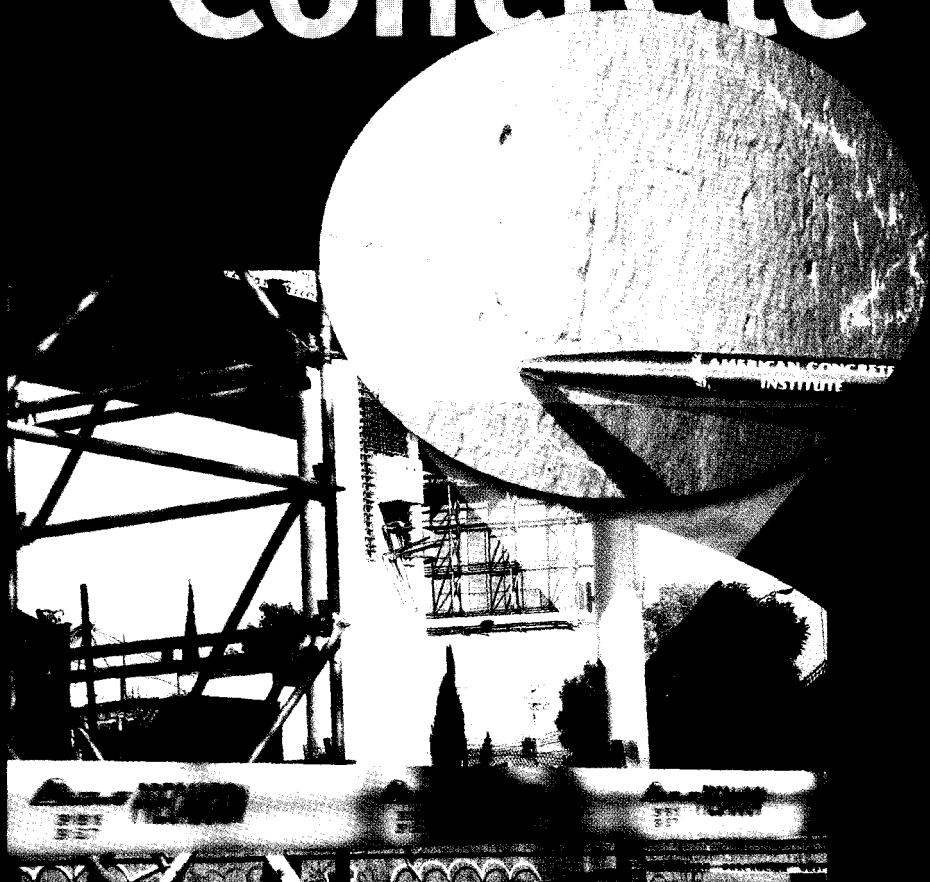


Shrinkage and Creep of Concrete



Co-Editors: N. J. Gardner and Jason Weiss

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Editors

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Jason Weiss



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SP-227

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Preface

Why should you be interested in the shrinkage and creep of concrete? To put this question into perspective, the load induced elastic (immediate) strains in concrete are of the order of 300 microstrain, depending upon stress level. The ultimate drying shrinkage can range from 400×10^{-6} to 900×10^{-6} . The creep strain can be 25% to 70% of the immediate strain after 24 hours and may be several multiples of the immediate strain after several years, depending on the relative humidity. If concrete is restrained against shrinkage it can crack.

Structural engineers are concerned with the consequences of shrinkage, creep and cracking on the serviceability and durability of their structures. Creep increases deflections, reduces prestress in prestressed concrete elements, and causes redistribution of internal force resultants in redundant structures. Shrinkage can cause warping of slabs on grade due to differential drying and increased deflections of non-symmetrically reinforced concrete elements. Materials scientists are concerned with understanding the basic phenomena and assessing new materials and the effects of admixtures on the mechanical behavior of concrete.

Concrete is an age stiffening material that has little tensile strength, shrinks, and exhibits creep in sealed conditions and additional drying creep in drying environments. Predicting the amount of shrinkage and deflection that may occur is not easy and is especially complicated in concrete that contains supplementary materials, chemical admixtures, and lightweight aggregates. Supplementary cementing materials and waste products are being used in increasing volumes in response to environmental concerns. Admixtures have been developed to modify the behavior of fresh and hardened concrete. Self consolidating concrete is being used in more applications. A recent development is the marketing of shrinkage reducing admixtures.

