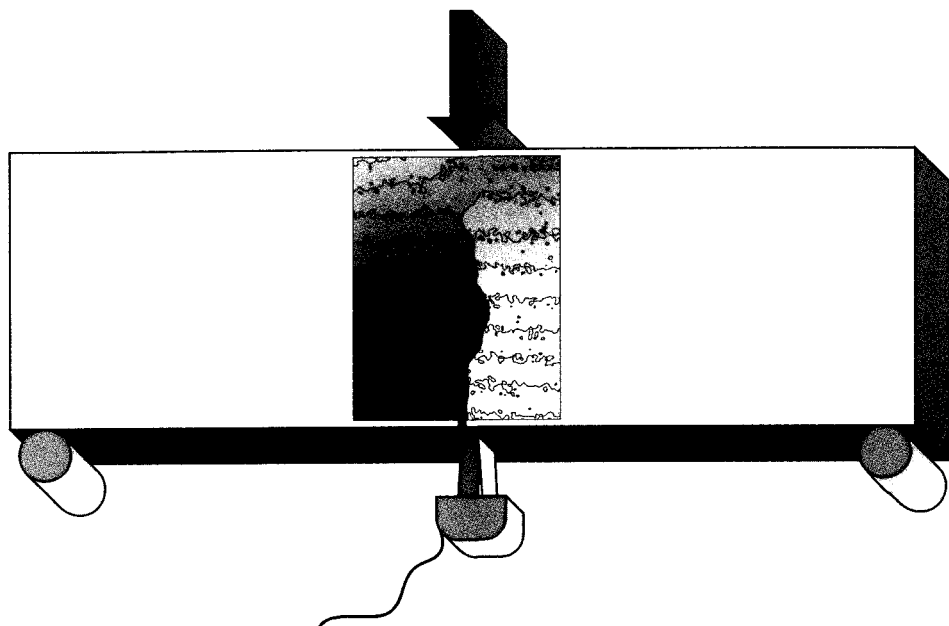


Fracture Mechanics for Concrete Materials:



Testing and Applications

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Full document details: ACI SP-201-01



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Printed in the United States of America

Editorial production: Bonnie L. Gold

Library of Congress catalog card number: 2001091922
ISBN: 0-87031-039-9

PREFACE

Fracture mechanics concepts are increasingly used in designing concrete structures and also in understanding the fundamental behavior of concrete-like materials. Fracture mechanics concepts have the potential for use in designing concrete-FRP composites and in repairing concrete structures. To review and discuss the current state-of-applications of fracture mechanics concepts to concrete materials, ACI Committee 446 sponsored two sessions at the ACI Spring 2000 Convention held in San Diego, California. One objective of this conference and the resulting publication is to provide an update on the use of fracture mechanics concepts to cement concrete, polymer concrete, and FRP-concrete materials.

This conference also provided an effective means of sharing recent technological advances, engineering applications, and research results among practitioners, researchers, and potential users from many parts of the world. Selecting the testing method to determine the fracture properties of concrete materials for various applications is an issue confronting the profession and was discussed during the sessions.

This publication contains the papers that were presented at the ACI Annual Convention held in San Diego, California, March 27, 2000. Also, some additional papers are included in this Special Publication. All the papers in this Special Publication were peer reviewed by at least two reviewers for content, quality, and for possible publication. All papers are eligible for discussion in the *ACI Materials Journal*.

The editors would like to thank the authors for their papers and the reviewers for the constructive and timely reviews of the papers. Thanks to the ACI staff for their role in completing this ACI Special Publication.

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Simplification of the Testing and Analysis Procedure for the Two Parameter Fracture Model

**by D. C. Jansen, W. J. Weiss,
and S. H. F. Schleuchardt**

Synopsis: This work describes a modification to the two-parameter fracture method's experimental procedure aimed at removing this operator/equipment dependence. With this method, three compliances are used to determine the focal point at which these compliances intersect. This focal point is then used to determine the slope of the unloading compliance that corresponds to the peak of the load vs. CMOD curve. The unloading compliance that corresponds to unloading at the peak load and initial compliance are then used to determine K_{IC} and $CTOD_C$ as normally done with the Two Parameter Fracture Model. Use of this method makes it possible to remove operator and machine dependence, especially if the materials are extremely brittle, such as in pastes or high strength concrete, thereby permitting the loading and unloading to be programmed using testing software removing the need for manual operator loading changes.

Tests on 15 mortar beams with 4 different notch lengths and initial unloading points ranging from 97% to 75% of maximum load are used to validate this approach. The experimental results are typically more consistent and better correlate to results from the peak load test method. These results indicate that utilizing the focal point correction typically reduces K_{IC} and $CTOD_C$ by 12% and 38% respectively for the mortar tested thereby causing the TPFM and peak load method results to coincide even more closely.

Keywords: compliance; concrete; cracking; effective crack; fracture mechanics; post-peak; test methods

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Stefan Schleuchardt is an engineer with Wörner und Partner in Frankfurt, Germany. In 1997, he was a visiting researcher at Tufts University during which time the experimental portion of this work was completed. He completed his Diplomingenieur in 1999 from Darmstadt Technical University.

INTRODUCTION

It is widely accepted that the fracture process zone that develops in cement-based materials is not typically negligible in comparison to the overall dimensions of the specimen tested. As a result, analysis using linear elastic fracture mechanics cannot be applied directly to cementitious systems and some modification is needed to account for the region of stable crack growth that occurs prior to the peak load. Over the last three decades several experimental procedures have been developed which provide a minimum of two parameters (i.e., K_{IC} and $CTOD_C$, G_F and c_f , f'_t and G , etc...) which have been used to characterize the behavior of the cementitious system.¹⁻⁷ Numerous testing procedures have been proposed for obtaining these parameters and research continues to develop a standard test procedure that is accurate, requires relatively few samples, can be accomplished in a relatively short time of testing, and does not require a great deal of specialty test equipment.⁸

In 1985 Jenq and Shah proposed one such method to account for the precritical crack growth that occurs prior to the maximum load.² This method, called the Two-Parameter Fracture Method (TPFM), is based on the simple premise that the change in compliance that occurs when the specimen is unloaded at the peak load can be used to calculate the precritical crack growth (ie., length of an effective crack). While this method is of great value since it can be used to determine fracture properties using one specimen it is logistically difficult to unload the specimens exactly at the peak load since the true peak load of a specimen is unknown before the test. For this reason it was recommended that unloading could be conducted when the specimen load decreased to 95% of its maximum load. This however is a frequently discussed as a point of concern citing that this introduces operator and machine dependence especially if the

materials are extremely brittle such as in pastes or high strength concrete. This paper will focus on a modification to the existing two-parameter model, currently a draft recommendation from RILEM,⁹ which will provide a systematic method for removing operator dependence and improving overall accuracy and reproducibility.

BACKGROUND

The Two Parameter Fracture Method

The two-parameter fracture method (TPFM), proposed by Jenq and Shah in 1985, is an effective crack model in which the length of the stable crack growth at peak is characterized to provide a method to account for non-linearities that exist in smaller specimen sizes.² To perform this test, the specimen is loaded and the compliance of the load versus crack mouth opening displacement (initial compliance, C_i) and initial crack length, a_0 , are used to determine the elastic modulus, E , of the concrete as shown in Figure 1. The length of the effective crack length, a_e , can be determined at any time during the test by unloading the specimen and using load-CMOD response (unloading compliance, C_u) in combination with the elastic modulus. Theoretically, the TPFM approach can be used to determine the length of the effective crack at any point along the Load-CMOD curve, however it is of practical interest to determine the length of crack growth exactly at the time the crack would begin to propagate unstably (i.e., the critical crack length, a_c , that occurs at the peak load). The fracture toughness, K_{IC} , and the critical crack tip opening displacement, $CTOD_C$, are determined from the critical crack length and the peak load. More detail of the analysis procedure along with the necessary equations is provided in Appendix A.

Determining the unloading compliance exactly at the peak load is impossible from an experimental viewpoint since one needs to go beyond the peak to know that it has been reached. As a result, the RILEM recommendation⁹ suggests that the specimen be unloaded at 95% of the peak load in the post peak region; however this point is still dependent on the operator and the speed at which the unloading can begin by the equipment from the time of the initial unloading request. This is especially critical in highly brittle materials, such as pastes or high strength concretes in which the rapid decrease in strength after the peak makes it extremely difficult to initiate unloading at 95% of the peak load.

More recently, Tang et al.⁴, proposed a method for determining K_{IC} and $CTOD_C$ requiring only the peak loads from two or more geometrically dissimilar specimens (ie, different notch lengths or loading conditions). With this method, a K_{IC} is determined which gives the smallest standard deviation of the $CTOD_C$. The elastic modulus can be taken from compression tests, from the initial compliance (if measured), or by estimation. The distinct advantage of the peak load method over the two parameter fracture method is that closed-loop test

equipment is not required to determine K_{IC} and $CTOD_C$. The disadvantage is that at least two tests must be performed instead of one and the elastic modulus may not be the same in tension as in compression due to bond between cement paste and aggregates.¹⁰

Crack Extension During the Loading Process and Its Influence on the TPFM

Numerous attempts have been made to characterize the size of the effective crack length or fracture process zone ahead of the crack experimentally including laser holography, microscopy, acoustic emission, dye penetration, re-cutting, epoxy impregnation, ultrasonic wave measurements, and X-rays. Holography, microscopy, and other surface imaging techniques provide a means of assessing the crack throughout the loading process, however these methods provide a measure only of the length of the crack at the surface of the specimen. Typically surface measurement techniques provide an overestimate of the average length of the crack since the length of the crack along the surface of the beam is longer than at the center of the beam indicating that the cracking profile does not propagate uniformly as the specimen cracks.^{11,12}

Alternative non-invasive test methods such as acoustic emission and ultrasonic wave propagation have been used to characterize the length of the crack inside the specimen. Figure 2 shows results from one such investigation where the relative change in compliance for a concrete slab was compared to the length of a crack measured using an elastic wave transmission measurement procedure for assessing crack depth.¹³ First it can be noted that the relative length of the crack growth corresponds well with the change in compliance that is observed as one would expect based on the Two Parameter Fracture Method. Second, it can be seen that this curve can be divided into three regions corresponding to load level. At low load levels before peak (i.e., 0%-60%), the change in compliance (and crack length) with load level has a relatively low slope, thus implying that only minimal crack growth is exhibited in this region. At higher loading levels, in both the pre- and post-peak region, the slope of the load level-crack length degradation is steep implying that more significant crack growth is occurring in this region. This is important because it illustrates that a relatively small change in load level from 95% (post-peak load) to 87% (post-peak load) corresponds to a change in crack length from 66 mm to 82 mm. This suggests that the length of the crack can undergo an increase in length between the peak load and the load level at which the specimen is unloaded, resulting in a slightly longer effective crack (i.e., a less brittle material) that is used in assessing the fracture parameters. Finally, at low load levels in the post-peak region the slope once again is reduced presumably due to crack arrest and from overall cross section reduction.

