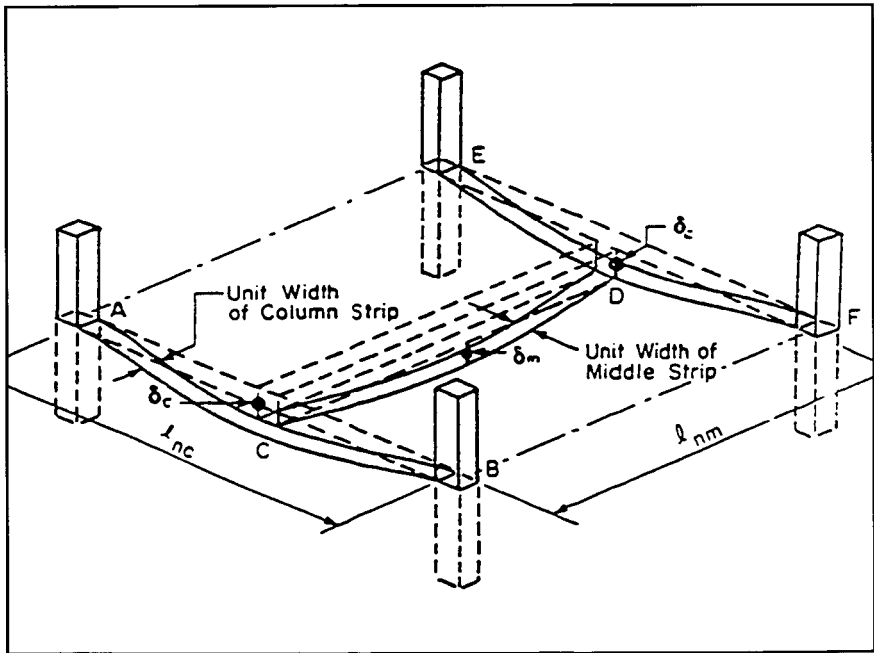


CODE PROVISIONS FOR DEFLECTION CONTROL IN CONCRETE STRUCTURES



Co-Editors: EDWARD G. NAWY
& ANDREW SCANLON

Code Provisions for Deflection Control in Concrete Structures

Co-Editors

Edward G. Nawy

Andrew Scanlon



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PREFACE

Design for serviceability as determined by deflection control is central to the work of structural engineers, code-writing bodies, and the users. The present era of high strength and high performance materials, exotic additives and limit states design, has necessitated more refined control of the design of more efficient constructed facilities. In this manner, more short-term and long-term performance can be achieved even if more slender members are used, and in many cases deflection control may be the governing design criterion.

Different codes differ in their approaches to the mitigation of deflection problems. Hence, serviceability provisions of various national codes are expected to differ significantly. With all the factors presented, ACI Committee 435, Deflection of Concrete Structures, generated a highly successful and fully subscribed technical session at the ACI San Diego Convention, entitled "Code Provisions for Deflection Control in Concrete Structures." It was co-chaired by the co-editors of this Special Volume and includes all the papers presented at the meeting, plus additional ones that further enhance the total body of knowledge on the subject of deflection.

Eleven papers are contained in this volume. The topics cover critical reviews of deflection control provisions of the ACI 318 Code, the latest Euro Code, the United Kingdom Code, the Australian Code, and the Brazilian Code, span/thickness limitations for deflection control, effect of flange width on deflection, deflection predictions, bending stiffness effects, time-dependent deflection and cracking effect on flat slabs, and comparison of deflection variability as affected by concrete strength in normal and high strength concrete structural elements.

This volume is a good companion to ACI Committee 435's extensive report "Deflection Control of Concrete Structures," which covers all aspects of deflection reduction and control in reinforced and prestressed concrete structural beams, and one-way and two-way slabs and plates. All the papers in this publication were reviewed by recognized experts in accordance with ACI review procedures, with each paper having been reviewed by no less than two reviewers. It is hoped that designers, constructors and codifying jurisdictions will be able to draw on the material presented in this volume and avail themselves of the benefit of exposure to provisions of codes from other countries and international entities, for better control of deflection performance of constructed facilities.

Co-Editors:

Edward G. Nawy, Past Chair, ACII Committee 435

Andrew Scanlon, Chair, ACII Committee 435

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ACI Code Requirements for Deflection Control: A Critical Review

by A. Scanlon, D. R. Cagley Orsak, and D. R. Buettner

Synopsis: ACI Building Code requirements for deflection control are critically reviewed. Provisions for minimum thickness, deflection computations, and permissible computed deflections are reviewed. Differences in the approaches to deflection control for one-way and two-way construction are identified. Limitations in the application of the prescribed deflection calculation method are discussed. Results of a survey of consulting firms concerning deflection control in design offices are presented. The paper concludes by suggesting possible directions for future changes in building code requirements for deflection control.

Keywords: beams; building code; cracking; deflection; minimum thickness; slabs

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INTRODUCTION

Deflection control is an important serviceability consideration in the structural design of concrete buildings. While provision of an adequate level of safety against collapse is the primary design consideration, the structural engineer must take into account possible adverse effects of excessive deflections on the performance of the structure at service load levels. Potential problems associated with excessive deflections are well known and include damage to nonstructural elements including partitions and windows, jamming of doors and windows, gaps between partitions and floors, and between columns and floors, improper operation of equipment, visual perception of sagging floors and ceilings, and the need to provide expensive floor leveling materials.

Some guidance is provided in Section 9.5 of the ACI Code (ACI 318-99) on design for deflection control of one-way and two-way nonprestressed construction, prestressed concrete construction, and composite construction. The general approach to deflection control in the code has remained essentially unchanged since 1971. This paper presents a review of the current provisions with some suggestions for improvement of these provisions.

DEFLECTION CONTROL AND THE BUILDING CODE

The introduction to the 1999 ACI Code contains the statement:

“A building code states only the minimum requirements necessary to provide for public health and safety. The code is based on this principle.”

Because deflection control is an issue which for the most part has no impact on public health and safety the question is sometimes raised as to why deflection control should be part of the building code. Some engineers have expressed the opinion that deflection control is an issue between the design engineer and client and should not be dealt with in the building code. A structure may have unacceptably large deflections and yet have an adequate margin of safety against collapse. Deflection control is an economic issue involving a balance between first cost and potential costs associated with maintenance, repair and other costs that might be incurred as a result of a problem related to deflection. There is also the question of the engineer's professional reputation and potential professional liability issues. Deflection control has traditionally been mentioned in the code even though it is not a safety issue. Engineers will continue to look to the code both for guidance and criteria for some measure of acceptability in deflection levels.

Because serviceability criteria are not as clearly defined as safety criteria and vary much more depending on requirements of particular projects, the question of establishing minimum requirements becomes quite difficult. The engineer may well have to exceed the minimum requirements given in the code to meet the requirements of the client. In many cases, the client will rely on the professional judgment of the engineer as to the needs for a particular project.

ACI 318 REQUIREMENTS

The ACI Code provides a two-tier approach to deflection control. Under certain circumstances deflection control requirements will be deemed to have been satisfied if the flexural member depth is greater than a minimum value expressed as a fraction of the span length. Otherwise the code requires that deflections be computed and compared with specified permissible values. Details of these provisions are outlined in the following sections.

Minimum Thickness Requirements

Minimum thicknesses for one-way slabs and beams are given in Table 9.5(a) of ACI 318-99. In this table span length is defined as clear span plus depth of member but need not exceed distance between centers of supports. The table is reproduced in Appendix A.

These minimum thicknesses can be used for members NOT supporting or attached to partitions or other construction likely to be damaged by large deflections. Members that do support or are attached to partitions or other

construction likely to be damaged by large deflections are not covered by Table 9.5 (a) and consequently, deflections should be computed for these members and compared to specified permissible values.

Minimum thicknesses for two-way slab systems are presented in Table 9.5 (c) and Eqs. 9-11 and 9-12 of ACI 318-99. In this case the span length to be used is defined as length of the clear span in long direction, measured face-to-face of supports. In contrast to one-way construction, no restrictions are placed on use of these minimum thickness values for members supporting partitions. However the commentary does note that "The minimum thicknesses in Table 9.5 (c) are those that have been developed through the years." and "These limits apply to only the domain of previous experience in loads, environment, materials, boundary conditions, and spans." This cautionary note in the commentary suggests that care should be taken in selecting the slab thickness for arrangements that fall outside the realm of previous experience.

