

Deflection Control for the Future



international[®]
SP-210

Editor:
N. J. Gardner
Documents
Document Sharing Club

Deflection Control for the Future

Editor

N. J. Gardner



DISCUSSION of individual papers in this symposium may be submitted in accordance with general requirements of the ACI Publication Policy to ACI headquarters at the address given below. Closing date for submission of discussion is October 2003. All discussion approved by the Technical Activities Committee along with closing remarks by the authors will be published in the January/February 2004 issue of either ACI Structural Journal or ACI Materials Journal depending on the subject emphasis of the individual paper.

The Institute is not responsible for the statements or opinions expressed in its publications. Institute publications are not able to, nor intended to, supplant individual training, responsibility, or judgment of the user, or the supplier, of the information presented.

The papers in this volume have been reviewed under Institute publication procedures by individuals expert in the subject areas of the papers.

Copyright © 2003
AMERICAN CONCRETE INSTITUTE
P.O. Box 9094
Farmington Hills, Michigan 48333-9094

All rights reserved, including rights of reproduction and use in any form or by any means, including the making of copies by any photo process, or by any electronic or mechanical device, printed or written or oral, or recording for sound or visual reproduction or for use in any knowledge or retrieval system or device, unless permission in writing is obtained from the copyright proprietors.

Front cover: This photo is reprinted from "Time-Dependent Deflection and Cracking of Reinforced Concrete in Flat Slabs" with permission of Ian Gilbert, Professor of Civil Engineering and Head of the School of Civil and Environmental Engineering at the University of New South Wales in Sydney, Australia.

Back cover: This photo is reprinted from "Deflections of Flat Plate Floors with Irregular Column Layout," which begins on page 37 of this book, with permission of S. Teng, Associate Professor at the School of Civil and Environmental Engineering at Nanyang Technological University (NTU) in Singapore, and Tavió, Ph. D. candidate at NTU and lecturer in the Department of Civil Engineering at Sepuluh Nopember Institute of Technology in Surabaya, Indonesia.

Printed in the United States of America

Editorial production: Bonnie L. Schmick

Library of Congress catalog card number: 2003101442
ISBN: 0-87031-107-7

PREFACE

This Special Publication contains the written versions of the papers presented at two sessions titled “Deflection Control for the Future” during the 2002 ACI Spring Convention in Detroit. The technical sessions were jointly sponsored by ACI Committees 435, Deflection of Concrete Building Structures, and 440, Fiber Reinforced Polymer Reinforcement. This volume includes six papers related to predicting the deflections of concrete members reinforced with fiber-reinforced polymer (FRP) reinforcement: one paper describing two field investigations, two unique laboratory investigations, a paper on estimating the errors in calculating deflections of HPC slabs, and a comparison of the ACI and EC2 deflection provisions. This volume should be read in association with Special Publication SP-203, *Code Provisions for Deflection Control in Concrete Structures*.

The increasing use of higher-strength concretes and reinforcing steels, and more effective design procedures, leads to shallower section members in reinforced concrete structures with attendant reductions in stiffness and, hence, larger deflections. The introduction of ACI 318-02, with lower load factors, will increase the service load/factored design load ratio and will tend to make more structures deflection critical.

FRP reinforcement is attractive for its resistance to corrosion. However, FRP reinforcement has a very high strength and a slightly lower modulus of elasticity than steel and limited ductility compared to steel. Consequently, the current methods of calculating deflections have to be examined for their applicability to FRP reinforced structures.

The object of structural design is to achieve acceptable probabilities that structures will perform satisfactorily over the intended lifetime of the structure. For safety, the structure must have adequate strength with a low probability of collapse. Although safety is the most important limit state, it is not sufficient without satisfying the requirements of serviceability. Service load deflections/curvatures may be excessive, or long-term deflections/curvatures due to sustained loads may cause damage to partitions, visual discomfort, and/or perception, etc.

Predicting the deflection serviceability of reinforced concrete members in service is fraught with uncertainties, which include imperfect knowledge of the limiting serviceability criteria, the material properties, and the load history including construction loads and the service load. The serviceability criteria can be immediate deflection/curvature or incremental deflection/curvature. Most codes offer two methods for control of deflections. The designer may choose to calculate the deflections and check that these computed deflections are less than specified allowable limits. Calculating the immediate deflections of reinforced concrete members is difficult due to the concrete cracking in the tension zones of such members. Calculating the additional deflections due to shrinkage, creep, and the consequent redistribution of stress is extremely difficult. Alternatively, the codes give specified maximum span/depth ratios for which serviceability can be assumed to be satisfied and deflections do not need to be calculated.

There appears to be agreement that the incremental deflection after construction of partitions and finishes is more critical than immediate deflection. There is also general agreement that the limiting incremental deflections are span/500 for brittle partitions otherwise span/250. However, there is less agreement over what loads should be used in the deflection calculation. Most codes prescribe calculating deflections for dead load plus some fraction, which depends upon the structure's usage of the specified live load. The characteristic live load, live load not exceeded 95% of the time, should be used for ultimate limit state calculations. However, experimental work indicates that an average live load of 50% of the extreme, assumed specified/characteristic, live load would be reasonable for serviceability limit state calculations. BS 8110-85 states that when calculating deflections, the portion of the load to be considered permanent should be 25 to 30% for office use but at least 75% for storage. AS 3600-1994 suggests that for deflection calculations the live load can be multiplied by 0.6 for offices (1.0 for storage) for immediate deflections and 0.25 for long-term deflections (0.5 to 0.8 for storage). Further, the expected, average, value of the concrete strength, not the lower-bound characteristic concrete strength, should be used in deflection calculations.

Code-recommended simplified procedures for calculating deflections can be separated into two groups: those based upon Branson's effective EI method and the CEB interpolated curvature method. Branson's effective EI can also be used to calculate curvatures and then deflections. Interestingly, the Australian Code uses Branson but incorporates shrinkage and compressive steel ratio into its calculation of M_{crack} . At the Detroit Convention, ACI Committee 435, Deflection of Concrete Building Structures, presented "A Guide to Calculate the Displacements of Concrete Structures" based on the CEB interpolated curvature method.

All papers in this publication were reviewed by recognized experts in accordance with ACI review procedures with each paper having been reviewed by no less than two reviewers.

Editor

N. J. Gardner

TABLE OF CONTENTS

Preface	iii
SP-210—1: Construction Loads and Serviceability Requirements: Deflection Control, Span/Thickness Limitations	1
by D. Kaminetzky and P. C. Stivaros	
SP-210—2: Influence of Reinforcement Size and Distribution on Deflections of RC Members	19
by M. A. Polak, K. G. Blackwell, and D. T. Killen	
SP-210—3: Deflections of Flat Plate Floors with Irregular Column Layout	37
by S. Teng and Tavo	
SP-210—4: Estimating the Error in Calculated Deflections of HPC Slabs: A Parametric Study Using the Theory of Error Propagation	65
by M. M. Reda Taha and M. A. Hassanain	
SP-210—5: A Comparison of the ACI and EC2 Codes Provisions for Flexural Deflection	93
by F. A. Malhas and A. Rahman	
SP-210—6: Rational Procedure for Calculating Deflections of RC Beams Strengthened with FRP	115
by H. A. Rasheed, H. Charkas, and H. G. Melhem	
SP-210—7: Design of FRP Reinforced Concrete for Serviceability	149
by D. Svecova, S. H. Rizkalla, and G. Tadros	
SP-210—8: Deflection Behaviour of Concrete Beams Reinforced with Carbon FRP Composite Bars	173
by C. Kassem, E. El-Salakawy, and B. Benmokrane	
SP-210—9: Rational Method for Calculating Deflection of Continuous FRP R/C Beams	191
by A. G. Razaqpur and O. B. Isgor	
SP-210—10: Flexural Stiffness of High Strength Concrete Beams Reinforced with GFRP Bars	209
by J. R. Yost, S. P. Gross, and D. W. Dinehart	
SP-210—11: Deflection Assessment of an FRP-Reinforced Concrete Bridge	225
by D. D. Kleinbans, A. Prota, and A. Nanni	
Index	249

Construction Loads and Serviceability Requirements: Deflection Control, Span/Thickness Limitations

by D. Kaminetzky and P. C. Stivaros

Synopsis: Throughout the history of concrete construction, numerous construction failures have occurred involving excessive deflections and cracking of the completed structure. This paper presents two building construction cases where concrete slabs developed extensive cracking and excessive deflections soon after the slab construction and formwork removal. The effects of the shoring-reshoring operations, the rate of concrete strength development, as well as the effects of design details on the slab cracking and deflections, are investigated. The ACI 318 requirements of minimum thickness and deflection control are applied to both construction cases, and the adequacy of these code requirements is discussed. Based on the findings of this work it was concluded that the ACI 318 long term creep and shrinkage deflection calculation method does not adequately account for the early-age high construction loads.

Keywords: code provisions; concrete slabs; cracking; deflection control; long-term deflection; reinforced concrete; serviceability

2 Kaminetzky and Stivaros

ACI Fellow Dov Kaminetzky is the president of Feld, Kaminetzky & Cohen, P.C., New York, N.Y. He has been involved in the investigation of failures and the evaluation of structures for more than forty years. He is currently a member of ACI Committees 347 - Formwork for Concrete; 350 - Environmental Engineering Concrete Structures; 364 - Rehabilitation; and 437 - Strength Evaluation of Existing Concrete Structures.

ACI member Pericles C. Stivaros is a Senior Associate with Feld, Kaminetzky & Cohen, P.C., Roslyn Heights, N.Y. His professional experience includes structural analysis and design of building systems, failure investigations, distress analysis, structural evaluation, and rehabilitation of buildings and other structures. He is a member of ACI Committees 347 - Formwork for Concrete; and 362 - Parking Structures.

INTRODUCTION

It has been well documented that failure of concrete structures during construction manifested by extensive cracking and large deflections most often occurs as a result of formwork failure, overloading, and/or lack of concrete strength at the time of the construction load application. Premature removal of formwork during the construction of multi-story reinforced concrete buildings has been the cause of some dramatic failures, as well as the source of many serviceability problems such as excessive deflections and cracking. The high construction loads that are imposed on immature slabs, in combination with long term creep and concrete shrinkage, seriously affect the serviceability of the concrete structure.

Deflection control, however, cannot be achieved with the use of proper construction procedures alone. The selection of adequate slab thickness, which is a measure of slab stiffness and resistance to deflection and cracking, is equally important. Most codes, including ACI 318-99 (1), offer the Engineer the choice of either calculating the expected deflections and limiting these deflections to some specified allowable limits, or using a minimum slab thickness to satisfy certain maximum span/thickness ratios.

According to the commentary in ACI 318-99 Code, the minimum thickness requirements are applicable only in situations consistent with "previous experience in loads, environment, materials, boundary conditions, and spans." It has been a matter of a long debate whether a structure during construction falls within the "previous experience" based on which code provisions were developed. A structure under construction is often subjected to heavy construction loads which sometimes exceed by far the design loads. Ambient environmental conditions directly affect an open wall structure during construction, while completed structures are usually protected from the elements. The properties and strength of early age concrete differ from

those of the cured and matured concrete. As a matter of fact, the early-age concrete strength is as difficult to predict as the prediction of the weather conditions which directly affect the concrete strength development. Furthermore, the boundary conditions and redundancy which affect the magnitude of deflections within a building under construction differ from those of a completed structure. Usually such conditions and peculiarities of buildings under construction are not accounted for during the design of the structure. Failure to comply with the ACI Code's minimum slab thickness requirements combined with problems during construction have resulted in excessive deflections.

Though serviceability failures in the form of excessive deflections are fairly common, they do not, however, generate as much interest and publicity as dramatic collapses. Deflection problems usually remain within the circles of the construction team "family" and rarely see the light of publicity. Engineering reports are generated usually to assist the opposing legal teams, but seldom attract media attention or become lesson for the public in avoiding similar failures in the future. The objective of this paper is to present the deflection and cracking problems of two separate construction cases, investigate the cause of these failures, and discuss the implications of the deflection requirements and the deflection calculation procedures of the ACI 318 code.

Construction Cases

The case studies presented in this paper involve two low-rise reinforced concrete buildings where the slabs developed extensive cracking and excessive deflections soon after the slabs construction and formwork removal. A common characteristic of both cases is that both were constructed during the winter months with prevailing low and sometimes freezing temperatures. The low temperatures which retarded the concrete strength development, combined with improper shoring procedures, resulted in excessive deflections and cracking. Inadequate slab thickness further exacerbated these conditions.

CASE STUDY I

Type of Construction

This building is a three-story flat slab reinforced concrete structure, approximately 180 ft. long by 150 ft. wide. It is an educational building consisting of offices, classrooms, and laboratories. The first floor and part of the second floor are 7-inch thick concrete slabs on grade. The second and third floors are elevated framed 9-inch thick, two-way flat slabs, with 8 ft. x 8 ft. by 5-1/2 inch deep drop panels. The roof slab is similar to the floor slabs except that an 8-inch thick flat slab was used. Floor areas adjacent to the elevator core are framed with one-way reinforced concrete slabs and beams. The flat plate slabs span 25 ft. in each direction and are supported

4 Kaminetzky and Stivaros

by 18-inch square reinforced concrete columns and by reinforced concrete spandrel beams along the periphery. All columns are continuous down to the foundation level and are supported on spread footings. The partition walls consist of 8-inch concrete masonry block walls bearing directly on the floor slabs. The partition walls are generally located along the column lines. The third floor plan is shown in Figure 1.