This volume contains papers presented during four sessions sponsored by ACI Committee 209, Creep and Shrinkage in Concrete, and ACI Committee 231, Properties of Concrete at Early Ages, held at the ACI Spring 2005 Convention. The subjects addressed by the authors are diverse and cover many aspects of shrinkage and creep. Some papers pay special attention to the development, use, and evaluation of models to predict shrinkage, creep, and deflection, while other papers consider the behavior of early age concretes that are restrained from shrinking, resulting in the development of residual stresses and cracking.

Papers are presented to evaluate models for predicting shrinkage and creep, which is especially crucial as ACI Committee 209 attempts to refine the models that will be presented in the *ACI Manual of Concrete Practice*. Several papers provide a field assessment of creep and shrinkage in concrete structures, thereby providing an explanation of how field data should be collected and how this data can be used to refine predictive models. Other papers deal with an improved understanding of early-age tensile stress development, including demonstrating the measurement of creep and

stress development. One paper presents experimental results to quantify differential drying and thermal deflections in slabs. This volume also recognizes that modern concretes are now no longer a simple mixture of water, cement, and aggregate. This volume discusses specific aspects of how the shrinkage of these modern concretes may differ from concrete made using conventional materials. Specifically, several papers deal with the shrinkage and curing of concrete containing supplementary cementing materials, which is especially important as the use of these materials is rapidly rising. The shrinkage and creep behaviour of high-strength, lightweight concrete is also evaluated and compared to existing models. One paper assesses the cracking behavior of self-consolidating concrete, a rapidly emerging material that is often suspected to be sensitive to shrinkage related concerns due to the higher paste volume that may exist in these mixtures. It is shown that self-consolidating concretes can be made to be resistant to cracking. Finally, this volume describes a study that is focused on the development of an approach to describe the shrinkage and early age stress development in concrete containing shrinkage reducing admixtures.

This volume includes significant contributions by many leading research and practicing engineers in the field of shrinkage and creep. The descriptions of field problems and the assessment of the accuracy of predictive models may assist practicing engineers. For researchers, the contents of this volume should be useful in the development and evaluation of future models and laboratory measurements. Finally, it should be noted that many people have contributed to the successful development of these proceedings, and the editors, and authors, thank all of the anonymous reviewers who assisted in reviewing the papers.

Co-Editors

N.J. Gardner
Jason Weiss

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Managing Deflection, Shortening and Cracking Arising from Restrained Contraction

by S. J. Alexander

Synopsis: Concrete shrinks. Steel doesn't, and the resistance of reinforcement to shrinkage causes deflection of slabs and beams and shortening of columns and walls. A simple visualization is given, and used to derive formulae for analysis. Current methods of calculating shrinkage curvature and deflection in reinforced sections are examined and compared, concluding that the ACI method appears realistic while the UK and European methods significantly over-estimate the deflection.

Restraint by differential contraction between an insitu concrete overlay and an older substrate produces tension in the overlay and curvature and deflection of the composite unit. A method for calculating this is given, and the resulting effects are found to be significant in certain circumstances. The method is extended to consider shrinkage in insitu slabs in steel-concrete composite construction. The deflection from shrinkage is found to be approaching $\text{span}/750$, and cracking in the slabs is predicted in some cases.

External restraint to contraction induces tensile stresses, and a rational approach to providing sufficient reinforcement to control cracking in direct tension is given. It is particularly relevant to elements which need to be water-resisting, and a case study of a basement is presented. The reinforcement needed to control cracking reliably is found to exceed most current US, UK and European recommendations.