The minimum thickness requirements for one-way and two-way construction appear to have developed independently and as a result a number of inconsistencies can be identified in comparing the two sets of requirements:

- a) Different definitions of span length: There appears to be no reason to use different definitions of span length. For members built integrally with supports, whether one-way or two-way, clear span seems to be the logical choice while for members not built integrally with supports, distance between edge of bearing seems to be a reasonable definition. The question also comes up in determining permissible computed deflections as discussed later.
- b) Different levels of conservatism for one-way and two-way slabs: If a two-way flat plate is designed based on minimum thickness and subsequently changed to a one-way system by addition of stiff beams or walls along column lines in the short span direction, an increase in minimum thickness or deflection calculations would be required. It is not logical to increase the slab thickness when stiffening elements have been added. Scanlon and Choi (1999) showed that the one-way slab minimum thickness values are generally conservative for typical building spans and recommended an alternative approach to minimum thickness. Proposals for revising minimum thickness of one-way or two-way construction have also been presented by Rangan (1982), Grossman (1981), Thompson and Scanlon (1988), and Gardner and Zhang (1995) among others.

- (c) Different treatment of partition damage: The minimum thickness for two-way construction is applicable even if the slab is supporting partitions, which is not the case for one-way slabs.

Computed and Permissible Deflections

The code prescribes a simplified methodology for computing deflections although more comprehensive methods are permitted as long as effects of cracking and reinforcement on member stiffness are considered for immediate application of load, and creep and shrinkage are taken into account when considering long-term deflection. The effective moment of inertia concept is used to allow for a gradual transition from uncracked to fully cracked stiffness as loading increases. Effective moment of inertia is given by,

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr}$$

(1)

Where,

$$M_{cr} = \frac{f_r I_g}{y_t}$$

(2)

and

$$f_r = 7.5 \sqrt{f_c}$$

(3)

A simple long-term multiplier is provided for the computing of long-term deflections. The multiplier is applied to the computed immediate deflection corresponding to the sustained load level considered. The multiplier is given by,

$$\lambda = \frac{\zeta}{1 + 50\rho} \quad (4)$$

Where ζ takes the following values

5 years or more.....	2.0
12 months.....	1.4
6 months.....	1.2
3 months.....	1.0

The compressive reinforcement ratio is included in Eq. 4 to account for the effect of reinforcement in the compressive zone which resists creep deformation. The commentary points out that long-term deflections are affected by many factors including temperature, humidity, curing, and age at time of loading. Many references are available for computing creep and shrinkage effects separately including those listed in the commentary and the ACI 435 report on Control of Deflection in Concrete Structures (ACI 435-95R). The simple multiplier is considered by ACI 318 to be satisfactory for use with the code procedures and the limits specified in Table 9.5 (b).

If minimum thickness cannot be used to satisfy deflection control requirements, deflections must be computed and compared to specified permissible deflections listed in Table 9.5 (b) of ACI 318-99 (See Appendix). The code requires two separate deflection calculations, one for immediate deflection due to live load, and one for that part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load). The latter will be referred to as the “incremental long-term deflection”.

The calculation of live load deflection depends on whether the live load is assumed to occur during first-cycle loading or subsequent loading. If first cycle loading is assumed and cracking occurs during loading, the live load deflection is taken as the difference between the total load deflection and the dead load deflection. Since the stiffness gradually decreases as loading increases due to progressive cracking, the stiffness under dead load is greater than the stiffness under total load. On the other hand, if it is assumed that the member has already experienced an application of live load, it should be recognized that the member cannot become uncracked after removal of the live load. In this case the stiffness for both dead load and total load is that corresponding to application of total load. This is illustrated in Figure 1. The definition of M_a in ACI 318-99 implies that first cycle loading should be considered since M_a is defined as the maximum

moment in the member at stage deflection is computed. Dead load deflection would then be calculated based on the dead load moment. In many concrete structures the highest loading is experienced during construction as a result of shoring, reshoring, and other construction loads. In such cases, the stiffness for all service load checks should be that associated with dead plus live load. This has the effect of decreasing the computed live load deflection but increasing the computed dead load deflection and consequently the incremental long-term deflection. In the event that restraint built into the structural system would produce shrinkage cracking then that should be taken into account in the design.

The effective moment of inertia procedure has now been a part of the code for almost 30 years and has generally been found to provide satisfactory results for members with medium to high reinforcement ratios. However some difficulties have been experienced in applying the method to members with low reinforcement ratios, particularly lightly reinforced slabs for which flexural stiffness is sensitive to cracking. The following factors contribute to poor correlation between computed and measured deflections in these cases.

1. The modulus of rupture specified as $7.5\sqrt{f'_c}$ represents a low estimate of the material property as determined from laboratory tests on small samples. ACI 209 reports that the typical range is about 6 to $12\sqrt{f'_c}$. On the surface it appears that a conservatively low estimate of the cracking moment is being used to compute deflections. However, cracking is affected not only by applied loads used to compute M_a , but also by restraint of shrinkage and temperature deformations. In some cases cracking can be detected before forms are removed and the member is subjected to dead load. The effect of restraint cracking should therefore be considered in deflection computations.
2. If incremental long-term deflection due to sustained load is computed based on first cycle loading, the stiffness will often be based on an uncracked section because the dead load moments may be less than the computed cracking moment. Recognizing that loads approaching the specified live load may be experienced at early age, the sustained load deflection should be computed based on the stiffness associated with dead plus live load. The problem may be exacerbated by the tendency for unanticipated overloading of slabs due to shoring and reshoring.
3. A further complicating factor comes into play for flat plates and flat slabs supported on columns. The elastic distributions of moments adjacent to columns produce locally high intensities that invariably initiate cracking in the negative moment regions around columns.

The cracking allows redistribution of these high moments but the stiffness has been reduced.

4. For members with low reinforcement ratios the rate at which the effective moment of inertia approaches the fully cracked moment of inertia is too low.

To account for the factors mentioned above, Scanlon and Murray (1982) recommended that a reduced effective modulus of rupture be used in computing deflections of two-way slab systems. A value of $4\sqrt{f'_c}$ has been found to produce satisfactory results in several case studies involving field measured deflections. A similar approach has recently been suggested by Gilbert (1999). Because of the many uncertainties and variabilities associated with estimating the factors that affect deflections a more precise estimate may not be worthwhile.

The historical development of deflection limits has been reported by Warwaruk (1979). It appears that deflection limits in use today have developed over the years based on experience with rules of thumb dating back at least to the 19th century. In the past thirty years or so a great deal of effort has been put into developing more refined algorithms for computing deflections. At the same time relatively little effort has gone into defining appropriate criteria for deflection limits (Scanlon and Pinheiro, 1992). Four deflection limits are specified in Table 9.5 (b), two for live load deflection, and two for incremental long-term deflection, depending on whether or not the deflection is likely to cause damage. These limits are specified as fractions of span length, $l/180$ and $l/360$ for live load deflection, $l/240$ and $l/480$ for incremental long-term deflection. The limit for incremental long-term deflection, $l/480$, which applies when damage to non-structural elements is likely, can be exceeded if special precautions are taken to prevent damage. The ACI code does not place a limit on total deflection as some codes do, however the other limits indirectly limit the permissible total deflection, immediate and long-term. While these deflection limits appear to cover a broad spectrum of design situations quite satisfactorily, it is recognized that there may be situations requiring more stringent limits. For example a limit of $l/1000$ has been suggested for members supporting brittle partitions such as unreinforced masonry. ACI Committee 435 (1963) provides suggested limits to cover some applications requiring more stringent limits than given in the code.

In computing the incremental long-term deflection the engineer needs to make an assumption regarding the time of installation of non-structural elements. In most cases this will be unknown at the design stage. It is probably not unreasonable to make the conservative assumption that installation will take place immediately in which case the full value of the multiplier would be applied. It is also necessary to make an assumption

regarding the portion of live load to be considered as sustained. Live load surveys for office and similar buildings have shown that the actual live load at any time is typically quite low compared to the design live load.

The span length to be used in applying Table 9.5 (b) to calculate the permissible deflection limit for two-way slab systems is not clearly defined. If the mid-panel deflection is computed it appears to be most logical to take the span length measured along the diagonal whereas if the column strip deflection is being calculated, the span length should be measured between the two columns on the column strip. It would also seem logical to use clear span for calculating permissible deflection limits.

In addition to the detailed treatment of deflection of one-way and two-way nonprestressed construction given in the code some general guidance is provided for deflection of prestressed and composite construction.

Construction Requirements

Deflection control is not simply a design issue. At least as important is the proper attention to construction procedures. Section 6.2 of ACI 318-99 addresses the question of removal of forms, shores, and reshoring, and includes the general statement: "Forms shall be removed in such a manner as not to impair safety and serviceability of the structure."

DEFLECTION CALCULATION PRACTICES IN THE U.S.