Construction of the building started in the spring of 1992. About ninety percent of the structural slabs were placed between October 1992 and February 1993. The remaining slab areas were completed by July 1993.

Problem Description

Soon after completion of the floor slabs, and before the construction of the interior partition walls, it was discovered that the slabs displayed extensive cracking and excessive deflection.

The masonry partition walls were constructed on the top of the deflected slabs by thickening the wall bedding joints at the deflected areas. After a few months time the wall bedding joints displayed further racking and separation from the slabs. This condition suggested that the slabs continued to experience further deflection after the erection of the masonry walls. The slab deflections have caused several problems with the partition walls, door jambs, laboratory equipment, and long counters resting on the floors.

Concern was raised for the structural integrity of the building and the long term serviceability. Our firm was retained by the owners to investigate the cause of the problem and assist them with the subsequent legal battle. The building was not occupied during our investigation.

Our investigation consisted of field, laboratory, and office studies. The field studies included floor deflection surveys, crack surveys, reinforcement location surveys (pachometer surveys), impact-echo testing to verify the slab thickness, and coring of the concrete slabs. The laboratory studies included compressive strength testing and petrographic examination of the concrete cores. The office studies included the structural analysis, review of the design of the slabs to verify their load carrying capacity at various stages of construction, construction load analysis, and deflection calculations. This paper presents only the parts of our investigation that are relevant to the construction loads and deflections.

Field Studies

Slab deflections were measured by taking survey readings along column lines and slab centerlines. The relative slab deflections at the middle of the slabs vary from 1-1/2" to 2" for the second floor, 1-1/2" to 2-7/8" for the third floor, and 1/4" to 2-3/8" for the roof. Figure 1 shows typical slab deflections of the third floor. Typical

Deflection Control for the Future 5

crack patterns consisted of radial cracks emanating from the columns and parallel cracks along the column lines. The crack widths vary from hairline to 1/8" and most extended through the slab full depth. Figure 2 shows typical slab cracking.

Impact-echo testing indicated that the actual slab thickness varied from 9" to 11" with an average thickness of 10". The design thickness was 9". Thus, the slabs had to support the additional dead load of the excess concrete. The flexural capacity of the slabs was not increased by the thicker slabs since the effective depth of the top reinforcement was not increased. Pachometer readings indicated that the top rebars were placed deeper into the thicker slab.

Laboratory Studies

Testing of concrete cores indicated that the in-place concrete strength at the time of coring (approximately 3 years after construction) exceeded the specified design strength f'_c of 4,000 psi. Examination of a core taken at a crack location indicated that the crack extended through several hard coarse aggregates. Such a crack is usually the result of structural overstress after hardening due to overloading.

Deflection Control

The flat slab design thickness was 9" with 5-1/2" drop panels. This slab thickness is within the ACI 318 (Section 9.5.3) requirements for minimum slab thickness. The immediate computed deflections based on the design loads vary between 0.25" and 0.4". The total long term deflections, including creep and shrinkage, are estimated in accordance with ACI 318 (Section 9.5.2.5) to be approximately one inch. According to Table 9.5d of ACI 318, the immediate live load deflections are limited to $L/360$ or 0.83" so that non-structural elements will not be expected to be damaged by large deflections. The long term deflections due to sustained loads and the immediate live load deflections are limited to $L/480$ or 0.63" when non-structural elements are expected to be damaged by large deflections. The calculated design deflections are within the limitations set by ACI 318. Therefore, this is a case where the slab thickness meets both the minimum slab thickness requirements, as well as the calculated deflection requirements as required by the ACI Building Code. However, the measured slab deflections were almost three times as much as the theoretical calculated deflections. The question is, what has gone wrong . . . the code requirements or the construction methods?

Slab Age and Construction Loads

In order to answer the above question, the construction loads and the slab concrete strength/capacity at the time of the construction load application must be investigated. Table 1 shows the construction operations, and the maximum construction load on each floor as well as the age of the slab when the load is applied. The construction loads are evaluated based on the elastic construction load

6 Kaminetzky and Stivaros

analysis by Grundy and Kabaila (2). In addition to the slab self weight, the slabs are assumed to carry a 50 psf construction live load, as recommended by ACI 347-92 (3).

The 50 psf live load is assumed to be applied only at the top floor during concrete placement. A 20 psf miscellaneous construction load is assumed during the shoring/reshoring removal and installation. Also, a 5 psf formwork load is used as suggested by the formwork drawings.

Slab Strength Analysis - Safety During Construction

The structural strength of a young slab depends primarily on the available concrete strength at the time of the early age loading. To ensure safe construction, the slab construction loads must be smaller than the available flexural, shear, and bond strength of the concrete members.

The early-age concrete strength can be estimated by testing of field-cured cylinders in combination with other available non-destructive methods such as penetration probes, pulse velocity measurements, and maturity methods. The specifications of this project called for field-cured cylinders to determine the in-place concrete strength.

For predicting the estimated in-place concrete strength, ACI 306 (4) "Cold Weather Concreting" recommends maturity based methods which must be confirmed by field testing. The strength-maturity relationship of concrete mixes was developed on the basis that the strength of concrete depends on the curing time-temperature history of concrete. Maturity represents quantitatively the cumulative effects of temperature and time up to any given age. It is assumed that every concrete mix possesses a unique strength-maturity relationship and is valid for any given temperature history. The maturity relationship for a proposed concrete mix can be developed experimentally by a laboratory controlled testing before starting the building construction. Also, field-cured or laboratory-cured cylinders of the actual concrete provided at the site can be used to develop the strength maturity relationship of the supplied concrete mix. The most common definition of maturity is:

$$M = \sum (T - T_o) \Delta t \quad (1)$$

where, M = maturity in °F-days

T = concrete temperature in °F during the particular time interval

T_o = datum temperature in °F, lowest temperature at which concrete ceases to gain strength with time.

Δt = time interval in days.

Deflection Control for the Future 7

The appropriate value for the datum temperature “To” depends on the type of cement, the type and quality of admixture, and the range of the curing temperature. For cold weather conditions, ACI 306 recommends a datum temperature of 23° F.

The relationship between the concrete strength and the maturity function is:

$$f_{ci} = [AM_i/(1 + AM_i)]f_{cu} \quad (2)$$

where, “fci” is the concrete strength at a given time, “fcu” is the limiting value of concrete strength as maturity approaches to infinity, “A” is the normalized slope of the strength-maturity curve divided by “fcu”, and “Mi” is the concrete maturity at a given time (F-Days).

Based on concrete cylinder strengths obtained from testing laboratory reports, the maturity relationships were developed for each placement of each floor level. The strength-maturity relationships can be used to determine the approximate concrete strength during construction, as long as the curing temperatures are known. Lacking records of construction daily temperatures, temperatures obtained from a nearby airport were used. The estimated maturities and predicted compressive strengths for each placement of each floor indicated that none of the predicted in-place concrete strengths reached the 4000 psi specified strength at the age of 28 days.

ACI 306R-88 recommends to keep the building heated to maintain a temperature of 50°F or more for several days after the concrete is placed. A review of the construction field reports showed that no adequate heat and protection of the freshly cast concrete was provided. The low and often freezing temperatures delayed the development of concrete strength significantly.

ACI 318 gives the minimum required strength limit state load capacity, U, as: $U = 1.4D + 1.7L$, where D and L are the service dead and live loads, respectively. It is reasonable to assume that the ultimate load capacity of an early-age slab is proportional to the ratio of the specified design concrete strength to the partially developed early strength, i.e. load capacity during construction is given by:

$$U_c = [\text{Load Capacity During Service}]f_c/f'_c$$

where, U_c is the available slab strength during construction, f_c is the partially developed concrete strength, f'_c is the specified design concrete strength at 28 days.

At the time when this investigation was performed, there was no existing design code specifying load factors to be used during construction. It was then reasonable to assume the same load factors specified for service loads. The upcoming ASCE-37 (5) publication provides load factors for construction loads.

8 Kaminetzky and Stivaros

Under normal design and construction circumstances, it is logical to presume that the building structure has been designed to satisfy the relevant code provisions for flexure and shear. The design loads are 121 psf slab dead load (for the 9" slab with 5-1/2" drop panels) and 60 psf live load, and about 50 psf superimposed partition, floor finishes, and miscellaneous dead load. The total factored design load is 341 psf for the second and third floor slabs.

Table 2 shows the estimated concrete compressive strength and the available slab load capacities along with the applied construction loads. The table shows that the slab construction loads exceed the available load capacity of the slabs during the various phases of construction. The overloading is more than two-hundred percent of the available slab capacity at the second floor and almost two-hundred percent overloading at the third floor.

Calculated Deflections

The total long term deflections, including creep and shrinkage, were estimated in accordance with ACI 318, Section 9.5.2.5. The long term deflections were calculated for a period of 2-1/2 to 3 years, which is the approximate time of measured deflections, thus comparisons could be made. The ξ value of equation 9-10 of the ACI 318 for this period is taken as 1.75. The sustained service loads for the long term deflection calculations include the slab self weight, masonry partition walls, and 50 percent of the live load. Table 3 shows the estimated and measured deflections for the above period for the various slab placement areas.

The immediate and long term deflections have been based on the cracked concrete section. The cracking of concrete is primarily related to the applied loads and the available concrete modulus of rupture. At this building, the maximum slab loads were applied during construction when the concrete had not fully developed its full strength and resistance to cracking. It is therefore appropriate to calculate the service load long term deflections based on the effective moment of inertia of the cracked section that occurred during construction. Once a slab is cracked during construction it does not revert to an uncracked condition during service condition unless crack repairs are performed.

The calculated total long term deflections for the various concrete placements of each slab range from 1" to 1.9" for the second floor slab, and 1" to 1.4" for the third floor slab. The maximum measured deflections for the same period of time range from 1.7" to 1.9" for the second floor and 2.2" to 2.9" for the third floor. The measured deflections are approximately double the theoretical estimated deflections.

ACI-318 Estimated Deflections vs. Measured Deflections

It is important to note that most of the estimated deflections for the various slab areas do not compare well with the measured deflections. Evidently, the

ACI 318 calculations procedure for the long term creep and shrinkage deflections generally underestimates the actual slab deflections when the slabs are heavily loaded at an early-age. It was concluded that the ACI 318 does not adequately account for the early-age high construction loads and the time-dependent cracking.

The ACI 435 Committee, "State-of-the-Art Report on Control of Two-Way Slab Deflections" (6) recognizes that the actual deflection may be larger than the ones calculated due to the restraint cracking caused by shrinkage and thermal contraction. ACI 435 cautions that for multistory slab construction, the extent of cracking is usually determined by the construction loads resulting from shoring and reshoring procedures. It further suggests to use high multipliers for the long term creep and shrinkage deflection estimation of slab systems.

Previous experimental and analytical studies (7-12) reported the inadequacy of the ACI 318 deflection procedure. These studies underlined the sensitivity of the slab deflections to early-age construction loads and suggested several methods to improve the ACI 318 procedure, including reduction of the concrete modulus of rupture, reduction of the effective moment of inertia, increased creep and shrinkage multipliers, introduction of the age of load and the construction-to-service load ratio into the deflection calculation, and other techniques. Most of these suggested methods yield approximately double the calculated deflections as compared with the present ACI 318 method. Indeed, had higher long term multipliers been applied in this case, the calculated deflections would have compared very well with the measured deflections. The calculated deflections of this case study based on the code provisions yielded deflections approximately half as much as the measured ones.

The question still remains - Is there a deficiency with the code provision, or with the construction method used for shoring and reshoring? However, before we attempt to answer this question, we should answer first another question - Why through the history of concrete construction have many floor slabs displayed deflections which were within acceptable code limits and why only a few slabs exhibited excessive deflections even though all slabs presumably have been designed using the same code requirements? The answer to the second question can be found within the different construction methods, distribution of construction loads, and environmental conditions that exist in the first few weeks of the life of the structure.

This case study is a typical case where the slabs have been overloaded during construction. Experience has shown that concrete loaded at an early age will have greater initial and long term deflections. Both the concrete strength and modulus of rupture were low when the construction loads were applied. Consequently, the concrete cracked and resulted in larger deflections. Furthermore, creep deflections are expected to be large since creep effects are dependent on the magnitude of the applied stress relative to the available concrete strength. Had the concrete been properly cured and protected from early overstress, it would have developed adequate

10 Kaminetzky and Stivaros

strength to resist cracking and deflections and the deflections would have been within acceptable limits as required by the code.

CASE STUDY II

Type of Construction

This building is a two-story reinforced concrete structure supported on spread footings and grade beams. This educational building consists of a library and laboratories. The building plan measurements are approximately 75 feet in the east-west direction by approximately 188 feet in the north-south direction. The building was designed and constructed in 1996/97.

Both the first floor level and the roof level are ten-inch normal weight concrete slabs reinforced as one-way slabs. The typical bay size is approximately 32 feet by 32 feet. A 20 foot wide, 20-inch deep slab band at the center column lines spans in the north-south direction. Typical floor plan is shown in Figure 3.

