



S. M. Uzumeri Symposium

**Behavior and Design of Concrete Structures
for Seismic Performance**



Editors:

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S. M. Uzumeri Symposium: Behavior and Design of Concrete Structures for Seismic Performance

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PREFACE

S. M. Uzumeri Symposium

The S. M. Uzumeri Symposium was held at the ACI Fall Convention in Toronto on October 16, 2000. It was organized by ACI-ASCE Committee 441, Reinforced Concrete Columns, and co-sponsored with ACI-ASCE Committee 352, Joints and Connections in Monolithic Concrete Structures, and ACI Committee 374, Performance-Based Seismic Design of Concrete Buildings. The symposium and this Special Publication volume celebrate and honor the distinguished career of Mike Uzumeri as an educator and an engineer. Mike Uzumeri started his academic career at the University of Toronto in 1958 and remained there until his retirement in 1993. Since then, he has remained active in teaching and research at the University of Toronto and the Middle East Technical University in Turkey.

During his 35 years at the University of Toronto, Mike Uzumeri worked tirelessly for the students and the faculty. He was the first Chairman of the Academic Board from 1988 to 1989, a member of the Governing Council from 1981 to 1989, and Chairman of the Department of Civil Engineering from 1984 to 1989. Between 1975 and 1981, Mike chaired ACI-ASCE Committee 442, Response of Concrete Buildings to Lateral Forces. He was also the Chairman of the Canadian National Committee on Earthquake Engineering from 1981 to 1990. His research interests mainly focused on the design of concrete structures for earthquakes; and in 1975, he and Professor Thomas Paulay were awarded the Gzowski Medal by the Engineering Institute of Canada for “the best structural engineering paper published in the *Canadian Journal of Civil Engineering*.”

The Uzumeri Symposium consisted of 12 papers presented at two sessions. The speakers at the symposium and the authors of the papers were mostly former students or colleagues of Mike Uzumeri, who traveled from around the world to be with Mike and express their appreciation for his distinguished career. Both sessions were very well attended, indicating the respect that Mike commands among ACI members and the appropriateness of the topics of his research. As part of the celebration, the Department of Civil Engineering at the University of Toronto invited former students, family members, colleagues, and close friends of Mike and Jean Uzumeri to a dinner on October 17, 2000, which provided them with an opportunity to be nostalgic about the good times and offer their best wishes to the Uzumeris. The editors of this Special Publication would like to express their gratitude to Mike for his guidance and sound advice over the years, and to all those who contributed in making the symposium and dinner a memorable event for Mike.

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Recently Identified Aspects of Ductile Seismic Torsional Response of Reinforced Concrete Buildings

by T. Paulay

Synopsis: With few exceptions, code provisions relevant to torsional phenomena in buildings subjected to seismic effects, are based on elastic structural behaviour. The key parameter is stiffness eccentricity. The appropriateness of this approach to the design of systems expected to respond in a ductile manner is questioned. The degree of restraint with respect to system twist, strength eccentricity and the pattern of element yield displacements are considered to be more important parameters. For the purposes of seismic design, bi-linear force-displacement approximations of the elasto-plastic behaviour of reinforced concrete systems and their constituent elements, are considered to be adequate. Strategies aiming at the elimination of undesirable effects of torsional phenomena in ductile systems are addressed.

The findings of this study are based on a re-definition of some common terms of structural engineering, such stiffness, yield displacement and displacement ductility relationships. Contradictions with corresponding terms applicable to elastic systems are demonstrated. The introduction of these features, relevant to bilinear modelling of reinforced concrete elements, precedes the examination of the designer's options for the control of earthquake-induced displacement demands resulting in system translations and twist.

Keywords: codes; deflections; ductility; earthquake-resistant structures; reinforced concrete; stiffness; strength; structural design; torsion in buildings

ACI Honorary Member Tom Paulay is Professor Emeritus at the University of Canterbury, Christchurch, New Zealand. His major research interest is the design and detailing of ductile reinforced concrete structures subjected to earthquakes. He is past president of the International Association for Earthquake Engineering.

INTRODUCTION

Studies of the assessment of the structural performance of existing buildings with earthquake risk (1) triggered inquiries addressing the likely response of buildings as constructed, rather than their compliance with a particular code. A major perceived need was the estimation of torsion-induced displacements of elements of ductile systems (2,3). In the process several issues emerged with apparent conflict with ingredients of existing design practice. The description of progressively emerging fallacies, firmly entrenched in widely accepted routine seismic design techniques (4), is the subject of this presentation.

The perceived need to address earthquake-induced displacements in ductile systems motivated this study. It led to the introduction of unfamiliar, yet not necessarily new, principles. The freedom available to the designer, when strength is to be assigned to elements of a system, is a particularly interesting feature of the conclusions. Identification of structural behaviour, rationale and transparency of a viable design strategy, combined with simplicity of application, were central issues of this motivation.

DESIGN CRITERIA

The primary purpose of this study was to address means by which performance criteria, conforming with the appropriate limit state, may be rationally executed. The criteria considered were:

- Expected earthquake-induced deformations, including those due to system twist, should be limited to ensure that the displacement ductility capacity of any element of the system, $\mu_{\Delta\text{imax}}$, will not be exceeded.
- Maximum interstorey displacements, to be expected at locations remote from the centre of mass, should not exceed those considered acceptable for buildings, typically 2-2.5% of the storey height.
- More restrictive performance criteria may require displacements associated with a specific limit state to be less than those allowed by the displacement ductility capacity of constituent elements.

TERMINOLOGY USED

- In the study of earthquake-induced displacement of buildings, reference will be made to the structural system.
- A structural system comprises lateral force-resisting elements generally arranged in two orthogonal directions. Due to torsional effects, elements of the system may be subject to different storey displacements. Typical elements are bents of ductile frames or interconnected walls in the same plane.
- A lateral force-resisting element may comprise several components. Components will be subjected to identical displacements. Typical components are beams or columns or walls.

Examples are illustrated in Fig. 1. In this study only simple mass systems are considered.

TRADITIONAL CONCEPTS OF THE THEORY OF ELASTICITY

The requirements for static equilibrium and deformation compatibility in a statically indeterminate structure, are well established. These principles are still widely used when strength is assigned to lateral force-resisting elements of a system. The procedure is consistent with the now abandoned principle, that the resulting stresses should be well within the elastic domain of material behaviour.

With the introduction of equivalent lateral static seismic design forces and the acceptance of ductile response, the same technique continued to be widely used. It implied the notion that strength assigned proportionally to element stiffness will eventually result in the simultaneous onset of yielding in all elements.

As a corollary it is generally assumed that components with traditionally defined stiffness, based on flexural rigidity, EI , will have increased yield displacements as their strength is being increased. The flexural rigidity is the product of the modulus of elasticity of the material, E , such as concrete, and I is the second moment of the cross sectional area of a prismatic member. Its value may be adjusted in recognition of the effects of cracking (5).

Requirements for static equilibrium and deformation compatibility in statically indeterminate structures, are well established. Strength allocation based on these principles implies that the intended strength of all components of a lateral force-resisting element will occur at the same displacement. Subsequent adjustments of strength, relying of strength redistribution in a partially nonlinear system, is assumed to result in corresponding changes in yield displacements. Some fallacies relevant to the traditional modelling of elasto-plastic behaviour, widely used in seismic design practice, have been identified (6,7,8).

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It is postulated that for the purposes of seismic design the simulation of the non-linear behaviour of a ductile system, can be adequately modelled with bi-linear force-displacement relationships. Implications in terms of component, element or system response, are briefly summarized. The findings enable displacement relationships, including those due to system twist, to be quantified in a simple form.

MODELLING OF DUCTILE COMPONENT BEHAVIOUR

The typical elasto-plastic behaviour of a reinforced concrete and steel component is shown by the full line curves in Fig.2. The relative unit strength shown corresponds to that developed when yield strain is first developed in the extreme tension fibre. The dashed line for the RC section illustrates the effects of crack developments during first loading. This is of no interest in seismic design based on ductile response.

The associated yield curvature of the critical section is readily determined (9). Even under reversing repeated loading, the stiffness of the component is adequately simulated by the slope of the force-displacement relationship up to the unit relative strength. Once the pattern of bending moments is known, the relevant displacement associated with the onset of yielding may also be determined. For a realistic estimate of this displacement the second moment of the section area, transformed into properties of the concrete, needs to be used. In routine design this and the strength at first yield, involve cumbersome computations, preferably to be avoided. Simpler approaches are suggested.

The evaluation of the nominal strength, V_{ni} , of the component, shown subsequently to be a choice of the designer, is an essential part of the design routine. As Fig.2 shows, a linear extension of the elastic response (Δ'_{ye}) allows the nominal or reference yield displacement, Δ_{ye} , to be determined. The dashed lines in Fig.2 show how the nonlinear response, for example of a RC component, can then be simulated. If desired, the designer may allow for post-yield stiffness of the component.

Yield Curvature

The first step in estimating the yield displacement, Δ'_{yi} , of a component is the evaluation of the yield curvature at the critical section of a potential plastic hinge. From the study of strain patterns it was found (10,11) that nominal curvatures linearly extrapolated, as in Fig. 2, from that at the onset of yielding at the extreme tension fibres of typical reinforced concrete sections subjected to flexure with moderate axial loads, are approximately constant. For the purposes of seismic design such estimates should be considered adequate. For example the yield curvature associated with bilinear modelling, as in Fig.2, of a typical rectangular wall section (10), is of the order of

$$\phi_y \approx 2\epsilon_y/\ell_w \quad (1)$$

where ϵ_y is the yield strain of the steel used and ℓ_w is the length of the wall (Fig.3(a)).

