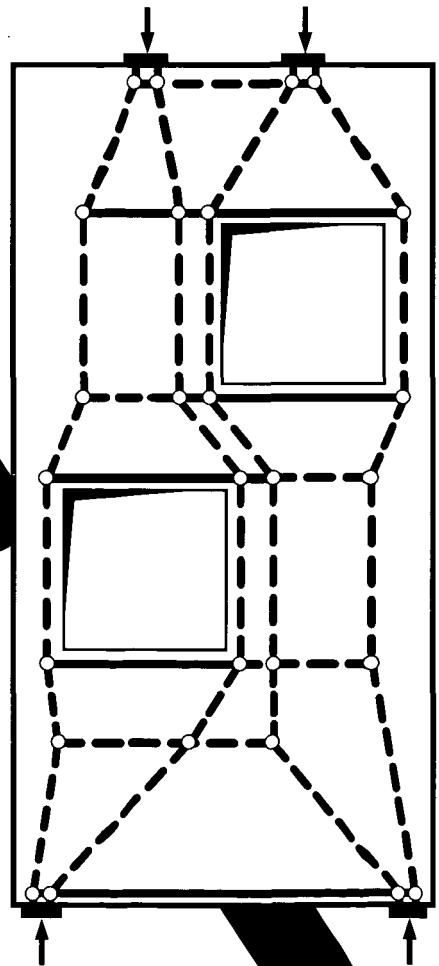
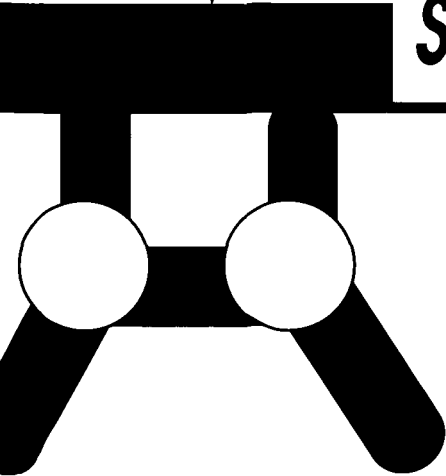


Examples for
the Design of
Structural Concrete with
Strut-and-Tie Models



Examples for the Design of Structural Concrete with Strut-and-Tie Models

Prepared by members of ACI Subcommittee 445-1, Strut and Tie Models, for sessions at the Fall Convention in Phoenix, October 27 to November 1, 2002, and sponsored by Joint ACI-ASCE Committee 445, Shear and Torsion, and ACI Committee 318-E, Shear and Torsion

Editor

Karl-Heinz Reineck



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Preface

The new Appendix A of ACI 318-2002 on strut-and-tie models provides an excellent tool for the design of structural concrete where sectional design procedures for flexure and shear do not apply. This is the case in discontinuity regions (D-regions), which if not properly detailed, may exhibit structural damage or even brittle failure. This special publication presents examples for the use of strut-and-tie models following Appendix A of ACI 318-2002 for designing D-regions such as corbels, deep beams with and without openings, dapped beam ends, beams with indirect supports, anchorage zones of prestressed members, shear walls with openings, bridge pier tables, and pile caps.

The contributions and the examples were prepared by members of *ACI Subcommittee 445-A Shear and Torsion: Strut-and-Tie Models* and were presented at two sessions at the ACI Fall Convention in Phoenix, October 27 – November 1, 2002, co-sponsored by the technical committee *ACI-ASCE Committee 445: Shear and Torsion* and *ACI Subcommittee 318E: Shear and Torsion*. The papers of this special publication were reviewed in accordance with American Concrete Institute policies. The cooperation of the authors in the preparation of the manuscripts and their revision considering comments by reviewers is greatly appreciated.

In addition, the efforts of the reviewers and of the ACI Headquarters staff, especially Mr. Todd Watson, Manager of Technical Documents, are appreciated.

The editor also acknowledges the efforts of several individuals at ILEK, University of Stuttgart, including Elfriede Schnee, Ali Daghighni, Hussein Coskun, and Angela Siller for their assistance in producing the document.

The editor finally wishes to thank Cathy French, Chair of ACI-ASCE Committee 445, and Jim Wight, former Chair of ACI Subcommittee 318E, for their continuous support of the work of ACI Subcommittee 445-A and of this special publication.

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Part 1

Introduction

Karl - Heinz Reineck

Karl-Heinz Reineck received his Dipl.-Ing. and Dr.-Ing. degrees from the University of Stuttgart. He is involved in both research and teaching at the Institute for Lightweight Structures Conceptual and Structural Design (ILEK), University of Stuttgart, and he is head of two research groups and managing director of the institute. His research covers the shear design of structural concrete, the design with strut-and-tie models and detailing of structural concrete and the design of high-performance concrete hot-water tanks. He is member of ASCE-ACI Committee 445 "Shear and Torsion" and chairs two subcommittees and also is a member of *fib* Task Group 1.1 "Practical Design".

Part 1: Introduction

1 Historical remarks

The last fifteen years has brought a major breakthrough in design methods for concrete structures which reflected in the terminology used. The term “structural concrete” was proposed as a unifying term for all kinds of applications of concrete and steel in order to overcome the traditional divisions between reinforced concrete, prestressed concrete and partially prestressed concrete and even externally prestressed concrete or unreinforced concrete. These differentiations were identified as artificial, leading to confusion in codes and in teaching as well as to unnecessary restrictions in practice, as pointed out at the IABSE Colloquium “Structural Concrete” April 1991 in Stuttgart [IABSE (1991 a, b)]. Shortly thereafter the American Concrete Institute renamed the ACI 318 code accordingly.

The limitations of purely empirical design procedures has been increasingly realized, driving the demand for the development of clear design models. The theory of plasticity was applied to the design of members under shear and torsion, especially by Thürlimann (1975, 1983) and Nielsen (1978, 1984) and their co-workers. This also formed the basis for strut-and-tie models after the works of Schlaich et al. (1987, 2001). Strut-and-tie models are a valuable design tool used since the beginning of reinforced concrete design, as demonstrated by the use of truss models for the shear design e.g. by Ritter (1899), Mörsch (1909, 1912, 1922), Rausch (1938, 1953) and others. This is especially true for discontinuity regions (D-regions), which have been poorly addressed in codes, even though improper design and detailing of D-regions led to structural damages including some failures [Breen (1991), Podolny (1985)]. The development of strut-and-tie models has brought the unique chance toward gaining consistency in the design concept covering D-regions and B-regions with similar models. Furthermore, the application of strut-and-tie models emphasizes the essential role of detailing in design. All this was pointed out in the State-of-the-Art Report on shear by the ASCE-ACI Committee 445 (1998).

The Appendix A of ACI 318-2002 consequently reflects this international development in research and thus is in line with some codes like the CEB-FIP Model Codes 1990, EC 2, Canadian Code and AASHTO, as well as the recent FIP Recommendations (1999) and the new German code DIN 1045-1 (2001-07).

2 Dimensioning procedures according to the present codes

The principles for the design are clearly defined in most codes, and they address the whole structure and not only sections when defining the requirements and principles for the design. Contrary to the principles, however, the dimensioning procedures and the checking procedures focus on sections, and separate checks are carried out for the different actions, such as moments and shear forces. In addition, the detailing rules given finally in codes are meant to ensure the overall safety of the structure.

The danger of a sectional design approach is that the overall flow of forces may be overlooked and that critical regions are not covered. Especially the regions with discontinuities due to the loading or/and the geometry, the D-regions, are not dimensioned but left to be covered by detailing rules, apart from some special cases (e.g. frame corners or corbels). All these considerations triggered discussions at the IABSE Colloquium “Structural Concrete” in 1991 and to the conclusions published thereafter [IABSE (1991 a, b)]. The demand for developing clear models, like strut-and-tie models, was expressed by Schlaich (1991) and Breen (1991). Most of these ideas were taken up by the FIP Commission 3 “Practical Design”, chaired by Julio Appleton, and one of its Working Groups developed the FIP Recommendations “Practical design of structural concrete” published in 1999 by *fib*. These recommendations are fully based on strut-and-tie models and show the direction for future developments. However, most codes still keep to the traditional concepts and only added a new chapter or appendix without integrating the new concept throughout the code. One exception is the case of the shear design where a truss model has been used for steel contribution for many years.

3 Aim and contents of this Special Publication

The implementation of strut-and-tie models in ACI 318-2002 with Appendix A is an important step in direction towards a more consistent design concept. Even more, it is a major achievement for the engineers in practice and should trigger efforts to apply strut-and-tie models in daily practice. Therefore, the main objective of this Special Publication is to show with design examples the application of strut-and-tie models according to the Appendix A of ACI 318-2002.

This Special Publication contains five parts. After the introduction (Part 1), Part 2 gives an insight into the development of Appendix A of ACI 318-2002 and of the discussions in ACI Committee 318 E “Shear and Torsion”. The scope and aim of the Appendix A is described and extensive explanations are given in addition to those already presented in the Commentary of Appendix A.

Part 3 presents a summary of important tests, which justify the use of strut-and-tie models for the design of structural concrete. Among the tests are the classical examples for D-regions like deep beams, corbels and dapped beam ends.

Part 4 forms the major part of this Special Publication presenting nine different examples designed with strut-and-tie models using Appendix A of ACI 318-2002. Most of these examples were taken from practice:

- The Example 1 (deep beam), Example 2 (dapped beam end) and Example 3 (double corbel and corbel at column) are classical D-regions, which have been designed with strut-and-tie models since long, and for which even tests were carried out, as described in Part 3.
- The Example 5 (beam with indirect supports) and Example 6 (prestressed beam) deal with well known D-regions of beams, which so far have mostly been dealt with in codes by rules for the shear design.
- The Examples 7 (pier table) and Example 9 (pile cap) deal with D-regions of 3D-structures, for the design which most codes give only rare information.

Some examples were selected to demonstrate the potential of strut-and-tie models to solve uncommon design problems, such as like Example 4 (deep beam with opening) and Example 8 (high wall with two openings).

All examples show the approach of finding a model, which is the first and an important step in a design with strut-and-tie models. The examples also point out where problems in dimensioning or in detailing and anchoring the reinforcement occur and how the design could be improved.