Problem Description

The first floor and the roof slabs were constructed in December 1996 and January 1997. All formwork, shores, and reshores were removed by March 1997. Soon afterwards, by the middle of March 1997, the contractor installed metal stud walls at the ground floor. After about a three month period, by June 1997, the contractor observed that the metal wall studs under the first floor slab started to buckle. The stud buckling was due to the excessive deflection of the first floor slab exerting compressive forces on the metal studs.

The first floor slab and the roof displayed excessive deflections and extensive cracking. Questions were raised concerning the structural integrity and long term serviceability of the structure. At the recommendation of the owners, our firm was retained by the contractor to investigate the cause of the problem. Our investigation consisted of field, laboratory, and office studies, similar to Case Study I.

Field Studies

Top of slab elevation surveys of the first floor showed that the relative slab deflection at the middle of the slabs varied from about $\frac{1}{2}$ " to 2". The survey was conducted approximately nine months after the slab construction. Typical crack patterns consisted of cracks along the column lines in the east-west direction. Some of these cracks appeared to be through the slab. Also, several diagonal cracks, i.e. along the diagonal long dimension of the slab, were observed at the underside of the first floor slab. The width of the cracks varied from hairline to approximately $1/16$ ". The cracking pattern of the top of the first floor is shown in Figure 4.

Deflection Control for the Future 11

The slab thickness was measured directly at several locations through sleeve openings within the slab. Also, the impact-echo testing was utilized to measure slab thickness at the middle of the bays. The slab thickness measurements showed an average uniform slab thickness of about 10-1/2". Pachometer readings indicated that the rebars as placed were in agreement with shop and design drawings.

Laboratory Studies

Concrete core testing indicated that the in-place concrete compressive strength at the time of coring was equal or exceeded the specified design strength of 4,000 psi. Petrographic examination indicated that the concrete met the project specifications. Additionally, the petrographic analysis did not identify any concrete freezing or curing issues during or after the concrete placement.

Deflection Control

Based on Table 9.5a of the ACI 318, the one-way slab thickness should have been a minimum $L/24$, or approximately 14 inches. A smaller slab thickness is acceptable provided deflection limits are checked. The slabs were designed as one-way slabs spanning in the east-west direction. A one-way slab construction will result in greater deflection than a two-way slab system, given a similar slab stiffness and geometrical arrangement.

Due to the actual slab geometry of this building, the load was transferred initially in a two-way manner causing negative movements along unreinforced column lines in the east-west direction. The absence of top reinforcement along these column lines facilitated the slab cracking. Such cracks caused a significant reduction of the slab stiffness resulting in increased deflections.

Although the cause of the problem was determined to be the design of the slabs, the construction methods were also investigated to determine the extent of construction methods contribution to the problem.

Construction Loads -Safety During Construction

A construction load analysis and evaluation was performed in a similar manner as for Case Study I. The maximum construction load was estimated to be approximately 76 percent of the design slab load applied on a six-day old slab. The construction load includes the slab dead load, formwork, and construction live load.

The available load carrying capacity of a six-day slab was estimated based on the concrete compressive strength at the time of the load application. Maturity relationships were developed for the concrete mix used based on laboratory cylinder tests. The strength-maturity relationships were used to determine the approximate concrete compressive strength during construction based on daily temperatures

12 Kaminetzky and Stivaros

recorded by the contractor. The approximate concrete strength of a six-day old slab was estimated to be 3,100 psi, or approximately 77 percent of the design strength. Assuming that the available load carrying capacity of the slab at this construction stage is approximately 77 percent of the design load, then the applied construction load (76 percent of design load) is within the available load capacity of the slab.

Calculated Deflections

As indicated earlier, the floor slabs were designed as one-way slabs, and initially they behaved as a two-way slab system. In order to take into consideration the two-way behavior of the slabs, the first floor slab was analyzed using finite element methods.

The calculations indicate that the design slab thickness of 10-inches is not adequate to keep deflection within the acceptable limits of the code. Computed total long term deflections, including immediate deflections, are approximately 4.0". The sum of the long term computed deflection due to all sustained loads and the immediate deflection due to additional 50 percent of the design live load, is estimated to be almost 3.0". Such a deflection is considerably larger than the acceptable ACI 318 limit of $L/480$, or 0.7", (ACI 318, Table 9.5b). The total long term deflections, including creep and shrinkage, are estimated in accordance with ACI 318, Section 9.5.2.5. Table 4 shows the expected slab deflections of the first floor slab at two points of time: 1) at nine months when the deflection surveys were conducted, and 2) at five years.

It is important to note that the effective moment of inertia due to service dead and live loads is smaller than the one due to construction loads. Therefore, the total long term deflections were calculated based on the concrete cracked section due to service loads.

Computed dead load deflections, including creep and shrinkage effects for the first nine months after the slab construction, are approximately 1.96 inch. This deflection compares favorably with the 2-inch measured deflections of the nine month old slabs. The deflection calculation is based on the effective moment of inertia of the cracked concrete section due to the application of construction loads.

The deflection calculations are based on a 135 psf design dead load and a 100 psf design live load. During the first nine months the slab loads consisted of the slab self weight and an assumed 20 psf construction live load.

ACI 318 Deflection Calculation Method

While the ACI 318 deflection calculation procedure did not accurately predict the measured slab deflection for the first case study, the ACI 318 method accurately predicted the measured slab deflections for the second case study. The difference is

that for the first case study the slabs were overloaded beyond the service load slab capacity during construction while for the second case study the slab construction load was kept within the service load slab capacity. The excessive deflections of the slabs of the second case study were primarily due to inadequate design slab thickness and improper reinforcing details.

CONCLUSIONS AND RECOMMENDATIONS

This paper presented the investigation of two construction cases of low-rise reinforced concrete buildings where the floor slabs exhibited excessive deflections and cracking soon after construction.

The major conclusions of this study can be summarized as follows:

1. High construction loads applied to immature early-age concrete slabs with low modulus of rupture and low elastic moduli will often cause excessive initial and long term creep deflections. Therefore, it is essential that the construction schedules for shoring and reshoring be carefully planned with the use of an engineering analysis in order to insure adequate strength and serviceability behavior.
2. The minimum slab thickness requirements in ACI 318 are not adequate as a safeguard against excessive deflections when high construction loads are applied to concrete slabs at an early-age. Overloading of slabs during construction produces more extensive cracking and larger immediate deflections as compared with cracking and deflections during service loads. Consequently, the long term deflections will also be larger.
3. The ACI 318 procedure for calculating long term creep and shrinkage slab deflections underestimates the actual deflections in cases where the slabs are heavily loaded at an early-age during construction. Long term creep deflections are expected to be large since creep effects are depending on the magnitude of the applied stress relative to the available concrete strength. The ACI method does not take into consideration the effect of the early-age construction loads and the resulting early-age cracking on the long term deflections.
4. Slabs with less than the minimum thickness required by ACI 318 will be expected to develop deflection problems even without high early-age construction loads. The excessive slab deflections will be further aggravated by high early-age construction loads.

Based on the above conclusions, the following recommendations are made:

14 Kaminetzky and Stivaros

1. The design process and construction methods are inseparable. The design engineers know best the strength and serviceability limitations of their structure. They should, therefore, provide the contractor with the design load schedules, and specify that the maximum construction loads permitted on the new slabs are related to the minimum acceptable concrete strength attained at the time of every construction operation. (i.e., stripping of forms and removal of shores). The contractor should develop its own construction schedules in order to meet the engineer's specifications. The ACI 318-99 requires the contractor to furnish to the building official, when so required, the structural calculations and concrete strength data used in planning and implementing shoring/reshoring operations. This is a step in the right direction.
2. Although the ACI code recognizes the importance of addressing the strength requirements of a structure under construction, still there is great latitude in incorporating serviceability requirements. The contractors are in effect on their own when it comes to deflection control for construction loads. Where speedy construction schedule causing high construction loads on early-age slabs is anticipated during the design period, the engineer should take into consideration the inadequacy of the code minimum slab thickness requirements for construction loads and provide larger slab thickness. Calculation of expected deflections are recommended and the slab thickness should be increased when required.
3. Both the immediate and long term creep and shrinkage deflections should be calculated based on the concrete cracked section that is expected to develop during construction. The same cracked section during construction should be used during service. The designers also should be aware that the ACI 318 deflection calculation method underestimates the actual deflections under construction loads. This is especially true for unusual long spans, single spans, and high sustained loads. It is therefore recommended to utilize a more conservative approach as suggested by ACI 435.
4. ACI 318 allows the use of member with thickness less than required by the minimum span/thickness limitation, provided the deflections are calculated and kept within specified limits. Taking into consideration the experience of deflection problems with slab systems, it is recommended that slabs with thickness smaller than that required by the span/thickness code limitations shall not be used. Moreover, the writers recommend that ACI 318 be changed and the provision permitting the use of smaller thickness than required by the minimum span/thickness limitation be eliminated.
5. Other important decisions made by the designer can affect the service load deflections. Such decisions include, but are not limited to, sizing of members for deflection rather than for strength only, providing member camber, providing compression reinforcement where possible to minimize long term

deflections, and specifying a concrete mix with low shrinkage characteristics, and specify high early strength mix to increase the stiffness of the early-age concrete. Other actions taken during construction in order to minimize deflections should include the review and approval of shoring/reshoring schedules, review curing procedures and concrete protection from inclement weather conditions, and the implementation of rigorous supervision at the site.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-99)", American Concrete Institute, Farmington Hills, Michigan, 1999.
2. Grundy, P., and Kabaila, A., "Construction Loads on Slabs with Shored Formwork in Multistory Building," ACI Journal, V.60, No. 12, December 1963, pp. 1729-1738.
3. ACI Committee 347, "Guide to Formwork for Concrete ACI 347-92," American Concrete Institute Farmington Hills, Michigan, 1992.
4. ACI Committee 306, "Cold Weather Concreting ACI 306-88", American Concrete Institute, Farmington Hills, Michigan, 1988.
5. ASCE-37 Committee, Structural Standard Division, "Proposed ASCE Standard for Design Loads on Structures During Construction," ASCE 2001.
6. ACI Committee 435.9R, "State-of-the-Art Report on Control of Two-Way Slab Deflections ACI 435.9R-91," American Concrete Institute, Farmington Hills, Michigan, 1991.
7. Tam, K.S.S., and Scanlon, A., "Deflection of Two-Way Slabs Subjected to Restrained Volume Change and Transverse Loads," ACI Journal, Proceedings, V.83, No. 5, 1986, pp. 737-744.
8. Sbarounis, J.A., "Multi-Story Flat Plate Buildings: Effect of Construction Loads on Long-Term Deflections," Concrete International, V.6, No. 2, February 1984, pp.62-70.
9. Graham, C.J., and Scanlon, A., "Long Time Multipliers for Estimating Two-Way Slab Deflections," ACI Journal, Proceedings, V. 83, No. 5, 1986, pp. 899-908.

16 Kaminetzky and Stivaros

10. Gardner, N.J., and Fu, H.C., "Effects of High Construction Loads on the Long-Term Deflections of Flat Slabs," ACI Structural Journal, July-August 1987, pp. 349-360.
11. Ofosu-Asamoah K., and Gardner, N.J., "Flat Slab Thickness Required to Satisfy Serviceability Including Early-Age Construction Loads," ACI Structural Journal, November-December 1997, pp. 700-708.
12. Gilbert, R.I., "Deflection Calculations for Reinforced Concrete Structures - Why We Sometimes Get It Wrong," ACI Structural Journal, November-December 1999, pp. 1027-1032.

