

PAPER NO. 1 Elements of a theory for the mechanical behavior of concrete are developed. A comparison of the stress-strain curves of sandstone, hardened cement paste, mortar and concrete suggests that the heterogeneity introduced by the aggregates is responsible for the inelastic behavior of concrete. A study of the mortar-aggregate interface behavior indicates that under compressive loading this interface is first to fail and that the cracked interface behaves essentially plastically even though the constituent materials are brittle. This pseudo-plastic behavior of the interface results in a gradually curving stress-strain diagram. An analytic model, based only on observed micro-behavior, measured materials properties, and a plausible statistical distribution of mortar strengths, permits a stress-strain curve to be calculated. This curve is strikingly similar to that of concrete, including a descending branch.

Inelastic Behavior and Fracture of Concrete*

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■ THIS STUDY PROVIDES INFORMATION on the nature of inelastic behavior of concrete by examining its micro-behavior under short-time loading. Based on this micro-behavior and on statistical concepts, the study attempts to develop elements of a unified theory for the response of concrete from zero load through the elastic, inelastic and descending ranges of the stress-strain curve to fracture.

ORIGIN OF INELASTIC DEFORMATIONS

Inelastic behavior of concrete is here defined as that behavior in which response of concrete to external loading is non-linear and irre-

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versible. Only the short-time loading in which the deformations due to creep are small compared to the instantaneous deformations caused by loading is considered.

To understand the response of concrete when it is loaded, it is desirable to study the behavior of each of its components. In Figure 1-1 are shown typical stress-strain relations of hardened cement paste, stone, mortar and concrete subjected to uniform compression up to failure of each material. Identical procedures were used for mixing and curing the paste, mortar and concrete specimens, and the methods of testing were the same for all four materials. The curves for paste, mortar and concrete, each represent the average of 8 dial gage readings, two on every specimen for four specimens each. These were prismatic specimens with flared ends and the strains were measured on a $5\frac{1}{2}$ in. length. The curve for stone is the average of 6 one-in. SR-4 gages, two on each of three prismatic specimens. The rate of loading was approximately the same for all four materials. Figure 1-1(a) shows:

1. Considering the two basic constituents of concrete, it is seen that the stress strain curve for stone is linear up to fracture, and that for hardened cement paste is nearly linear.

2. For mortar, the stress-strain relation is definitely curved, much more so than those of the two constituents.

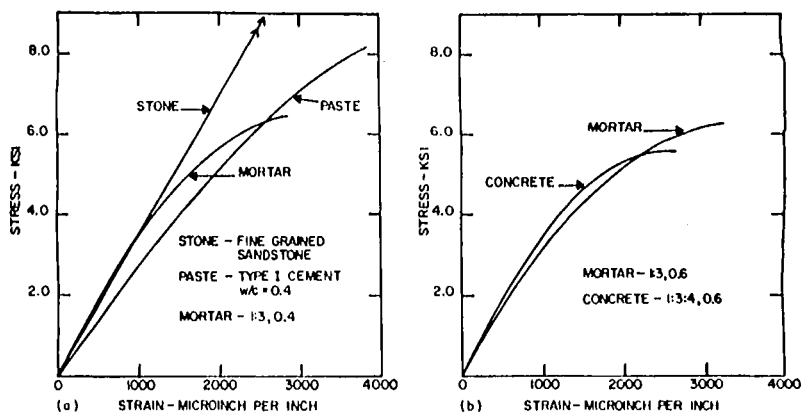


Figure 1-1 Compression stress-strain diagrams for stone, paste, mortar and concrete

3. For concrete, in turn, the diagram is more strongly curved than that for mortar.

4. The compressive strength of mortar is significantly less than that of paste.

5. The compressive strength of concrete in turn is less than that of mortar.

Note that paste and mortar of the same w/c -ratio are compared and, in turn, mortar and concrete of the same w/c -ratio are used.

These observations suggest that the inelasticity in concrete is caused by the introduction of aggregates and by the attendant heterogeneity.

The fact that the stress-strain curve of paste is almost linear has also been observed by Ruetz¹ and Gilkey.² Ruetz found that the stress-strain curves for the paste of water-cement ratios of 0.30 and 0.65 are very nearly straight lines. Gilkey plotted the stress-strain curves of concrete, mortar and paste in non-dimensional form and observed that the curve for the paste is nearly linear while those for mortar and concrete are successively more curvilinear.

The relation between inelasticity and the presence of aggregates in concrete is also indicated by Kaplan's observation that the higher the percentage of aggregates (with the same w/c ratio), the lower the strain at which the stress-strain curve deviates from a straight line.³

Since mortar is weaker than either stone or cement paste alone, and since concrete is weaker than mortar, the bond between cement paste and aggregate is likely to be a weak link in the heterogeneous concrete system. Consequently, one might suspect that the presence of aggre-

gates produces inelasticity in concrete as a result of relative weakness of the interfaces between paste or mortar and aggregates. This idea seems to be supported by the microcrack investigations at Cornell University⁴ and later at the Cement and Concrete Association, London,⁵ by the work of Jones,⁶ and Dantu.⁷

The microcrack investigation at Cornell showed that cracks between the aggregate and mortar exist before any load is applied (see also Ref. 5). These interfacial cracks, which are called bond cracks, began to increase noticeably at about 30 to 50 percent of ultimate load and were predominant throughout the entire loading. By studying the ultrasonic-pulse velocity, Jones found that "the onset of cracking arises mainly from spread of microcracks along the boundaries of the coarse aggregate. . . ." Dantu used photo-elastic coatings on the surface of the compression specimens and found that the strain at the mortar-aggregate interface was 4 to 6 times the average strain. All this evidence strongly suggests that the bond between mortar and aggregate is a weak link in the heterogeneous concrete system, and that this interfacial bond is mainly responsible for the inelastic behavior of concrete under short-time loading.

STRESS-STRAIN BEHAVIOR OF THE INTERFACE

Since the behavior of concrete under an external short-time load seems to be significantly influenced by the interfacial bond, the authors decided to study the stress-strain behavior of the interface between mortar and aggregate. In actual concrete, there are innumerable interfaces. To study what happens at the interface by directly loading concrete would be very difficult, but the interface can be isolated and studied by itself.

Experimental Investigation

The behavior of the interface was examined by testing specimens as shown in Figure 1-2. They consist of an inclined stone slab, $\frac{1}{2}$ in. thick, about $3\frac{1}{2}$ in. long and 2 in. wide, and the mortar surrounding the stone. The stone was fine grained sandstone with a compressive strength of 27,000 psi. The mortar had a sand/cement ratio of 3.0 and a water/cement ratio of 0.5. The cement was portland, Type III.

Three different inclinations of the stone, ϕ , were selected, 23, 42 and 50 deg. There was a minimum of three specimens for each angle.

Structurally speaking, the specimen can be divided into zones. The center zone contains the stone slab and represents the influence of the interface. The exterior zones of homogeneous mortar simulate the effect of confinement to which an interface is subjected in the concrete mass. The specimens were loaded to failure and the strains and loads were

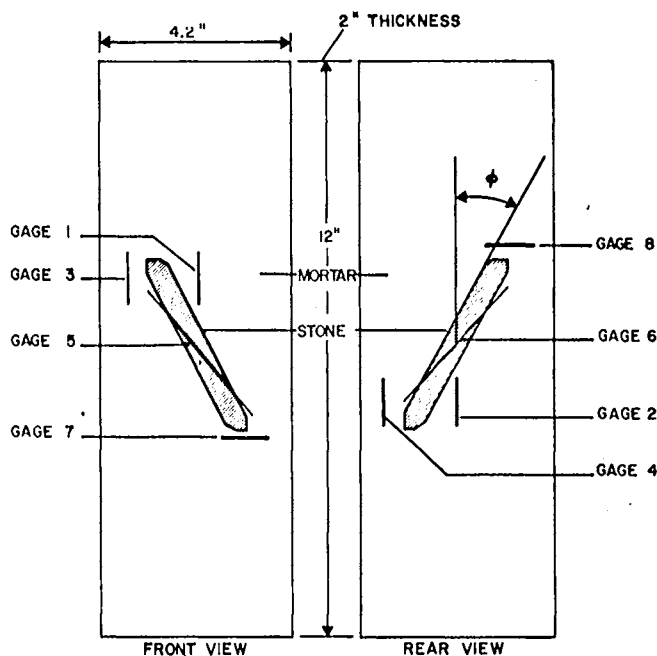


Figure 1-2 The specimen for the interfacial study

recorded at regular intervals. The testing time for each test was about one-half hour. The arrangement of the SR-4 strain gages is shown in Figure 1-2.

RESULTS AND DISCUSSION

The load-strain curves up to failure obtained from the various gages for one each of the specimens with the stone inclined at 23, 42, and 50 deg are shown in Figures 1-3, 1-4, and 1-5 respectively. For the sake of brevity, only the results of the specimens with the stone inclined at 23 deg will be discussed in detail. The following observations are made from Figure 1-3.

1. Up to a load of 12 kips, the strains in gage 1, representing the center zone, and gage 3, representing mortar in the exterior zones, are nearly alike and linear. This means that up to 12 kips, the interface was intact and as a result the specimen behaved homogeneously and elastically.

2. For loads beyond 12 kips, and up to failure, the center strain gage 1 did not show any appreciable increase in strain at all. This

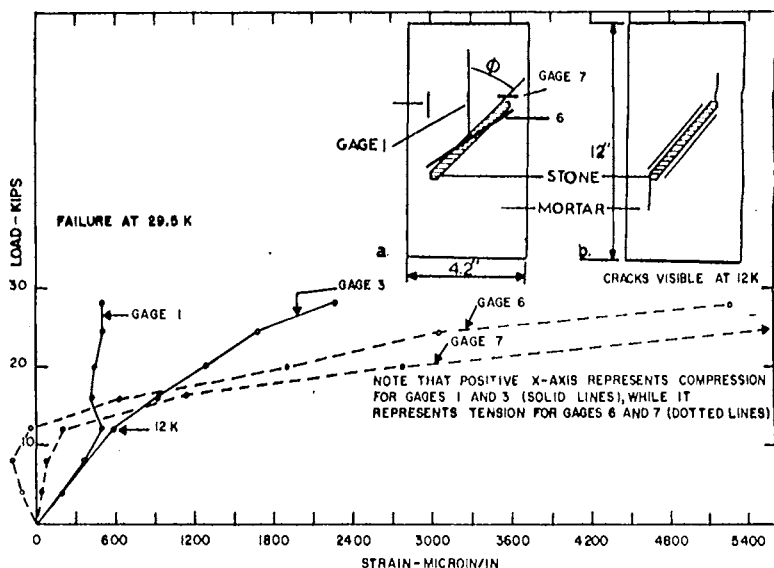


Figure 1-3 Behavior of restrained interface. Specimen, cracks and strains for $\phi = 23$ deg

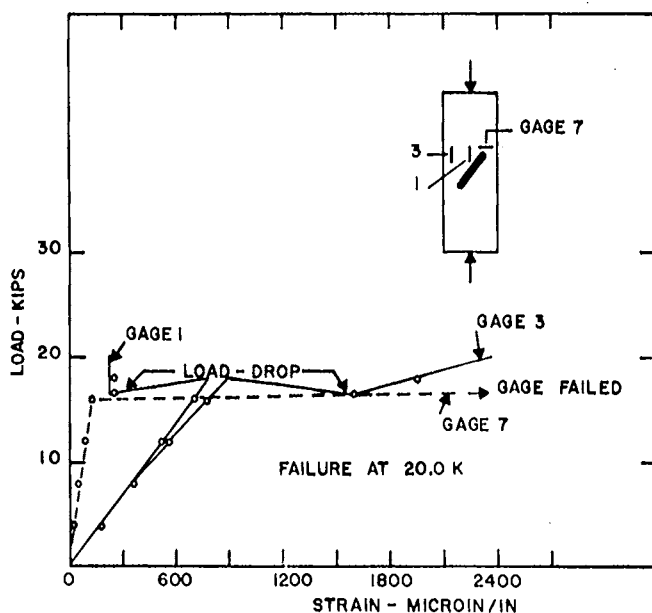


Figure 1-4 Load-strain relations for specimen W-8, $\phi = 42$ deg

means that the center portion, the one containing the aggregate, did not take any additional stress beyond that which was present at a load of 12 kips. The increase in load above 12 kips, therefore, must have been resisted by the mortar in the exterior zones. This is clearly indicated by the increased rate of strain of gage 3 at 12 kips.

3. At about 12 kips, cracks at the interface or bond cracks were observed visually. This indicates that at that load, the unconfined bond strength of this interface was exceeded. Since these inclined bond cracks did not open until after final failure, it seems likely that the contact between the stone and the mortar was maintained. This means that the cracked interface can continue to carry a share of the total load through friction. Thus, at 12 kips, only the cohesion between the stone and mortar was lost.