Part 5 gives a summary and discusses some issues which are either common for all examples or turned up in several examples. After a brief review of the procedures for finding a model, the uniqueness of a model is discussed and why different models may be selected by several engineers. The other issue addresses the transition between D- and B-regions of beams and is of general importance for many examples, because many D-regions are part of a larger structure and have to be "cut out" of it, i.e. the correct actions and forces have to be applied at the border of the D-region. Finally in Part 5 the importance of detailing is pointed out as it was demonstrated in several examples.

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Part 2

Derivation of strut-and-tie models for the 2002 ACI Code

James G. MacGregor

Synopsis

This paper documents the decisions made by ACI Committee 318 to introduce strut-and-tie models into the 2002 ACI Code. Sections 3 and 4 of this paper review code statements concerning the layout of strut-and-tie models for design. The format and values of the effective compression strength of struts are presented in Sec. 5. The first step was to derive an effective compression strength which gave the same cross-sectional area and strength using Appendix A as required by another code for the same concrete strength and same unfactored loads. The final selection of design values of the effective compression strength considered test results, design values from the literature, values from other codes, and ACI Code design strengths for similar stress situations. A similar derivation of the effective compression strengths of nodal zones is summarized in Sec. 6 of the paper. The description of the geometry of nodal zones in code language proved difficult. The design of ties is described in Sec. 7 of this paper and requirements for nominal reinforcement are in Sec. 8. Nominal reinforcement is provided to add ductility, to improve the possibility of redistribution of internal forces, and to control cracks at service loads.

James G. MacGregor received a B.Sc. in Civil Engineering from the University of Alberta, Canada in 1956, and a Ph.D. from the University of Illinois in 1960. He joined the Department of Civil Engineering at the University of Alberta in 1960, employed there until 1993. Dr. MacGregor serves on ACI technical committees on shear and torsion, columns, and the ACI Code committee. He chaired Canadian code committees on reinforced concrete design, and structural design. In 1992-93 he was president of ACI. Dr. MacGregor is an Honorary Member of ACI, a Fellow of the Royal Society of Canada, and a Fellow of the Canadian Academy of Engineering. In 1998 he received an honorary Doctor of Engineering degree from Lakehead University, and in 1999, an honorary Doctor of Science degree from the University of Alberta.

1 Introduction

The 2002 ACI Code (2002) includes a new Appendix A, "*Strut-and-Tie Models*" and changes to a number of code sections to allow the use of strut-and-tie models (STM) in design. In developing Appendix A, concepts were drawn from the AASHTO LRFD Specification (1998), the CEB/FIP Model Code (1993) as interpreted in the FIP Recommendations (1999), and the Canadian concrete design code, CSA A23.3-94 (1994). Research reports [ACI Committee 445 (1997)] also provided some of the basis for the appendix. This paper, in combination with the ACI 318 Commentary for Appendix A [ACI (2002)] explains the decisions and assumptions made in the development of Appendix A.

2 Research significance

This paper documents major decisions made in the development of Appendix A, "*Strut-and-Tie Models*" in the 2002 ACI Code.

3 What are strut-and-tie models?

3.1 B-Regions and D-Regions

Concrete structures can be divided into beam-like regions where the assumption of the straight line strain distribution of flexure theory applies, and disturbed regions, adjacent to abrupt changes in loading at concentrated loads and reactions; or adjacent to abrupt changes in geometry such as holes or changes in cross section. In the latter regions the strain distributions are not linear. These different portions are referred to *B-regions* and *D-regions*, respectively.

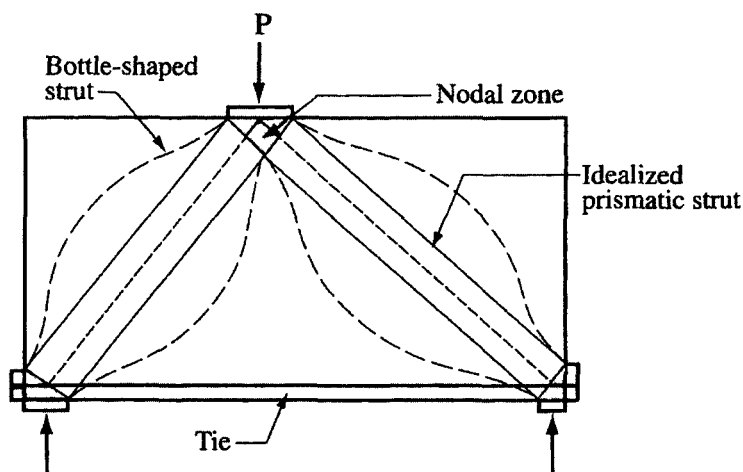


Fig. 1: A strut-and-tie model for a deep beam.

The traditional theory of flexure for reinforced concrete, and the traditional ($V_c + V_s$) design approach for shear apply in B-regions. In D-regions, on the other hand, a major portion of the load is transferred directly to the supports by in-plane compressive forces in the concrete and tensile forces in reinforcement and a different design approach is needed. D-regions may be modeled using hypothetical trusses consisting of concrete *struts* stressed in compression, steel *ties* stressed in tension, joined together at joints referred to as *nodes*. These trusses are referred to as *strut-and-tie models* (STM's). The strut and tie model of a single span deep beam shown in Fig. 1 is composed of two inclined struts, and a horizontal tie joined together at three nodes [ACI 318 (2002)]. The nodes are enclosed within *nodal zones* which transfer forces from the struts to the ties and reactions. Strut-and-tie models are assumed to fail due to yielding of the ties, crushing of the struts, failure of the nodal zones connecting the struts and ties, or anchorage failure of the ties. The struts and the nodal zones are assumed to reach their capacities when the compressive stresses acting on the ends of the struts or on the faces of the nodal zones, reach the appropriate *effective compressive strength*, f_{cu} .

De St. Venant's principle and elastic stress analyses suggest that the localized effect of a concentrated load or a geometric discontinuity will die out *about one member depth* away from the load or discontinuity. For this reason, D-regions are assumed to extend *approximately* one member depth from the load or discontinuity. The words "about" and "approximately" are emphasized here because the extent of D-regions can vary from case to case (see ACI Sec. A.1.).

If two D-regions each of length d or less, come together and touch or overlap, they are considered in Appendix A to act as a combined D-region. For a shear span in a deep beam the combined D-region has a depth of d and a length up to $2d$ one way or two ways from the disturbance. This establishes the smallest angle between a strut and a tie attached to one end of the strut as $\arctan(d / 2d) = 26.5^\circ$, rounded off to 25° (see ACI Sec. A.2.5.)

Figure 2, reproduced from "*Prestressed Concrete Structures*" [Collins and Mitchell (1991)], compares the experimental shear strengths of simply supported beams with various shear-span-to-depth ratios, a/d , from 1 to 7. B-region behavior controlled the strengths of beams with a/d greater than 2.5 as shown by the approximately horizontal line to the right of $a/d = 2.5$. D-region behavior controlled the strengths of beams with a/d ratios less than about 2.5 as shown by the steeply sloping line to the left of $a/d = 2.5$ in Fig. 2.

ACI Committee 318 limited the maximum lengths of isolated D-regions to d , and to $2d$ for overlapping D-regions. Strut-and-tie models can also be used in the design of B-regions [Marti (1985)]. However, the V_c term in the traditional ACI shear strength equation is not included.

Two-dimensional strut-and-tie models are used to represent planar structures such as deep beams, corbels and joints. Three-dimensional strut-and-tie models are used for structures such as pile caps for two or more rows of piles.

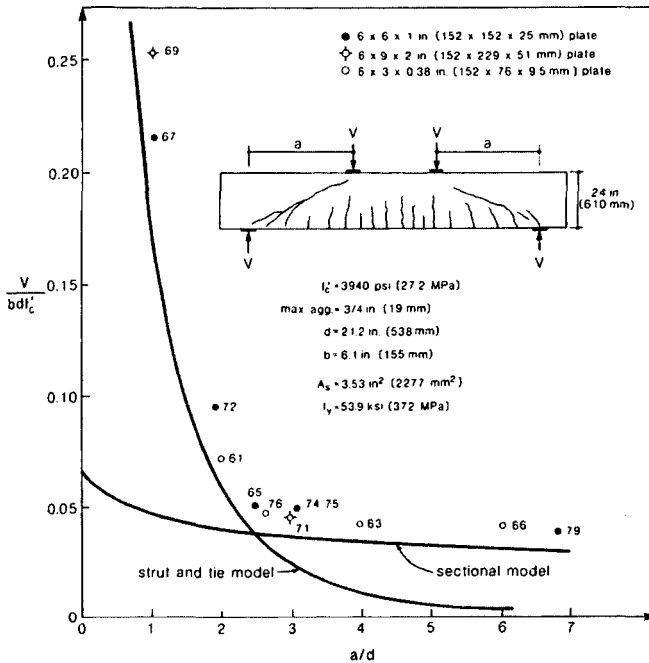


Fig. 2: Strengths of concrete beams failing in shear for various a/d ratios.

3.2 Decisions necessary to develop design rules for strut-and-tie models

To codify strut-and-tie models for design, the major items to be defined and specified are:

- the geometric layout of strut-and-tie models,
- what effective concrete strengths and ϕ factors should be used,
- the shape and strength of the struts,
- the arrangement and strength of the nodal zones,
- the layout, strength, and anchorage of the ties, and
- the detailing requirements.

The definitions of these items differ considerably in various codes and other design documents. The effective concrete strength and the strength reduction factors in Appendix A were originally derived using the load and resistance factors in Chapter 9 of the 1999 ACI Code. The changes necessitated by the new load and resistance factors in the 2002 ACI Code are presented briefly.

3.3 Geometric layout of strut-and-tie models

A strut-and-tie model is a hypothetical truss that transmits forces from loading points to supports. The selection of STM's, the calculation of the forces in the struts and ties, and the design of reinforcement in the ties are discussed, with examples, in *"Toward a Consistent Design of Structural Concrete"* [Schlaich, Schäfer and Jennewein (1987)] *"Prestressed Concrete Structures"* [Collins and Mitchell (1991)], ., and *"Basic Tools of Reinforced Concrete Beam Design"* [Marti (1985), and *[Reinforced Concrete: Mechanics and Design, MacGregor (1997)]*.

