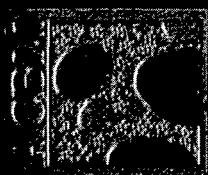
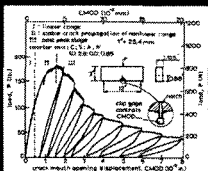
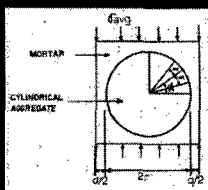


CONCRETE: MATERIAL SCIENCE TO APPLICATION

A Tribute to Surendra P. Shah



Editors:
P. Balaguru
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Concrete: Material Science to Application

A Tribute to Surendra P. Shah

Editors

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Printed in the United States of America

Editorial production: Bonnie L. Gold

Library of Congress catalog card number: 2002103339
ISBN: 0-87031-075-5

PREFACE

Surendra (Suru) Poonamchand Shah, the son of a Gujarati businessman, grew up in Mumbai (Bombay), India and attended Modern School and Elphinstone College where he became President of the Literary Society. While in Engineering College, he played the leading role in a drama that traveled to New Delhi and was performed in front of the then Prime Minister, Jawaharlal Nehru. For Suru, this tour of India was a memorable introduction to the delights of travel, an aspect of his academic career that he genuinely enjoys. Among Suru's other passions are exploring cuisine and wines in restaurants worldwide, investigating archaeological and architectural wonders of the world, analyzing theatre, opera, and film productions, and entertaining visitors to Chicago and Evanston, as well as working with students, post-docs, and colleagues.

After earning a B.E. at Sardar Vallabhbhai Vidyapath Gujarat University, Suru came to the United States for graduate study at Lehigh University where he completed a Master of Science degree. For two years, he worked as a design engineer for Modjeski and Masters, a bridge design firm in Harrisburg, Pennsylvania, where he met Dorothea Crispell, whom he married before he enrolled in the Ph.D. program in civil engineering at Cornell University. There, Professor George Winter, Professor Richard White, and Professor Floyd Slate were his research mentors.

In 1965, Suru joined the faculty of the Materials Engineering Department at the University of Illinois in Chicago. In addition to teaching a variety of civil engineering classes, he developed a state-of-the-art research laboratory and built a graduate program. He was rapidly promoted to associate, then full, professor. In 1981, he joined the faculty at Northwestern University where he holds the Walter P. Murphy Chair in Civil Engineering.

Suru was key in establishing the pioneering National Science Foundation Science and Technology Center for Advanced Cement Based Materials and has served as director since its inception in 1989. The ACBM Center is a multi-disciplinary consortium comprised of five institutions: Northwestern University, University of Illinois at Urbana-Champaign, University of Michigan at Ann Arbor, Purdue University, and the National Institute of Science and Technology. The goal of the center is to improve knowledge of cement and concrete so that stronger, tougher, and more-durable materials are economically produced for infrastructure facilities and other structures. With ACBM Center colleagues, post-docs, and graduate students, Suru continues his tireless pursuit of fundamental understanding of cement-based materials and related fiber-reinforced cement composites. Currently, a consortium of world-class industrial partners provide strong support for the ACBM Center and assist the center in promoting improved teaching of the materials science of concrete.

Suru's accomplishments are too numerous to list here; suffice it to say that anyone involved in the materials science of concrete, fracture mechanics, high-performance concrete, or fiber-reinforced concrete has encountered and utilized his contributions. He has co-authored two textbooks, edited over a dozen books, and published more than 400 papers. Suru has received many major professional and technical awards, including the ACI Arthur R. Anderson Award, the RILEM Gold Medal, the Swedish Concrete Award, the Alexander von Humboldt Award, Engineering Newsletter Award, ASCE-CERF Charles Pankow Award, and ASTM Thompson Award. He has been the principal advisor of almost 100 graduate students and supervised the work of over 60 post-docs and visiting scholars. Contributors to this symposium in his honor include many of his former students. A list of former students and post-docs appear in the Appendix.

Suru and Dorothie Shah have two married sons, Byron and Daniel, and a grandson, Kian, born in 1999.

It is with great respect, admiration, and affection that members of the organizing committee of this symposium dedicate this volume in tribute to their colleague, mentor, and friend, Surendra P. Shah.

The Organizing Committee

April 2002

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ADVANCES IN FATIGUE AND FRACTURE

Failure Mechanism of Reinforced Concrete Under Cyclic Loading

by T. C. Hsu and M. Y. Mansour

Synopsis: The load-deformation response of R/C membrane elements (panels) subjected to reversed cyclic shear shows that the orientation of the steel bars with respect to the principal coordinate of the applied stresses has a strong effect on the “pinched shape” of the post-yield hysteretic loops. When the steel bars in a panel are oriented in the coordinate of the applied principal stresses, there is no “pinching effect,” and the panel exhibits ductile behavior and high capacity of energy dissipation. Whereas, when the steel bars are oriented at an angle of 45° to the applied principal stresses, severe pinching effect is observed and the panel becomes more brittle.

This paper presents concisely a rational theory, called the *Cyclic Softened Membrane Model* (CSMM). This new rational theory is capable of predicting the entire history of the hysteretic loops (pre- and post-yield); can explain the mechanism behind the “pinching effect”; and can elucidate the failure mechanism that causes the deterioration of reinforced concrete structures under cyclic loading.

Keywords: cyclic loading; mechanism; pinching; reinforced concrete; shear; strain; steel; stress; softened truss models

2 Hsu and Mansour

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M. Y. Mansour is Assistant Professor in the Dept. of Civil & Construction Engineering at Bradley University, Peoria, IL. He received his B.E. and M.E. from the American University of Beirut, Lebanon, and his Ph. D. from University of Houston in 2001. His research interests include design of concrete structures, analytical modeling, and experimental testing.

INTRODUCTION

Under earthquake condition, the behavior of wall-type reinforced concrete structures (such as shear walls) can each be visualized as assemblies of membrane elements subjected to in-plane cyclic stresses. The key to rational analysis of these structures is to study and thoroughly understand the behavior of isolated reinforced concrete elements, separated from the complications of whole-structure behavior. By studying panel elements' behavior under cyclic loading, rational analytical models are developed to predict their cyclic behavior. Then by incorporating these models into a finite element program, one can predict the behavior of whole structures under earthquake loading.

Structures located in high earthquake regions are designed to withstand not only moderate seismic loading within the elastic range, but must also be able to absorb the energy of high seismic loading in the inelastic range. Thus, it is necessary to evaluate the inelastic response of structures under high earthquake loading. When the shear force governs the response, as in the case of low-rise shear walls, the effect of shear on the panel response was thought to be responsible for the "pinching effect" in the hysteretic loops. The "pinching effect" which results in the degradation of stiffness and the reduction of energy dissipation capacity, was also surmized to be caused by the bond slips between concrete and steel bars.

Since 1997 a total of 15 panels had been subjected to reversed cyclic loading at the University of Houston using the Universal Panel Tester (Hsu, Belarbi and Pang, 1995), (1), equipped with a servo-control system that was capable of conducting strain-control tests (Hsu, Zhang and Gomez, 1995). Using this strain-control feature, the panels were loaded beyond the yield point up to 10 times the yield strain. It was found that the primary factor affecting the pinched behavior of hysteretic loops was the orientation of the steel bars with respect to the applied principal stresses. When the steel bars were oriented in the principal coordinate of the applied stresses (i.e. at a fixed angle α_2 of 90°), there was no pinching effect. When the steel bars orientation was $\alpha_2 = 45^\circ$ or 68.2° to the principal coordinate, there was severe pinching effect.



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Based on the testing of panels during the past 13 years, a series of theories were developed at UH to predict the monotonic behavior of cracked RC membrane elements. These theories include the rotating-angle softened truss models (RA-STM) (Hsu, 1993; Belarbi and Hsu, 1994, 1995; Pang and Hsu, 1995), (3,4,5,6), the fixed-angle softened truss model (FA-STM) (Pang and Hsu, 1996; Hsu and Zhang, 1997, Zhang and Hsu, 1998), (7,8,9), and the softened membrane model (SMM) (Zhu, 2000; Hsu & Zhu, 2001; Zhu and Hsu, 2002; Hsu and Zhu, 2002), (10,11,12,13). The SMM is an extension of the FA-STM with two improvements: First, SMM can predict the entire load-deformation history of panel behavior, including the post-peak descending branches, because the Hsu/Zhu ratios (or Poisson effect) are taken into account. Second, the complicated and empirical shear modulus of concrete in FA-STM is replaced by a simple and rational shear modulus in SMM. As a result, the solution algorithm of SMM is considerably simpler and the prediction more accurate than those of FA-STM.

In order to predict the behavior of membrane elements under cyclic loading, SMM for monotonic loading was extended for application to cyclic loading by adding the constitutive models of materials (concrete and embedded steel bars) in the unloading and reloading regions (Mansour, 2001; Mansour, Lee and Hsu, 2001; Mansour, Hsu and Lee, 2001), (14,15,16). This paper presents a new model, called the “Cyclic Softened Membrane Model (CSMM),” that is capable of predicting the entire cyclic history of load-deformation relationship, including the post-yield hysteretic loops and the pinching effect.

Applying the CSMM to predict the behavior of two panels CA3 ($\alpha_2 = 45^\circ$) and CE3 ($\alpha_2 = 90^\circ$), this paper discusses rationally the presence and absence of the pinching effect in the hysteretic loops. The comparison of the behavior of these two panels not only reveals the mechanism inherent in the pinching phenomenon, it also elucidates the failure mechanism of reinforced concrete composites under cyclic loading.

TESTS OF R/C SHEAR PANELS CA3 AND CE3

The two test panels, CA3 and CE3, have a size of 1398 mm x 1398 mm x 178 mm, and the steel bars are placed at angles α_2 of 45 and 90 degrees, respectively, as shown in Fig. 1, to form orthogonal steel grids. The reinforcing ratios of panels CA3 and CE3 are 1.7% and 1.2%, respectively, in each direction. The material properties for these two panels are summarized in Table 1.

The panels were subjected to reversed cyclic stresses in the horizontal and vertical directions using the Universal Panel Tester. When these two principal applied stresses were equal in magnitude and opposite in direction, a state of

pure shear stress τ_{45° was created at the 45° direction to the applied principal stresses. The testing facility was equipped with a servo-control system capable of switching from load-control mode to strain-control mode as the yielding load was approached. In the strain-control mode, the shear strain (i.e. the algebraic sum of the horizontal and vertical strains), which followed a specified strain history, was used as an input signal to control the horizontal principal stress. The horizontal principal stress was, in turn, used to control the vertical principal stress such that they were always equal in magnitude and opposite in direction.

The hysteretic loops of the two panels, CA3 and CE3, are shown in Fig. 2 (a) and (b), respectively. In these figures, the vertical and horizontal axes represent the shear stress τ_{45° and the shear strain γ_{45° at 45° degrees to the principal coordinate of applied stresses. In Fig. 2 (a), the hysteretic loops of panel CA3 displayed a highly pinched shape that are generally associated with shear dominated behavior. The envelope curve of this panel also exhibited a distinct descending branch indicating a severe strength degradation of the panel with increasing shear strain magnitude. In contrast, no pinching effect was observed in Fig. 2 (b) for the hysteretic loops of panel CE3, with its steel grid parallel to the applied principal stresses. The envelope curve of panel CE3 did not have a descending branch and the strength deterioration was not noticeable.