Consulting engineers typically design structures (and particularly concrete structures) using purchased software. There are a number of relatively inexpensive and readily available software packages for design of concrete beams, slabs and two-way systems. One can argue that the design practice for deflections is whatever procedures are built into commonly used commercial software.

ACI Committee 435 conducted a survey of consulting firms in 1999. The results of that survey are most interesting. All respondents use one or more commercial software packages. That software in turn incorporates the basic code prescribed procedures for deflections. Essentially the $I_{\text{effective}}$ concept is used along with all code prescribed procedures for long term deflection calculation. The deflection is compared with permissible limits (e.g. $L/360$) and member sizes are increased until the deflection criteria is satisfied.

The software does not include options to enable the designer to account for restraint (and the effect this would produce on cracking and, in turn, on

stiffness). Software incorporates $7.5\sqrt{f'_c}$ as the modulus of rupture and designer input on this parameter is not available.

According to the survey, engineers are generally comfortable with purchased software and feel it adequately predicts deflections. If any calculation problems are perceived to exist, engineers feel that the deflection predictions for two-way systems (flat slabs and flat plates) are the most unreliable and that the limited deflection related problems that have been experienced in structures have occurred in two-way systems.

FUTURE CODE DEVELOPMENTS

The deflection control provisions given in the ACI code appear to have resulted in satisfactory performance of concrete structures over the years. While deflection at service load levels is understood to have no impact on structural safety, it is expected that deflection control will continue to be an integral part of the code, to provide guidance to engineers in design for serviceability. This paper has pointed out a number of areas where improvement and clarification of the code provisions are possible. These include a more unified treatment of one-way and two-way construction, and additional guidance on deflection calculations for two-way systems. Computers are becoming increasingly used in structural design practice and future developments of the code provisions should keep this in mind.

As new developments are made through research involving different reinforcing materials, the calculations for deflection of members using these materials will have to be reviewed. The new materials have different properties than the Grade 60 reinforcement on which the current deflection calculations have been based.

The code should continue to provide guidance to designers on deflection limits, but perhaps some of the limiting conditions could be better defined.

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Appendix: Minimum Thickness and Permissible Deflection Tables from ACI Code

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

		Minimum thickness, h		
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member.	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$l/180^a$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$l/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) ^b	$l/480^a$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$l/240^a$

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS

Yield strength, f_y , psi ^a	Without drop panels [†]		With drop panels [†]		
	Exterior panels		Interior panels	Exterior panels	
	Without edge beams	With edge beams [‡]		Without edge beams	With edge beams [‡]
40,000	$\frac{l}{33}$	$\frac{l}{36}$	$\frac{l}{36}$	$\frac{l}{36}$	$\frac{l}{40}$
60,000	$\frac{l}{30}$	$\frac{l}{33}$	$\frac{l}{33}$	$\frac{l}{33}$	$\frac{l}{36}$
75,000	$\frac{l}{28}$	$\frac{l}{31}$	$\frac{l}{31}$	$\frac{l}{31}$	$\frac{l}{34}$

Note: Span length, l (center-to-center span) is defined in ACI 318, Section 8.7.

Span length, l_n (clear span) is defined in ACI 318, Section 9.0.

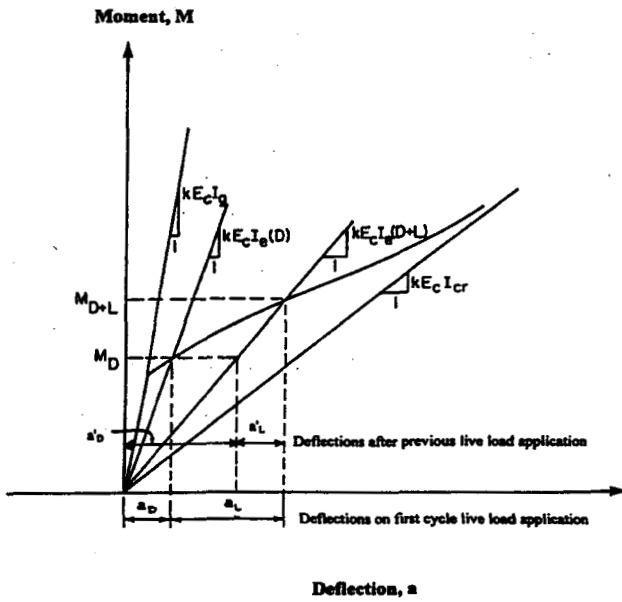


Figure 1. Effect on Loading Cycle on Dead and Live Load Deflections.

Serviceability Provisions in the New Eurocode for the Design of Concrete Structures

by A. W. Beeby

Synopsis:

This paper provides an outline of the provisions for design for serviceability given in the current version of Eurocode 2; the Eurocode for the design of concrete structures. The provisions for stress limitation and crack control are outlined briefly together with some of the reasoning behind them. A more detailed description of the background and content of the clauses dealing with the control of deflection are given.

Keywords: beams; code provisions; deflection control; Eurocode; serviceability; stress checks

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INTRODUCTION

The purpose of this paper is to provide an outline of the current thinking within the European code drafting community on the design of reinforced and prestressed concrete structures for serviceability. It must be said at the outset that the provisions for serviceability design in the Eurocode have not been finalised and there are still areas of debate. This paper therefore provides only an interim picture of the state of affairs.

Though it is likely to be well known to many readers of this paper, it may be useful at the start of the paper to have an outline of the Eurocode project generally and the current state of development.

The Treaty of Rome has, as one of its aims the removal of barriers to the free movement of goods and services across the national boundaries within the European Union. The great range of different design codes within the EU was seen to constitute a barrier to the free movement of services and so a decision was made (in around 1980) to develop a uniform and co-ordinated set of design codes for structural design. This has turned out to be a far greater, and more ambitious, task than the European Commission initially envisaged and it is still far from complete, despite 20 years of effort. Early work was carried out by committees appointed directly by the European Commission but they eventually handed the work over to the European Standards Organisation, CEN

There have probably been two major difficulties in the development of the Eurocodes. The first has arisen from the attempt to produce a uniform basis for structural design applicable to all structural design in any material. The second is the inevitable difficulties of obtaining agreement between different countries on the detailed design rules for any particular material. It is doubtful if a completely coherent and generally accepted set of theories exist describing the behaviour of any structural material. As a result, the design rules for any material in any particular country have developed from the theories accepted in that country, moderated by many years of experience. The designers in any country have developed design methods which experience has shown to work. These methods are enshrined in that country's design codes but are likely to be different to those in other countries. In the absence of a generally accepted and complete theory of behaviour, how can one decide which country's procedures are the best?



The Eurocodes currently under preparation are the following:

Eurocode 0	Basis of design
Eurocode 1	Actions on structures
Eurocode 2	Design of concrete structures
Eurocode 3	Design of steel structures
Eurocode 4	Design of composite steel and concrete structures
Eurocode 5	Design of timber structures
Eurocode 6	Design of masonry structures
Eurocode 7	Geotechnical design
Eurocode 8	Earthquake resistant design of structures

Eurocode 2, the code of practice for the design of concrete structures, is one of the most advanced. A complete version was published at a Pre-Standard (ENV) in 1991 (1). In this form it may be used for design in any of the member countries of CEN but is not mandatory. After having been published for 3 years, an ENV must be the subject of an international enquiry and vote on its future. This has taken place for Eurocode 2 and the result was that it should be converted to a full European Standard, taking account of comments made by the member countries. These comments were extensive and a new project group was set up to produce a revised draft. This group is currently about half way through developing a final draft, which is expected to be the subject of a final vote in 2002. If accepted, the code will then replace competing documents in all CEN member countries, though there will be an overlap permitted for some years between the Eurocode and the national codes it replaces.

The problem of developing an international consensus on design procedures has been made easier for concrete design by the work of the Comité Euro-International du Béton (CEB) and the Fédération International de la Précontrainte (FIP). These two international bodies, working together, drafted a number of Model Codes for concrete design. The 1978 Model Code (2) formed the basis for early drafts of the Eurocode and the current revision will be influenced to a major degree by the more recent 1990 Model Code (3). The CEB and the FIP have recently merged to form the Fédération International du Béton (*fib*).

An increasingly important aspect of concrete design is design for serviceability and this paper will outline the serviceability provisions as they appear in the prestandard version of Eurocode 2. These provisions may change significantly before publication of the final Standard and the debates on these issues currently going on will be considered.

GENERAL INTRODUCTION TO THE SERVICEABILITY PROVISIONS

Eurocode 2 contains detailed provisions for three specific conditions:

1. The limitation of stresses in steel and concrete under service loads

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2. The limitation of cracking
3. The limitation of deflections.

