updating of model parameters is particularly important for high-strength concretes and other special concretes containing various admixtures, superplasticizers, water-reducing agents and pozzolanic materials. For the updating of shrinkage prediction, a new method in which the shrinkage half-time is calibrated by simultaneous measurements of water loss is presented. This approach circumvents the large sensitivity of the shrinkage extrapolation problem to small changes in the material parameters. The new model allows a more realistic assessment of the creep and shrinkage effects in concrete structures, which significantly affect the durability and long-time serviceability of civil engineering infrastructure.

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**Zdeněk P. Bažant**, F.ACI, was born and educated in Prague (Ph.D. 1963). He joined Northwestern University faculty in 1969, became Professor in 1973, was named to the distinguished W. P. Murphy Chair in 1990, and served during 1981-87 as Director of the Center for Concrete and Geomaterials. In 1996 he was elected to the *National Academy of Engineering*. He has authored almost 400 refereed journal articles and published books on *Stability of Structures* (1991), *Fracture and Size Effect* (1997), *Concrete at High Temperatures* (1996) and *Creep of Concrete* (1966). He served as Editor (in chief) ASCE J. of Engrg. Mech. (1988-94) and is Regional Editor of Int. J. of Fracture. He was founding president of IA-FraMCoS, president of Soc. of Engrg. Science (SES), and chairman of SMiRT Div. H. He is an Illinois Registered Structural Engineer, and chaired various techn. committees in ASCE, RILEM, and ACI. His honors include: *Prager Medal* from SES; *Warner Medal* from ASME; *Newmark Medal*, *Croes Medal* and *Huber Prize* and *T. Y. Lin Award* from ASME; *L'Hermite Medal* from RILEM; *Humboldt Award*; *Honorary Doctorates* from CVUT, Prague, and from Universitat Karlsruhe; *Guggenheim*, *NATO*, *JSPS*, *Ford* and *Kajima Fellowships*; etc. He is a Foreign Member of Academy of Engrg. of Czech Rep. and a fellow of Am. Academy of Mechanics, ASME, ASCE and RILEM.

**Sandeep Baweja**, M. ACI, earned a Ph.D. in structural engineering from Northwestern University in 1996. He is now a Senior Software Engineer at EA Systems, Inc., Alameda, California, and serves as a member of ACI Committee 209, Creep and Shrinkage. His research interests include constitutive modeling of structural materials, especially concrete, computational mechanics, and computer aided engineering.

**Keywords:** concrete; creep; design; drying; extrapolation of short-time data; mathematical models; moisture effects; prediction; shrinkage; statistical variations; viscoelasticity

# Chapter 1

## Description of Model B3 and Prediction Procedure

### 1.1 Introduction

During the last two decades, significant advances in the understanding of creep and shrinkage of concrete have been achieved. They include: (1) vast expansion of the experimental data base on concrete creep and shrinkage; (2) compilation of a computerized data bank; (3) development of computerized statistical procedures for data fitting and optimization; and (4) improved understanding of the physical processes involved in creep and shrinkage, such as the aging, diffusion processes, thermally activated processes, microcracking and their mathematical modeling. These advances have made possible the formulation of the present model, which represents an improvement compared to the model in ACI 209. The new model (representing the third major update<sup>2</sup> of the models<sup>3,4</sup> developed at Northwestern University) is labeled Model B3.

In Chapter 1 of this paper the model is formulated succinctly, without any explanations, justifications, extensions and refinements. These are relegated to Chapter 2 and to the Commentary at the end of Chapter 1 of this paper, which does not have to be read by those who merely want to apply the model and do not have time for curiosity about its justification. The methods and typical examples of structural analysis for creep and shrinkage will not be discussed in this paper. They are treated in Chapters 3-5 of ACI 209, other articles and some books.

A background at the level of standard undergraduate courses in mechanics of materials, structural mechanics, engineering mathematics and concrete technology is expected from the user.

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The present paper presents an improved alternative to Chapter 2 of ACI 209. Chapters 3-5 of that report, dealing with the structural response, remain applicable to the present paper. The improvement means that the coefficient of variation of the errors of the predictions of creep and shrinkage strains are 23 % for creep (basic and with drying) and 34% for shrinkage for the present model, while those for the model from Chapter 2 of ACI 209 are 58% for basic creep, 45% for creep with drying and 55% for shrinkage. The penalty is modest reduction in simplicity of the model. The user should decide what accuracy he needs depending on the sensitivity of the structure defined in Section 1.2.1 .

## 1.2 Applicability Range

### 1.2.1 Levels of Creep Sensitivity of Structures and Type of Analysis Required

Accurate and laborious analysis of creep and shrinkage is necessary only for some special types of structures. That depends on the sensitivity of the structure. Although more precise studies are needed, the following approximate classification of sensitivity levels of structures can be made on the basis of general experience<sup>C1,\*</sup>.

**Level 1.** Reinforced concrete beams, frames and slabs with spans under 65 ft (20 m) and heights of up to 100 ft (30 m), plain concrete footings, retaining walls.

**Level 2.** Prestressed beams or slabs of spans up to 65 ft. (20 m), high-rise building frames up to 325 ft (100 m) high.

**Level 3.** Medium-span box girder, cable-stayed or arch bridges with spans of up to 260 ft (80 m), ordinary tanks, silos, pavements.

**Level 4.** Long-span prestressed box girder, cable-stayed or arch bridges; large bridges built sequentially in stages by joining parts; large gravity, arch or buttress dams; cooling towers; large roof shells; very tall buildings.

**Level 5.** Record span bridges, nuclear containments and vessels, large off-shore structures, large cooling towers, record-span thin roof shells, record-span slender arch bridges.

The foregoing grouping of structures is only approximate. If in doubt to

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\*The superscripts preceded by 'C' refer to the comments listed in the Commentary in Section 1.7.8

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which level a given structure belongs one should undertake an accurate analysis of the creep and shrinkage effects in a given structure (such as maximum deflection, change of maximum stress and crack width) and then judge the severity of the effects compared to those of the applied loads.

The full B3 model presented in this paper is necessary for levels 4 and 5. It is also preferable but not necessary for level 3. For level 3 the short form of model B3 presented also in this volume is appropriate. For level 2 and as an approximation also for level 3, simpler models are adequate including the model in Chapter 2 of ACI 209.

A refined model such as that presented here ought always to be used for structures analyzed by sophisticated computer methods, particularly the finite element method (because it makes no sense to input inaccurate material properties into a very accurate computer program). The error in maximum deflections or stresses caused by replacing an accurate analysis of creep and shrinkage effects with a simple but crude estimation is often larger than the gain from replacing old fashioned frame analysis by hand with a computer analysis by finite elements.

The age-adjusted effective modulus method (ACI 209) is recommended for levels 3 and 4. The effective modulus method suffices for level 2. For level 1, creep and shrinkage analysis of the structure is not needed but a crude empirically based estimate is desirable. Level 5 requires the most realistic and accurate analysis possible, typically a step-by-step computer solution based on a general constitutive law, coupled with the solution of the differential equations for drying and heat conduction.

The creep and shrinkage deformations invariably exhibit large statistical scatter. Therefore a statistical analysis with estimation of 95% confidence limits is mandatory for level 5. It is highly recommended for level 4. For lower levels it is desirable but not necessary, however, the confidence limits for any response  $X$  (such as deflection or stress) should be considered, being estimated  $\bar{X} \times (1 \pm 1.96\omega)$  where  $\bar{X}$  = mean estimate of  $X$  and  $\omega$  is taken same as in Eq. (1.25).

Analysis of temperature effects and effects of cycling of loads and environment ought to be detailed for level 5 and approximate for level 4. It is not necessary though advisable for level 3 and can be ignored for levels 1 and 2 (except perhaps for the heat of hydration effects).

### 1.2.2 Parameter Ranges

The prediction of the material parameters of the present model from strength and composition is restricted to Portland cement concrete with the following

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parameter ranges:<sup>C2</sup>

$$0.35 \leq w/c \leq 0.85, \quad 2.5 \leq a/c \leq 13.5 \quad (1.1)$$

$$\begin{aligned} 2500 \text{ psi} \leq \bar{f}_c \leq 10,000 \text{ psi}, \quad 10 \text{ lb/ft}^3 \leq c \leq 45 \text{ lbs/ft}^3 & \quad \text{inch-pound system} \\ 17 \text{ MPa} \leq \bar{f}_c \leq 70 \text{ MPa} \quad 160 \text{ kg/m}^3 \leq c \leq 720 \text{ kg/m}^3 & \quad \text{S.I.} \end{aligned} \quad (1.2)$$

$\bar{f}_c$  is the 28 day standard cylinder compression strength of concrete (in psi (inch-pound system) or MPa (S.I.) units),  $w/c$  is the water-cement ratio by weight,  $c$  is the cement content (in lb/ft<sup>3</sup> (inch-pound system) or kg/m<sup>3</sup> (S.I.) units) and  $a/c$  is the aggregate-cement ratio by weight. The formulae are valid for concretes cured for at least one day<sup>C3</sup>. If the model parameters are calibrated by tests, the present model is applicable to any portland cement concrete including light-weight and high-strength concrete except that the autogenous shrinkage of the latter may need a more detailed formulation.

## 1.3 Definitions, Basic Concepts and Overview of Calculation Procedures

The present prediction model is restricted to the service stress range<sup>C4</sup> (or up to about  $0.45\bar{f}_c$ , where  $\bar{f}_c$  = mean cylinder strength at 28 days). This means that, for constant stress applied at age  $t'$ ,

$$\epsilon(t) = J(t; t')\sigma + \epsilon_{sh}(t) + \alpha\Delta T(t) \quad (1.3)$$

in which  $J(t, t')$  is the compliance function = strain (creep plus elastic) at time  $t$  caused by a unit uniaxial constant stress<sup>C5, C6</sup> applied at age  $t'$ ,  $\sigma$  = uniaxial stress,  $\epsilon$  = strain (both  $\sigma$  and  $\epsilon$  are positive if tensile),  $\epsilon_{sh}$  = shrinkage strain (negative if volume decreases),  $\Delta T(t)$  = temperature change from reference temperature at time  $t$ , and  $\alpha$  = thermal expansion coefficient (which may be approximately predicted according to ACI 209<sup>5</sup>).

The compliance function may further be decomposed as

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \quad (1.4)$$

in which  $q_1$  = instantaneous strain due to unit stress,  $C_0(t, t')$  = compliance function for basic creep (creep at constant moisture content and no moisture movement through the material), and  $C_d(t, t', t_0)$  = additional compliance function due to simultaneous drying<sup>C7</sup>.

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The creep coefficient  $\phi(t, t')$ , which represents the most convenient way to introduce creep into simple structural analysis, must be calculated as<sup>C8</sup>

$$\phi(t, t') = E(t')J(t, t') - 1 \quad (1.5)$$

where  $E(t')$  = (static) modulus of elasticity at loading age  $t'$ . Various definitions of  $E$  exist. Nevertheless, since only the values of  $J(t, t')$  matter for structural analysis, any reasonable estimate of  $E(t')$  can be used<sup>C7</sup> provided that  $\phi(t, t')$  is calculated from Eq. (1.5) –for example, the result of the ASTM standard test or the value estimated from strength  $\bar{f}_c(t')$  at age  $t'$  by ACI formula, valid if  $\bar{f}_c$  and  $E$  are in psi (if in MPa, replace 57000 with 4734):

$$E(t') = 57000\sqrt{\bar{f}_c(t')} \quad (1.5a)$$

The relative humidity in the pores of concrete is initially 100%. In the absence of moisture exchange (as in sealed concrete), a gradual decrease of pore humidity, called self-desiccation, is caused by hydration<sup>C9</sup>. Exposure to the environment engenders a long-term drying process (described by the solution of the diffusion equation), which causes shrinkage and additional creep<sup>C10</sup>. In the absence of drying there is another kind of shrinkage, called autogeneous shrinkage, which is caused by the chemical reactions of hydration. This shrinkage is usually small for normal concretes (not for high-strength concretes) and can usually be neglected<sup>C11</sup>.

In the following sections, the expressions for individual terms in Eq. (1.3)–(1.4) will be presented first. The formulae to predict the coefficients of these equations, statistically derived from calibration with the data bank, will be given next. Two examples of the calculation procedure will then be given. Estimation of the statistical scatter of the predicted shrinkage and creep values due to parameter uncertainties will be discussed next. Finally, a method of improving the predictions of the model by extrapolation of short-time test data will be presented.