CYCLIC SOFTENED MEMBRANE MODEL (CSMM)

In the basic concept of CSMM, the cracks are smeared throughout the reinforced concrete elements, and the reinforcing bars are uniformly distributed in two orthogonal directions (ℓ and t). The behavior of the panels is, therefore, formulated in terms of smeared (average) stresses and smeared (average) strains, and the continuum mechanics can be applied. The equilibrium equations, compatibility equations, and the material constitutive models are summarized in this paper:

Equilibrium Equations:

$$\sigma_\ell = \sigma_v^c \cos^2 \alpha_2 + \sigma_h^c \sin^2 \alpha_2 + \tau_{vh}^c 2 \sin \alpha_2 \cos \alpha_2 + \rho_t f_t \quad (1)$$

$$\sigma_t = \sigma_v^c \sin^2 \alpha_2 + \sigma_h^c \cos^2 \alpha_2 - \tau_{vh}^c 2 \sin \alpha_2 \cos \alpha_2 + \rho_\ell f_\ell \quad (2)$$

$$\tau_{\ell t} = (-\sigma_v^c + \sigma_h^c) \sin \alpha_2 \cos \alpha_2 + \tau_{vh}^c (\cos^2 \alpha_2 - \sin^2 \alpha_2) \quad (3)$$

Compatibility Equations:

$$\varepsilon_\ell = \varepsilon_v \cos^2 \alpha_2 + \varepsilon_h \sin^2 \alpha_2 + \frac{\gamma_{vh}}{2} 2 \sin \alpha_2 \cos \alpha_2 \quad (4)$$

$$\varepsilon_t = \varepsilon_v \sin^2 \alpha_2 + \varepsilon_h \cos^2 \alpha_2 - \frac{\gamma_{vh}}{2} 2 \sin \alpha_2 \cos \alpha_2 \quad (5)$$

$$\frac{\gamma_{th}}{2} = (-\varepsilon_v + \varepsilon_H) \sin \alpha_2 \cos \alpha_2 + \frac{\gamma_{vH}}{2} (\cos^2 \alpha_2 - \sin^2 \alpha_2) \quad (6)$$

where the symbols are given in the list of Notations.

The constitutive laws of steel bars and concrete are summarized in the following sections.

Constitutive Relationships of Steel Bars

The proposed cyclic stress-strain relationship of steel bar embedded in concrete is shown by the solid curves in Fig. 3. This solid curves can be divided into two groups: the backbone envelope curves and the unloading and reloading curves. The figure also gives the monotonic stress-strain curves of bare steel bars as shown by the dotted lines.

Backbone Envelope Curves (Stage 1, Stage 2T and Stage 2C) - The monotonic tensile stress-strain curve of embedded steel bars proposed by Belarbi and Hsu (8) can be used to approximate the backbone envelope curve of the cyclic tensile stress-strain curves of steel bars. This monotonic, bilinear stress-strain relationship, which was adopted for Stage 1 and Stage 2T, is expressed as:

$$\text{(Stage 1)} \quad f_s = E_s \varepsilon_s \quad (\varepsilon_s \leq \varepsilon_n) \quad (7a)$$

$$\text{(Stage 2T)} \quad f_s = f_y \left[(0.91 - 2B) + (0.02 + 0.25B) \frac{\varepsilon_s}{\varepsilon_y} \right] \quad (\varepsilon_s > \varepsilon_n) \quad (7b)$$

where f_s and ε_s are the average stress and strain of mild steel bars, respectively; f_y and ε_y are the yield stress and strain of bare mild steel bars, respectively; E_s is the modulus of elasticity of steel bars; and $\varepsilon_n = \varepsilon_y (0.93 - 2B)$. The parameter B is given by $B = (f_{cr} / f_y)^{1.5} / \rho$, where ρ is the reinforcement steel ratio and $\geq 0.5\%$. f_{cr} is the cracking strength of concrete given by $f_{cr} = 0.31 \sqrt{f'_c} \text{ (MPa)}$.

If the steel stress progresses in the compression region, the average maximum stress f_s is limited to the compressive yield stress $-f_y$, Eq. (7c), as indicated in Stage 2C.

$$\text{(Stage 2C)} \quad f_s = -f_y \quad (f_s \leq -f_y) \quad (7c)$$

Unloading and Reloading Curves (Stage 3 and Stage 4) - The cyclic stress-strain relationship for steel bars subjected to reversed cyclic loading must include the unloading and the reloading branches. In this analysis, the unloading and reloading stress-strain relationships are expressed by the Ramberg-Osgood type of equations:

$$\text{(Stage 3 and Sage 4)} \quad \varepsilon_s - \varepsilon_i = \frac{f_s - f_i}{E_s} \left[1 + A^{-R} \left| \frac{f_s - f_i}{f_y} \right|^{R-1} \right] \quad (7d)$$

where f_s and ε_s are the smeared stress and smeared strain of an embedded steel bar; f_i and ε_i are the smeared stress and smeared strain of steel bars at the load reversal point.

The coefficients A and R in Eq. (7d) were determined from the reversed cyclic loading tests at the University of Houston (Mansour, Lee and Hsu, 2000), (15), to best fit the test results: $A = 1.9k_p^{-0.1}$, $R = 10k_p^{-0.2}$. The parameter in the coefficients A and R is the plastic strain ratio k_p which is defined as the ratio $\varepsilon_p/\varepsilon_n = (\varepsilon_i - \varepsilon_n)/\varepsilon_n$. In this expression ε_p is the plastic strain, and ε_n is the initial yield strain.

Constitutive Relationships of Concrete

The cyclic stress-strain curves of concrete in the CSMM are shown in Fig. 4. The curves are divided into three groups: the compressive backbone envelope curves, the tensile backbone envelope curves, and the unloading and reloading curves. In addition, a rational shear modulus of concrete is adopted in CSMM. This shear modulus is simply a function of the stress-strain relationships of concrete in compression and in tension.

Compressive Backbone Envelope Curves (C1 and C2) - As pointed out by Mansour, Lee and Hsu (2001), (15), the backbone envelope curves for the cyclic compressive stress-strain curves of concrete can be expressed by a small modification of the monotonic compressive stress-strain curve of concrete proposed by Belarbi and Hsu (1995), (5):

$$\text{(Stage C1)} \quad \sigma_c = D(\zeta'f'_c - f'_cT4) \left[2\left(\frac{\varepsilon_c}{\zeta'\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\zeta'\varepsilon_o}\right)^2 \right] + f'_cT4 \quad \varepsilon_o \leq \varepsilon_c < 0 \quad (8a)$$

$$\text{(Stage C2)} \quad \sigma_c = D\zeta'f'_c \left[1 - \left(\frac{\varepsilon_c/\varepsilon_o - 1}{4/\zeta - 1} \right)^2 \right] \quad \varepsilon_c < \varepsilon_o \quad (8b)$$

where σ_c is the smeared stress of concrete; ε_c is the smeared strain of concrete; f'_c and ε_o are the maximum concrete compressive cylinder strength and the peak cylinder compressive strain, respectively; f'_{cT4} is the stress at point TD (Fig. 4) between Stage C1 and Stage T4.

The softening coefficient ζ in Eqs. (8a) and (8b) was given by Zhang and Hsu (1998), (9):

$$\zeta = \frac{5.8}{\sqrt{f'_c (MPa)}} \frac{1}{\sqrt{1 + k\varepsilon_1}} \leq 0.9 \quad (8c)$$

where ε_1 is the tensile strains, either ε_H or ε_V in cyclic loading; k is the loading coefficient taken as $400/\eta$ for proportional loading, where $\eta=1$ for the case of pure shear and equal volume of steel in the ℓ and t directions.

The damage coefficient, D , takes into account the effect of concrete damage in one direction on the concrete strength in the perpendicular direction:

$$D = 1 - 0.4 \frac{\varepsilon_{mc}}{\varepsilon_o} \quad (8d)$$

where ε_{mc} is the maximum compression strain occurring in the immediate opposite direction and perpendicular to the compression strain being considered. ε_o is the strain corresponding to the peak cylinder strength. The damage coefficient, which is not considered in Mansour, Hsu and Lee (2001), (16), and Mansour, Lee and Hsu (2001), (15), does take care of the descending branches of the backbone envelope curves, as shown in Fig. 2 (a) for panel CA3.

Tensile Backbone Envelope Curves (T1 and T2) - The envelope curves for the cyclic tensile stress-strain curves of concrete can be expressed by the monotonic tensile stress-strain curve of concrete proposed by Belarbi and Hsu (1994), (4):

$$\text{(Stage T1)} \quad \sigma_c = E_c \varepsilon_c \quad 0 \leq \varepsilon_c \leq \varepsilon_{cr} \quad (8e)$$

$$\text{(Stage T2)} \quad \sigma_c = f_{cr} \left(\frac{\varepsilon_{cr}}{\varepsilon_c} \right)^{0.4} \quad \varepsilon_c > \varepsilon_{cr} \quad (8f)$$

where E_c is the modulus of elasticity of concrete taken as $3875\sqrt{f'_c}(\text{MPa})$; f_{cr} is the cracking stress of concrete taken as $0.31\sqrt{f'_c}(\text{MPa})$; and ϵ_{cr} is the cracking strain of concrete taken as 0.00008.

Unloading and Reloading Curves - A linear expression is proposed for the unloading and reloading curves as follows:

$$\sigma_c = \sigma_{ci} + E_{cc}(\epsilon_{ci} - \epsilon_c) \quad (8g)$$

where σ_{ci} and ϵ_{ci} are concrete stress and strain at the load reversal point “i” or at the point where the stages change; E_{cc} is the slope of the linear expression and is taken to be:

$$E_{cc} = \frac{\sigma_{ci} - \sigma_{ci+1}}{\epsilon_{ci} - \epsilon_{ci+1}} \quad (8h)$$

where σ_{ci+1} and ϵ_{ci+1} are the concrete stress and strain at the end of the stage under consideration. The points for ϵ_{ci} , σ_{ci} , ϵ_{ci+1} and σ_{ci+1} are specified in Fig. 4. Notice that the concrete stress of point TD is $1.5f_{cr} + 0.8f'_{cT2}$, rather than $f_{cr} + 0.8f'_{cT2}$ given in Mansour, Hsu and Lee (2001), (16) and Mansour, Lee and Hsu (2001), (15). This slight modification is taken to improve the agreement of crack closing strains between the CSMM predictions and the experimental results for all the test panels.

Constitutive Relationship of Concrete in Shear - Zhu, Hsu and Lee (2001), (17), showed that a rational relationship exists between the shear stress and the shear strain of concrete. This rational relationship in the (H,V) system of axes is given by the following expression:

$$\tau_{vH}^c = \frac{\sigma_H^c - \sigma_V^c}{2(\epsilon_H - \epsilon_V)} \gamma_{vH} \quad (9)$$

Poisson Effect – the Poisson effect of cracked reinforced concrete subjected to monotonic loading is characterized by two Hsu/Zhu ratios based on the smeared crack concept (Zhu, 2000; Hsu and Zhu, 2001), (10,11). The Hsu/Zhu ratio ν_{CT} (compression strain caused by tensile strain) was found to be zero for the entire post-cracking range. The Hsu/Zhu ratio ν_{TC} (tensile strain caused by compression strain), however, was found to be a function of the maximum steel strain and varied from 0.2 to 1.9. Before the Hsu/Zhu ratios

of cracked reinforced concrete subjected to cyclic loading can be determined, an average value of $\nu_{TC} = 1.0$ is assumed in this study.

COMPARISON OF CSMM PREDICTIONS WITH EXPERIMENTS

Pinching Effect

The CSMM-predicted hysteretic loops of the two panels CA3 and CD3 are plotted in Fig. 2 (a) and (b), together with the experimental curves. It can be seen that the CSMM is capable of predicting the pinched shape of the hysteretic loops of panels CA3 as well as the fully-rounded hysteretic loops of panel CE3.

Panel CA3 ($\alpha_2 = 45^\circ$) - The first cycle of the hysteretic loops beyond yielding for panel CA3 is plotted in Fig. 5(a). Four points A, B, C, and D are chosen in Fig. 5(a) to illustrate the *presence* of the pinched shape. Point A is at the maximum shear strain of the first cycle beyond yielding. Point B is at the stage where the shear stress is zero after unloading. Point C with a very low shear stress is taken in the negative shear strain region at the end of the low-stress pinching zone just before the sudden increase in stiffness. Point D is at the maximum negative shear strain of the first negative cycle. The three segments of curves from point A to point D in Fig. 5(a) clearly define the pinched shape of the hysteretic loops.

Panel CE3 ($\alpha_2 = 90^\circ$) - The first cycle of the hysteretic loops beyond yielding for panel CE3 is plotted in Fig. 5(b). Four points A, B, C, and D are chosen in Fig. 5(b) to illustrate the *absence* of the pinched shape. Points A, B and D correspond to the same three points in Fig. 5(a). However, point C with a high shear stress is taken in the negative shear strain region when the compression steel reaches yielding. The three segments of curves from point A to point D in Fig. 5(b) clearly show the absence of pinching.