Table 1 - Construction Load Analysis - Case I

FLOOR LEVEL	CONSTRUCTION OPERATION	SLAB AGE (DAYS)	CONSTRUCTION LOAD (PSF)		
			D.L.	L.L.	FORM
ROOF	REMOVAL OF SHORES AT THIRD FLOOR	22	108	0	0
THIRD FL.	CONCRETE PLACEMENT AT ROOF	20	266	50	5
SECOND FL.	REMOVAL OF SHORES AT FIRST FLOOR	2	133	20	10

Table 2 - Slab Load Capacities - Case I

FLOOR LEVEL	SLAB AGE (DAYS)	AVAILABLE CONCRETE STRENGTH (PSI)	SLAB CAPACITY (PSF)	MAXIMUM CONSTRUCTION LOAD (PSF)	LOAD COMPARISONS
ROOF	22	1400	89	151	170% Over
THIRD FL.	20	2750	235	464	197% Over
SECOND FL.	2	1300	109	234	214% Over

Table 3 - Estimated and Measured Deflections - CASE I

SLAB PLACEMENT AREAS	ESTIMATED DEFLECTIONS (IN)	MEASURED DEFLECTIONS (IN)
SECOND FLOOR		
PLACEMENT A	1.03	1.92
PLACEMENT B	1.17	1.74
PLACEMENT C	1.85	1.80
THIRD FLOOR		
PLACEMENT A	1.11	2.28
PLACEMENT B	1.38	2.22
PLACEMENT C	1.41	2.88

Table 4 - Estimated Deflections - Case II

SLAB AGE MONTHS	SERVICE LOAD IMMEDIATE DEFLECTIONS (IN)		SUSTAINED LOAD LONG TERM DEFLECTIONS (IN)		ACI LIMIT L/480 DEFLECTIONS (IN)	TOTAL DEFLECTION (IN)
	DL	LL	DL	DL + % LL		
9	0.73	0.12 (20% LL)	0.95 ($\lambda = 1.3$)	1.11 DL + 20% LL ($\lambda = 1.3$)	--	1.96
60	0.85	0.63 (100% LL)	1.70 ($\lambda = 2$)	2.33 DL + 50% LL ($\lambda = 2$)	2.96	3.81

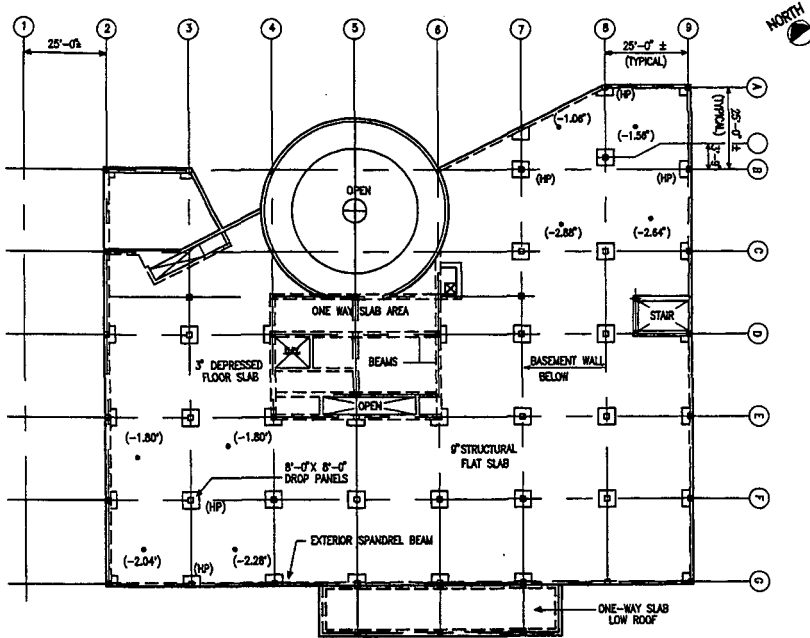


Figure 1. Case Study - I Third Floor Plan

• Denotes Center Of Slab Elevation
(Deflection) Relative To High Point
(HP) Of Slab At Column Locations Of Each Bay.

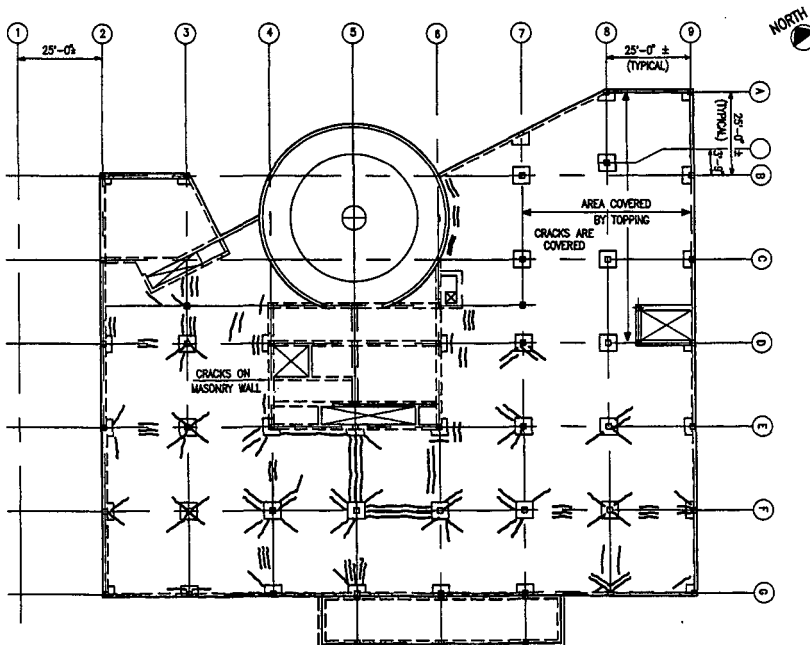


Figure 2. Case Study - I Third Floor Plan - Top Surface Crack Pattern

18 Kaminetzky and Stivaros

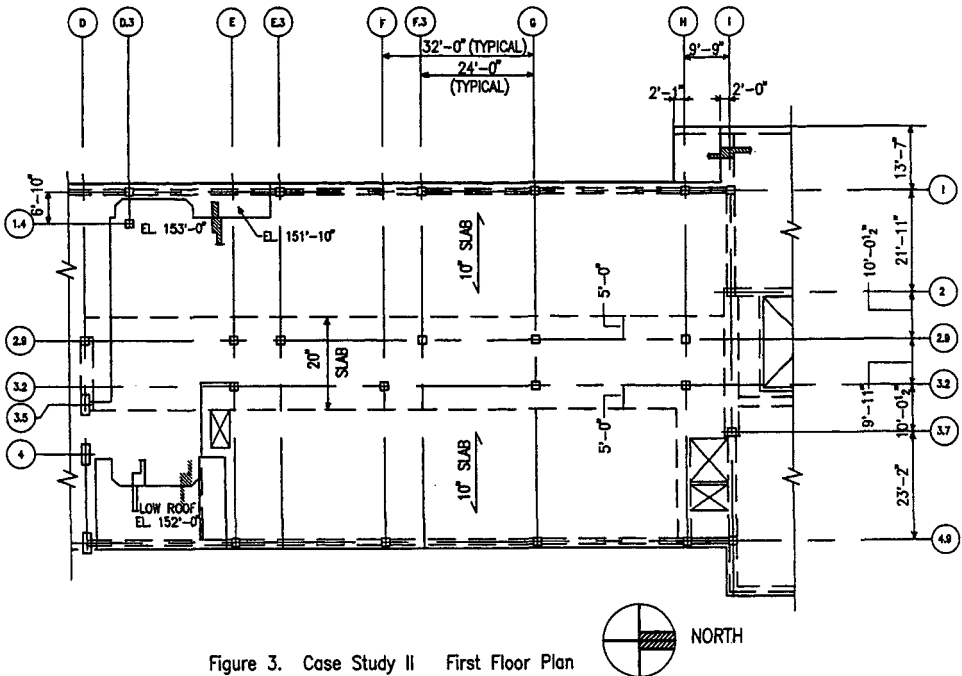


Figure 3. Case Study II First Floor Plan

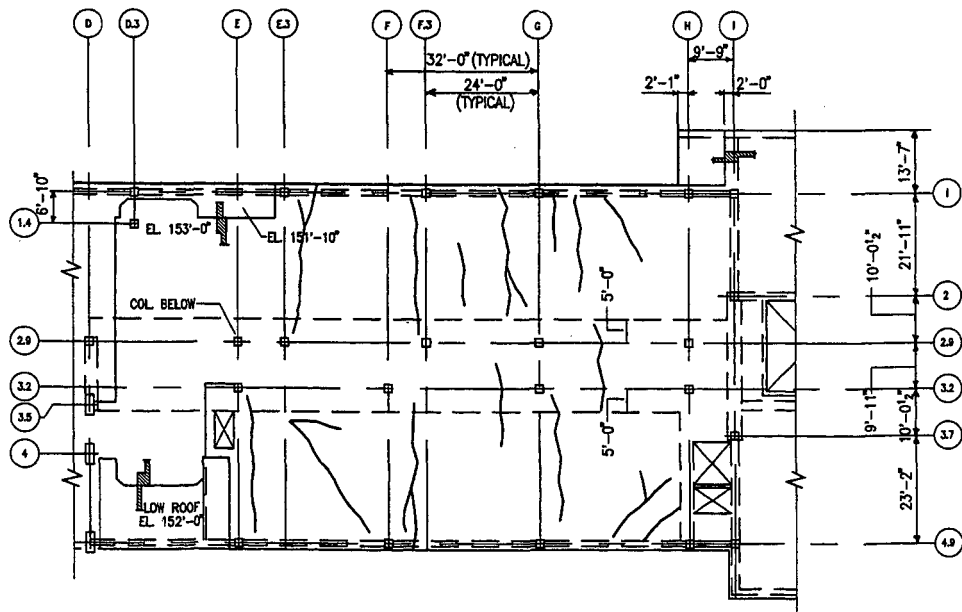


Figure 4. Case Study II First Floor Plan - Top Surface Crack Pattern

Influence of Reinforcement Size and Distribution on Deflections of RC Members

by M. A. Polak, K. G. Blackwell, and D. T. Killen

Synopsis: The paper presents analysis of results from fourteen full-size tests on reinforced concrete members subject to different ratios of bending and in-plane loads. The tests examined the influence of bond between concrete and reinforcement on the deflections and cracking of the members. The parameters in the test program included reinforcing bar diameter, ratio of in-plane to bending loadings, reinforcement ratio and concrete cover. Two types of specimens were involved: one set with a larger number but smaller diameter bars, and one with a smaller number but larger diameter bars. Since reinforcement sizes varied, the bond characteristics of these bars varied, leading to different load-deflection responses. The test results showed that different longitudinal reinforcement configurations, for the same reinforcement ratio and the same loading and boundary conditions, result in different cracking loads, post-cracking stiffness and deflections.

Keywords: bond; crack spacing; deflections; reinforcement size; tests

20 Polak et al.

ACI member Maria Anna Polak is an associate professor of civil engineering at the University of Waterloo, Waterloo, Canada. She is a member of ACI committees 435 Deflections of Concrete Building Structures and 445 Shear and Torsion. Her research interests are in nonlinear structural analysis, material modelling, punching shear in reinforced concrete slabs and characterization of civil engineering materials.

Kevin G. Blackwell is a senior project engineer with Morrison Hershfield in Burnaby, British Columbia Canada. He obtained his M.A.Sc degree for the Department of Civil Engineering, University of Waterloo.

David T. Killen is an engineer with BTS Consulting Engineers in Windsor, Ontario, Canada. He obtained his M.A.Sc degree for the Department of Civil Engineering, University of Waterloo.

INTRODUCTION

Deflections of reinforced concrete members at service load level depend to a large extent on the ability of cracked reinforced concrete to carry tensile forces. Due to bond between concrete and reinforcement, cracked concrete can carry certain amount of tensile forces thus contributing to the increased overall flexural stiffness of the cross section. This phenomenon is often referred to as "tension stiffening" because the presence of cracked concrete adds to the stiffness of the reinforced concrete member. Understanding this phenomenon and its proper modelling is very important for predicting deflections and also for analyzing response of reinforced concrete structures subjected to e.g., thermal and shear loads.

The behavior of reinforced concrete beams is influenced by the longitudinal reinforcement configuration (Clark and Spiers 1978) and thus by bond between concrete and reinforcement. This paper presents results from research which was designed to study deformations of members subjected to bending and in-plane loads. The total bond force depends on the type of reinforcing bar (deformed or smooth), and the surface area attached to concrete. It is logical to assume that in the case of larger surface area of reinforcement there is more tension stiffening in a member and thus the deflections of the member are be smaller. Therefore, a test program was designed in which the major test parameter was the reinforcing bar diameter and thus the total side area of reinforcing bars.

EXPERIMENTAL PROGRAMME

Several parameters influence immediate deflection of reinforced concrete members, namely the reinforcement ratio, the reinforcing bar diameter and type, the concrete cracking strength and the type of loading. The experimental programme was designed to study these parameters. The emphasis was placed however on the amount of transfer forces between concrete and reinforcement. Considering that all

reinforcing bars used in the tests were deformed bars, this transfer force (bond) depended primarily on the area of the bars exposed to concrete. Therefore, the test specimens were designed in pairs; each specimen in a pair had identical reinforcement ratio and location but one had more but smaller bars and the other less larger bars. Each specimen in a pair was subjected to the same loading, but different ratios of bending to in-plane loading were used for different pairs.

TEST SPECIMENS

The presented test program included fourteen reinforced concrete slab-strips subject to different ratios of in-plane loading to bending ($N/M = 0, 2, 4, 6$ and -3 (1/m)) (Fig. 1 and Table 1) (Polak and Killen 1998, Polak and Blackwell 1998-a). The specimens had two different cross sectional dimensions; Series DB had a cross section of 300 (height) x 500 (width) and series KB had cross-sectional dimensions of 250 (height) x 500 (width) mm (Figure 2). The width and the height of the specimens were chosen to allow proper distribution of the longitudinal reinforcement and to be able to accommodate the test parameters. Because the tests involved in-plane loading, the specimens were symmetrically reinforced (top and bottom) to allow application of pure tension or compression at the ends of the specimens (tension was applied by pulling on the reinforcing bars).

The specimens were cast in pairs (Table 1). Each pair of specimens was cast on the same day and from the same batch of concrete. The concrete compressive strength for all specimens was approximately 50 MPa. Each specimen in a pair had identical properties (dimensions, reinforcement ratio, concrete compressive strength), but they contained different numbers of different reinforcing bars.