Keywords: composite beams; cracking; creep; deflection; design; early age contractions; overlays; reinforced concrete; restraint; serviceability; shortening; shrinkage; structural analysis

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INTRODUCTION

Concrete contracts and shrinks. The main effects to be considered in design are early age contraction, temperature drop and long-term drying shrinkage. Restraint arises in a number of ways. Embedded reinforcement causes deflection of slabs and beams and contributes to shortening of columns and walls. Casting concrete against a previous pour or as an overlay on top of an older substrate induces tensile stresses that can cause cracking. So do metal decking and supporting beams in steel-concrete composite construction. Similarly, external restraint from rigid elements such as in-plane walls, piles and pile caps, and even friction from the underlying soil, can cause cracking. Shrinkage is also significant in concrete roads and industrial ground floors, but the paper is limited to typical structural elements in buildings.

The paper considers how to quantify these effects, and makes recommendations for managing them.

LIFE CYCLE OF CONCRETE

Concrete typically goes through the following life cycle:

Early age contractions. In the first three to six days, the hydration process first causes the concrete to heat up – from a casting temperature that is usually already a few degrees above ambient. During this period, it behaves in a plastic manner so the only result is an increase in volume. However, when it cools to ambient temperature (Figure 1) it has hardened enough to go into tension and may crack if it is restrained. At the same time, shrinkage (a complex combination of actions, see Altoubat and Lange (1)) also occurs, adding to the contractions.

Strength gain. In the following three to four weeks (longer in mixtures with flyash or slag), both compressive and tensile strength increase.

Temperature. In the first year, it will be subjected to the seasonal temperature and humidity cycle if it is exposed, or to indoor conditions if it is enclosed. Daily variations will also occur.

Shrinkage. Over a longer period, perhaps two to ten years, long-term drying shrinkage will gradually occur.

It is important to recognise the cumulative effect of all the contractions, that is early age contractions (partially relieved by creep, see below) plus temperature drop plus shrinkage, if they are restrained. Thus only a small temperature drop or amount of shrinkage can be enough to initiate a crack in concrete with locked-in tensile stress from early age contraction. However, note that concrete and steel can usually be assumed to have the same coefficient of thermal expansion, so that embedded reinforcement does not

form a restraint to thermal movements. Also, if the only restraint present is that to early age contraction from a recent adjacent pour, shrinkage and temperature drop can usually be assumed to apply equally to the whole structure and any long-term differential therefore ignored.

Alleviation by creep

Some alleviation of sustained tensile stresses, that is those arising from early age contractions and long-term shrinkage but only seasonal temperature, is provided by creep. Altoubat and Lange found that early age contractions were reduced very rapidly by as much as 50 %. Interestingly, the same reduction is included in BS 8007 (2) where it is explained as a 'restraint factor'.

European guidance on creep and shrinkage is published in BS 5400-4 (3), but appears to be largely additional to early age effects. Figure 2 is derived from BS 5400-4, and shows the reduction over time of both early thermal contraction and shrinkage. Early thermal contraction stresses are reduced quite rapidly. The figure is based on fairly conservative assumptions (loading at 3.5 days, section 300 mm [12 in] thick exposed on one side only, r_h 80 %), yet the reduction is estimated to be 30 % at four months and 60 % at two years. Long-term shrinkage is different. Although each increment is lessened by creep, the shrinkage builds up quite slowly so the effect of creep is even more delayed. The figure shows that the effect is hardly significant up to one year, although ultimately the reduction is about one-third.

Note that these reductions apply to the tensile stress in the concrete. Thus if it is cracked the initial stress will be the stress at which it cracked and the reduction should be applied to that stress.

RESTRAINT FROM EMBEDDED REINFORCEMENT

The easiest way to understand the restraining effect of reinforcement is to envisage a unit length taken from a member. Imagine firstly that the reinforcement is disconnected from the concrete, for example by being greased. When shrinkage takes place, the ends of the reinforcement will be left projecting from the face. The reinforcement is then pushed back into the concrete and at the same time the concrete is pulled out so that the surface is flush once again (Figure 3).

Symmetrically reinforced sections

If the reinforcement is symmetrical and the compressive strain in it is ϵ_s , the total force in the reinforcement is $\epsilon_s E_s A_s$. The tensile strain in the concrete is $\epsilon_{cs} - \epsilon_s$ where ϵ_{cs} is the free (ie unrestrained) shrinkage. A straightforward method of estimating ϵ_{cs} taking relevant parameters into account is given in ACI 209R-92 (4) and is similar in principle to the European method in BS 5400-4. The tensile force in the concrete is $(\epsilon_{cs} - \epsilon_s) E_c A_c$. Equating these two forces gives the restrained shrinkage as

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$$\varepsilon_{cr} = \varepsilon_s = \varepsilon_{cs} / (1 + m \rho_n) \quad (1)$$

where m is the modular ratio ($= E_s / E_c$) and ρ_n is the ratio of reinforcement area to concrete area.