Stresses and Strains Represented by Mohr Circles

The smeared strains, the applied stresses, the smeared concrete stresses, and the smeared steel stresses at the four points (A, B, C and D) chosen in Fig. 5 (a) and (b) are represented by Mohr circles as shown in Figs. 6 and 7 for panels CA3 and CE3, respectively. Using the Mohr circles has two advantages: First, Mohr circles represent the entire stress or strain state in an element, i.e. stresses or strains in all directions. Second, Mohr circles are the best means to help illustrate the mechanism behind the pinching effect and the failure mechanism of reinforced concrete elements under cyclic loading. To better visualize what is happening in the two panels, CA3 and CE3, subjected to reversed cyclic loading, we will examine the stresses and the strains as the applied shear stresses change from point A to point D.

Panel CA3 ($\alpha_2 = 45^\circ$) - At point A in the first post-yield cycle, Fig. 6, the maximum applied shear stress, τ_{45° , of 6.84 MPa produces a large shear strain, γ_{45° , of 0.00828 (twice the number shown in the Mohr circle because the vertical axis represents $\gamma/2$). To resist this applied shear stress, the concrete is subjected to a maximum vertical compressive stress, σ_v^c , of 13.64 MPa, and a maximum smeared steel stresses, ρf_s , of 6.80 MPa in both the longitudinal and transverse directions.

When the panel is unloaded from point A to point B, the applied shear stress of 6.84 MPa is reduced to virtually zero. From equilibrium, the compressive stress in the concrete and the tensile stress in the steel are also approaching zero. Correspondingly, the shear strain is reduced from 0.00828 at point A to a value of 0.00288 at point B. This unloading process produces an almost linear shear stiffness due to the normal relaxation of steel and concrete. These nearly proportional reductions of stresses in the concrete and steel are also related to the closing of cracks in the vertical direction. The horizontal strain, ϵ_H , decreases from 0.00762 at point A to 0.00278 at point B, but remains in tension with significant size of crack width. The compressive strain in the vertical direction, ϵ_v , decrease from -0.00067 at point A to -0.00011 at point B.

When the positive shear strain of 0.00288 at point B is reversed to become a negative shear strain of -0.00268 at point C, the vertical strain increases to a tensile strain of 0.00273, while the horizontal strain further decreases to a small tensile value of 0.000039 (not in compression). This large change of shear strain through the origin, however, is not accompanied by a corresponding change in the applied shear stress, meaning that the shear stiffness in the BC region is very small. This is because at point C the vertical cracks are not fully closed, and the concrete compressive stress cannot be developed. Without forming an effective set of concrete compressive struts, the stresses in the steel bars also cannot be developed. Hence, both the concrete stress and steel stress remain small at point C.

When the negative shear strain reaches -0.00862 at point D, the vertical cracks are fully closed ($\epsilon_H = -0.00067$) and the concrete compressive struts are fully formed. The concrete struts can now resist a compressive stress of 13.50 MPa, and the smeared steel in both the longitudinal and transverse directions are resisting a stress of 6.59 MPa. Correspondingly, the element is resisting an applied shear stress of 6.91 MPa. In other words, in the reloading CD range the shear stiffness is restored to its normal magnitude.

The pinched shape of the hysteretic loop of panel CA3, Fig. 2 (a), is formed by a small shear stiffness in the BC region, sandwiched between two large shear stiffnesses in the AB and the CD regions.

smear strain, however, continues to increase. At point D the shear strain is – 0.00386, and the vertical strain reaches a value of 0.00400.

Physical Visualization

Panel CA3 ($\alpha_2 = 45^\circ$) - The presence of the pinching mechanism in panel CA3 can be explained intuitively by examining a cracked element with 45° steel bars as shown in Fig. 8(a). A state of pure shear in the 45° direction of this element is equivalent to applying a horizontal compressive stress σ_H and a vertical tensile stress σ_V of equal magnitude. In the reverse loading stage from point B to point C (Fig. 5(a)), which defines the region where pinching occurs, both the vertical and the horizontal cracks are open. The concrete struts have not yet been formed, and the applied stresses σ_H and σ_V must be resisted by the two 45° steel bars. Separate the effect of σ_H and σ_V as shown in Fig. 8(b) and Fig. 8(c), respectively. The horizontal compressive stress σ_H induces a compressive stress in the two 45° steel bars, Fig. 8(b), while the vertical tensile stress σ_V induces a tensile stress of equal magnitude in the same two 45° bars, Fig. 8(c). These two stresses in the two 45° steel bars cancel out each other. As a result, the element offers no shear resistance to the applied shear stress τ_{45° in the 45° direction, while the shear strain γ_{45° increases rapidly due to the opening of the horizontal cracks and the closing of the vertical cracks. The resulting near-zero shear stiffness in the BC regions creates the pinched shape in the hysteretic loops of the shear stress vs. shear strain curves.

Panel CE3 ($\alpha_2 = 90^\circ$) - The absence of pinching mechanism in panel CE3 can be intuitively visualized by considering a cracked element with 90° steel bars as shown in Fig. 9(a). This element is also subjected to a pure shear state in the 45° direction, which is equivalent to applying a horizontal compressive stress σ_H and vertical tensile stress σ_V of equal magnitude. In the reverse loading stage from point B to point C (Fig. 5(b)) both the vertical and horizontal cracks are open. Consequently, the horizontal compressive stress σ_H is resisted by the compressive stress in the horizontal steel bar (Fig. 9(b)), while the vertical tensile stress σ_V is resisted by a tensile stress in the vertical bar (Fig. 9(c)). Both the compressive stress in the horizontal steel bar and the tensile stress in the vertical steel bar contribute to the shear stress τ_{45° in the 45° direction. As a result, the shear stress increases proportionally to the shear strain, and the shear stiffness in the BC region becomes large. This large stiffness creates a smooth and robust hysteretic loop without the pinched shape.

FAILURE MECHANISM UNDER CYCLIC LOADING

Structures Subjected to Static Loading

When a reinforced concrete structure is subjected to static loading, the principal compression stresses in the structure can be resisted by concrete, but cracks will occur due to principal tensile stresses, because concrete is very weak in tension. To resist additional loads, steel bars must be added to resist the principal tensile stresses. Thus, reinforced concrete can be visualized as a truss consisting of concrete struts in compression and steel ties in tension. This truss is capable of resisting high magnitude of applied static loads. In this strut-and-tie model, equilibrium condition is satisfied by maintaining force equilibrium at the joints (or nodes) among struts and ties.

As we know, this strut-and-tie concept can be used to design all reinforced concrete structures under static loads. The discussion below addresses the question whether this concept is applicable to structures subjected to reversed cyclic loading.

Structures Subjected to Cyclic Loading or Earthquake Loading

Comparison of the cyclic behavior of panel CE3 ($\alpha_2 = 90^\circ$) and panel CA3 ($\alpha_2 = 45^\circ$) clearly shows that the strut-and-tie model is no longer valid for application to cyclic loading and earthquake. When reinforced concrete structures are resisting cyclic loading beyond the yielding of steel, crack widths increase in both directions with each cyclic of loading (elements are expanding). Crack widths will not close after unloading and the concrete struts cannot be formed to resist the reversed loading while the cracks are still open. Therefore, the steel bars must be designed to resist principal compression as well as principal tension. Panel CE3 ($\alpha_2 = 90^\circ$), which is designed based on this principle, does perform very well. It does not have the pinching problem, and it exhibits ductile behavior and high capacity in energy dissipation.

When the steel bars are oriented at an angle of 45° to the principal stress coordinate as in panel CA3 ($\alpha_2 = 45^\circ$), steel bars in both directions are always in tension. Then the cracks are forced to close in order to form the concrete struts required in establishing a truss of struts and ties. This forced closing of cracks and the subsequent reopening of cracks in each cycle of loading represent a very destructive failure mechanism. This mechanism leads to the weakening of concrete in compression and the rapid deterioration of bond between the concrete and the steel bars. This menacing failure mechanism is responsible for the pinching effect in the hysteretic loops, the early arrival of the descending branch, and the low capacity in energy dissipations.

CONCLUSION

The analysis of the two panels CA3 ($\alpha_2 = 45^\circ$) and CE3 ($\alpha_2 = 90^\circ$) clearly reveal the working mechanism of cracked reinforced concrete elements under cyclic shear. It is found that the "pinching effect" in the post-yield hysteretic loops is not an inherent behavior attributed to shear per se. Also, it is not caused by the bond slips between concrete and steel bars as has often been mistaken as the main reason. This pinching phenomenon is actually caused by the orientation of the steel bars with respect to the principal coordinate of the applied stresses, and can be predicted by a rational theory.

Whereas the strut-and-tie concept can be used to design reinforced concrete structures subjected to static loading (i.e., the concrete bears compression and steel bars resist tension), it cannot be applied to concrete structures subjected to cyclic or earthquake loading. This is because cracked concrete cannot be relied upon to resist compression after the yielding of steel. Thus, the steel bars must be designed to take on the role of resisting principal compression, in addition to principal tension. This design principle for cyclic loading is applicable to shear as well as bending actions.

REFERENCES

- (1) Hsu, T. T. C., Belarbi, A., and Pang, X. B. "A Universal Panel Tester," *Journal of Testing and Evaluations*, ASTM, Vol. 23, No. 1, 1995, pp. 41-49.
- (2) Hsu, T. T. C., Zhang, L. X. and Gomez, T. "A Servo-Control System for Universal Panel Tester," *Journal of Testing and Evaluations*, ASTM, Vol. 23, No. 6, 1995, pp. 424-430.
- (3) Hsu, T. T. C., *Unified Theory of Reinforced Concrete*, CRC Press Inc., Boca Raton, FL, 1993, 329 pp.
- (4) Belarbi, A. and Hsu, T. T. C. "Constitutive Laws of Concrete in Tension and Reinforcing Bars Stiffened by Concrete", *Structural Journal of the American Concrete Institute*, V. 91, No. 4, July-Aug., 1994, pp. 465-474.
- (5) Belarbi, A. and Hsu, T. T. C. "Constitutive Laws of Softened Concrete in Biaxial Tension-Compression", *Structural Journal of the American Concrete Institute*, V. 92, No. 5, Sept.-Oct., 1995, pp. 562-573.
- (6) Pang, X. B. and Hsu, T. T. C. "Behavior of Reinforced Concrete Membrane Elements in Shear," *Structural Journal of the American Concrete Institute*, Vol.92, No.6, Nov.-Dec., 1995, pp.665-679.
- (7) Pang, X. B. and Hsu, T. T. C. "Fixed-Angle Softened-Truss Model for Reinforced Concrete," *Structural Journal of the American Concrete Institute*, Vol. 93, No. 2, Mar.-Apr., 1996, pp. 197-207.
- (8) Hsu, T. T. C. and Zhang, L. X. "Nonlinear Analysis of Membrane Elements by Fixed-Angle Softened-Truss Model," *Structural Journal of the*

- American Concrete Institute*, Vol. 94, No. 5, Sept.-Oct., 1997, pp. 483-492.
- (9) Zhang, L. X. and Hsu, T. T. C., "Behavior and Analysis of 100 MPa Concrete Membrane Elements," *Journal of Structural Engineering*, ASCE, Vol. 124, No. 1, Jan. 1998, pp. 24-34.
 - (10) Zhu, R. R. H. "Softened Membrane Model of Cracked Reinforced Concrete Considering Poisson effect," *Ph.D. Dissertation*, Department of Civil Engineering, University of Houston, Houston, TX, 2000.
 - (11) Hsu, T. T. C. and Zhu, R. R. H. "Post-yield Behavior of Reinforced Concrete Membrane Elements - the HSU/ZHU ratios," *ASCE Publication: Modeling of Inelastic Behavior of RC Structures Under Seismic Loads*, American Society of Civil Engineers, Reston, VA, April 2001, pp. 139-157.
 - (12) Zhu, R. R. H., and Hsu, T. T. C. "Poisson Effect in Reinforced Concrete Membrane Elements," *Structural Journal of the American Concrete Institute* (submitted for publication in 2002).
 - (13) Hsu, T. T. C. and Zhu, R. R. H. "Softened Membrane Model for Reinforced Concrete," *Structural Journal of the American Concrete Institute* (submitted for publication in 2002).
 - (14) Mansour, M. Y. "Behavior of Reinforced Concrete Elements Under Cyclic Shear: Experiments to Theory" *Ph.D. Dissertation*, Department of Civil Engineering, University of Houston, Houston, TX, 2001.
 - (15) Mansour, M. Y., Lee, J. Y., and Hsu, T. T. C. "Cyclic Stress-Strain Curve of Concrete and Steel Bars in Membrane Elements", *Journal of Structural Engineering*, ASCE, Vol. 127, No. 12, Dec. 2001, (scheduled).
 - (16) Mansour, M. Y., Hsu, T. T. C., and Lee J. Y. "Pinching effect in Hysteretic Loops of R/C Shear Elements", *ACI Special Publication*, American Concrete Institute (accepted for publication in 2001).
 - (17) Zhu, R. R. H., Hsu, T. T. C. and Lee, J. Y. "Rational Shear Modulus for Smeared Crack Analysis of Reinforced Concrete," *Structural Journal of the American Concrete Institute*, Vol. 98, No. 4, July-Aug. 2001, pp.443-450.