For each of these, the code has to define:

- Criteria defining the limits to satisfactory performance,
- A definition of the design loading under which the criteria should be checked
- A definition of the assumptions that should be made about the material properties
- The specification of an appropriate behavioural model which may be used to predict the behaviour.

The definition of criteria is recognised to be probably impossible in any general way. The objective of design for serviceability is to ensure that the behaviour of the structure is *not* such as to impair the proper functioning or appearance of the structure under the expected service conditions. Designers wish to have clear rules specified which will allow them to demonstrate clearly that their design meets the requirements of the code. Code drafters are well aware that this is impossible and that what will impair the proper functioning of a structure will depend upon the precise purpose of the structure and can only be established by the client and the designer coming to an understanding of the exact nature of the required function of the structure and the economic consequences of varying the criteria. The code makes clear that the designer should agree the service criteria with the client but recognises that it cannot avoid specifying limits that are believed to be generally acceptable. Furthermore, the general principle adopted in the drafting is that there should be simple methods which permit serviceability to be checked without detailed calculation, backed up by more rigorous procedures for use in special cases. Simple methods, such as permissible span/depth ratios for the control of deflection, can only be derived on the basis of defined criteria.

Three levels of service load are defined, the use of which depends upon what is being checked;

1. The Characteristic or Rare combination of loads. This corresponds to a loading which may be expected to occur sometime during the life of a structure, but only rarely.
2. The Frequent combination.
3. The Quasi-Permanent combination of loads, which is used to establish long term effects.

Each of the aspects of behaviour listed above will now be considered in turn.

STRESS CHECKS.

The Eurocode proposes limits to the stress under service load in both reinforcement and concrete.

The limiting stress in ordinary reinforcement is $0.8f_{yk}$ where f_{yk} is the characteristic strength of the reinforcement. Some limit on the service stress in the reinforcement seems reasonable since any inelastic deformation of the reinforcement is likely to lead to large, permanently open cracks or possibly excessive deflections. Cold worked steels will deform inelastically at stresses below the specified characteristic proof stress so a limit below the expected yield seems appropriate. Whether, however, this limit should be $0.8f_{yk}$ or, say, $0.9f_{yk}$, remains a matter for debate. This is checked under the Rare combination of loads.

There are two limits to stress in the concrete in compression. These are;

$0.6f_{ck}$, where f_{ck} is the characteristic compressive strength of the concrete, under the rare combination of loads where the member may be exposed to aggressive environments. This limit is specified to avoid the possible formation of cracking parallel with the applied compression. It is suggested that this cracking could lead to a reduction in durability.

$0.45f_{ck}$ under the Quasi-Permanent combination of loads in circumstances where it is considered that excessive creep could result in unacceptable deformations.

These limits to the compressive stresses in the concrete have been, and still are, the subject of much debate. A considerable proportion of the member countries of CEN do not currently carry out any checks for stresses under the service loads and see no reason introduce them. Other countries, however, have traditionally designed to meet limiting permissible stresses under service loads rather than using ultimate load approaches and do not wish to see their traditional design approaches abandoned.

The stresses in steel and concrete may be checked on the assumption that the concrete and steel are both elastic and that concrete carries no tension. There is a let-out clause which says that, for reinforced concrete, the stresses can be assumed to be within the limits provided that design for the ultimate loads has been carried out according to the code. Unfortunately, this is not strictly true and direct calculation of the stresses will show that the compressive stresses will control the design of columns.

CRACK CONTROL

The Eurocode proposes the limits to crack widths given in Table 1.

Two methods are given to enable these limits to be satisfied in design: direct calculation of crack widths or a check on bar arrangements. For either of these two methods to work, it is essential that at least a minimum amount of reinforcement is provided sufficient to ensure that the reinforcement does not yield when the first crack forms. If insufficient reinforcement is provided then when the first crack forms the steel will yield and this will limit the tensile force that can develop to a level below that necessary to form further cracks. All the subsequent deformation will be concentrated in this first crack. This issue is not generally a problem where a member is stressed exclusively by applied loading since the reinforcement will have been designed to carry the design ultimate load and, if this is less than the minimum, it simply means that the cracking strength is greater than the design ultimate load and cracking will not occur under service conditions. It is, however, a problem if the member is restrained against contraction due to shrinkage or early thermal contraction. In these cases, if the stresses induced by restrained shortening exceed the tensile strength of the concrete then cracking will occur independently of any applied load. It is often difficult to predict the degree of restraint provided or the possible shortening and so it is generally prudent to provide at least the proposed minimum reinforcement. This minimum is given by the formula:

$$A_s \geq k_c k_{ct-eff} A_{ct} / \sigma_s$$

This requirement is not found to be particularly onerous for bending but can require considerable areas of reinforcement in members subjected to pure tension.

Crack widths may be calculated from the formula:

$$w_k = \beta s_{rm} \epsilon_{sm}$$

where s_{rm} is the average final crack spacing and is given by the formula:

$$s_{rm} = 50 + 0.25 k_1 k_2 \phi / \rho_r$$

The effective reinforcement ratio is the ratio of the area of the reinforcement to the area of concrete immediately surrounding the bars. For beams this area will be given by the breadth of the tension zone multiplied by $2.5(h - d)$ where h is the overall depth of the section and d is the effective depth.

This formula is empirical and has been shown from extensive tests to give a reasonable prediction in most normal situations. The prediction of crack widths is, however, well known to be highly uncertain and it was felt by the drafting committee that the use of simple detailing rules should normally be an adequate method of complying with the crack width provisions. Parameter studies showed that practical limitation of

crack widths could be achieved by controlling either the bar diameter or the bar spacing as a function of the steel stress. Tables are given for both these possibilities.

CONTROL OF DEFLECTIONS

The code states that, in normal circumstances, deflections should not impair the functioning of the structure if the overall deflection is limited to span/250 and, where brittle finishes or partitions are used, the deflection occurring after installation of the finishes and partitions is limited to span/500. Generally, it is satisfactory to check the deflection under the quasi-permanent load. The code makes clear, however, that the designer should satisfy himself that these limits are appropriate for the particular structure considered and its function.

The calculation of the deformations in a cracked reinforced member is done applying a simple conceptual model of behaviour. This is illustrated in Figure 1 which shows a short element of a cracked member subjected to a uniform moment. The stress in the reinforcement at each crack will be that calculated if the concrete is assumed to carry no tension (a fully cracked section). At increasing distance from the crack, the stress in the reinforcement decreases and tension is transferred to the concrete until, if the cracks are fairly widely spaced, the stress distribution will become that in an uncracked section where the concrete is fully mobilised. When calculating the deformations, what is required is the integral of the local deformations over the length considered. This can be conveniently done by establishing an average deformation. This average deformation could be obtained by either integrating the strains over the length between cracks or the same result could be obtained by considering part of the beam to be fully cracked and part to be uncracked, as shown by the broken line in Figure 1. Using this idealisation, it can be seen that the average steel stress (say) is given by the relation:

$$\sigma_{sm} = \zeta \sigma_{s2} + (1 - \zeta) \sigma_{s1}$$

It should be able to be seen that this relationship can be used for any stress or deformation parameter. For example, the concrete strain at the compression face could be calculated by the equation simply by substitution concrete strain for steel stress. Similarly for curvature. Nor is the expression necessarily limited to beams; for example, it could be applied equally validly to tension members.

The remaining question is the definition of the coefficient ζ . Clearly, a function is required which gives a value for ζ of zero when the moment is equal to the cracking moment and increases towards 1.0 as the moment increases. The following empirical equation is given in the code for ζ .

$$\zeta = 1 - \beta_1 \beta_2 (\sigma_{sr} / \sigma_{s2})^2$$

Long term deformation are calculated by using an effective modulus of elasticity which allows for the effects of creep and the long term value of β_2 . The use of the steel stress in the formula is done to make the formula as general as possible. σ_{sr} and σ_{s2} can be calculated for a reinforced beam, a tension member, a column or a prestressed beam. The formulae thus provide a completely general approach to the estimation of the conditions of stress and deformation in a cracked section. For a reinforced concrete beam, it will be seen that the equation for ζ for members using deformed bars and short term loading may be rewritten as:

$$\zeta = 1 - (M_{cr}/M)^2$$

It may be seen that there are distinct similarities with the ACI 318 method for calculating deflections. Both methods give a deflection equal to the uncracked deflection at the cracking moment and approach the fully cracked deflection as the applied moment is increased above the cracking moment. Figure 2 compares the two methods for a beam where the uncracked second moment of area is five times the cracked second moment. It will be seen that at moments up to about three times the cracking moment, the Eurocode approach will lead to deflections slightly greater than the ACI approach but that at higher moments the two methods are indistinguishable. The differences between the two approaches are probably too small for it to be possible to establish which method gave the closer agreement with experimental data. It might be reasonable to conclude that, while the Eurocode method was more general, the ACI method was slightly easier to use.