This simple relationship shows that yield deformations are sensitive to the grade of reinforcement used and are inversely proportional to the overall depth of a component. Yield curvatures for components with different lengths cannot be identical, as assumed when using conventional analyses based on elastic behaviour. An important feature of yield curvature is, that for the purposes of seismic design, it is not affected by the strength assigned to the component.

Yield Displacements

As previously stated, once the nominal yield curvature, ϕ_{yi} , at the critical section or sections of a component is established, the yield displacement, used in the elasto-plastic bi-linear modelling, is readily evaluated. For example, yield displacements of reinforced concrete structural walls, subjected to a particular pattern of lateral forces are found to be

$$\Delta_{yi} = C\phi_{yi} h_{wi}^2 \approx (2C h_{wi}^2 \epsilon_y)/\ell_{wi} \propto 1/\ell_{wi} \quad (2)$$

where C is a coefficient which quantifies the pattern of lateral forces, and h_{wi} is the height of the wall. When wall heights are identical and the grade of reinforcement used is the same in one building, the bracketed term in eq.(2) is a constant. Therefore, in these common cases yield displacements of wall components, such as shown in Fig.3(a), are inversely proportional to the length of the walls, ℓ_{wi} .

The simple relationship (eq.(2)) can be conveniently used in seismic design whenever relative values, for example for the estimation of displacement ductilities, are sufficient. Equation (2) is fundamental in establishing displacement relationships between components, elements and the entire system (Fig.1). Implications of these relationships have been previously reviewed (7,8,12). The important conclusion to be drawn is the fact that yield displacements are functions of geometric and material properties and are independent of the nominal strength of components.

A RE-DEFINITION OF STIFFNESS

When seismic design is based on bi-linear modelling of ductile behaviour, as shown in Fig.2, the stiffness of the component is simply

$$k_i = V_{ni}/\Delta_{yi} \quad (3)$$

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where V_{ni} is the nominal strength assigned to the component and Δ_{yi} is a geometry-dependent predetermined property, as defined in eq.(2). It is seen that, contrary to traditional assumptions which are extensively used in seismic design, stiffness is proportional to strength assigned to the component by the designer. An example to be presented in Fig.3, where the bi-linear force-displacement behaviour of components with different lengths compared, will offer further explanations.

ASSIGNMENT OF STRENGTHS

As the bi-linear modelling in Fig. 2 implies, components will develop their nominal strength, V_{ni} , when their nominal yield displacement is imposed. Therefore, components with different yield displacements can never yield simultaneously. Correspondingly, components of an element, such as seen in Fig.1, will develop their nominal strength in a given sequence until all components have yielded, dictated by the attainment of their yield displacements. From this it may be concluded that, within rational limits, strength to components may be assigned arbitrarily. In this and subsequent examples it is assumed that the total nominal strength, V_E , is unity.

The principle offers great possibilities to the designer to exploit this freedom of choice in nominal strengths. Thereby more rational and economic solutions may be achieved. It is a fundamental tool aiding the mitigation or even elimination of the detrimental effects of torsional phenomena in ductile systems.

STRENGTH, STIFFNESS AND DISPLACEMENT RELATIONSHIPS

The relevance of these relationships, so important in seismic design, will be illustrated with an example presented in Fig.3. The relative lengths of four interconnected rectangular reinforced concrete cantilever walls with identical width, are such that the traditionally defined second moment of sectional area of the components bear the ratio of 1, 2, 4 and 8, respectively, to each other. Conventional design will assign lateral strength, V_{ni} , in these proportions.

The bilinear simulation of the components and the element, based on this traditional distribution of strengths, is shown in Fig.3(b). The relative strength of component (4) with the greatest length, is thus $8/15 = 0.53$, while its relative yield displacement is according to eq.(2) $\Delta_{y4} = \frac{1}{2} = 0.5$. Therefore, its relative stiffness (eq.(3)) is $k_4 = 0.53/0.5 = 1.06$. The superposition of bilinear component responses results in the total response of the element. This can be again simulated by simple bi-linear modelling. No post-yield stiffness was assumed in this example. The superposition relevant to this traditional strength distribution, shown in Fig.3(b), allows the total

translational stiffness of the 4-component element to be defined as:

$$k_e = \Sigma k_i = \Sigma (V_{ni} / \Delta_{yi}) \quad (4)$$

This in turn enables the nominal yield displacement of the element to be quantified as

$$\Delta_y = \Sigma V_{ni} / \Sigma k_i = V_E / k_e \quad (5)$$

The element nominal yield displacement, so derived and shown as 0.58 displacement units in Fig.3(b), is the weighted average of the component yield displacements. It is seen that when this displacement is imposed, some components will have yielded while some others would not. The purpose of the element yield displacement, given by eq.(5), is to allow the element displacement ductility capacity to be defined.

It is evident that, when the displacement ductility capacity of the components is specified, for example, as $\mu_{\Delta_{imax}} = 5$, the element displacement at the ultimate limit state must be limited to $\Delta_u \leq 5\Delta_{yimin}$. In the example the smallest yield displacement is that of component (4), i.e., 0.5 displacement units. Therefore, the displacement ductility demand on this element, controlled by component (4), should be limited to $\mu_{\Delta} \leq 5 \times 0.5/0.58 \approx 4.3$.

The results of an entirely different assignment of component strengths, the aim of which will be referred to subsequently, is illustrated in Fig.3(c). With different strengths, component stiffness are also different. The reduced total stiffness in this case, defined by eq.(4), resulted in a small increase of the nominal element yield displacement to 0.66 units and a corresponding reduction of the element displacement ductility capacity to 3.79.

A similar procedure can be used when the corresponding properties of the entire building system, comprising a number of parallel elements is determined.

TORSIONAL PHENOMENA IN DUCTILE SYSTEMS

The Origin of Torsional Actions

The purpose of considering the effects of torque and consequent twist on a system, is to estimate displacements imposed on elements, additional to that which would occur at the centre of the mass. The study is based on the usual assumption that lateral force resisting elements are interconnected by infinitely rigid floor diaphragms.

Under the action of a base shear force, the static torque, resulting from stiffness or

strength eccentricities, may be readily determined. The possibility of resisting such a torque during the elastic or ductile response needs to be studied first. Whenever diaphragm rotation, i.e., system twist, occurs, a dynamically induced torque, due to the rotary inertia of the distributed mass, will also be introduced. This is difficult to predict. However, the conditions under which a static and/or a dynamic torque can develop may be identified. This is subsequently illustrated.

Eccentricities

The cause of torsional phenomena is eccentricity. To define this, familiar locations within the plan of a building need to be identified. With reference to Fig.4 the centre of the distributed mass is shown as CM. For the assessment of elastic response the centre of element stiffness, commonly referred to as the centre of rigidity, CR, is of importance. After the elements have entered the inelastic domain of response and the base shear capacity, V_E , of the system is developed, the associated centre of stiffness becomes meaningless. At this stage the important location is the centre of resistance, i.e., that of the nominal strengths of all the yielding elements, CV. Simple equilibrium criteria enables the location, CV, of the corresponding resultant forces, generated by displacements in either of the principal directions, larger than that causing yielding in the element with the smallest length, to be readily determined. A typical example is shown in Fig. 4.

At certain instants of a seismic event, some elements may be subjected to displacements which are less than the relevant yield displacement. At such a stage the full base shear capacity, V_E , will not be developed. Moreover, maximum displacements of the yielding elements are not likely to approach their displacement capacity, unless an extremely large angle of twist is imposed by the earthquake. As a general rule, the maximum displacement ductility demand on any element, which is of major interest to the designer, can be expected to occur with the ductile response of all elements, when the full base shear capacity, V_E , is developed.

Torsional phenomena will arise whenever stiffness or strength eccentricities with respect to the base shear force, V_{Ey} , acting at the centre of mass, CM, denoted as e_{rx} and e_{vx} , respectively, exist. Both can be readily determined. The definition by eq.(3) of the strength-dependent element stiffness implies that the two types of eccentricities are related to each other. This feature is not recognised in current codified design procedures. The key ingredient of the design strategy, relevant to torsional phenomena, is the ability of the astute designer to assign strengths to lateral force-resisting elements in such a way as to obtain a suitable location for the centre of strength, CV.

In the study of the influence of stiffness (e_{rx}) and strength (e_{vx}) eccentricities, the following four classes of structures are distinguished: (a) $e_{rx} = e_{vx} = 0$, (b) $e_{vx} = 0$ but $e_{rx} \neq 0$, (c) $e_{rx} = 0$ but $e_{vx} \neq 0$, (d) $e_{rx} \neq 0$ and $e_{vx} \neq 0$. Case (a) is the condition for, what is commonly referred to as, a torsionally balanced system. In these torsional

phenomena do not arise, unless rotary motions are introduced at the foundations. Case (d) represents the majority of real structures. If desired, the designer may readily achieve conditions (b) or (c) above.