4. Inclined bond cracks were shortly followed by vertical cracks in the mortar at the top and bottom corners of the stone slab (see Fig. 1-6). As the loading proceeded, these vertical cracks did not perceptibly increase in length until final failure.

If the interface had not been confined by the mortar in the exterior zones, there would have been sliding along the interface when the load in the central zone was carried by friction alone. This is because the angle of inclination of the interface from the horizontal, 67° , is greater than the angle of friction which is 30 to 40° at the interface. The confinement, or prevention of sliding provided by mortar produces tensile stresses in the mortar as shown in Figure 1-7(a). This is evident by the readings of gage 7 in Figure 1-3. When these tensile

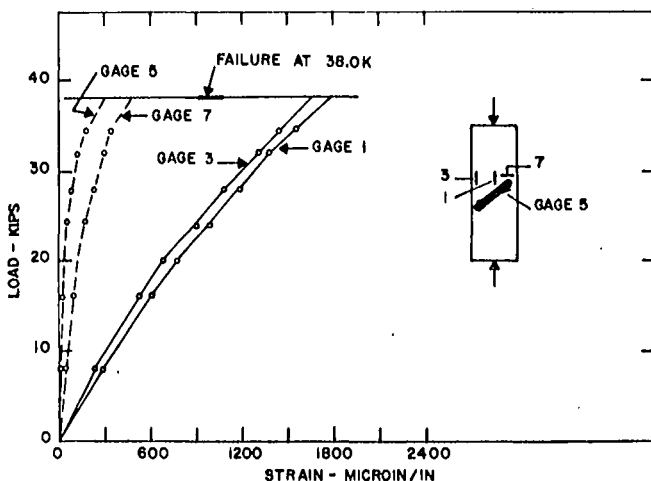


Figure 1-5 Load-strain relations for specimen W-11, $\phi = 50^\circ$

stresses exceed the strength of mortar, local vertical mortar cracks form.

5. After the formation of local vertical mortar cracks, the central zone can be strained longitudinally without necessarily increasing the stresses in that zone. The opening of local vertical mortar cracks permits sliding at the interface so that the central zone deforms longitudinally. Readings of gages 6 and 7 show that there was sliding at the interface and that the local vertical mortar cracks did widen (Fig. 1-3).

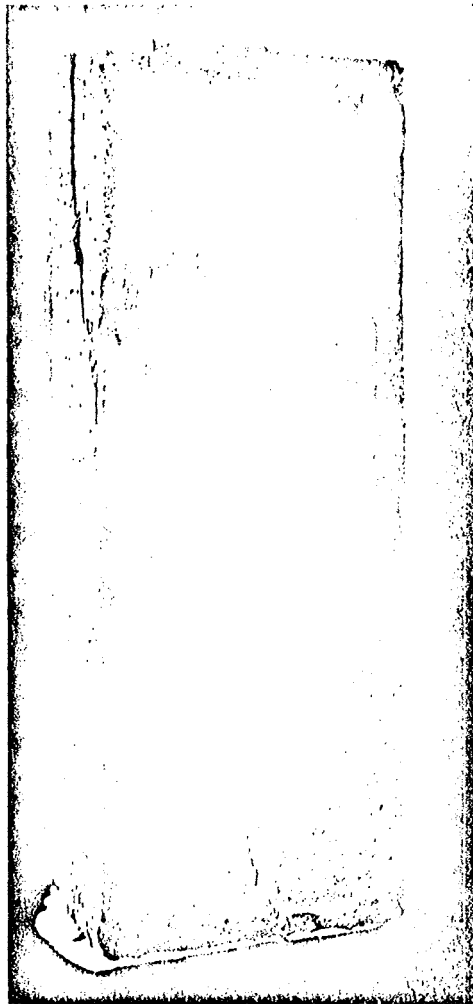


Figure 1-6 Photograph of failed specimen W-5

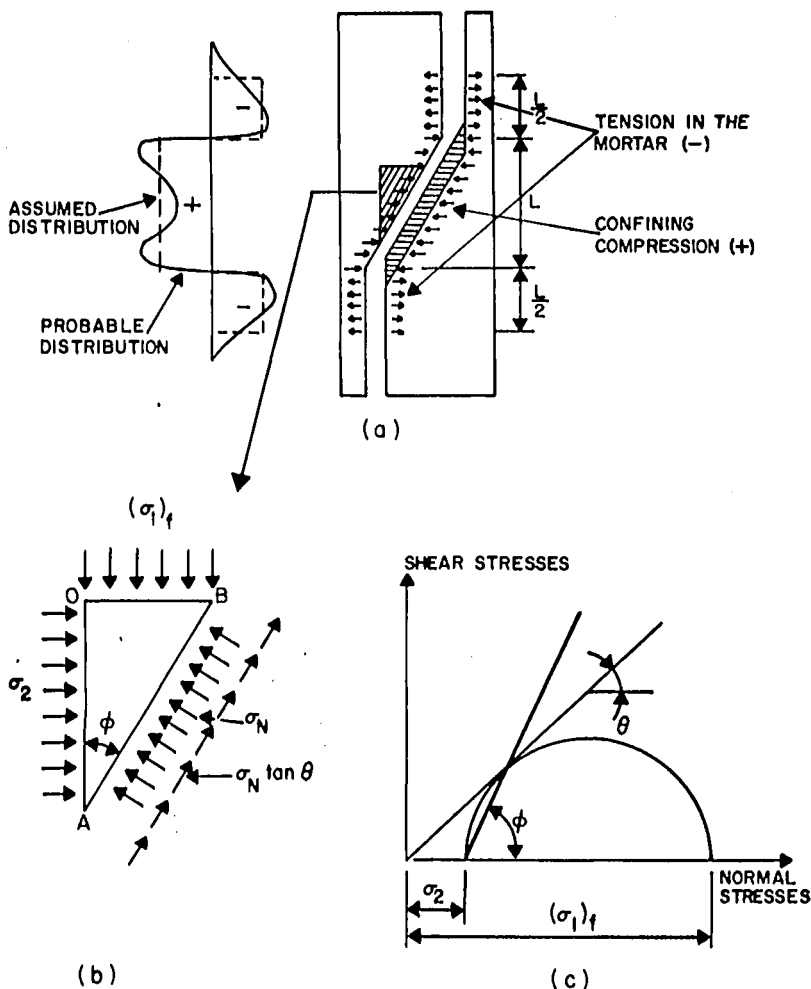


Figure 1-7 Stress-distribution in the interfacial specimen after the cohesion between the stone and the mortar is lost

Thus, the formation of mortar cracks provided the interface with a yielding type of confinement, i.e., a confinement which furnished a reasonably constant horizontal restraining force as further limited sliding proceeded.

The net effect was that for loads above 12 kips, the center zone or the cracked interface behaved essentially plastically (gage 1 on Fig. 1-3), even though there is no flow process involved in these brittle materials.

6. At the final failure of the specimen, crushing of the mortar in the exterior zones was observed (Fig. 1-6). This is because beyond the load of 12 kips the increase in load was carried entirely by the exterior zones. Therefore, the specimen failed when the mortar in these zones could not carry any additional load.

Numerical confirmation

When a compressive load is applied to the specimen, there are normal (σ) and shearing (τ) stresses acting on the plane of the interface between stone and mortar. If σ_1 is the applied vertical compressive stress on the specimen, then the stresses acting on the interface inclined at an angle ϕ to the vertical are given by:

$$\tau = \sigma_1 \sin 2\phi / 2 \quad (1)$$

$$\sigma = \sigma_1 \sin^2 \phi \quad (2)$$

The relation between the normal and shear stresses acting on the interface at failure is given by the Coulomb-Mohr theory:

$$\tau = (\tau)_{\text{lim}} = c + \sigma \tan \theta \quad (3)$$

where c is the cohesion and θ the angle of friction for the interface (Ref. 8). By combining (1), (2), and (3), the maximum vertical compression that an unconfined interface inclined at an angle ϕ can support is given by:

$$(\sigma_1)_{\text{u.b.}} = c [\cot \phi + \tan(\theta + \phi)] \quad (4)$$

$(\sigma_1)_{\text{u.b.}}$ is called the unconfined bond strength of the interface. The calculated values of $(\sigma_1)_{\text{u.b.}}$ in terms of load for all three angles of the interface tested is given in the first row of Table 1-1. The values of c and θ correspond to the type of mortar and stone used and are taken from a prior and independent investigation (Ref. 8). Since these values can be determined only approximately, two sets of the values of c and θ , differing from each other by about 10 percent, are used. The calculated values of $(\sigma_1)_{\text{u.b.}}$ in terms of load agree well with the observed values for the appearance of bond cracks.

After the appearance of bond crack, the interface carries its share of total load entirely through friction. $(\sigma_1)_f$ represents the maximum stress the interface can carry through friction so that the necessary horizontal restraint is no greater than the tensile strength of the mortar. $(\sigma_1)_f$ was calculated approximately by assuming the stress distributions of Figure 1-7. That is, the confining compression (σ_2) which is necessary to keep the interface from sliding is assumed to be uniformly distributed and is equal in magnitude to uniformly distributed tensile stresses in the mortar. Then $(\sigma_1)_f$ can be approximately calculated from equilibrium,

TABLE 1-1 PERFORMANCE OF SPECIMENS OF FIGURE 1-2

Load-Stage	$\phi = 23^\circ$			$\phi = 42^\circ$			$\phi = 50^\circ$		
	Observed	Calculated (1)*	Calculated (2)†	Observed	Calculated (1)	Calculated (2)	Observed	Calculated (1)	Calculated (2)
Appearance of interfacial cracks	12.0 ^k	11.3 ^k	13.2 ^k	17.9 ^k	14.3 ^k	18.3 ^k	38.0 ^k	23.5 ^k	37.8 ^k
Appearance of local vertical mortar cracks	12.0 ^k	11.4 ^k	12.6 ^k	15.8 ^k	13.8 ^k	17.0 ^k	—	21.2 ^k	—
Failure of specimens	28.0 ^k	20.0 ^k	20.4 ^k	20.0 ^k	23.9 ^k	25.4 ^k	38.0 ^k	25.0 ^k	37.8 ^k

(1)* Calculated using: $c = 350$ psi, $\theta = 32^\circ$, $f_t = 400$ psi, $f_m = 4500$ psi.

(2)† Calculated using: $c = 400$ psi, $\theta = 35^\circ$, $f_t = 400$ psi, $f_m = 4500$ psi.

Figure 1-7(b) either by using the Mohr-circle shown in Figure 1-7(c) or by the following formula:

$$\frac{(\sigma_1)_t - f_t}{2} \sin 2\phi = \tan \theta [(\sigma_1)_t \sin^2 \phi + f_t \cos^2 \phi] \quad (5)$$

where f_t is the tensile strength of the mortar. The calculated values of $(\sigma_1)_t$, in terms of total load agree well for all three angles with the observed values for the appearance of local vertical mortar cracks (Table 1-1).

The failure load of the specimen was calculated as the sum of the plastic load of the interface $(\sigma_1)_t$ and the compressive strength of the mortar (f_m) in the exterior zones. The last row of Table 1-1 shows that the failure loads of the specimens have been fairly well estimated. The numerical values of f_m and f_t were taken from experimental results of Refs. 8 and 9 respectively, which were obtained independently on presumably identical material.

IMPLICATIONS

The study of the specimens was helpful in approximately simulating the behavior of real stone-mortar interface. The major implications of this study can be summarized as follows:

1. In concrete, the interfaces are located at various angles, and therefore bond cracks will form at different loads at these interfaces. Since bond cracks behave plastically, a gradual curving of the stress-strain curve of concrete results. This provides an explicit explanation of the fact that the introduction of the aggregates makes the stress-strain curve of concrete deviate from a straight line.

2. The sliding of the interface is accompanied by widening of local vertical mortar cracks at the ends of the interface. This phenomenon probably accounts for the volume of expansion of concrete which is observed near the ultimate load.

3. The pseudo-plastic behavior of pre-existing no-load bond cracks⁴ explains the moderate non-linearity of the stress-strain curve of concrete even at very low loads.

4. The Coulomb-Mohr theory applied to the behavior of the interface, combined with the maximum normal stress theory applied to both tension and compression failure of the mortar satisfactorily predicts the strength and behavior of the confined interface.

ELEMENTS OF A THEORY OF MECHANICAL BEHAVIOR OF CONCRETE

Evidence was presented which indicates that the bond between aggregate and mortar is a weak link in the heterogeneous concrete system. Under compressive short-time loading, this bond is first to break. The broken bond or bond crack behaves pseudo-plastically under further load, and appears to be responsible for initiating mortar cracks.

On the basis of this experimentally observed microbehavior of concrete, an attempt was made to formulate analytically a behavior theory of concrete, which is intended to explain the performance of concrete under short-time load up to and including failure. The hypothesis mainly involves the integration of the observed microbehavior to obtain a reasonable picture of macrobehavior.