Accuracy of deflection calculations.

There has been a considerable amount of discussion on the usefulness of calculating deflections. This has arisen because it was felt that calculation of deflections was inherently very inaccurate and that the results were therefore misleading rather than useful. Proponents of this view believe that some simple method, such as span/depth ratios provide as good a guidance on deflection performance as it is reasonable to make use of. The basis for this argument can be illustrated using an example. Deflections have been calculated for a 200 mm deep simply supported slab of 6 m span reinforced with 12 mm bars at 150 mm centres. A concrete compressive strength of 40 N/mm² has been assumed and the quasi-permanent id span moment is 25 kNm per metre. The code gives an elastic modulus for the concrete of 35 kN/mm². The major source of variability in the deflection of lightly reinforced members such as slabs is the tensile strength of the concrete. What is specified for a structure is normally the compressive strength. The tensile strength can be estimated from the compressive strength but only inaccurately. Furthermore, a stronger concrete than specified may be used, possibly to enable shorter formwork striking times or tensile stresses may develop due to restraint of shrinkage or thermal

movements. The tensile strength cannot therefore be known at the design stage with any precision. The Eurocode gives a range of tensile strengths for any given value of compressive strength. A mean strength is given together with a lower and upper characteristic value. The characteristic values vary by $\pm 30\%$ from the mean value. Figures 3 and 4 show the calculated short term and long term deflections respectively calculated for the three specified tensile strengths. A creep coefficient of 2.5 has been assumed. It will be seen that the variation in tensile strength leads to a much greater variation in the deflection. This variation is likely to be an underestimate of the actual variability since no account has been taken of variability in the elastic modulus of the concrete or the creep coefficient or inherent inaccuracies in the prediction method. Table 2 below summarises the calculated deflections under the quasi-permanent load.

It can be seen that the ratio of the largest to the smallest short term deflection is 4.25 while the ratio for the long term deflection is 1.7 and that, for the long term increment, it is 1.6. All these are large variations and, as mentioned above, are likely to be an underestimate of the actual variations. The view that a calculated deflection can give a highly misleading impression of what may actually occur can thus be understood. This led to the deflection calculation method not appearing in the main body of the code but being included in an appendix. This problem of accuracy of deflection calculation is not peculiar to the Eurocode but is inherent in all attempts to calculate deflections at the design stage, whatever method is used.

Span/ depth ratios.

Codes recognise that it would be an unnecessary waste of design effort to require calculations for all members and provide alternative simple checks for the control of deflection. By far the most usual approach is to define permissible ratios of span to overall depth or effective depth which, provided they are not exceeded can be deemed to satisfy the deflection criteria. The Eurocode follows this approach and uses ratios of span to effective depth. In many cases such limits have been derived from considerations of experience but, in the case of the Eurocode, the specified values were obtained from a parameter study using the calculation method described above. The procedure used can be expressed in simplified terms as follows.

The deflection is given by:

$$a = k_a L^2 (1/r)$$

The curvature can be calculated from:

$$(1/r) = M_{ser}/E_{c,eff}I_{eff}$$

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It is convenient to work in terms of non-dimensional parameters such that:

$$\alpha = M_{ser}/bd^2 \text{ and } \beta = I_{eff}/bd^3$$

This results in the expression for $(1/r)$ as:

$$(1/r) = bd^2\alpha/E_c bd^3\beta \text{ or } (d/r) = \alpha/E_c\beta$$

Substituting this into the expression for deflection gives:

$$a = kL^2(d/r)E_{c,eff}\beta/\alpha$$

Assuming a constant value for the modulus of elasticity, $E_{c,eff}$, combining all the various constants into a single constant, K and rearranging gives:

$$a/L = K(L/d)(d/r) \text{ or } (L/d) = (a/L)/K(d/r)$$

a/L can be calculated from the deflection limit (for example, if the limiting deflection is span/250, then $a/L = 1/250$). K and (d/r) can be calculated for the particular beam considered and hence L/d can be calculated. If L/d is calculated in this way for beam sections with a range of reinforcement ratios, a result is obtained as shown in Figure 5. This is for simply supported rectangular section beams with a concrete strength of 40 N/mm² and with a maximum stress in the reinforcement calculated on the basis of a cracked section of 200 N/mm².

It is found more convenient, rather than to produce separate charts for beams with different support conditions, to produce a set of basic ratios for beams with different support conditions and a further table of multipliers which are a function of reinforcement ratio and steel stress. This was done for the UK code, BS8110 (4) except that it was found more convenient operationally to produce the table in terms of M/bd^2 instead of reinforcement ratio. Such a set of tables was proposed for the Eurocode but was discarded as being too complicated. A simpler approach was therefore included where there are span/effective depth ratios given for two situations: members with 0.5% and 1.0% of tension reinforcement. This table is reproduced below. Interpolation between 0.5 and 1% of reinforcement is permitted. It is also permitted to assume that "concrete lightly stressed" covers slabs and "concrete highly stressed" covers beams.

FUTURE DEVELOPMENTS

It has been explained that, at present, a project group is working on producing a revised draft of the Eurocode taking account of national comments on the prestandard version. It is not at present clear what the final form of this document will take but there will certainly be some

changes to the serviceability provisions. There is still argument about the provisions for stress checks. Also, the provisions for crack control will require some revision as a result of experience in drafting the Eurocodes for the design of concrete bridges and for the design of liquid retaining and containment structures. The change which is almost certain to be made is to the span/effective depth provisions. Contrary to the opinions expressed at the time that the clauses were originally drafted, it is now the view from the national comments that the current span/effective depth ratio provisions are too simplistic and that something similar to the original proposal of a continuous function between reinforcement ratio and span/effective depth ratio appears likely to be included. Furthermore, the Eurocodes will in future take account of concrete strengths up to 100 N/mm^2 . High strength concrete can give significant advantages for deflection control because of their higher cracking moment and it is likely that this will be included in the forthcoming revisions.

CONCLUDING SUMMARY

The Eurocode for the design of concrete structures, Eurocode 2, is probably the most advanced of the family of Eurocodes and should appear as a full European Standard by about the end of 2002. The provisions for serviceability design cover checks for stresses in the concrete in compression and the reinforcement in tension under service loads; checks on crack widths and checks on the deflections. All these checks are drafted at two levels: a simplified approach requiring no or minimal calculation and a more rigorous approach requiring direct calculation of the behaviour of the structure and comparison with specified criteria defining the limits of acceptable performance. There is still debate about some aspects of these provisions and the ones described in this paper may not represent the final form of the document.

NOTATION

A_s	=	Area of reinforcement
A_{ct}	=	the area of the tension zone of the member immediately before formation of the first crack.
$E_{c,eff}$	=	the effective elastic modulus including allowance for creep
I_{eff}	=	the effective second moment of area of the section
L	=	the span of a beam
M	=	the moment considered
M_{cr}	=	the cracking moment
M_{ser}	=	the service load moment
a	=	the deflection
b	=	the breadth of a section
d	=	the effective depth of a section
f_{ck}	=	the characteristic compressive strength of the concrete
f_{ct-eff}	=	the tensile strength of the concrete at the time



cracking is expected to occur. This should not generally be taken as less than 3 N/mm^2 .

- f_{yk} = the characteristic yield strength of the reinforcement
- h = the overall depth of a section
- k_c = a coefficient which takes account of the nature of the stress distribution; $k_c = 1$ for pure tension and 0.4 for pure bending.
- k = a coefficient which takes account of the effect of non-uniform self-equilibrating stresses; $k = 0.8$ for members less than 300 mm deep and 0.5 for members greater than 800 mm deep. Linear interpolation may be used for intermediate values.
- k_1 = a coefficient taking account of the bond properties of the bars. $k_1 = 0.8$ for plain bars and 0.4 for deformed bars.
- k_2 = a coefficient which takes account of the nature of the stress distribution across the section. $k_2 = 1$ for pure tension and 0.5 for bending.
- k_a = a constant depending on the form of the bending moment diagram.
- $(1/r)$ = the curvature at the critical section
- s_{rm} = the average final crack spacing
- β = a constant relating the characteristic crack width to the average value. It usually takes a value of 1.7
- β_1 = a coefficient which takes account of the bond properties of the bars. $\beta_1 = 0.5$ for plain bars and 1.0 for deformed bars
- β_2 = a coefficient which takes account of the duration of the loading. $\beta_2 = 1.0$ for short term loading and 0.5 for long term or repeated loading.
- ϵ_{sm} = the average strain in the reinforcement allowing for tension stiffening.
- ϕ = the bar diameter
- ρ_r = the effective reinforcement ratio
- σ_{sm} = the average stress in the reinforcement
- σ_s = the maximum stress permitted in the reinforcement immediately after formation of the first crack. This may normally be taken as the yield strength of the reinforcement.
- σ_{s1} = the stress in the reinforcement in an uncracked member
- σ_{s2} = the stress in the reinforcement calculated assuming a fully cracked section
- ζ = a coefficient which defined the proportion of the element which can be considered

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Table 1. Crack width limits (in mm)

	Reinforced concrete	Post-tensioned members	Pre-tensioned members
load combination	Quasi-Permanent	Frequent	Frequent
Exposure condition			
Dry	*	0.2	0.2
Humid	0.3	0.2	Decompression
Humid with frost & salt	0.3	Decompression	Decompression
Seawater	0.3	Decompression	Decompression
* Crack width will depend upon appearance and widths greater than 0.3 mm may be acceptable.			
'Decompression' requires that the prestressing tendons are at least 25 mm within compressed concrete.			