Mass eccentricity, e_{mx} , with respect to the geometric centre of the plan, may also exist. This, however, will affect only the rotary inertia of the mass, to be taken into account in the evaluation of dynamic response, a subject absent in current design practice and beyond the scope of this presentation.

Degree of Torsional Restraint

It has been suggested (13) that, as part of routine structural design, when unidirectional seismic attack in the principal directions of the building are considered separately, two types of torsional mechanisms should be distinguished. Special features of the behaviour of each of these are briefly reviewed subsequently.

Torsionally unrestrained systems - When only one element, transverse to the direction of the base shear, V_{Ey} , and concurrent with CM, is present, as shown in Fig.5(a), no torque can be introduced to the system after the translatory elements have entered the inelastic domain. This feature is not recognized in existing code provisions (14). Therefore, in terms of rotary motions, the ductile system is unrestrained. Element (3) is effective only in resisting earthquake-induced forces in the x direction.

Figure 5(b) shows a similar example where the sole two-component transverse element is eccentric with respect to CM. In terms of a static base shear, V_{Ey} , this ductile system is also torsionally unrestrained. However, during dynamic response system twist will introduce displacements in the x direction to the transverse components (3) and (4) and to CM. The acceleration of the mass in the x direction will thus introduce an inertia force at CM. An equal and opposite force will then be developed in the transverse element, leading to a dynamically induced torque.

To illustrate relationships between stiffness and strength eccentricities, the simple specific model comprising rectangular cantilever walls, as shown in Figs 5(a) and 6, will be considered. The two elements (1) and (2) are required to sustain a base shear in the y direction, V_{Ey} . Equilibrium criteria suggest that the nominal strength of element (1) should be twice that of element (2). However, this condition will not be achieved in real buildings because compliance with existing code requirements, and inevitable variance of the strengths of the elements, as constructed, will lead to some strength eccentricity. For example the nominal strength of element (2) may be exactly as intended, i.e., $V_{n2} = V_{Ey}/3$. However, the reinforcement of element (1) is such that its nominal strength turned out to be greater than that required, i.e., $V_{n1} = \lambda_1 (2V_{Ey}/3)$, where $\lambda_1 \geq 1.0$. Similarly situations may be considered when inevitably element (2) may have some excess strength, quantified by the parameter λ_2 .

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Figure 6 shows how strength eccentricities, expressed as the ratio e_{vx}/D , vary with increasing excess strength of one or the other of the two elements. With increased values of λ , the total base shear capacity of the system will also increase. This is shown by the dashed straight lines in Fig.6.

In a previous section it was demonstrated by eq.(3), that the stiffness of an element with given sectional dimensions is proportional to its nominal strength. Hence the stiffness eccentricities corresponding with the excess strength of the elements of the model in Fig.5(a) are readily determined. These are shown, again in terms of e_{rx}/D , in Fig.6.

The purpose of presenting this example is to illustrate that:

- Stiffness and strength eccentricities are interdependent.
- For a wide range of the variation of the excess strength of elements, differences between the respective eccentricities, i.e., $|e_{rx} - e_{vx}|$, remain essentially constant for systems with a given geometry.
- The designer, having full control over strength eccentricity, thereby also controls stiffness eccentricity. According to current design practice the latter is only a geometry-dependent property of the system, beyond the control of the designer.
- Classes of systems characterized by the relations of the two types of eccentricities, previously presented as (a) to (d), are readily identified. Because in this example structure (Fig. 5(a)) the geometry of the two elements is different, a torsionally balanced condition can never be achieved.
- In terms of dynamic response, a particularly favourable situation arises in this structure when $\lambda_2 \approx 1.2$, i.e., when, contrary to indications of statics, the nominal strength of element (2) is made 60%, rather than 50%, of that of element (1). In this case CR and CV are approximately equidistant on opposite sides of CM. In attempting to reduce the adverse effects of torsional phenomena, the designer can readily approach this condition.

Torsionally restrained systems - When transverse elements in at least two planes are provided, as in Fig. 4, a torque, for example due to strength eccentricity, can be resisted after all translatory elements have yielded. Thereby rotational deformations of the diaphragm, i.e., twist, are restrained. This is particularly the case when the transverse elements remain elastic while they resist the torque generated. Designers are aware that such systems are preferable. Both models, shown in Fig. 5, are torsionally restrained in terms of a base shear, V_{Ex} .

Specific examples will be used to illustrate how the designer can influence effects of torsional phenomena on inelastic element displacement demands.

To facilitate meaningful comparisons, three alternatives of an example structure are presented. For these, important design quantities, such as eccentricities, total translatory system stiffness, system yield displacement and the attainable maximum

system displacement ductility capacity, are distinctly recorded under each of the relevant figures.

An example based on the geometric stiffness-proportional distribution of element strengths - As an illustrative example, consider the previously studied structure shown in Fig.3(a). However, the walls, instead of being components of an element, will now be used as elements of a system. The corresponding layout of these elements is shown in the plan of Fig.7(a). The conventional assignment of l_{wi}^3 proportional translatory strengths to elements (1) to (4), recorded in Fig.3(b), results in this case in a stiffness eccentricity of $0.188A$, where A is the distance between elements (1) and (4), as shown in Fig. 7(a). Because strengths have been made proportional to geometric stiffness, (EI) , the centres of rigidity, CR , and strength, CV , coincide, as seen in the plan, Fig.7(a). The translatory stiffness and yield displacement of this system, shown in Fig. 7(c), are the same as those of the corresponding element, presented in Fig. 3(a).

In the absence of the (T) type transverse elements, the resulting torque, $M_{et} = 0.188AV_E$, could be resisted only within the elastic domain of the 4 wall elements. Therefore, in routine design, a torsional analysis of the elastic system is used with the aim to eliminate strength eccentricity and hence the need to sustain a static torque. This scenario is of no further interest.

After the translatory elements, (1) to (4), have yielded, the torque resulting from the strength eccentricity, $e_{vx} = 0.188A$, or any other cause, could be resisted only by the transverse elements. It is assumed that the relevant properties of the two identical (T) type transverse elements are: $l_{wt} = 1.73$, hence from eq. (2) $\Delta_{yt} = 1/1.73 = 0.58$ and $V_{nt} = 0.5V_E$. The relative stiffness is thus $k_t = V_{nt}/\Delta_{yt} = 0.5/0.58 = 0.86$ when $V_E = 1.000$. Therefore, the static torque-induced forces in the transverse elements are $V_t = \pm 0.188A/0.5A = 0.376 < 0.5$, i.e., less than the strength of these elements. The corresponding displacements of the (T) elements would be $\Delta_t = V_t/k_t = 0.376/0.86 = 0.44$ displacement units, resulting in an angle of twist of $\theta_t = 2 \times 0.44/0.5A = 1.76/A$. This is shown in the displacement profile within Fig.7(b).

It is assumed again that the ultimate displacement of the critical element (4) is limited by $\mu_{\Delta i} = 5.0$, as in Fig.3(b), to 2.5 displacement units. The corresponding displacement profile is shown in Fig.7(b). The displacement demands associated with the static torque indicate that the system displacement ductility capacity of the system, compared with that in Fig.3(a) ($\mu_{\Delta} = 4.31$) could increase to $\mu_{\Delta} = (2.5 + 0.433A \times 1.76/A)/0.58 = 5.62$, without the displacement ductility limit of 5 being exceeded in any element. This example shows that twisting of the system could in fact better utilize the displacement capacity of elements. Nevertheless, for reasons explained subsequently, this approach is not recommended.

Equilibrium and deformation compatibility requirements, embodied in current codes for elastic systems, would not allow such a distribution of strengths to be

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used. Irrespective of the presence or absence of the (T) type transverse elements, the conventionally defined torsional stiffness would be used to determine the torque-induced forces. These would radically change the strength of elements (1) to (4). Hence, their stiffness would no longer correspond to the true values, defined by eq. (3).

The example presented in Fig. 7 intends to illustrate primarily the ambiguity associated with the requirement that element strengths be made proportional to geometry- based stiffness.

An example based on an arbitrary assignment of element strengths resulting in zero strength eccentricity - Common sense, as well as standard design practice, would suggest that a promising distribution of element strengths should result in little or no strength eccentricity. This could be achieved in a variety of ways.

One arbitrary choice of strength distribution, among the four walls, resulting in zero strength eccentricity, is that shown in Fig.3(c). As the strengths of the four elements are now different, their stiffness have also changed. The increased yield displacement (Fig. 3(c)) and corresponding system stiffness are reproduced in Fig. 8 (c). As the plan in Fig.8(a) shows, this results in a much reduced stiffness eccentricity, $e_{rx} = 0.07A$. This important feature is ignored in current codified procedures. In the presence of the (T) type transverse elements this relatively small stiffness eccentricity, relevant primarily to elastic behaviour, is likely to result in only a small twist of the system. The worst scenario, associated with zero twist, depicted in Figs.3(c) and 8(b), would then limit the system displacement capacity to $\mu_{\Delta} = 3.79$. Some twist that may be introduced due to the stiffness eccentricity, e_{rx} , during the dynamic response of this ductile system, shown by the dashed line profile, is likely to increase its displacement ductility capacity.