The additional contribution of the crack growth between the peak load and the point of unloading may be of practical interest when results of this test method are compared with the Peak Load approach. In the Peak-Load method, K_{IC} and

CTOD_C are determined based on only the peak loads from two or more beams with different notch lengths and does not use the unloading compliance; thus the results are not subject to effective crack between the point of peak load and unloading. Figure 3 illustrates that the critical stress intensity factor (K_{IC}) and the critical crack tip opening displacement (CTOD_C) determined using the conventional TPFM method divided by the parameters determined using the peak load approach for normal and high strength concrete tested at different ages.¹⁴ It can be seen that the ratio of these values is typically slightly greater than one. This result would be consistent with the observation that the conventional TPFM method uses an effective crack length that is slightly longer than the effective crack at peak due to crack growth while reaching load reduction to 95% of peak load. Further confirmation of this can be seen in comparing the values obtained from normal and high strength concrete. It would be expected that the critical effective crack length would be smaller as the strength of the concrete increases.¹⁵ As such, the growth of the length of the crack between peak and unloading loads would have a more significant impact on the determination of properties of more brittle materials, thereby accounting for higher ratio of the CTOD_C values for the higher strength concrete.

The Focal Point for Multiple Unloading Curves

Recently, Lee and Willam¹⁶ re-illustrated the idea that if multiple lines, which define unloading stress strain curves, are extended, then they will meet at a unique focal point. Lee and Willam's tests were performed on concrete cylinders tested in compression, and the identified focal point could be used for assessing the change in stiffness using this point and the load displacement curve. Using these results, Tasdemir et al.¹⁷ demonstrated that multiple unloading compliances from notched beam tests also meet at a mutual focal point similar to that as shown in Figure 4. This implies that changes in stiffness (alternatively compliance) can be determined for a specimen at any point thereby implying that the length of the effective crack can be determined at any point along a load-deflection curve provided the focal point of the material is known.

PROPOSED MODEL

This section outlines a procedure for determining the focal point and how this focal point is used to determine the fracture properties of a given material. The proposed method is to find the focal point based on the unloading compliances for the Load-CMOD response. A line is extended from the focal point back to the peak load. This new line provides the 'true' unloading compliance corresponding to the peak load, C_{uc} . If three unloading compliances, shown in Figure 4, are used and the lines are defined by¹⁸

$$P = \frac{1}{C_{u1}} \text{CMOD} + B_1 \quad (1)$$

$$P = \frac{1}{C_{u2}} \text{CMOD} + B_2 \quad (2)$$

$$P = \frac{1}{C_{u3}} \text{CMOD} + B_3 \quad (3)$$

Using error minimization to find the closest point to the lines defined by equations 1-3, the coordinates of the focal point, as shown in Figure 5, can be determined:

$$\text{CMOD}_f = \frac{1}{3} \left[\frac{(B_1 - B_3)}{\left(\frac{1}{C_{u3}} - \frac{1}{C_{u1}}\right)} + \frac{(B_2 - B_1)}{\left(\frac{1}{C_{u1}} - \frac{1}{C_{u2}}\right)} + \frac{(B_3 - B_2)}{\left(\frac{1}{C_{u2}} - \frac{1}{C_{u3}}\right)} \right] \quad (4)$$

$$P_f = \frac{1}{3} \left[\frac{1}{C_{u3}} \frac{(B_1 - B_3)}{\left(\frac{1}{C_{u3}} - \frac{1}{C_{u1}}\right)} + \frac{1}{C_{u1}} \frac{(B_2 - B_1)}{\left(\frac{1}{C_{u1}} - \frac{1}{C_{u2}}\right)} + \frac{1}{C_{u2}} \frac{(B_3 - B_2)}{\left(\frac{1}{C_{u2}} - \frac{1}{C_{u3}}\right)} + B_1 + B_2 + B_3 \right] \quad (5)$$

The unloading compliance which corresponds to the peak load, C_{uc} , is defined by the inverse of the slope of the line which extends from the focal point (CMOD_f, P_f) to the critical (peak) load (CMOD_c, P_c):

$$C_{uc} = \frac{\text{CMOD}_c - \text{CMOD}_f}{P_c - P_f} \quad (6)$$

In the same manner as the TPFM,^{2,9} the elastic modulus is determined from the initial compliance and initial notch length. With the focal point method, the critical crack length, a_c , which truly corresponds to the peak load, is found using the new compliance, C_{uc} , and the elastic modulus. Using the new a_c and the load at peak, the toughness, K_{IC} , and critical crack tip opening displacement, CTOD_c , can be determined using the same equations as with the TPFM.

EXPERIMENTAL VERIFICATION

Experimental Program

To verify the model, 15 mortar notched beam specimens were tested. In addition, 5 cylinders, 102 mm diameter by 203 mm length, were tested in compression. The water:cement:sand ratio for the mortar was 0.50 : 1.00 : 3.00. All specimens were tested between 32 and 33 days after casting. From the cylinder tests, the compressive strength of the mortar was determined to be 41.4 ± 1.4 MPa and the modulus of elasticity, E_c , was 25.4 ± 0.7 GPa.

The beams which were tested to evaluate the fracture model were 559 mm long by 127 mm high by 50.4 mm thick. Notches were cast into the beams, and 4 different initial notch lengths were used (12.7 mm, 25.4 mm, 43.1 mm, and 50.8 mm) corresponding to 10% of the beam depth ($\alpha = a_0/d = 0.1$), 20% of the beam depth ($\alpha = 0.2$), 30% of the beam depth ($\alpha = 0.3$), and 40% of the beam depth ($\alpha = 0.4$).

($\alpha = 0.4$), respectively. The notches were 2.38 mm thick and were tapered to a point at the tip of the notch.

The notched beams were tested in three point bending with an overall span length of 508 mm using a 100 kN closed-loop servo-hydraulic test machine. To measure the crack mouth opening displacement, two ± 0.5 mm LVDTs were attached to the sides of the specimen level to the crack mouth. The average signal from the two LVDTs was used as the feedback control and recorded as CMOD. The tests were performed at a CMOD rate of 0.0004 mm/second, and the points at which unloading occurred were manually signaled to the test machine.

Each specimen was unloaded 3 times. For each notch length, 2 specimens were unloaded at 95%, 85%, and 75% of the peak load (specimens M*A and M*D), one was unloaded at 85%, 75%, and 65% of the peak load (specimen M*B), and one was unloaded at 75%, 65%, and 55% of the peak load (specimens M*C). A summary of the specimens and notch lengths is provided in Table 1. Data was recorded at a rate of 1 Hz throughout the test.

Data Analysis and Results

The data was analyzed to determine K_{IC} and $CTOD_C$ by three methods: 1) the two parameter fracture method^{2,9}, 2) the peak load method⁴, and 3) the focal point method. With the two parameter fracture method, only the compliance from the first unloading was used, and the actual equations along with a more detailed description of the analysis method can be found in Appendix A. The results of the analysis are given in Table 2; the mean and standard deviations were calculated only from the results of specimens where the first unloading compliance started near 95% of the peak load (specimens M*A and M*D). With the peak load method, all specimens were included in the analysis, and K_{IC} and $CTOD_C$ were determined to be 23.9 N/mm^{3/2} and 0.0057 mm, respectively. With the focal point method, for each specimen the focal point of three unloading compliances was determined, from which the unloading compliance corresponding to the line extending from the focal point to the peak load was calculated. The coordinates of the focal point and the unloading compliance from the peak load are given in Table 1. K_{IC} and $CTOD_C$ for each specimen determined from the focal point method are given in Table 2.

Comparison of Results

To further illustrate that K_{IC} and $CTOD_C$ increase significantly as the point at which the initial unloading continues along the post-peak of the Load vs. CMOD curve with the two parameter fracture method, Figures 6a and 7a show K_{IC} and $CTOD_C$ versus percent of peak load at which first unloading took place.

It should be understood that unloading should be done at least at 95% of the peak load and the tests where unloading took place at loads less than this value should not be considered as correct.^{2,9} The mean and standard deviation for K_{IC} and $CTOD_C$ given in Table 2 for the TPFM are determined only using specimens M*A and M*D. This is not unexpected, as discussed earlier, the crack extends significantly during the post-peak, and the TPFM uses this effective crack length when unloading takes place, a_e , as the critical crack length in calculating K_{IC} and $CTOD_C$. The longer crack lengths result in high reported values for K_{IC} and $CTOD_C$. With the focal point correction, K_{IC} and $CTOD_C$ are as significantly affected by the point at which the initial unloading takes place as shown in Figures 6b and 7b, although a consistent and decreasing trend is observed for the specimens unloaded at 75% of peak load (specimens M*C). Furthermore, K_{IC} and $CTOD_C$ determined by the focal point method are reduced by 12% and 38%, respectively, in comparison to the TPFM. The results from the focal point method also correspond more closely to the results from the peak load method than do the results obtained using the two parameter fracture method, as illustrated in Figure 8.

DISCUSSION AND CONCLUSIONS

This paper has illustrated that a focal point can be used to determine the fracture properties of concrete correcting for crack extension which occurs after the peak load. Although in this work the focal point method was determined from the unloading at 3 points along the postpeak, the focal point can also be found from the intersection of two unloadings or even from a single unloading compliance and the initial compliance.

The focal point method has the distinct advantage over the TPFM in that the first unloading does not need to take place while at least 95% of the load carrying capacity of the beam remains, but can take place at nearly any point along the postpeak. By implementing this type of approach, determining the fracture properties of cementitious materials can be automated since a operator controlled unloading is no longer needed, instead preset points of CMOD can be used to signal when unloading is to occur. This implies that operator/equipment response dependence can be eliminated.

ACKNOWLEDGEMENTS

Special thanks go to Dr. John Popovics from the Department of Civil and Architectural Engineering at Drexel University for supplying the data in Figure 2. The third author wishes to thank the Department of Civil and Environmental Engineering at Tufts University for partial support during his visit at Tufts from May to December 1997.

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APPENDIX A. Equations for Determination of E , K_{IC} , and $CTOD_c$

The Jenq-Shah model^{2,9} is a two parameter fracture model allowing a nonlinear quasi-brittle material to be replaced with an equivalent elastic crack model. The measured material properties which define the fracture properties of concrete are the fracture toughness, K_{IC} , and the critical crack tip opening displacement, $CTOD_c$. Following are equations used to determine K_{IC} and $CTOD_c$ from experimental data.^{2,9} The modulus of elasticity from the fracture beam test is

$$E = \frac{6S a_0 V_1(\alpha)}{C_i b d^2} \quad (7)$$

$$V_1(\alpha) = 0.76 - 2.28\alpha + 3.87\alpha^2 - 2.04\alpha^3 + 0.66/(1-\alpha)^2 \quad (8)$$

Where:

S = specimen span length

b = specimen width

d = specimen depth

a_0 = initial notch length

$\alpha = a_0/d$

C_i is initial compliance from the Load vs. CMOD

Once the modulus of elasticity is determined, the effective critical crack length, a_e , may be evaluated iteratively by rearranging equation (7):

$$a_e = \frac{E C_u B W^2}{6 S V_1(\alpha_e)} \quad (9)$$

where C_u is the unloading compliance from the measured Load versus CMOD curve and $\alpha_e = a_e/d$.

The fracture toughness, K_{IC} , is taken at the critical condition (ie. peak load, at which point $a_e = a_c$ and $\alpha_e = \alpha_c$):

$$K_{IC} = \frac{6 Y(\alpha_c) M_c \sqrt{a_c}}{b d^2} \quad (10)$$

where,

$$Y(\alpha_c) = \frac{1.99 - \alpha_c (1 - \alpha_c) (2.15 - 3.93\alpha_c + 2.7\alpha_c^2)}{(1 + 2\alpha_c)(1 - \alpha_c)^{3/2}} \quad (11)$$

and $M_c = (P_c S/4 + p S^2/8)$, P_c = load at peak and p is the distributed load due to self weight of the beam.