Appendix A is formulated on the assumption that strut-and-tie models will be used in design. It is not intended that design be reduced to equations for the shear resisted by struts and web reinforcement. The selection of the model and the production of a drawing of the model are integral parts of

Commentary RA.2.1 in Appendix A [ACI(2002)] gives a step-by-step procedure for laying out a strut-and-tie model. Various authors have suggested methods of doing this [ACI (1999)], Collins and Mitchell (1991), Schlaich, Schäfer and Jennewein (1987) and MacGregor (1997). The starting point is usually the computation of the reactions for the structure and loads. Generally speaking, a strut-and-tie model which minimizes the amount of reinforcement approaches the ideal model. For two-dimensional structures some researchers [Schlaich, Schäfer and Jennewein (1987)] recommend using a finite element analysis to determine the stress trajectories for a given load case. The struts are then aligned within $\pm 15^\circ$ of the resultant compression forces from such an analysis, and the ties within $\pm 15^\circ$ of the resultant tensile forces.

On the other hand, the Canadian code [CSA(1994)] requires that finite element analyses be checked by independent analyses satisfying equilibrium, and suggests using strut-and-tie models for this check.

In developing a strut-and-tie model for a given application it is frequently useful to select initial trial locations for the nodes and use these in the initial cycle of calculations of the member forces. If pictures of the cracking pattern in similar structures are available, the location of the struts and ties can be arranged within the structure such that struts fall between cracks. Struts should not cross cracked regions.

Section A.2 of the 2002 ACI presents several major requirements that must be satisfied by a strut-and-tie model:

1. *First and foremost, the strut-and-tie model must be in equilibrium with the factored applied loads and factored dead loads (ACI Sec. A.2.2). The calculation of the reactions and strut and tie forces satisfies statics. It therefore produces a statically admissible force field.*
2. *The strengths of the struts, ties, and nodal zones must equal or exceed the forces in these members. (ACI Sec. A.2.6). If the strength at every cross section equals or exceeds the strength required by the analysis in item 1, the structure is said to have a safe distribution of strengths.*
3. *In the early stages in the design of a D-region it may be sufficient to consider only the axes of the struts and ties when laying out a strut-and-tie model. It is generally necessary, however, to consider the widths of the struts, ties, and nodal zones and support regions when laying out a strut-and-tie model (Sec. A.2.3).*
4. *Struts must not cross or overlap each other (Sec. A.2.4). The widths of struts are chosen to carry the forces in the struts using the effective strength of the concrete in the struts. If struts overlapped, the overlapping parts of the struts would be overstressed.*
5. *Ties are permitted to cross struts or other ties. (Sec. A.2.4)*
6. *The smallest angle between a strut and a tie that are joined at a node is set at 25°. (Sec. A.2.5).*

A structural design that is both statically admissible and safe, satisfies the requirements of a lower bound solution in the theory of plasticity. This implies that the failure load computed by a strut-and-tie model underestimates the actual failure load. For this to be true, the structure must have enough ductility to accommodate any needed redistribution of forces.

4 Forces in struts and ties, and strength reduction factors, ϕ

After the initial strut-and-tie model has been selected, the reactions to the applied loads and self weight loads are computed. Once the reactions have been computed, the forces, F_u , in all the struts, ties, and nodal zones loads are computed using truss analysis. The struts, ties and nodal zones are then proportioned based on:

$$\phi F_n \geq F_u \quad (1)$$

where F_u is the force in the member (strut, tie, or nodal zone) due to the factored loads, F_n is the nominal strength of the member, and ϕ is a strength reduction factor. The nominal strengths of struts, ties, and nodal zones are F_{ns} , F_{nt} , and F_{nn} , respectively.

Equation 1 includes the factored resistance ϕF_n . In the CEB/FIP Model Code (1993) the Canadian Code [CSA(1994)], and the FIP Recommendations (1999), material strength reduction factors, ϕ_c and ϕ_s , or γ_c and γ_s , are applied to the concrete and steel strengths, f'_c and f_y . The 1999 ACI Code used different strength reduction factors for each type of structural resistance, $\phi_f = 0.9$ for flexure and $\phi_v = 0.85$ for shear in beams, corbels, and deep beams. (Subscripts have been added to the ϕ factors in this paper to indicate the structural action corresponding to the various ϕ factors.)

In the 2002 ACI Code, the load combinations and ϕ factors in Appendix C of ACI 318-99, were interchanged with those in Chapter 9 of ACI 318-99. In the 2002 Code, ϕ_v and ϕ_{STM} were changed to 0.75 for the design of strut-and-tie models using the load factors and strength reduction factors in Chapter 9 of the 2002 ACI Code. A re-evaluation of the ϕ factor for flexure indicated ϕ_f could remain equal to 0.90.

ACI 318-99 included a new Sec. 18.13 on the design of post-tensioned tendon anchorage zones, based in part on strut-and-tie models. That code specified $\phi_{pA} = 0.85$ in the design of post-tensioned anchorage zones to be used with a load factor of 1.2 (Sec. 9.2.8) on the prestressing force. Strut-and-tie models of prestress anchorage zones retain this ϕ factor and load factor because the tendon forces and the load factor on the tendon forces are unchanged.

5 Struts

5.1 Types of struts

Struts vary in shape. In strut-and-tie models, they are generally idealized as prismatic or uniformly tapered members as shown by the straight sides of the idealized prismatic struts in the shear spans of the deep beam shown in Fig. 1. In that strut-and-tie model, the concrete in the beam webs adjacent to a strut is stressed by the lateral spread of the strut stresses into the adjacent concrete in the shear spans. If there is room for this spread to occur, the struts are said to be *bottle-shaped*. Most struts in two-dimensional strut-and-tie models will be bottle-shaped.

5.2 Design of struts

Struts are designed to satisfy Eqs. 1 to 4. The factored strength of a strut is computed as:

$$F_{ns} = f_{cu} A_c \quad (2)$$

where f_{cu} is the *effective compressive* strength of the concrete in a strut, taken equal to:

$$f_{cu} = v f_c' \quad (3)$$

or:

$$\phi f_{cu} = \phi v f_c' = \phi_{STM} \alpha_1 \beta_s f_c' \quad (4)$$

where v (nu) is called the effectiveness factor, A_c is the end area of the strut acted on by f_{cu} . ϕ_{STM} is the value of ϕ for struts, ties, and nodal zones in strut-and-tie models, α_1 is the 0.85 factor in ACI Sec. 10.2.7.1 and β_s is the effectiveness factor for a strut. If f_{cu} is different at the two ends of a strut, the strut is idealized as being uniformly tapered. The term v is introduced as an intermediate step in the derivation of Eq. 4 because different codes and researchers include different factors in their definitions of the effective compressive strength.

5.3 Effective compressive strength of struts, f_{cu}

5.3.1 Factors affecting the effective concrete strength of struts

The stress acting in a strut is assumed to be constant over the cross-sectional area at the end of the strut. Three major factors that affect the effectiveness factor are given in the following paragraphs. Depending on the emphasis placed on each factor when deriving values of the effectiveness factor, the values of f_{cu} differ from code to code.

(a) Load Duration Effects. The effective strength of the struts is given by Eqs. 1 and 4 where $v = \alpha_1 \beta_s$, and α_1 is the 0.85 factor defined in ACI Sec. 10.2.7.1, explained in various references as accounting for load duration effects, or accounting for different stress regimes in the cylinders and the flexural stress blocks, or accounting for the vertical migration of bleed water. In Eq. 4, α_1 was taken equal to 0.85 from ACI Sec. 10.2.7.1. This factor probably should be a function of f'_c , decreasing as f'_c increases [Ibrahim and MacGregor (1997)]. Recently, several relationships have been suggested as replacements for the $\alpha_1 = 0.85$ in ACI Sec. 10.2.7.1. In the event that one of these proposed revisions is accepted, the α_1 in Eq. 4 would be modified to agree. The subscript "s" in β_s refers to strut.

(b) Cracking of the struts. Typically, the struts develop axial, diagonal, or transverse cracks. The resulting reduction in the compressive strength of the struts is explained in the following paragraphs.

(i) Bottle-shaped struts. Struts frequently are wider at midlength than at their ends because the width of concrete that the strut stresses can spread into is greater at midlength than at the ends of the strut. The curved, dashed outlines of the struts in Fig. 1 represent the effective boundaries of a typical strut. Such a strut is said to be *bottle-shaped*. In design, bottle-shaped struts are idealized as the prismatic struts shown by the straight, solid-line boundaries of the struts in Fig. 1.

The divergence of the forces along the length of the strut tends to cause longitudinal splitting near the ends of the strut as shown in Fig. 3. In the absence of reinforcement to confine this splitting, these cracks may weaken the strut. Schlaich et al. (1987) have analyzed this type of cracking and predict that it will occur when the compressive stress on the end of the strut exceeds approximately $0.55f'_c$. Schlaich et al. and Breen et al. (1994) suggest that the diverging struts in Fig. 3 have a slope of 1:2 as shown.

(ii) Cracked struts. The strut may be crossed by cracks which will tend to weaken the strut [Schlaich, Schäfer and Jennewein (1987)]. The effective compressive strengths given in Appendix A and Sec. 5.3.5 of this paper reflect this concept.

(iii) Transverse tensile strains. Tensile strains perpendicular to the axis of a cracked strut connected to a tie may reduce the compressive strength of the strut [Vecchio and Collins (1982)]. In the Canadian Code [CSA(1994)] and the AASHTO Specifications (1998) it is assumed that the strength of a strut is a function of the transverse tensile

strain in the strut due to the tie attached to one or other end of the strut, computed as a function of the angle between the axis of the strut and the axis of the tie. Tests of uniformly-stressed, square concrete panels by Vecchio and Collins (1982) have given rise to the values of f_{cu} given by Eqs. 11 and 12 in Sec. 5.3.3.

(c) Confinement from surrounding concrete. In three-dimensional concrete structures like pile caps, the compressive strength of a strut may be increased by the confinement resulting from the large volume of concrete around the strut. Adebar and Zhou (1993) have proposed equations for the effective compressive strength for use in designing pile caps.