An example structure with moderate strength eccentricity - The optimal static strength eccentricity would result in the full utilization of the displacement ductility capacity of at least two of the four elements, seen in Fig.9(a). In the example structure this hypothetical situation would involve simultaneous ductility demands of 5 on elements (3) and (4). This limit is shown by the dashed displacement profile passing through two circled points in Fig.9(b). The extremely large twist, $\theta_{10} = 1.95/A$, would allow in this model structure the largest displacement of CM and hence system displacement ductility capacity of $\mu_{\Delta} = 5.39$ to be mobilized. The (T) type transverse elements would be close to yielding. This hypothetical situation intends to show only the limits of achievable system twist due to a static torque, and its benefits.

A more realistic and promising approach to the assignment of strengths to the four elements in Fig. 9(a) might be one which results in moderate strength eccentricity, such as say $e_{vx} = 0.08A$. It may be shown that the corresponding strength-dependent stiffness eccentricity is $e_{rx} = 0.16A$. This static strength eccentricity would result

in a system twist, controlled by the elastic transverse elements, of only $0.74/A$. The corresponding achievable system displacement ductility would be $\mu_{\Delta} = (2.5 + 0.433A \times 0.74/A)/0.62 = 4.55$.

During the dynamic response of the system, mass rotation may impose additional system twist. However, as the displacement profile in Fig.9(b) shows, this deliberately chosen position of the centre of strength (CV) resulted in an extremely tolerant structure. Any possible increase of torque, generated by mass rotations, increasing the angle of twist by a factor as much as 2.6, will deliver increased energy dissipation of the system without exceeding the specified displacement capacity of any of the elements.

The message of this example is, that the elimination or drastic reduction of eccentricities in systems, comprising elements with different yield displacements, will not necessarily result in improved ductile performance.

The predictable magnitude of twist due to a static strength eccentricity only must, however, be kept to moderate values. This will make allowances for significant variations in additionally imposed twists during the dynamic response of the ductile system. This is the reason why the static torque-generated twist shown in Fig.7(b), although only 90% of the optimum value, should not be relied on.

Directional Effects

In conformance with code [14] requirements, generally only unidirectional seismic attacks, independently in the two principal orthogonal directions of a building, are considered by designers.

An earthquake-generated instantaneous base shear may, however, act in any direction. Researchers prefer to use the term "bi-directional effects". A study of this involves usually the evaluation of simultaneous orthogonal dynamic responses of ductile systems to a selected suit of earthquake records. This technique cannot be considered presently as a viable tool in routine design. The term preferred here [15] is "skew seismic attack". It is postulated that an in-depth study of unidirectional ductile response, independently in the principal orthogonal directions, can usually provide sufficient information with respect to torsional mechanisms and consequent displacement demands under a skew attack. Details are not considered in this paper.

A CRITICAL REVIEW OF EXISTING CODE PROVISIONS

With priority placed on the designers intention to control earthquake-induced displacements in reinforced concrete structural systems, certain conflicts with widely accepted torsional provisions emerge. A brief review of these is presented here.

- (i) Relevant world wide code provisions [14] are based on elastic structural behaviour. Yet the vast majority of buildings in seismic regions is designed for ductile seismic response.
- (ii) Strength assignment to elements is made proportional to traditionally defined, i.e., geometry-based, (EI), element stiffness. The displacements of elements with such predetermined stiffness, are assumed to vary with the actual strength assigned to them. This is then fallaciously taken as the yield displacement of the element on which displacement ductilities are based.
- (iii) It is now widely recognised that a critical issue of the seismic design of ductile systems, is the estimation of element displacements associated with either performance or ultimate limit state criteria. Some codes also impose limits on the twist of elastic systems. Corresponding inelastic deformations are not addressed.
- (iv) Emphasis in codes is placed on providing torsional and enhanced translational strengths, with disregard of the ensuing effects on displacements.
- (v) No reference to the estimation of yield displacement of elements and that of the entire system is made in codes. Thereby the differences in displacement ductility demands on various elements, particularly those on critical edge elements, cannot be evaluated.
- (vi) The fact that stiffness is strength-dependent, is not recognised. Thereby, when element dimensions are set, stiffness eccentricities, based on traditional strength-independent definition of element stiffness, are fixed. Subsequent changes of element strengths, often radical, may significantly affect stiffness eccentricity. This remains unrecognised.
- (vii) Computed stiffness eccentricities are manipulated, using additional nominal eccentricities, in order to arrive at increased strength demands on elements, considered to be adversely affected. Strength eccentricities, which are affected by such manipulations, are not considered.
- (viii) No attention is given to the facts that the astute designer can control strength eccentricity, and that stiffness eccentricity is a simple and quantifiable function of the strength eccentricity.

These comments should not be construed as a critique addressed to those researchers who, over several decades of dedicated work, developed techniques meeting the envisaged needs in seismic design. Some of the manipulations with nominal eccentricities, extensively covered in the relevant literature, were intended,

by means of parametric studies, to cater for inelastic displacement demands. Unfortunately these intentions are not transparent in current design provisions. Existing code recommendations grew out of the consideration of elastic behaviour only. As such they were widely endorsed by the design profession, including the author (5). The comments, and indeed the major part of this paper, address issues which emerged from recently identified fallacies in seismic design (6), and from corresponding promising exploration of seismic design strategies (7,8,13).

CONCLUSIONS

A definition of component yield displacement enables the properties of appropriate bi-linear approximations of force-displacement relationships for reinforced concrete components, to be defined. These lead to a redefinition of component stiffness.

Within rational limits, the nominal strengths of components, satisfying the strength requirements for the relevant element, may be freely chosen by the designer.

The summation of the stiffness and nominal strengths of components of an element is used to define its reference or nominal yield displacement. A similar summation of element properties enables to nominal yield displacement of the system, at the centre of mass, to be defined. These nominal yield displacements serve to establish displacement ductility relationships between components, elements and the system.

The order of twist resulting from static strength eccentricity and the likely effects of the rotary inertia of the mass during dynamic response, enables displacements of critical elements at the ultimate limit state to be gauged. In general, critical elements are those having the smallest yield displacement.

Because stiffness is postulated to be strength-dependent, stiffness eccentricities are related to strength eccentricities. The latter, however, are under the control of the designer.

The displacement at the appropriate limit state of the centre of mass may be based on a displacement profile of the system, controlled by the acceptable displacement of the critical element and the expected system twist. This in turn will define the displacement ductility capacity of the system, generally used in force-based seismic design strategies to estimate the required base shear strength.

The elimination of eccentricities does not necessarily result in more efficient system response.

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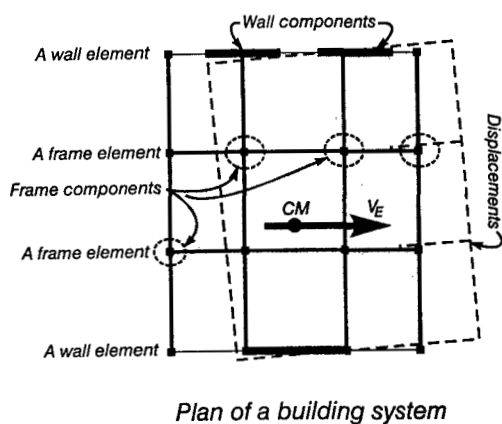


Fig.1- Components and elements of a ductile system

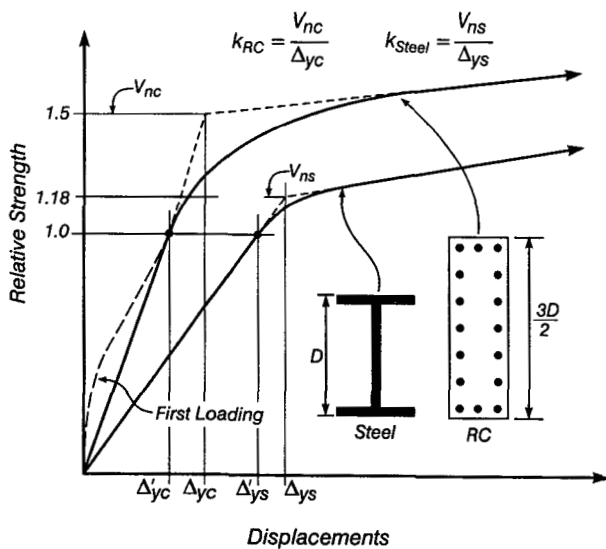


Fig.2- Bi-linear idealization of ductile component behaviour

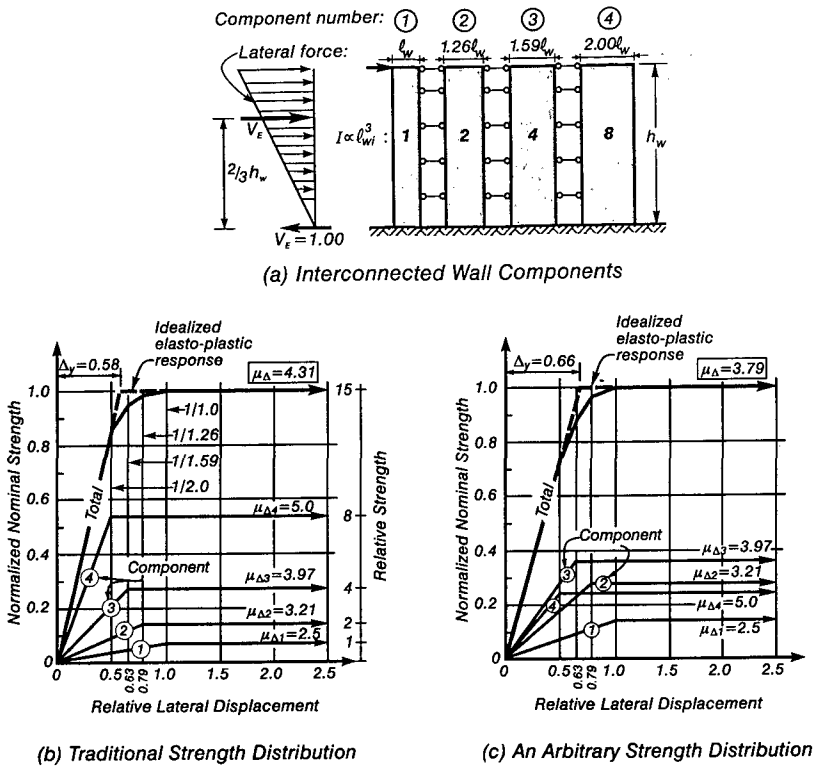


Fig.3- Idealized force-displacement relationships for an element with wall components