The critical crack tip opening displacement, $CTOD_c$, is,

$$CTOD_c = \frac{6 P_{\max} S a_c}{E b d^2} V_1(\alpha_c) \sqrt{(1-\beta)^2 + (-1.149\alpha_c + 1.081)(\beta - \beta^2)} \quad (12)$$

where $\beta = a_0/a_c$.

Table 1. Test Parameters and Measured Values

Specimen I.D.	Initial Notch Length $\alpha = a_0/d$	Peak Load, P_c (kN)	Unloading (% of Peak Load)			Compliances				Focal Point		
			1 st Unloading	2 nd Unloading	3 rd Unloading	Initial Loading C_i (mm/kN)	1 st Unloading C_{u1} (mm/kN)	2 nd Unloading C_{u2} (mm/kN)	3 rd Unloading C_{u3} (mm/kN)	CMOD _f (mm)	Load _f (kN)	C_{uc} (mm/kN)
M1A	0.1	3.53	94%	84%	74%	0.00211	0.00436	0.00662	0.00866	-0.0027	-1.58	0.00325
M1B	0.1	3.60	84%	74%	64%	0.00220	0.00709	0.00995	0.0144	-0.0021	-1.30	0.00349
M1C	0.1	3.63	74%	60%	50%	0.00209	0.0109	0.0159	0.0212	0.0034	-1.12	0.00242
M1D	0.1	3.70	94%	84%	75%	0.00227	0.00484	0.00643	0.00873	-0.0025	-1.44	0.00342
M2A	0.2	2.46	99%	85%	75%	0.00554	0.0128	0.0202	0.0252	-0.0069	-1.25	0.01100
M2B	0.2	2.39	84%	75%	65%	0.00474	0.0175	0.0228	0.0307	-0.0062	-1.13	0.01040
M2C	0.2	2.86	74%	59%	50%	0.00431	0.0188	0.0263	0.0337	-0.0041	-1.39	0.00533
M2D	0.2	2.77	93%	84%	75%	0.00460	0.00843	0.0113	0.0154	-0.0048	-1.30	0.00651
M3A	0.3	2.04	98%	84%	75%	0.00794	0.0139	0.0213	0.0274	-0.0058	-0.94	0.01100
M3B	0.3	2.13	85%	70%	60%	0.00779	0.0198	0.0287	0.0361	-0.0061	-0.85	0.01230
M3C	0.3	2.01	75%	65%	54%	0.00776	0.0265	0.0325	0.0398	-0.0040	-0.88	0.01070
M3D	0.3	1.94	92%	85%	75%	0.01000	0.0170	0.0213	0.0277	-0.0089	-1.03	0.01390
M4A	0.4	1.61	95%	80%	70%	0.01320	0.0267	0.0349	0.0427	-0.0106	-0.88	0.02000
M4B	0.4	1.59	84%	75%	65%	0.01290	0.0299	0.0354	0.0440	-0.0109	-0.85	0.01940
M4C	0.4	1.55	75%	65%	55%	0.01460	0.0356	0.0469	0.0628	-0.0071	-0.84	0.01760

Table 2. Calculated Test Results

Specimen I.D.	Two Parameter Fracture Method		Focal Point Method	
	K_{IC} (N/mm ^{3/2})	CTOD _C (mm)	K_{IC} (N/mm ^{3/2})	CTOD _C (mm)
M1A	28.5	0.0103	25.0	0.0064
M1B	(36.1)*	(0.0194)*	26.1	0.0072
M1C	(46.2)*	(0.0319)*	23.1	0.0031
M1D	31.2	0.0120	26.8	0.0069
M2A	30.4	0.0185	28.1	0.0149
M2B	(37.3)*	(0.0278)*	28.4	0.0144
M2C	(48.1)*	(0.0365)*	25.4	0.0055
M2D	29.5	0.0128	26.1	0.0081
M3A	28.7	0.0122	25.4	0.0079
M3B	(37.3)*	(0.0214)*	28.8	0.0104
M3C	(40.8)*	(0.0295)*	24.8	0.0075
M3D	27.6	0.0138	24.9	0.0094
M4A	31.5	0.0170	26.7	0.0105
M4B	(34.5)*	(0.0196)*	27.0	0.0098
M4C	(35.1)*	(0.0232)*	23.5	0.0061
Mean	29.6	0.0138	26.0	0.0085
St. Dev.	1.5	0.0029	1.7	0.0032

*Specimens M*B and M*C were not used to calculate the mean and standard deviation for the Two Parameter Fracture Model because the first unloading did not occur near 95% of peak load.

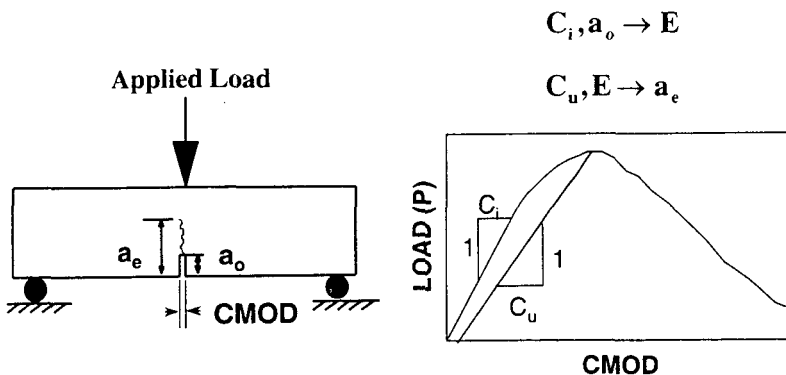


Figure 1: Illustration of the Two Parameter Fracture Method and its use to Determine the Length of the Effective Crack in three point bending²

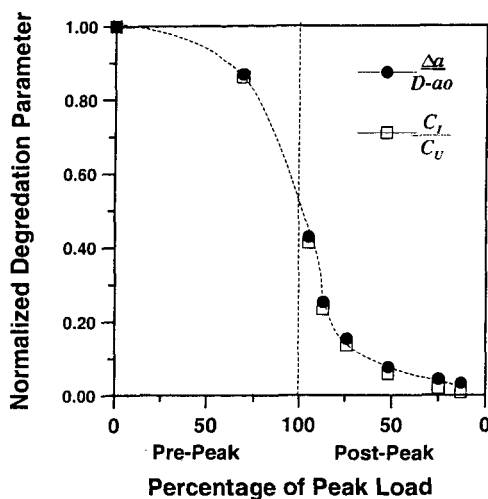


Figure 2: Effective Crack Extension and Compliance Change in the Pre- and Post-peak

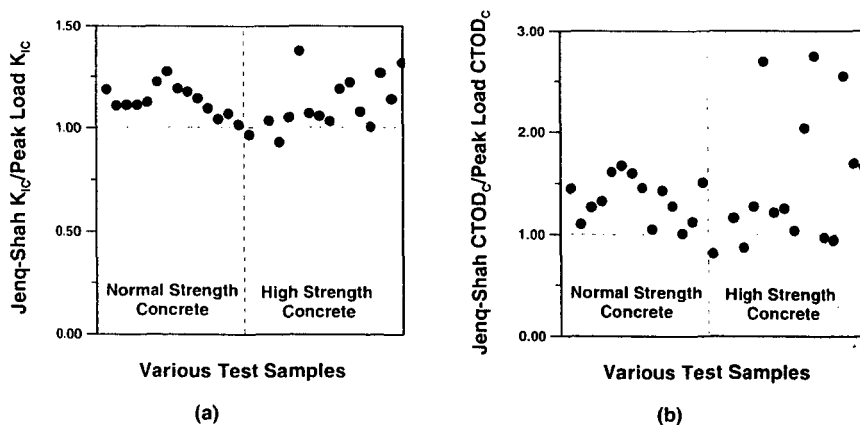


Figure 3: (a) Critical Stress Intensity Factor Values and (b) Critical Crack Tip Opening Displacement Values determined Using the Two Parameter Fracture Method^{2,9} and the Peak Load Method⁴ for Normal and High Strength Mixtures