NOTATIONS

- A = coefficient in the unloading and reloading equation of a steel bar, given as $1.9k_p^{-0.1}$.
- B = parameter equal to $(f_{cr}/f_y)^{1.5}/\rho$.
- D = damage coefficient for concrete in compression.
- E_c = modulus of elasticity of concrete.
- E_{cc} = slope of the linear unloading and reloading expression of concrete.
- E_s = modulus of elasticity of a bare steel bar.
- f'_c = maximum compressive strength of a concrete cylinder.



- f_{cr} = cracking stress of concrete.
- f_ℓ = smeared stress of steel bars embedded in concrete in the longitudinal direction.
- f_s = smeared stress of steel bars embedded in concrete.
- f_t = smeared stress of steel bars embedded in concrete in the transverse direction.
- f_y = yield stress of bare steel bars.
- k = coefficient in the equation for softened coefficient ζ , taken as 400 for proportional loading.
- k_p = plastic strain ratio given as ϵ_p / ϵ_n .
- R = coefficient in the unloading and reloading equation of a steel bar, given as $10k_p^{-0.2}$.
- ϵ_1 = tensile strain in 1 – direction, could be ϵ_H or ϵ_V under cyclic loading.
- ϵ_c = concrete strain.
- ϵ_{comp} = maximum compression strain occurring in the immediate opposite direction and perpendicular to the compression strain being considered.
- ϵ_{cr} = cracking strain of concrete taken as 0.00008.
- ϵ_{ci} = concrete strain at the load reversal point “i”.
- ϵ_{ci+1} = concrete strain at the end of a stage.
- ϵ_H = smeared strain in horizontal direction.
- ϵ_ℓ = smeared strain in longitudinal direction.
- ϵ_n = yield strain of steel bars embedded in concrete.
- ϵ_p = plastic strain of steel bars embedded in concrete.
- ϵ_s = smeared strain of steel bars embedded in concrete.
- ϵ_t = smeared strain in transverse direction.
- ϵ_V = smeared strain in vertical direction.
- ϵ_y = yield strain of a bare steel bar.
- ϵ_0 = concrete strain at maximum compressive cylinder strength.
- ζ = stress or strain softening coefficient
- ρ = reinforcement steel ratio.
- ρ_ℓ = reinforcement ratio in longitudinal direction.
- ρ_t = reinforcement ratio in transverse direction.
- σ_c = concrete stress.
- σ_{ci} = concrete stress at the load reversal point “i”.

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σ_{ci+1} = concrete stress at the end of a stage.

σ_ℓ = longitudinal applied stress.

σ_t = transverse applied stress.

σ_H^c = concrete stress in horizontal direction.

σ_V^c = concrete stress in vertical direction.

τ_{45° = shear stress at 45° to the horizontal direction.

$\tau_{\ell t}$ = shear stress in $(\ell - t)$ coordinate system.

τ_{VH}^c = concrete shear stress in (H-V) coordinate system.

γ_{45° = shear strain at 45° to horizontal direction.

$\gamma_{\ell t}$ = shear strain in $(\ell - t)$ coordinate system.

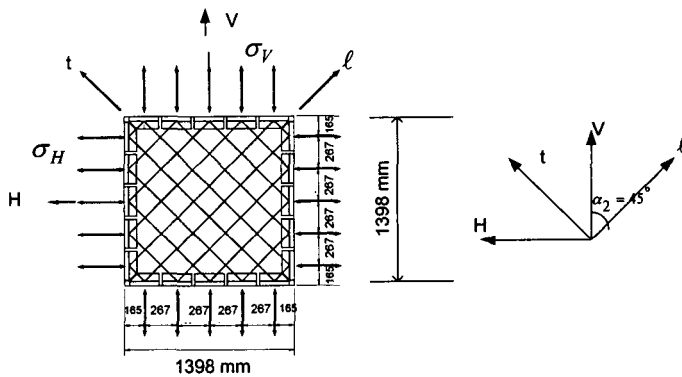
γ_{VH}^c = concrete shear strain in (H-V) coordinate system.

ν_{CT} = Hsu/Zhu ratio that takes into account the effect of tensile strain on compression strain.

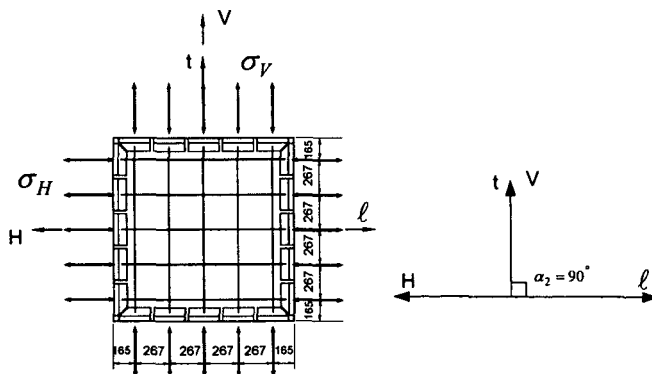
ν_{TC} = Hsu/Zhu ratio that takes into account the effect of compression strain on tensile strain.

Table 1 Material properties of test specimens

Panel	Concrete		Steel in ℓ direction			Steel in t direction		
	f'_c (MPa)	ϵ_o	Spacing of #6 bars	ρ_ℓ	f_{ty} (MPa)	Spacing of #6 bars	ρ_t	f_{ty} (MPa)
CA3	45	0.0025	189 mm	1.7%	428	189 mm	1.7%	428
CE3	50	0.0023	267 mm	1.2%	428	267 mm	1.2%	428



(a) Panel CA3 ($\alpha_2 = 45^\circ$).



(b) Panel CE3 ($\alpha_2 = 90^\circ$).

Fig. 1 Steel layout and dimensions of test specimens.

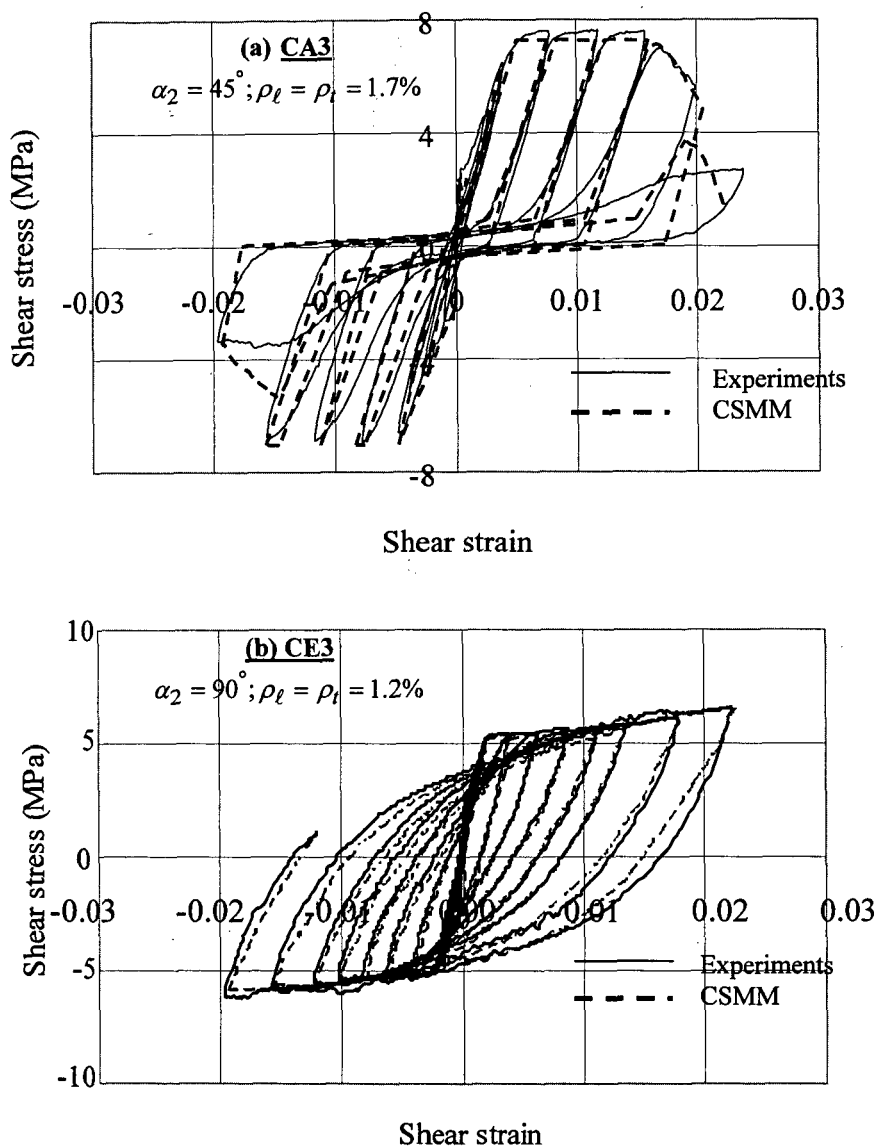


Fig. 2 Experimental and analytical shear stress-strain curves of panels CA3 and CE3.

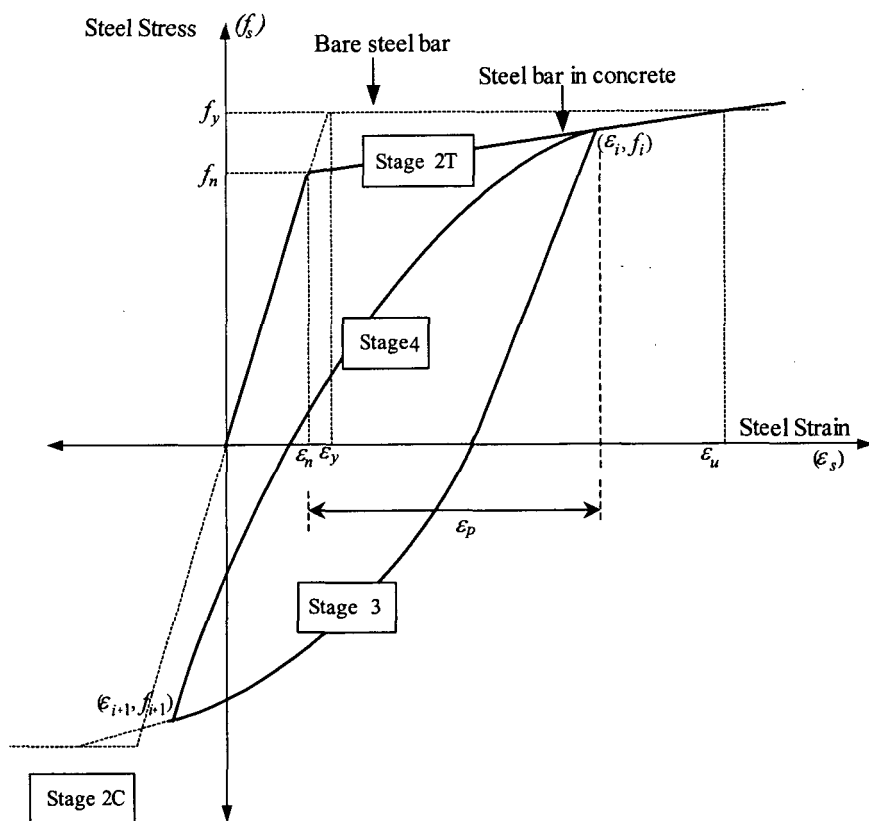


Fig. 3 Average cyclic stress-strain curves of steel bars in concrete.