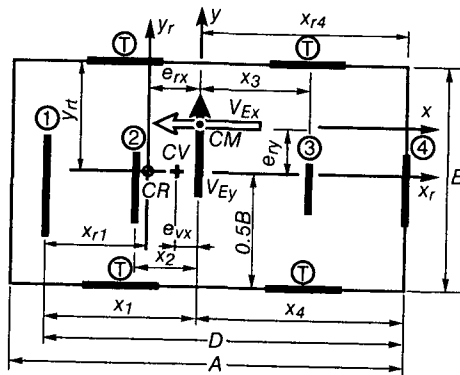


Fig.4- Eccentricities arising in system with wall elements

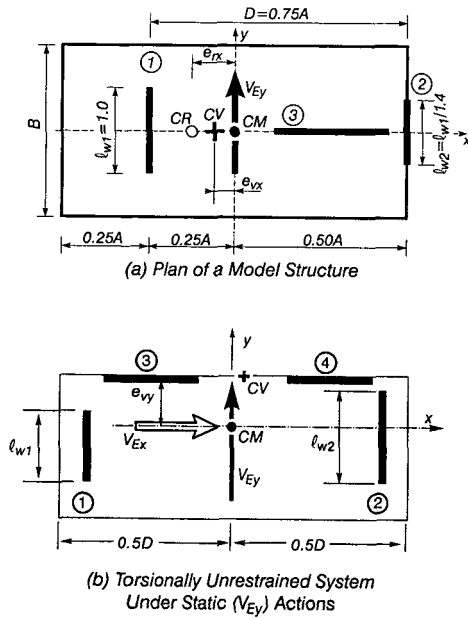


Fig.5- Torsionally unrestrained systems

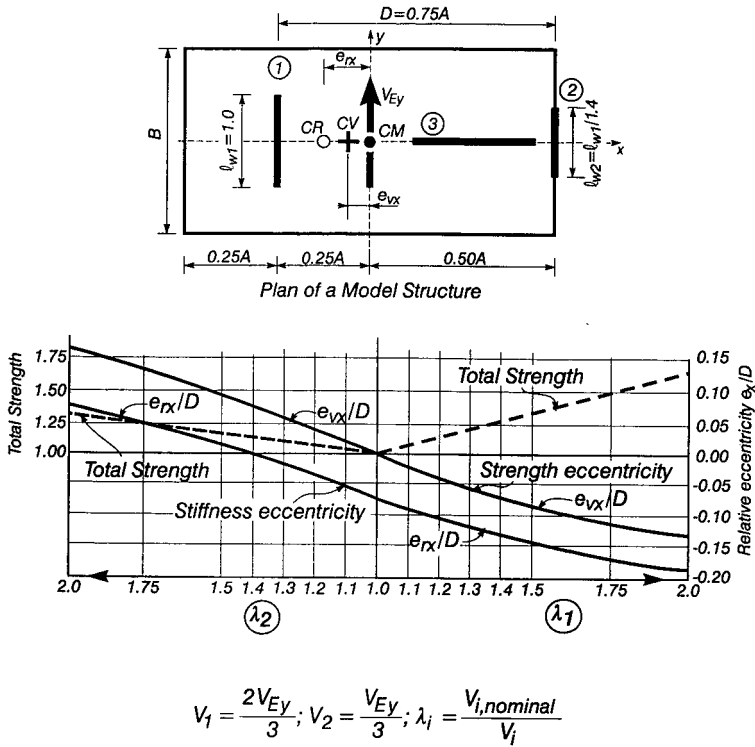


Fig-6- Eccentricities in a two-element torsionally unrestrained system

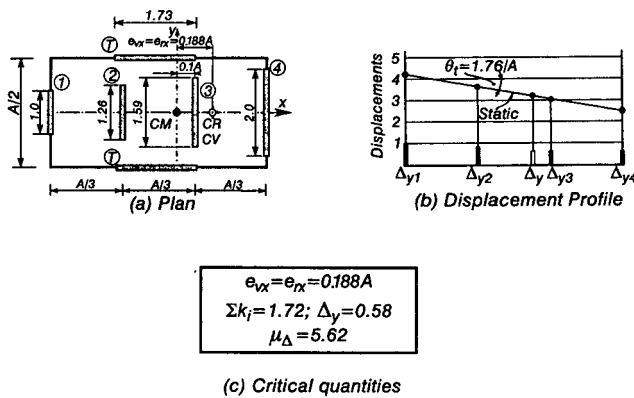


Fig.7- Effects of stiffness-proportional distribution of element strength

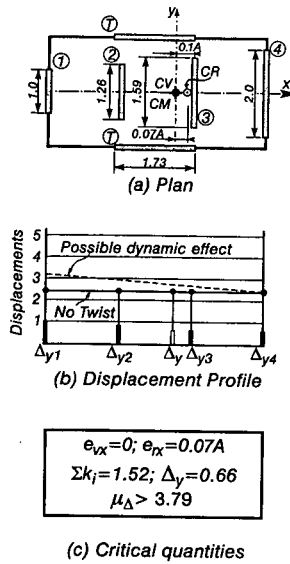


Fig.8- Effects of zero strength eccentricity

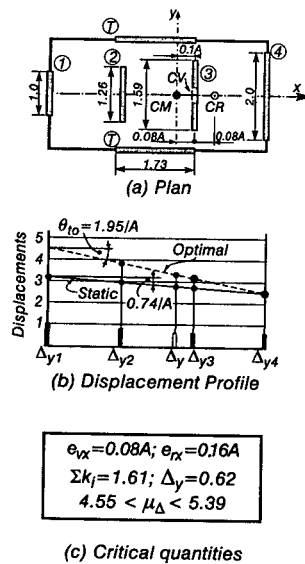


Fig.9- A system with moderate strength eccentricity

Remembering the Column Analogy

by M. A. Sozen

Synopsis:

To my friend and hero Professor Şükrü Muvaffak Üzümeri, a bouquet of thoughts and an invention of another hero, Professor Hardy Cross.

Keywords: column analogy; Hardy Cross; theory

Mete A. Sözen, Kettelhut Distinguished Professor of Structural Engineering at Purdue University, specializes in teaching and research related to concrete building and massive structures subjected to static and dynamic loads. Currently he chairs the National Research Council Committee for Oversight and Assessment of Blast-effects and Related Research and the National Science Foundation U.S.-Japan Technical Coordination Committee for Cooperative Research in Urban Earthquake Disaster Mitigation.

It has been said that it took an age to understand Aristotle and another age to forget him. Hardy Cross may have been forgotten even before he was understood. If his name is mentioned at all today, he is remembered through his contribution to the solution of bending moments in structural frames and flow in networks. Sometimes he is even dismissed as a sleepwalker who stumbled on the relaxation method for solution of linear simultaneous equations. Actually, his close friends and students have said that Cross tended to advise strenuously against the use of the moment distribution method because he considered it to be too exact for inexact structures. Cross's influence on the profession of structural engineering is indelible and awesome. At the same time, it is subtle and easy to overlook. To get a flavor of his approach, consider his class notes for a course on indeterminate structures. Before he goes into explanations, he questions:

What is theory? It is perhaps worthwhile to call attention to the double use of the word "theory" in scientific discussions. In some cases it is used to mean a body or group of facts the truth of which is not questioned, in others it means a hypothesis which has strong evidence in its favor though its truth is still open to some question. Thus the theory of elasticity is a group of geometrical relations which are not open to debate, but the idea that time yield of the concrete will delay failure from temperature stresses in a concrete arch is a theory in quite a different sense. Other debatable points in indeterminate structures are not theories at all, but merely convenient assumptions: thus no one holds any theory that the modulus of elasticity is constant throughout an arch ring, the only question being whether such variations as do occur produce an important effect on the results.

Theory, to Cross, is the axiom. He does not think that plane geometry needs to be proven for plane continua. But the student is cautioned against mixing "theory" and "theory in a quite different sense." All that is based on the observed is refutable. He expands on it.