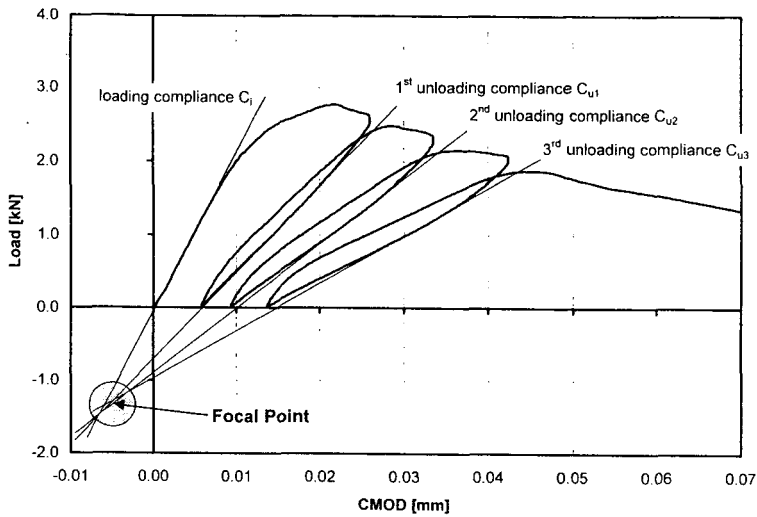


Figure 4: Load vs. CMOD, Unloading Compliances, and Focal Point

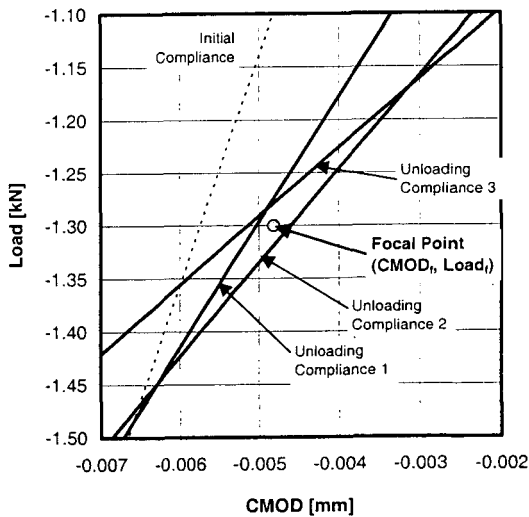


Figure 5. Location of Focal Point

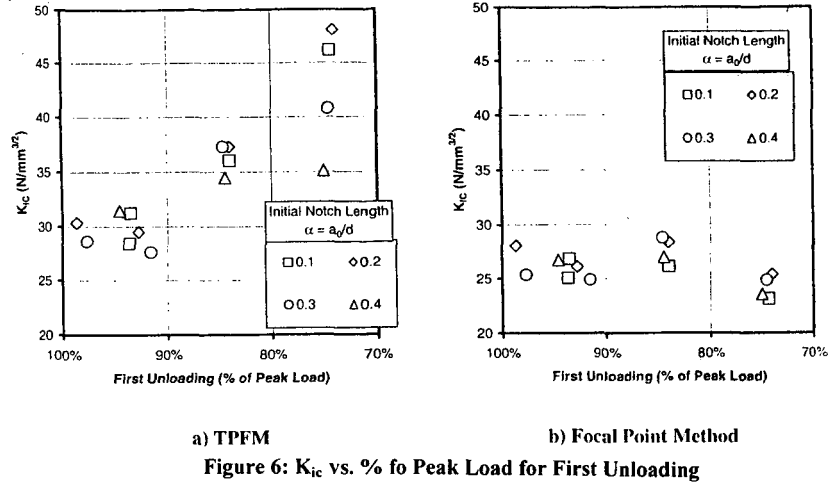


Figure 6: K_{Ic} vs. % of Peak Load for First Unloading

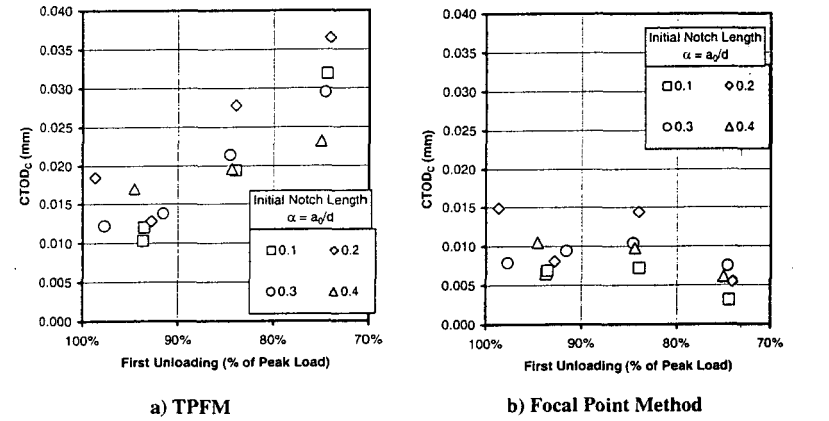


Figure 7: $CTOD_c$ vs. % of Peak Load for First Unloading

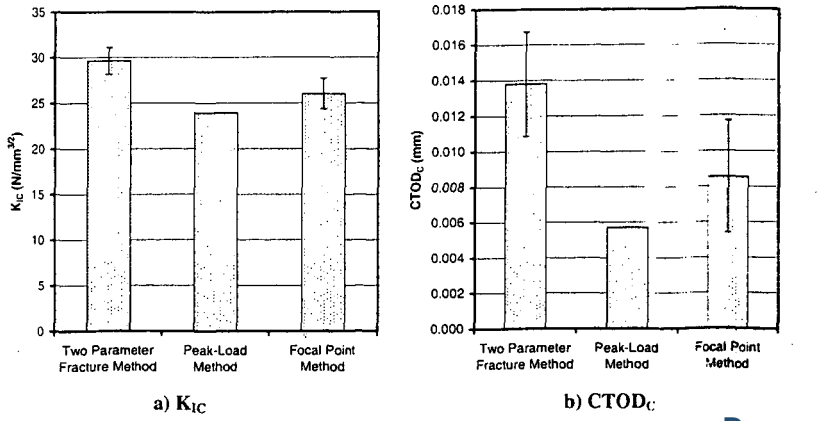


Figure 8. Comparison of Results Obtained by the TPFM, Peak Load Method, and the Focal Point Method

Using Fracture to Predict Restrained Shrinkage Cracking: The Importance of Specimen Geometry

by W. J. Weiss, W. Yang, and S. P. Shah

Synopsis:

Early-age cracking can occur in concrete if free shrinkage is prevented by the surrounding structure. This paper highlights recent findings to illustrate that shrinkage cracking is influenced by the geometry of the structure. A series of experimental results are presented from three different ring specimen geometries to illustrate that although these specimens had the same residual strain level (and similar residual stress), the age of cracking varied with specimen geometry. A second series of experiments was performed to illustrate that a geometry dependence also exists in specimens with moisture gradients.

This paper describes how fracture mechanics concepts can explain this geometry dependent behavior under a uniform moisture distribution. Residual stress levels are computed, non-linear fracture mechanics failure criterion is applied to develop the time and geometry dependent tensile stress resistance curves, and the age of cracking is predicted. The theoretical simulations were found to compare reasonably with the experimental observations. A discussion is provided to illustrate how these considerations may be extended to the specimens with moisture gradients.

Keywords: cracking; early-age; geometry; restrained shrinkage; R-curve; shrinkage; testing

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INTRODUCTION

It is known that when concrete dries it shrinks. If concrete is prevented from shrinking freely, residual tensile stresses develop which, if high enough, can result in cracking. This can be especially problematic at early-ages while the concrete is gaining strength slowly. Historically, models have been proposed to predict the age of (or potential for) cracking based on comparing the time dependent tensile strength of concrete with the residual stress level that develops as a result of restrained shrinkage as shown in Figure 1. In this approach stress is assumed to develop due to a combination of shrinkage stress and stress relaxation. Cracking is assumed to occur when the stress that develops exceeds the tensile strength of concrete. While this method provides a valuable phenomenological description of shrinkage cracking, the accurate prediction of cracking can be more complicated for several reasons as described in the following section.

JUSTIFICATION FOR A FRACTURE MECHANICS APPROACH

If tensile strength based approaches have been used in the past, why advocate the use of a fracture mechanics approach? This question can be answered by considering three primary factors. First, material performance may not be

accurately represented by a sole criterion of tensile strength. This point can be illustrated by considering conventional fiber reinforced concrete (low volume percentages) which typically does not have a significantly different strength than that of plain concrete. However, fibers will influence the crack opening resistance (crack bridging stress) and overall material toughness. As a result, an approach based only on tensile strength does not provide any description of the ability of the fibers to delay the age of cracking or to bridge the cracks when they develop. The second concern that arises with the use of a tensile strength based criterion deals with how accurately this strength can be determined. The following paragraph describes this problem in greater detail. Third, the use of a strength-based criterion does not account for any size or geometry dependence of cracking in field structures. The remainder of the paper will provide proof-of-concept results to illustrate that a failure criterion should be chosen that demonstrates the ability to predict geometry dependence for the development of predictive models.

As previously suggested, it may be difficult to accurately determine the 'true' tensile strength of concrete and over the years numerous tests procedures have been proposed as methods to measure tensile strength. Direct tensile strength tests offer the most straight-forward specimen geometry, however these tests can be difficult to perform due to complexities associated with avoiding eccentricity and providing appropriate end restraint. The time and effort required eliminating these difficulties may make the direct tensile test too cumbersome for use in everyday quality control procedures. To overcome this problem several other specimen geometries have been used to indirectly assess tensile strength including: the pressurized ring [1], tensile-splitting (Brazilian split cylinder), and flexural strength (third point loading). While the values obtained from these tests may commonly be referred to as tensile strength it should be remembered that these are approximations of the 'true' direct uni-axial tensile strength of concrete. Recently, it has been shown that the experimentally measured values for tensile-splitting and flexural strength are dependent on the size of the specimen tested as shown in Figure 2 [2,3,4,5]. This is especially true for relatively small specimen geometries, such as those commonly used for materials characterization and quality assurance testing since the size of the fracture process zone is not negligible in comparison to the overall specimen dimensions. For example, the smaller split cylinder tests typically demonstrate a higher strength which decreases as specimen size increases until it finally begins to increase again for larger specimen dimensions. This behavior has been attributed to the influence of boundary conditions and stress distribution at the loading point [2,3]. In addition, smaller flexural specimens typically demonstrate a higher flexural strength due to the development of the micro-cracking region at the tensile side of the beam that allows some stress redistribution [6]. It has been suggested that the differences between the measured strength of different size and geometry

specimens may be explained with the use of fracture mechanics considerations [7,8,9,10].

It should also be noted that even if the 'true' tensile strength of concrete could be determined easily (and accurately), a strength-based failure criterion may not be appropriate in predicting the failure of restrained shrinkage in structures since the mechanical performance of concrete structures may be size/geometry dependent as well [2,3]. This paper describes two sets of recent experiments that show different specimen geometries may exhibit a dramatically different restrained shrinkage cracking response. The first set of experiments considers the role of geometry under a uniform moisture gradient while the second series illustrates the influence of geometry in the presence of a moisture profile. This paper provides an explanation for these observations in the case of a uniform moisture profile using a fracture mechanics based failure criterion. This approach will be used to show the geometry dependence of the structure on the failure process providing 'proof-of-concept' information to indicate that the accurate prediction of cracking requires fracture based concepts as opposed to a failure criterion based solely on tensile strength.

EXPERIMENTAL PROGRAM

Two concrete mixtures are described in this paper, a normal strength concrete (plain) and a normal strength concrete with a shrinkage-reducing admixture (SRA). Both mixtures consisted of 65% aggregate by volume with equal components of coarse (9 mm) and fine (3 mm) aggregate with a liquid to binder ratio (approximately water-to-cement ratio) of 0.5. The first mixture (plain) was a conventional composition while the second mixture (SRA) had 4% of the water replaced with a glycol-based shrinkage-reducing admixture (SRA). After mixing, the concrete was placed in the forms, rodded, finished with a steel trowel, placed under a plastic sheet to prevent moisture loss, and maintained at 30°C for 24 hours. The specimens were demolded at an age of 24 hrs and placed in a 30°C, 40% RH environment for the remainder of the experiment.

Age-dependent fracture and compressive properties were also measured in accordance with the procedures outlined in the RILEM recommendations [11], ASTM C-39, and C-469 respectively using three specimens at each age. Free shrinkage (ϵ_{SH}) was measured using three 30x100x400 mm³ prisms that were permitted to dry only from the top and bottom surface (100x400mm²; note that the volume to surface ratio is identical to that of the uniform moisture gradient rings). Figure 3 summarizes the average age-dependent mechanical properties of these mixtures. While it can be seen that the mechanical behavior of the plain and SRA mixtures is relatively similar, Figure 4 illustrates that the free shrinkage measured on the concrete containing the shrinkage-reducing admixture (SRA) resulted in a substantial reduction in free shrinkage at both

early and later ages. Further information on the testing procedures or results that are obtained are available elsewhere in literature [12].

Nine ring specimens were tested for each mixture and moisture (drying) condition (3 specimens tested for 3 different geometries). Each ring specimen consisted of a concrete annulus that was cast around a solid steel cylindrical plate with a radius of 150 mm. For the uniform moisture condition the top and bottom surfaces of the ring were permitted to dry while the outer circumference of the ring was sealed to prevent moisture loss. By allowing uniform moisture loss from the top and bottom surfaces, uniform shrinkage strains are developed in the radial direction. The depth of the rings was relatively short (30 mm) to minimize the influence of moisture gradients. Three different wall thicknesses were selected for the concrete rings, 30, 75, and 150 mm to produce specimens with a ratio of the outer radius to inner radius (R_o/R_i) of 1.2, 1.50, and 2.00 as shown in Figure 5 to illustrate the influence of geometry. In addition, a second series of specimen were cast as shown in Figure 5b. While the dimensions of the ring were identical (except for depth which was 150 mm), drying was permitted from the circumferential direction in the second series by sealing the top and bottom surfaces. The ring specimens were monitored every 12 hours for the development of visible cracks and the average age at which cracking was observed in the three specimens is noted in Figure 6. It can be seen that as the wall thickness of the ring increases, the potential for cracking is reduced in both cases. For the uniform moisture condition the specimens should exhibit identical free shrinkage since all the specimens have the same surface to drying ratio. Calculation of the maximum tensile strength indicated that the smallest ring had a stress that was only approximately 4% higher than the thicker ring however differences in the age of cracking were observed. The smallest plain concrete rings cracked on average at approximately 8 days, whereas the medium thickness ring cracked at an average of 10.67 days, and the largest ring had not cracked when the testing was stopped at 70 days. For specimens containing moisture gradients a geometry dependence was also observed as shown in Figure 6b. Again the smallest size specimen cracked earliest while the larger specimen cracked later. In addition, the use of the shrinkage-reducing admixture (SRA) can be seen to delay or prevent cracking in all the rings.