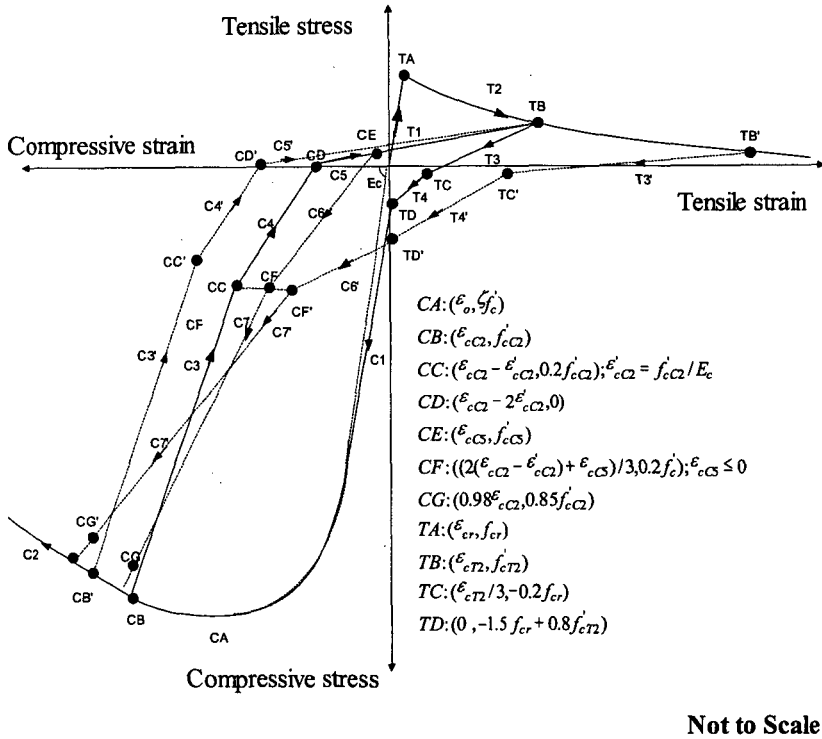


Fig. 4 Proposed model for cyclic stress-strain curves of concrete.

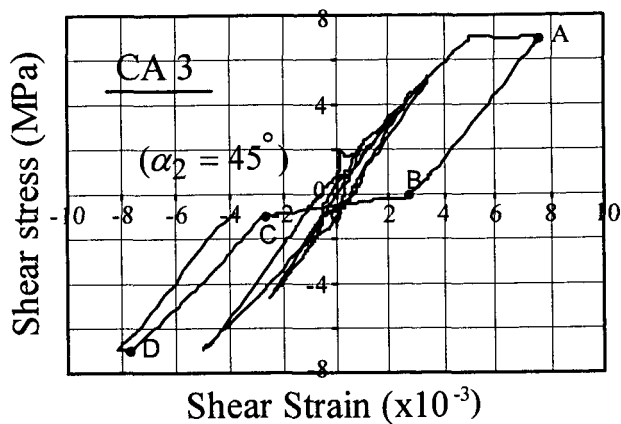


Fig. 5 (a) Pinching mechanism for panel CA 3.

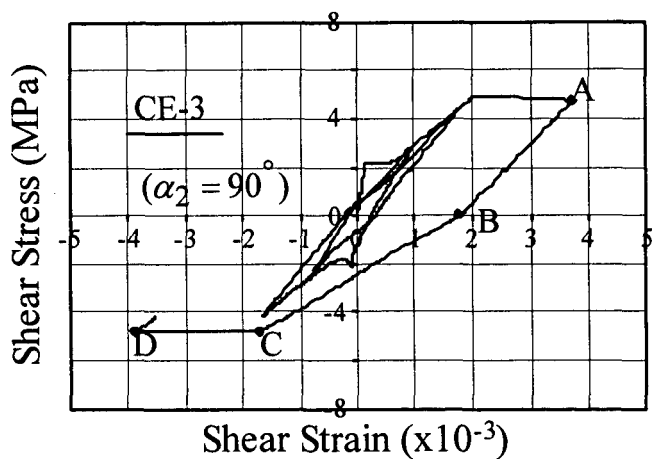


Fig. 5 (b) Absence of pinching mechanism for panel CE 3.

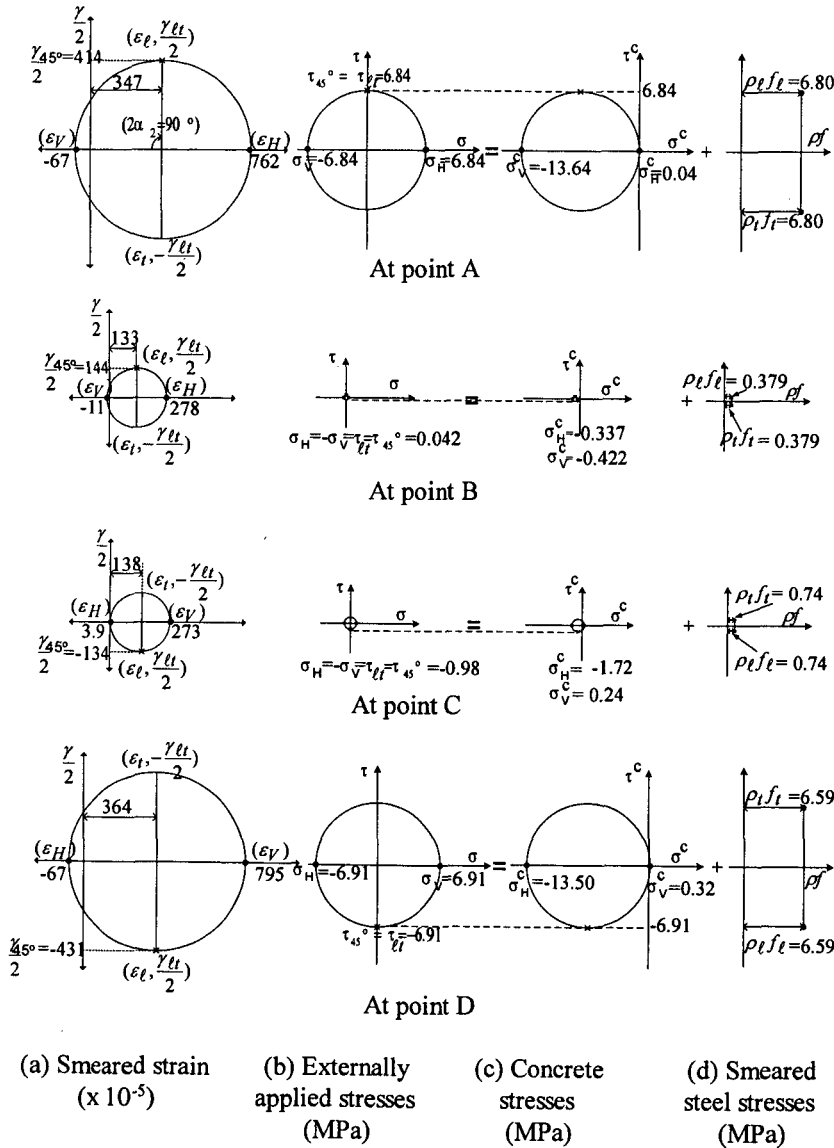


Fig. 6 Mohr's circles of smeared strains and stresses at points A, B, C, and D for panel CA3 ($\alpha_2 = 45^\circ$).

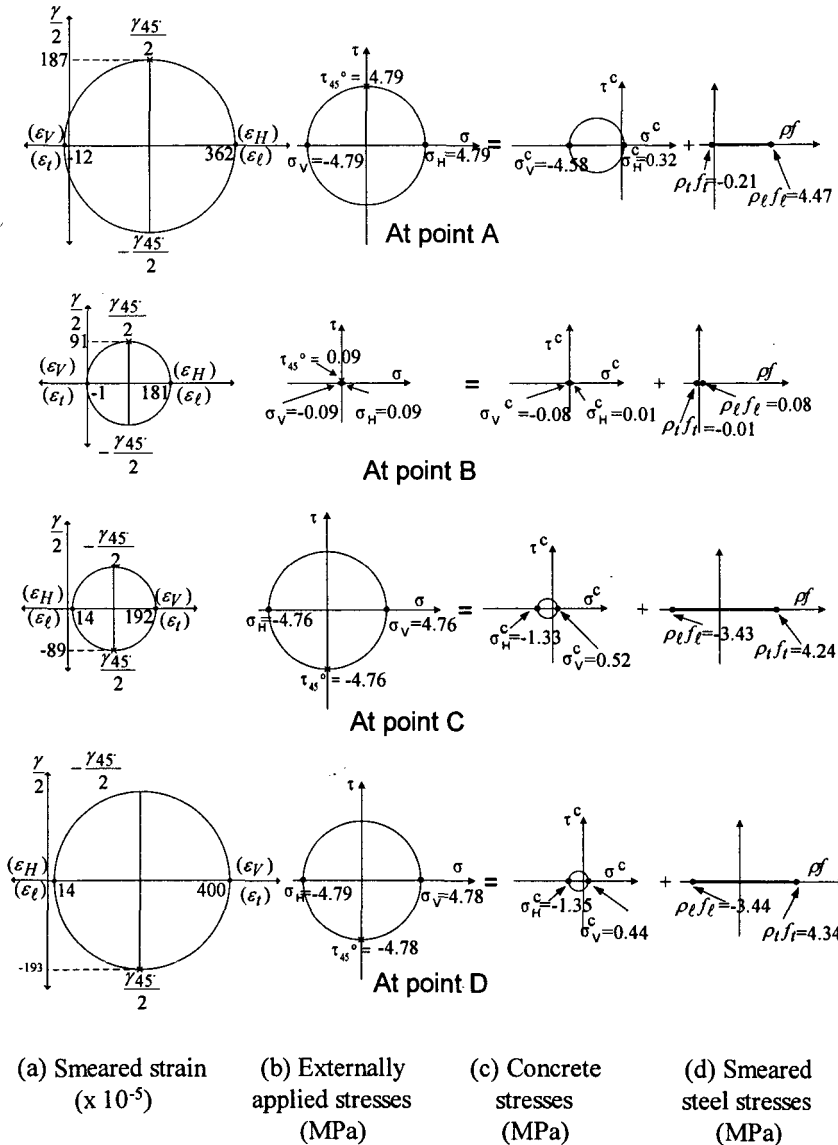


Fig. 7 Mohr's circles of smeared strains and stresses at points A, B, C, and D for panel CE3 ($\alpha_2 = 90^\circ$).

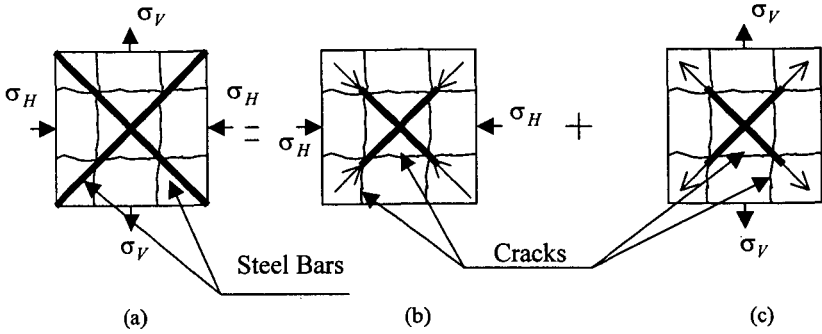


Fig. 8 Cracked R/C element with 45° steel bars.