The DB tests specimens had low reinforcement ratio of 0.44% per layer of reinforcement ($\rho = A_s/bd$), with the cross sectional area of reinforcement being 600 mm² per layer. Three of the specimens (DB2, DB3, DB5) were reinforced using two No. 20M (nominal diameter 19.5 mm) reinforcing bars (Fig. 2) per reinforcement layer and three (DB1, DB4, DB6) were reinforced using six No. 10M (nominal diameter 11.3 mm) reinforcing bars per reinforcement layer. A relatively small concrete cover of 24 mm for 10M bars and 20 mm for 20M bars was used. The specimens with No. 10 M reinforcement had a total reinforcing bar side area bonded to concrete equal to 2128920.0 mm²/m and the specimens reinforced with No. 20M bars had a total side area of approximately 122460 mm²/m. The ratio between the two side areas is equal to 1.7.

The KB tests consisted of two series, each with different reinforcing ratio. The KB- I test series included four specimens (two pairs) with 1000 mm² per layer of reinforcement and $\rho = 0.94\%$. The specimens in test series I contained either ten No.10M reinforcing bars or two No.25M (nominal diameter 25.2 mm) bars. The side reinforcement area for ten 10M bars is approximately 354820.0 mm²/m and for

22 Polak et al.

two 25M bars it is $158256.0 \text{ mm}^2/\text{m}$. The ratio between these two side areas is 2.2. Test series KB-II included four specimens with 600 mm^2 per layer of reinforcement and reinforcing ratio of 0.6%. The specimens contained either six No.10M bars or two No.20M bars. The side areas were the same as for DB tests.

All specimens were cast using a normal density concrete purchased from a local ready-mix supplier. It had a maximum aggregate size of 20 mm, Aggregate/Cement ratio of 6.5, water/cement ratio of 0.55, gravel/sand ratio of 1.15 and the compressive strength of around 50 MPa. The reinforcing bars used had slightly different yield stresses (Table 1) however this did not influence pre-yield load-displacement responses.

Each pair of specimen was subjected to the same loading. The variables were the ratio of in-plane load to bending moment. The ratios used were 0, 2, 4 and 6 for tension in-plane forces and -3 for compression force. The loading is specified in Table 1.

EXPERIMENTAL SETUP AND PROCEDURE

An experimental setup consisted of two frames: vertical frame used to apply bending moment and horizontal self-reacting frame (Fig. 1, 3 and 4). The vertical load was applied using a spreader beam. Constant bending moment was achieved by subjecting a simply supported beam to a two-point load (Fig 1). The in-plane tension and compression was applied through stiff end plates attached to the reinforcement (Polak and Killen 1998). The transfer zones at the ends of the specimens were large enough to ensure uniform in-plane stresses in the test zone (the zone with constant bending moment, Fig 1 a).

Vertical displacements of the beams were measured by DCDTs. Strains in the reinforcement were monitored using strain gauges in tension and compression zones. The deformations of the specimens were measured in terms of the surface strains. These were measured at the discrete load stages using a Digital Surface Strain Indicator (DSSI). The DSSI measured the change in length between the targets, using a gauge length of 200 mm (Polak and Killen 1998). The targets were attached to the surface of the specimens: on the top (compression) and the bottom (tension) faces, and on the sides of the specimens at the locations of tension and compression reinforcements (Fig. 4).

DEFLECTIONS

Deflections of the specimens are presented in Figures 5 and 6. In all cases there was a visible difference between the deflections of specimens in each pair; more than 50 % for all tested pairs. The difference in deflections was largest between the cracking load and half of the yielding load, which means around the service load level. Reinforcing bar size and distributions was the most important

parameter influencing the deflections. For all concrete covers and for all reinforcement ratios the same pattern in behavior was observed (e.g. behavior of pairs KB-a and KB-c). Smaller reinforcement ratio did make the effect a little more visible. (e.g. behavior of pairs KB-a and KB-c). In -plane load did not substantially influence the difference in deflections. For both in-plane tension and compression, the same trend was observed. The result for pair KB-d, tested in compression and bending suggests that reinforcement size influence also compressive response of reinforced concrete member (the specimens were reinforced symmetrically in tension and compression zones).

Moment curvature for all specimens was obtained by analyzing surface strain readings at discrete load stages. These results are presented in Figures 7 and 8. Also shown in the Figures are theoretical predictions using the developed model for tension in reinforced concrete (Polak and Blackwell, 1998b). Moment curvature relationships are directly related to the moment-deflection relationships. The moment curvature responses confirm the trends shown by deflections. Specimens with more smaller bars were stiffer and has smaller curvatures.

CRACKING AND YIELDING

During the DB tests, the recorded cracking loads corresponded to the point on the load deflection curves when the members changed the stiffness. However, it was noticed that cracks in the specimens appeared at loads lower than this cracking load. Therefore, appearance of the first cracks and the corresponding bending moments were monitored for KB series (KB tests were done after DB tests) The first appearance of cracks did not directly correspond to a noticeable change in stiffnesses (from uncracked to cracked) of the specimens. Table 2 gives two values for the cracking moment: moment I, which corresponds to the first appearance of cracks, and moment II, which corresponds to the change in flexural stiffness. Both cracking moments I and II were always higher for the specimens with more but smaller diameter reinforcing bars. The specimens with more but smaller bars had 25 to 100% higher cracking moments than their counterparts with less but larger diameter reinforcing bars.

Crack spacing observations also provided important information. For all load cases and for all specimen pairs, the crack spacing was smaller for specimens with more smaller reinforcing bars (Fig.9 and 10). The stabilized crack spacing (at load levels between service and yield) are given in Tables 2 and 3. The tables also show the predicted crack spacing for these specimens according to the CEB code (1978).

The experimental and theoretical cracking and yield moments are compared in Tables 2 and 3. The predicted values are based on a cross-sectional, layered analysis (Collins and Mitchell 1991). The theoretical cracking moments were calculated assuming that concrete cracks at the strain (due to applied stress) of 0.06 millistrain.

24 Polak et al.

The yield moments were based on the assumption that tensile steel yields at the strain of $f_y/200000.0$ (MPa)

The yielding moments of the specimens showed a similar tendency as the cracking moments: higher values of the yield moment for specimens with more but smaller diameter bars. However, the smaller bars had a higher yield stress than the larger bars (Table 1). The ratios between the predicted and observed yield moments remains independent of the reinforcing bar diameter suggesting that the configuration of longitudinal steel had little influence on the flexural capacity of members.

THEORETICAL MODELLING

The experimental results prompted the development of a theoretical formulation. Other formulations exist for tension stiffening (e.g., Gupta and Maestrini 1990, Chan et. al 1993, Vecchio and Collins 1986, Massicotte et. al 1990, Izumo et. al 1992, Floegl and Mang 1982, Gajer and Dux 1990, Clark and Spiers 1978, Gerstle et. al 1992) but the formulation developed as part of this program put special emphasis on bond as the main parameter influencing the response of a member.

The developed procedure is for the analysis of cross-sections subjected to bending and in-plane loads. The procedure incorporates constitutive models for concrete in tension and compression. The constitutive model for concrete in compression used in the procedure is by Thorenfeldt, Tomaszewicz and Jensen (1987) and later calibrated by Collins and Porasz (1989). The constitutive model for tension is a new model, which takes into account bond properties of reinforcing steel. The main feature of the developed tension model is that the tensile force in the cracked portion of the concrete is limited to the maximum bond force that the given type of reinforcing bar can develop. The maximum bond force depends on the type of reinforcing bars (diameter, distribution and geometry of lugs on a bar), their location in a member and the crack spacing. Thus the amount of tension stiffening and overall flexural stiffness of a member are dependent on the crack spacing and the reinforcement type. Within the developed tensile formulation, models for bond and for crack spacing have the most significant influence on results. The crack spacing was predicted using the equation from the CEP code (1978). The bond model assumed that bond is the result of bearing of concrete on the bar lugs and that this bearing stress changes linearly between the cracks. The details of the developed model are given by Polak and Blackwell (1998-b).

The analyses results are presented in a form of moment curvature graphs in Figures 7 and 8. The predictions compare well with the experimental results, and what is most important; the trends in the behavior are predicted well. In Figure 8, the results are also shown for predictions obtained when actual crack spacing

(recorded from testing specimens DB6 and DB5) was used. Using the actual crack spacing resulted in improvement in the accuracy of predictions.

CONCLUSIONS

The results of the presented tests indicate that the size and distribution of flexural reinforcement has a substantial effect on the deflections of reinforced concrete members. For the same reinforcement ratio, members with more, but smaller bars are stiffer, have higher effective cracking moment and thus smaller deflections. This is especially true in the load range between cracking and service load.

The observed cracking moments were always smaller for specimens with less but larger bars. It is possible that this is due to the slip between concrete and reinforcement, right after first cracking, which was much more pronounced when less bond area (larger bars) was available. Also the observed crack spacing and crack size were important. Visibly larger cracks were observed for specimen with less bond area.

The final conclusion that can be offered is that deflection analysis of reinforcing concrete should incorporate the size and distribution of reinforcement in a rational manner. The difference in deflections can be quite large, 50% to 100% , for member which, from the strength point of view, are designed to be identical.

ACKNOWLEDGEMENTS

The financial support for the described research program was provided by the Natural Sciences and Engineering Research Council of Canada (NSERC, Grant No. OGP0138111) and the University of Waterloo. The authors would like to express their gratitude for the support received.

REFERENCES

1. Clark, L.A., Spiers, D.M., (1978), " Tension Stiffening in Reinforced Concrete Beams and Slabs Under Short term Load", Cement and Concrete Association, Technical Report 42.521, July, 19 pp.
2. Polak, M.A., Killen, D.T., (1998), "The Influence of the Reinforcing Bar Diameter on the Behavior of R.C. Members in Bending and In-Plane Tension", ACI Structural Journal, 95(5) , 471-479.
3. Polak, M.A., Blackwell, K.G., (1998-a), "Modeling Tension in Reinforced Concrete Members Subjected to Bending and Axial Load", ASCE Journal of Structural Engineering, 124(9), 1018-1024.

26 Polak et al.

4. Polak, M.A., Blackwell, K.G., (1998-b), " Reinforced Concrete Members Subjected to Bending and In-Plane Loading ", ACI Structural Journal, 95(6), 740-748.
5. CEB-FIP Model Code for Concrete Structures, Third Edition, CEB, Paris. 1978, 348 pp.
6. Collins, M.P., Mitchell, D.,(1991), "Prestressed Concrete Structures", Prentice Hall, Englewood Cliffs, New Jersey.
7. Gupta, A.K., Maestrini, S.R. (1990), "Tension Stiffness Model for Reinforced Concrete Bars", ASCE Journal of Structural Engineering, 116(3), pp.769-790.
8. Chan, H.C., Cheung, Y.K., Huang, Y.P., (1993), "Analytical Crack Model for Reinforced Concrete Structures", Journal of Structural Engineering, ASCE, Vol. 119, No.5, pp. 1339-1357.
9. Vecchio, F.J. and Collins, M.P., (1986) "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear", Journal of the American Concrete Institute, Vol. 83, No. 2. pp. 219-231.
10. Massicotte, B., Elwi, A.E. and MacGregor, J.G., (1990) ,"Tension-Stiffening Model for Planar Reinforced Concrete Members", Structural Journal, ASCE, November, pp. 3039-3058CEB-FIP Model Code for Concrete Structures, Third Edition, CEB, Paris. 1978, 348 pp.
11. Izumo, J., Shin, H., Maekawa, K. and Okamura, H., (1992), "An Analytical Model for RC Panels Subjected to In-Plane Stresses", Concrete Shear in Earthquake, Elsevier Applied Science, London and New York, pp. 206-215.
12. Floegl, H., Mang, H.A, (1982), " Tension Stiffness Concept Based on Bond Slip", ASCE Journal of Structural Division, Proceedings of the American Society of Civil Engineers, 108(12), pp.2681-2701.
13. Collins, M.P., Porasz, A. (1989), "Shear Strength for High Strength Concrete", Bulletin D'Information, No. 193, "Design Aspects of High Strength Concrete", CEB, pp.75-83.
14. Gajer, G., Dux, P.F., (1990), "Crack Band Based Model for FEM Analysis of Concrete Structures", Journal of Structural Engineering, ASCE, Vol. 116, No.6, pp. 1696-1714.
15. Gerstle, W.H., Dey, P.P., Prasad, N.N.V., Rahulkumar, P., Xie, M., (1992), " Crack Growth in Flexural Members – A Fracture Mechanics Approach", ACI Structural Journal, Nov.-Dec., pp.617-625.
16. Thorenfeldt, E., Tomaszewicz, A. and Jensen, J.J., (1987), "Mechanical Properties of High-Strength Concrete and Application in Design", Proceedings of the Symposium "Utilization of High-Strength Concrete", Stavanger, Norway, June, Tapir Trondheim.
17. Collins, M.P., Porasz, A. (1989), "Shear Strength for High Strength Concrete", Bulletin D'Information, No. 193, "Design Aspects of High Strength Concrete", CEB, pp.75-83.