Table 2. Calculated deflections under the quasi-permanent load

Tensile strength N/mm ²	Short term deflection mm	Long term deflection mm	Long term deflection as fraction of span	Long term deflection increment mm	Increment as fraction of span
2.5 (lower characteristic)	17	27	span/219	10	span/581
3.5 (mean value)	7	23	span/265	16	span/384
4.6 (upper characteristic)	4	16	span/386	12	span/520

Table 3. Span/effective depth ratios from Eurocode 2

Structural System	Concrete highly stressed (1% reinforcement)	Concrete lightly stressed (0.5% reinforcement)
Simply supported beam, one or two way spanning simply supported slab	18	25
End span of continuous beam or one way continuous slab or two way spanning slab continuous over one long side	23	32
Interior span of beam or one or two way spanning slab	25	35
Slab supported on columns without beams (Flat Slab)(based on longer span)	21	30
Cantilever	7	10

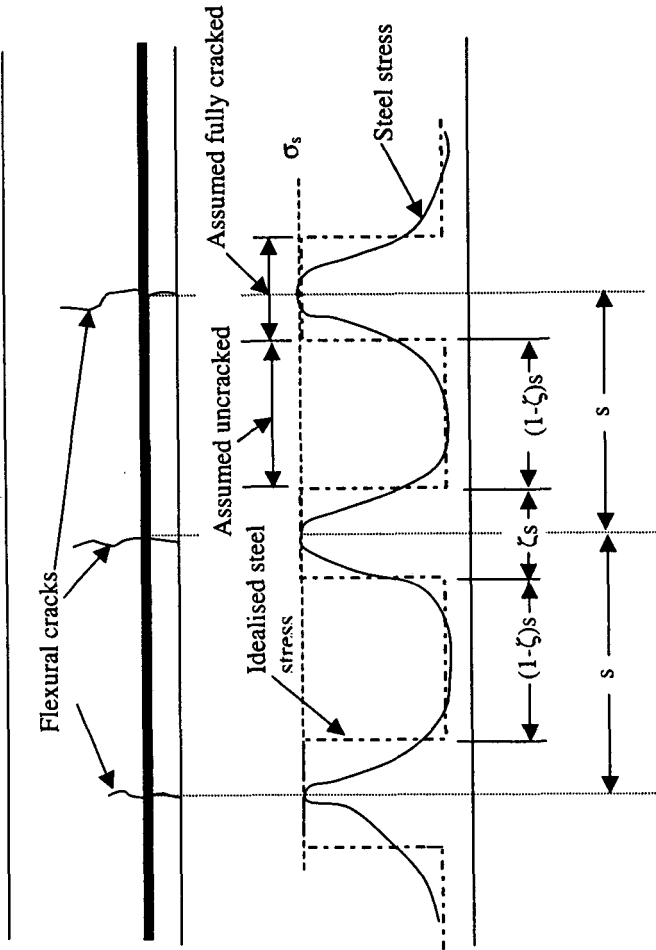


Figure 1. Conceptual model used in the derivation of the Eurocode deflection calculation method

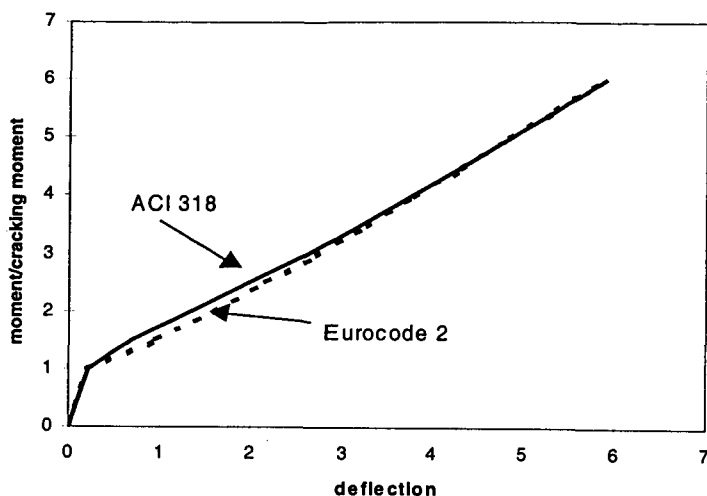


Figure 2. Comparison of Eurocode and ACI formulae for deflection calculation

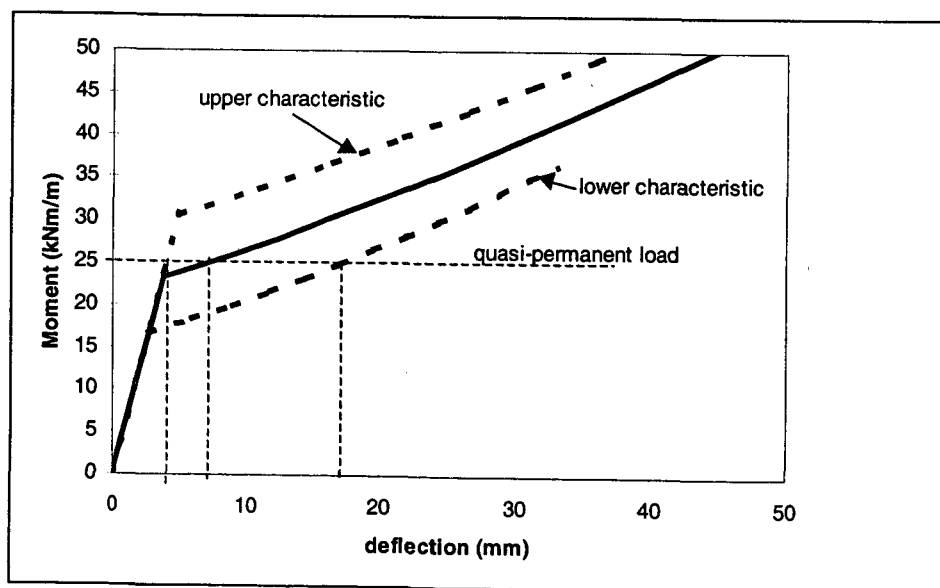


Figure 3. Short term deflection calculated for three levels of tensile strength

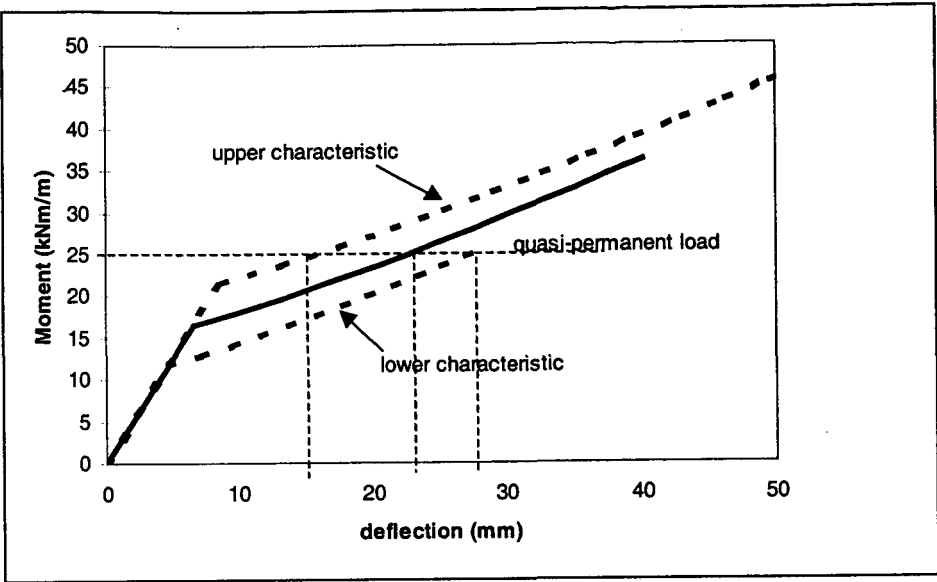


Figure 4. Long term deflections calculated for the same material properties as used in Figure 3