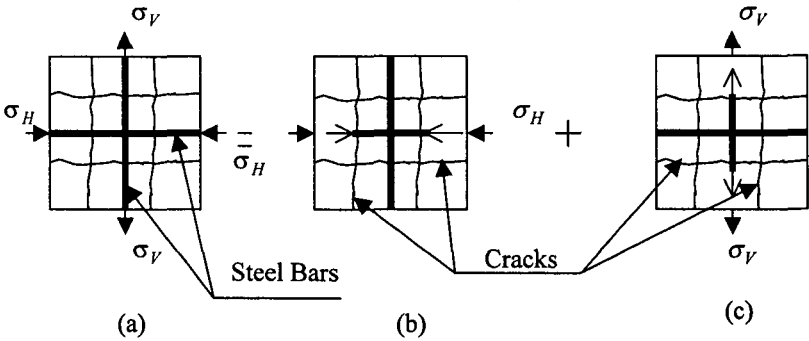


Fig. 9 Cracked R/C element with 90° steel bars.

Fatigue Fracture and Crack Propagation in Concrete Subjected to Tensile Biaxial Stresses

by K. V. Subramaniam, J. S. Popovics, and S. P. Shah

Synopsis: The objective of this paper is to characterize the quasi-static and fatigue response of concrete subjected to biaxial stresses in the t-C-T region, where the principal tensile stress is larger in magnitude than the principal compressive stress. An experimental investigation of material behavior is conducted. The failure of concrete in the stated biaxial region is shown to be a local phenomenon under both quasi-static and fatigue loading, wherein the specimen fails owing to a single crack. The crack propagation is studied using the principles of fracture mechanics. It is observed that crack growth in constant amplitude fatigue loading is a two-phase process: a deceleration phase followed by an acceleration stage. The quasi-static load envelope is shown to predict the crack length at fatigue failure. A fracture-based fatigue failure criterion is proposed, wherein the fatigue failure can be predicted using the critical mode I stress intensity factor obtained from the quasi-static response. A material model for the damage evolution during fatigue loading of concrete in terms of crack propagation is proposed. The model parameters obtained from uniaxial fatigue tests are shown to be sufficient for predicting the biaxial fatigue response.

Keywords: biaxial; concrete; crack; damage; failure; fatigue; fracture; uniaxial

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INTRODUCTION

The Compression-Tension (C-T) region within the biaxial stress space is the area in which the principal stresses are of opposite signs. The biaxial C-T space can be further divided into two regions: (a) the tensile-Compression-Tension (t-C-T) region, where the magnitude of the principal tensile stress is greater than or equal to that of the principal compressive stress, and (b) the compressive-Compression-Tension (c-C-T) region, where the principal compressive stress has a larger magnitude. The t-C-T response of the material represents the material behavior subjected to combined shear and tension and is investigated in this paper.

Different methods have been tried to test concrete subjected to a biaxial state of stress. These include: subjecting cylindrical specimens to hydrostatic pressure in radial and axial directions (1); subjecting hollow cylinders to torsion and axial compression or to internal pressure and axial compression (2,3,4); and subjecting concrete plates to in-plane loading for different biaxial stress combinations (5,6). In these investigations the strength of concrete subjected to different ratios of principal stresses was determined and the failure surface of concrete in the biaxial stress space was established. However, no data are available in the published literature for the behavior of concrete subjected to biaxial loading after the peak stress. From the modes of failure reported in the literature (5), the damage localizes into a single crack in the t-C-T region. The predominant mode of failure seems to be the formation and propagation of a single crack. Controlled experiments are needed to obtain the post peak part of the response in the t-C-T region. Application of fracture mechanics can be

explored to explain the behavior of concrete in the post peak part of the response.

A distinction is generally made between low-cycle, high-amplitude fatigue and high-cycle, low-amplitude fatigue. The former involves a few load cycles of high stress (earthquakes, storms, etc.) while the latter is characterized by a greater number of cycles of low stress (wind and wave loading). A comprehensive review of the fatigue behavior of concrete subjected to uniaxial fatigue is provided in committee reports of RILEM and ACI (7,8,9). Few experimental results on the response of concrete subjected to repeated biaxial loading are available in the literature. All the results in the literature pertain to cyclic biaxial compression (where both principal stresses are compressive) (10,11,12). From their experimental investigation considering four different principal stress ratios, Su and Hsu [1988] established the S-N curve for concrete subjected to biaxial compression. The S-N relationship was found to be non-linear instead of being a straight line. The observed S-N curve can be idealized by two straight lines that have significantly different slopes. The slope for the low-cycle fatigue was seen to be several times higher than that for high cycle fatigue. Su and Hsu also concluded that the fatigue strength of concrete under biaxial compression is greater than that under uniaxial compression for any given number of cycles. Observation of the failure modes indicated that the failure mode under monotonic and fatigue loadings was identical. The S-N curve was also observed to be a curve in the case of high strength concrete (12). No reference concerning fatigue behavior of concrete subjected to fatigue loading in the C-T region was found in the literature.

OBJECTIVES AND SCOPE

An experimental investigation to characterize the material response in biaxial t-C-T region is conducted in this paper. The main objective is to understand the response of concrete subjected to high-amplitude biaxial fatigue loading in the tensile-compression-tension (t-C-T) stress space. Characterization of the material response under quasi-static loading and establishment of the mechanism of failure under such loading is the essential first step in the process. A predictive model for response of concrete under fatigue loading is also proposed.

EXPERIMENTAL AND ANALYTICAL PROCEDURES

In the experimental program two different test configurations were used. Concrete beam specimens were tested in a three-point bend configuration to obtain the tensile material properties. The experimental setup for applying biaxial t-C-T stresses consisted of applying combined torsional and axial

loading to hollow cylindrical concrete specimens. Torsion introduces a state of pure shear stress in the material which, when resolved in terms of principal stresses corresponds to a biaxial state of stress wherein the two principal stresses are equal in magnitude but of opposite signs. The ratio of the two principal stresses can be changed by superposition of axial load over the torsional load. The entire C-T region of the biaxial stress space can be spanned by varying the magnitude of the applied axial load, the limit corresponding to uniaxial tension and compression. The entire process is illustrated schematically in Figure 1.

Experimental evidence suggested that the decrease in stiffness of a specimen tested in the t-C-T region is associated with the growth of a single crack. Further, the decrease in the stiffness of the specimen in constant amplitude fatigue loading in the t-C-T region was also observed to be due to an increase in the crack length.

To understand the material response to t-C-T loading, it is hence important to obtain information about the crack growth resulting from load application. The problem can then be framed in terms of fracture mechanics, and a fracture-based crack growth criterion can be established for such loading. The evolution of damage under fatigue loading can then be studied and a fracture based fatigue damage law can be developed.

In concrete, the non-linear process zone ahead of crack tip is relatively large and its effects cannot be neglected as in linear elastic fracture mechanics (LEFM). One approach to account for the non-linear fracture process zone ahead of the crack tip is using the concept of effective crack. It is a first order correction to LEFM. In this approximation, the crack in a concrete structure surrounded by a large non-linear zone is replaced with an effective elastic crack that gives the same compliance of the structure as the actual crack. Effective crack is the traction free crack that gives the same compliance as the true crack. Crack growth in the concrete structure is framed in terms of the effective traction free crack and principles of LEFM are applied to study crack propagation (13).

Closed form solutions that relate the crack length with the compliance of a concrete beam are available in the literature (14). For a notched beam specimen tested in three-point bend configuration, the Young's modulus (E) of the material is calculated as

$$E = \frac{6Sa_0V_1(\alpha)}{C_i b d^2} \quad \text{where } V_1(\alpha) = 0.76 - 2.26\alpha + 3.87\alpha^2 - 2.04\alpha^3 + \frac{0.66}{(1-\alpha)^2} \quad (1)$$

C_i is the compliance (inverse of the stiffness) calculated from the load-crack mouth opening displacement (CMOD) curve, a_0 the initial notch length, S the span of the beam, b and d are the thickness and depth of the beam respectively and $\alpha = a_0/d$. The procedure for determining the crack length from the observed change in compliance associated with crack growth using Equation 1 is described by Jenq and Shah (14). The Mode I stress intensity factor (K_I), at a

given point on the quasi-static envelope and for the given crack length can be computed using equation 2

$$K_I = 3P \frac{S(\pi a_e)^{1/2} F(\alpha)}{2bd^2} \text{ where } F(\alpha) = \frac{1.99 - \alpha(1 - \alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\pi^{1/2}(1 + 2\alpha)(1 - \alpha)^{3/2}} \quad (2)$$

$\alpha = a_e/d$ and P is the load at the given point in the post-peak response.

The effective crack length in a cylinder subjected to torsion can also be similarly estimated from the change in rotational stiffness (or compliance) of the specimen. Unfortunately, closed-form solutions for prediction of the stiffness as a function of crack length are not available for this particular specimen geometry. Numerical analyses were performed for a cylinder with cracks of different lengths to determine the influence of crack length on the rotational stiffness (inverse of compliance) and the stress intensity factors at the crack tip (15).

From the results of the numerical analysis it was observed that the mode I stress intensity factor (K_I) is considerably larger than the stress intensity factors for modes II and III (K_{II} and K_{III}) at all non-trivial crack lengths (15). Through regression analysis of the numerical data the expression for computing K_I for a given crack length and applied torque is given as

$$K_I = (2.6296 \times 10^{-6}(a) + 7.2064 \times 10^{-6}) * \text{Torque} \quad (3)$$

where K_I is in $\text{N/mm}^{3/2}$, Torque is in N-mm , and a is in mm . Similarly, from the numerical analysis an expression that relates the decrease in rotational stiffness of the cylinder with the increase in crack length was obtained. The expression given in Equation 4 that relates the crack length with the observed decrease in rotational stiffness was obtained.

$$S = -0.0353a^2 - 0.0208a \quad (4)$$

where a is the crack length in mm , and S is the percentage decrease in rotational stiffness with respect to an uncracked cylinder.

EXPERIMENTAL RESULTS

Under quasi-static loading it was observed that a crack initiated in the pre-peak region of the response under both the flexural and torsional loading (15,16,17). The crack propagation under quasi-static loading was studied by unloading and reloading the specimen at several points on the quasi-static load envelope. Typical cyclic-quasi-static test response obtained by unloading the specimen at different points on the load envelope in flexure is shown in Figure 2(a). The crack length at these points in the quasi-static load response was

determined from the decrease in unloading stiffness. Using the results of the previous section, the crack lengths were determined by matching the percentage drop in the stiffness with respect to the pristine specimen with the numerical computed change in stiffness associated with an increase in the effective crack length. The K_I corresponding to the given crack length is then determined using the expressions that relate crack length and applied loading to K_I (such as Equations 2 and 3). The value of K_I computed at different points on the post-peak quasi-static load envelope are shown plotted in Figure 2(b). It has been previously established that the value of K_I corresponding to the peak load of the specimen is a material constant referred to as the critical stress intensity factor, K_{IC} (14). This analysis shows that the stress intensity factor during crack growth in the post peak part of the quasi-static response stays constant and is equal to K_{IC} . An average value of $40 \text{ N/mm}^{3/2}$ is obtained for K_{IC} . Similar response was also obtained from the torsional specimens, wherein a constant value of K_I equal to $40 \text{ N/mm}^{3/2}$ was obtained in the post-peak part of the torsional response. The crack growth along the quasi-static post-peak envelope can hence be characterized by a fracture condition

$$K_I = \text{critical value of } K_I = K_{IC} \quad (5)$$

Constant amplitude, fatigue tests were conducted in both flexure and torsion at three different load ranges (15,16,17). In all the fatigue tests the lower load level in a fatigue cycle was kept constant, equal to 5% of the average quasi-static peak load. Three different upper load levels corresponding to 75%, 85%, and 95% of the average quasi-static peak load were used in the study. The crack length during fatigue loading was determined from the observed change in compliance (or stiffness) of the specimen using the compliance calibration curves.

The plot showing the rate of crack growth versus crack length for all the specimens tested in flexure at maximum load in a fatigue cycle equal to 85% of the average quasi-static peak load is shown in Figure 3. Similar responses were obtained at all the load ranges tested and for both flexure and torsional loading. The rate of crack growth follows a two-stage process; a deceleration stage followed by an acceleration stage up to failure. In the deceleration stage there is a decrease in the rate of fatigue crack growth with an increase in crack length. The deceleration stage typically lasts for the first 30-40% of the fatigue life of the specimens. In the acceleration stage the rate of fatigue crack growth increases continuously. There is a distinctive bend-over point (a_{bendover}) that corresponds to the crack length where the rate of crack growth changes from deceleration to acceleration.