Much confusion of thought has come from misuse of this term [theory]. We may further cite: as groups of facts not open to experimentation or debate, the theory of the elastic arch, the theory of continuous girders, the theory of deflection; as hypotheses strongly supported but as yet not fully proved. The theories of fatigue failure, the theory of earth pressure, the theory that the strength of concrete in a structure is the same as that shown by a cylinder in a testing machine or that rate of application of load is a negligible factor in producing failure, and finally, as misuses of the word, the “theory” that the moment of inertia of a concrete beam varies as bd^3 , that the tension rods do not slip in concrete beams, that there is no distortion due to shear. The first group of “theories” is not debatable, the second depend usually on experimental verification, while in the case of the third the important question is how significant is the error. The data often needed in the third group are elementary: when these are available, deductive processes furnish a definite answer as to the importance of the error.

Taxonomy of the conceptual models was not his sole concern. His preoccupation was with engineering thinking and design in general. He never expressed it that way, but his constant quest was to determine whether or in which case an exact analysis of an approximate model was an approximate analysis of the exact model.*

He sought simplicity: “The analysis of a structure for continuity should be less complicated than the determination of anchorage and stirrup spacing....”** It is ironical that his wish came true, not because continuity analysis was simplified but because the determination of anchorage and stirrup spacing was made more difficult by illuminati who preferred the rigidity of rules to the flexibility of principles.

He revered statics: No indeterminate analysis – no structural analysis of any kind – is complete until the computer has satisfied himself

- (1) that the forces balance, at least within the accuracy of the computation used.
- (2) that he has not overlooked any forces.***

In our time, this wish was also fulfilled with the exception that tragically “himself” became “itself” and “he” became “it.” The following paraphrase from an announcement by an institution that prides itself on being at the cutting edge of

* All analyses are based on some assumptions which are not quite in accordance with the facts. From this, however, it does not follow that the conclusions of the analysis are not very close to the facts. (from Ref. 2, p. 3)

** Ref. 2, p. 2.

*** Ref. 2, p. 3

knowledge captures the intellectual fashion: “The advanced experimental capabilities will enable us to test and validate more complex and comprehensive analytical and computer numerical models to improve design and performance.” In the complex and comprehensive environment envisioned, will the computer (it, he, or she) check simple equilibrium? Fat chance!

Traduttori traditori. It is unfair to Cross to pretend to synthesize his view with a few quotations misplaced in time. The reader is urged to read references 1 through 4. If he/she has already done so, he/she is urged to return to them. They will give him/her different insights, always valuable, at different times. In the text below, a conceptual invention of his is discussed primarily to illustrate Cross’s creativity. How he arrived at his moment-distribution method can be understood, if with difficulty, in terms of deformations and the stiffness method. But his “Column Analogy” can only be classed as an artistic leap of imagination.

The Column Analogy

It is very interesting that Professor Cross started his lectures (Ref. 1) on indeterminate structures by referring to “three easily established principles:”

1. Column Analogy
2. Distribution of Moment
3. Virtual Work

Of the three principles he emphasized, the moment distribution survives, sometimes for the wrong reasons. Virtual work, being a theory and not developed but elegantly defined by Cross, has been a perennial. But the column analogy has been lost. It deserves recycling. The column analogy is essentially a theorem for finding indeterminate moments in a one-span restrained beam. What is important and useful about it is that it applies to straight and curved beams. It can be a very useful tool for determining flexural stiffness properties of nonprismatic beams.

To appreciate Cross’s leap of imagination, let us examine the simplest column-analogy application.

Consider a prismatic beam with fixed ends over a span L . It is loaded at mid-span by a concentrated load P . What are the restraining moments at the ends?

To solve the problem, Cross takes us to an imagined world. In that world, the beam is represented by a section (section of an imagined or analogous column) with depth L and thickness I/EI where E is the Young’s modulus for the material of the beam and I is its moment of inertia. This imaginary section responds linearly to an imagined load represented by the angle-change diagram, M/EI , distributed over the section just as the moment, M , is distributed over the span of a simply-supported beam.

The unit stresses at the ends, in the imagined world, are the moments sought:

$$M_{end} = \frac{P_{analog}}{A_{analog}} = \frac{PL^2}{8EI} * \frac{1}{L} * \frac{EI}{1} = \frac{PL}{8}$$

P_{analog} : Total Load on the analogous section on $(1/2) (PL/4) (L)$

A_{analog} : Area of analogous section or L/EI

If the load on the beam is uniform,

$$M_{end} = \frac{2}{3} \frac{wL^3}{8EI} * \frac{EI}{L} = \frac{wL^2}{12}$$

The operation is so simple that the correctness of the results appears to be coincidence, but it is not. Plates I through V present examples.

Plate I contains the column-analogy solution for a concentrated load at any distance αL from one end of a prismatic beam with fixed ends. The extension of this model to a similar beam with uniform load is described in Plate II. It is to be noted that the stresses in the imagined world are calculated from the familiar expression

$$\alpha = \frac{P}{A} \pm \frac{Mc}{I}$$

hence the term “column analogy.” The column analogy has its best use in determining fixed-end moments and stiffnesses for nonprismatic beams. An application is demonstrated in Plate III.

The solution, given in Plate IV, for a concentrated load on a prismatic beam with one end fixed and the other free, takes us down the rabbit-hole to the Queen of Hearts. Cross wants us to imagine a section which has an infinite width over an infinitesimal length. The only thing that the observer can say is that she has seen the unimaginable and it works. The application is also a witness to Cross’s rare ability to associate images.

The example in Plate V is simply to show an application involving a frame. Though it is simple, its use cannot be recommended vis a vis other current methods. But it does demonstrate that Cross’s Analogy can be used for “one cell” frames, arches, and curved beams.

Concluding Remarks

Cross's written works are replete with jewels of thought. Some, contained in references 1 and 2, are reasonably well known. But it is not a waste of print to revisit his judgment about the results of analysis rendered in relation to stresses computed in arches in reference 3 (paper #8).

The investigations here recorded indicate:

- (1) that for a large part (over one-half) of the stresses in an arch there can be practically no uncertainty arising from assumptions involved in the method of analysis used.
- (2) that for the flexural stresses due to live load the true stresses cannot be predicted with absolute precision, because the stresses are a matter of chance.
- (3) that the departure of the stresses existing in any arch from the values given by the usual methods of analysis can scarcely be greater than the variations in the quality of the concrete, and will most probably be very much less.

Beyond this it does not seem wise or profitable to draw conclusions, though others are apparently indicated by the data. The important fact is that any wide departure from the predicted values of the moments and thrusts in a concrete arch is not possible unless the variation in the properties of concrete is much greater than is commonly supposed. Within a narrow zone of uncertainty, then, the maximum moments and thrusts due to loads in a concrete arch are given without possible question by the "geometrical" (elastic) analysis. The zone of such uncertainty seems to have a width of about ± 10 per cent; the zone of probable uncertainty seems to have a width of about ± 5 per cent. The terms "true value" and "real value," however, are meaningless except as applied to a given arch under a given condition of loading and a given atmospheric condition; otherwise the reactions are a matter of chance.

The geometrical theory of analysis for arch reactions appears more dependable than the theory of flexure used to compute the fiber stresses produced by these reactions, and much more dependable than the concrete itself. It is not exact or precise, but it is a safe and convenient guide in design.

Socrates is supposed to have told Antisthenes the Cynic, "Your pride shows through the holes of your rags." Is it intellectual arrogance, confidence, sensibility, or humility to have spent years in building, analyzing, testing concrete arches and then to arrive at such a modest set of conclusions? Whichever it is, Cross ought to be a model to current and future writers.

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HARDY CROSS

Brief Professional Record

1902	BA	Hampden-Sydney College, Virginia
1903	BS	Hampden-Sydney College, Virginia
1908	BS in CE	Massachusetts Institute of Technology, Cambridge
1911	MS in CE	Harvard University, Cambridge
1908-1910	Asst. Eng.,	Bridge Department, Missouri-Pacific Railway
1912-1918	Asst. Prof. of Civil Eng.,	Brown University
1918-1921	Practice in	New York City and Boston
1921-1937	Professor of Structural Engineering,	Univ. of Illinois, Urbana
1938-1951	Head, Department of Civil Eng.,	Yale University

List of Publications (1936)

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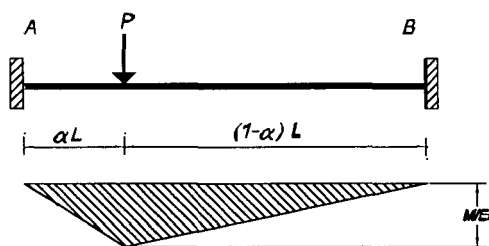
PRISMATIC BEAM WITH CONCENTRATED LOAD AND FIXED ENDS

The operation of determining the fixed-end moments in a prismatic beam subjected to a concentrated load is simple. It involves three steps.

- Step 1. Determine the "load" on the analogous column.
- Step 2. Determine the section properties of the analogous column.
- Step 3. Determine the unit normal stresses in the analogous column.