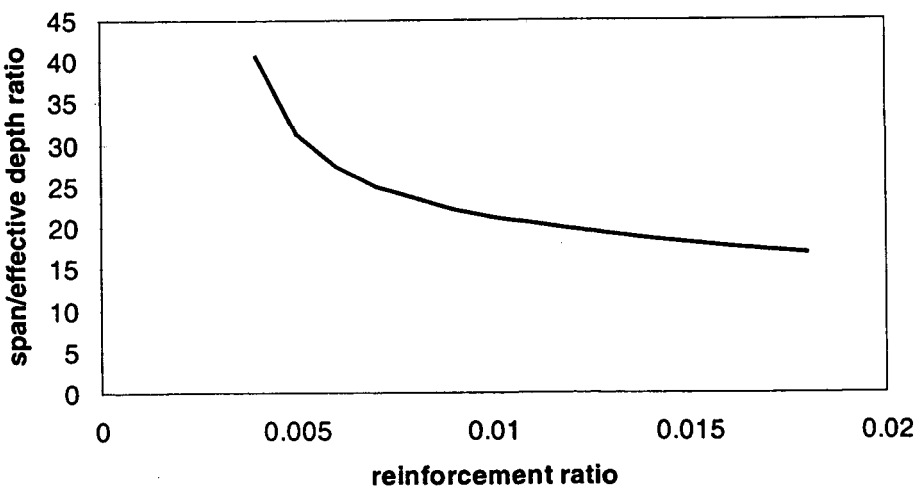


Figure 5. Influence of reinforcement ratio on the permissible span/effective depth ratios

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UK Code Requirements for Deflection Control

by R. H. Scott

Synopsis: A brief history is given of UK Code requirements for deflection control since 1948. Current requirements of BS8110 are then presented followed by a summary of developments concerning high strength concrete. Finally, future developments involving Eurocode 2 are indicated.

Keywords: codes and standards; deflection control; structural concrete

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INTRODUCTION

The first formal Code of Practice in the UK concerned with the design of concrete structures was CP114, *The Structural Use of Reinforced Concrete in Buildings*, published in 1948 (1). This was followed by CP115, *The Structural Use of Prestressed Concrete in Buildings* (2), and CP116, *Structural Use of Precast Concrete* (3). These were all essentially permissible stress codes although CP114 later allowed resistance moments of beams and slabs to be calculated by the load-factor method.

CP114, CP115 and CP116 were all revised at regular intervals but in 1972 a major development occurred with the publication of CP110, *Structural Use of Concrete* (4). This covered reinforced, prestressed and precast concrete in a single document and introduced limit state design to UK design codes. It was, however, published in three parts, Part 1 presenting design rules supported by Parts 2 and 3 which contained design charts. CP110 received a major revision in 1985 when it was republished as BS8110, *Structural Use of Concrete* (5). This is also in three parts but arranged differently, Part 1 now being for "normal" design situations and Part 2 for "special circumstances". Design charts are only in Part 3. BS8110 has been amended at regular intervals since 1985 and Part 1 was revised and reprinted in 1997. It is this 1997 edition of Part 1, with the 1985 editions of Parts 2 and 3, which, together, form the current document for structural concrete design in the UK.

BRIEF HISTORY OF UK DEFLECTION CONTROL

CP114(1948) was succinct concerning deflection control. It said (Clause 306(a)) that:

"Reinforced concrete subject to bending action in a building should possess adequate stiffness to prevent such deflection or deformation as might impair



the strength or efficiency of the structure, or produce cracks in finishes or in superimposed partitions, etc."

This statement is, of course, as appropriate now as it was then, but CP114(1948) gave no further guidance as to how it should be implemented. In the 1957 edition, however, a straightforward table of Span/Effective Depth ratios was included, covering both beams and slabs, compliance with which was deemed to ensure that a member possessed sufficient stiffness for all normal cases. Modification factors were specified to reduce the permissible values in situations of high steel and/or concrete stress.

The basic approach of using Span/Effective Depth ratios (rather than the US practice of using minimum thicknesses), first adopted in CP114(1957), has persisted through CP110 and BS8110 (Part 1) although it has become considerably more sophisticated, as will be indicated shortly. Additionally, however, CP110 gave recommendations for the *calculation* of deflections which have been continued in BS8110 (Part 2). There are thus two approaches to deflection control currently permitted in the UK by BS8110. It is anticipated that in all normal circumstances deflection control will be achieved through the use of Span/Effective Depth ratios but there is also the alternative of calculating deflections when conditions are in any way unusual. Further details of both procedures are given below.

SPAN/EFFECTIVE DEPTH RATIOS

Beams

BS8110 anticipates that, in the majority of design situations, satisfaction of Span/Effective Depth ratios will ensure adequate deflection control, thus avoiding the more lengthy procedure of calculating actual deflection values. The intention is to limit TOTAL deflection to SPAN/250 which, it is anticipated, will normally ensure that the part of the deflection occurring after the construction of finishes and partitions will be limited to SPAN/500 or 20mm, whichever is the lesser, for spans up to 10m.

Basic Span/Effective Depth ratios are tabulated in Table 3.9 of BS8110 which is reproduced in this paper as Fig. 1 (see *Acknowledgements* for licence details). Values are given for cantilevers, simply supported beams and continuous beams. These values are then multiplied, if appropriate, by (250/NEW LIMIT) where deflection limits other than Span/250 are required, or by (NEW DEFLECTION/20) for deflection limits other than 20mm. For spans greater than 10m, the basic ratio is multiplied by (10/SPAN).

If beam behaviour was elastic then it would be satisfactory (and rigorous) to use ratios of span to *overall* depth. However, the actual stiffness of a reinforced concrete beam depends on the steel percentage and state of cracking and so some means of accommodating actual beam behaviour must be devised. This is partly achieved in BS8110 (and in all the British Codes which preceded it) by using ratios of span to effective depth rather than span to overall depth, as indicated above, and further achieved by the introduction of modifying factors, as described below.

BS8110 requires that the basic Span/Effective Depth ratios be modified to allow for the percentage of tension reinforcement and the percentage of compression reinforcement. The multiplication factors for tension reinforcement are tabulated in Table 3.10 of BS8110, reproduced here as Fig. 2, and are related to M/bd^2 (where M is the design ultimate moment at the centre of the span) and the service stress in the tension steel. For more heavily loaded situations the coefficients are < 1 but for lightly loaded beams they are > 1 to reflect the beneficial contribution of tension stiffening. The multiplication factors for compression steel are tabulated in Table 3.11 of BS8110 (Fig. 3) and are all > 1 (i.e. beneficial). They are related to percentage of compression steel actually provided. Permissible Span/Effective Depth ratios obtained from this procedure take account of normal creep and shrinkage deflection.

The above procedure for determining the "final" Span/Effective Depth ratio is summarised in Fig. 4.

Solid Slabs

Normally Span/Effective Depth Ratios, as summarised above, are used for deflection control in both one-way and two-way spanning slabs (the shorter span being used in the latter case). As before, the appropriate ratio is obtained from Table 3.9 (Fig. 1) and modified by the coefficient obtained from Table 3.10 (Fig. 2). (The compression steel modifier is normally inappropriate). Only the conditions at the centre of the span in the width of slab under consideration should be considered to influence deflection.

Rules for flat slabs are generally similar to those for one-way and two-way slabs provided that the drops (if present) have a gross width, in both directions, at least equal to $1/3$ of the respective spans. Otherwise, a multiplier of 0.9 must be applied to the Span/Effective Depth ratios. The check should be performed for the more critical direction.

CALCULATION OF DEFLECTIONS

Part 2 of BS8110 gives guidance for calculating the deflection of a reinforced concrete member which may be followed as an alternative to using Span/Effective Depth ratios. The procedure, in its most general form, involves numerical integration of the following equation relating deflected shape and curvature:-

$$\frac{1}{r_x} = \frac{d^2 a}{dx^2} \quad [1]$$

where $1/r_x$ is the curvature at x
 a is the deflection at x

As an alternative, a simplified approach may be used as given below:-

$$a = Kl^2 \frac{1}{r_b} \quad [2]$$

where $1/r_b$ is the curvature at mid-span (or support for cantilevers)

K is a constant that depends on the shape of the bending moment diagram

Values of K for common shapes of bending moment diagram are given in Table 3.1 of BS8110 Part 2.