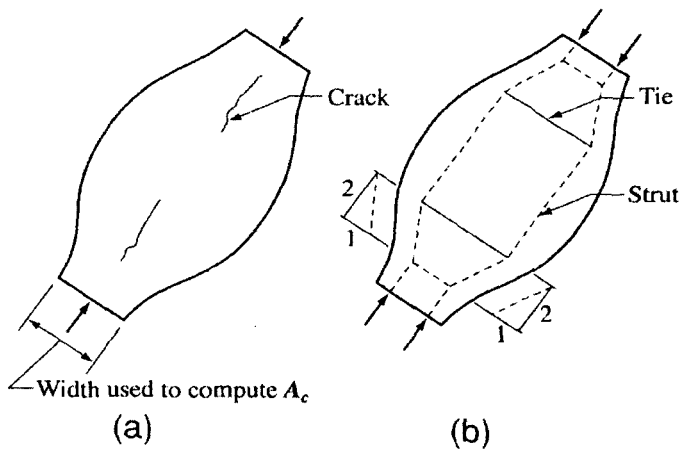


Fig. 3: Splitting of a bottle-shaped strut.

5.3.2 Effective compressive strength of struts—from tests and literature

Various researchers have proposed values of the effectiveness factor v . A few comparisons are made here. Many more references are listed in a Bibliography on Strut-and-Tie Models, collected by the ACI-ASCE committee on Shear and Torsion [ACI 445(1997)].

- Based on tests of 24, one- and two-span deep beams, Rogowsky and MacGregor (1986) proposed $f_{cu} = v f_c' = 0.85 f_c'$. They observed that the selection of an appropriate truss was more important than the choice of v .
- Ricketts (1985) reported ratios of test to calculated strengths of six two-span continuous deep beams. The failure loads predicted using strut-and-tie models, had a mean ratio of test / calculated strengths of 0.96 for $v = 1.0$. When v was taken as 0.6, the mean test to calculated strength ratio increased to 1.13. This indicates that v was closer to 1.0 than to 0.6.
- Rogowsky (1983) stated that $v = 1.0$ conservatively predicted the strengths of the corbels tested by Kriz and Raths (1965). Virtually all of the corbel tests had effectiveness factors, v , between 1.0 and 1.3.
- Ramirez and Breen (1991) proposed a relationship between v and $\sqrt{f_c'}$ with v ranging from 0.55 to 0.39 for f_c' ranging from 3000 to 6000 psi.
- Bergmeister et al. (1991) related v and f_c' with v ranging from 0.77 to 0.69 for concrete strengths ranging from 3000 psi to 6000 psi.
- Schlaich et al. (1987) recommended values of v similar to those given in Section 5.3.5 of this paper.
- Marti (1985) suggested a constant value of $v = 0.6$.

5.3.3 Effective concrete strength of struts—from other codes

The values of β_s from other codes cannot be used directly in Appendix A because other codes have different load factors, different resistance factors, and different ways of specifying the concrete strength. A further complication is introduced by the fact that the load and resistance factors in Chapter 9 of the 2002 ACI Code differ from those in Chapter 9 of the 1999 ACI Code.

Initial values of $\phi_{STM} \alpha_1 \beta_s$ corresponding to the 1999 ACI Code load factors and the ACI definition of f_c' were derived by calibration to the FIP Recommendations (1999). The cross-sectional areas of struts required by the FIP Recommendations, for example, were computed for the axial forces in a hypothetical strut due to assumed combinations of dead and live load, based on the load factors in the FIP Recommendations and for the corresponding concrete strengths, f_c' .

The values of $\phi_{STM} \alpha_1 \beta_s$ needed in the ACI Code so that the strut area from the two codes give the same load capacities were then computed using ..

$$\phi_{STM} = 0.85 \text{ and } \alpha_1 = 0.85.$$

FIP Recommendations. The FIP Recommendations (1999) give the cross-sectional area of a strut required for a given set of loads as:

$$A_{c,FIP} = \frac{F_{uFIP}}{f_{cdeff}} \quad (5)$$

where $A_{c,FIP}$ is the cross-sectional area of the strut computed using the FIP Recommendations, $F_{u,FIP}$ is the strut force due to the sum of the factored loads acting on the strut., and $f_{cd,eff}$ is the effective concrete strength from FIP. For unfactored strut loads of 100 kips dead load and 50 or 200 kips live load, values of $A_{c,FIP}$ were computed using Eq. 5 for specified concrete strengths, f'_c of 3000 psi and 6000 psi. These were set equal to the corresponding areas, $A_{c,ACI}$, from Appendix A:

$$A_{c,ACI} = \frac{F_{uACI}}{\phi_{STM} \alpha_1 \beta_s f'_c} \quad (6)$$

where $A_{c,ACI}$ is the area of the strut or the compression stress block for a member designed using the ACI Code, and $F_{u,ACI}$ is the force in the strut due to ACI factored loads on the strut-and-tie model.

Setting $A_{c,FIP}$ equal to $A_{c,ACI}$, assuming that the unfactored loads are defined in the same way in both the ACI and FIP, and rearranging, gives:

$$\phi_{STM} \beta_s = \frac{F_{uACI} f_{cdeff}}{\alpha_1 f'_c F_{uFIP}} \quad (7)$$

where α_1 is the factor from ACI Sec. 10.2.7.1.

Load Factors and Loadings. The basic load case considered is dead load plus live load ($D + L$), where $D = 100$ kips, with L either 50 kips (0.5D) or 200 kips (2.0D). It is reasonable to assume that D and L are defined in a similar fashion in both American and European codes.

In FIP, the basic load case is $U = 1.35D + 1.5 L$, where U stands for ultimate load. For $L = 0.5D$, $U = 2.10D$ (210 kips) and for $L = 2D$, $U = 4.35D$ (435 kips).

In Sec. 9.2.1 of the 1999 ACI Code, $U = 1.4D + 1.7L$. For $L = 0.5D$, $U = 2.25D$ (225 kips) and for $L = 2D$, $U = 4.8D$ (480 kips).

In Sec. 9.2.1 of the 2002 ACI Code, the basic load combination changes to $U = 1.2D + 1.6L$, giving $U = 200$ kips and 440 kips for the two loading cases.

Concrete Strengths. Two concrete strengths were considered, 3000 and 6000 psi.

3000 psi Concrete. Consider 3000 psi (20.7 MPa) concrete with a standard deviation of 450 psi (3.1 MPa), and a coefficient of variation = 0.15. From ACI Sec. 5.3.2.1, the required mean strength, $f_{cr}' = 3000 + 1.34 \times 450 = 3600$ psi. FIP defines concrete strength using the characteristic or 5th %ile strength, f_{ck} , which is a lower fractile than used in ACI to define f_c' . For concrete with an ACI specified strength of 3000 psi,

$$f_{ck} = 3600 (1 - 1.645 \times 0.15) = 2710 \text{ psi (18.7 MPa).}$$

The design strength for uniaxial compression from FIP is:

$$f_{lcd} = \alpha \times \frac{f_{ck}}{\gamma_c} \quad (8)$$

where $\alpha = 0.85$ is similar to the $\alpha_1 = 0.85$ in ACI 10.2.7.1 and Eq. 4, and γ_c is a resistance factor for concrete, equal to 1.5. For $f_c' = 3000$ psi:

$$f_{lcd} = \frac{0.85 \times 2710}{1.5} = 1540 \text{ psi}$$

6000 psi Concrete. Consider 6000 psi (41.4 MPa) concrete with a standard deviation of 600 psi (4.14 MPa), and a coefficient of variation = 0.10. From ACI Sec. 5.3.2.1, the required mean strength, $f_{cr}' = (6000 + 2.33 \times 600) - 500 = 6900$ psi. The FIP uses the 5th % fractile strength, $f_{ck} = f_{cr}'(1 - 1.645 \times 0.10) = 5760$ psi (39.8 MPa). From Eq. 8 the design strength for uniaxial compression from FIP is $f_{lcd} = 3260$ psi.

In the FIP Recommendations, the strength of the concrete in a strut is taken as the smaller of

$$f_{cd,eff} = v_1 f_{lcd} \quad (9)$$

or

$$f_{cd,eff} = v_2 f_{lcd} \quad (10)$$

where Eq. 9 is applicable only in the compression zones for beams or axially loaded columns that are assumed to be uniformly stressed with $f_{cd,eff} = v_1 f_{lcd}$ acting over the distance c from the neutral axis to the extreme compression fiber. Because this locates the resultant compression force at $c/2$ from the extreme compression fiber, rather than at $a/2$ as assumed in the ACI Code rectangular stress block, values of $f_{cd,eff}$ based on v_1 will not be considered.

FIP defines v_2 in Eq. 10 using the following descriptions:

5.3.2(3) a) Uncracked struts with uniform strain distribution. $v_2 = 1.00$.

For 3000 psi concrete, $f_{cd,eff}$ is 1.00×1540 psi = 1540 psi and, for $\phi_{STM} = 0.85$, β_s ranged from 0.76 to 0.83 for the 1999 load factors and $\phi_{STM} = 0.85$. For the 2002 load and resistance factors β_s ranged from 0.77 to 0.86 .

5.3.2(3) b) Struts with cracks parallel to the strut and bonded transverse reinforcement; The reduction in strut strength is due to the transverse tension and to the disturbances by the reinforcement and the irregular crack surfaces. FIP gives $v_2 = 0.80$. For the 1999 ACI load factors $\alpha_1 v \beta_s \phi = 0.75$, and the values of β_s ranged from 0.61 to 0.63. For the 2002 load and ϕ factors, the range was 0.64 to 0.66.

5.3.2(3) c) Struts transferring compression across cracks with normal crack widths, e.g. in webs of beams. FIP gives $v_2 = 0.60$. For 1999 Code load and ϕ factors, β_s ranged from 0.46 to 0.50. For the 2002 load and resistance factors, the range was 0.46 to 0.52.

5.3.2(3) d) Struts transferring compression across large cracks, as e.g. in members with axial tension or in tension flanges. FIP gives $v_2 = 0.45$. For 3000 psi concrete, the corresponding values of β_s ranged from 0.34 to 0.37 for the 1999 Code. For the 2002 Code the range is 0.35 to 0.39.

These values are listed in Table 1 for comparison with values of β_s derived from other codes, and those proposed for Appendix A. Because the verbal descriptions used in ACI Appendix A differ from those given in the FIP Recommendations, some FIP cases and ACI cases overlap and are listed two or more times.