Step 1: Load and Moment Acting on Section of Analogous Column

The entity defined by the area of the M/EI diagram for a simply supported beam is P_{analog} , the "load" acting on the section of the analogous column.



$$P_{analog} = \frac{P \cdot L^2}{2EI} \cdot \alpha \cdot (1 - \alpha)$$

The centroid of the load represented by the M/EI diagram is at

$$x = \frac{L}{3} \cdot (1 + \alpha)$$

from end A . The counterclockwise moment acting on the analogous column is

$$M_{analog} = P_{analog} \cdot \left(\frac{L}{2} - x \right)$$

PLATE I

substituting for P_{analog}

$$M_{analog} = \frac{1}{12} \cdot \frac{P \cdot L^3}{E \cdot I} \cdot \alpha \cdot (1 - \alpha) \cdot (1 - 2 \cdot \alpha)$$

Step 2: Section Properties of Analogous Column

$$A_{analog} = \frac{1}{E \cdot I} \cdot L$$

$$I_{analog} = \frac{1}{12} \cdot \frac{1}{E \cdot I} \cdot L^3$$

Step 3: Normal Stresses on Section of Analogous Column

The normal unit stress σ_A at end A for the load on the analogous column section corresponds to the moment at in a beam with a concentrated load at αL from end A.

$$\sigma_A = \frac{P_{analog}}{A_{analog}} + \frac{M_{analog} \cdot c}{I_{analog}}$$

where $c=L/2$.

Substituting for P_{analog} , M_{analog} , A_{analog} , I_{analog} and c ,

$$\sigma_A = P \cdot L \cdot \alpha \cdot (\alpha - 1)^2$$

To obtain a solution for the variatio of th coefficients for end moments at A and B , we set

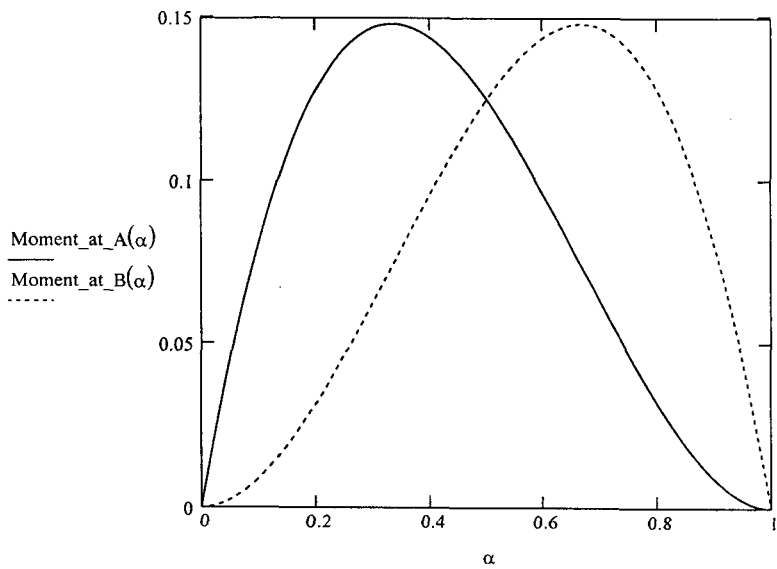
$$P := 1 \quad L := 1$$

$$\text{Moment_at_A}(\alpha) := P \cdot L \cdot \alpha \cdot (1 - \alpha)^2$$

Similarly,

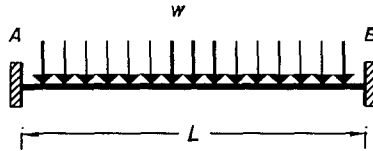
$$\text{Moment_at_B}(\alpha) := P \cdot L \cdot \alpha^2 \cdot (1 - \alpha)$$

PLATE I



Variation of Fixed-End Moment, Coefficient of PL, with α

FIXED-END MOMENT FOR UNIFORMLY LOADED PRISMATIC BEAM



Start with end moment at A for concentrated load P at distance αL from end A

$$\frac{M_A}{PL} = \alpha \cdot (\alpha - 1)^2$$

M_A = Moment at end A for a concentrated load at αL from end A

P = Concentrated load at αL from end A

L = Beam span

α = Ratio of distance to concentrated load from end A to beam span

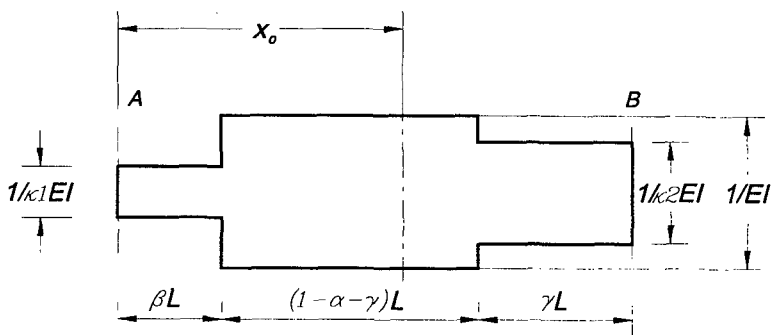
Assume unit load $w = P/dL$ and integrate over beam span

$$M_A = w \cdot L^2 \int_0^1 \alpha \cdot (\alpha - 1)^2 d\alpha$$

$$M_A = \frac{1}{12} \cdot w \cdot L^2$$

FIXED -END MOMENTS IN A NONPRISMATIC BEAM
Three Segments with Different EI's

Centroid of Analogous Column Section



Cross Section of Analogous Column

Definitions and Default Values

Young's Modulus	$E := 1$
Standard Beam Moment of Inertia (Segment 2)	$I := 1$
Beam Span	$L := 1$
First Segment, Ratio of Length of Segment to Beam Span	$\beta := 0.2$
Third Segment, Ratio of Length of Segment to Beam Span	$\gamma := 0.3$
First Segment, Ratio of Moment of Inertia to That of Second Segment	$\kappa_1 := 4$
Third Segment, Ratio of Moment of Inertia to That of Second Segment	$\kappa_2 := 2$
Unit Load	$w := 1$

PLATE III

Determination of Centroidal Distance from End A

Segment β

Area

$$A_{\beta} := \frac{1}{\kappa_1 \cdot E \cdot I} \cdot \beta \cdot L$$

First Moment about End A

$$M_{\beta} := A_{\beta} \cdot \frac{\beta}{2} \cdot L$$

Segment Standard

Area

$$A_s := \frac{1}{1 \cdot E \cdot I} \cdot (1 - \beta - \gamma) \cdot L$$

First Moment about End A

$$M_s := A_s \left[(\beta) + \left(\frac{1 - \beta - \gamma}{2} \right) \right] \cdot L$$

Segment γ

Area

$$A_{\gamma} := \frac{1}{\kappa_2 \cdot E \cdot I} \cdot \gamma \cdot L$$

First Moment about End A

$$M_{\gamma} := A_{\gamma} \cdot \left(1 - \frac{\gamma}{2} \right) \cdot L$$

Centroid

$$x_o := \frac{(M_{\beta} + M_s + M_{\gamma})}{A_{\beta} + A_s + A_{\gamma}}$$

$$x_o = 0.51$$

Determination of Moment of Inertia**Segment 1**

$$I_{\beta 1} := \frac{1}{12} \cdot \frac{1}{\kappa 1 \cdot E \cdot I} \cdot (\beta \cdot L)^3 \quad I_{\beta 1} = 1.67 \times 10^{-4}$$

$$I_{\beta 2} := A_{\beta} \cdot \left(\frac{\beta \cdot L}{2} - x_0 \right)^2 \quad I_{\beta 2} = 8.43 \times 10^{-3}$$

Segment 2

$$I_{s1} := \frac{1}{12} \cdot \frac{1}{1 \cdot E \cdot I} \cdot [(1 - \beta - \gamma) \cdot L]^3 \quad I_{s1} = 1.04 \times 10^{-2}$$

$$I_{s2} := A_s \cdot \left[\beta \cdot L + \frac{(1 - \beta - \gamma) \cdot L}{2} - x_0 \right]^2 \quad I_{s2} = 1.84 \times 10^{-3}$$

Segment 3

$$I_{\gamma 1} := \frac{1}{12} \cdot \frac{1}{\kappa 2 \cdot E \cdot I} \cdot (\gamma \cdot L)^3 \quad I_{\gamma 1} = 1.12 \times 10^{-3}$$

$$I_{\gamma 2} := A_{\gamma} \cdot \left[\left(1 - \frac{\gamma}{2} \right) \cdot L - x_0 \right]^2 \quad I_{\gamma 2} = 1.73 \times 10^{-2}$$

Moment of Inertia and Area for Analogous Column Section

$$I_{\text{analog}} := I_{\beta 1} + I_{\beta 2} + I_{s1} + I_{s2} + I_{\gamma 1} + I_{\gamma 2}$$

$$I_{\text{analog}} = 0.039 \quad 1/EI \quad x_0 = 0.51$$

$$A_{\text{analog}} := A_{\beta} + A_s + A_{\gamma}$$

$$A_{\text{analog}} = 0.7 \quad 1/EI$$

PLATE III

"Load" on Analogous Column Section

$$P_a := \frac{1}{2\kappa l} \left[\int_0^\beta (\alpha - \alpha^2) d\alpha \right] + \frac{1}{2} \left[\int_\beta^{1-\gamma} (\alpha - \alpha^2) d\alpha \right] + \frac{1}{2\kappa 2} \left[\int_{1-\gamma}^1 (\alpha - \alpha^2) d\alpha \right] \quad wL/EI$$