Long-term deflections are dealt with by using an effective modulus of elasticity which has a value of $1/(1+\phi)$ times the short-term modulus, where ϕ is a creep coefficient. Section 7.3 of BS8110 Part 2 gives a procedure for determining ϕ .

PRESTRESSED CONCRETE

The use of Span/Effective Depth ratios is inappropriate due to the major influence of the level of prestress. As a consequence BS8110 requires that deflections be calculated and gives guidance concerning this. Deflection limits are similar to those for reinforced concrete beams but, in addition, limits to upward deflections are prescribed (BS8110, Part 2, Clause 3.2.1.2).

DEVELOPMENTS

High Strength Concrete

Tables 3.9 and 3.10 of BS8110 (Figs. 1 and 2) are effectively based on a characteristic compressive *cube* strength of 35MPa. However, in recent years developments in concrete technology have rendered strengths considerably in excess of this to be easily obtainable. Important advantages of high strength concrete when considering deflection control are its higher Young's modulus and, more importantly, its higher tensile strength which gives a higher cracking moment. Consequently, a revised Table 3.10 has been published by the Concrete Society in the UK in their Technical Report No. 49 (6). Modification factors for tension reinforcement are presented for a range of characteristic compressive cube strengths up to, and including, 100MPa. The report shows that modification factors can be more than doubled with a consequent doubling of the Span/Effective Depth ratio. This obviously has significant implications for deflection control and it is to be hoped that the results of this work can be incorporated into Codes and Standards in due course.

Eurocode 2 (EC2)

The drive for harmonisation of technical standards across the European Union is leading to the development of Eurocodes which will eventually replace national codes and standards. The first of these, *Eurocode 2: Design of Concrete Structures Part 1: General Rules for Buildings* (7), was published in 1992. At present, this version of EC2 is classed as a draft for development but a revised document is under development which, when published in 2001/2, will run in parallel with national standards, such as BS88110, before superseding them around 2006/7. Consequently, a brief description of the provisions for deflection control in EC2 are appropriate here since, in due course, they will become UK practice.



EC2, like BS8110, permits deflection control to be achieved either through the use of Span/Effective Depth ratios or by calculation. Deflections for quasi-permanent loads are limited to Span/250 and deflections of elements, such as finishes, which could suffer damage are limited to Span/500.

Basic Span/Effective depth ratios are tabulated in EC2 for beams and slabs for two levels of concrete stress under serviceability conditions (but textbooks usually tabulate values for nominally reinforced sections as well). These ratios are then modified for spans longer than 7m (8.5m for flat slabs), for characteristic steel strengths other than 400MPa and for situations where the area of tension reinforcement provided exceeds the area actually required. Guidance is given for checking deflections by calculation.

Procedures in the current draft of the 2001 revision to EC2 are very similar. One detail difference is the recasting of the Span/Effective Depth ratios for a characteristic steel strength of 500MPa.

At present, there is no provision in EC2 for modifying Span/Effective Depth ratios to take account of the beneficial effects of compression reinforcement. Overall, current rules for deflection control in EC2 are less sophisticated than those in BS8110.

CONCLUSIONS

1. UK Code requirements for deflection control date from 1948.
2. Current requirements in BS8110 permit deflection control of reinforced concrete beams and slabs to be achieved either by using Span/Effective Depth ratios (BS8110, Part 1) or by calculation (BS8110, Part 2). It is anticipated that Span/Effective Depth ratios will be the method normally used.
3. Deflection control of prestressed concrete beams in BS8110 is achieved by calculation.
4. Recent developments indicate that the use of high strength concrete can significantly improve Span/Effective Depth ratios compared with those achieved with normal strength concrete.
5. The provisions for deflection control in Eurocode 2 follow a broadly similar approach to those in BS8110.

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Extracts from the British Standard BS8110, Part 1, 1997, are produced with the permission of BSI under licence number 2000SK/0223. British Standards can be obtained by post from BSI Customer Services, 389 Chiswick High Road, London, W4 4AL, UK (Tel. +44 20 8996 9001). The author also acknowledges the assistance of Arup Research and Development, London.

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Table 3.9 Basic span/effective depth ratio for rectangular or flanged beams		
Support conditions	Rectangular sections	Flanged beams with $\frac{b_w}{b} \leq 0.3$
Cantilever	7	5.6
Simply supported	20	16.0
Continuous	26	20.8

Figure 1. Basic span/effectie depth ratios (BS8110, Part 1, Table 3.9)

Table 3.10 Modification factor for tension reinforcement									
Service stress	M/bd^2								
	0.50	0.75	1.00	1.50	2.00	3.00	4.00	5.00	6.00
$(f_y = 250)$	100	2.00	2.00	2.00	1.86	1.63	1.36	1.19	1.01
	150	2.00	2.00	1.98	1.69	1.49	1.25	1.11	1.01
	167	2.00	2.00	1.91	1.63	1.44	1.21	1.08	0.99
	200	2.00	1.95	1.76	1.51	1.35	1.14	1.02	0.94
	250	1.90	1.70	1.55	1.34	1.20	1.04	0.94	0.87
	300	1.60	1.44	1.33	1.16	1.06	0.93	0.85	0.80
$(f_y = 460)$	307	1.56	1.41	1.30	1.14	1.04	0.91	0.84	0.79
NOTE 1. The values in the table derive from the equation: Modification factor = $0.55 + \frac{(477 - f_s)}{120 \left(0.9 + \frac{M}{bd^2}\right)} \leq 2.0$ equation 7									
where M is the design ultimate moment at the centre of the span or, for a cantilever, at the support.									
NOTE 2. The design service stress in the tension reinforcement in a member may be estimated from the equation: $f_s = \frac{2f_y A_{s, req}}{3A_{s, prov}} \times \frac{1}{\beta_b}$ equation 8									
NOTE 3. For a continuous beam, if the percentage of redistribution is not known but the design ultimate moment at mid-span is obviously the same as or greater than the elastic ultimate moment, the stress f_s in this table may be taken as $2/3f_y$.									

Figure 2. Modification factors for tension reinforcement (BS8110, Part 1, Table 3.10)

Table 3.11 Modification factor for compression reinforcement	
$100 \frac{A'_s \text{ prov}}{bd}$	Factor
0.00	1.00
0.15	1.05
0.25	1.08
0.35	1.10
0.50	1.14
0.75	1.20
1.0	1.25
1.5	1.33
2.0	1.40
2.5	1.45
≥ 3.0	1.50

NOTE 1. The values in this table are derived from the following equation:
 Modification factor for compression reinforcement =

$$1 + \frac{100A'_s \text{ prov}}{bd} \left(3 + \frac{100A'_s \text{ prov}}{bd} \right) \leq 1.5$$
 equation 9

NOTE 2. The area of compression reinforcement A used in this table may include all bars in the compression zone, even those not effectively tied with links.

Figure 3. Modification factors for compression reinforcement (BS8110, Part 1, Table 3.11)

PROCEDURE	COMMENTS
"Final" SPAN/EFFECTIVE DEPTH RATIO = BASIC RATIO X 250/NEW LIMIT or NEW DEFLECTION/20 X 10/SPAN X MODIFICATION FACTOR FOR TENSION REINFORCEMENT X MODIFICATION FACTOR FOR COMPRESSION REINFORCEMENT	<i>BS8110 Table 3.9 (Fig. 1)</i> <i>Modifiers for non-standard deflection limits</i> <i>Modifier for spans greater than 10m</i> <i>BS8110 Table 3.10 (Fig. 2)</i> <i>BS8110 Table 3.11 (Fig. 3)</i>

Figure 4 : Summary of procedure for determining span/effective depth ratios in beams

Deflection Calculation and Control – Australian Code Amendments and Improvements

by R. I. Gilbert

Synopsis: This paper describes the behavior of reinforced and prestressed concrete flexural members under sustained service loads and outlines recent developments in the design of concrete structures for the serviceability limit states, particularly with regard to deflection and crack control. The effects of concrete cracking, creep and shrinkage on cross-sectional stresses and deformation are demonstrated and discussed for a wide range of actions and reinforcement layouts. Recent amendments to the serviceability provisions of the Australian Standard for Concrete Structures AS3600 are presented and the background to, and reasons for, the proposed changes are explained. The paper also highlights the inadequacies of the existing deflection calculation procedure in ACI 318M-99 and suggests ways to improve it. A method is proposed for calculating the time-dependent deflection of reinforced and prestressed concrete members taking into account the time-dependent effects of creep and shrinkage, including the loss of stiffness caused by shrinkage induced cracking and the breakdown of tension stiffening with time. The method is illustrated by several examples.

Keywords: cracking; creep; deflection; design; prestressed concrete; reinforced concrete; serviceability; service loads; shrinkage; structural behavior; time-dependent behavior