THEORETICAL PREDICTION OF CRACKING POTENTIAL UNDER UNIFORM MOISTURE GRADIENTS

Stress Development

The residual stress that develops in concrete as a result of restraint can not be computed by directly multiplying the free shrinkage by the elastic modulus since the concrete is sensitive to stress relaxation (creep). As a result the residual stress that develops in a thick walled ring specimen over time can be

written in terms of the radial (σ_r) and circumferential (σ_θ) component using the following equations. (Note: This derivation assumes uniform shrinkage in the radial direction. A full derivation of the stress computation procedure can be found elsewhere in literature [12]):

$$\sigma_\theta = \int_0^t A(\xi) \left(1 + \frac{R_o^2}{r^2} \right) d\xi \quad (1)$$

$$\sigma_r = \int_0^t A(\xi) \left(1 - \frac{R_o^2}{r^2} \right) d\xi \quad (2)$$

in which R_o represents the outer radius, r represents the radial coordinate, and $A(\xi)$ represents a time constant of integration which can be obtained by solving equation 3 which satisfies the condition that zero displacement is permitted at the internal boundary.

$$\int_0^t \left[C(t, \xi) \right] \left[(1 - \nu) + (1 + \nu) \frac{R_o^2}{R_i^2} \right] A(\xi) d\xi + \varepsilon_{SH}(t) = 0 \quad (3)$$

where R_i denotes the inner radius, ν is Poisson's ratio, ε_{SH} is free shrinkage, and $C(t, \xi)$ is the creep compliance (in this work $C(t, \xi)$ is considered using the approach described by Muller [13]). Using this approach and incorporating the age dependent material properties the maximum tensile stress (circumferential σ_{θ, R_i} at $r = R_i$) that develops can be predicted. Once the level of residual stress is determined, an appropriate failure criterion can be applied to assess whether the ring can be expected to crack.

Failure Criterion

The failure criterion used in this model relates the fracture resistance (R-Curve) of a material to the energy consumed in propagating (driving) a crack (G-Curve). The material is thought to reach its peak load (failure) when the energy rates are equal ($G=R$), as are their derivatives ($dG/da=dR/da$) implying the G-Curve is at the outer edge of the stable failure region as shown schematically in Figure 7.

Resistance Curve - The resistance curve (R-Curve) used in this investigation is based on the derivation of Ouyang and Shah [14] with the form shown in equation 5

$$R = \beta \psi (a - a_0)^{d_2} \quad (5)$$

where a is the crack length, a_0 is the initial crack length (assumed in this case to be 2 mm), and β , ψ , and d_2 can be determined from the critical crack length

(a_c), critical stress intensity factor (K_{IC}), and elastic modulus (E) using the equations shown below.

$$d_{1,2} = \frac{1}{2} + \frac{\alpha - 1}{\alpha} \pm \left[\frac{1}{4} + \frac{\alpha - 1}{\alpha} - \left(\frac{\alpha - 1}{\alpha} \right)^2 \right]^{1/2} \quad (6)$$

$$\beta = \frac{K_{IC}^2}{E(\alpha a_0 - a_0)^{d_2} \left[1 - \frac{(d_2 \alpha - \alpha + 1)}{(d_1 \alpha - \alpha + 1)} \right]} \quad (7)$$

$$\underline{\psi} = 1 - \frac{(d_2 \alpha - \alpha + 1)}{(d_1 \alpha - \alpha + 1)} \left[\frac{\alpha a_0 - a_0}{a - a_0} \right]^{d_2 - d_1} \quad (8)$$

In these expressions α denotes the ratio of the critical crack length (a_c) and the initial crack length (a_0) (i.e., $\alpha = a_c/a_0$). To determine the value for the critical crack length, Shah et al. [15] used a procedure where the theoretical expressions for a tensile loaded single edge-notched (SEN) specimen was set equal to the critical material properties (critical stress intensity factor and the critical crack tip opening displacement). The advantage of this approach is that the material properties could be determined experimentally using another specimen geometry.

Closed-form solutions for the crack tip opening displacement (CTOD) were unavailable for each ring geometry. Therefore to determine the R-Curve for the ring geometries used in this investigation, a weighting function approach [16] was used to approximate the crack tip opening displacement for each geometry as shown in Figure 9a. It should be noted that the values shown in Figure 9a were computed for specimens with the same size initial crack length ($a_0 = 2$ mm for a 12 inch diameter ring irrespective of the wall thickness). Figure 9b shows the R-Curves that result for specimens with different wall thicknesses using this approach. It can be seen that the R-Curve for finite sized specimens is very similar to that of the infinite specimen geometry up until the point at which the peak load is reached, (i.e., the point at which the R-Curve response flattens). This finding confirmed that the R-Curve response of the SEN specimen could be used to describe the response of all the ring specimen geometries.

Crack Driving Energy - The G-Curve represents the energy that is available to cause the crack to grow (i.e., drive the crack). The shape of the G-Curves change as specimen wall thickness increases as shown in Figures 7 and 8. As the wall thickness of the specimen increases the G-Curve becomes flatter and eventually approaches a straight line. This signifies that a larger value of the

G-Curve would be necessary in a larger specimen to cause it to come in contact with the R-Curve, thereby implying that a higher stress level is required to cause a thicker specimen to fail. Mathematically the G-Curve is written using the classical fracture definition that describes the energy that is supplied to propagate the crack (G-Curve) as,

$$G = \frac{(\sigma \sqrt{\pi a} f(\varphi_w))^2}{E} \quad (9)$$

in which E is the elastic modulus, σ is the maximum tensile stress level (in this case circumferential stress at the inner radius $\sigma_{\theta-R_I}$), a is the crack length, and $(f(a/w))$ is the geometry correction factor. This geometry factor is unity in the case of an infinite body with an internal crack and 1.12 in the case of semi-infinite specimen with an edge crack. In this work, the expressions for the geometry factor were obtained from Wu and Carlsson [16] as listed below.

$$\begin{aligned} f(\varphi_w)_{1.25} &= 1.12 - 0.0887(\varphi_w) + 3.8572(\varphi_w)^2 - 0.0006(\varphi_w)^3 - 0.7679(\varphi_w)^4 \\ f(\varphi_w)_{1.50} &= 1.12 - 0.6648(\varphi_w) + 5.3858(\varphi_w)^2 - 6.7727(\varphi_w)^3 + 3.6573(\varphi_w)^4 \\ f(\varphi_w)_{2.00} &= 1.12 - 1.5692(\varphi_w) + 5.5320(\varphi_w)^2 - 7.8713(\varphi_w)^3 + 4.5378(\varphi_w)^4 \end{aligned} \quad (10)$$

Computation of Geometry Dependent Failure Strength - The critical resistance stress level (geometry dependent tensile strength), $\sigma_{\theta-R_I}$, was calculated for various ages of drying using age-dependent material properties (Figure 3) with the R-Curve approach. This was done by incrementally increasing the stress level until the failure criterion is met ($G=R$ and $dG/da=dR/da$). Figure 10 illustrates the geometry dependent tensile strength (stress resisted) of the different specimens. It can be seen that as the wall thickness of the specimen becomes larger, (increasing R_O/R_I) the specimen becomes more resistive to cracking (i.e. the tensile strength increases). This is significant in that it differs from a maximum strength based approach in which a unique tensile strength exists that is independent of specimen geometry.

Comparison of Experiments and Theoretical Predictions

Figures 10 and 11 show the age of cracking that is defined as the age when the residual stress that develops due to restraint is equal to the strength predicted using the fracture approach. Figure 10 illustrates that the combination of the lower strength of the smaller specimen and the slight increase in stress level combine to result in a significant difference in the age of cracking. Comparing the experimental observation with the predicted values as shown in Figure 11 shows that while the fracture approach predicts a slightly earlier age of cracking the general trend is relatively similar to the behavior observed in the experiments (i.e., geometry dependence). However, the tensile strength prediction (based on a percentage of the compressive strength) would predict a

more similar age of cracking for all the mixtures and the only differences that are observed are attributed to the difference in residual stress level. The degree to which the geometry dependence exists is dependent on the material composition and a reduction in brittleness corresponds with stronger geometry dependence.

DISCUSSION OF CRACKING WITH MOISTURE GRADIENTS

While the stress field that develops in the uniform moisture case always has the same form (i.e. $1/r^2$), the stress field that develops in specimens with moisture gradients differs as a function of drying time due to the influence of the moisture gradient. At early-ages the gradient is extremely sharp, however over time as the specimen dries the profile eventually softens and develops into a more uniform gradient across the cross section. For this reason the geometric correction factor which is used for both the crack opening displacement and stress intensity factor (i.e., crack driving energy and crack resistance curve) is more difficult to compute for these specimens. The geometry factor is based on the relative shape of the stress field and since the shape of the stress field varies as a function of drying time so will the geometry correction factor. In addition the high concentration of stresses at the specimen surface probably results in micro-cracking which will relieve the high stress levels that develop. While research continues to incorporate these considerations into the modeling approach described in the previous section it is clear from Figure 6b that the age of cracking increases with specimen thickness.

Figure 11 illustrates a simplified approach that can be used to estimate the residual stress development in thick sections [17,18]. In this approach the structure is discretized into a series of fictitious elements that were stacked in the direction of the member thickness. The humidity and depth dependent strain function can then be used to determine the residual stress level that develops. Using this type of approach it can be shown that the total load that is generated at early-ages by specimens of various thickness is typically fairly similar since the interior of the section is saturated and shape of the humidity and shrinkage profile is relatively similar near the surface. As a result the local stresses that develop at early-ages are also fairly similar. This implies that using a maximum stress or load criteria would predict the same age of cracking (especially at early-ages), however this behavior is not observed. This implies that the failure of specimens with non-uniform moisture conditions also demonstrates thickness dependent behavior, however further modeling is required to confirm this hypothesis.

SUMMARY AND CONCLUSIONS

In conclusion, this paper has illustrated that an ultimate strength criterion may be inappropriate to accurately predict the potential for restrained shrinkage cracking due to geometry considerations. Experimental results showed that the age at which shrinkage cracking occurs (or potential for shrinkage cracking) can be influenced by the specimen dimensions. Residual stress development was calculated and a non-linear elastic fracture mechanics theory was utilized to predict the age of cracking for specimens of different geometry. A favorable correlation was observed between the theoretically predicted age of cracking and the experimentally observed age of cracking.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the center for Advanced Cement-Based Materials (ACBM) and Grace Construction Co. – Conn.