Canadian Concrete Design Code and AASHTO LRFD Specification. These two codes define f_{cu} based on transverse tensile strains in the struts. One definition of f_{cu} is assumed to apply for all types of struts.

In the Canadian code [CSA(1994)] design is carried out using a factored concrete strength $\phi_c f'_c$ and a factored steel strength $\phi_s f_y$ where $\phi_c = 0.60$ and $\phi_s = 0.85$. The strength of concrete is defined in the same way as in ACI 318. The load factors in the Canadian Code are $U = 1.25D + 1.5L$. The effective compressive strength of the concrete in struts is:

$$f_{cu} = \frac{f'_c}{0.80 + 170\varepsilon_1} \leq 0.85f'_c \quad (11)$$

where

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \theta_s \quad (12)$$

ε_s = tensile strain in the tie.

θ_s = the smallest angle between the axis of the compressive strut and the axis of the tie attached to one end of the strut.

In the strut-and-tie model in Fig. 1, the strut is anchored by the longitudinal tie and is crossed by stirrups or minimum reinforcement. Here, θ_s is taken as the angle between the axis of the strut and the tie which is limited in ACI Sec. A.2.5 to 25° and by implication to an upper limit of $90 - 25 = 65^\circ$ relative to the tie. The stirrups and minimum surface reinforcement are ignored when computing θ_s and ε_1 .

TABLE 1—Values of β_s for Struts in Strut-and-Tie Models. $\phi f_{cu} = \phi_{STM} \alpha_1 \beta_s f'_c$

Case	Code	β_s for $\phi = 0.85$, $\alpha_1 = 0.85$, 1999 load factors		β_s for $\phi = 0.75$, $\alpha_1 = 0.85$, 2002 loads factors	
		Values	Chosen	Values	Chosen
A.3.2.1—Struts in which the area of the mid-strut cross section is the same as that at the nodal zones, such as the compression zone of a beam. • Uncracked strut with a uniform strain distribution, $v_2 = 1.00$ • Compression zone in a beam • Compression zone in a tied column	ACI App. A		1.0		1.0
	FIP 5.3.2(3)a)	0.76-0.83		0.77-0.86	
	ACI 10.2.7	1.06		1.20	
	ACI Chapter 10	0.82		0.87	
A.3.2.2—Struts located such that the width of the mid-section of the strut is larger than the width at the nodal zones (Bottle-shaped struts) (a) with reinforcement satisfying A.3.3 • Struts with cracks and bonded transverse reinforcement, $v_2 = 0.80$ • Struts transferring compression across cracks of normal width, $v_2 = 0.6$ • Struts in STM's of post-tensioned anchorage zones • Struts crossed by reinforcement at an angle θ with the axis of the strut - CSA $\theta = 60^\circ$ 0.683 $\theta = 45^\circ$ 0.513 $\theta = 30^\circ$ 0.293 • Back calculated from tests of one and two span deep beams	ACI App. A		0.75		0.75
	FIP 5.3.2(3)b)	0.61-0.66		0.61-0.69	
	FIP 5.3.2(3)c)	0.46-0.50		0.46-0.52	
	ACI 18.13	0.82λ			
	CSA $\theta = 60^\circ$	0.683			
	$\theta = 45^\circ$	0.513			
	$\theta = 30^\circ$	0.293			
	Ref. 21	0.95			

TABLE 1—(Continued)

Case	Code	β_s for $\phi = 0.85$, $\alpha_1 = 0.85$, 1999 load factors		β_s for $\phi = 0.75$, $\alpha_1 = 0.85$, 2002 loads factors	
		Values	Chosen	Values	Chosen
(b) without reinforcement satisfying A.3.3 • Struts transferring compression across cracks of normal width, $v_2 = 0.60$	ACI App. A		0.60λ		0.60λ
	FIP 5.3.2(3)c)	0.46-0.50		0.46-0.52	
A.3.2.3—Struts in tension members, or the tension flanges of members • Struts transferring compression across large cracks	ACI App. A		0.40		0.40
	FIP 5.3.2(3)d)	0.34-0-0.37		0.35-0.39	
A.3.2.4—For all other cases	ACI App. A		0.60		0.60

For $\theta_s = 60^\circ$ and $\epsilon_s = \epsilon_y = 0.002$, Eq. 11 from the Canadian Code [CSA(1994)] gives $f_{cu} = 0.73f_c'$.

For $\theta_s = 45^\circ$ and $\epsilon_s = 0.002$, Eq. 11 gives $f_{cu} = 0.55f_c'$.

For $\theta_s = 30^\circ$, and $\epsilon_s = 0.002$, $f_{cu} = 0.31f_c'$.

Assuming $\phi f_{cu} = \phi_{STM} \alpha_1 \beta_s f_c'$, the values of β_s for these angles θ are 0.68, 0.51, and 0.29 respectively, based on $\alpha_1 = 0.85$ and $\phi_{STM} = 0.85$.

For strut-and-tie models, the AASHTO LRFD Specification [AASHTO(2002)] gives the following values of ϕ :

- for compression in strut-and-tie models . . . 0.70
- for compression in anchorage zones:
 - normal weight concrete 0.80
 - lightweight concrete 0.65

For strut-and-tie models of prestress anchorage zones AASHTO gives

$$f_{cu} = 0.7 \phi f_c'$$

except that, in areas where the concrete may be extensively cracked at ultimate due to other force effects, or if large inelastic rotations are expected, the factored effective compressive strength is limited to $0.6 \phi f_c'$.

Setting ϕf_{cu} equal to $\phi \alpha_1 \beta_s f_c'$ and neglecting differences in the load factors gives $\beta_s = 0.82$ and 0.71 for $\phi f_{cu} = 0.7 \phi f_c'$ and $0.6 \phi f_c'$, respectively, with $\alpha_1 = 0.85$.

5.3.4 Effective Concrete Strength of Struts - From Other Sections of ACI 318-99

The effective concrete strength of a strut is given as the product $\phi f_{cu} = \phi_{STM} \alpha_1 \beta_s f_c'$. It would be desirable that $\phi v f_c'$ agree with $\phi_{STM} \alpha_1 \beta_s f_c'$ for the following three cases to minimize the differences at the interface between B-regions designed using the traditional flexure and shear design theory, and D-regions designed using strut-and-tie models.

The Flexural Stress Block in Beams. For flexure by the 1999 ACI Code, $\phi_f = 0.90$, and $\alpha_1 = 0.85$, the flexural compressive force, C_u , acting on the height, $a = \beta_{1c}$, of the rectangular stress block is:

$$C_{u,flex} = \phi_f \alpha_1 f_c' a b \quad (13)$$

If we assume the compressive force in a strut-and-tie model of the same beam is also equal to C_u :

$$C_{u,STM} = \phi_{STM} \alpha_1 \beta_s f_c' a b \quad (14)$$

Setting these equal and substituting $\phi_{STM} = 0.85$ we get $\beta_s = \phi_f / \phi_{STM} = 1.06$.

For the same case using $\phi_{STM} = 0.75$ and $\phi_f = 0.90$ from the 2002 ACI Code, $\beta_s = 0.90/0.75 = 1.20$.

The Flexural Stress Block in Columns Failing in Compression. The ϕ factor for tied columns is $\phi_{tc} = 0.70$ and for spiral columns $\phi_{sc} = 0.75$. For the 1999 ACI Code, $\phi_{tc} v f_c' = \phi_{STM} \alpha_1 \beta_s f_c'$. The corresponding value of β_s is $0.70/0.85 = 0.82$ for tied columns and 0.88 for spiral columns. This value for the compression zone in a tied column is lower than the value of β_s for the compression zone in a beam because the ACI ϕ factors for columns were arbitrarily set below the ϕ for flexure to account for the more serious and more brittle nature of column failures.

Prestressed Tendon Anchorage Zones--ACI 318-99 Section 18.13.4.2 In 1999, ACI 318 included a new Section 18.13 on tendon anchorage zones based in large part on strut-and-tie models. This revision used $f_{cu} = 0.7 \lambda f_c'$ and $\phi = 0.85$. Rewriting the expression for f_{cu} as $\phi f_{cu} = \phi_{STM} \alpha_1 \beta_s f_c'$ gives $\beta_s = 0.7 \lambda / \alpha_1 = 0.824 \lambda$.

5.3.5 Selection of f_{cu} for Struts for Appendix A

The values of f_{cu} presented in ACI Code Appendix A were chosen to satisfy four criteria:

- (a) Simplicity in application.
- (b) Compatibility with tests of D-regions, such as deep beams, dapped ends or corbels.
- (c) Compatibility with other sections of ACI 318.
- (d) Compatibility with other codes or design recommendations.

Because these four criteria lead to different values of f_{cu} for a given application, judgement was required in selecting the values of f_{cu} .

Values of β_s are summarized in Table 1. The bold headings numbered A.3.2.1 through A.3.2.4 are the descriptions of the types of struts used in Appendix A of ACI 318-02. Values of β_s for related cases from other documents are also listed in each section. Two different methods of specifying f_{cu} are given in the various codes cited: (a) FIP uses descriptions of the cracking of the struts to select the applicable values of v . (b) CSA and AASHTO base f_{cu} on Eqs. 11 and 12 which require ϵ_s to be computed. The first option depends on finding unambiguous descriptions of the state of cracking in the member. The second, depends on being able to compute a poorly defined strain in the web of the member. In Appendix A, the first option, verbal descriptions was adopted. The italicized words in the following paragraphs are directly from ACI 318-02.

A.3.2.1 For a strut of uniform cross-sectional area over its length, . . . $\beta_s = 1.0$.

Table 1 lists values of β_s ranging from 0.76 to 1.20 for related cases. β_s was taken equal to 1.0, or $\beta_s = 0.85$ has been proposed to correspond to the rectangular stress block for flexure. In making this choice, evidence of β_s approaching 1.0 in tests was also strongly weighted.

A.3.2.2 For struts located such that the width of the mid-section of the strut is larger than the width at the nodes (bottle-shaped struts):

(a) with reinforcement satisfying A.3.3 $\beta_s = 0.75$.

Other codes give β_s ranging from 0.46 to 0.824. The CSA code values are based on a different concept and will be disregarded. Experiments gave β_s as high as 0.94.