The theory is based on an analytic model, termed the structural unit (Fig. 1-8). The unit consists of a single piece of aggregate embedded in a prism of mortar. A circular cylindrical aggregate was selected because it seems that in a unit volume of concrete all the interfacial angles are equally represented. Thus, a circular cylinder seems appropriate and mathematically convenient to represent an average condition of the interface in bulk concrete.

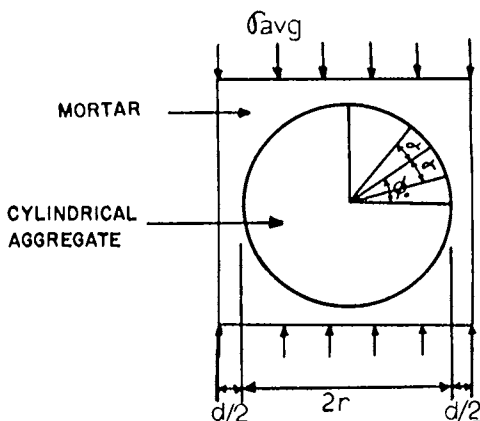


Figure 1-8 The structural unit

The ratio of the thickness of the surrounding mortar to the radius of the aggregate, d/r , depends primarily on the relative amounts of aggregate and mortar in a given mix. This ratio can be approximated by the method used by Hsu.¹⁰ This method assumes that all the aggregates are spherical balls of equal radius, and requires the knowledge of the volume of aggregates in a given mix and their volume in a fully compacted state. The larger the amount of mortar relative to stone, the higher the ratio d/r .

Stress-strain relationship for the structural unit

The behavior of this structural unit under axial compression can be calculated analytically from equilibrium, compatibility of deformations and from experimentally verified, though simplified, assumptions as follows:

i. The three-phase system: paste, sand and gravel is replaced by a two-phase system consisting of the matrix (i.e. paste or mortar) and the aggregate.

ii. Both component phases, the matrix and the aggregate are assumed to have linear stress-strain relations (Fig. 1-1); for analytical simplicity they are also assumed to have identical moduli of elasticity.

iii. The portion of the cylindrical interface over which the stress exceeds the unconfined bond strength is assumed to behave in the same manner as obtained experimentally for the specimens of Figure 1-2.

It is believed that these assumptions, while somewhat idealizing the system and making it amenable to simple analytical treatment, are in basic agreement with independent experimental evidence and retain all the essential features of the real, more complex multi-phase material.

On this basis, the structural unit shows the following behavior under increasing axial compression:

1. Because the stone and mortar are assumed to have linear stress-strain curves and identical moduli of elasticity, the structural unit will behave linearly elastically as long as the interface between aggregate and mortar is intact.

2. At some load or stress, the unconfined bond strength will be exceeded over some angular portion of the interface and a bond crack will have formed. The relation between the unconfined bond strength (σ_1)_{u.b.} and the angle of the interface, for given set of c and θ , is given by Equation (4) or by the Mohr-circle shown in Figure 1-9. In Figure 1-9, the broken line is the bond strength line given by the Coulomb-Mohr failure theory for given c and θ . As the axial load is applied to the unit, the normal and shear stresses occurring anywhere can be described by the Mohr circle. Increasing the load increases the diameter of the circle. At some load, the Mohr circle will just touch the bond strength line. The compressive stress at this point is denoted

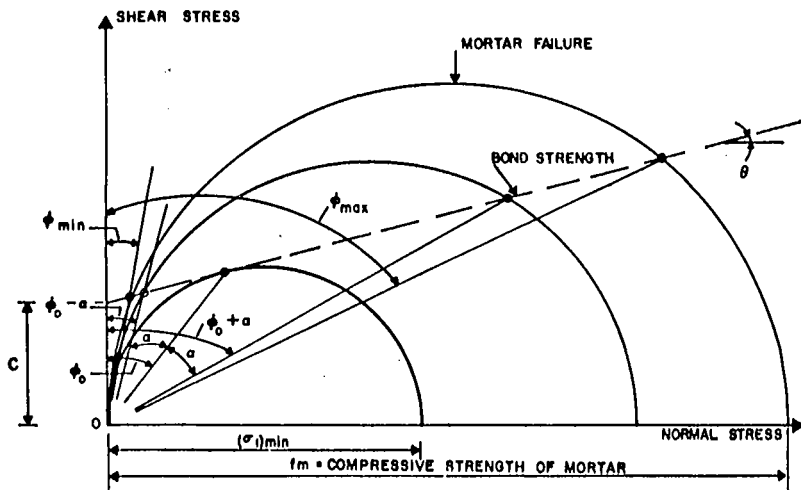


Figure 1-9 Mohr-Circles representing different stages of interfacial failures or bond crack propagation

as $(\sigma_1)_{\min}$ and the angle of the interface at which the first bond crack will form is defined as ϕ_0 and is given by $\phi_0 = 45 \text{ deg} - \frac{\theta}{2}$. As the load increases, additional bond cracks will form. Eq. (4) can be rewritten in terms of ϕ_0 as:

$$(\sigma_1)_{u.b.} = \frac{2c \sin 2\phi_0}{\cos 2(\phi - \phi_0) - \cos 2\phi_0} \quad (6)$$

It can be seen from this equation and Figure 1-9 that propagation of bond cracking will be symmetrical about the angle ϕ_0 . In other words, at some stress greater than $(\sigma_1)_{\min}$, the portion of the interface between the angles $\phi_0 + \alpha$ and $\phi_0 - \alpha$ will have failed. Substituting $\alpha = \phi_0 - \phi$ in Eq. (6) yields

$$(\sigma_1)_{u.b.} = \frac{2c \sin 2\phi_0}{\cos 2\alpha - \cos 2\phi_0} \quad (7)$$

With the increasing load on the unit, α and the diameter of the Mohr circle increase. At some load, the Mohr circle will coincide with the circle representing failure in the mortar and the unit will fail.

3. The cracked interface behaves pseudo-plastically. This plastic load or stress is the maximum stress supported by friction and is limited by the tensile strength of the mortar. This stress, $(\sigma_1)_t$, can be calculated by making the same assumptions that were made for the interfacial

specimen (Fig. 1-7), and is given by Eq. (5) which can also be expressed as

$$(a_1)_t = 2 \frac{\frac{\cos 2\phi_0}{3} [(a_1)_t + f_t]}{\cos 2\alpha - \frac{\cos 2\phi_0}{3}} + f_t \quad (8)$$

4. The stress-strain curve of the unit below the compressive stress of $(\sigma_1)_{\min}$, or before bond cracking, is a straight line which can be obtained by knowledge of the modulus of elasticity of mortar. The stress-strain curve of the unit above the stress of $(\sigma_1)_{\min}$ was calculated as follows:

Let the applied load on the unit be such that the extent of bond cracking is $2a$. That is, the portion of the interface having angles between $\phi_0 + a$ and $\phi_0 - a$ is behaving plastically while the remaining portion is still elastic. The vertical stress in an elastic element is designated as σ_γ , while the vertical stress in a plastic element is designated as σ_β , (Fig. 1-10). The stress in any plastic element may be formulated from Eq. (8) as:

$$\sigma_\beta = 2 \frac{\frac{\cos 2\phi_0}{3}(\sigma_\beta + f_t)}{\cos 2\beta - \frac{\cos 2\phi_0}{3}} + f_t \quad (9)$$

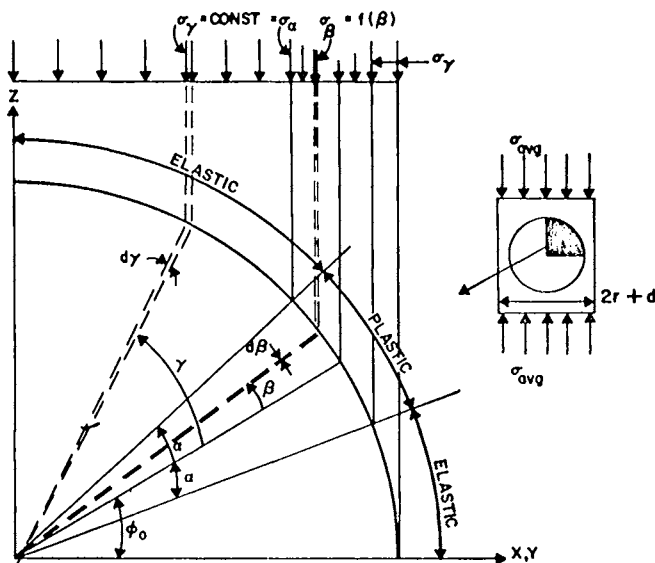


Figure 1-10 Definitions of α , β , γ , σ_α , σ_β , and σ_γ . Note that for a given α , β and γ are variables of integration.

The stresses in all elastic elements are equal and are determined by the stress in that element which is just being transformed into the plastic stage:

$$\sigma_{\gamma} = \frac{2c \sin 2\phi_0}{\cos 2\alpha - \cos 2\phi_0} = \sigma_a$$

From equilibrium of vertical forces,

$$\begin{aligned} & \int_{\phi_0}^{\alpha} \sigma_{\beta} r \sin(\phi_0 + \beta) d\beta + \int_{\phi_0}^{-\alpha} \sigma_{\beta} r \sin(\phi_0 + \beta) d\beta \\ & + \sigma_a \int_{-\phi_0}^{-\alpha} r \sin(\gamma + \phi_0) d\gamma + \sigma_a \int_{\alpha}^{\frac{\pi}{2} - \phi_0} r \sin(\gamma + \phi_0) d\gamma \quad (10) \\ & + \sigma_a \frac{d}{2} = \sigma_{nvk} \left(r + \frac{d}{2} \right) \end{aligned}$$

where σ_{nvk} is the average stress for the whole unit. Carrying out the integrations indicates that σ_{nvk} can be expressed in terms of α . For selected values of α , σ_{nvk} can be explicitly calculated.

The strain in the unit can be derived by assuming that plane sections remain plane. In that case, the strain of the whole unit is the same as that in the elastic elements which is equal to

$$\frac{\sigma_a}{E} = \frac{2c \sin 2\phi_0}{E(\cos 2\alpha - \cos 2\phi_0)} = \epsilon_{nvk} \quad (11)$$

where E is the modulus of elasticity of mortar. Thus, one can also calculate the strain explicitly for selected values of α .

Thus, for any given value of α , the corresponding stress and strain of the structural unit can be explicitly calculated from the Equations (10) and (11). In this manner, one can calculate point by point the complete stress-strain curve of the unit. For details of analysis, see Ref. 11.

From this, it follows that the quantities which determine the stress-strain curve of the structural unit of a given mix are the following properties of the materials involved: degree of compaction of the aggregates, cohesion and angle of friction of the interface, compressive and tensile strength of the mortar, and modulus of elasticity of mortar and aggregate. It is significant that all these quantities can be, and have been, experimentally measured, independently and *a priori*.

A stress-strain curve of the structural unit calculated from the analysis described here and for a realistic set of material constants is shown

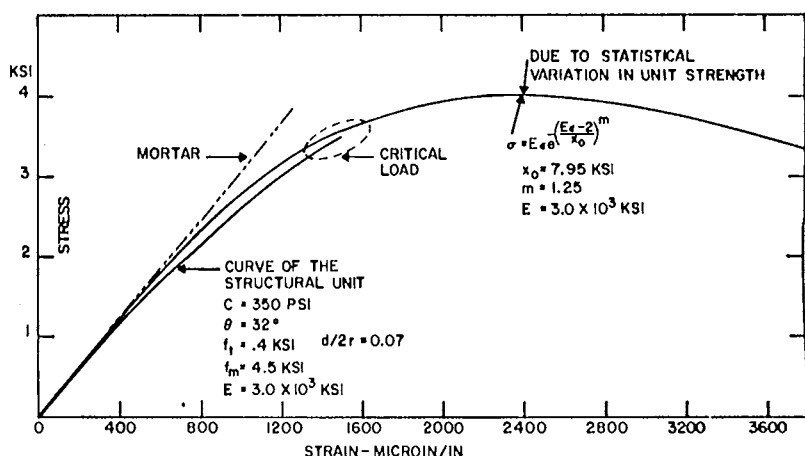


Figure 1-11 Calculated stress-strain curves

in Figure 1-11. This stress-strain curve of the unit slowly deviates from a straight line. It is important to note that this deviation is caused entirely by the separately observed behavior of the interface since it was assumed that both the aggregate and mortar are ideally elastic. The inelastic behavior of the structural unit is caused by the weak bond between the aggregate and mortar and does not involve any real flow processes of cement gel or other materials. The unit, therefore, does exhibit inelasticity of the type found in concrete. However, the actual stress-strain curve of concrete bends more sharply toward the horizontal at about 90 percent of the ultimate and has a peak and a descending branch. It seems that the stress-strain curve of a single unit does not reflect the inelastic behavior which appears in the concrete near the ultimate load.