## 1.4 Calculations of Creep and Time Dependent Strain Components

### 1.4.1 Basic Creep (Material Constitutive Property)

The basic creep compliance is more conveniently defined by its time rate than its value:

$$\dot{C}_0(t, t') = \frac{n(q_2 t^{-m} + q_3)}{(t - t') + (t - t')^{1-n}} + \frac{q_4}{t}, \quad m = 0.5, n = 0.1 \quad (1.6)$$

in which  $\dot{C}_0(t, t') = \partial C_0(t, t') / \partial t$ ,  $t$  and  $t'$  must be in days,  $m$  and  $n$  are empirical parameters whose value can be taken the same for all normal concretes and are indicated above; and  $q_2, q_3$  and  $q_4$  are empirical constitutive parameters which will be defined later<sup>C12</sup>. The total basic creep compliance is obtained by integrating Eq. (1.6):

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$$C_0(t, t') = q_2 Q(t, t') + q_3 \ln[1 + (t - t')^n] + q_4 \ln\left(\frac{t}{t'}\right) \quad (1.7)$$

in which  $Q(t, t')$  is given in Table 1.1 and it can also be calculated from an approximate explicit formula given by Eq. (1.35) in the Appendix to this chapter<sup>C13</sup>. Function  $Q(t, t')$ , of course, can also be easily obtained by numerical integration (see Section 1.8.1 in the Appendix).

Table 1.1: Values of function  $Q(t, t')$  for  $m = 0.5$  and  $n = 0.1$

|            | log t' |        |        |         |         |         |         |          |          |
|------------|--------|--------|--------|---------|---------|---------|---------|----------|----------|
| log (t-t') | 0.0    | 0.5    | 1.0    | 1.5     | 2.0     | 2.5     | 3.0     | 3.5      | 4.0      |
| -2.0       | 0.4890 | 0.2750 | 0.1547 | 0.08677 | 0.04892 | 0.02751 | 0.01547 | 0.008699 | 0.004892 |
| -1.5       | 0.5347 | 0.3009 | 0.1693 | 0.09519 | 0.05353 | 0.03010 | 0.01693 | 0.009519 | 0.005353 |
| -1.0       | 0.5586 | 0.3284 | 0.1848 | 0.1040  | 0.05846 | 0.03288 | 0.01849 | 0.01040  | 0.005846 |
| -0.5       | 0.6309 | 0.3571 | 0.2013 | 0.1133  | 0.06372 | 0.03583 | 0.02015 | 0.01133  | 0.006372 |
| 0.0        | 0.6754 | 0.3860 | 0.2185 | 0.1231  | 0.06929 | 0.03897 | 0.02192 | 0.01233  | 0.006931 |
| 0.5        | 0.7108 | 0.4125 | 0.2357 | 0.1334  | 0.07516 | 0.04229 | 0.02379 | 0.01338  | 0.007524 |
| 1.0        | 0.7352 | 0.4335 | 0.2514 | 0.1436  | 0.08123 | 0.04578 | 0.02576 | 0.01449  | 0.008149 |
| 1.5        | 0.7505 | 0.4480 | 0.2638 | 0.1529  | 0.08727 | 0.04397 | 0.02782 | 0.01566  | 0.008806 |
| 2.0        | 0.7597 | 0.4570 | 0.2724 | 0.1602  | 0.09276 | 0.05239 | 0.02994 | 0.01687  | 0.009494 |
| 2.5        | 0.7652 | 0.4624 | 0.2777 | 0.1652  | 0.09708 | 0.05616 | 0.03284 | 0.01812  | 0.01021  |
| 3.0        | 0.7684 | 0.4656 | 0.2808 | 0.1683  | 0.1000  | 0.05869 | 0.03393 | 0.01935  | 0.01094  |
| 3.5        | 0.7703 | 0.4675 | 0.2827 | 0.1702  | 0.1018  | 0.06041 | 0.03541 | 0.02045  | 0.01166  |
| 4.0        | 0.7714 | 0.4686 | 0.2838 | 0.1713  | 0.1029  | 0.06147 | 0.03641 | 0.02131  | 0.01230  |
| 4.5        | 0.7720 | 0.4692 | 0.2844 | 0.1719  | 0.1036  | 0.06210 | 0.03702 | 0.02190  | 0.01280  |
| 5.0        | 0.7724 | 0.4696 | 0.2848 | 0.1723  | 0.1038  | 0.06247 | 0.03739 | 0.02225  | 0.01314  |

The terms in Eq. (1.7) containing  $q_2$ ,  $q_3$  and  $q_4$  represent the aging viscoelastic compliance, non-aging viscoelastic compliance, and flow compliance, respectively, as deduced from the solidification theory<sup>6</sup>.

### 1.4.2 Average Shrinkage and Creep of Cross Section at Drying

#### Shrinkage

*Mean shrinkage strain in the cross section:*

$$\epsilon_{sh}(t, t_0) = -\epsilon_{sh\infty} k_h S(t) \quad (1.8)$$

*Time dependence:*

$$S(t) = \tanh \sqrt{\frac{t - t_0}{\tau_{sh}}} \quad (1.9)$$

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*Humidity dependence:*

$$k_h = \begin{cases} 1 - h^3 & \text{for } h \leq 0.98 \\ -0.2 & \text{for } h = 1 \text{ (swelling in water)} \\ \text{linear interpolation} & \text{for } 0.98 \leq h \leq 1 \end{cases} \quad (1.10)$$

*Size dependence:*

$$\tau_{sh} = k_t(k_s D)^2 \quad (1.11)$$

where  $v/s$  = volume to surface ratio of the concrete member,  $D = 2v/s$  = effective cross-section thickness which coincides with the actual thickness in the case of a slab,  $k_t$  is a factor defined by Eq. (1.20) and  $k_s$  is the cross-section shape factor:

$$k_s = \begin{cases} 1.00 & \text{for an infinite slab} \\ 1.15 & \text{for an infinite cylinder} \\ 1.25 & \text{for an infinite square prism} \\ 1.30 & \text{for a sphere} \\ 1.55 & \text{for a cube} \end{cases} \quad (1.12)$$

The analyst needs to estimate which of these shapes best approximates the real shape of the member or structure. High accuracy in this respect is not needed and  $k_s \approx 1$  can be assumed for simplified analysis.

*Time-dependence of ultimate shrinkage:*

$$\epsilon_{sh\infty} = \epsilon_{s\infty} \frac{E(607)}{E(t_0 + \tau_{sh})}; \quad E(t) = E(28) \left( \frac{t}{4 + 0.85t} \right)^{1/2} \quad (1.13)$$

where  $\epsilon_{s\infty}$  is a constant (given by Eq. 1.19). This means that  $\epsilon_{s\infty} = \epsilon_{sh\infty}$  for  $t_0 = 7$  days and  $\tau_{sh} = 600$  days<sup>C14</sup>.

### Additional Creep Due to Drying (Drying Creep)

$$C_d(t, t', t_0) = q_5 [\exp \{-8H(t)\} - \exp \{-8H(t'_0)\}]^{1/2}, \quad t'_0 = \max(t', t_0) \quad (1.14)$$

if  $t \geq t'_0$ , otherwise  $C_d(t, t', t_0) = 0$ ;  $t'_0$  is the time at which drying and loading first act simultaneously; and

$$H(t) = 1 - (1 - h)S(t) \quad (1.15)$$

Fig. 1.1 shows the typical curves of basic creep, shrinkage and drying creep according to the present model.

### 1.4.3 Prediction of Model Parameters

Some formulae that follow are valid only in certain dimensions. Those are given both in inch-pound system units (psi, in.) and in SI (metric) units (MPa, m). The units of each dimensional quantity are also specified in the list of notations (Appendix to Chapter 1)<sup>C15</sup>.

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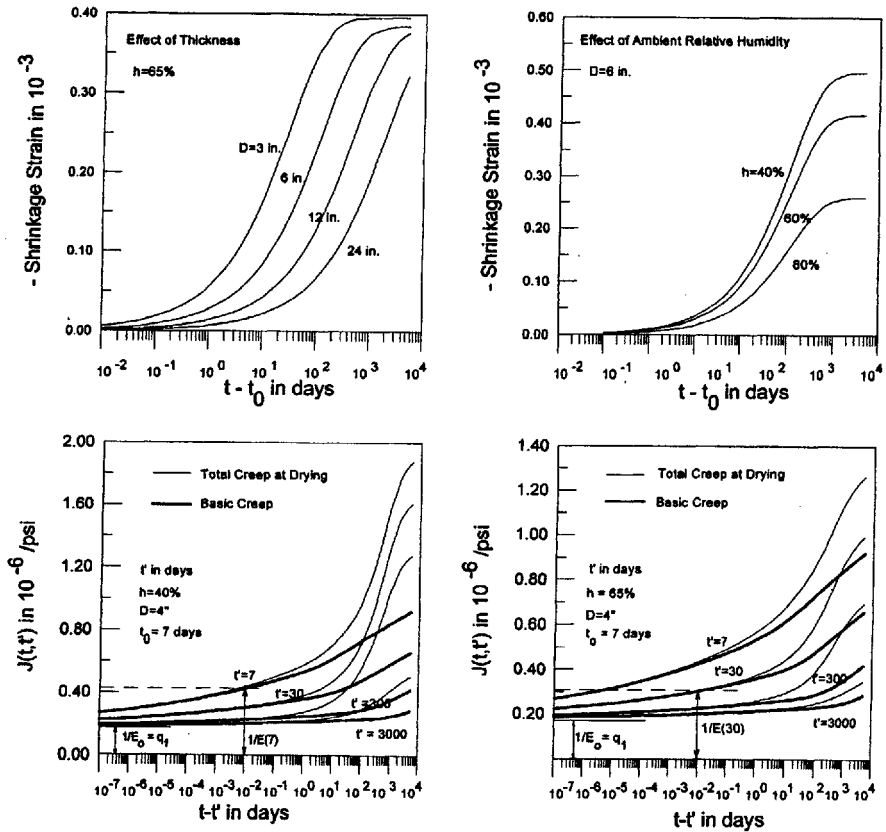


Figure 1.1: Typical creep and shrinkage curves given by Model B3

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### 1.4.4 Estimation from Concrete Strength and Composition

In the following  $q_1, q_2, q_3, q_4$  and  $q_5$  are in the units of  $(10^6 \text{ psi})^{-1}$  or  $(\text{MPa})^{-1}$ ,  $\bar{f}_c$  and  $E_{28}$  in the units of psi or MPa and  $\epsilon_{s\infty}$  and  $\epsilon_{sh\infty}$  in the units of  $10^{-6}$ .

#### Basic Creep

$$\begin{aligned} q_1 &= 0.6 \times 10^6 / E_{28}, & E_{28} &= 57000 \sqrt{\bar{f}_c} & \text{inch-pound system} \\ q_1 &= 0.6 \times 10^6 / E_{28}, & E_{28} &= 4734 \sqrt{\bar{f}_c} & \text{S.I.} \end{aligned} \quad (1.16)$$

$$\begin{aligned} q_2 &= 451.1 \sqrt{c} \bar{f}_c^{-0.9}, & q_4 &= 0.14 (a/c)^{-0.7} & \text{inch-pound system} \\ q_2 &= 185.4 \sqrt{c} \bar{f}_c^{-0.9}, & q_4 &= 20.3 (a/c)^{-0.7} & \text{S.I.} \end{aligned} \quad (1.17)$$

$$q_3 = 0.29 (w/c)^4 q_2 \quad (1.18)$$

#### Shrinkage

$$\begin{aligned} \epsilon_{s\infty} &= -\alpha_1 \alpha_2 [26 w^{2.1} \bar{f}_c^{-0.28} + 270] & (\text{in } 10^{-6}) & \text{inch-pound system} \\ \epsilon_{s\infty} &= -\alpha_1 \alpha_2 [1.9 \times 10^{-2} w^{2.1} \bar{f}_c^{-0.28} + 270] & (\text{in } 10^{-6}) & \text{S.I.} \end{aligned} \quad (1.19)$$

and

$$\begin{aligned} k_t &= 190.8 t_0^{-0.08} \bar{f}_c^{-1/4} \text{ days/in}^2 & \text{inch-pound system} \\ k_t &= 8.5 t_0^{-0.08} \bar{f}_c^{-1/4} \text{ days/cm}^2 & \text{S.I.} \end{aligned} \quad (1.20)$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement,} \\ 0.85 & \text{for type II cement,} \\ 1.1 & \text{for type III cement} \end{cases} \quad (1.21)$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam-curing} \\ 1.2 & \text{for sealed or normal curing in air with initial protection}^* \\ 1.0 & \text{for curing in water or at 100\% relative humidity} \end{cases} \quad (1.22)$$

*Creep at Drying* (Same in both inch-pound system and S.I. units)

$$q_5 = 7.57 \times 10^5 \bar{f}_c^{-1} |\epsilon_{sh\infty}|^{-0.6} \quad (1.23)$$

*When only strength  $\bar{f}_c$  is known:* In that case typical values for  $c$  and  $a/c$  should be assumed and the  $w/c$  should be determined from its relation to  $\bar{f}_c$ , e.g.  $w/c = [(\bar{f}_c/f_w) + 0.535]^{-1}$  with  $f_w = 22.8 \text{ MPa (3307 psi)}$  which describes the graph in Fig. 5.6 of Neville<sup>59</sup>.