Moment about End A on Analogous Column Section

$$M_A := \frac{1}{2\kappa l} \cdot \int_0^\beta \alpha \cdot (\alpha - \alpha^2) d\alpha + \frac{1}{2} \left[\int_\beta^{1-\gamma} \alpha \cdot (\alpha - \alpha^2) d\alpha \right] + \frac{1}{2\kappa 2} \left[\int_{1-\gamma}^1 \alpha \cdot (\alpha - \alpha^2) d\alpha \right] \quad wL^2/EI$$

Clockwise Moment about Section Centroid

$$M_a := M_A - P_a \cdot x_o$$

Fixed-End Moment at A

$$M_A := \frac{P_a}{A_{\text{analog}}} - \frac{M_a \cdot \frac{L}{2}}{I_{\text{analog}}}$$

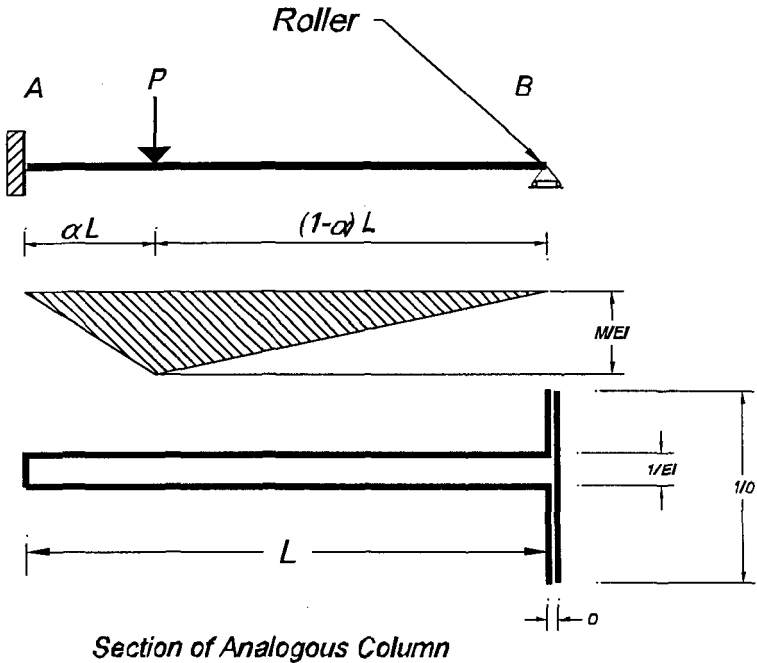
Fixed-End Moment at B

$$M_B := \frac{P_a}{A_{\text{analog}}} + \frac{M_a \cdot \frac{L}{2}}{I_{\text{analog}}}$$

$$M_A = 0.111 \quad wL^2$$

$$M_B = 0.083 \quad wL^2$$

PRISMATIC BEAM WITH FIXED AND SIMPLE SUPPORTS



The area of the M/EI diagram for a simply supported beam is P_{analog} , the "load" acting on the section of the analogous column.

$$P_{analog} = \frac{P \cdot L^2}{2E \cdot I} \cdot \alpha \cdot (1 - \alpha)$$

The centroid of the load represented by the M/EI diagram is at

$$x = \frac{L}{3} \cdot (1 + \alpha)$$

from end A . The counterclockwise moment acting on the analogous column is

PLATE IV

$$M_{\text{analog}} = P_{\text{analog}} \cdot (L - x)$$

substituting for P_{analog}

$$M_{\text{analog}} = \frac{1}{6} \cdot \frac{PL^3}{E \cdot I} \cdot \alpha \cdot (1 - \alpha) \cdot (2 - \alpha)$$

Step 2: Section Properties of Analogous Column

$$A_{\text{analog}} = \infty$$

Because the centroid of the analogous-column section is at B

$$I_{\text{analog}} = \frac{1}{3} \cdot \frac{1}{E \cdot I} \cdot L^3$$

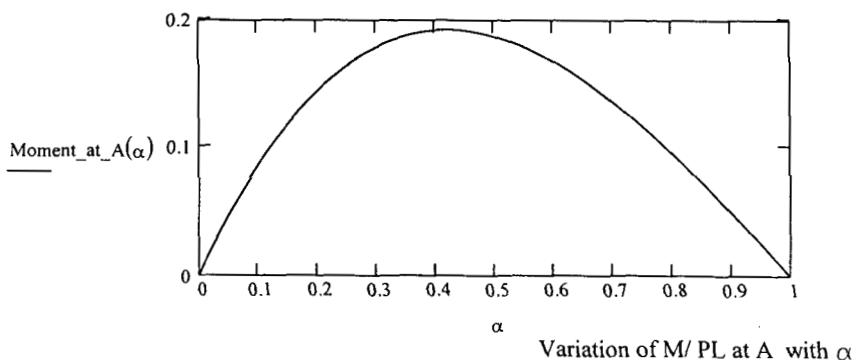
Step 3: Normal Stresses on Section of Analogous Column

Because $A_{\text{analog}} := \infty$ the normal-stress term disappears,

$$\sigma_A = \frac{M_{\text{analog}} \cdot L}{I_{\text{analog}}} \quad \sigma_A = \frac{1}{2} \cdot P \cdot L \cdot \alpha \cdot (1 - \alpha) \cdot (2 - \alpha)$$

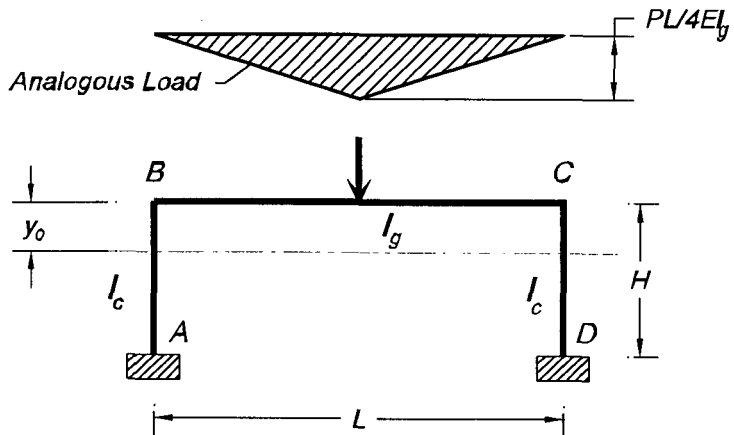
for $P := 1 \quad L := 1$

$$\text{Moment_at_A}(\alpha) := \frac{1}{2} \cdot P \cdot L \cdot \alpha \cdot (1 - \alpha) \cdot (2 - \alpha)$$



FRAME WITH CONCENTRATED LOAD

Prismatic Columns and Girder



Definitions and Default Values

Young's Modulus	$E := 1$
Moment of Inertia of Columns	$I_c := 1$
Moment of Inertia of Girder	$I_g := 1$
Span	$L := 2$
Height	$H := 1$
Load Applied at Center of Girder Span	$P := 1$

PLATE V

Properties of Analogous Section

Centroidal Distance from BC

$$y_o := \frac{2 \cdot \frac{H}{E \cdot I_c} \cdot \frac{H}{2}}{2 \cdot \frac{H}{E \cdot I_c} + \frac{L}{E \cdot I_g}} \quad y_o = 0.25$$

Moment of Inertia of Analogous Column Section

$$I_{\text{analog}} := 2 \cdot \frac{1}{12 \cdot E \cdot I_c} \cdot H^3 + 2 \cdot \frac{H}{E \cdot I_c} \cdot \left(\frac{H}{2} - y_o \right)^2 + \frac{L}{E \cdot I_g} \cdot y_o^2 \quad I_{\text{analog}} = 0.42$$

Cross-Sectional Area of Analogous Column

$$A_{\text{analog}} := \frac{2 \cdot H}{E \cdot I_c} + \frac{L}{E \cdot I_g} \quad A_{\text{analog}} = 4$$

"Load" on Analogous Section

$$P_{\text{analog}} := \frac{1}{8} \cdot \frac{P \cdot L}{E \cdot I_g} \quad P_{\text{analog}} = 0.25$$

$$M_{\text{analog}} := \frac{1}{8} \cdot \frac{P \cdot L}{E \cdot I_g} \cdot y_o \quad M_{\text{analog}} = 0.063$$

$$\text{Moment at B} \quad \sigma_B := \frac{P_{\text{analog}}}{A_{\text{analog}}} + \frac{M_{\text{analog}} \cdot y_o}{I_{\text{analog}}} \quad \sigma_B = 0.1 \quad wL^2$$

$$\text{Moment at A} \quad \sigma_A := \frac{P_{\text{analog}}}{A_{\text{analog}}} + \frac{M_{\text{analog}} \cdot (y_o - H)}{I_{\text{analog}}} \quad \sigma_A = -0.05 \quad wL^2$$

Shear Strength of Circular Reinforced Concrete Columns

by M. P. Collins, E. C. Bentz, and Y. J. Kim

Synopsis:

Considering the very large number of circular concrete columns used to support buildings and bridges and the critical importance of ensuring that the shear strength of these members is sufficient to survive a possible earthquake, relatively few studies have been conducted on the shear strength of circular reinforced concrete columns. This paper summarizes the results of three experimental investigations in which a total of 15 large circular specimens were tested in shear. The paper also explains how analytical models based on the modified compression field theory can be used to predict the shear response of circular reinforced concrete columns.

Keywords: circular columns; ductility; high strength reinforcement; seismic resistance; shear strength