(b) *without reinforcement satisfying A.3.3.* $\beta_s = 0.60 \lambda$

The term λ for cracking of lightweight concrete was included in the β_s value for A.3.2.2(b) because the stabilizing effect of reinforcement transverse to the struts is not present and failure is assumed to occur shortly after cracking.

A.3.2.3 *For struts in tension members or tension flanges of members.* $\beta_s = 0.40$.

The similar case from FIP corresponds to $\beta_s = 0.34$ to 0.37 .

A.3.2.4 *For all other cases.* $\beta_s = 0.60$

The selected values of β_s are generally higher than those from other codes because more weight was given to values of f_{cu} corresponding to related design cases in the ACI Code and values of f_{cu} from tests, than was given to f_{cu} from other codes.

6 Nodes and nodal zones

6.1 Classification of nodes and nodal zones

It is desirable to distinguish between nodes and nodal zones. *Nodes* are the points where the axial forces in the struts and ties intersect, and *nodal zones* are the regions around the joint areas in which the members are connected. For vertical and horizontal equilibrium at a node, there must be a minimum of three forces acting on the node in a planar structure like a deep beam.

Nodes are classified by the types of forces that meet at the node. Thus a C-C-C node anchors three struts, a C-C-T node anchors two struts and one tie, a C-T-T node anchors one strut and two ties. Appendix A assumes the faces of the nodal zone that are loaded in compression have the same width as the ends of the struts. The width of the faces anchoring ties will be discussed more fully in Sec. 7.1.

6.2 Types of nodal zones and their use in strut-and-tie models

The literature on nodes in strut-and-tie models is based on two quite different concepts.

Hydrostatic Nodal Zones. Originally, nodal zones were assumed to have equal stress on all in-plane sides. Because the Mohr's circle for the in-plane stresses acting on such nodal zones plots as a point, this class of nodes was referred to as *hydrostatic nodal zones*. If the stresses were equal on all sides of the nodal zone, the ratio of the lengths of the sides of a hydrostatic nodal zone, $w_{n1} : w_{n2} : w_{n3}$ are in the same proportions as the forces, $C_1 : C_2 : C_3$ acting on the sides.

Hydrostatic nodal zones were extended to C-C-T or C-T-T nodes by assuming the ties extended through the nodal zones to be anchored on the far side by hooks or bond on the tie reinforcement beyond the nodal zone. This concept is represented using a *hypothetical* anchor plate behind the joint. The area of the hypothetical anchor plate is chosen so that the bearing pressure on the plate was equal to the stresses acting on the other sides of the nodal zone. The effective area of the tie is the tie force divided by the permissible bearing stress for the struts meeting at a node. The requirement for equal stresses on all faces of a hydrostatic nodal zone tends to complicate the use of such nodal zones.

Extended Nodal Zones. These are nodal zones bounded by the outlines of the compressed zones at the intersection of:

- (a) the struts,
- (b) the reactions, and
- (c) the assumed widths of ties including a prism of concrete concentric with the ties.

This is illustrated in Fig. 4(a) where the darker shaded area is the hydrostatic nodal zone and the total shaded zone is the extended nodal zone. The extended nodal zone falls within the area stressed in compression due to the reactions and struts. The compression stresses assist in the transfer of forces from strut to strut, or strut to tie. In general, Appendix A uses extended nodal zones in place of hydrostatic nodal zones.

Relationships between the dimensions of a nodal zone. Equations can be derived relating the widths of the struts, ties, and bearing areas if it is assumed that the stresses are equal in all three members meeting at a C-C-T nodal zone.

$$w_s = w_t \cos \theta + \ell_b \sin \theta \quad (15)$$

where w_s is the width of the strut, w_t is the effective width of the tie, ℓ_b is the length of the bearing plate, and θ is the angle between the axis of the strut and the horizontal axis of the member. This relationship is useful for adjusting the size of nodal zones in a strut-and-tie model. The strut width can be adjusted by changing w_t or ℓ_b , one at a time. Once this has been done, it is necessary to check the stresses on all the faces of the nodal zone. The accuracy of Eq. 15 decreases as the stresses on the sides become more and more unequal. Equation 15 was included in Fig. 4 (ACI Fig. RA.1.6), but not in the code itself. Future code committees should consider adding such equations to the Commentary.

Resolution of forces acting on nodal zones. If more than three forces act on a nodal zone in a two-dimensional structure, it is frequently necessary to resolve some of the forces to end up with three intersecting forces. Alternatively, the nodes in the strut-and-tie model acted on by more than three forces could be analyzed assuming that all the strut and tie forces act through the node, with the forces on one side of the nodal zone being resolved into a single resultant strut during the design of the nodal zone. This concept is illustrated in ACI Commentary Fig. RA.2.3.

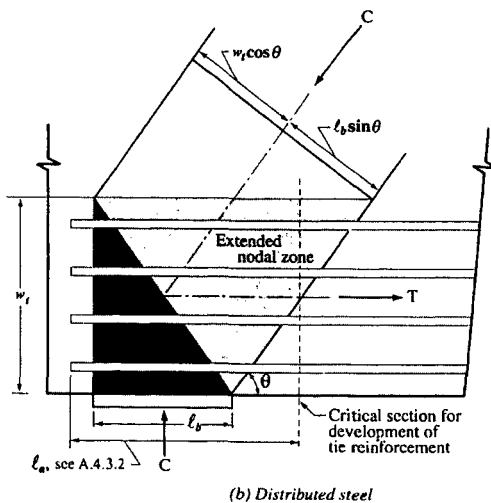
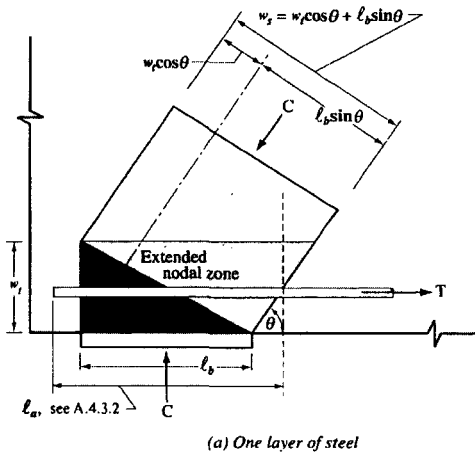


Fig. 4: An extended nodal zone.

6.3 Effective compressive strength of nodal zones

6.3.1 Effective compressive strength of nodal zones from tests

Very few tests of the strength of nodal zones are available. Tests of ten isolated C-C-T and nine C-T-T nodal zones reported by Jirsa et al. (1991) indicate that $f_{cu} = 0.80f'_c$ can be developed in such nodal zones if properly detailed. For C-C-T nodal zones the average test / calculated strengths were 1.17 with a standard deviation of 0.14. For C-T-T nodal zones the mean test / calculated strength was 1.02. Assuming $\phi = 1.0$ for comparison with the strength of test specimens and $\alpha_1 = 0.85$, this corresponds to $\beta_n = 0.94$.

6.3.2 Effective compressive strength of nodal zones from other codes

6.3.2.1 FIP Recommendations "Practical design of structural concrete"

The FIP Recommendations limit f_{cu} on nodal zones to the following values:

- for node regions anchoring one or more ties: $0.85 f_{lcd}$ where f_{lcd} is given by Eq. 8.
For $\alpha_1 = 0.85$ and $\phi_{STM} = 0.85$ this corresponds to $\beta_n = 0.65$ to 0.71 .
- for biaxially- or triaxially-loaded C-C-C nodal zones anchoring only struts:
 - biaxial stress state: up to $1.20 f_{lcd}$ ($\beta_n = 0.91$)
 - triaxial stress state: up to $3.88 f_{lcd}$ ($\beta_n = 2.9$)

6.3.2.2 Canadian Code and AASHTO LRFD Specification

The Canadian Code (1994) defines the effective compressive strength of nodal zones, ϕf_{cu} , as:

- for nodal zones loaded by compressive struts and bearing areas: $0.85 \phi_c f'_c$
- for nodal zones anchoring only one tie: $0.75 \phi_c f'_c$
- for nodal zones anchoring more than one tie: $0.65 \phi_c f'_c$,

where $\phi_c = 0.60$ in the Canadian code.

The β_n values are intended to reflect the weakening effect of the discontinuity of strains introduced when ties stressed in tension are anchored in a node stressed in compression. The values from the Canadian code are equivalent to β_n equal to 0.80, 0.70 and 0.61 in ACI.

The AASHTO LRFD Specification uses similar values of ϕf_{cu} .

6.3.3 Selection of the Effective Compression Strength of Nodal Zones, f_{cu} .

The values of f_{cu} for nodal zones from other codes are summarized in Table 2.

For the 1999 Code, the strength reduction factor for nodal zones was taken as $\phi_{STM} = 0.85$. The effective compressive strength, f_{cu} , will be based on the AASHTO and CSA values, modified to agree with the ACI load factors, the α_1 factor, and $\phi_{STM} = 0.85$ using Eq. 4.

The following values were selected for inclusion in the 2002 ACI Code:

- for C-C-C nodal zones bounded by compressive struts and bearing areas $\beta_n = 1.0$
- for C-C-T nodal zones anchoring only one tie $\beta_n = 0.80$
- for C-T-T or T-T-T nodal zones anchoring ties in more than one direction $\beta_n = 0.60$.

Table 2 shows these are a reasonable fit to the β_n values for each type of nodal zone.

7 Ties

7.1 Ties in strut-and-tie models

In strut-and-tie models based on hydrostatic nodal zones, the tie reinforcement is spread over the height of the tie computed as

$$w_t = \frac{F_u / \phi}{f_{cu} b_w} \quad (16)$$

The tie is assumed to consist of the reinforcement and a hypothetical prism of concrete concentric with the axis of the tensile force. Thus, ACI Sec. A.4.2 requires the tie reinforcement to be distributed approximately uniformly over the width of the tie, w_t . This may entail putting the reinforcement in several layers as shown in Fig. 4(b), rather than concentrating it near the tension face of the beam as shown in Fig. 4(a).

If extended nodal zones are being used, the lower extreme value of the height of the tie corresponds to the steel being placed in one layer with w_t taken equal to the diameter of the bars plus twice the cover to the bars as in Fig. 4(a). (See also Section RA.4.2 of the Commentary to Appendix A.)