\* against drying

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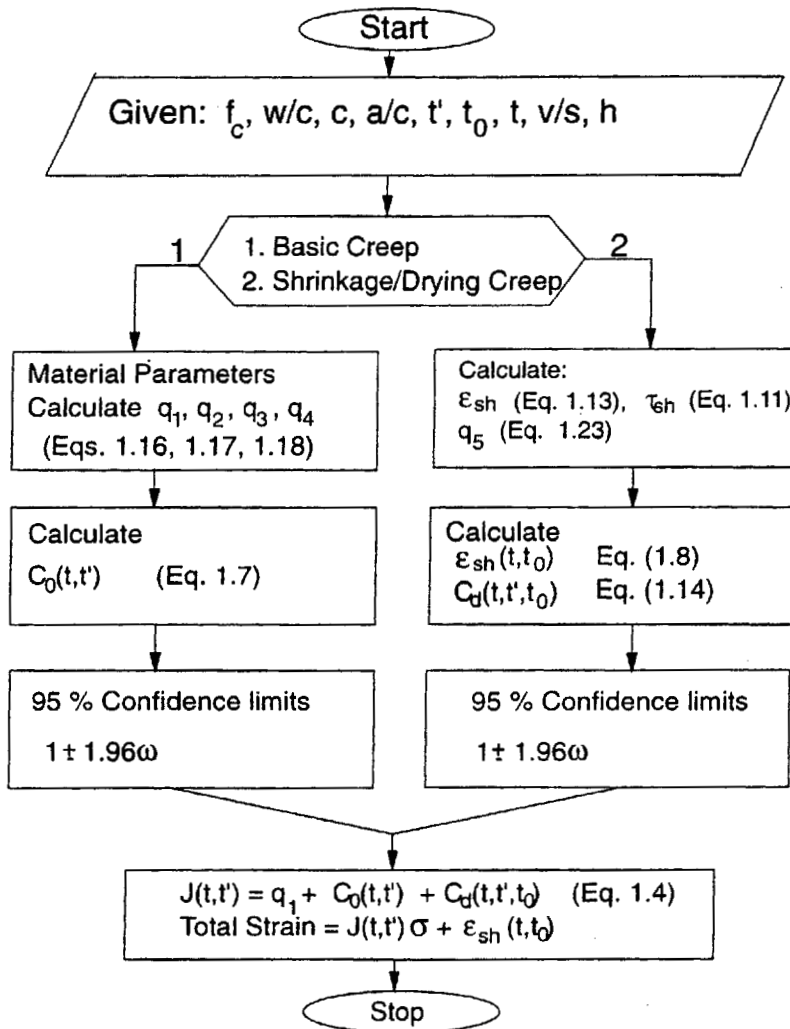


Figure 1.2: Flow chart showing the calculation procedure for Model B3

### 1.4.5 Example of Calculation of Model B3

The user may check the correctness of his implementation of Model B3 by a comparison with the following examples. The calculations are made using the inch-pound system formulas. As such the values of material parameters  $q_1, q_2, q_3, q_4$  and  $q_5$  as well as  $J(t, t')$  are in the units of  $10^{-6}$  /psi. The time required for these calculations using a hand held calculator was 55 minutes.

#### Input parameters:

##### Case 1. All Parameters Given

1) age of concrete  $t = 18250$  days; 2) age at loading  $t' = 28$  days; 3) age when drying begins  $t_0 = 14$  days; 4) relative humidity  $h = 60\%$ ; 5) volume to surface ratio  $v/s = 3$  in.; 6) cylinder compression strength  $\bar{f}_c = 5200$  psi; 7) Type I cement concrete; 8) cement content  $c = 19$  lb/ft<sup>3</sup>; 9) water-cement ratio  $w/c = 0.50$ ; 10) aggregate-cement ratio  $a/c = 7.0$ ; 11) applied stress (40 % of  $\bar{f}_c$ )  $\sigma = -2000$  psi.

We can calculate water content of concrete  $w = (w/c) \times c = 9.5$  lb/ft<sup>3</sup>;

#### Compliance function:

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \quad (\text{Eq. 1.4})$$

$$q_1 = 0.6 \times 10^6 / E_{28} \quad (\text{Eq. 1.16})$$

$$E_{28} = 57000(\bar{f}_c)^{0.5} = 57000 \times (5200)^{0.5} = 4,110,328 \text{ psi} \quad (\text{Eq. 1.16})$$

$$q_1 = 0.6 \times 10^6 / 4,110,328 = 0.146 (\times 10^{-6} / \text{psi})$$

$$C_0(t, t') = q_2 Q(t, t') + q_3 \ln[1 + (t - t')^n] + q_4 \ln(t/t') \quad (\text{Eq. 1.7})$$

$$q_2 = 451.1c^{0.5}(\bar{f}_c)^{-0.9} = 451.1 \times \sqrt{19} \times 5200^{-0.9} = 0.89 (\times 10^{-6} / \text{psi}) \quad (\text{Eq. 1.17})$$

$$q_3 = 0.29(w/c)^4 q_2 = 0.29 \times 0.5^4 \times 0.89 = 0.016 (\times 10^{-6} / \text{psi}) \quad (\text{Eq. 1.18})$$

$$q_4 = 0.14(a/c)^{-0.7} = 0.14 \times 7^{-0.7} = 0.036 (\times 10^{-6} / \text{psi})$$

By interpolation from Table 1.1<sup>C16</sup>:  $Q(t, t') = 0.183$

$$C_0(t, t') = 0.89 \times 0.183 + 0.016 \times \ln[1 + (18250 - 28)^{0.1}] + 0.036 \times \ln(18250/28) = 0.42$$

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$$C_d(t, t', t_0) = q_5 [\exp\{-8H(t)\} - \exp\{-8H(t')\}]^{1/2} \quad (\text{Eq. 1.14})$$

$$\alpha_1 = 1.0, \alpha_2 = 1.0 \quad (\text{Eq. 1.21-1.22})$$

$$\epsilon_{s\infty} = \alpha_1 \alpha_2 [26w^{2.1}(\bar{f}_c)^{-0.28} + 270] = 1.0 \times 1.0 \times [26 \times 9.5^{2.1} \times 5200^{-0.28} + 270] = 537.74 \approx \epsilon_{sh\infty} \quad (\text{Eq. 1.19})$$

$$q_5 = 7.57 \times 10^5 (\bar{f}_c)^{-1} | \epsilon_{s\infty} |^{-0.6} = 3.348 (\times 10^{-6} / \text{psi}) \quad (\text{Eq. 1.23})$$

$$k_s = 1.00 \quad (\text{Eq. 1.12})$$

$$k_t = 190.8 t_0^{-0.08} \bar{f}_c^{-1/4} = 18.192 \quad (\text{Eq. 1.20})$$

$$D = 2V/S = 6 \text{ in.}$$

$$\tau_{sh} = k_t (k_s D)^2 = 18.192 \times (1 \times 36) = 654.9 \text{ days} \quad (\text{Eq. 1.11})$$

$$S(t) = \tanh[(t - t_0)/\tau_{sh}]^{0.5} = 1 \quad (\text{Eq. 1.9})$$

$$S(t') = \tanh[(t' - t_0)/\tau_{sh}]^{0.5} = 0.1449$$

$$H(t) = 1 - (1 - h)S(t) = 1 - (1 - 0.6) \times 1 = 0.6 \quad (\text{Eq. 1.15})$$

$$H(t') = 1 - (1 - h)S(t') = 1 - (1 - 0.6) \times 0.1449 = 0.942$$

$$C_d(t, t', t_0) = 3.348 \times [\exp(-8 \times 0.6) - \exp(-8 \times 0.94)]^{1/2} = 0.293$$

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \\ = (0.146 + 0.42 + 0.293) \times 10^{-6} = 0.86 \times 10^{-6} / \text{psi}$$

$$\text{Total strain} = J(t, t')\sigma = 0.86 \times 2000 = 1720 \times 10^{-6}$$

Choice of elastic modulus (for  $\bar{f}_c(t') = \bar{f}_c = 5200 \text{ psi}$ ):

$$E(t') = 57000\sqrt{5200} = 4.110 \times 10^6 \text{ psi} \quad (\text{Eq. 1.5a}),$$

Corresponding creep coefficient:

$$\phi(t, t') = (4.110 \times 10^6)(0.86 \times 10^{-6}) - 1 = 2.53 \quad (\text{Eq. 1.5}).$$

$$\text{Humidity effect on shrinkage: } k_h = 1 - 0.6^3 = 0.784 \quad (\text{Eq. 1.10})$$

$$\text{Shrinkage} = \epsilon_{sh}(t, t_0) = \epsilon_{sh\infty} k_h S(t) = 537.7 \times 0.784 = 421.5 \times 10^{-6}$$

**Case 2 Concrete Mix Parameters Unknown:** Only strength  $\bar{f}_c = 5200 \text{ psi}$  is given. The water cement ratio may then be estimated as  $w/c = [(\bar{f}_c/f_w) + 0.535]^{-1} = (5200/3307 + 0.535)^{-1} = 0.474$ . Parameters no. 7-11 must be estimated as typical values for the concretes in the geographical region. The subsequent calculation is the same as above. In the absence of availability of information on typical values of parameters 7-11, values around the middle of the parameter ranges of the applicability of the model (Eq. 1.1-1.2) may be used for rough estimates on creep and shrinkage values.

## 1.5 Parameter Uncertainties and Statistical Range of Predicted Creep and Shrinkage Values

The parameters of any creep model must be considered as statistical variables. The preceding formulae predicting the creep and shrinkage parameters from the concrete composition and strength give the mean value of  $J(t, t')$  and  $\epsilon_{sh}$ . To take into account the statistical uncertainties, the parameters  $q_1, q_2, q_3, q_4, q_5, \epsilon_{sh\infty}$  ought to be replaced by the values

$$\psi_1 q_1, \psi_1 q_2, \psi_1 q_3, \psi_1 q_4, \psi_1 q_5, \psi_2 \epsilon_{sh\infty} \quad (1.24)$$

in which  $\psi_1$  and  $\psi_2$  are uncertainty factors for creep and shrinkage, which may be assumed to follow roughly the normal (Gaussian) distribution with mean value 1. According to the statistical analysis of the data in the RILEM data bank, the following coefficients of variation of these uncertainty factors should be considered in design<sup>C17</sup>:

$$\begin{aligned} \omega(\psi_1) &= 23\% & \text{for creep, with or without drying} \\ \omega(\psi_2) &= 34\% & \text{for shrinkage} \end{aligned} \quad (1.25)$$

This means that, if the statistical distribution is approximated as Gaussian (normal), the 95% confidence limits for  $\psi_1$  are  $1 \pm 1.96 \times 0.23 = 1 \pm 0.45$ , and for  $\psi_2$  are  $1 \pm 1.96 \times 0.34 = 1 \pm 0.67$ .

Some other input parameters of the model are also statistical variables. At least, one should consider in design the statistical variations of environmental humidity  $h$  and of strength  $\bar{f}_c$ . This can be done by replacing them with  $\psi_3 h$  and  $\psi_4 \bar{f}_c$  where  $\psi_3$  and  $\psi_4$  are uncertainty factors having a normal distribution with mean 1. In the absence of other information and sophisticated statistical analysis, the following coefficients of variation may be considered for these uncertainty factors<sup>a</sup>:

$$\begin{aligned} \omega(\psi_3) &\approx 20\% & \text{for } h \text{ replaced by } \psi_3 h \\ \omega(\psi_4) &\approx 15\% & \text{for } \bar{f}_c \text{ replaced by } \psi_4 \bar{f}_c \end{aligned} \quad (1.26)$$

Factor  $\psi_3$  is statistically independent of  $\psi_1, \psi_2$ , and  $\psi_4$ . As an approximation, all the factors may be assumed mutually statistically independent<sup>C18, C19</sup>.

If the structure is exposed to a climate that is permanently hot or has prolonged high temperature extremes, it is advisable to take into account the temperature effect according to the Appendix to Chapter 1. If this is not done, it is recommended to increase the aforementioned coefficients of variation  $\omega$  by 10% of  $\omega^{16}$ .

## 1.6 Improved Estimation: Updating Based on Short-Time Tests

A method to improve the prediction based on short-time tests<sup>2</sup> will now be described<sup>C20</sup>. Its use is mandatory for highly creep sensitive structures characterized as level 5 in the Section 1.2.1. It is advisable for level 4, and is useful for level 3. It is not needed for levels 1 and 2.