Table 1 Specimen Properties

Test Series [mm]	Pair	Specimen	f_c (MPa)	f_y (MPa)	No./type of top/bot. bars	Reinf. Ratio (%)	N/M (1/m)	Cover (mm)	d'/d (mm)
DB-I 300x500	DB-a	DB1	49	488	6-10M	0.44	0	24	30/270
		DB2	49	475	2- 20M	0.44	0	20	30/270
	DB-b	DB4	53	488	6-10M	0.44	4	24	30/270
		DB3	53	475	2-20M	0.44	4	24	30/270
	DB-c	DB6	54	488	6- 10M	0.44	6	24	30/270
		DB5	54	475	2-20M	0.44	6	20	30/270
KB -I 250x500	KB-a	KB1	50	507	10-10M	0.94	4	20	37/213
		KB2	50	492	2-25M	0.94	4	24	37/213
	KB-b	KB3	56	507	10-10M	0.94	2	20	37/213
		KB4	56	492	2-25M	0.94	2	24	37/213
KB-II 250x500	KB-c	KB5	54	507	6-10M	0.6	4	44	50/200
		KB6	54	475	2-20M	0.6	4	40	50/ 00
	KB-d	KB7	49	507	6-10M	0.6	-3	44	50/200
		KB8	49	475	2-20M	0.6	-3	40	50/200

1 MPa = 145 psi

1mm = 0.0394 in

Table 2 Experimental And Theoretical Results – DB Tests

Specimen	DB1	DB2	DB4	DB3	DB6	DB5
Pair	DB-a		DB-b		DB-c	
Reinforcement type	10 - 10M	2 - 25M	10 - 10M	2 - 25M	6 - 10M	2 - 20M
N/M (1/m)	0	0	4	4	6	6
Experimental Results						
Cracking Moment I [kNm]	--	--	--	--	--	--
Cracking Moment II [kNm]	27	24	21	25	16	21
Crack spacing[mm]	150	220	200	230	220	220
Yield Moment[kNm]	72	64	52	45	43	38
Theoretical Results						
Crack.Moment[kNm]	20.14	20.14	17.96	17.96	16.64	16.64
Crack spacing (CEB)	146	261	148	267	148	270
Yield Moment [kNm]	73.63	71.84	48.77	47.47	41.6	40.5
Ratio: Experimental/Theoretical						
Cracking moment	--	--	--	--	--	--
Cracking moments II	1.34	1.2	1.17	1.4	0.96	1.26
Yield moments	0.97	0.89	1.07	0.95	1.03	0.94

1 mm - 0.039 in., 1kNm = 0.738ft.kipf.

Table 3 Experimental And Theoretical Results - KB Tests

Specimen	KB1	KB2	KB3	KB4	KB5	KB6	KB7	KB8
Pair	KB-a		KB-b		KB-c		KB-d	
Reinforcement type	10 - 10M	2 - 25M	10 - 10M	2 - 25M	6 - 10M	2 - 20M	6 - 10M	2 - 20M
N/M (1/m)	4	4	2	2	4	4	-3	-3
Experimental Results								
Cracking Moment I [kNm]	15	--	15	10	10	8	12	6
Cracking Moment II [kNm]	17	8.5	17.5	11	12.5	10.7	16	15
Crack spacing[mm]	142	189	127	160	154	260	144	185
Yield Moment[kNm]	68	65	76	75.6	36	32.5	62	58
Theoretical Results								
Crack.Moment[kNm]	7.28	7.28	8.45	8.45	6.98	6.98	8.74	8.74
Crack spacing (CEB)	109	230	109	230	189	287	183	277
Yield Moment [kNm]	69	67	80.7	78.3	39.7	37.2	76.2	71.5
Ratio: Experimental/Theoretical								
Cracking moment	2.06	--	1.78	1.18	1.43	1.15	1.37	0.69
Cracking moments II	2.33	1.17	2.07	1.30	1.79	1.53	1.83	1.72
Yield moments	0.98	1.01	0.94	0.97	0.91	0.87	0.81	0.81

1 mm - 0.039 in., 1kNm = 0.738ft.kipf.

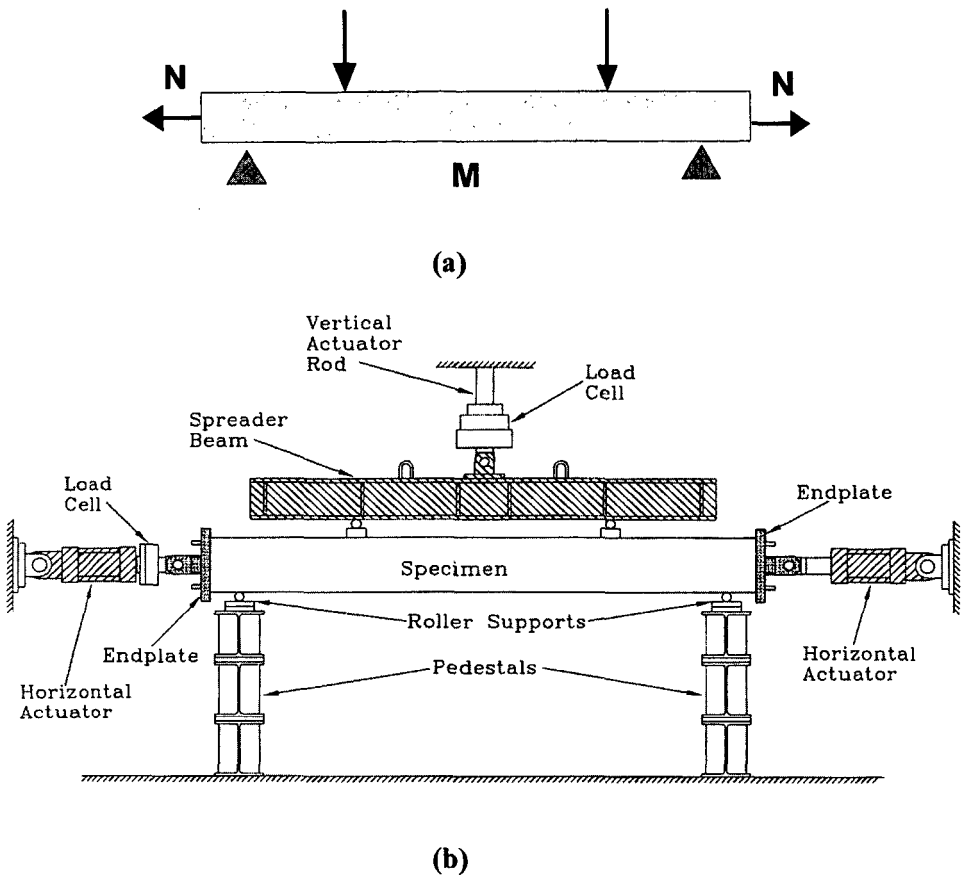


Figure 1: Specimens a) application of loads, b) experimental setup.

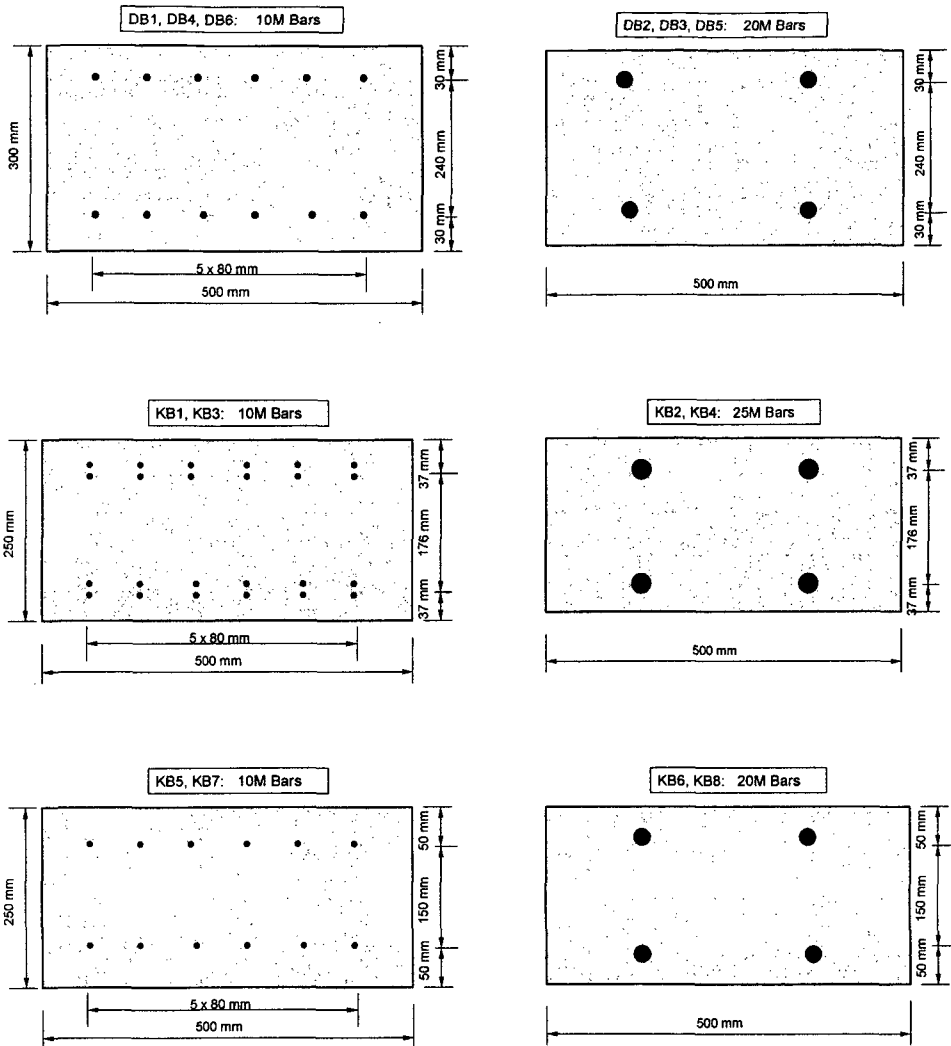


Figure 2: Cross sectional properties of the specimens in DB and KB series.

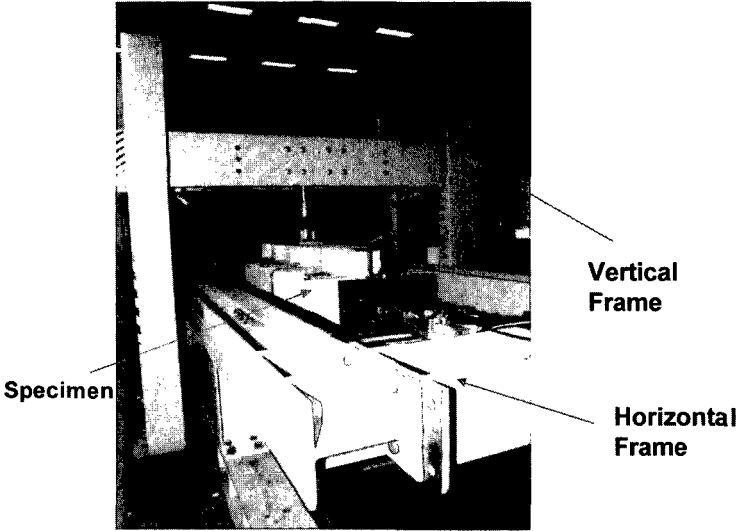


Figure 3: Testing frames.

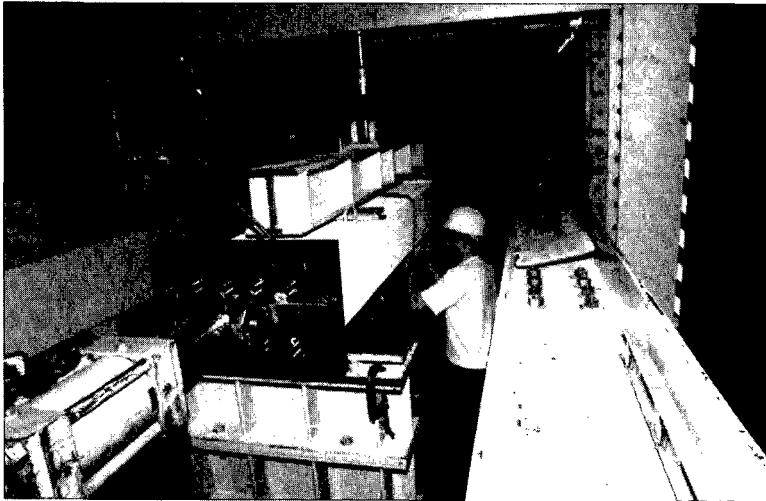


Figure 4: Specimen during testing; taking the surface deformation readings.

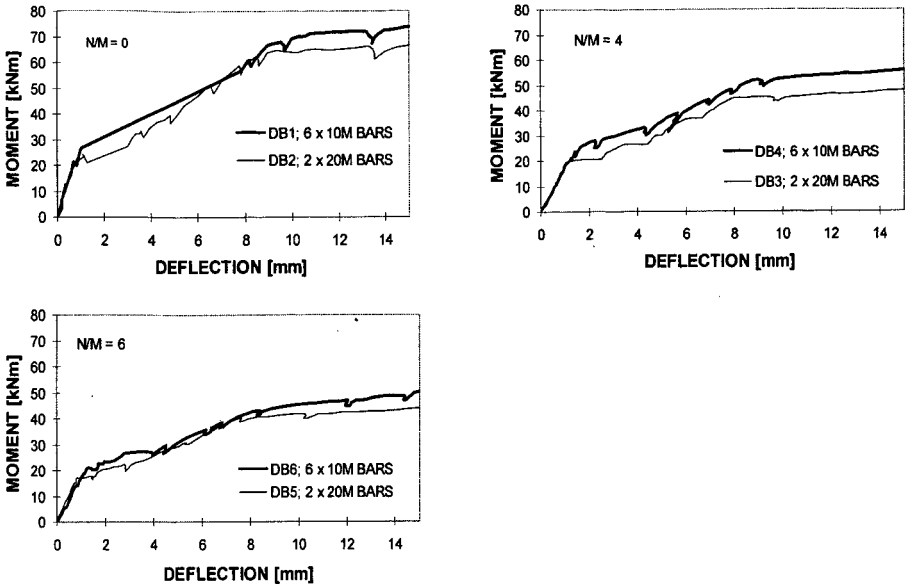


Figure 5: Moment-central deflection curves for DB series specimens.