7.2 Strength of ties

A non-prestressed tie is assumed to reach its capacity when the force in the tie reaches

$$T_n = A_s f_y \quad (17)$$

A second term is added for prestressed ties, ACI Eq.A-6 assumes Δf_p equals 60 ksi. This is a reasonable approximation to the change in stress in the prestressed reinforcement as a member is loaded to failure. Generally the major problem in the design of ties is the anchorage of a tie in a nodal zone. The hypothetical concrete prism concentric with the tie does not resist any of the tie force. In serviceability checks the reduced strain in the tie due to this concrete may reduce the elongation of the tie, leading to less deflection of the member.

Table 2: Recommended values of β_n for nodal zones in strut-and-tie models,
 $\phi = 0.85$ and 1999 load factors and $\phi = 0.75$ and 2002 load factors
 $\phi f_{cu} = \phi \alpha_1 \beta_n f_c'$

Case	Code	Range of β_n	β_n^1 and 1999 Load Factors	Range of β_n	β_n^2 and 2002 Load Factors
C-C-C Nodal zones • Nodal zones anchoring only compression struts.	CSA A23.3 FIP	0.79 - 0.80 0.91 - 1.00	1.0	0.76-0.83 0.92-1.03	1.0
C-C-T Nodal zones • Nodal zones anchoring one or more ties.	CSA A23.3 FIP	0.70-0.71 0.65 - 0.71	0.80	0.67-0.73 0.65-0.73	0.80
C-T-T Nodal zones • Nodal zones anchoring one or more ties. Tests of Nodal zones $\phi = 1.0$ for tests.	CSA A23.3 FIP Ref. 15	0.61 0.65 - 0.71 0.94	0.60	0.58-0.63 0.65-0.73	0.60

Notes: ¹ The first set of recommended values of β_n are for use with the 1999 load factors assume $\alpha_1 = 0.85$ and $\phi_{STM} = 0.85$.

² The second set of recommended values of β_n are for use with the 2002 load factors assume $\alpha_1 = 0.85$ and $\phi_{STM} = 0.75$.

7.3 Anchorage of ties

ACI Secs. A.4.3.1, A.4.3.2 and A.4.3.3 require that the anchorage of the tie forces be achieved completely by the time the geometric centroid of the bars in a tie leaves the extended nodal zone. This point is shown in Fig. 4. Hooks or mechanical anchorages may be required. ACI Section A.4.3 itemizes other requirements for anchoring ties. At nodal zones in beam-like structures where diagonal struts are anchored by stirrups, the change in the tie forces at the truss node must occur within the width of the nodal zone.

8 Reinforcing Requirements

8.1 Effect of minimum confinement reinforcement from tests

Grids of reinforcement are desirable in the side faces of D-regions to restrain the splitting cracks near the ends of bottle-shaped struts, and to give some ductility to the struts.

Three, two-span continuous deep beams tested by Rogowsky (1983) and Ricketts (1985), with clear shear span to depth ratios of 0.8 with vertical stirrups giving a stirrup reinforcement ratio, ρ_v , of 0.0015 and no horizontal web reinforcement, failed after the positive moment (bottom) reinforcement yielded but before the top steel did, and as a result, these two beams did not develop their full plastic load capacity. This ρ_v did not allow adequate moment redistribution.

Three similar beams with vertical stirrup ratios in the order of 0.0035, again without horizontal web reinforcement, had varying distributions of flexural reinforcement. In these beams the relative amounts of top and bottom steel from an elastic analysis of a slender beam were 4 top bars and 3 bottom bars. The three beams varied the amounts of top and bottom bars from 2 top bars and 5 bottom bars, 4 top and 3 bottom bars in the second beam, and 5 top and 2 bottom bars in the third. All three beams failed after both top and bottom steel yielded. This required some moment redistribution. The trends from these tests and others with varying web reinforcement ratios suggested that in beams with vertical stirrups only, the full plastic capacity would be reached with a stirrup reinforcement ratio of 0.0025.

8.2 Minimum reinforcement required in bottle-shaped struts.

ACI Secs. A.3.2.2(a) and (b) allow the use of $\beta_s = 0.75$ when computing the effective compression strength of bottle-shaped struts with reinforcement satisfying ACI Sec. A.3.3. The value of β_s drops to 0.60 λ if this reinforcement is not provided. ACI Sec. A.3.3 requires reinforcement transverse to the axis of the strut proportioned to resist the tensile force lost when the strut cracks due to the lateral spread of the strut forces. ACI Sec. A.3.3 allows the designer to compute the necessary steel either from an assumed localized strut-and-tie model of the strut as shown in Fig. 3(b), or, for beams with a concrete strength of 6000 psi or less,

ACI Sec. A.3.3.1 allows the results of the strut-and-tie model to be approximated using the empirical equation, Eq. A-4. This equation was derived assuming the normal stress, σ_1 , acting on the crack resulting from a layer of confinement steel is

$$\sigma_1 = \frac{A_{s1} f_{s1}}{b s_1} \sin \gamma_1 \quad (18)$$

where A_{s1} is the area of the bars in one direction and the angle γ_1 is the angle between the crack and the component of the force in the bar in question. The direction of the bar is chosen so that a tensile bar force in the bar causes a compressive force in the concrete perpendicular to the crack. It has been written without the term f_{s1} to simplify the presentation.

A.3.3.1 allows this requirement to be satisfied using layers of steel crossing the strut which satisfy

$$\sum \frac{A_{si}}{b s_i} \sin \gamma_i \geq 0.003 \quad (19)$$

where A_{si} is the total area of reinforcement at a spacing s_i in a layer of bars at an angle γ_i to the axis of the strut. ACI Sec. A.3.3.1 states that this steel generally is placed in grids in two orthogonal directions in each face, but allows it to be placed in one layer in cases such as corbels.

8.3 Minimum shear reinforcement in deep beams

There is a major discontinuity in the amount of shear reinforcement required at the limit between the deep beams and shallow beams in the 1999 and earlier codes. The FIP Recommendations (1998) provide a transition from a D-region to a B-region for a/d ratios close to the limit of 2. For shear spans with a/z between 0.5 and 2, where $z = jd$ is the internal lever arm between the resultant tensile and compressive forces in flexure, FIP requires that a portion, V_2 , of the total shear, V , on the shear span be transferred to the supports by an inclined strut, and the rest, $V_1 = V - V_2$, be transferred by vertical web reinforcement.

For a/z between 0.5 and 2.0, the amount V_1 is given by:

$$V_1 = V \frac{\left(\frac{2a}{z} - 1 \right)}{3} \quad (20)$$

At the limits, $a/z = 0.5$ and 2.0, this equation gives $V_1 = 0$, and $V_1 = V$. Thus, for $a/z = 0.5$, FIP recommends that all of the shear V be transferred by the inclined strut, and for $a/z = 2$, FIP recommends that all the shear should be resisted by stirrups. This, or a similar provision, should be considered by ACI Committee 318 Sub E as a possible addition to Appendix A. Such a provision prevents strut-and-tie models with struts which are at too small an angle with the longitudinal axis of the member. ACI Sec. A.2.5 limits the angle to 25° for the same reason.

8.4 Minimum web reinforcement from codes

- The 1994 CSA Code requires an orthogonal grid of reinforcing bars near each face with a ratio of total steel area to gross concrete area in each direction of at least 0.002. This is referred to as crack control reinforcement. The portion of this reinforcement parallel to a tie and located within the assumed cross-sectional area of the tie can be considered as part of the tie reinforcement.
- AASHTO (2002) has similar requirements to CSA but the minimum total reinforcement ratio is set at 0.003 in each direction. The Commentary to the AASHTO LRFD Specification states that, in thinner members, there should be a grid of bars at each face, while for thicker members it may be necessary to have multiple grids throughout the thickness.
- The FIP Recommendations (1999) suggest that deep beams should have a minimum reinforcement ratio of 0.001 in each direction, in each face, for a total of not less than 0.002 each way.
- Sections 11.8.4 and 11.8.5 of ACI 318-99 require minimum ratios of vertical and horizontal web reinforcement of $0.0015b_{ws}$ and $0.0025b_{ws2}$, respectively in deep beams.

These minimum amounts provide a considerable shear capacity. Each $0.0010b_{ws}$ of vertical web reinforcement corresponds to a shear stress resistance, V_s corresponding to $v = 60$ psi, calculated using ACI Eq. 11–15. Horizontal web reinforcement is much less efficient in transferring shear. The additional capacity provided by the vertical web reinforcement is not implicitly included in the strength calculation using ACI Eqs. A–1 and A–2. It is accounted for by the increase in β_s .

For some D-regions, such as those in a deep beam, it is feasible to provide orthogonal grids of steel near the side faces of the deep beam. In other cases, such as corbels or dapped ends, it is easier to place the web steel in one direction, horizontal in the case of a corbel. ACI Sec. A.3.3.2 allows uni-directional confining steel in corbels or similar cases. If in one layer, the steel is placed in one direction at an angle of at least 40° with the axis of the strut.

In some structures, such as pile caps for more than two piles, it is frequently not feasible to place web steel in the three-dimensional strut-and-tie model. A reduced strut strength is imposed by ACI Sec. A.3.2.2(b) in such cases.

8.5 Selection of minimum web reinforcement.

The two structural functions of minimum web steel in a D-region are to resist the transverse tension in the bottle-shaped areas near the ends of the strut after splitting cracks occur, and to provide ductility to the struts and nodal zones by confinement. The minimum steel was given in Eq. A–4 in terms of equivalent amounts perpendicular to the axis of the strut.

In the Rogowsky (1983) and Ricketts (1985) tests quoted earlier, the full plastic capacity was obtained with a ratio of vertical steel equal to 0.0035.

These beams did not have horizontal web steel. The strut-and-tie model of the failure region indicated that the critical strut had a slope of about 55° with the vertical steel.

The critical web steel ratio taken perpendicular to the strut is $\rho_v \sin \gamma$, where γ is the angle between the reinforcement and the axis of the strut. In the Rogowsky and Ricketts tests quoted above, γ was 55°, and the steel area provided was equivalent to

$$\sum \frac{A_{s_i}}{bs_i} \sin \gamma_i = 0.0029$$

In design this crack restraint is achieved by providing minimum web steel satisfying:

$$\sum (\rho_{vi} \sin \gamma_i) \geq 0.003 \quad (21)$$

where ρ_{vi} is the reinforcement ratio for web reinforcement at an angle γ_i to the axis of the strut.