Statistical analysis

The end point of the calculated curve of the structural unit is determined by the selected strength of the mortar. In a given mass of concrete, there are probably significant variations in the mortar strength and in aggregate spacing, represented by d/r in the structural unit. Bulk concrete, therefore, can be thought of as consisting of large numbers of structural units whose ultimate strengths show a certain amount of scatter. Thus, even when a number of lower strength units fail, load can still be resisted by the remaining stronger units, which are decreasing in number with an increasing load. In what follows, it will be shown that this type of behavior, resulting from the statistical distribution of

the strengths of the units, can give a stress-strain curve with a descending branch.

It is assumed that a given cross section of a concrete specimen under uniaxial compression is made up of n structural units. The ultimate strengths of these units are assumed to be distributed according to a distribution function $G(x)$, where x denotes the variable strength in ksi of each unit. $G(x)$ is the probability that the strength of a unit has a value greater than or equal to x . It is not possible to experimentally observe $G(x)$. However, it is possible to measure the distribution of the compressive strengths of the whole specimens, e.g., cylinders. The problem, then, is to determine the relation between the known distribution of the strengths of the specimens and the unknown distribution of structural units. To find such a relationship, the following assumptions were made:

1. The strength of each unit is independent of others.
2. Hooke's law is valid for each unit and all units have the same modulus of elasticity.
3. Plane sections remain plane.
4. As soon as the ultimate strength of one of these units is exceeded, that unit is completely destroyed and its load bearing capacity is reduced to zero. This causes redistribution of, and an increase in stresses in other units which have so far remained whole. Total failure does not occur so long as the load-bearing capacity of the unimpaired part is not exceeded.

If $(\sigma)_{avg}$ is the average stress on a cross section containing n structural units, then from equilibrium,

$$(\sigma)_{avg} = G(x) \cdot x \quad (12)$$

The maximum strength or the ultimate strength of the cross section, $(\sigma)_c$, corresponds to that value of x , x_{max} , which satisfies the equation

$$\frac{d}{dx} [G(x) \cdot x] = 0 \quad (13)$$

Daniels (Ref. 12) has studied the limiting distribution of $(\sigma)_c$ for large values of n . He showed that if $G(x)$ approaches zero at a higher rate than $1/x$, the distribution function for $(\sigma)_c$ for large values of n approaches the normal distribution. Furthermore, in that case, the mean value of $(\sigma)_c$, M , is

$$M = x_{max} \cdot G(x_{max}) \quad (14)$$

and the coefficient of the variation of $(\sigma)_c$, V , is

$$V = \sqrt{\frac{1 - G(x_{max})}{nG(x_{max})}} \quad (15)$$

The above relations are valid provided the strengths of the specimens, $(\sigma)_u$, are distributed normally. Since the strengths of the concrete cylinders can be regarded as being normally distributed, these relations are applicable to concrete. A suitable function for $G(x)$ is still to be assumed.

A. I. Johnson¹³ has shown that for a brittle material like concrete the most suitable distribution of the strength of structural units, $G(x)$, is perhaps the Weibull type of distribution.¹⁴ Accordingly, the following distribution of the strengths of the units was assumed:

$$G(x) = e^{-\left(\frac{x-x_1}{x_0}\right)^m} \quad (16)$$

where x_1 is the lowest possible strength of the structural units and x_0 and m are constants, which can be determined from the Eqs. (14) and (15).

Tests of 32 four-by-eight cylinders showed that the coefficient of variation of the compressive strengths, V , was 0.071 and the mean compressive strength, M , was 4.0 ksi. The lowest possible value of the ultimate strengths of the units, x_1 , was taken as 2.0 ksi. The number n was assumed to be 150. Substituting these values and the Eq. (16) into the Eqs. (14) and (15), the values of x_0 and m were found to be 7.95 and 1.25. The distribution function of the strength of the units was thus found to be:

$$G(x) = e^{-\left(\frac{x-2}{7.95}\right)^{1.25}} \quad (17)$$

The average stress-strain relationship for the concrete can now easily be obtained. When the stress in the units which are still intact is equal to x , then the average stress $(\sigma)_{avg}$ is given by

$$\sigma_{avg} = xG(x) = xe^{-\left(\frac{x-2}{7.95}\right)^{1.25}} \quad (18)$$

The strain ϵ is simply given by

$$\epsilon = \frac{x}{E} \quad (19)$$

where E is the modulus of elasticity of the unit, which was assumed to be 3.0×10^3 ksi. Combining (18) and (19) shows the relation between average stress and strain for the entire cross section.

$$\sigma_{avg} = E\epsilon e^{-\left(\frac{E\epsilon-2}{7.95}\right)^{1.25}} \quad (20)$$

This statistically calculated relation is plotted on Figure 1-11.

The statistically calculated curve has a peak and a descending branch similar to the compression curve of concrete (Fig. 1-11). Thus, it seems that the scatter in the strength of the mortar and of aggregate spacing can strongly affect the inelastic behavior of concrete in the near and post-ultimate range of loading.

In the statistical analysis it was assumed that individual units behave elastically. This was done to simplify the analysis. In actual concrete, both the inelasticity of the units which was analyzed in the previous section and is due to presence of interfaces, and the inelasticity resulting from the statistical variations in the strengths of the units occur simultaneously.

From the microcrack investigation¹⁵ it was observed that mortar cracking increases sharply at about 90 percent of ultimate load. This load was termed critical load. Even though bond cracking also increases at the critical load, it is significant throughout the entire loading range. This suggests that bond cracking plays a relatively major role in causing inelastic behavior of concrete below the critical load. The critical load is reached when the mortar cracking becomes sufficiently extensive to alter the character of the material from an essentially continuous to an essentially discontinuous material. Beyond the critical load, significant additional inelasticity is introduced by the statistical distribution of the strength of the mortar, which played a relatively minor role below the critical load.

In summary, the stress-strain curve, calculated on the basis of the experimentally investigated effects of microcracking and a reasonably assumed mortar strength distribution within bulk concrete, has all the well-known characteristics of a typical compression stress-strain curve of concrete. This suggests that for concrete which is composed of two almost ideally elastic materials (aggregate and hardened paste), its inelastic behavior can be explained as resulting from substantially brittle microcracking combined with a generally accepted type of distribution of materials strength properties.

Application of the analysis

The elements of a behavior theory developed here can be useful in explaining some other observed phenomena of concrete, in addition to its inelastic behavior. For example:

1. Under biaxial or triaxial compression, the unconfined bond strength of the interface will be greater than under uniaxial compression, since this will increase the normal compression forces acting on the interface. In addition, the extra confinement will increase the effective tensile strength of the mortar. Thus, the hypothesis will correctly predict a higher ultimate strength for biaxial and triaxial than uniaxial compression.
2. The hypothesis also shows that the compressive strength of concrete

is much more sensitive to the strength of mortar and the relative amounts of mortar and aggregate than to the properties of the interface. This is because interfacial failures are critical only for a portion of the total available interface. In Figure 1-9, only those portions of the interface having angles of inclinations between $\phi_{\max}$ and $\phi_{\min}$ will eventually fail. For the remaining portions of the interface, and also for the mortar between aggregate, it is failure in the mortar which is critical.

3. Since higher strength concretes generally have lower aggregate-to-cement ratios, the theory explains why the higher strength concretes tend to show less inelastic deformation than those of lower strength.

Simplifications

The primary purpose of the behavior theory presented here is to show that one of the significant and perhaps the primary cause of the inelastic behavior of concrete is the nature of the interface between the aggregate and mortar. It is felt that the structural unit and the assumptions involved in developing this theory represent essential features of actual concrete. However, because of the complex nature of the problem, several simplifications had to be made. Some of these are briefly discussed here.

1. It was assumed that the mortar is a perfectly elastic material. This simplification can be justified if mortar is regarded as an intermediate material between hardened paste and concrete and so that the inelasticity of mortar, in turn, is due to the behavior of the interface between sand and paste. Thus, mortar can be replaced by paste in the analysis, and the latter can be considered as a perfectly elastic material (Fig. 1-1).

2. The differences between the moduli of elasticity of mortar and aggregate were neglected. It has been reported that the moduli of mortar at 28 days and of normal aggregate are indeed often fairly close,^{16, 17, 18} so that differences in moduli, where they exist, seem to play no decisive role in concrete behavior.

3. The hypothesis predicts that the higher the percentage of coarse aggregate, the lower the d/r ratio and, therefore, the lower the compressive strength of the concrete. This is known to be true for the range of structural concretes used commercially. However, continuing research at Cornell seems to indicate that for the impractically low percentages of volume of aggregates, increasing the amount of aggregates increases the strength rather than decreasing it. This is not reflected in the hypothesis, probably because the effect of the addition of aggregates on the void-to-volume ratio was neglected.

CONCLUSIONS

1. Comparison of stress-strain curves in short-time compression of stone, paste, mortar and concrete show that the first two are nearly

linear while concrete's curve is most curvilinear. This indicates that inelasticity in concrete is connected with the heterogeneity produced by the presence of aggregates.

2. A detailed study of the interface between the aggregate and mortar indicates that when external load is applied to concrete, interfacial surfaces are first to fail. That is, when the unconfined bond strength of the interface is reached, a bond crack is formed; the cohesion between the aggregate and mortar is lost, and thereafter the interface carries its share of the total load entirely through friction.

3. Under increasing load, the interface where a bond crack exists behaves pseudo-plastically. It is conjectured that the loss of cohesion produces the observed local vertical mortar cracks which make possible the plastic behavior of the cracked interface.

4. The plastic behavior of the interface and the resulting redistribution of the internal stresses cause the overall stress-strain curve of concrete to deviate gradually from a straight line.

5. The unconfined bond strength of the interface can be determined fairly accurately by the Coulomb-Mohr failure theory. The confined strength of the interface is limited by and related to the tensile strength of the mortar.

6. Based on this experimentally observed microbehavior, an attempt has been made to formulate, analytically, the elements of a behavior theory for concrete under short-time load up to and including failure. This theory gives a stress-strain curve which has all the shape characteristics of that of concrete up to the critical load (about 85 to 90 percent of ultimate). It is significant that the only quantitative information necessary to obtain this theoretical stress-strain relationship are the values of basic material properties of the constituents of concrete. These quantities have been obtained experimentally and independently.

7. The hypothesis explains that the mechanical behavior of concrete below the critical load is governed chiefly by the interfacial behavior.

8. The critical load is that at which mortar cracking has become sufficiently extensive to cause volume expansion and to change the character of the material from an essentially continuous to an essentially discontinuous material.

9. The shape of the peak region and the descending branch of concrete's stress-strain curve is caused primarily by statistical variations in the ultimate strength of the mortar and in aggregate spacing throughout bulk concrete.

10. The hypothesis correctly predicts higher biaxial and triaxial than uniaxial compressive strength of concrete in contrast to some theories which assume highly indeterminate space-frames as models for concrete behavior.

11. Even though the constituent materials of concrete (aggregate and hardened paste) are almost ideally elastic and brittle, the behavior theory demonstrates that the significant inelasticity which exists in concrete can be explained on the basis of brittle micro-behavior without assuming any experimentally unobserved flow processes.

12. It is noted that parts of that portion of the proposed behavior theory which relates to the "structural unit," developed step by step from experimental evidence of microcrack research at Cornell and elsewhere, strikingly confirms some of the concepts advanced by Brandtzaeg¹⁹ forty years earlier and without comparable experimental basis.

ACKNOWLEDGMENTS

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PAPER NO. 2 Fracture of plain concrete under biaxial tensile and compressive stress is investigated. The stress states at inception of microcracking, or discontinuity, and strength are found. Such properties as strength, stiffness and mode of breaking alter with the extent of microcracking in a specimen; the extent of microcracking depends on loading program. Explanations of fracture which overlook these subjects by considering only mean stress at breaking may be faulty. The curve of discontinuity is suggested as a simple, safe criterion of strength for some problems of engineering design.

Properties of Concrete Change When Microcracking Occurs

By J. ISENBERG

■ THE PROCESS OF FRACTURE IN CONCRETE under uniaxial tensile or compressive stress begins with the spreading of microcracks,^{1, 2, 3} most of which originate in the coarse-aggregate mortar interface.^{4, 5} The present experiment⁶ defines the inception of microcracking, or discontinuity,⁷ under biaxial tensile and compressive stress. Complete breaking develops from the microcracking process.

Theories of strength, such as the octahedral shear stress theory,^{8, 9} have been proposed both as engineering design criteria and to explain the fracture process. It is a common fault of these theories to assume that strength can be forecast exclusively in terms of mean stresses and that stiffness is a permanent property. Repeated¹⁰⁻¹³ and sustained¹ loading tests illustrate the fault by showing that strength and stiffness are not permanent properties, but change as microcracking develops; further illustrations are cited below.