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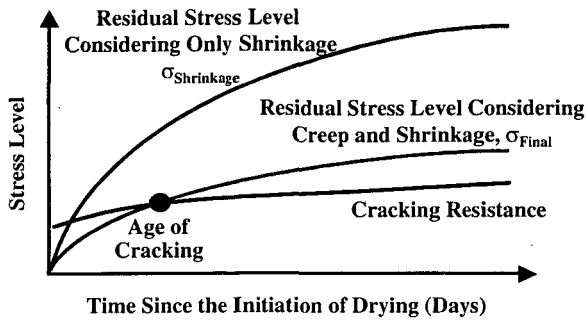
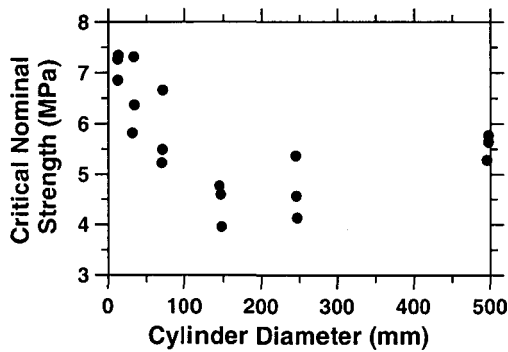
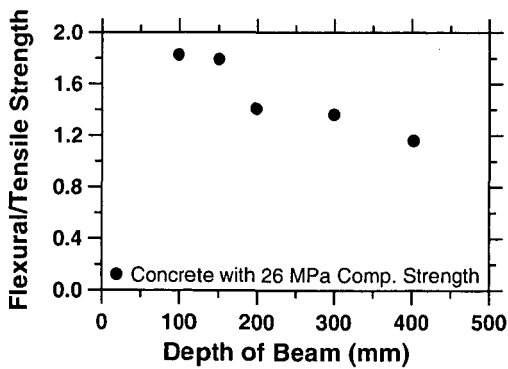


Fig. 1: Stress Development and Early-Age Cracking Potential



(a)



(b)

Fig. 2: Size Dependence in (a) Splitting Tensile Strength (After [4]) and (b) Flexural Strength (After [5])

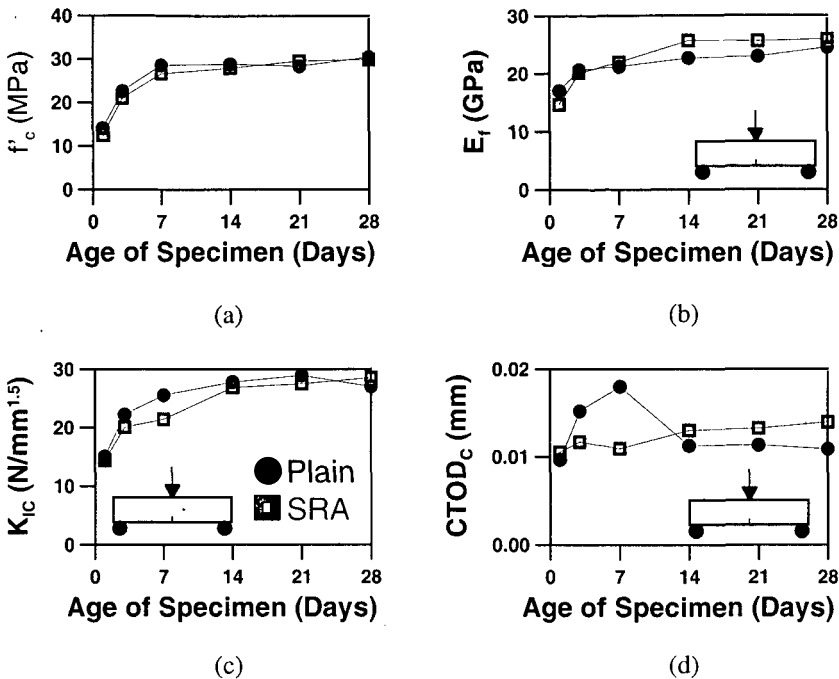


Fig. 3. Age Dependent Material Property Development for the Plain and SRA Mixtures (a) Compressive Strength, (b) Elastic Modulus, (c) Critical Stress Intensity Factor, and (d) Critical Crack Tip Opening Displacement

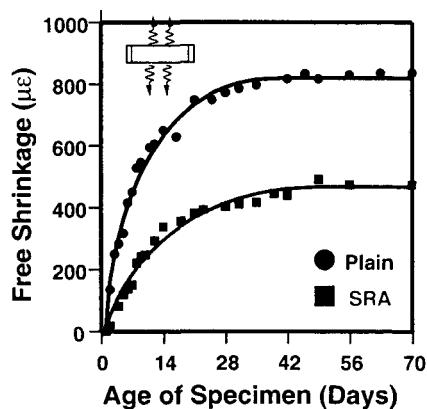
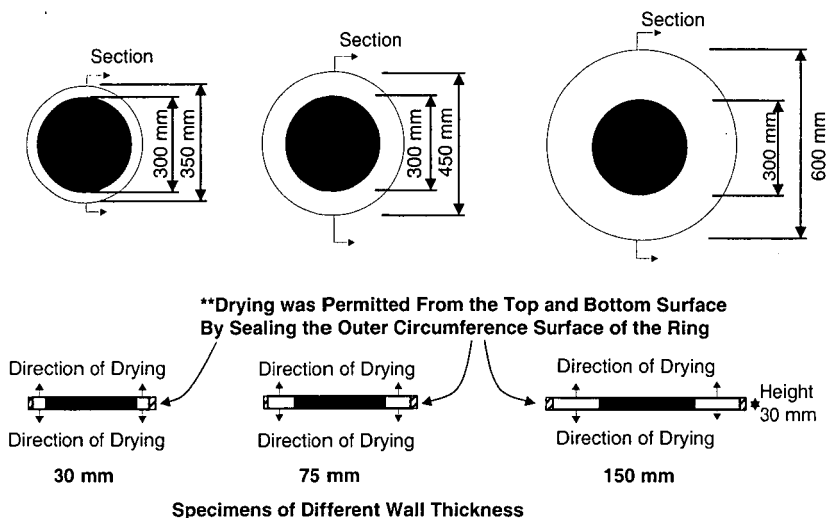
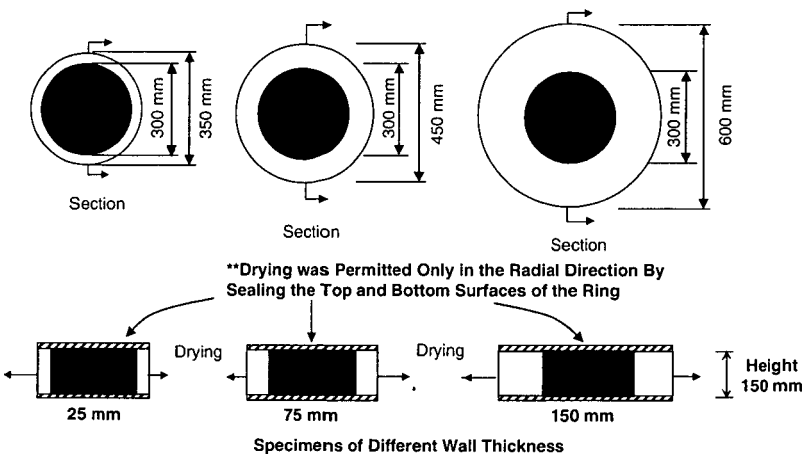


Fig. 4: Free Shrinkage Measured on a 30 mm x 100 mm x 400 mm Section Sealed on the Sides for Normal Strength Concrete With (SRA) and Without (Plain) a Shrinkage Reducing Admixture

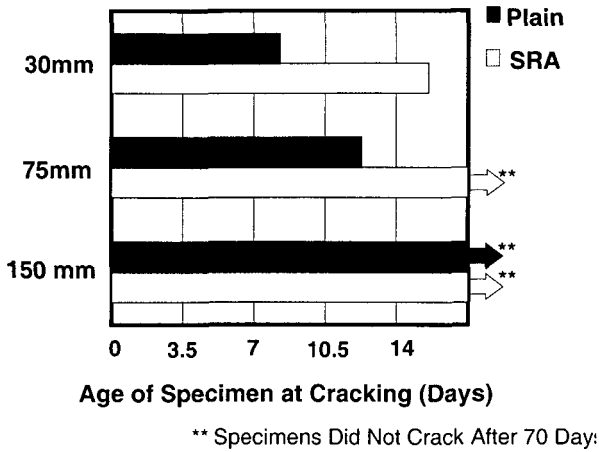


(a)

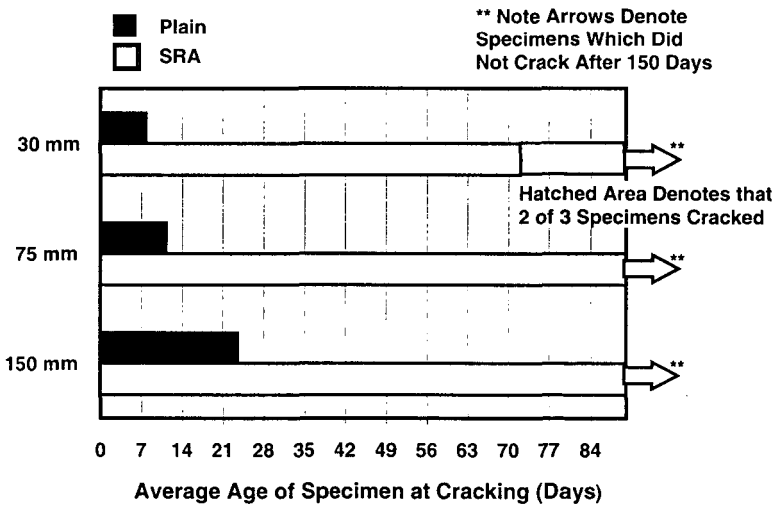


(b)

Fig. 5. Specimen Geometry: (a) Specimens with a Uniform Moisture Profile and (b) Specimens with Moisture Gradients



(a)



(b)

Fig. 6. Average Age of Cracking for Rings with Different Wall Thickness: (a) Specimens with a Uniform Moisture Gradient and (b) Specimens with Moisture Gradients