### *Updating Creep Predictions*

The procedure will be explained by considering as an example, the data for basic creep by L'Hermite et al.<sup>9</sup>, for which the present formulae for the effect of composition and strength do not give a good prediction, as is apparent from Fig. 1.3 (for information on these data see Ref. 3 and 4). We now pretend we know only the first 5 data points for the first 28 days of creep duration, which are shown by the solid circles. We consider that the compliance function is updated by only two update parameters  $p_1$  and  $p_2$ , introduced as follows:

$$J(t, t') = p_1 + p_2 F(t, t') \quad (1.27)$$

in which

$$F(t, t') = C_0(t, t') + C_d(t, t', t_0) \quad (1.28)$$

Function  $F(t, t')$  is evaluated according to the model, using the formulae for the effect of composition parameters and strength (Eq. 1.16–1.23). If the data agreed with the form of Model B3 exactly, the plot of  $J(t, t')$  versus  $F(t, t')$  would have to be a single straight line for all  $t, t'$  and  $t_0$ . The vertical deviations of the data points from this straight line represent errors that may be regarded as random and are to be minimized by least-square regression<sup>C21</sup>. So we consider the plot of the known (measured) short-time values  $Y = J(t, t')$  (up to 28 days of creep duration) versus the corresponding values of  $X = F(t, t')$ , calculated from Model B3, and pass through these points the regression line  $Y = AX + B$ . The slope  $A$  and the  $Y$ -intercept  $B$  of this line give the values of  $p_1$  and  $p_2$  that are optimum in the sense of the least-square method;  $A = p_2$  and  $B = p_1$ . According to the well-known normal equations of least-square linear regression,

$$p_2 = \frac{n \sum (F_i J_i) - (\sum F_i)(\sum J_i)}{n \sum (F_i^2) - (\sum F_i)^2}, \quad p_1 = \bar{J} - p_2 \bar{F} \quad (1.29)$$

where  $\bar{J}$  = mean of all the measured  $J_i$  values, and  $\bar{F}$  = mean of all the corresponding  $F_i$  values. Using the values  $p_1 = B$  and  $p_2 = A$ , the updated  $J(t, t')$  for the concrete tested may be obtained using Eq. 1.27. This updating

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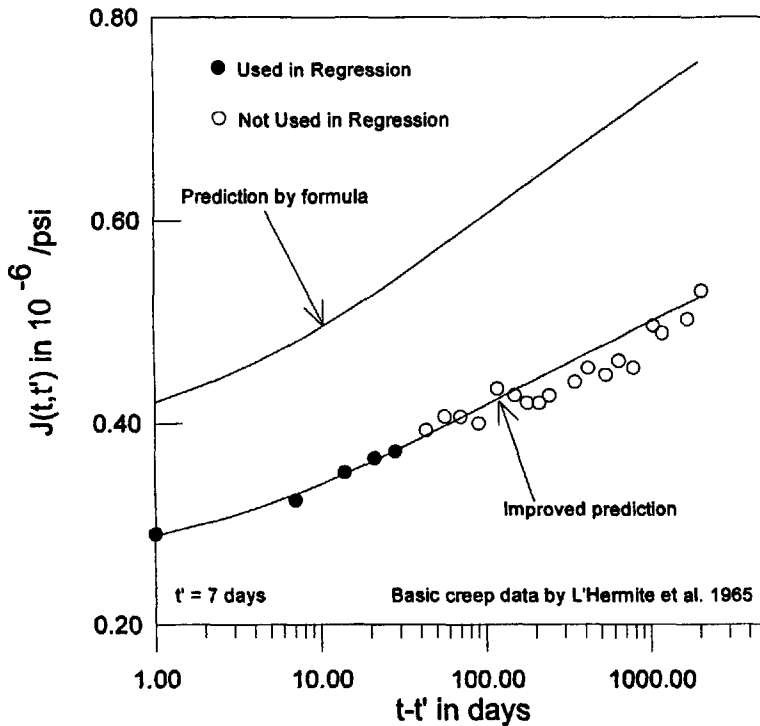


Figure 1.3: Example of improving the prediction of creep by the use of short-time test data

is equivalent to replacing the values  $q_1, q_2, q_3, q_4, q_5$  calculated from Eq. 1.16, 1.17, 1.23 by the values  $q_1^* = p_1 q_1, q_2^* = p_2 q_2, q_3^* = p_2 q_3, q_4^* = p_2 q_4, q_5^* = p_2 q_5$ . For the real structure, the ages at loading other than the  $t'$ -value used for the test specimens may be needed and the effective thickness, environmental humidity, etc., may be different. To obtain the values of  $J(t, t')$  for the real structure, one simply evaluates  $J(t, t')$  from Eq. (1.4) and (1.6)–(1.15), using parameters  $q_1^*, q_2^*, q_3^*, q_4^*, q_5^*$  instead of  $q_1, q_2, q_3, q_4, q_5$  in Eq. 1.4, 1.6–1.15.

As seen in Fig. 1.3, the improvement of long-time prediction achieved by short-time measurements is in this example very significant. The well-known formulae of linear regression<sup>10</sup> also yield the coefficients of variation of  $p_1$  and  $p_2$ , which in turn provide the coefficient of variation of  $J(t, t')$  for any given  $t$  and  $t'$ . (However these formulae might not suffice since the uncertainty of

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long-time predictions obtained by updating from the short-time data should properly be handled by the Bayesian statistical approach<sup>11,12</sup>.)

### *Updating Shrinkage Predictions*

The shrinkage predictions cannot be successfully updated solely on the basis of short-time shrinkage tests alone because of a certain special problem with the shrinkage tests<sup>C22</sup>. This problem can be circumvented if the shrinkage specimens are weighed to determine their relative water loss  $w$  (as a percentage of the weight of the concrete), and if also the final relative water loss is estimated by heating the specimens at the end of the shrinkage test in an oven to about 105°C. The water loss should preferably be measured directly on the shrinkage specimens. Anyway, if the water loss is measured on companion specimens, they must be identical and have the same environmental exposure all the time. The following procedure, developed in Ref. 2, is recommended (for a justification, see Sec. 2.3.2–2.3.3 of Chapter 2):

#### 1) Calculate

$$\Delta w_{\infty}(h) \approx 0.75 \left[ 1 - \left( \frac{h}{0.98} \right)^3 \right] \Delta w_{\infty}(0) \quad (\text{for } 0.25 \leq h \leq 0.96) \quad (1.30)$$

where  $\Delta w_{\infty}(h)$  is the estimate of the final relative water loss for drying at given environmental relative humidity  $h$  (and room temperature), and  $\Delta w_{\infty}(0)$  is the final water loss that would occur at environmental relative humidity 0, which is taken the same as the average final water loss measured upon heating the specimens to about 105°C.

#### 2) Calculate the auxiliary values

$$\psi_j = \left[ \tanh^{-1} \left( \frac{\Delta w_j}{\Delta w_{\infty}(h)} \right) \right]^2 \quad (j = 1, 2, \dots, m) \quad (1.31)$$

where  $\Delta w_j$  are the measured values of relative water loss at times  $t_j$  ( $j = 1, 2, \dots, m$ ), which should be spaced approximately uniformly in the scale of  $\log(t - t_0)$ .

#### 3) Instead of Eq. (1.11), obtain the improved estimate, $\bar{\tau}_{sh}$ , of shrinkage half-time from the equation:

$$\bar{\tau}_{sh} = 1.25\tau_w, \quad \tau_w = \frac{\sum_j (t_j - t_0)\psi_j}{\sum_j \psi_j^2} \quad (1.32)$$

- 4) Denote  $\epsilon'_{sh_i} = \epsilon'_{sh}(t_i, t_0)$  = measured short-time values of shrinkage at times  $t_i$  ( $i = 1, 2, \dots, n$ ), which should be spaced at approximately constant intervals in the scale of  $\log(t - t_0)$  and should preferably coincide with times  $t_j$  of water loss measurements. Also denote  $\bar{\epsilon}_{sh} = \bar{\epsilon}_{sh}(t, t_0)$  = shrinkage values calculated from Model B3 using  $\bar{\tau}_{sh}$  instead of Eq. (1.11). Calculate the values  $\bar{\epsilon}_{sh_i} = \bar{\epsilon}_{sh}(t_i, t_0)$  predicted for the times  $t_i$  from Model B3 on the basis of  $\bar{\tau}_{sh}$  instead of Eq. (1.11). Then calculate the scaling parameter:

$$p_6 = \frac{\sum_i \epsilon'_{sh_i} \bar{\epsilon}_{sh_i}}{\sum_i \bar{\epsilon}_{sh_i}^2} \quad (1.33)$$

- 5) The updated values of shrinkage prediction for any time  $t$  then are

$$\epsilon_{sh}^*(t, t_0) = p_6 \bar{\epsilon}_{sh}(t, t_0) \quad (1.34)$$

The last equation is equivalent to Eq. (1.8) with (1.9) for the shrinkage of the test specimen if  $k_h \epsilon_{sh\infty}$  is replaced by  $\epsilon_{sh\infty}^* = p_6 \epsilon_{sh\infty}$  and  $\tau_{sh}$  is replaced by  $\bar{\tau}_{sh}$ . To obtain the updated values of  $\epsilon_{sh}(t, t_0)$  for the real structure for which  $t_0, D$  and  $h$  may be different, Eq. (1.19)–(1.20) for  $\epsilon_{s\infty}$  and  $k_t$  are disregarded. Using the updated value  $\bar{\tau}_{sh}$  (instead of the original predicted value  $\tau_{sh}$ ) for the short-time test specimen, Eq. (1.11) is solved for  $k_t$ . This yields the updated value,  $k_t^*$ , replacing  $k_t$ , i.e.,  $k_t^* = \bar{\tau}_{sh}(k_s D)^{-2}$ . Next Eq. (1.13) is used with the updated value  $\epsilon_{sh\infty}^*$  (instead of the original value  $\epsilon_{sh\infty}$ ) for the test specimen to solve for the updated value  $\epsilon_{s\infty}^*$  that should replace  $\epsilon_{s\infty}$ , i.e.,  $\epsilon_{s\infty} = \epsilon_{sh\infty}^* E(t_0 + \bar{\tau}_{sh}) / E(607)$ . With the updated values,  $k_t^*$  and  $\epsilon_{s\infty}^*$ , and the values of  $t_0, D$ , and  $h$  for the structure, Eq. (1.8)–(1.13) are used to obtain the updated values of  $\epsilon_{sh}(t, t_0)$  for the structure.

The updated  $\bar{\tau}_{sh}$  should also be used in the calculation of the drying creep term from Eq. (1.14), which modifies the function  $F(t, t', t_0)$  in Eq. (1.28). This improves the previously indicated procedure.

#### Example of Shrinkage Updating

To illustrate the shrinkage updating procedure, consider the recent shrinkage and water-loss data obtained by Granger<sup>14</sup> (Fig. 1.4). They measured shrinkage on cylinders with a 16 cm diameter and 100 cm length over a gauge length of 50 cm. The relative weight loss was measured on cylinders with a 16 cm diameter and 15 cm length. The tests were carried out in an environment of 50% relative humidity. The ends of both the shrinkage and the weight-loss specimens were sealed to ensure radial drying. The concrete (Civaux BHP) had an average cylindrical compressive strength of 64.3 MPa. The composition of concrete (quantities in kg/m<sup>3</sup>) was cement (266), water

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(161), silica fume (40.3), fine aggregate (782), coarse aggregate (1133), filler (57), and additives (9.98).

We now pretend that we know only the first nine points of data (up to drying duration of 64 days) which are shown as the solid black points in Fig. 1.4. To estimate the final relative water loss,  $\Delta w_\infty(0)$ , at  $h \approx 0$ , we assume that the water used up in the hydration reaction is about 20 %, by weight, of the cementitious materials.  $\Delta w_\infty(50)$  is calculated using Eq. (1.30). Using this value of  $\Delta w_\infty(50)$  and the first nine points of the weight-loss data, we determine  $\bar{\tau}_{sh} = 1.25\tau_w$  from Eq. (1.31)–(1.32). This value of  $\bar{\tau}_{sh}$  and the drying shrinkage data up to 57 days duration are used in Eq. (1.33)–(1.34) to determine the updated values of shrinkage prediction. Curves, *a* and *b* in Fig. 1.4, showing the prediction by formula and the updated prediction, confirm that a significant improvement is achieved.

Fig. 1.4 also shows curve *c* obtained by trying to match the short-time data solely by vertical scaling of curve *a*, which is what one would have to do if the water-loss data were unavailable. Although this curve also matches the short-time data quite well, the long-time prediction would in this case become even worse than with curve *a* before updating. Clearly, updating of  $\tau_{sh}$  by water-loss data is essential for achieving improved predictions.

Now consider, for example, that the designer needs the value of  $\epsilon_{sh}(t, t_0)$  for  $D = 25$  cm,  $t_0 = 10$  days, and  $h = 65\%$ . Table 1.2 shows the calculations described in the preceding paragraphs for updating the shrinkage predictions on the basis of the short-time tests and for obtaining, the update parameters for the real structure. Note that the updated material parameters  $k_i^*$  and  $\epsilon_{s\infty}^*$  are the same for both the specimen and the structure. These update material parameters are used in Eq. (1.8)–(1.13) along with the given values of  $D$ ,  $t_0$ , and  $h$  for the real structure to obtain the updated shrinkage values for the real structure.