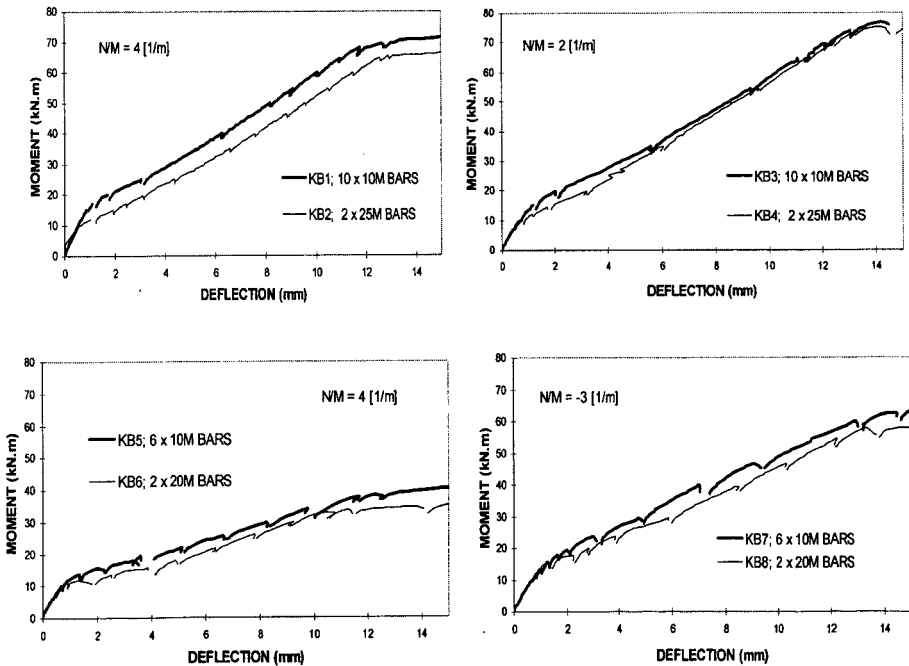


Figure 6: Moment-central deflection curves for KB series specimens.

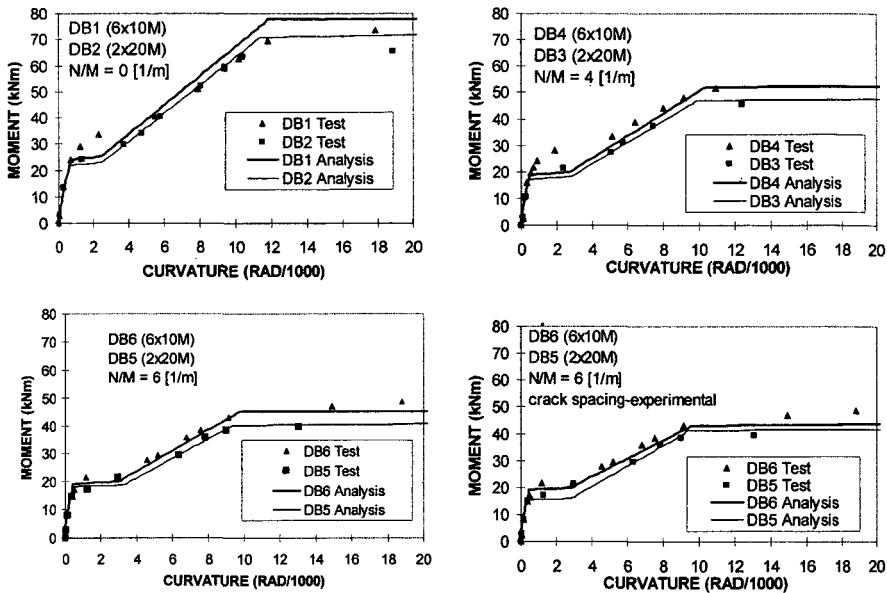


Figure 7: Moment curvature relationship for specimen in DB series: experimental results and theoretical predictions.

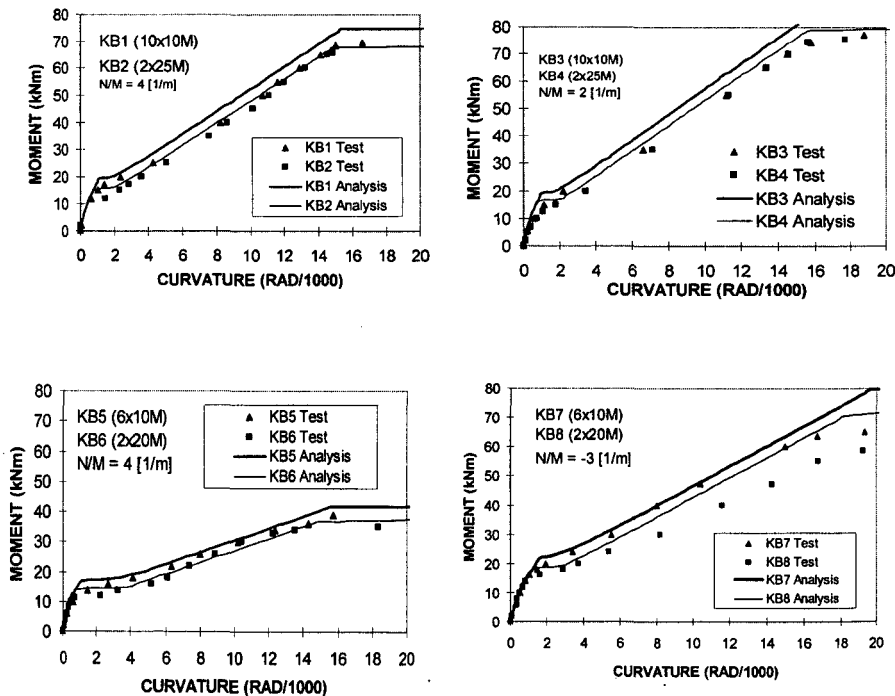
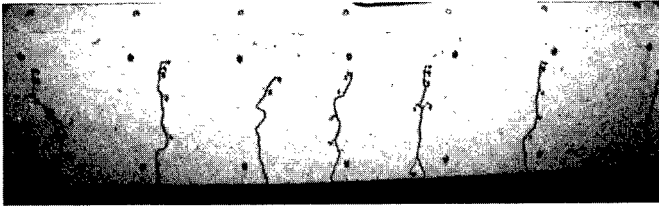
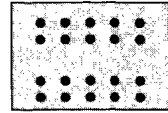


Figure 8: Moment curvature relationship for specimen in KB series: experimental results and theoretical predictions.



KB 3
(10M bars)



KB 4
(25M bars)

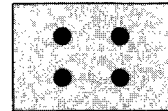
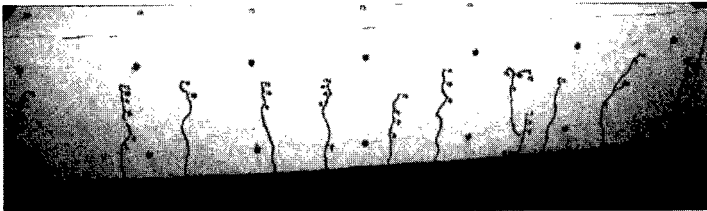
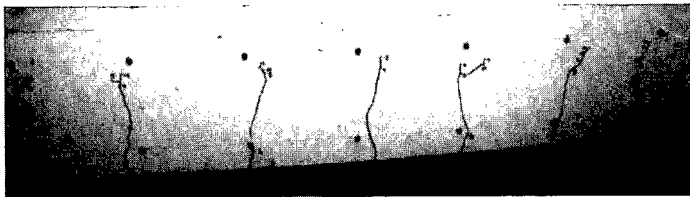
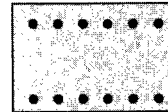


Figure 9: Crack spacing for specimens KB 3 and KB 4.
Moment = 65 kNm, axial force = 130 kN. $N/M = 2$



KB 7
(10M bars)



KB 8
(20M bars)

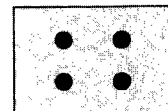


Figure 10: Crack spacing for specimens KB 7 and KB 8.
Moment = 47.3 kNm, axial force = -142 kN. $N/M = -3$

Deflections of Flat Plate Floors with Irregular Column Layout

by S. Teng and Tavio

Synopsis: A procedure for calculating the instantaneous deflections of irregular reinforced concrete flat plate floors is presented. The method is applicable throughout the entire loading stages from the uncracked state to the fully cracked state and even up to the ultimate limit state.

Based on the familiar effective moment of inertia approach, the proposed procedure also uses the ACI Direct Design Method with some generalization. The total static moment in a particular design strip will be distributed to the midspan and support sections according to the distribution coefficients of the ACI Direct Design Method, taking into account column sizes and irregularity of slab geometry around the edge slab-column connections. At high loads, the modulus of elasticity of the concrete can be reduced to better reflect the nonlinear behavior of concrete slabs. The definitions of effective span, width of design strips, torsional rigidity of transverse torsional members, and column sizes and orientations will be generalized as far as practical.

The accuracy of the proposed procedure has been compared with measured deflections of two multiple-panel irregular flat plate floors tested at Nanyang Technological University – Singapore. The comparison between the calculated deflections and the experimental values shows that the proposed procedure is reasonably accurate.

Keywords: column strip; concrete slabs; deflection; design strip; flat plates; floors; irregular column layout; middle strip

38 Teng and Tavio

ACI Member Susanto Teng is an Associate Professor in the School of Civil and Environmental Engineering, Nanyang Technological University (NTU), Singapore. He received his Ph.D. from the University of Iowa prior to joining NTU. His current research interests include behavior of flat plate floors, initial and time-dependent deflections, shear strength of deep beams, and structural connections.

Tavio is a Ph.D. candidate in the School of Civil and Environmental Engineering, Nanyang Technological University (NTU), Singapore. He is a lecturer in the Department of Civil Engineering, Sepuluh Nopember Institute of Technology (ITS) – Surabaya, Indonesia. His current research interests include analytical and numerical modeling of irregular flat plate structures.

INTRODUCTION

The flat plate floor system has been increasingly used for modern high-rise residential buildings due to its economic advantages and speed of construction. However, modern architectural and functional requirements dictate the use of an irregular column layout and elongated column cross-sections. The currently available methods for flat plate design are applicable only to flat plates with regular column layouts (rectangular column grids) and very little information is available on the design of irregular flat plate floors [1-3]. Naturally, typical design issues related to flat plate design, such as punching shear and deflection, become more complex in irregular flat plates. This paper will concentrate on the instantaneous or initial deflections of irregular flat plate floors and present a possible method for computing the deflections.

To calculate deflection, the amount of steel reinforcement should be known. Thus, the flexural design has to be performed first. A possible way for designing an irregular flat plate floor is to use the elastic finite element method or one of the simplified design approaches [1-4]. Whichever design method is used, it is necessary to identify the column strips as well as the middle strips in the floor slab (Fig. 1). The column strip will require more flexural reinforcement compared to the middle strip. In addition, the column strip would normally require negative (top) reinforcement. Due to the irregularity of the slab, it is also very likely for the main orthogonal flexural reinforcement to be directed at other directions rather than along the span direction [4]. Once the design strips have been chosen and the amount of reinforcement has been calculated, the midspan deflection of a particular span can then be computed. Deflection along a column strip can be computed directly by assuming that the column strip is a wide beam while the deflection in the middle of a panel or in the middle strip can be obtained by summing the average column strip deflections in one direction with the middle strip deflection in the other direction.

The proposed procedure is based on the idea introduced by Nilson and Walters [5] for flat plates with regular column layout (see also Ref. [6]), except that the ACI Direct Design Method is used rather than the Equivalent Frame Method.

The total static moment in a particular design strip will be calculated and distributed to the midspan and support sections according to the distribution coefficients of the ACI Direct Design Method, taking into account column sizes and irregularity of slab geometry around the edge slab-column connections. The ACI effective moment of inertia [7, 8] is adopted after cracking occurs. If desired, at higher load levels, the modulus of elasticity of the concrete can be reduced to better reflect the nonlinear behavior of concrete slabs. The definitions of effective span, width of design strips, torsional rigidity of transverse torsional members, and column sizes and orientations will be generalized as far as practical. Other details of the method that are not mentioned in this paper follow those of the ACI Direct Design Method.

RESEARCH SIGNIFICANCE

A relatively simple and effective method for computing the instantaneous or initial deflections of irregular flat plate floors is introduced. The method uses Branson's effective moment of inertia approach combined with the ACI Direct Design Method and is applicable to load cases well beyond the serviceability limit state. So far, very little research has been done on the behavior of irregular flat plate floors. So, the authors hope that this research can be useful to design engineers and researchers alike.