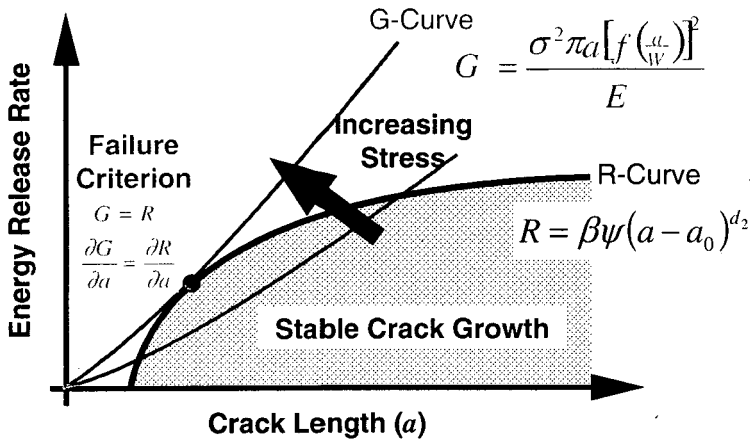


Fig. 7. A Conceptual View of the R-Curve

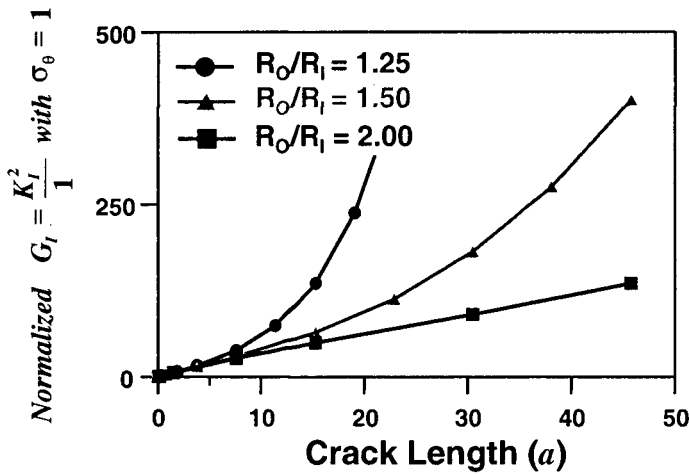


Fig.8. The Geometry Dependent G-Curve Response

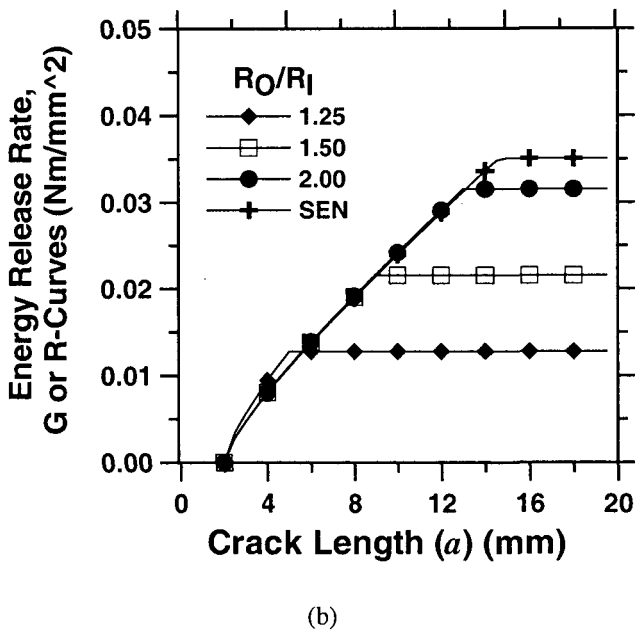
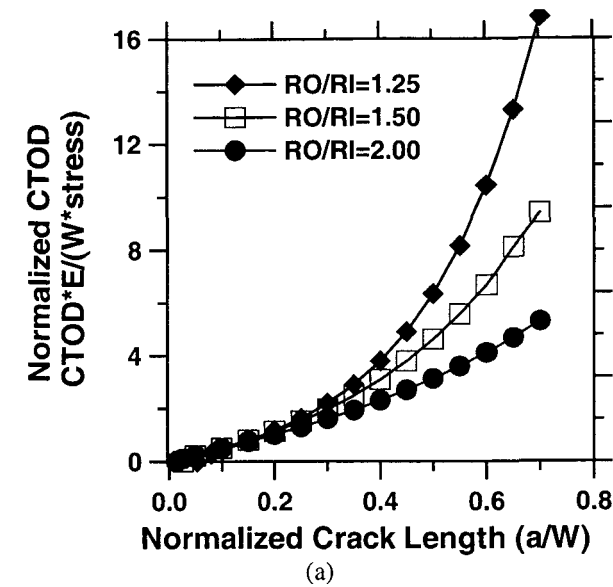


Fig. 9. (a) Geometry Dependent Crack Tip Opening Displacement Functions and (b) the R-Curves Computed Using These Geometry Dependent Crack Tip Opening Displacement Functions

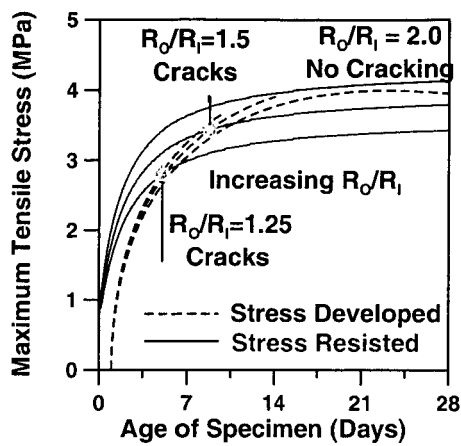


Fig. 10. Residual Stress and Geometry Dependent Strength Development

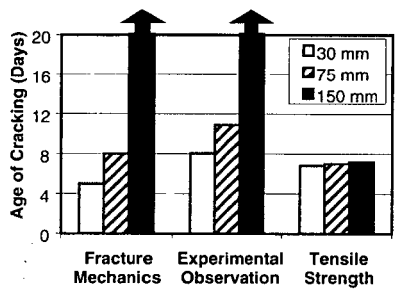


Fig. 11. Age of Cracking Predicted Using the Fracture Mechanics Approach Compared With Experimental Observation

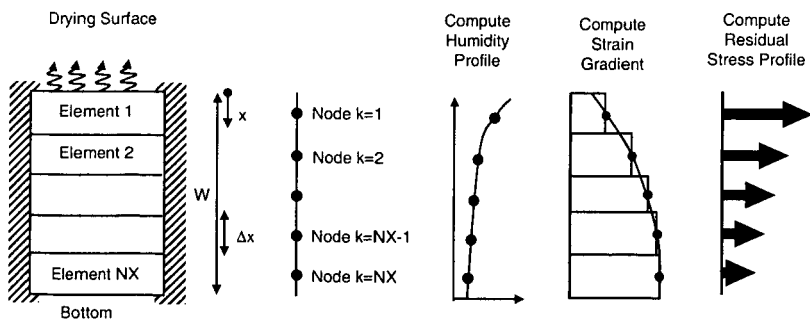


Fig. 12. A Conceptual View of the Prediction Procedure for Assessing the Cracking Potential of Thick Sections

The Fracture Surface Roughness of Concrete with Different Aggregate Sizes and Loading Rates

by Y.-S. Roh and Y. Xi

Synopsis:

Fracture surface provides valuable information on internal structure and mechanical behavior of composite materials. Loading rate affects the roughness of the fracture surface of composites. A higher loading rate, in general, results in a smoother fracture surface. Similarly, aggregate size influences the roughness of the fracture surface. Larger aggregates cause a rougher fracture surface under the same loading rate. The roughness of the fracture surface of concrete is experimentally studied using concrete specimens made of the different aggregate sizes under different loading rates. Fractal dimension is used to evaluate the surface roughness of concrete specimens. A new fractal fracture model is developed which correlates the fractal dimension with concrete mix design parameters, such as volume fraction and size of aggregate, as well as loading rate. The model prediction agrees with test data very well.

Keywords: aggregate size; fractal; fracture; loading rate; roughness

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INTRODUCTION

Fracture properties of concrete have not been very well characterized by conventional linear elastic fracture mechanics. The nonlinear and inelastic behaviors have been attributed to irregularity of crack surfaces and nonhomogeneity of internal structure of the material^{1,2}. For these reasons, quantitative description of fracture surface and correlation of the features of the internal structure and the fracture surface have been of great interest in recent years^{1,2,3,4}. Different fracture processes are reflected by different fracture surface topographies. The quantitative characterization of fractography not only enables us to identify the micro-mechanisms, but also provides us with valuable information about energy absorption in the process of fracture, which bridges the gap between micro-mechanical and the corresponding macroscopic fracture behavior of quasibrittle solids, such as concrete.

Using a three-point bend specimen, Strange and Bryant¹⁰ found an increase in fracture toughness with an increase in aggregate size. For the same concrete mixes, a crack meets an aggregate particle, it passes along the matrix-aggregate interface, and then re-enters the matrix. Larger aggregate particles result in a greater increase in crack surface than smaller particles and thus, require more energy for the crack propagation. Nallathambi et al.¹¹ showed that fracture toughness increases with an increase in the maximum size of coarse aggregate. They stated that microcracking and matrix-aggregate debonding during the process of crack propagation consume considerable energy. Therefore, the larger the aggregate, the larger the crack area, increasing the energy demand required by the crack growth. Perdikaris and Romeo¹² indicated that aggregate size has a considerable influence on the fracture energy of concrete. For both the normal and the high strength concrete with 25 mm (1 in.) aggregate, the fracture energy of the concretes was about twice the fracture energy of the concrete containing 6 mm (0.25 in.) aggregate. They concluded that, for concrete with the larger aggregate, there is a higher degree of matrix aggregate interlock, resulting in an increase in the energy required for crack propagation. In short, it is well accepted that the crack that develops in concrete as it is loaded to failure is affected significantly by the aggregate particles, especially by the coarse aggregate¹³.

The main purpose of this paper is to establish a quantitative correlation among the fractality (fractal dimension, or roughness) of the fracture surface, the degree of heterogeneity (size of the inclusion), and the mechanical effect (loading rate). A carefully designed testing program was conducted and a theoretical model was developed based on the obtained test data.

EXPERIMENTAL PROGRAM

The experimental investigation was carried out to examine the effects of aggregate size and loading rate on the strength of concrete. Four different aggregate sizes were used in the study. With different aggregates, the concrete mixes were identified in the study as hardened cement paste, mortar, concrete with fine aggregate, and concrete with coarse aggregate. Two different loading rates were used: static loading rate (0.00001 to 0.000002 in/sec) and moderately fast loading rate (0.1 to 0.0625 in/sec.). 4 in. by 8 in. concrete cylinders were used in the study. The main testing variable was the average or maximum aggregate sizes (d_{avg} , d_{max}). Testing parameters are depicted in Table 1. The water-cement ratio was kept as a constant at 0.5 except for the cement paste specimens, in which $w/c = 0.45$ was used. The ratio of total aggregate volume fraction to total volume of concrete, a/T , was 0.7.

Since the fracture surfaces from tensile failure and compressive failure could be different, two different loading configurations were selected: direct compression test and splitting tension test. It should be noted that fracture surface due to tensile failure can also be characterized by the three point bending test. The splitting tension test was selected in the present study for convenience (the same set of compressive platen can be used both for the tension and the compression test. The failure surfaces from the direct compression test may not be well defined, but there is a lack of better alternative for investigating the failure surface due to compressive failure. In total, 51 specimens in compression and 52 specimens in splitting tension were tested.

FRACTAL SURFACE PROFILES

Previous researches showed that a crack surface profile can be effectively described in terms of fractal geometry^{5,6,7,8}. There are numerous devices and methods for tracing and recording the profile of a fracture surface. A three-dimensional scanner was used in the present study for evaluating surface profiles. Basically, a mechanical probe is stretched on to a crack surface, and elevations (distance from the original probe position to the crack surface) are measured at a regular interval along the crack profile with 25 μm resolution. The maximum

scanning length is $0.1m$. The topological information provided by the 3D scanner is then used to determine surface roughness number (RN), which is defined as $RN = A_a(\delta) / A_o(\delta)$, where A_a and A_o are the actual surface area and the projected surface area, respectively, and δ is the grid size or measurement interval. The sum of all elemental areas provides an approximation for the actual surface. Thus, the roughness number of a fracture surface can be calculated as $RN = \Sigma \Delta A_a(\delta) / \Sigma \Delta A_o$.

From the experimentally determined RN, fractal dimension of a surface, D , can be evaluated, which is commonly defined by the expression $A_a(\delta) = A_o r^{-(D-d)}$, where d is the topological dimension ($d = 1.0$ for the line profile estimation and $d = 2.0$ for the area estimation), and r is a unit length scale. The fractal dimension, D , can be evaluated by plotting the roughness number versus the measurement size in a Richardson plot¹⁷. The slope of the linear region in the log-log plot can be given by the relationship $D = d - slope$.

Figs. 1 to 4 show the fracture surfaces obtained by the 3-dimensional scanner from direct compression specimens with different aggregate sizes at the static loading rate. The scale of measurement (resolution) was from $25\mu m$ to $0.1m$. Comparing Figs. 2 and 4, one can see that a carefully selected range of measurement scale is very important for evaluating RN. For instance, the surface roughness of the mortar (Fig. 2) may be higher than the roughness of the concrete surface (Fig. 4) if one uses very small scale. This is because only a very small local surface area in Fig. 4 (e.g. a small part of aggregate surface) would be measured, which does not represent the global surface profile of the concrete. Hence, the measurement scale that is larger than the maximum aggregate size should be included in the range of the scale.