## 1.7 Appendix to Chapter 1

### 1.7.1 Approximate Formula for $Q(t, t')$ and Numerical Integration

Instead of Table 1.1, the values of this function can also be obtained from the following approximate formula (derived by Bažant and Prasannan, 1989<sup>6</sup>) which has an error of less than 1% for  $n = 0.1$  and  $m = 0.5$ ;

$$Q(t, t') = Q_f(t') \left[ 1 + \left( \frac{Q_f(t')}{Z(t, t')} \right)^{r(t')} \right]^{-1/r(t')} \quad (1.35)$$

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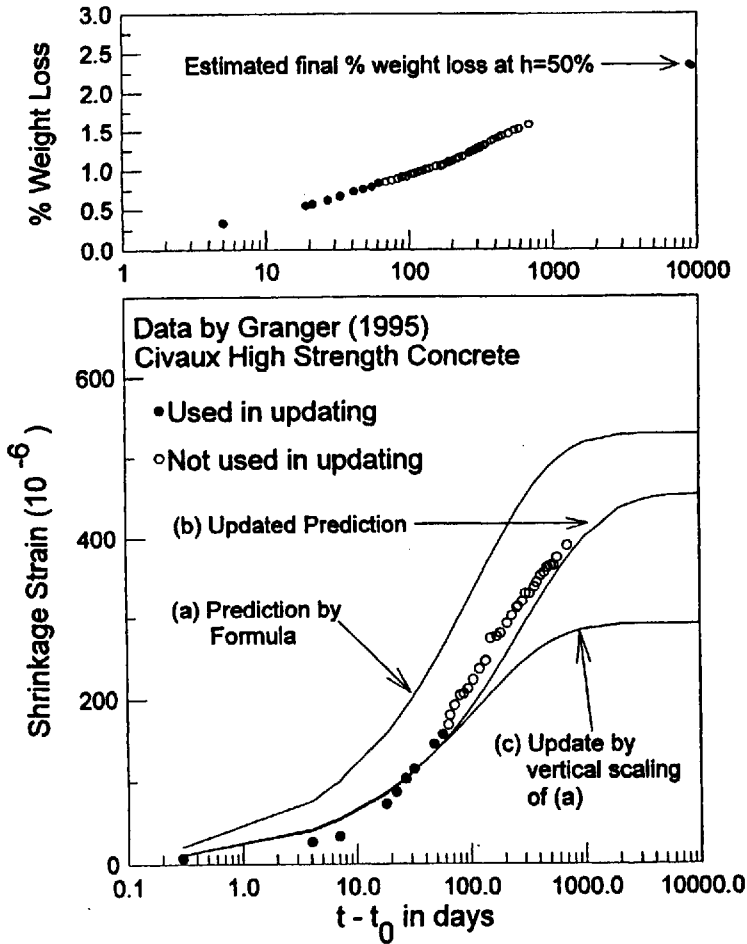


Figure 1.4: Example of updating shrinkage prediction using short-time data and estimated final water-loss

Table 1.2: Example of Shrinkage Updating Calculations

| Parameter                                                                                  | Value  |
|--------------------------------------------------------------------------------------------|--------|
| <b>Shrinkage Specimen</b> $D = 2V/S = 8\text{cm}$ , $t_0 = 28$ days, $h = 50\%$            |        |
| $\tau_{sh}$ (Eq. 1.11)                                                                     | 195.8  |
| $\epsilon_{sh\infty}$ (Eq. 1.13)                                                           | 605.2  |
| $\tau_w$ (Eq. 1.32)                                                                        | 414.5  |
| $\bar{\tau}_{sh} = 1.25 \times \tau_w$                                                     | 518.1  |
| $\bar{\epsilon}_{sh\infty}$                                                                | 601.5  |
| $p_6$ (Eq. 1.33)                                                                           | 0.86   |
| $\epsilon_{sh\infty}^* = p_6 \bar{\epsilon}_{sh\infty}$                                    | 517.0  |
| $k_t^* = \bar{\tau}_{sh} (k_s D)^{-2}$                                                     | 39.5   |
| $\epsilon_{s\infty}^* = \epsilon_{sh\infty}^* E(t_0 + \bar{\tau}_{sh}) / E(607)$           | 516.7  |
| <b>Real Structure</b> , ( $D = 25\text{cm} = 10\text{in.}$ , $t_0 = 10$ days, $h = 65\%$ ) |        |
| $\tau_{sh} = k_t^* (k_s D)^2$                                                              | 5221.5 |
| $\epsilon_{sh\infty} = \epsilon_{s\infty}^* E(607) / E(t_0 + \tau_{sh})$                   | 514.9  |
| $\epsilon_{sh}(1010, 10)$ (Eq. 1.8)                                                        | 153.7  |

in which

$$r(t') = 1.7(t')^{0.12} + 8, \quad Z(t, t') = (t')^{-m} \ln[1 + (t - t')^n]$$

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} \quad (1.36)$$

Those who wish to implement numerical integration of Eq. (1.6) on their computer should note that  $\lim_{t \rightarrow t'} \dot{C}(t, t') = \infty$  for  $t \rightarrow t'$ . Consequently the integral over the first time interval  $\Delta t = t_1 - t'$  should be evaluated analytically<sup>6</sup>. Use  $\Delta t \leq 10^{-4}t'$  and  $Q(t_1, t') = (t')^{-m} \ln[1 + (\Delta t)^n]$ .

### 1.7.2 Extension to Basic Creep at Constant Elevated Temperature

Eq. (1.6) for the rate of basic creep compliance function is generalized as follows:

$$\dot{C}_0(t, t', T) = R_T \left[ \left( q_2 \frac{\lambda_0}{t_T} + q_3 \right) \frac{n \xi^{n-1}}{\lambda_0(1 + \xi^n)} + \frac{q_4}{t_T} \right] \quad (1.37)$$

where  $\xi = t_T - t'_e$  at current time  $t$  and  $T$  = absolute temperature. In the foregoing equation, the age at loading and the stress duration are replaced

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by the following equivalent age and equivalent stress duration:

$$t'_e = \int_0^{t'} \beta_T(t'') dt'', \quad t_T - t'_e = \int_{t'}^t \beta'_T(t') dt' \quad (1.38)$$

The temperature dependent coefficients are defined by equations<sup>C23</sup>: The temperature dependent coefficients are defined by equations<sup>C23</sup>:

$$\beta_T = \exp \left[ \frac{U_h}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \quad \beta'_T = \exp \left[ \frac{U_c}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (1.39)$$

$$R_T = \exp \left[ \frac{U'_c}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (1.40)$$

$T_0$  = reference absolute temperature (for all the data fits,  $T_0 = 293^\circ K$ );  $U_h$  = activation energy of cement hydration;  $U_c, U'_c$  = activation energies of creep describing the acceleration of creep rate and magnification of creep due to temperature increase; and  $R$  = gas constant. Integration of Eq. (1.37) yields the basic creep compliance function:

$$C_0(t, t', T) = R_T \left\{ q_2 Q(t_T, t'_e) + q_3 \ln[1 + (t_T - t'_e)^n] + q_4 \ln \left( \frac{t_T}{t'_e} \right) \right\} \quad (1.41)$$

Fitting of the available test data indicates the following parameter values:

$$\begin{aligned} U_c/R &= 110w^{-0.27} \bar{f}_c^{0.54} \quad (\text{inch-pound system}) = 3418w^{-0.27} \bar{f}_c^{0.54} \quad (\text{S.I.}) \\ U_h/R &= 5000^\circ K, \quad U'_c = 0.18U_c \end{aligned} \quad (1.42)$$

### 1.7.3 Further Refinements for Highly Sensitive Structures

In the case of structures that are highly sensitive to creep (levels 5 and 4), such as nuclear reactor structures and large span bridges or shells, several other influences on creep and shrinkage should be taken into account in order to reduce the uncertainty of prediction. However extreme caution is necessary for such structures and updating of the model parameters by testing is required.

#### Creep at Elevated Temperature with Drying

The formulation given in Ref. 4 (part IV, Eq. 9-17) consists of relatively simple explicit expressions. However, it must be warned that explicit formulae can never be very accurate for drying concrete when temperature varies. Only integration of the differential equations of the problem can fulfill such expectations.

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These influences can be approximately described by simple explicit expressions given in Ref. 4 (part V, Eq. 1-10).

#### 1.7.4 Uncertainties Due to Time-Variation of Stress and Method of Structural Analysis

The stresses in concrete structures often vary significantly in time. The cause can be the load changes. But even at constant applied loads significant changes of stress can be caused by the stress relaxation due to imposed displacements, by sequential construction with joining of previously disconnected load-carrying members, and by changes of restraints of the structure. Significant stress variation can also be caused by differences in shrinkage and creep of joined structural parts of different age, of parts made of different materials (different concretes or concrete and steel), of parts of different thickness or different hygrothermal conditions, etc.<sup>15-18</sup>.

In such cases the effect of stress variation is calculated according to the principle of superposition. This causes additional *errors of two kinds*: (1) an error of the principle of superposition per se, and (2) an error in the approximate method of analysis compared to the exact solution according to the principle of superposition<sup>15</sup>. Estimation of these errors is a complex question that needs to be researched more deeply but the following simplified approach is better than ignoring these errors altogether.

Let  $\omega_X$  be the coefficient of variation of some response like the deflection or the maximum stress in the structure. Let  $\lambda_L = \max(\Delta P/P_{\max}, \Delta\sigma/\sigma_{\max})$  where  $\Delta P = P_{\max} - P_{\min}$  and  $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$  where  $P_{\max}$ ,  $P_{\min}$ ,  $\sigma_{\max}$ ,  $\sigma_{\min}$  are rough estimates of the maximum and minimum values of load  $P$  and stress  $\sigma$  during the lifetime of the structure. For a constant load,  $\lambda_L = 0$ . Then one should replace  $\omega_X$  by  $\omega_X + \Delta\omega_X$  where the increase  $\Delta\omega_X$  of the coefficient of variation may be taken approximately as follows:

$$\begin{aligned}\Delta\omega_X &= 0.05\lambda_L && \text{for class I methods} \\ &= 0.07\lambda_L && \text{for class II methods} \\ &= 0.25\lambda_L && \text{for class III methods}\end{aligned}\tag{1.43}$$

Coefficient  $\lambda_L$  takes into account errors of the first kind and the classes of methods take into account the errors of the second kind. The class I methods are the computer methods of structural creep and shrinkage analysis that solve the problem accurately according to the principle of superposition (using either integral equations or the Maxwell or Kelvin chain approximations,

solved in small time steps). The class II methods are the simplified methods of good accuracy, such as the age-adjusted effective modulus method. The class III methods are crude simplified methods such as the effective modulus method, the rate-of-creep (Dischinger) method and the rate-of-flow method.

Approximate knowledge of the coefficient of variation provides a rational basis to the designer for deciding how sophisticated a method should be used. Simple but crude methods for predicting creep and shrinkage of concrete can be used but the important point is that their coefficients of variation should be considered. If the designer regards the coefficient of variation (or the 95% confidence limits) of the deflection or stress obtained for the effective modulus method as acceptable (not uneconomic), he can use that method and need not bother using a more complicated method of structural analysis. The coefficient of variation depends on the type of structure and type of response. For example, the deflection of a small-span nonprestressed reinforced concrete beam is a problem relatively insensitive to creep, and a very simple estimation based on the effective modulus method is adequate, as documented by a small coefficient of variation and small mean values of deflections. On the other hand, the deflection of a large-span prestressed box girder bridge is a creep-sensitive problem, which is manifested by a high coefficient of variation of the deflection or maximum stress. For such problems it pays to use the most sophisticated method.

### 1.7.5 Free Shrinkage and Thermal Strain as a Constitutive Property

Some sensitive structures are analyzed by layered beam finite element programs or by two and three dimensional finite element programs. In such programs, the material properties used in each finite element must be the constitutive properties, independent of the cross-section dimensions and shape as well as the environmental conditions (which represent the boundary conditions for partial differential equations<sup>19</sup>). At drying, the constitutive properties cannot be measured directly, but they have been identified by fitting with a finite element program the overall deformation measurements on test specimens<sup>20–23</sup>. For this kind of analysis, only Eq. (1.3), (1.4) and (1.6) [or (1.11)] are retained, while Eq. (1.8)–(1.15) and (1.37)–(1.42) must be deleted and replaced by the following constitutive relation<sup>20</sup> for the strain that must be added to the basic creep strain (with elastic deformation):

$$\dot{\epsilon}_{s,ij} = \epsilon_s^0 \frac{E(t_0)}{E(t)} [\delta_{ij} + s_h(r\sigma_{ij} + r'\sigma^v\delta_{ij})] (\dot{h}_l + a_T \dot{T}_l) \quad (1.44)$$

Subscripts  $i, j$  refer to Cartesian coordinates  $x_i$  ( $i, j = 1, 2, 3$ ); superimposed dots denote time rates (i.e.,  $\partial/\partial t$ );  $\delta_{ij}$  = Kronecker delta;  $\sigma_{ij}$  = stress tensor;  $\sigma^v$  = volumetric (or mean) stress;  $\epsilon_s^0$  = final shrinkage (at material points), which has a similar but not exactly the same value as  $\epsilon_{sh\infty}$  for the cross-section average;  $\dot{h}_i$  and  $\dot{T}_i$  = rates of local relative humidity and temperature in the pores of concrete (which must be obtained by solving the diffusion equations);  $s_h$  = sign of  $(\dot{h}_i + a_T \dot{T}_i)$ , which is 1 or  $-1$ ; and  $a_T$  = coefficient relating the stress-induced thermal strain and shrinkage. When Eq. (1.44) is used, the cracking or fracture must also be included in the analysis.