Different rules apply to the complete breaking of a specimen, according to whether the breaking mode is tensile cleavage or compressive crushing. Cleavage begins when the mean tensile stress is sufficiently magnified at the tip of a crack to break bond between cement gel particles, and

when the energy supplied by the applied forces or their stress field is at least as large as the energy absorbed in extending the crack.¹⁴⁻¹⁶ The stability of the crushing mode, which involves extensive microcracking, depends on the flexibility of the test machine.^{17, 18}

The present experiment illustrates that it is reasonable to design some structures such that substantial microcracking never occurs. This is because the ability of concrete to resist tensile stress is not a permanent property; it depends partly on the extent of microcracking, which is a highly uncertain factor if discontinuity develops at any time during the life of the structure. Hence, discontinuity should be considered as a design criterion for such structures as beams under combined bending and shear which may collapse due to excessive principal tensile stress.

TEST TECHNIQUE

Apparatus

Combinations of torsion and compression were applied to tubular specimens. Apparatus is shown in Figure 2-1. Torsion was applied by turning the screw jack A1-3. The resulting rotation of the arm, B, on its self-aligning roller race was transmitted to the specimen through shear pins. Rotation at the top was resisted by the leaf spring, F1. The spring accommodated changes in length of the specimen without applying appreciable direct force.

Twist was measured as the relative displacement of two arms. The movement of each was recorded on micrometers. Contact, which was sensitive to unit twist 4×10^{-6} radians/in., was sensed by the establishment of an electric circuit and the lighting of a bulb.

Direct force was applied through the screw jack, J1,2. Direct force was transmitted to the bottom of the specimen through a central post-tensioning strand. Direct stress, p , is computed from direct force.

Axial strain, ϵ , was measured with a mechanical extensometer sensitive to average strain of 10×10^{-6} in./in.¹⁷ The distribution of ϵ around the circumference of the specimen was checked with ERS gauges, and

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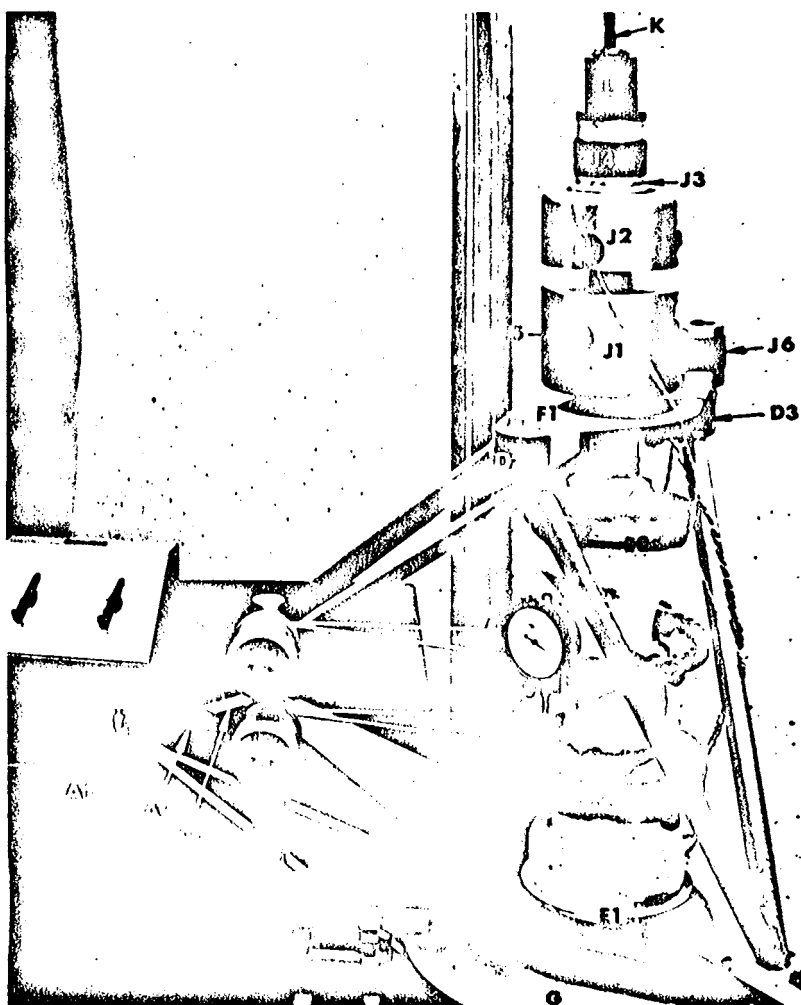


Figure 2-1 Apparatus to apply torsion and compression

found to be uniform within ± 4 percent. Circumferential strain ϵ_1 was measured with ERS gauges.

Shear stress, τ , is computed as if it were uniform in the wall of the specimen; this enables shear stress at the middle radius to be calculated within ± 2 percent from torque at all stages of inelastic deformation. The assumption of a linear shear-stress/strain relation, used by others,⁸ gives an accurate result only for loading programs in which the axial

stress is small at breaking. Their approximation is doubtful at high levels of axial stress because an inelastic component of twist of the same order as the elastic part can develop. Shear strain at the middle radius, γ , is computed from unit twist by assuming that shear strain is proportional to radius. Principal stresses, p_r and p_t and strains, e_r and e_t , are computed from τ , p , γ , e and e_t by well-known formulas.¹⁰

Manufacture of specimens

A specimen is shown in Figure 2-2. The mold and external dimensions are described elsewhere.¹⁰ The wall was $0.33 \pm .005$ in. thick; the wall thickness:radius ratio was 1:3.82. The maximum size of coarse aggregate particle was $\frac{3}{16}$ in. Biaxial stress/strain relations were consistent among many specimens, which indicates that the wall thickness, 1.78 times larger than the largest stones, was adequate for the present experiment. If the experiment had required the third principal strain, in the radial direction, to be measured then a thicker wall would have been needed in order to make measurements representative of structural concrete.

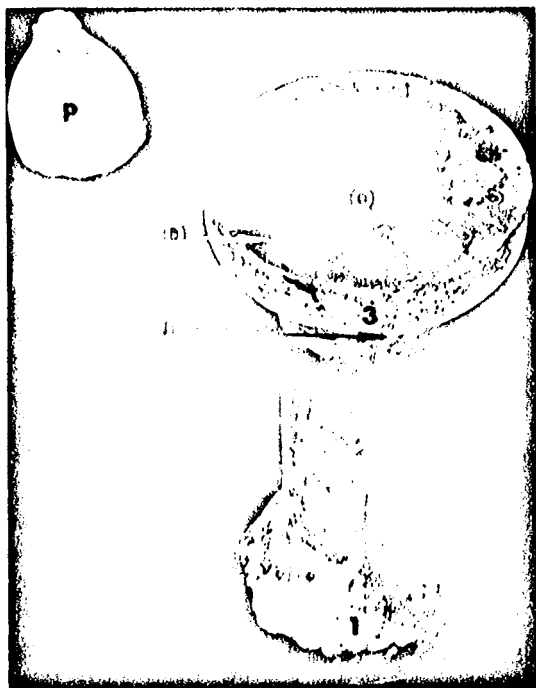


Figure 2-2 A specimen ready for sealing



Documents

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The chief difficulty in casting the specimens was to prevent cracking in the waisted portion due to restraint of shrinkage by the external mold. End one was made of a rich mix. Central portion two was made of the mix to be tested. Concrete was placed to the top of the straight portion of the mold; a steel cylinder formed the central cavity Q. The concrete was allowed to set for three hours. Then, a thin ring of rich mortar, J, was placed and unrestrained shrinkage was permitted for 12 hours more. At that stage, the mortar ring was scraped and moistened, and end three was cast with a rich mix.

The steel cylinder was extracted at age 24 hours. The polystyrene form, P, was inserted into the open end which was then sealed with concrete. Brass sleeves, D, which were cast with the fresh concrete, accommodated shear pins projecting from the end plates. A fuller description of the method is given elsewhere.⁶ The mix proportions were: W/C = 0.71; A/C = 6.0. The specimens were immersed for 9 weeks at 19 deg C; then they were stored in air at 60 percent relative humidity until testing at age 12 weeks. The average strength of companion three-in. cubes was 6250 psi; the coefficient of variation among the strength of cubes was 5.0 percent.

TEST PROGRAM

One object of the tests was to find curves of strength and discontinuity in principal stress plane for two types of loading programs. Test Series D was made in proportional loading $\frac{p_c}{p_t} = k$. Test Series DD was made in compression followed by torsion; additional compression was applied in Tests DD 2,3 after reaching discontinuity. The principal stress/strain curves were measured in loading and unloading.

A second object was to determine the effect of severe microcracking on the properties of a specimen in a subsequent test. Some specimens of Test Series D were loaded up to the maximum stress and then quickly unloaded before breaking in the compressive crushing mode. The results illustrate how part of a structure might behave if it is loaded up to maximum stress, unloaded, and subsequently reloaded. The Test Series are summarized in Table 2-1.

TEST RESULTS

In each test, discontinuity is recognized by a sensible departure from linearity in the τ/γ curves. The method of Least Squares is used to fit a line of intercept τ_0 and slope G to data from early load stages which are obviously before discontinuity. The value of τ_0 is always less than 1 percent of maximum τ . Student's t-test is used to determine the highest load stage at which the quantity $\gamma'' = \gamma - \left(\frac{\tau - \tau_0}{G}\right)$ can be attributed to experimental error. This is defined as the stage of discontinuity. The

TABLE 2-1

TEST SERIES D				
First Loading			Second Loading	
Specimen	k	Mode of Breaking or Impending Breaking	k	Mode of Breaking
1	1	fast cleavage		
2	1	fast cleavage		
3	4.5	fast cleavage		
4	6	fast cleavage		
5	10.5	cleavage		
6	11.5	slow cleavage		
7	17	transition	1	stable cleavage
8	17	transition	1	stable cleavage
9	25	crushing		
10	25	crushing		
11	38	_____		
12	38	_____		
13	38	crushing		
14	39	crushing		
15	39	crushing	1	slow cleavage
16	300	crushing		
17	300	crushing		
18	520	crushing		
19	∞	crushing	1	slow cleavage

TEST SERIES DD			
Specimen	Initial Direct Stress (psi)	τ Applied to (psi)	Additional Direct Stress
1	505	715	no
2	800	595	yes
3	1300	660	yes
4	2000	940	no

upper graphs in Figure 2-3 show γ^p as ordinate and e_t as abscissa. The arrows indicate the stage of discontinuity according to the previous definition. The definition of discontinuity in terms of γ^p is easy to apply consistently, but it should not be inferred that discontinuity occurs at a particular load stage. The lower curves in Figure 2-3 show p_t/e_t relations; these also reflect discontinuity and indicate that serious cracking develops over several load stages.

Figure 2-4 shows that a rule of constant p_t does not predict discontinuity. A rule of constant e_t is also inapplicable; e_t at discontinuity

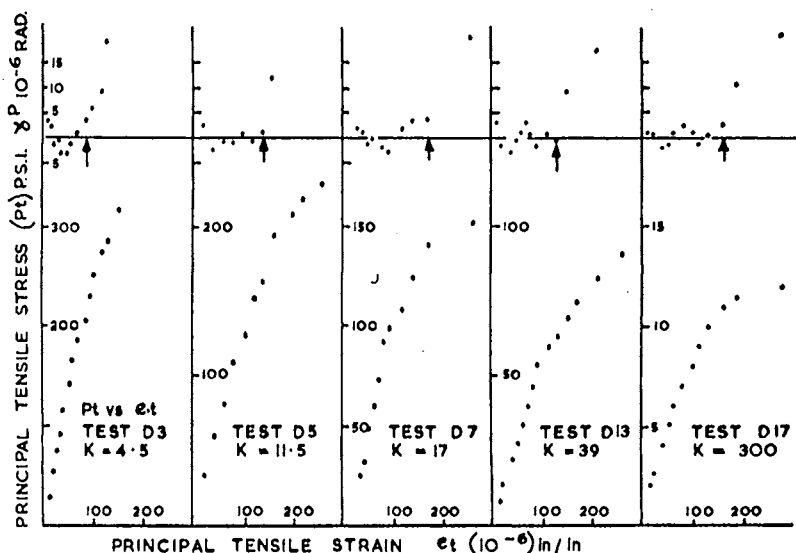


Figure 2-3 Arrows indicate discontinuity which is defined by and confirmed by p_t/ϵ_t curves

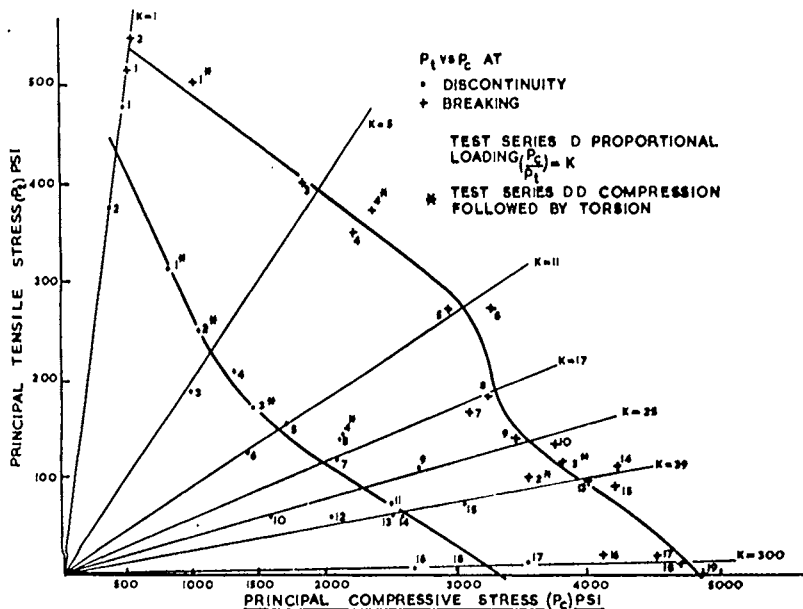


Figure 2-4 p_c versus p_t at discontinuity and breaking

increases approximately linearly with increasing p_c from 90 to 160×10^{-6} in./in.