THE EXPERIMENTAL RESULTS

Fig. 5 shows maximum aggregate size versus compressive strength and splitting tension strength of the concretes. One can see that both the compressive strength and the tensile strength increase with increasing aggregate size. As shown in Fig. 6, the fractal dimension also increases with increasing aggregate size. In both figures, the ranges of the increases in strengths and fractal dimension are larger for compressive specimens than for tensile specimens. The present experimental results are consistent with other test results^{13,14,15,16}.

Fig. 7 shows the effect of loading rate on concrete strength. One can see that the compressive strength is increased at higher loading rate, as expected. While the tensile strength does not change much with increasing loading rate, which is mainly due to the limited range of the loading rate used in our study (the high loading rate used is the upper limit of our MTS machine).

Bentur and Mindess¹³ compared crack patterns in different types of plain concrete subjected to bending as a function of loading rate. They observed that cracks in normal strength concrete tend to form around the aggregate particles, passing along the matrix-aggregate interface. At a higher loading rate (10 in./min.), cracks propagate through the aggregate particles, resulting in straight crack paths. Their study of crack propagation was carried out with contoured double cantilever beams. From the experimental results obtained in the present study, a similar observation can be made. The fractured surfaces are smooth when the fast loading is applied, while the surfaces are rather rough when the slow loading is applied. A qualitative explanation of this behavior is that when the loading energy is introduced into the system in a very short period of time, cracks are forced to develop along shorter paths, thus, resulting in crack propagation through aggregate particles. The difference of the surface roughness due to loading rate can be further characterized quantitatively by fractal dimension. As shown in Fig. 8, the fractal dimensions for both compression and tension decrease with increasing loading rate.

Fractal dimension obtained by the two-dimensional analysis in this study was between 2.05 to 2.21 in the compression test and 2.03 to 2.08 in the tension test. In the literature, most of reported results were based on one-dimensional analysis and thus the fractal dimension was in the range of 1.1 to 1.2. Saouma and Barton¹⁴ utilized the box counting method to characterized concrete fracture profiles and concluded that fractal dimension, D , was 1.20 at the microscopic scale and 1.10 at the macroscopic scale. Also they concluded that the roughness of the fracture surface does not vary significantly with the direction of crack propagation. Issa and Hammad¹⁵ reported that D was around 1.10 and indicated that the fractal dimension as well as the profile roughness increased slightly with crack propagation. Whalley and Orford¹⁶ reported a fractal dimension of 1.06 to 1.13. Although other researchers used one-dimensional analysis and did not consider the difference between the crack surfaces of compression and tension, the reported results are in good agreement with our results.

FORMULATION OF A NEW FRACTAL MODEL

There are many different scenarios of crack development in cementitious materials. Among these different scenarios, four fracture types are selected in this study. These four fracture types are then modeled based on fractal theory³. Fractal dimensions of the four fractal models are derived. Three of the fractal models simulate a crack going around an aggregate particle, and one of them simulates a crack going through an aggregate. The major advantage of the proposed new fractal models is that the fractal dimension of a fracture surface can be correlated to the volume fraction and the size distribution of the aggregate. The shape of aggregate can also be simulated. The hexagon is selected as an example in this paper to demonstrate the formulation of the new model. In the

same manner, aggregate of any shape can be simulated by using a general polygon.

In the case of static loading, most crack propagates along the aggregate boundaries. A unit element box of the size $L_o \times w_o$ can be used to represent a section of a crack path. The same unit element can be duplicated along the direction of crack path, say y direction. In the out of plane direction (direction z), it is assumed that the unit element does not change so that the one-dimensional profile can be expanded to a two-dimensional surface. If, for example, the fractal dimension $D = 1.2$ in one direction, then the fractal dimension of a two-dimensional surface composed of the unit element becomes $D = 2.2$. It should be mentioned that, in the proposed fractal model, the difference of fractal characteristics between the direction of crack propagation and the transverse direction has not been considered. The following is a brief derivation of the four fractal models.

Fractal fracture type A

The fracture type A consists of a hexagon (aggregate) exposing two sides to the crack path as shown in Fig. 9. The image shown in Fig. 9 illustrates two aggregates exposed to the crack surface, which is similar to the type A fractal model. Now, let us consider N be the number of crack segments, $N = L_t / \varepsilon_i$, $r = \varepsilon_i / L_o$, where, L_t is the total length of the crack path in the type A. L_o has been chosen so that ε_i is constant for all three segments. The number of crack segments is the summation of the crack in the matrix as well as in the aggregate-matrix interface, $N = N_{matrix} + N_{agg}$. r is defined as a ratio of similarity, and it is a function of relative aggregate size ε_i . From the basic fractal theory, r can be written as:

$$r = \frac{\varepsilon_i}{L_o} = \frac{1}{1 + \sqrt{3}} \quad (1)$$

where ε_i is the length of the crack segment or one side of the aggregate (called relative aggregate size of the type A; and L_o is the height of the unit element. w_o is the width of the unit. The fractal dimension is defined by:

$$D = \frac{\log N}{\log(1/r)} = 1.0931 \quad (2)$$

The volume fraction of the aggregate can be calculated by:

$$V = \frac{V_{\text{aggregate}}}{V_{\text{total}}} = \frac{3\sqrt{3}}{2} \frac{\varepsilon_i^2}{w_o L_o} = 0.3481 \frac{L_o}{w_o} \quad (3)$$

Fractal fracture type B

In the case that more aggregate surface is exposed in a crack path, Type B fractal model can be used to simulate the fracture surface, as shown in Fig. 10. A similar procedure can be used to derive fractal parameters for the type B, which are $N = 4$, $r = \frac{1}{3}$, $D = 1.262$, and $\bar{V}_{\text{agg.}} = 0.2887 \frac{L_o}{w_o}$. Comparing the fractal dimensions of the type A and the type B, one can see that the larger the portion of aggregate exposed to the crack, the higher the fractal dimension. The fracture type B contains less aggregate volume fraction compared to the type A, because ε_{Ai} (ε_{Ai} for the fracture type A) has a larger length at the fixed L_o and w_o .

Fractal fracture type C

Fractal fracture type C also has 50% of aggregate exposed against a crack path like fracture type B, however, it is rotated 90° with respect to the longer aggregate axis, as shown in Fig. 11. This particular crack path has a dramatic change of crack angle (90°) when a crack deflects from the matrix to the aggregate. In this case the number of crack segments is 8 due to the special geometry of the crack path. The other fractal parameters are $r = \frac{1}{2 + 2\sqrt{3}}$, $D = 1.2245$ and $\bar{V}_{\text{agg.}} = 0.3481 \frac{L_o}{w_o}$.

Fractal fracture type D

Instead of going around aggregates, a crack can also go through aggregates as shown in the image of Fig. 12, in which the crack path breaks three aggregates into fragments. The type D fractal model is designed for this fracture scenario, which is more complex than the cases that a crack goes around aggregates. This is because the crack orientation may change randomly throughout the crack surface, that is, the crack may deflect inside the aggregate. In order to simulate the change of crack orientation, the deflection angle α is introduced inside the aggregate (see Fig. 12). The deflection angle affects the fractal dimension of the model, which means that fractal parameters of the model depend on the deflection angle α . Like all other fracture types, the type D has the same length of crack segment ε_i . Table 2 shows fractal dimensions and volume fractions of the type D with different angle α . In the case that α is zero,

the crack has no deflection inside the aggregate, going straightly through the aggregate. Hence, the fractal dimension is simply 1.0 that corresponds to a flat surface.

Combinations of the fractal fracture types

A real fracture surface is a combination of many possible crack patterns. In order to have a more realistic simulation of a fracture surface, one can combine the four fracture types in many different ways. The following is an example for a combination of A_2 & B_2 . The subscript 2 means a certain size of aggregate, and this subscript varies from one to n for different sizes of aggregate. So, in the present model, the fractal dimension for each fractal type is also a function of aggregate size. In this particular combination, ε_i is kept the same for all types and element width is a constant. The fractal parameters for this combination can be calculated as following

$$N = 7, L_o = (4 + \sqrt{3})\varepsilon_i, r = \frac{1}{(4 + \sqrt{3})} \text{ and } D = \frac{\log 7}{\log(4 + \sqrt{3})} = 1.1145$$

Table 3 shows the fractal dimensions for different combinations of the fractal types. It is possible to combine all fractal fracture types with different sizes of aggregates. In this way, a real fracture surface can be simulated by a proper combination of fractal fracture types.

In Figs. 14 and 15, the type B and type C give similar fractal dimensions for the fracture surfaces obtained from the compression test in both the slow and fast loading cases. Our calculation shows that the crack patterns in the compression test seem to be similar to the type B and type C rather than the type A or the type D. From visual inspection, more of the type B and C were observed on the failure surfaces of the compression test. On the other hand, the type A and D can be used to simulate fracture surfaces of the tension test that had smaller fractal dimensions than that of the compression test (see Figs. 16 and 17). More of the type A and D were observed on the failure surfaces of the tension test.

CORRELATION BETWEEN FRACTAL DIMENSION AND LOADING RATES

The fractal theory can be applied even further to consider the effect of loading rate on fracture surfaces. The fractal crack length has been defined as $L_i = N\varepsilon_i$, which N is the number of segments (or crack segments), ε_i is the measurement scale. The number of segments N can be modified as N' , in order to include the effect of loading rate, $N' = N \cdot \psi$, where ψ is the segment density factor that indicates crack segment density and can be expressed as:

$$\psi = b^{1-\xi} \quad (5)$$

in which b = the ratio of loading rate, which may be assumed as the ratio between an elemental component of crack L_o and its growth increment ΔL_o ; and ξ is the loading rate coefficient. Thus, Eq. (5) becomes the following empirical relationship:

$$\psi = \left(\frac{L_{oi}}{\Delta L_o} \right)^{1-\xi} \quad \text{where } 1 < \psi \leq b \quad (6)$$

There were two loading rates applied in this study. The ratio of the segment density factors between the two different loading rates can be obtained. The faster loading rate used in this study can be considered as the reference loading rate (0.1 in/sec); therefore, the segment density factor, ψ , may be taken as one. The general trend is that the segment density factor increases as the loading rate decreases (i.e. at a very fast loading rate, the segment density factor reduces to one, which corresponds to a flat fracture surface). Of course, more experimental studies are needed using various loading rates in order to obtain the proper values of parameter ξ , and the segment density factor, ψ .

Using Eq. (6), the decrease of fractal dimension as loading rate increases can be simulated for the cases of mortar and coarse aggregate concrete, the results are shown in Fig. 18.

CONCLUSIONS

The fracture surface of concrete can be quantitatively characterized by the fractal theory. Fractal dimensions of concrete specimens with different aggregate sizes under different loading rates were measured. The difference of the fractal dimension for crack patterns under direct compression and splitting tension was experimentally studied.

1. The strength of concrete and the fractal dimension of fracture surface of concrete increase with increasing aggregate sizes.
2. The compressive strength of concrete is increased with higher loading rate. The strength enhancement due to high loading rate does not appear in the tension test in the range of the loading rates used in this study.
3. The fractal dimension decreased with increasing loading rate.

A novel fractal fracture model is proposed in which, the fractal parameters can be correlated to the intrinsic parameters of the microstructure, such as sizes and volume fraction of aggregates and macro-mechanics quantities, such as