The constitutive relation (1.44) is much simpler than Eq. (1.8)–(1.15) and (1.19)–(1.23) which it replaces. However, at present there are not enough data to predict the values of  $\epsilon_s^0$ ,  $r$ ,  $r'$  and  $a_T$  from the composition and strength of the concrete. They must be identified by fitting the given data for drying creep, shrinkage and thermal expansion.

### 1.7.6 Simplified Approximate Method of Structural Analysis for Creep and Shrinkage

Theoretically exact solutions according to the principle of superposition lead to integral or differential equations. Although computer methods for such equations are accurate and effective, for most practical problems it suffices to use a much simpler approximate method—the age-adjusted effective modulus method, which converts the problem to elastic structural analysis. This method was recommended in ACI 209 and its basic applications were described in Chapters 3–5 of that report. These descriptions remain valid except the table listing the values of the aging coefficient,  $\chi$ , (Table 5.1.1) because  $\chi$  depends on the creep model, which is different in this paper. A list of values of the aging coefficient  $\chi$  for the present model for one set of typical values of constitutive parameters  $q_1$ ,  $q_2$ ,  $q_3$ ,  $q_4$  is given in Table 1.3. The aging coefficient can be easily calculated using the relation:

$$\chi(t, t') = \frac{E(t')}{E(t') - R(t, t')} - \frac{1}{\phi(t, t')} \quad (1.45)$$

where  $R(t, t')$  is the relaxation function, and  $\phi(t, t')$  is the creep coefficient. For the relaxation function,  $R(t, t')$  the following approximation can be used<sup>24</sup>:

$$R(t, t') = \frac{0.992}{J(t, t')} - \frac{0.115}{J_0} \left[ \frac{J(t' + \xi, t')}{J(t, t - \xi)} - 1 \right]; \quad J_0 = J(\xi + t', \xi + t' - 1) \quad (1.46)$$

where  $\xi = (t - t')/2$ . For the compliance function considered in Ref. 24,  $J_0 = J(t, t - 1)$ , as recommended in ACI 209-R92. However the expression

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Table 1.3: Values of the Aging coefficient,  $\chi$  for a set of typical values of material parameters.

| $q_1 = 0.2, q_2 = 0.4, q_3 = 0.02, q_4 = 0.07 (\times 10^{-6}/\text{psi})$ |                 |       |       |       |
|----------------------------------------------------------------------------|-----------------|-------|-------|-------|
|                                                                            | $t - t'$ (days) |       |       |       |
| $t'$                                                                       | 10              | 100   | 1000  | 10000 |
| 1                                                                          | 0.462           | 0.445 | 0.490 | 0.547 |
| 10                                                                         | 0.706           | 0.588 | 0.593 | 0.634 |
| 100                                                                        | 0.877           | 0.709 | 0.625 | 0.643 |
| 1000                                                                       | 0.942           | 0.887 | 0.706 | 0.640 |

for  $J_0$  in Eq. (1.46) was found to give more accurate results for the present compliance function.

For more detailed information on the age-adjusted effective modulus method see Ref. 25 and 26.

### 1.7.7 Notation

$a/c$  = aggregate-cement ratio, by weight;

$c$  = cement content of concrete in  $\text{lb}/\text{ft}^3$  (in  $\text{kg}/\text{m}^3$  for the S.I. version,  $1 \text{ lb}/\text{ft}^3 = 16.03 \text{ kg}/\text{m}^3$ );

$C_0(t, t')$  = compliance function for basic creep only;

$C_d(t, t', t_0)$  = compliance function for additional creep due to drying;

$D = 2v/s$  = effective cross section thickness in inches (in mm for the S.I. version,  $1 \text{ inch} = 25.4 \text{ mm}$ );

$E(t)$  = conventional Young's elastic modulus at age  $t$ ;

$E_{28} = E(t)$  for  $t = 28$  days;

$\bar{f}_c$  = mean 28-day standard cylinder compression strength in psi (in MPa for the S.I. version,  $1 \text{ psi} = 6895 \text{ Pa}$ ) (if only design strength  $f'_c$  is known, then  $\bar{f}_c = f'_c + 1200 \text{ psi}$ );

$F(t, t')$  = function used in creep updating;

$h$  = relative humidity of the environment (expressed as a decimal number, not as percentage)  $0 \leq h \leq 1$ ;

$H$  = spatial average of pore relative humidity within the cross section,  $0 \leq H \leq 1$ ;

$J(t, t')$  = compliance function = strain (creep plus elastic) at time  $t$  caused by a unit uniaxial constant stress applied at age  $t'$  (always given in  $10^{-6}/\text{psi}$ , the S.I. version of the formulae give  $J(t, t')$  in  $10^{-6}/\text{MPa}$ , 1

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- $k_t$  = parameter used in calculation of  $\tau_{sh}$ ;  
 $k_h$  = humidity correction factor for final shrinkage;  
 $p_1, p_2$  = parameters used in creep updating;  
 $p_6$  = parameter used in shrinkage updating;  
 $q_1, q_2, q_3, q_4, q_5$  = empirical material constitutive parameters given by formulae based on concrete strength and composition;  
 $R(t, t')$  relaxation function;  
 $S(t)$  = time function for shrinkage;  
 $t$  = time, representing the age of concrete, in days;  
 $t'$  = age at loading, in days;  
 $t_0$  = age when drying begins, in days;  
 $t'_0 = \max(t', t_0)$  = time when the drying and loading first act simultaneously;  
 $v/s$  = volume-to-surface ratio in inches;  
 $w/c$  = water-cement ratio, by weight;  
 $w = (w/c)c$  = water content of concrete mix in lb/ft<sup>3</sup> (in kg/m<sup>3</sup> for the S.I. version);  
 $\chi(t, t')$  aging coefficient;  
 $\Delta w(h)$  = relative water (weight) loss at relative humidity  $h$ ;  
 $\epsilon_{sh}, \epsilon_{sh\infty}$  = shrinkage strain and ultimate (final) shrinkage strain;  $\epsilon_{sh\infty} \geq 0$  but  $\epsilon_{sh}$  is considered negative (except for swelling, for which the sign is positive); always given in  $10^{-6}$ ;  
 $\bar{\epsilon}_{sh}, \epsilon'_{sh}$  = shrinkage values given by the formula and measured shrinkage values used in the updating procedure;  
 $\phi(t, t')$  creep coefficient;  
 $\tau_{sh}$  = shrinkage half-time in days;  
 $\bar{\tau}_{sh}$  = updated value of  $\tau_{sh}$ ;  
 $\theta$  = relative water content;  
 All symbols marked with an asterisk represent the corresponding parameters updated using short-time tests.

### 1.7.8 Commentary and Explanations

- <sup>C1</sup> A more precise classification of the level of sensitivity of a structure could be made on the basis of the coefficients of variation of responses such as deflection or stress or even better on the basis of sensitivity analysis. But such an approach might be cumbersome for practice and would anyway require further research.  
<sup>C2</sup> The numbers 0.85 for  $w/c$  ratio and 10 lbs/ft<sup>3</sup> or 45 lbs/ft.<sup>3</sup> for cement content are of course outside the range of good concretes in today's practice.

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- <sup>C3</sup> Formulae predicting model parameters from the composition of concrete have not been developed for special concretes containing various admixtures, pozzolans, microsilica, and fibers. However, if the model parameters are not predicted from concrete composition and strength but are calibrated by experimental data, the model can be applied even outside the range of applicability in section 1.2, for example, to high-strength concretes, fiber-reinforced concretes, and mortars.
- <sup>C4</sup> This means that creep may be assumed to be linearly dependent on stress.
- <sup>C5</sup> When stresses vary in time, the corresponding strain can be approximately calculated from Eq. (1.3) according to the principle of superposition as recommended by RILEM committee guidelines<sup>1</sup>. Simplified design calculations can be done according to the age-adjusted effective modulus method<sup>25,26</sup>, which allows quasi-elastic analysis of the structure.
- <sup>C6</sup> Generalization to multiaxial stress may be based also on the principle of superposition. The creep Poisson ratio may be assumed to be constant and equal to the instantaneous Poisson ratio  $\nu = 0.18$ . (Tensile microcracking can cause the apparent Poisson ratio to be much smaller, but this is properly taken into account by a model for cracking.)
- <sup>C7</sup> The instantaneous strain, which is the same as in previous models BP and BP-KX<sup>3,4</sup>, may be written as  $q_1 = 1/E_0$  where  $E_0$  is called the asymptotic modulus.  $E_0$  should not be regarded as a real elastic modulus but merely as an empirical parameter that can be considered as age-independent. The age-independence of  $E_0$  is demonstrated by the experimental fact (reported in 1976 by Bažant and Osman<sup>27</sup>) that the short-time creep curves for various  $t'$ , plotted as  $J(t, t')$  versus  $(t - t')^n$  (with  $n \approx 0.1$ ), appear approximately as straight lines all of which meet for  $t - t' = 0$  approximately at the same point, regardless of  $t'$  (see Fig. 2.8d,e in Chapter 2). As a rough estimate,  $E_0 \approx 1.5E$ . The use of  $E_0$  instead of the conventional static modulus  $E$  is convenient because concrete exhibits pronounced creep even for very short load durations, even shorter than  $10^{-4}$  s. The usual static elastic modulus  $E$  normally obtained in laboratory tests and used in structural analysis corresponds to

$$E(t') = 1/J(t' + \Delta, t') \quad (1.47)$$

in which the stress duration  $\Delta = 0.01$  day gives values approximately agreeing with ACI formula,  $E(t) = 57,000\sqrt{\bar{f}_c(t)}$ , with  $E$  and  $\bar{f}_c$  in psi (or  $E(t) = 4734\sqrt{\bar{f}_c(t)}$  with  $E$  and  $\bar{f}_c$  in MPa). This formula uses the average compressive strength  $\bar{f}_c$ ; it is important that the

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design strength  $f'_c$  not be used in this formula). For short-time loading ( $t - t' \ll t'$ ), Eqs. (1.47), (1.4) and (1.7) give the age dependence:

$$E(t') = \frac{1}{A_0 + A_1/\sqrt{t'}} \quad (1.48)$$

where  $A_0 = q_1 + q_3 \ln(1 + \Delta^n)$ ,  $A_1 = q_2 \ln(1 + \Delta^n)$ ,  $A_0, A_1 = \text{constants}$  and  $\Delta$  is in days.  $q_1, q_2, q_3$  are material constitutive parameters given by formulae based on the concrete strength and composition. This age dependence agrees with the test data on  $E(t')$  even better than the current ACI formula,  $E(t) = E(28)[t/(\alpha + \beta t)]^{1/2}$ , in which  $\alpha, \beta$  are constants (see Fig. 2.10), but the difference is unimportant. For  $\Delta = 10^{-7}$  day, Eq. (1.48) gives also realistic values of the dynamic modulus of concrete and its age dependence. The graphical meaning of the values of  $q_1 = 1/E_0$  and  $1/E$  is explained in Fig. 1.1. Note that for structural analysis it is not important which value of  $\Delta$  corresponds to  $E(t')$  in Eq. (1.5), and not even whether some other definition of  $E$  is used in Eq. (1.47). One can use the ACI formula

$$E(t) = 57000\sqrt{\bar{f}_c(t)}$$

with  $E$  and  $\bar{f}_c$  in psi (or  $E(t) = 4734\sqrt{\bar{f}_c(t)}$  with  $E$  and  $\bar{f}_c$  in MPa) or Eq. (1.47) for any value of  $\Delta \leq 0.1$  day. For the results of structural analysis of creep and shrinkage (for  $t - t' \geq 1$  day), the only important point is that  $E(t)$  and  $\phi(t, t')$  together give correct total compliance

$$J(t, t') = [1 + \phi(t, t')]/E(t')$$

of Model B3. This is guaranteed by calculating the creep coefficient from the compliance and chosen  $E$ -value according to Eq. (1.5).