Fatigue failure criterion

A comparison of K_I obtained from post-peak quasi-static response and constant amplitude fatigue loading for flexural and torsional tests is shown in Figure 4. For fatigue loading, the K_I corresponding to crack lengths at failure

have been computed using the maximum load in the fatigue cycle. The values of K_I at fatigue failure are seen to compare closely in flexural and torsional loading. Further, comparable critical values of K_I are obtained from quasi-static post-peak and fatigue loadings for flexure and torsion. It can be concluded that the load-deformation response obtained from quasi-static loading therefore acts as the failure envelope curve for fatigue loading when framed in terms of crack lengths. Further, the condition, $K_I = K_{IC}$, can be used to predict fatigue failure.

ANALYTICAL MODEL

A schematic representation of the results obtained from the quasi-static and fatigue loading of concrete in the t-C-T region is attempted in this section. The load response of a specimen (Figure 5(a)) can be transformed to the equivalent load – effective crack length space, as shown in Figure 5(b). In Figure 5(b), the origin, O' , is located at the initial flaw, a_0 . As the load is increased there is no crack growth till a certain load is reached (point O). The region below the threshold value O in the equivalent load-effective crack length space corresponds with the linear portion of the load response. The region OB represents stable crack growth in the quasi-static monotonic response up to peak load. After peak load, there is a decrease in the load with an increase in crack length (region BD). Region BD corresponds with the post-peak part of the load response.

The fatigue response, which exhibits crack growth due to repeated loading between two fixed load levels, is also shown schematically by line AC in Figure 5(b). From the previous analysis, it has been demonstrated that the quasi-static response acts as the failure envelope to the fatigue response when framed in terms of crack length. Point A represents the crack length at the beginning of the first cycle of fatigue loading and point C represents the crack length at fatigue failure. The post peak part of the quasi-static response (region BD in Figure 5(b)) can be predicted using the failure condition, $K_I = K_{IC}$, given in Equation (5). Using the failure envelope concept in terms of effective crack length, the same conditions also applies to fatigue failure. It is of interest to predict the crack growth in terms of load cycles.

Mechanistic Understanding of Crack Growth

To predict the fatigue crack growth it is important to understand the various mechanisms that influence the crack growth in concrete. There are two competing mechanisms involved in crack growth process in concrete and are illustrated schematically as the G and R curves in Figure 6. The R-curve of an infinite-sized specimen is shown by the curve OCF and the R-curve for the finite-sized specimen is given by a constant value after the critical crack length, curve OCE. The R curve represents the energy absorbed due the growth of the inelastic fracture process zone during crack growth and the G curve is the crack

driving force resulting from elastic unloading of the body during crack growth. The quasi-static load response of the specimen can be approximately predicted using G and the R-curves. The process is shown schematically in Figure 6. For the given specimen, the intersection of the G curves starting at the same initial notch length a_0 (for different stress levels) and the R-curve generates the quasi-static response that is shown on load effective crack plot in Figure 6(b). For a given stress level, the G-curve intersects the R-curve at two points. The first point of intersection is in the pre-peak portion while the second point is in the post-peak of the quasi-static response. By varying the stress, the entire load-deformation response of the specimen can be approximately predicted from the intersection of G and R-curves. On increasing σ , there comes a point where the G-curve intersects the R-curve tangentially. This corresponds to the peak load of the specimen and the crack length (a_{crit}). The constant value of the R-curve after a_{crit} corresponds to the observed constant value of K_{IC} in the post-peak part of the quasi-static response.

A simple phenomenological model for fatigue-crack growth is described in this section. Constant amplitude fatigue is a process of stable crack growth between two fixed load (stress) levels up to failure. The upper load (stress) level in the load cycle governs the energy release rate for unit-crack extension during this process. For a given upper load (stress) level during fatigue loading, (say σ_1 in Figure 6), the energy release rate with crack growth follows the G-curve from point A to point E in Figure 6(a); Point A represents the crack length during the first cycle of loading and point E represents crack length at failure given by the failure condition, $K_I = K_{IC}$. With an increase in crack length there is an increase in the stress intensity factor, K_I , at the crack tip, which provides a larger crack driving force. However, as the crack length increases the resistance to crack propagation (energy required for crack propagation) also increases along the path ACE on the R-curve (Figure 6(a)). The resistance to crack growth increases up to point C and then remains constant along CE. The point C corresponds to the critical crack length (a_{crit}) at the peak load of the quasi-static response.

The fatigue crack growth experiences increasing resistance up to a_{crit} . It can be therefore be hypothesized that the deceleration stage in the fatigue crack growth ends at a_{crit} . A comparison of the critical crack length at the peak load of the quasi-static response (a_{crit}) and the crack length at the bend-over point, where the rate of crack growth changes from deceleration to acceleration ($a_{bendover}$) is shown in Figures 7 (a) and (b) for flexure and torsion specimens respectively. Each point on the graph corresponds to one specimen tested in fatigue. The X-axis corresponds to the maximum applied load in a fatigue-load cycle as a percentage of the average maximum quasi-static monotonic peak load. The Y-axis of the graph is the ratio of $a_{bendover}$ to the average a_{crit} obtained by testing six specimens under cyclic quasi-static loading. From the figure it can be seen that $a_{bendover}$ in the fatigue response at load ranges used in this study is the same magnitude as a_{crit} of the quasi-static monotonic response. Hence, $a_{bendover}$ in constant-amplitude fatigue, for the load ranges considered, can be predicted from a_{crit} obtained from the quasi-static response.

PROPOSED MODEL FOR FATIGUE CRACK GROWTH

A proposed model for fatigue-crack growth rate is based on the following assumptions:

1. The crack growth rate in the deceleration stage is governed only by the increasing resistance, R .
2. The crack growth rate in the acceleration stage is governed only by the stress intensity factor, K_I .

In the deceleration stage, increasing K_I with increasing crack has a minor influence. This effect is ignored in this simplified model. In the deceleration stage, the reduction in the crack growth rate per unit crack advance is assumed to be proportional to the increasing resistance (rising R-curve) experienced by the crack. The resistance is proportional to the total increase in crack length from the beginning of loading ($Da = a - a_0$). For unnotched specimens a_0 can be taken equal to 2mm (18). It is proposed that the rate of crack growth in this stage can be represented by the expression

$$\frac{\Delta a}{\Delta N} = C_1 (Da)^{n_1} \quad (6)$$

where C_1 and n_1 are the constant and the exponent respectively, to be determined from the experimental data.

In the acceleration stage, it is assumed that the crack growth rate can be predicted by Paris law, rewritten here as

$$\frac{\Delta a}{\Delta N} = C_2 (\Delta K_I)^{n_2} \quad (7)$$

where C_2 and n_2 are the constant and exponent of the Paris law respectively, to be determined from the experimental data.

From the fatigue test data of three-point bending configuration, the obtained values of the coefficients, $\log(C_1)$ and n_1 of Equation 6, were -2.05 and -1.7 respectively, when the maximum stress in the fatigue cycle is below 95% of the peak quasi-static stress. For the very high maximum fatigue load case the coefficients were found to be -1.0 and -1.5 respectively. This difference in the very high load ranges was attributed to the significant influence of increasing K_I with increasing crack length. Similarly, the Paris law exponents of Equation 7, $\log(C_2)$ and n_2 , were determined from the flexural tests to be 28.6 and 17.33 respectively.

Equations 6 and 7 therefore provide a convenient approach for modeling fatigue crack growth that can be implemented relatively easily; material constants determined from the uniaxial tests can be used to predict the structural response in the t-C-T region with sufficient accuracy. The fatigue

crack growth process simulated using Equations 6 and 7 for flexure and torsion is shown in Figures 8 (a) and (b). The material constants obtained from the flexure tests were used in both simulations.

The analytical curves were obtained by numerically integrating Equations 6 and 7. The first step in the procedure consists of numerically determining a_{bendover} . The rate of crack growth predicted by Equations 6 and 7 (shown in Equation 8) are equated and the crack length solved iteratively.

$$\text{Log}(C_1) + n_1 \text{Log}(Da) = \text{Log}(C_2) + n_2 \text{Log}(\Delta K_I) \quad (8)$$

The procedure for obtaining the simulated crack fatigue growth now consists of determining computing the number of cycles (ΔN_i) for a chosen crack increment (Δa_i) and maximum and minimum load levels (P_{max} and P_{min}) as shown below

deceleration stage: ($a \leq a_{\text{bendover}}$)

1. set $a = a_0$ and $N = 0$
2. $a = a + \Delta a_i$, $Da = a - a_0$
3. $\Delta N_i = \frac{\Delta a_i}{C_1 (\Delta a_i)^{n_1}}$ and $N = N + \Delta N_i$
4. repeat steps 2-3 till $a = a_{\text{bendover}}$

acceleration stage: ($a_{\text{bendover}} < a$)

5. set $a = a_{\text{bendover}}$
6. $a = a + \Delta a_i$, $\Delta K_I = K_I(a, P_{\text{max}}) - K_I(a, P_{\text{min}})$
7. $\Delta N_i = \frac{\Delta a_i}{C_2 (\Delta K_I)^{n_2}}$ and $N = N + \Delta N_i$
8. repeat steps 6-7 till $\Delta N_i = 0$

Typical experimental results are also plotted in the same figure for comparison. It can be seen that the S-shaped trend observed in the fatigue crack growth is well captured by the simplified model consisting of Equations 6 and 7. Also, it can be seen that the fatigue crack growth in torsion is accurately predicted using coefficients obtained from flexural tests.

CONCLUSIONS

Based on the results presented in this paper the following conclusions can be drawn.

1. The load-deformation response obtained from quasi-static loading acts as the failure envelope curve for fatigue loading when framed in terms of crack length. The condition, $K_I = K_{IC}$, can be used to predict failure in high-amplitude fatigue.

2. Fatigue-crack growth is characterized by two distinct stages of crack growth rate: (a) a deceleration stage, where the rate of crack growth decreases as the crack grows; and (b) an acceleration stage, where there is a steady increase in crack growth rate to failure. The acceleration stage follows the deceleration stage.
3. There are two competing mechanisms that influence the crack growth rate during constant amplitude fatigue loading: (a) increasing resistance to crack growth owing to the rising R-curve, and (b) increasing K_I with crack length that accelerates the crack growth. The resistance to crack growth increases up to the crack length at the peak load of the quasi-static response and then assumes a constant value. The crack length where the rate of crack growth changes from deceleration to acceleration can hence be predicted from the crack length at the peak load in quasi-static loading.

ACKNOWLEDGEMENTS

This paper was prepared from a study conducted in the Center of Excellence for Airport Pavement Research. Funding for the Center of Excellence was provided in part by the Federal Aviation Administration under Research Grant Number 95-C-001. The Center of Excellence is maintained at the University of Illinois at Urbana-Champaign and is in partnership with Northwestern University and the Federal Aviation Administration. The authors would also like to acknowledge support from the NSF Center for Advanced Cement Based Materials, Northwestern University, during the course of this study.

REFERENCES

1. Karman, T., "Tests on Materials under Triaxial Compression," Verein Deutscher Ingenieure (Berlin), No. 42. 1911.
2. Bresler, B., and Pister, K., "Strength of Concrete under Combined Stresses," *ACI Journal Proceedings*, Vol. 55, No. 3, Sep. 1958, pp. 321-345.
3. Goode, C.D., and Helmy, M.A., "The Strength of Concrete under Combined Shear and Direct Stresses," *Magazine of Concrete Research*, vol. 19, No. 59, June 1967, pp. 105-112.