8.6 Comparison of strut-and-tie models to tests of deep beams.

A range of test results were considered in developing the ACI Code provisions for strut-and-tie models as reported in Section 5.3.2 of this paper. In addition, the final code statements were compared to tests of each end of six simple-span, deep beams tested by Rogowsky and MacGregor (1986). These beams were relatively large, having overall depths of 500 to 1000 mm, and spans of 2000 mm, and a/d ratios of 1.03, 1.86 and 2.21.

Each beam was assumed to have five failure limit states, ranging from yield of the ties, to crushing of each end of each strut, and crushing of the nodal zones at each end of each strut. The strength of each shear span was taken to be the lowest of the five shear strengths for that shear span. The ratios of the lowest test/calculated shear strength ranged from 0.96 to 2.14, with an overall mean of 1.54, with a coefficient of variation of 0.247. The mean test/calculated ratios decreased from 2.0 for beams with $a/d = 1.06$, to 1.32 for beams with $a/d = 2.0$. This is reasonable agreement for shear test data. Further checks against test data are presented by D. Mitchell in another paper in this symposium.

9 Other details

9.1 Changes to 1999 ACI Code sections. 10–7.1 and 11–8

When strut-and-tie models were introduced, it was necessary to change the definitions of deep beams in 1999 ACI Code Secs. 10.7.1 and 11.8.1. The new definitions are based on the definition of deep beams in ACI 2002 Sec. A.1.

The 1999 and earlier ACI Code design equations for deep beams were based on the $(V_c + V_s)$ approach to shear design. The 1999 Code Eqs. 11–29 and 11–30 for V_c and V_s were deleted, without replacements in Chap. 11, because these equations did not reflect the true behavior of deep beams. This is especially true for continuous deep beams where the moment approaches zero at the critical section defined in ACI Sec. 11.8.5. As a result, the second term on the right-hand side of ACI Eq. 11–29 for V_c approaches zero. In addition, there was a major discontinuity in the magnitude of V_s from 1999 ACI Eq. 11 – 30. At $\ell_n / d = 5.0$, the V_s provided by horizontal stirrups dropped step-wise from $0.5 A_v f_y$ at $\ell_n / d = 4.99$ to zero at $\ell_n / d = 5.01$. In the same region, the V_s provided by vertical stirrups increased step-wise from 0.5 to 1.0 $A_v f_y$.

ACI Sec. 11.8, "Deep Beams" was shortened by omitting these sections and design was permitted either by non-linear analysis or by strut-and-tie models.

9.2 Pile caps

Revisions were made to ACI Sec. 15.5.3 to allow the use of strut-and-tie models in the design of pile caps. In a pile cap the strut-and-tie model is three-dimensional. The nodal zones are also three-dimensional with the struts from the various piles converging on the nodal zone at the base of the column. Section A.5.3 is intended to simplify the design of three-dimensional strut-and-tie models by requiring that the individual faces of the nodal zone have the areas computed from Eq. A.—7, but not requiring an exact match between the shape of the end of the strut and the corresponding shape of the face of the nodal zone acted on by the strut, provided that enough bearing area is provided.

9.3 Serviceability

The last paragraph of RA.2.1 mentions serviceability. Deflections can be estimated using an elastic frame analysis of the strut-and-tie model. The axial stiffness of the ties can be modeled as cracked regions with axial stiffnesses of $A_s E_s / \ell_{cr}$ and uncracked regions with axial stiffnesses of $A_c E_c / \ell_{uncr}$, where ℓ_{cr} and ℓ_{uncr} are the portions of the length of tie which can be assumed to have a cracked section stiffness, and the fraction having an uncracked stiffness respectively. This allows the elongation of the tie to be modelled in a simple manner. Further study is needed in this area.

10 Summary

The development of Appendix A of the 2002 ACI Code and the selection of the ϕ factors and effective concrete strengths is explained. The layout of strut-and-tie models is reviewed.

11 Acknowledgements

ACI Code Appendix A was developed by ACI Committee 318, Subcommittee E, *Shear and Torsion*, chaired by J.O. Jirsa, who was succeeded as chair by J.K. Wight during the period in which the Appendix was letter balloted. Extensive assistance and a number of widely differing design checks were provided by ACI Committee 445–A, *Strut-and-Tie Models*, chaired by K-H Reineck. Design checks and comparisons to tests were carried out by members of ACI 445.A.. Appendix A was reviewed by ACI Committee 318 and approved over the course of four letter ballots.

12 Notation

The following notation is used in this paper:

w_{n1}, w_{n2}, w_{n3} = lengths of the sides of a nodal zone.

A_c = smaller end area of a strut.

A_v = area of vertical web reinforcement within a spacing s .

b_w = width of the web of a beam.

f'_c = specified concrete compressive strength.

$F_{cd\text{ eff}}$ = strength of the concrete in a strut—FIP.

f_{cu} = effective compression strength of struts or nodal zones—ACI.

f_{1cd} = design concrete strength for uni-axial compression—FIP.

$F_n, F_{nn}, F_{ns}, F_{nt}$ = nominal strength, the nominal strength of a nodal zone, the nominal strength of a strut, and the nominal strength of a tie.

F_u = force in a strut, tie, or node due to the factored loads.

jd = internal lever arm between the resultant compressive and tensile forces in a beam. from ACI Sec. 10.2.7.

s = spacing of vertical web reinforcement.

- V_c = shear strength provided by concrete.
 V_s = shear strength provided by stirrups.
 w_t = height of a tie and its surrounding concrete.
 z = internal lever arm between the resultant compressive and tensile forces in a beam.
 α_1 = ratio of the average stress in the rectangular stress block to the concrete strength.
 β_s, β_n = ratio of the design strength of the concrete in a strut or node to the specified strength.
 ϵ_1 = principal tension strain in the concrete.
 ϵ_2 = strain in a tie.
 $\phi, \phi_f, \phi_v, \phi_{STM}$ = strength reduction factor, strength reduction factors for flexure, for shear, for strut-and-tie models.
 γ = angle from axis of strut and axis of a layer of confining reinforcement.
 λ = correction factor for the strength of lightweight concrete.
 v = effectiveness factor.
 ρ_v = reinforcement ratio for vertical web reinforcement = $\frac{A_v}{b_w s}$

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Part 3: Experimental verification of strut-and-tie models

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Synopsis

Results of experiments on a corbel, deep beams and a beam with dapped ends are presented to illustrate how strut-and-tie models are applied to these cases and to provide some experimental verification of the accuracy of the predictions. Both simple strut-and-tie models, as well as refined strut-and-tie models are presented.

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1 Introduction

Although truss models have been used since the turn of the century [Ritter (1899) and Mörsch (1909)], they have gained increased popularity in recent years. The reason for this increased popularity is that they provide the designer with a simple, yet powerful tool, for the design of complex regions in reinforced concrete. This increased popularity is due to the fact that the designer can model the flow of the forces with struts and ties, even for complex design situations. Thus, rather than using empirically based design approaches, the designer can apply a strut-and-tie model, which not only illustrates the flow of the forces, but also provides a clear understanding of the various resisting elements. Design approaches using strut-and-tie models have been codified in Appendix A of the 2002 ACI Code (2002), as well as in the 1984 and 1994 CSA Standards (1984, 1994) and the FIP Recommendations (1999). Design using strut-and-tie models provides an alternative to the empirically based code approaches for disturbed regions, such as brackets, corbels and deep beams. Empirically based approaches are not only limited in their applicability, but do not provide the designer with insight into the actual behavior. An additional advantage of using strut-and-tie models is that sketching the flow of the forces within a member highlights the need for careful details of the reinforcement in key regions. This paper illustrates some simple strut-and-tie models and compares these predictions with experiments. In selecting the experiments only well-instrumented, full-size tests were chosen. Walraven and Lehwalter (1994) have summarized the importance of size effects not only for slender beams, but for deep beams as well. The testing of geometrically similar deep beams demonstrated that, compared to larger beams, smaller beams exhibit less severe crack propagation and as a result exhibit higher stresses for the crushing of the concrete near the bearing plates. Therefore, the response of small-scale specimens may not be representative of the response of actual structural elements.

2 Corbel

Figure 1 illustrates the strut-and-tie model for a double-sided corbel tested by Cook and Mitchell (1988). Figure 1(a) shows the details of the reinforcement and the dimensions of the test specimen. During testing, the corbel was subjected to a vertically applied load, V , and an outwards horizontal load that was kept equal to $0.2V$. This horizontal force of $0.2V$ represented the minimum horizontal design force required in the ACI Code (Clause 11.9.3.4), unless special provisions are made to avoid tensile forces. The 50 mm-300 mm (2 in.-11.8 in.) bearing plate was 25 mm (1 in.) thick and was welded to four No. 15 (16 mm dia. (0.63 in.)) weldable grade bars. The concrete strength at the time of testing was 40.4 MPa (5860 psi). Figure 1(c) shows the strut-and-tie model for this corbel. This simplified model used the conservative assumption that only the main tie reinforcement contributed to the strength of the corbel. The forces shown in Fig. 1(c) are those resulting from yielding of the four No. 15 bars. The total yield force is $A_s f_y = 4.200 \text{ mm}^2 \cdot 444 \text{ MPa} = 355 \text{ kN}$ (79.8 kips). In order to predict the capacity, it was necessary to find the geometry of the strut-and-tie model. While the ACI Code and CSA Standard do not recommend strut-and-tie models for different cases, the FIP Recommendations (1999) provide guidance on suitable strut-and-tie models for different types of regions. For a corbel, the inclined struts are equilibrated by vertical struts with dimensions of a_1 by b in the column and a horizontal strut with dimensions of a_2 by b near the bottom of the corbel (see Fig. 1(c)), where b is the width of the column and corbel. In solving for the geometry, a uniform stress of $0.85 f'_c$ was assumed in the stress blocks. From equilibrium, a_1 was calculated to be 39.2 mm (1.54 in.) and a_2 was 21.7 mm (0.85 in.). The corresponding predicted capacity, V , was 471 kN (106 kips) which is 94% of the measured failure load, V , of 502 kN (113 kips).