- <sup>c8</sup> Note that if a prediction model would specify  $\phi(t, t')$  instead of  $J(t, t')$ , there would be the danger of combining  $\phi(t, t')$  with some incompatible value of  $E(t)$  which would result in wrong  $J(t, t')$  values. There are many combinations of  $\phi(t, t')$  and  $E(t)$  that give the same  $J(t, t')$ . What usually matters for structural creep calculations is only the values of  $J(t, t')$ , and not the values of  $\phi$  and  $E$  that yield  $J(t, t')$ . Care in this regard must also be taken when updating the model parameters from some test data for which only the values of  $\phi(t, t')$  were reported.  $J(t, t')$  cannot be calculated from such data using a definition of  $E$ . For example,  $E = 57000\sqrt{\bar{f}_c}$ , may not give values compatible with these  $\phi$ -values and may give a  $J(t, t')$  disagreeing with Eq. (1.5). Conversions of such data from  $\phi$  to  $J$ -values must be based on short-time strains or  $E$ -values measured on the creep specimens themselves, or else such data cannot be used.

- <sup>c9</sup> In normal concretes this decrease is small (to about 96%-98%).

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- <sup>C10</sup> This means that the normal strain  $J(t, t')\sigma$ , representing the sum of the elastic and creep strains, is measured by subtracting the deformations of a loaded specimen and a load-free companion. For shear creep this is not necessary because shrinkage is strictly a volume change.
- <sup>C11</sup> Autogenous shrinkage terminates if the relative humidity in the pores drops below approximately 85%. Further shrinkage (or expansion) may be caused by various chemical reactions, for example carbonation. But in good concretes, carbonation occurs only in a surface layer a few millimeters thick and can be neglected for normal structures. For concrete submerged in water ( $h=100\%$ ), there is positive  $\epsilon_{sh}$ , that is, swelling, which is approximately predicted by the present model upon substituting  $h=100\%$ .
- <sup>C12</sup> Note that the computer solutions of structural creep problems in small time steps require only the rate of compliance  $\dot{J}(t, t')$ , not the total value  $J(t, t')$ .
- <sup>C13</sup>  $Q(t, t')$  is a binomial integral, that cannot be expressed analytically.
- <sup>C14</sup> Note that  $\epsilon_{sh}$  is relatively insensitive to the precise definition of  $E$  and either formula for  $E(t)$  yields about the same result<sup>13</sup>. For simplified analysis one can assume  $\epsilon_{sh\infty} \approx \epsilon_{s\infty}$ . The typical values of  $\epsilon_{sh\infty}$  according to Eq. (1.13) range from  $300 \times 10^{-6}$  to  $1100 \times 10^{-6}$ . The value of elastic modulus  $E(t)$  can be expressed either from Eq. (1.47) of the present model or approximately from the equation  $E(t) = E(28)[t/(4 + 0.85t)]^{1/2}$ . This equation (which is used in ACI 209), has been adopted for the present fitting of data; it does not fit the data on the age-dependence of  $E$  as well as Eq. (1.48) but the difference is unimportant. Because  $\tau_{sh} \propto D^2$ , Eq. (1.13) also approximately describes the effect of  $D$  on  $\epsilon_{sh\infty}$ , which is caused by the fact that thicker specimens reach a higher degree of hydration because their core remains wet for a longer time, and the fact that they undergo more microcracking, which reduces shrinkage. The present model does not describe separately the autogenous shrinkage which occurs in sealed specimens (or mass concrete). Such shrinkage is caused by volume changes during the chemical reactions of hydration and is independent of the size of the specimen. Except for high-strength concrete, this shrinkage is usually much smaller than the drying shrinkage. In exposed specimens there is some autogenous shrinkage too, but is still smaller because: (1) most of it occurs before stripping the forms, and (2) after stripping the forms it occurs only in the core and only until the drying front reaches the core. This part of autogenous shrinkage is included in the present model because the model was fitted to the

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total shrinkage data for drying specimens.

- <sup>C15</sup> Predicting the creep and shrinkage properties of concrete from the composition of concrete mix and the strength of concrete is an extremely difficult problem for which no good theory has yet been developed. The present formulae which are partly empirical and partly reflect trends theoretically deduced from the understanding of physical mechanisms, were calibrated by statistical analysis of the data in a computerized data bank involving about 15,000 data points and about 100 test series.
- <sup>C16</sup> Alternatively, from the approximate formulae,  $Z(t, t') = (t')^m \ln[1 + (t - t')^n] = 0.1775$ ,  $Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} = 0.1818$ , and  $Q(t, t') = Q_f(t') \{1 + [Q_f(t')/Z(t, t')]^{r(t')}\}^{-1/r(t')} = 0.1818 \times \{1 + [0.1818/0.1785]^{10.5358}\}^{-0.0949} = 0.1681$ .
- <sup>C17</sup> The present paper does not rule out using the previous model in Chapter 2 of ACI 209. However, if this is done, one must consider the predicted creep and shrinkage to have coefficients of variations of 58%.
- <sup>C18</sup> The coefficients of variation in Eq. (1.26-1.27) can result in similar or very different (much smaller or much larger) coefficients of variation of structural response such as the predicted maximum deflection or the maximum stress in the structure. Creep sensitive structures, and especially those that can collapse due to creep (long-time creep buckling of shells), should not be designed for the mean effects of creep and shrinkage. Rather they should be designed for the response (deflection, stress, strain) values representing the 95% confidence limits. This means that, if 20 identical structures were built and subjected to the same loading and environment, only one of them would be likely to suffer intolerable deflections or cracking damage, whereas the design for mean response means that 10 of them would be likely to suffer such a fate. One may assume the response values to have a normal (Gaussian) distribution, and then the 95% confidence limits may be estimated as  $(\text{mean of } X) \times (1 \pm 1.96\omega_X)$ ;  $\omega_X$  can be calculated by generating, in a proper way<sup>26</sup>, about 10 random samples of material parameters and then running for each sample a deterministic structural analysis<sup>8</sup>.
- <sup>C19</sup> The large uncertainty in the prediction of creep and shrinkage of concrete, reflected in the values of the coefficients of variation in Eq. (1.26), is caused mainly by the effect of the composition and strength of concrete. This effect is very complicated and is handled at present empirically because its mathematical theory is not yet available. At present, the only way to reduce the uncertainty is to conduct short-

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time tests and use their results to update the values of the material parameters in the model. This approach is particularly effective for creep but is more difficult for shrinkage<sup>13</sup>.

<sup>C20</sup> The largest source of uncertainty of creep and shrinkage prediction model is the dependence of the model parameters on the composition and strength of concrete. This uncertainty can be greatly reduced by carrying out, on the given concrete, short-time creep and shrinkage measurements (of duration less than 1 to 3 months) and adjusting the values of the model parameters accordingly. This is particularly important for special concretes such as high-strength concretes or fiber-reinforced concretes. Various types of admixtures, high-range water reducers and pozzolanic ingredients used in these concretes have been found to have a significant effect<sup>28,29</sup>. Empirical formulae for the effects of all these ingredients on the model parameters would be very difficult to formulate because of the great variety of additives and their combinations. For the planning of short-time creep measurements, note that the prediction improvement based on short-time data is more successful if the creep measurements begin at very short times after the loading (and likewise for shrinkage, if the measurements begin immediately after the stripping of the mold). Also, the measurements should be taken at approximately constant intervals in log-time, e.g. at 30, 100, 300, 1000, 3000... seconds. The reason is that the creep curves rise smoothly through the entire range from 0.0001 s to 30 years. In our example, the first reading was taken as late as 1 day after loading, as is often done, and therefore as many as 28 days of creep data were needed for prediction improvement. In a similar example using a different data set, it was shown<sup>12</sup> that if the first reading is taken as soon as possible after loading (within 1 minute) and about six readings are taken during the first two days of load duration (uniformly spaced in log-time), a similar improvement can be achieved using those readings only. Thus the required duration of the short-time test could be reduced if the readings begin immediately after loading. Anyhow, to reliably predict creep values for over five years of creep duration, it is recommended to carry out short-time tests of at least 28 day duration (with the first reading immediately after the loading and further readings equally spaced on the logarithmic scale of creep duration).

<sup>C21</sup> An important advantage of the present model is that all the free parameters for creep with elastic deformation, that is,  $q_1, q_2, q_3, q_4, q_5$ , are contained in the formulae linearly. Therefore, linear regression based

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on the least-square method, which minimizes the value of  $\bar{\omega}_{all}^2$ , can be used to identify these parameters from test data. The same is true of parameter  $\epsilon_{sh\infty}$  for shrinkage. Thus the only nonlinear parameter of the entire Model B3 is the shrinkage half-time  $\tau_{sh}$  but it can also be determined by linear regression from weight loss measurements.

<sup>C22</sup> The reason is that the shrinkage half-time  $\tau_{sh}$  is involved nonlinearly and in such a manner [seen from Fig. 2.11 (a,b)] that very different values of  $\tau_{sh}$  can yield almost equally good fits given short-time data (except if the data reach well beyond the time at which the slope of  $\epsilon_{sh}$  versus  $\log(t - t_0)$  begins to level off).

<sup>C23</sup> These equations represent Arrhenius equations based on the activation energy theory.

### 1.7.9 Remark on Large-Scale Computer Analysis

Efficiency of large-scale step-by-step finite element analysis requires avoiding the storage of stress history and evaluation of history integrals. This is achieved by representing the compliance function  $J(t, t')$  with an equivalent Kelvin chain<sup>1,16</sup>. To determine Kelvin chain moduli  $E_\mu$  ( $\mu = 1, \dots, N$ ), tedious least-square fitting of the compliance function is in general necessary<sup>1,16</sup>. However, among various existing formulations, Eq. (1.6) of B3 model has the advantage that the spring moduli  $E_\mu$  can be simply evaluated from an explicit formula, which reads:

$$\frac{1}{E_\mu(t)} = q_2 (\ln 10) \log \tau_\mu \frac{n(1-n)(3\tau_\mu)^n}{1 + (3\tau_\mu)^n} \left[ \left( \frac{\lambda_0}{t} \right)^m + \frac{q_2}{q_3} \right] \quad (1.49)$$

where  $\lambda_0 = 1$  day (see Z.P. Bažant and Y. Xi, J. of Engrg. Mech. ASCE 121 (2), 1995, 281–288).  $\tau_\mu$  are the retardation times which must be chosen to cover the time range of interest (under the restriction  $\tau_{\mu+1}/\tau_\mu \leq 10$ ). The dashpot viscosities are  $\eta_\mu(t) = \tau_\mu E_\mu(t)$ . A spring of modulus  $E_0 = 1/q_1$  and an aging dashpot of increasing viscosity  $\eta(t) = t/q_4$  must be attached in series to the Kelvin chain. The bulk moduli and shear moduli of a triaxial rate-type aging viscoelastic constitutive relation based on Kelvin chain model are  $K_\mu(t) = E_\mu(t)/3(1 - 2\nu)$  and  $G_\mu(t) = E_\mu(t)/2(1 + \nu)$ .

The additional creep due to drying (Eq. 1.14 with  $q_5$ ) need not be represented by a Kelvin chain model. It is better taken into account by stress-induced shrinkage according to Eq. (1.44) and direct modeling of tensile cracking, or even more realistically by the microprestress solidification theory (Bažant et al., ASCE J. of Engrg. Mech. 123 (11), 1997, 1188–1201).

## Chapter 2

# Justification and Refinements of Model B3

### 2.1 Introduction

Realistic prediction of concrete creep and shrinkage is of crucial importance for durability and long-time serviceability of concrete structures, and in some cases also for long-time stability and safety against collapse. Mispredictions of this phenomenon, which contribute to excessive deflections and cracking, have been one of the reasons for problems with longevity of the civil engineering infrastructure in all countries. The errors in the prediction of concrete creep and shrinkage have generally been larger than the errors caused by simplifications in the methods of structural analysis. It is now clear that, for creep sensitive structures, it makes little sense to use finite element analysis or other sophisticated computational approaches if a realistic model for creep and shrinkage is not introduced in the input. If a simplistic and grossly inaccurate prediction model for creep and shrinkage is used for a creep sensitive structure, one can hardly justify anything more than simple hand calculations of stresses and deformations in structures. In such a case, it makes no sense for the analyst to spend weeks on the computer analysis of the structure while spending half an hour to determine creep and shrinkage properties to use as the input. The design will be better if more time is devoted to the latter than the former.

Realistic prediction of creep and shrinkage of concrete is a formidably difficult problem because the phenomenon is a result of several interacting physical mechanisms and is influenced by many variable factors. In view of this fact, it is not surprising that improvements have been only gradual and slow. No sudden breakthrough has occurred. However, the accumulated

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advancement of knowledge since the early systematic researches in the 1930's, and especially during the last two decades, has been enormous. It is now possible to formulate a much better prediction model than two decades ago.