4. McHenry D., and Karni J., "Strength of Concrete under Combined Tensile and Compressive Stresses," *ACI Journal Proceedings*, Vol. 54, No. 10, Apr. 1958, pp. 829-840.
5. Kupfer, H., Hilsdorf, H., and Rusch " Behavior of Concrete Under Biaxial Stresses," *ACI Journal proceedings*, Vol. 66, No. 8, August 1969, pp. 656-666.
6. Kupfer H. B., and Gerstle K.H., " Behavior of Concrete under Biaxial Stresses", *Journal of Engineering Mechanics*, ASCE, Vol. 99, No. EM4, 1973, pp. 853-866.
7. Hawkins N.M., and Shah S.P.,: American Concrete Institute Considerations for Fatigue. IABSE Colloquium, "Fatigue of Steel and Concrete Structures," Lousanne, March 1982, *Proceedings IABSE Reports*, Vol. 37, Zurich, 1982, pp. 41-50.
8. RILEM Committee 36-RDL, "Long Term Random Dynamic Loading of Concrete Structures", *RILEM Materials and Structures*, Vol. 17, No. 97, 1984, pp. 1-28.
9. ACI Committee 215: Fatigue of Concrete Structures, (Editor Shah, S.P.), *American Concrete Institute*, Publication SP-75, Detroit 1982.
10. Buyukozturk O., and Tseng T., "Concrete in Biaxial Compression", *Journal of Structural Engineering*, ASCE, vol. 110, No. 3, March 1984, pp. 461-476.
11. Su E.C.M., and Hsu T.T.C., "Biaxial Compression Fatigue and Discontinuity of Concrete", *ACI Materials Journal*, May-June 1988, pp. 178-188.
12. Nelson E.L., Carrasquillo R.L., and Fowler D.W., "Behavior and Failure of High-Strength Concrete Subjected to Biaxial-Cyclic Compression Loading", *ACI Material Journal*, July-Aug 1988, pp. 248-253.
13. Shah, S.P., Swartz, S.E., Ouyang, C., "Fracture Mechanics of Concrete", Wiley Interscience, New York, 1995.
14. Jenq, Y.S., and Shah, S.P., "Two Parameter Fracture Model for Concrete," *Journal of Engineering Mechanics*, ASCE, Vol. 111, No. 10, 1985, pp. 1227-1241.
15. Subramaniam Kolluru V., (1999), "Fatigue of Concrete Subjected to Biaxial Loading in the Tension Region," PhD dissertation, Northwestern University.

16. Subramaniam, K.V., O'Neil, E., Popovics, J.S., and Shah, S.P. (2000), "Flexural Fatigue of Concrete: Experiments and Theoretical Model," *J. Engrg. Mech., ASCE*, vol 126(9), pp.891-898.
17. Subramaniam, K.V., Popovics, J.S., and Shah, S.P., (1999) "Fatigue Behavior of Concrete subjected to Biaxial Stresses in the C-T Region," *Mat. Jrnl., ACI*, Vol 96, No.6, pp. 663-669.
18. Ouyang, C., and Shah, S.P., " Geometry Dependent R-Curve for Quasi-Brittle Materials," *Journal of American Ceramic Society*, Vol 74, No. 11, 1991, pp. 2831-2836.

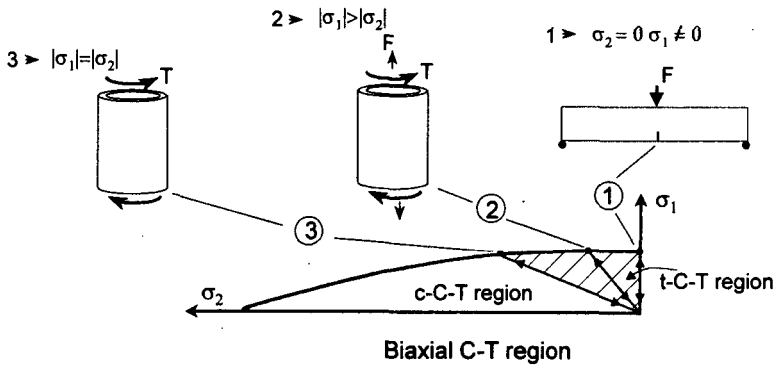


Figure 1: Schematic representation of the test configurations in the biaxial t-C-T stress-state.

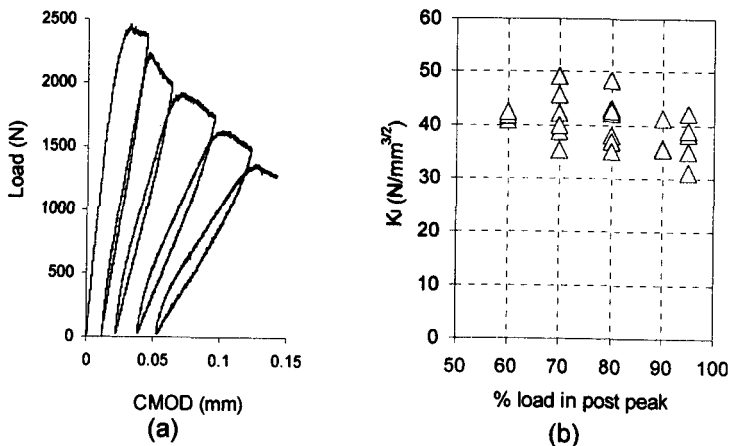


Figure 2: (a) Typical cyclic quasi-static response from a flexure test, (b) The K_I stress intensity factor at different points in the post-peak of quasi-static response.

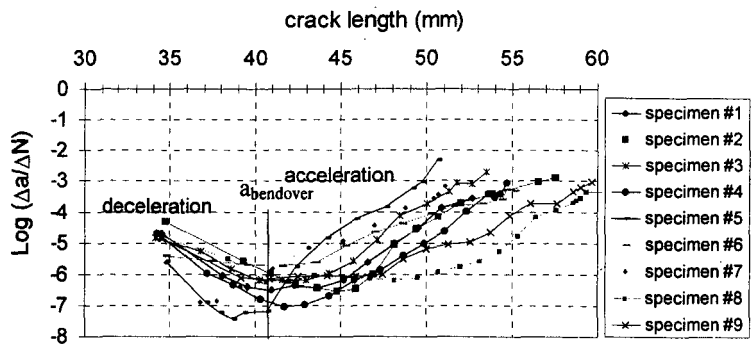


Figure 3: Rate of fatigue crack growth as a function of crack length during constant amplitude flexural fatigue loading.

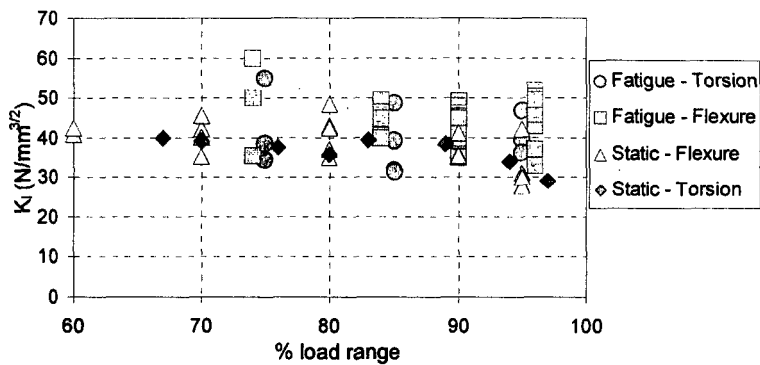


Figure 4: K_I at fatigue failure and in the quasi-static post peak of specimens tested in flexure and torsion.

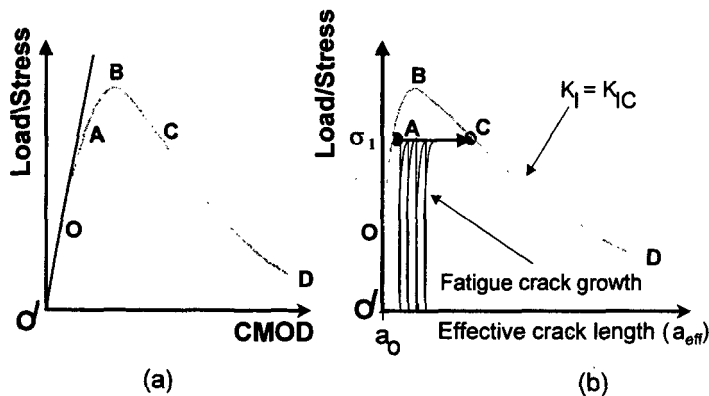


Figure 5: Quasi-static and fatigue crack propagation (a) Load-CMOD response (b) equivalent load-effective crack representation.

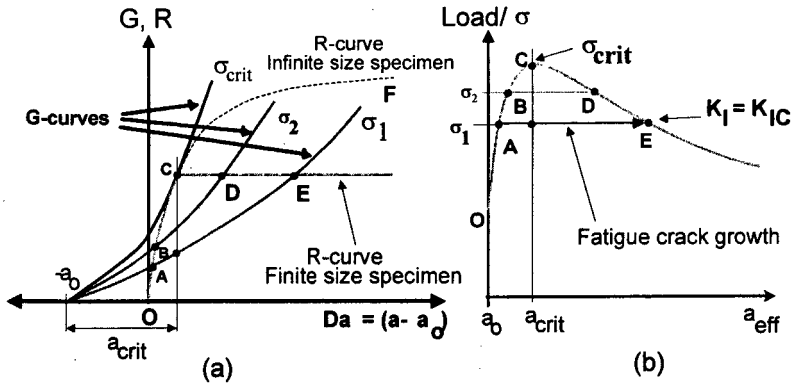


Figure 6: Crack growth analysis using R-curve concept (a) G and R-curves showing the energy release rates at different applied stress levels and the resistance to crack growth with increasing crack length (b) load-effective crack representation of the load-deflection response.

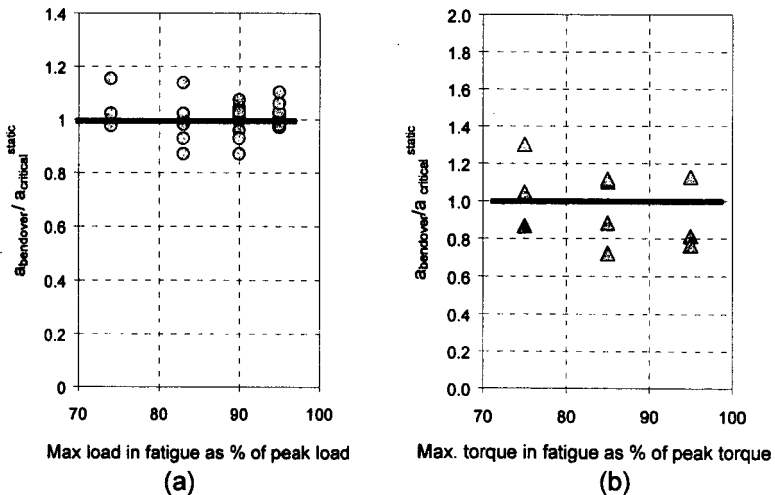


Figure 7: Ratio of the crack length at which the rate of crack growth changes from deceleration to acceleration ($a_{bendover}$) to crack length corresponding to the peak load ($a_{critical}^{static}$) in quasi-static loading (a) flexure (b) torsion.

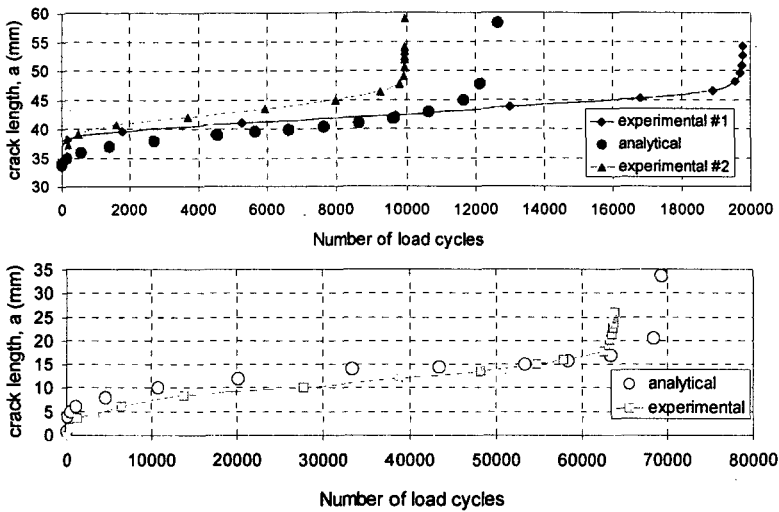


Figure 8: Analytical prediction of fatigue crack growth using the simplified model (a) flexural fatigue with maximum load equal to 95% of quasi-static peak load (b) torsional fatigue with maximum torque equal to 75% of the quasi-static peak torque.