The shape of the outer curve in Figure 2-4, which represents strength, is consistent with the results of previous investigators.^{8, 20} It appears that the different loading programs of Series D and DD have little effect on stress at breaking or discontinuity in virgin specimens.

ANALYSIS

The fracture process in virgin specimens

The fracture process in virgin specimens is evaluated by the amount of irrecoverable work done in forming and extending crack surfaces. The amount of work per unit volume of specimen or specific work W_{pt} , up to maximum stress is an index of the extent of microcracking and hence of change in specimen properties. The following analysis shows that specimen properties change concurrently with the transition in fracture mode as k is changed from 1 to ∞ .

Analysis is made of Test Series D. It is assumed that W_{pt} is equal to the area enclosed by the stress/strain curves of loading up to maximum stress and unloading. Part of the inelastic energy is due to viscous effects; this part is assumed to be small since the tests were completed within about four hours. In addition, the relative values of W_{pt} are important. It is reasonable to assume that, if tests at $k = 11$ and $k = 25$ are carried out at approximately equal strain rates, the amount of work due to viscous effects is about equal; hence the difference in inelastic energy may be attributed to cracking.

Unloading curves for Specimens D1-6, which fractured by cleavage, were drawn parallel to the initial tangents to the loading curves; independent experiments show that stiffness is virtually unchanged by loading at $k \leq 12$. The unloading curves for specimens D7-10, 13-19 were drawn straight between the stress and strain points which were measured at maximum and at zero stress. This overlooks some curvature in the unloading relation in the sense opposite to the loading relation. If this curvature were considered, the value of W_{pt} would be slightly higher than the value shown below, and the transition would be sharper.

The relation between W_{pt} and k is shown in Figure 2-5. Slightly erratic values of W_{pt} are due to unloading at e_c between $1800-2000 \times 10^{-6}$ in./in., whereas maximum stress would probably have been reached at about $e_c = 2000 \times 10^{-6}$ in./in., in each of Tests D9, 10, 13-19; a better estimate of W_{pt} for these tests is 4 psi.

The transition in W_{pt} signifies replacement of the cleavage breaking mode by the crushing mode at about $k = 17$. The transition also indicates that two initially identical specimens acquire different properties due

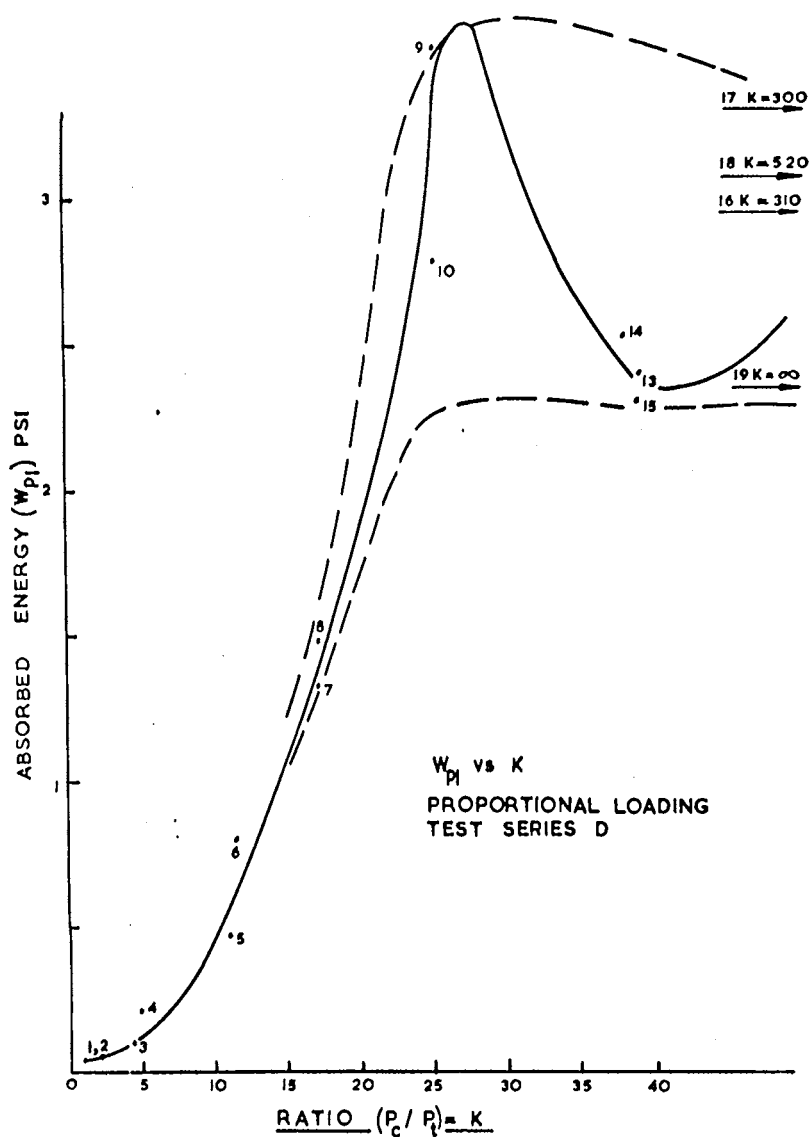


Figure 2-5 Specific inelastic energy at breaking depends on loading program

to microcracking if one is loaded at $k = 12$ and the other at $k = 25$. For example, the ratio of specimen stiffness at breaking to initial stiffness is about 1.0 for the cleavage regime, $k \leq 12$; for the crushing regime, $k \geq 25$, the average ratio is about 0.80 in compression and 0.50 in tension.

The distinction between the two modes of fracture extends into the breaking stage of the fracture process. The irrecoverable specific work of a three-inch cleavage crack in the present experiment is about 0.02 psi.^{14, 15} In tests under uniaxial compression in a stiff machine where maximum $\epsilon_c = 3800 \times 10^{-6}$ in./in., the irrecoverable work done after maximum stress is approximately 9 psi. In the latter case, this shows that many of the cracks which are formed during the first part of the fracture process participate in crushing, whereas only one of a few existing cracks participate in cleavage.

The fracture process in specimens containing extensive microcracking

Some specimens which were first tested at $k \geq 17$ were re-tested to determine the effect of extensive microcracking on such properties as the

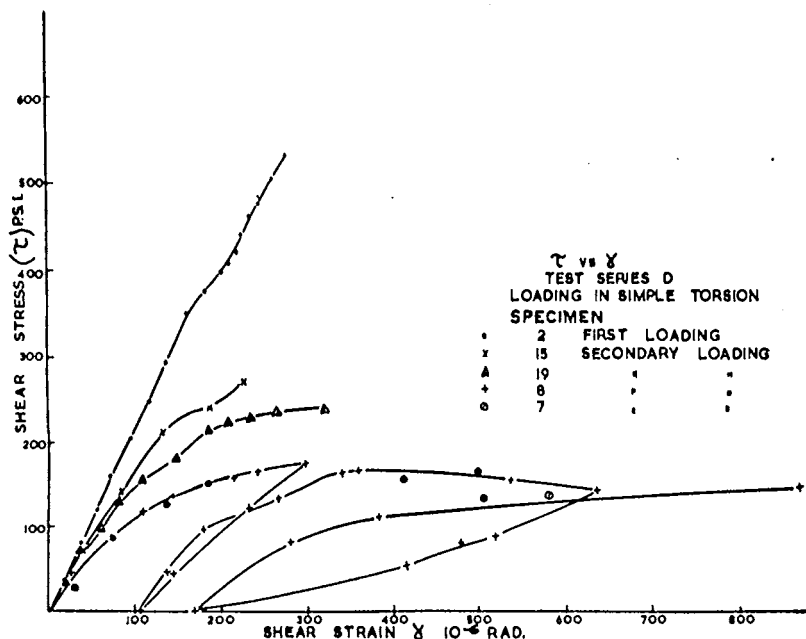


Figure 2-6 Shear stress/strain curves show effect of microcracking on stiffness and strength.

strength and stiffness during a second loading program. None of the specimens which were re-tested contained cracks visible to the unaided eye; the extent of microcracking in each is reflected in the magnitude of W_{pi} , Figure 2-5. The experiment illustrates that different design methods are needed according to whether a structure is to withstand a single application of load or several applications of load near to maximum.

Figure 2-6 shows that extensive microcracking causes strength and stiffness in second loading at $k = 1$ to be reduced. A virgin specimen is shown for comparison. In the reloading tests, the condition of stress magnification was met at a low value of p_i or τ and stable spreading of a cleavage crack began. The crack spread stably through Specimens D7,8; however, in Specimens D15,19, the condition of energy release was also met and unstable cleavage occurred. These tests illustrate that strength and stiffness of specimens and in some cases the stability of cleavage cracks depend not only on the mean stress, but also on the extent of microcracking present in the specimen.

Previous microcracking had negligible effect on strength when reloading was made at $k \geq 17$. The substantial reduction in stiffness, noted above, is consistent with previous tests¹³ in repeated loading under uniaxial compressive stress.

A specimen which contains extensive microcracking is weaker than a virgin specimen when tested in the cleavage regime because the size and number of cracks capable of initiating cleavage are greater. Strength in the crushing regime is affected less because the fracture mode naturally involves extensive microcracking. Microcracking reduces stiffness partly by permitting relaxation of stress in the stiff coarse aggregate particles which is transferred to the more flexible mortar.

CONCLUSIONS

A criterion of strength for engineering design should recognize that some properties of concrete change when microcracking occurs. In simple cases, such as uniaxial compression, harmful changes can usually be predicted and avoided, although experiments in sustained and fatigue loading indicate that even this is difficult. Some reloading tests in the present experiment illustrate that design rules for more complicated states of stress may be unsafe if based on the strength and stiffness of a virgin specimen in short-term loading. Since danger or uncertainty is due to spreading of microcracks, it appears that confidence would be increased in some cases if the curve of discontinuity replaced the curve of short-term strength as a criterion of strength. Further research is required, however, before recommending a practical method of predicting the curve of discontinuity for different mix proportions, coarse aggregate type and moisture content.

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NOTATION

- e_c = principal compressive strain
 e_t = principal tensile strain
 G = shear modulus
 k = ratio of principal stresses $\frac{p_c}{p_t}$
 p_c = principal compressive stress
 p_t = principal tensile stress
 W_{pl} = average inelastic energy per unit volume absorbed up to maximum stress
 γ = shear strain at middle radius of specimen
 γ^p = difference between observed value of γ and the value predicted by linear theory of elasticity
 τ = shear stress at middle radius of specimen
 τ_0 = intercept of τ/γ relation determined by Least Squares procedure

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PAPER NO. 3 Crack formation and propagation were studied experimentally and theoretically for concrete submitted to static compressive load, to sustained load, and to pulsating load. Prior to first loading, shrinkage cracks were not observed in the cement-aggregate bond phase or in the mortar matrix of the concretes studied. A few well-defined bond cracks developed when the specimens were loaded to about half the compressive strength. These cracks propagated through the cement mortar matrix to other aggregate particles as load was increased, and they ultimately caused the specimens to fail. There was no evidence of gradual deterioration due to widespread formation of cracks in the material structure when the load was increased to failure. Stability of the cracks under sustained and pulsating load was studied. Fracture mechanism of hardened cement paste is discussed.

Cracking and Fracture of Concrete and Cement Paste

By TORBEN C. HANSEN

■ TINY CRACKS DEVELOP IN CONCRETE at stresses far below ultimate failure. Formation and propagation of such cracks have been studied indirectly by sonic and acoustic methods and by measuring surface strains on concrete specimens. Robinson¹ has observed formation and propagation of cracks by x-ray methods. Due to limitations in the technique employed, the detection of cracks was somewhat uncertain. Hsu *et al.*² also studied internal cracking by x-ray techniques. In addition, they examined cross sections of concrete, sliced perpendicular to the direction of applied load, after the specimens had been submitted to loads of different magnitudes, and subsequently unloaded. The cracks were detected by treating the cut surfaces with a colored dye. The dye supposedly penetrated and discolored the cracks, but not the uncracked material. Kaplan,³ Romualdi and Batson,⁴ and Glucklich⁵

have presented theories of crack growth and fracture of concrete, based on Griffith's hypothesis for fracture of brittle materials.⁶

According to Hsu *et al.*,² differential shrinkage between the cement mortar matrix and aggregate particles produces bond cracks even when a concrete is not submitted to any external load. When load is applied, stress concentrations around aggregate particles augment and result in more extensive cracking. The increase in cracking is slight for loads below 30 to 50 percent of the ultimate. Beyond this limit, bond cracks increase rapidly in length, width, and number, the sonic velocity of concrete decreases markedly, the stress-strain diagram deviates from linearity, and there is an increase in the lateral expansion of the concrete. At higher loads, bridging through the mortar matrix between individual bond cracks creates a continuous crack pattern. As the number of cracks increases with increasing load, the concrete gradually crumbles due to general disintegration of the matrix.