CLEAR SPAN AND COLUMN DIMENSIONS

Some of the important issues that arise in computing deflections of irregular flat plate floors include how to determine the clear span and the equivalent column cross-section. Guidance given in the ACI Code [7] for computing deflections is limited to floors with regular column layout. Thus, it cannot be applied directly to irregular flat plate floors shown in Fig. 1 or similar floor geometry. New and general definitions of the clear span and equivalent column cross section are needed.

A column can have an elongated cross section and it can also be oriented in any direction with respect to a particular span direction (Fig. 1). For these irregular cases, the following definitions of column dimensions and clear span are introduced. The longer dimension of a column is defined as b_1 and the shorter dimension as b_2 (Fig. 2). If the column is oriented at an angle α with respect to the span direction, modified column dimensions need to be used for analysis. The dimensions of the modified column cross section are c_1 along the span direction

40 Teng and Tavio

and c_2 in the transverse direction. The value of c_1' is obtained by multiplying the value of c_1'/b_1 from Eq. (1), to the longer column dimension b_1 while c_2' is obtained by dividing the shorter column dimension b_2 by the value of b_2/c_2' from Eq. (1). Both values c_1'/b_1 and b_2/c_2' can be obtained from the following expression (see Fig. 2):

$$\frac{c_1'}{b_1} = \frac{b_2}{c_2'} = \left(\frac{b_2}{b_1} \right)^{\sin^2 \alpha} \quad (1)$$

Note that Eq. (1) is essentially an interpolation function to convert the column cross section (oriented at an angle α from the span direction) to become a cross section of the same area but oriented along the span direction (see Fig. 2).

Fig. 3 shows a dimensionless chart that can be used to obtain the value of Eq. (1). Fig. 2 also shows the clear span l_n' , which can be determined by the following expression:

$$l_n' = l_1 - 0.5[c_{1(A)}' + c_{1(B)}'] \quad (2)$$

where l_1 is length of the span measured from the center of column A to the center of column B. Note that the modified column cross section has the same area as the original one. When applied to flat plate floors with a regular column layout, the new general definitions of the clear span and column dimensions along the span or transverse directions will naturally return to the current respective ACI definitions of l_n , c_1 and c_2 .

WIDTHS OF DESIGN STRIPS

Several ways are possible to define the transverse width of a design strip. The following definitions are essentially similar to those in various codes of practice, including the ACI Code [7]. The transverse width of the design strip, l_2 can be defined as follows (Fig. 4):

- the mid-span (average) width of the strip bounded laterally by the centerlines of adjacent panels on each side of the column line.
- for an edge or perimeter strip (or exterior strip), l_2 should be taken as the mid-span width between the edge of panel to panel centerline.

The definitions above are essentially generalization of the ones for flat plates with regular column layout. Thus, there is no difference between an interior

strip or an exterior or perimeter strip, except for the width of the design strip.

The width of the column strip can also be taken approximately as half the width of the design strip.

STIFFNESS OF TRANSVERSE TORSIONAL MEMBERS

The magnitude of the negative moment in the slab over a column support is influenced by a number of parameters, including the flexural stiffness of the supporting column. The effective flexural stiffness of the column at a slab-column connection is influenced by the torsional stiffness K_t [7, 9] of the transverse torsional member. The value of K_t is in turn influenced by the geometry of the slab around the connection. For irregular slab geometry, the ACI formula for K_t will have to be modified to take into account the irregular geometry, especially at edge slab-column connections. Fig. 5 shows a possible geometry of an irregular, edge slab-column connection. For this case, the current expression for K_t may result in a significantly higher value than anticipated because the torsional stiffness of the transverse torsional member may not contribute fully to the equivalent stiffness of the corresponding equivalent column. Therefore, a reduction factor α is introduced, depending of the slab geometry, to modify the torsional stiffness of the transverse torsional members.

Based on comparative studies using NTU experimental results (see Experimental Verification below), the coefficient α listed in Table 1 was found to be very reasonable as a reduction factor for K_t . The modified torsional stiffness of the transverse torsional members K'_t can then be computed from the following expression:

$$K'_t = \sum \alpha_i K_{ti} \quad (3)$$

where: $K_t = \frac{9E_c C}{l_2 (1 - c_2/l_2)^3}$; $C = \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3}$ (see Ref. [7, 9]); E_c is elastic

modulus of the concrete, c_2 is the width of equivalent rectangular support transverse to the span direction, x and y is shorter and longer dimensions of the transverse torsional member based on the modified column area.

DISTRIBUTION OF NEGATIVE AND POSITIVE MOMENTS

The basic ACI values of the negative and positive moments for an interior span are $0.65M_0$ for the negative moment over a support and $0.35M_0$ for the positive moment in the mid-span, where M_0 is the total static moment. These distributions of moments are maintained even for irregular flat plate floors.

42 Teng and Tavio

For an end span, the values of the moments are influenced by the relative flexural stiffness of the edge column, taking into account the contribution of the transverse torsional members. This can be handled by using ACI coefficient α_{ec} as shown by Eqs. (4) to (6). The use of these ACI 1977 equations [10] is preferable for irregular flat plate floors compared to the tabulated values in ACI 318-99 [7] because they allow the influence of the column stiffness to be included. The distribution of the end span moments is thus expressed as

$$M_{in} = M_0 \left[0.75 - \frac{0.10}{1 + (1/\alpha_{ec})} \right] \quad (4)$$

$$M_m = M_0 \left[0.63 - \frac{0.28}{1 + (1/\alpha_{ec})} \right] \quad (5)$$

$$M_{ex} = M_0 \left[\frac{0.65}{1 + (1/\alpha_{ec})} \right] \quad (6)$$

where M_{in} is negative interior support moment, M_m is midspan positive moment, M_{ex} is negative exterior support moment, and $\alpha_{ec} (= K_{ec}/K_s)$ is the equivalent relative stiffness of the exterior column. The equivalent column stiffness K_{ec} is governed by the following relationship [7, 9]

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \quad (7)$$

where K_t is the reduced torsional stiffness of the transverse torsional members; ΣK_c is the sum of the flexural stiffnesses of the original columns above and below the slab about an axis perpendicular to the span direction; and K_s is the flexural stiffness of the slab.

For a span bounded by two exterior supports, the distribution of the three moments over exterior support 1, midspan, and exterior support 2 can be obtained as follows

$$M_{ex1} = M_0 \left[\frac{0.65}{1 + (1/\alpha_{ec1})} \right] \quad (8)$$

$$M_m = M_0 - (M_{ex1} + M_{ex2})/2 \quad (9)$$

$$M_{ex2} = M_0 \left[\frac{0.65}{1 + (1/\alpha_{ex2})} \right] \quad (10)$$

MOMENTS IN PERIMETER STRIPS WITH AN IRREGULAR END SUPPORT

In an irregular flat plate floor, it is very likely that one end panel of a perimeter strip has an irregular end support (Fig. 6). When the angle θ is 0° , the support condition is commonly known as an edge (or corner) support. When θ is 90° , the support is an interior support. In an irregular flat plate floor, the end support condition can be irregular, that is, the angle θ can vary from 0° to 90° . This condition requires new distribution coefficients for moments in the end span of the perimeter strip.

A simple approach using an interpolation factor, α_{ir} , can work well and is introduced here. The minimum negative moment at a support occurs when the support is a corner (exterior) support where $\theta = 0^\circ$ in Fig. 6, while the maximum negative moment occurs at an interior support ($\theta = 90^\circ$). It can be expected that the moment over an irregular support should lie in between these two lower and upper limits. Thus, for a span with irregular end conditions, the distribution of the three moments over support 1, mid-span, and support 2 can be expressed by the following equations

$$M_{sp1} = M_{ex1} + \alpha_{ir} (M_{in1} - M_{ex1}) \quad (11)$$

$$M_m = M_0 - (M_{sp1} + M_{sp2})/2 \quad (12)$$

$$M_{sp2} = M_{ex2} + \alpha_{ir} (M_{in2} - M_{ex2}) \quad (13)$$

where the subscripts 1 and 2 refer to the first and second supports; M_{in} is the negative moment over a support assuming that the corresponding support is an interior support ($\theta = 90^\circ$); M_{ex} is the negative moment over a support assuming that the corresponding support is an exterior (edge) support with $\theta = 0^\circ$. The values of α_{ir} for various values of θ are listed in Table 2. The values of the moments obtained from Eqs. (11) to (13) compared satisfactorily with experimental results on irregular flat plate floors [4, 11], indicating that the values of α_{ir} as shown in Table 2 are reasonable.

DISTRIBUTION OF MOMENTS TO COLUMN AND MIDDLE STRIPS

The ACI Code distributes the negative and positive moments to the column strip and middle strip according to the condition of the support. Over an

44 Teng and Tavio

interior support ($\theta = 90^\circ$ in Fig. 6), 75% of the negative moment will be distributed to the column strip while the remaining 25% to the middle strip. Over an edge or exterior support ($\theta = 0^\circ$), 100% of the negative moment will have to be distributed to the column strip alone. Naturally, for an irregular support condition (θ between 0° and 90°), the percentage of negative moments to be distributed to the column strip should lie between 75% and 100%. The values in Table 3 are suggested and used here. The distribution of the positive span moment into the column and middle strips follows the ACI values of 60% and 40%, respectively. The distributed moments using coefficients in Table 3 had also been checked against experimental results [4, 11] and were found to be satisfactory.

EFFECTIVE MOMENT OF INERTIA

Before cracking, the moment of inertia of the slab section is essentially equal to its gross moment of inertia, I_g . After cracking, the moment of inertia reduces to its effective value, I_e . The ACI formula for the effective moment of inertia I_e as proposed by Branson [8, 12] is currently the most widely used and is also adopted in this analysis. The I_e formula for a simple flexural member is given below by

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \quad (14)$$

where M_{cr} is the cracking moment, given by $f_r I_g / y$, M_a is maximum applied moment in the span, I_{cr} is the fully cracked transformed moment of inertia, f_r is the modulus of rupture, and y is the centroidal distance to the outermost tension fiber.

Note that I_{cr} will have to be computed by taking into consideration the orientation of the slab reinforcement; that is, if the reinforcement is not directed along the span direction, the reinforcement area will have to be projected into the span direction first before the I_{cr} is computed. The I_e formula has also been implemented in several finite element formulations [13].

For computing deflection, the effective stiffness $E_c I_e$ is commonly used as one quantity even though it actually varies along the span. Thus, for flexural members with monolithic or continuous supports such as flat plate floors, an average effective stiffness can be used. One general formula for effective stiffness $E_c I_{e_avg}$ has been recommended by Branson [12] as shown in Eq. (15).

$$E_c I_{e_avg} = E_{cm} I_m \left[1 - \left(\frac{M_{e1} + M_{e2}}{2M_0} \right)^2 \right] + \frac{E_{c1} I_{e1} + E_{c2} I_{e2}}{2} \left(\frac{M_{e1} + M_{e2}}{2M_0} \right)^2 \quad (15)$$

where I_m is the effective moment of inertia at midspan; I_{e1} and I_{e2} are the effective

moments of inertia at ends 1 and 2 of the span, respectively; M_{e1} and M_{e2} are, respectively, the negative moments at ends 1 and 2 of the span. The effective moments of inertia are to be computed using Eq. (14) with M_a at the section considered. An effective modulus of rupture discussed below is to be used as well in computing the cracking moment. Eq. (15) has also been generalized here to include the possibility of using different modulus of elasticity at different section. This is to facilitate the use of reduced E_c or E_r for loading beyond the serviceability limit state (see Eq. (18)).

Once the I_{e_avg} has been computed, the magnitude of deflection in the middle of the span can be obtained by the elastic formula

$$\Delta_m = \frac{5L^2}{48E_c I_{e_avg}} \left(M_m + \frac{1}{10}(M_{e1} + M_{e2}) \right) \quad (16)$$

EFFECTIVE MODULUS OF RUPTURE OF CONCRETE

It has been known [14-16] that the cracking moment in slabs is affected or reduced by restraint stresses arising from restrained shrinkage, especially in those areas near large supports. A practical way to consider the reduced cracking moment is by reducing the modulus of rupture f_r into its effective value f_e .

Scanlon and Murray [15] suggested a useful formula for f_e as shown in Eq. (17). The value of f_e obtained from Eq. (17) will be used in the calculation of the cracking moment in this paper.

$$f_e = 0.33\sqrt{f'_c} \text{ (MPa)} \quad (17)$$

REDUCED MODULUS OF ELASTICITY OF CONCRETE

In computing deflection, the concrete modulus of elasticity is normally kept constant while the effective moment of inertia of the section reduces as the load increases. Using this combination, the effective moment of inertia approach has been shown [17] to work well for beams up to about 85% of the failure load. For concrete slabs, however, the load-deflection curves tend to bend relatively earlier than those of beams. This suggests that the concrete modulus of elasticity E_c may have to be reduced for higher loads.

As shown later in the comparative study, the deflection at the service load level up to about two times M_{cr} or 60% of ultimate load can be computed reasonably accurately using a constant E_c . When the deflection beyond the serviceability limit state is required, more accurate results can be obtained by reducing the E_c . In reinforced concrete slabs, the reduced E_c can be expected to