Four major advances have made a significant prediction improvement possible:

1. Improved theoretical understanding, for example, of mathematical modeling of the solidification process of cement, diffusion processes, thermally activated processes, cracking damage, residual stresses, and non-uniformity of stress and pore-humidity profiles.
2. Gradual accumulation of test data and formulation of an extensive computerized data bank, which started with the Northwestern University data bank in the 1970's<sup>3</sup> having over 10,000 data points. Subsequently, in collaboration with ACI and CEB, this developed into the RILEM Data Bank, compiled by Subcommittee 5 of RILEM Committee TC-107 (chaired by H. Müller). (This data bank now comprises about 600 measured time curves from about 100 test series from various laboratories around the world, with about 15,000 data points.)
3. Progress in statistical evaluation of test data, optimization of the creep and shrinkage prediction model and optimization that minimizes the sum of the squares of errors. This task has been facilitated by the computerized form of the aforementioned RILEM data bank.
4. Numerical studies of the response of test specimens and structures (especially by finite elements) and their comparisons with observed behavior, which has shed light on various assumptions on the material model used in the input.

Much of the complexity and error of the prediction model is caused by the fact that the design offices still analyze most structures according to beam theory, which requires the average material characteristics for the cross section of the beam as a whole. Because the material creep and shrinkage properties inevitably become non-uniform throughout the cross section (due to diffusion phenomena, residual stresses, cracking, damage localization and fracture), the model for the average material properties in the cross section is not a material constitutive model. It depends on many more influencing factors than the constitutive model, such as the shape of the cross section, environmental history, ratios of the bending moment, normal force and shear force.

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For this reason, a really good model for the prediction of the average shrinkage properties and average creep at drying in the cross section under general loading and environmental conditions will never be possible. One must accept significant errors, increased complexity, and a greater number of empirical parameters if one insists on characterizing the behavior of the cross section as a whole by its average properties. In the future, the design approach should move away from characterizing the cross section creep and shrinkage properties of the cross section as a whole, and toward an approach in which the cross sections are subdivided into individual small elements. However, for the time being, an integral prediction model for the cross section as a whole is needed. In the special case of constant temperature and constant moisture content, the average characterization of creep in the cross section is identical to a constitutive model for a material point. In that case, the model becomes far simpler and more accurate.

The present model, which for brevity, is labeled Model B3 and is based on report<sup>2</sup>, is the third major refinement in a series of models developed at Northwestern University, including the BP model<sup>3</sup> and BP-KX model<sup>4</sup>. Model B3 is simpler, better supported theoretically and as accurate as these previous models. Research progress will not stop, and no doubt further improved versions will become possible in the future. Compared to the latest BP-KX model, the improvement in Model B3 consists in simplification of the formulae achieved by sensitivity analysis, incorporation of a theoretically derived rather than empirical expression for the drying creep, and calibration of the model by an enlarged data set including the data published in the last few years.

Model B3 conforms to the guidelines that have recently been formulated by RILEM Committee TC-107, as a refinement and extension of the conclusions of a preceding RILEM Committee TC-69<sup>1</sup>. These guidelines summarize the basic properties of creep and shrinkage that have been well established by theoretical and experimental researches and represent the consensus of the committee. The existing prediction models of major engineering societies violate many of these guidelines. The present model must nevertheless be regarded as only an example of a model satisfying these guidelines because formulation of a model based on the guidelines is not unique and conceivably partly different models satisfying these guidelines could be formulated, too.

The purpose of this Chapter 2 is to provide justification as well as some refinements of Model B3. All the notations from Chapter 1 are retained.\*

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\*The same model as described here has been proposed to the RILEM committee TC-107.

## 2.2 Unbiased Statistical Evaluation Based on Computerized Data Bank

Development of a data bank comprised of practically all the relevant test results on creep and shrinkage of concrete obtained in various countries and laboratories up to the present time facilitates evaluation and calibration of creep and shrinkage prediction models. No longer tediousness limits such evaluation to a few subjectively selected test data. It is important to use the complete set of available test data, because subjective selections of some data for verification of a creep model have been shown to be capable of greatly distorting the conclusions.

The statistical evaluation and optimization of Model B3 has been carried out in the same manner as for the preceding models BP and BP-KX (See Ref. 3-Part 6; Ref. 4 and Ref. 13). Optimum values of the model parameters that minimize the sum of squared deviations from the data in the data bank have been determined. The deviations of the model from the test data (errors) have been characterized by their coefficient of variation  $\bar{\omega}$  which is defined for the data set number  $j$  as

$$\bar{\omega}_j = \frac{s_j}{\bar{J}_j} = \frac{1}{\bar{J}_j} \sqrt{\frac{1}{n-1} \sum_{i=1}^n (w_{ij} \Delta_{ij})^2}, \quad (2.1)$$

in which

$$\bar{J}_j = \frac{1}{n} \sum_{i=1}^n w_{ij} J_{ij}, \quad w_{ij} = \frac{n}{n_d n_1} \quad (2.2)$$

Here  $J_{ij}$  are the measured values (labeled by subscript  $i$ ) of the compliance function in the data set number  $j$ ;  $n$  is the number of data points in the data set number  $j$ ;  $\Delta_{ij}$  is the deviation of the value given by the model from the measured value;  $s_j$  is the standard deviation of  $\Delta_{ij}$  values for set  $j$ ;  $w_{ij}$  are the weights assigned to the data points;  $n_d$  is the number of decades on the logarithmic scale spanned by measured data in data set number  $j$ ; and  $n_1$  is the number of data points in one decade.

The weight assigned to a data point in a decade on the logarithmic scale is taken as inversely proportional to the number of data points,  $n_1$ , in that decade and the weights are normalized such that  $\sum_i w_{ij} = n$ . The use of  $n-1$  in (2.1) is required for an unbiased estimate of  $s_j$ . The overall coefficient of variation of the deviations of the model from the measured values for all the

data sets in the data bank has been defined as

$$\bar{\omega}_{all} = \sqrt{\frac{1}{N} \sum_{j=1}^N \bar{\omega}_j^2} \quad (2.3)$$

in which  $N$  is the number of data sets in the bank (in Table 2.1, the  $\bar{\omega}_j$  values are those on line 1 to 17, in Table 2.2 those on lines 1 to 21, etc). Similar expressions, with  $J$  replaced by shrinkage strain  $\epsilon_{sh}$ , have been used for shrinkage. Eq. (2.3) represents the standard deviation of the population of relative errors  $\Delta_{ij}/\bar{J}_j$  of all the test series put together, with the same weight on each test series rather than on each reading. One might be tempted to use the alternative formula  $\bar{\omega}_{all} = [\sum_j s_j^2/N]^{1/2}/\bar{J}$  with  $\bar{J} = \sum_j \bar{J}_j/N$ , which is based on the population of actual rather than relative errors, or the formula  $[\sum_i \sum_j (w_{ij} \Delta_{ij})^2 / Nn - 1]^{1/2}/\hat{J}$  with  $\hat{J} = \sum_i \sum_j J_{ij}/Nn$ , which is based on the population of  $\Delta_{ij}$  of all the test series put together. But the former would give more weight to weaker concretes (because they creep more) and the latter to those test series for which more readings were taken (the number of readings is partly a subjective choice).

The statistics of the errors of Model B3 in comparison to the test data sets in the RILEM data bank are given in Tables 2.1–2.3 for basic creep, shrinkage and creep at drying. For comparison, the statistics of errors of the current ACI model<sup>5</sup>, developed in the mid-1960's by Branson et al., and the latest CEB model<sup>30</sup> are also shown (for a comparison to another recently proposed model<sup>31</sup> see the discussion of Ref. 31). Table 2.4 presents the statistics of the extension of the model to basic creep at constant elevated temperatures. Figs. 2.1–2.4 show some typical comparisons with selected important test data<sup>33–44</sup> from the data bank.

In evaluating a creep and shrinkage model, it is important to avoid subjective bias. This has not been true of the evaluations of some other models recently presented in the literature. For example, consider that there are over 1,000 data points for creep durations under 100 days, and only 10 data points in excess of 1,000 days. If all these data were used in the statistical evaluation with the same weight, the error in predictions for times over 1,000 days would obviously have a negligible influence on the resulting statistics. Thus, a very low coefficient of variation of errors would be obtained if the model fitted the data well only in the range of up to 100 days. Yet predictions of the long-time behavior are most important.

A similar problem arises when the data bank has many values for loading ages under 100 days and very few over 1,000 days. However, in long-time relaxation, significant stress changes can occur in structures even after 1,000

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Table 2.1: Coefficients of variation of errors (expressed as percentage) of basic creep predictions for various models.

| Model                                           | B3             | ACI            | CEB            |
|-------------------------------------------------|----------------|----------------|----------------|
| Test data                                       | $\bar{\omega}$ | $\bar{\omega}$ | $\bar{\omega}$ |
| 1. Keeton                                       | 19.0           | 37.5           | 42.8           |
| 2. Kommendant et al                             | 15.3           | 31.8           | 8.1            |
| 3. L'Hermite et al.                             | 49.4           | 133.4          | 66.2           |
| 4. Rostasy et al.                               | 15.2           | 47.6           | 5.0            |
| 5. Troxell et al.                               | 4.6            | 13.9           | 6.2            |
| 6. York et al.                                  | 5.6            | 37.7           | 12.8           |
| 7. McDonald                                     | 6.9            | 48.4           | 22.2           |
| 8. Maity and Meyers                             | 33.8           | 30.0           | 15.7           |
| 9. Mossiossian and Gamble                       | 18.6           | 51.5           | 47.3           |
| 10. Hansen and Harboe et al. (Ross Dam)         | 14.1           | 51.2           | 31.1           |
| 11. Browne et al. (Wylfa vessel)                | 44.7           | 47.3           | 53.3           |
| 12. Hansen and Harboe et al. (Shasta Dam)       | 22.7           | 107.8          | 43.1           |
| 13. Brooks and Wainwright                       | 12.6           | 14.9           | 15.4           |
| 14. Pirtz (Dworshak Dam)                        | 12.5           | 58.2           | 32.5           |
| 15. Hansen and Harboe et al. (Canyon ferry Dam) | 33.3           | 70.2           | 56.9           |
| 16. Russell and Burg (Water Tower Place)        | 15.7           | 19.3           | 31.5           |
| 17. Hanson                                      | 14.1           | 63.3           | 12.1           |
| $\bar{\omega}_{all}$                            | 23.6           | 58.1           | 35.0           |

days of age, and if the model does not predict the creep well for such large ages at loading, the stress relaxation cannot be correctly predicted from the principle of superposition. This point is important to realize since some recent comparisons with test data suffered from this kind of bias. Therefore one must ensure that the measured data within each decade of load or shrinkage duration (in logarithmic time scales) have equal weights. If possible this should also be ensured for the age at loading. Ideally, equal weights would be achieved and subjective bias eliminated if the measured data had equal spacing in the logarithmic scales of load or drying duration and age at loading, and if they covered the entire range. But unfortunately creep measurements have not been made in this way.

In a previous study<sup>3,4</sup>, the subjective bias was eliminated by hand-smoothing in log-time scale the experimentally measured curves and then placing points equally spaced in the log-scale on such hand-smoothed curves. However, there

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Table 2.2: Coefficients of variation of errors (expressed as percentage) of shrinkage predictions of various models.

| Model                                    | B3             | ACI            | CEB            |
|------------------------------------------|----------------|----------------|----------------|
| Test data                                | $\bar{\omega}$ | $\bar{\omega}$ | $\bar{\omega}$ |
| 1. Hummel et al.                         | 27.0           | 30.0           | 58.7           |
| 2. Rüsch et al.(1)                       | 31.1           | 35.2           | 44.8           |
| 3. Wesche et al.                         | 38.4           | 24.0           | 36.1           |
| 4. Rüsch et al.(2)                       | 34.7           | 13.7           | 27.8           |
| 5. Wischers and Dahms                    | 20.5           | 27.3           | 35.9           |
| 6. Hansen and Mattock                    | 16.5           | 52.9           | 81.5           |
| 7. Keeton                                | 28.9           | 120.6          | 48.3           |
| 8. Troxell et al.                        | 34.1           | 36.8           | 47.4           |
| 9. Aschl and Stökl                       | 57.2           | 61.3           | 44.2           |
| 10. Stökl                                | 33.0           | 19.5           | 29.6           |
| 11. L'Hermite et al.                     | 66.7           | 123.1          | 69.4           |
| 12. York et al.                          | 30.6           | 42.8           | 8.9            |
| 13. Hilsdorf*                            | 11.7           | 24.7           | 29.6           |
| 14. L'Hermite and Mamillan               | 46.1           | 58.7           | 45.5           |
| 15. Wallo et al.                         | 22.0           | 33.0           | 55.6           |
| 16. Lambotte and Mommens                 | 39.1           | 30.7           | 31.3           |
| 17. Weigler and Karl                     | 31.3           | 29.6           | 21.3           |
| 18. Wittmann et al.                      | 23.7           | 65.4           | 40.0           |
| 19. Ngab et al.                          | 20.4           | 45.3           | 64.6           |
| 20. McDonald                             | 5.1            | 68.8           | 21.4           |
| 21. Russell and Burg (Water Tower Place) | 38.5           | 51.0           | 58.1           |
| $\bar{\omega}_{all}$                     | 34.3           | 55.3           | 46.3           |

\*Hilsdorf H.K., "Unveröffentlichte Versuche an der MPA München," private communication (1980).