In order to apply to concrete the refined theories of crack formation and propagation based on energy considerations, as attempted by Kaplan, Romualdi and Batson, and Glucklich, it is essential that the exact nature of the internal cracking of concrete be established. It was therefore decided to verify the conclusions presented by Hsu *et al.*, by direct microscopic observation of formation and propagation of cracks in concrete specimens during compressive loading. The results obtained did not agree with some of the conclusions arrived at by Hsu *et al.* In addition, new information was obtained on the behavior of internal cracks in concrete submitted to sustained and to pulsating load. Some exploratory investigations of the fracture mechanism of neat cement paste were also carried out.

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FRACTURE OF CONCRETE UNDER STATIC COMPRESSIVE LOAD

Procedures and equipment

The mechanism of crack initiation and propagation in concrete under static compressive load was studied. Specimens, 7.5 by 7.5 cm in cross section and 40 cm long were prepared from concrete having a water-cement ratio of 0.52 by weight, and 325 kg Swedish Standard Portland cement per cu m. High quality granite gravel aggregate, 16 mm max. size, was used. The specimens were stored for 28 days in water at 20 deg C, and then sliced in sections, 7.5 by 7.5 by 2.5 cm. The cut surfaces, displaying cross sections through the material structure, were polished and the specimens were dried in the laboratory for several years. Then, the polished surfaces were examined microscopically at 1000 power magnification in order to determine the extent of possible bond cracking due to shrinkage.

The specimens were placed on edge between the platens of a 20 t Amsler compression testing device. A microscope was attached to the device, so that the polished surfaces could be examined while the load was being applied (Figure 3-1). The microscope was interchangeable with a camera by means of which the crack patterns could be recorded (Figure 3-2). The transducers of a Cawkell Ultrasonic Materials Tester were positioned on the two opposite exposed edge surfaces of the specimens, and the sonic velocity was continuously recorded throughout the experiments, parallel to the plane of the polished surfaces and perpendicular to the direction of applied load. The onset of cracking is indicated by a "marked drop" in velocity of the sonic pulse. The drop in velocity is defined as "marked" when it exceeds the reading accuracy of the apparatus. By using an improved technique developed independently by the author, and Reiding,⁷ it is possible to obtain a reproducibility of 0.02 μ sec. in reading the sonic velocity on the Ultrasonic Tester. After a specimen had been positioned in the testing device, the compressive load was applied in small increments and the polished surfaces were examined continuously for cracks. Crack propagation was recorded after each load increment. Optimum results and convenience were obtained by using 50 power microscopic magnification and 4 power magnification from object to film. Nothing was gained by increasing the microscopic magnification, as all cracks were easily detected under low magnification.

Results

After several years exposure to dry atmosphere, there was no evidence of shrinkage cracks, neither in the cement-aggregate bond phase, nor

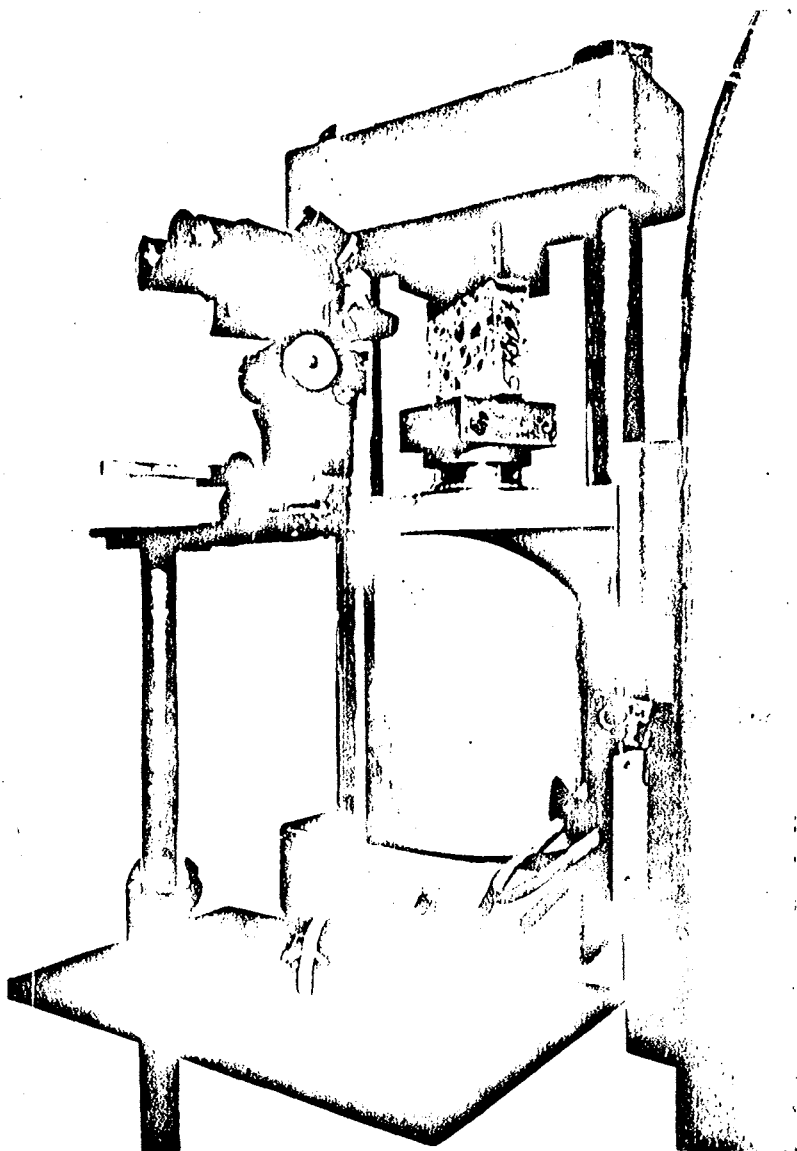


Figure 3-1 Equipment for study of internal cracking of concrete under compression

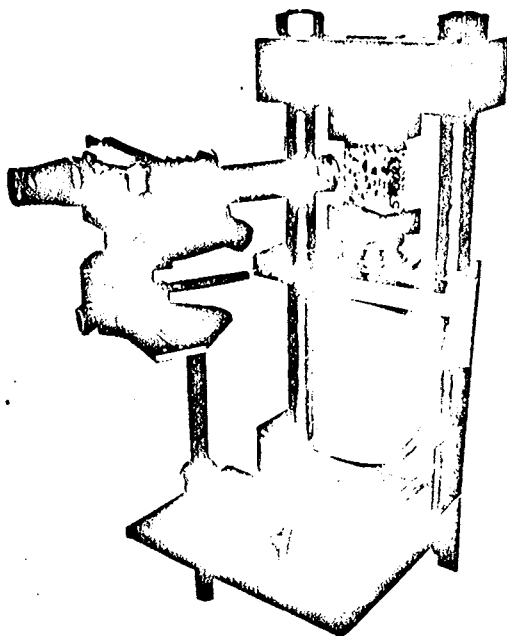


Figure 3-2 Equipment for recording of internal cracking of concrete under compression

in the mortar matrix of the concretes studied, prior to first loading. This is contrary to what has been suggested by Robinson¹ and Hsu *et al.*²

When the specimens were loaded more or less instantaneously, the first crack usually developed at 45 to 55 percent of the compressive strength. This crack appeared abruptly and was accompanied by an immediate drop in the sonic velocity. The crack was always clearly visible at low microscopic magnification, and usually developed midway between the platens of the testing device and in the center third of the specimen. It always developed in the bond phase between the mortar matrix and one of the larger aggregate particles and extended in the direction of applied load. Typically, the crack passed through the mortar matrix to join an air void or another aggregate particle located immediately in its vertical plane, as illustrated by a-a in Figure 3-6. At this stage, the crack was usually 0.01 to 0.02 mm wide (see Figure 3-3 which is a close-up of the small framed portion of Figure 3-6). The

initial length of the crack was one fourth to one third of the height of the specimen. It never extended all the way to the platens, even in two cases where an 0.5 mm thick rubber membrane was placed between the specimen and the platens in order to eliminate lateral restraint from the platens.

The crack usually extended clear through the specimen and could be observed on both opposite surfaces. Observation of the specimens under high microscopic magnification at this stage provided clear evidence that the crack was an isolated phenomenon and not an indication of

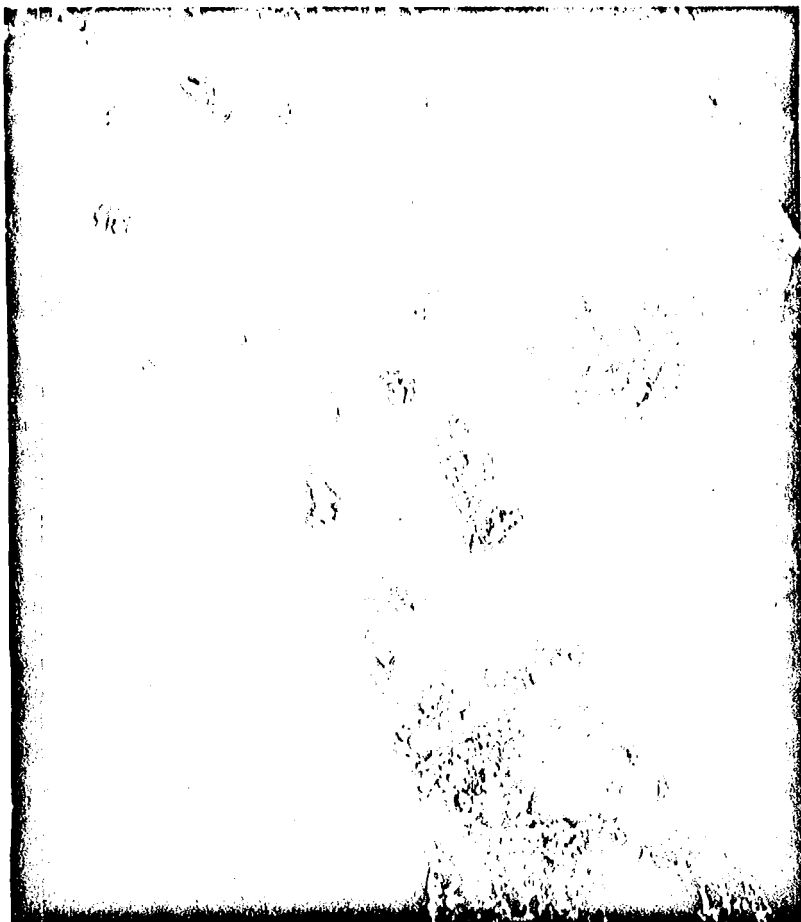


Figure 3-3 Crack in concrete at onset, close-up of Figure 3-6



Figure 3-4 Crack in concrete moments before ultimate failure, close-up of Figure 3-7

more wide-spread disintegration of the material structure. When the specimens were unloaded, the crack almost closed up. After unloading, it was seldom more than 0.005 mm wide. Upon repeated loading and unloading, the crack opened and closed in what appeared to be an almost reversible, elastic manner.

When the load was increased, the crack widened and propagated in the direction of applied load. At 70 to 80 percent of the compressive strength another similar crack occasionally developed at some distance from the first crack. The second crack also continued to propagate

with increasing load, but the material structure remained intact up to failure. Some specimens were examined at high microscopic magnification after loading to 90 percent of their compressive strength. There was no indication of any general disintegration of the matrix.

No more than two independent cracks were ever observed before ultimate failure occurred.

At loads near ultimate failure the cracks would typically be 0.1 mm wide midway between the platens, and more narrow near the platens (see Figure 3-4, which is a close-up of the small framed portion of Figure 3-7 just before failure occurred). Development of one or two 0.1 mm wide cracks in a 7.5 cm wide specimen provides sufficient explanation for the large increase in the transverse expansion, over and beyond the normal elastic dilatation, which is observed in all concrete specimens loaded beyond the level of first cracking.

In specimens loaded directly against the platens, the cracks never extended all the way to the platens. As the load was increased, they stopped growing in length but not in width, and the typical "hour-glass" shaped shear failure pattern eventually developed. In specimens loaded against a rubber membrane, the cracks first split the specimens into two or three separate columns, and ultimate failure then took place due to buckling of the individual columns or to shear failure along inclined surfaces of larger aggregate particles. The experimental technique employed in this investigation did not permit a detailed study of the ultimate failure process. Highspeed photography should be useful for this purpose.

Discussion of results

In order to explain why bond cracks are formed in concrete under compressive load it is expedient to consider the stress distribution around a single elastic particle located in a continuous elastic matrix of infinite extension and submitted to a uniaxial compressive stress.

According to Goodier⁸ the stresses in the elastic matrix surrounding an elastic inclusion can be determined from Eq. 1, 2, 3.

$$\sigma_{rr} = 2E_1 \left\{ \frac{2A}{r^3} - \frac{2\nu_1}{1-2\nu_1} \frac{C}{r^3} + 12 \frac{B}{r^5} + \left[-\frac{2(5-\nu_1)}{(1-2\nu_1)} \frac{C}{r^3} + 36 \frac{B}{r^5} \right] \cos 2\theta \right\} \quad (1)$$

$$\sigma_{\theta\theta} = 2E_1 \left\{ -\frac{A}{r^3} - \frac{2\nu_1}{1-2\nu_1} \frac{C}{r^3} - 3 \frac{B}{r^5} + \left[\frac{C}{r^3} - 21 \frac{B}{r^5} \right] \cos 2\theta \right\} \quad (2)$$

$$\sigma\psi\psi = 2E_1 \left\{ -\frac{A}{r^3} - \frac{2(1-\nu_1)}{1-2\nu_1} \frac{C}{r^3} - 9 \frac{B}{r^5} + \left[3 \frac{C}{r^3} - 15 \frac{B}{r^5} \right] \cos 2\theta \right\} \quad (3)$$

where E_1 = modulus of elasticity of the matrix material
 r = distance from center of particle
 ν_1 = Poisson's ratio of matrix material
 $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\psi\psi}$ = stresses in matrix material defined by Figure 3-5
 ψ, θ = angles defined by Figure 3-5

A, B, C are constants which can be determined from the following equations:

$$\begin{aligned} \frac{A}{a^3} = & -\frac{T}{8E_1} \frac{E_1 - E_2}{(7 - 5\nu_1)E_1 + (8 - 10\nu_1)E_2} \\ & \times \frac{(1 - 2\nu_2)(6 - 5\nu_1)2E_1 + (3 + 19\nu_2 - 2\nu_1\nu_2)E_2}{(1 - 2\nu_2)2E_1 + (1 + \nu_2)E_2} \\ & + \frac{T}{4E_1} \frac{[(1 - \nu_1)\frac{1 + \nu_2}{1 + \nu_1} - \nu_2]E_2 - (1 - 2\nu_2)E_1}{(1 - 2\nu_2)2E_1 + (1 + \nu_2)E_2} \end{aligned} \quad (4)$$

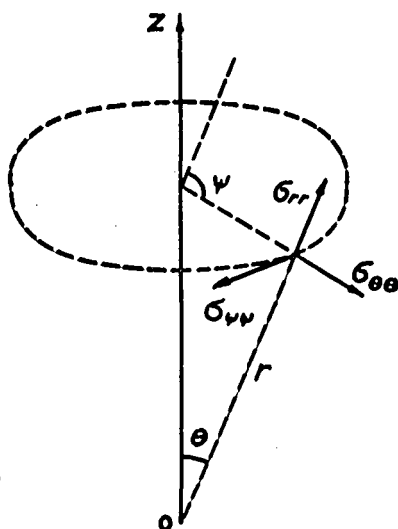


Figure 3-5 Notation used in analysis of stresses around spherical inclusion in matrix of infinite extension

$$\frac{B}{a^5} = \frac{T}{8E_1} \frac{E_1 - E_2}{(7 - 5\nu_1)E_1 + (8 - 10\nu_1)E_2} \quad (5)$$

$$\frac{C}{a^3} = \frac{T}{8E_1} \frac{5(1 - 2\nu_1)(E_1 - E_2)}{(7 - 5\nu_1)E_1 + (8 - 10\nu_1)E_2} \quad (6)$$

where T = applied external tensile stress, positive for tension

ν_2 = Poisson's ratio of particle material

E_2 = modulus of elasticity of particle

a = radius of particle

A spherical cavity and a spherical rigid inclusion occur as limiting cases. For the cavity ($E_2 = 0$) we obtain

$$\frac{A}{a^3} = - \frac{T}{8E_1} \frac{13 - 10\nu_1}{7 - 5\nu_1} \quad (7)$$

$$\frac{B}{a^5} = \frac{T}{8E_1} \frac{1}{7 - 5\nu_1} \quad (8)$$

$$\frac{C}{a^3} = \frac{T}{8E_1} \frac{5(1 - 2\nu_1)}{7 - 5\nu_1} \quad (9)$$

For the rigid inclusion ($E_2 = \infty$) we obtain

$$\frac{A}{a^3} = \frac{T}{4E_1} \left[\frac{1 - \nu_1}{1 + \nu_1} + \frac{3}{16 - 20\nu_1} \right] \quad (10)$$

$$\frac{B}{a^5} = - \frac{T}{8E_1} \frac{1}{8 - 10\nu_1} \quad (11)$$

$$\frac{C}{a^3} = - \frac{T}{8E_1} \frac{5(1 - 2\nu_1)}{8 - 10\nu_1} \quad (12)$$

The question arises whether it is warranted to draw any conclusions concerning the stress distribution around particles in a closely packed system like concrete, on the basis of an analysis of the stress distribution around an isolated particle in a matrix of infinite extension. The single particle model has been shown to give reasonable results when used in the analysis of elasticity and shrinkage of concrete (Hansen^{9, 10}). This is an indication that it may also be applicable in the analysis of internal stresses. Nielsen¹¹ has measured the stresses on cylindrical stress gages embedded in concrete under uniaxial compressive load. The stiffness of the gages was equivalent to the stiffness of various types of aggregate material used. Hast¹² has shown that the compressive stress on one isolated, elastic, cylindrical particle embedded in an elastic matrix of

Infinite extension, with the axis parallel to the direction of applied load, can be calculated according to Eq. 13:

$$\sigma_2 = \left[\frac{E_2[1.125(1 - \nu_1^2)\pi R + l]}{1.125(1 - \nu_1^2)\pi R E_2 + l E_1} \right] \sigma_1 \quad (13)$$

where σ_2 = axial compressive stress on cylindrical particle

σ_1 = applied compressive stress

E_2 = modulus of elasticity of particle

E_1 = modulus of elasticity of matrix

ν_1 = Poisson's ratio of matrix

R = radius of particle

l = height of particle

Table 3-1 shows that there is reasonably good agreement between particle stresses in concrete measured by Nielsen and the stresses calculated according to Eq. 13 for a single particle model. It may therefore be assumed that Eq. 1 through Eq. 12 apply to concrete. The stress in the bond phase between an infinitely rigid particle and the matrix phase, perpendicular to an applied compressive load of magnitude $-T$, can be determined from Eq. 1 when inserting Eq. 10, 11, 12, $r = a$, and $\theta = 90$ deg. We find:

$$\sigma_{rr} = - \left(\frac{1 - \nu_1}{1 + \nu_1} - \frac{5 - 5\nu_1}{8 - 10\nu_1} \right) T \quad (14)$$

For real materials, the numerical value of σ_{rr} depends on the modulus of elasticity and Poisson's ratio of the component materials. When the modulus of elasticity of an inclusion is a few times higher than

TABLE 3-1 CALCULATED AND MEASURED STRESSES ON CYLINDRICAL STRESS GAGES, EMBEDDED IN CONCRETE, LOADED IN COMPRESSION.

Gage No.	Length of gage cm	Diameter of gage cm	Modulus of elasticity of gage $E_2 \times 10^{-3}$ kg/cm ²	Modulus of elasticity of matrix $E_1 \times 10^{-3}$ kg/cm ²	Externally applied stress σ_1 kg/cm ²	Calculated stress on gage σ_2 , calc kg/cm ²	Measured stress on gage σ_2 , meas kg/cm ²
A ₁	4.40	2.00	170	300	134	95	84
A ₂	4.40	2.00	193	300	134	95	112
B ₁	4.40	2.16	448	300	134	164	220
C ₁	3.70	2.16	408	300	134	154	98
C ₂	3.70	2.16	394	300	134	154	116
D ₁	9.00	2.11	523	300	134	192	200
D ₂	9.00	2.11	530	300	134	192	216
E ₁	9.00	1.87	201	300	134	98	98
E ₂	9.00	1.87	252	300	134	99	76

the modulus of elasticity of the matrix material, it can be shown that the stresses around the inclusion approximate the stresses around an infinitely rigid particle. (See for instance, Coutinho.¹³)

Provided certain assumptions are made, the magnitude and sign of σ_{rr} can be estimated. Assuming that the matrix material is cement paste for which the modulus of elasticity is considerable lower than for the aggregate particles, and assuming that $\nu_1 = 0.3$, we find from Eq. 14 a tensile stress $\sigma_{rr} = 0.16T$, acting on the bond phase, perpendicular to the direction of applied load. The tensile strength of the bond phase is determined by the chemical composition and texture of the particle surface, by the water-cement ratio, and by the conditions under which the concrete is cured and stored. If we assume that the tensile strength of cement paste is approximately one tenth of the compressive strength of the paste, and according to Hsu *et al.*, that the bond strength is approximately 65 percent of the tensile strength of cement paste, bond cracks would develop along an aggregate particle at a stress corresponding to 40 percent of the compressive strength. This value is of the right order of magnitude.

The particle size does not affect the magnitude of the stress in the bond phase. However, according to Hsu *et al.*, the strength of the bond between cement mortar and large aggregate particles is somewhat less than the strength of the bond between cement paste and sand grains. Therefore, it can be expected that cracks first appear along the larger aggregate particles.

Formation of cracks in concrete does not take place at random. Once the first crack has formed along an aggregate particle, tensile stresses in the material structure are relieved and subsequent cracks cannot develop parallel to the first crack for a distance of several times the diameter of the particle. Moreover, it is apparent from energy considerations, that it is easier to extend an already existing crack, than it is to form a new crack. This explains why no more than two cracks were ever observed in the test specimens, and why the second crack always developed at a somewhat higher load than the first. The number of cracks formed in a specimen is determined by the size of the specimen, and the distance between the individual cracks is probably determined by the maximum size of aggregate particles.

From the experimental results, it may be concluded that compressive failure of concrete, made with dense aggregate, takes place in two stages. At a comparatively low stress, bond cracks develop parallel to the direction of applied load. Provided there is no friction between the specimen and the platens of the testing device, these cracks continue to grow until the specimen is split into a number of structurally independent columns which ultimately fail in buckling or shear, resulting in complete collapse of the material structure. When there is con-

siderable friction between the platens and the specimen, the cracks never extend all the way to the platens before the specimen fails in shear, producing the well-known "hour-glass" shaped failure pattern.

Although the mechanisms of initial cracking and ultimate failure are different, they are not completely independent. If the strength of the bond between aggregate and cement mortar is low, the compressive strength of the concrete is low, and vice versa. This was shown by Dantinne¹⁴ who tested two series of concrete specimens, one series made with regular aggregate, and the other series made with the same aggregate, but all particles coated with a thin layer of bitumen which prevented any bond from developing. Dantinne found that the strength of the concrete made with coated aggregate was considerably lower than the strength of comparable concrete made with clean aggregate. Other investigators have found that concrete made with cement clinker aggregate, with excellent bonding qualities, has exceptionally high compressive strength. Alexander, Wardlaw, and Gilbert¹⁵ have attempted to correlate bond strength and compressive strength of concrete.

Let us now consider the stress distribution around a void embedded in a cement paste matrix. Inserting Eq. 7, 8, and 9, $r = a$, and $\theta = 0$ in Eq. 2, and assuming that the material is submitted to an externally applied compressive load, we find a tensile stress above and below the void and perpendicular to the direction of applied load, of magnitude:

$$\sigma_{\theta\theta} = \left(\frac{3 + 15\nu_1}{14 - 10\nu_1} \right) T \quad (15)$$

Making the same assumptions as before, concerning the properties of the cement paste, we find that the tensile strength of the paste is exceeded above and below the void, when the externally applied compressive stress is 15 percent of the compressive strength. Chao and Van Vlack¹⁶ observed cracks above and below soft particles embedded in a sintered steel matrix under uniaxial compressive load. At low compressive stresses, such cracks were never observed around air voids in our specimens. However, at higher compressive loads, the cracks would often pass from the side of a large aggregate particle to the top or bottom of an air void located in its path.

Experimental evidence that submicroscopical cracks may form in concrete at a very low level of compressive stress was obtained by Kaiser¹⁷ who studied changes in amplitude of ultrasonic vibrations transmitted through concrete specimens submitted to progressively higher compressive loads. Kaiser found that the amplitude of such vibrations increased rapidly when concrete was loaded from 0 to 5 percent of the compressive strength, but dropped markedly when the load was further increased from 5 to 10 percent. If microscopic cracks