Each increment of shrinkage occurs at a different age before being subsequently relieved by creep, so it is not immediately obvious how to evaluate m . A simple spreadsheet analysis by the author has shown that the 30-years effect is given quite accurately by imposing the total shrinkage in one step at an age of 150 days. For a typical structure indoors ($rh = 45\%$, $f_{cu} = 28/35$ MPa [4000 psi], $h_c = 250$ mm [10 in]), this gives the creep coefficient $\nu = 1.6$, whence $E_{eff} = 11.5$ GPa [1670 ksi] and $m = 17.5$.

Shrinkage deflection in reinforced members

If the section is reinforced asymmetrically – as is usual in a beam or slab – the restraint provided by the reinforcement is lopsided. The shortening described above for symmetrical sections takes place, but in addition the asymmetry produces curvature, which leads to deflection.

Once again, the strain ε_s in the reinforcement is used to define the force, and the stress in the concrete is calculated by the conventional formula $F/A + My/I$. The resulting strain at the level of the reinforcement is equated to $\varepsilon_{cs} - \varepsilon_s$, giving the uncracked curvature as:

$$\kappa_{un} = \varepsilon_{cs} m S_s / (I_g + m \rho_n I_c) \quad (2)$$

where S_s is the net first moment of area of the reinforcement about the neutral axis of the concrete section (ie deducting S of any compression reinforcement), I_c is the second moment of area of the concrete section, and I_g is the second moment of area of the gross section, ie including the reinforcement multiplied by $(m-1)$.

This differs from the British code of practice BS 8110-2 (5) in that the second term in parentheses is excluded and the points about calculating ν at 150 days and deducting the S of compression reinforcement are not made. More importantly, both BS 8110-2 and the forthcoming Eurocode EN 1992-1-1 (6) state that the cracked section properties should be used if the section is cracked under load. This needs further consideration.

At a crack, the concrete below the neutral axis cannot transmit shrinkage strain, and it can be shown that the fully cracked curvature is

$$\kappa_{cr} = \varepsilon_{cs} / (d - c/2) \quad (3)$$

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Compression reinforcement can be allowed for by reducing ϵ_{cs} by $1 + m \rho'$ where ρ' is $A_s' / b c$ and m is determined as above. The actual behaviour is presumably an alternation between cracked and uncracked zones as shown in Figure 4.

The method recommended in the US is equation 2.26 in ACI 435R-95 (7):

$$\delta_{sh} = k_{sh} (A_{sh} \epsilon_{sh} / h) L^2 \quad (4)$$

where $A_{sh} = 0.7 \rho^{1/3}$ for $\rho' = 0$ (modified for $\rho' > 0$). The curvature is the term in parentheses.

In order to compare the shrinkage curvatures obtained by different methods, they have all been reduced to the form

$$\kappa = k_p \epsilon_{cs} / d \quad (5)$$

and plotted in Figure 5 against values of $\rho_d = 100 A_s / b d$. Where a method did not lend itself to this formula directly, values were derived from the model of a 300 mm (12 in) thick solid slab, taking $d = 0.9 h$ and $m = 17.5$. The tension reinforcement is assumed to be the amount required for ultimate load, ie stressed to 300 MPa [105 ksi] under service load.

It can be seen that the ACI method lies 25-30 % of the way between k_{un} and k_{cr} while the BS and EN cracked methods lie 75-80 % of the way between k_{un} and k_{cr} , although this would effectively reduce to 65-70 % when used with the incorrect deflection coefficient 0.104 (see below). Examination of figure 4 suggests that the ACI method is realistic, and that the UK and EN methods significantly over-estimate shrinkage deflection.

Compression reinforcement A_s' has been taken as zero, but similar graphs can be produced incorporating different values of A_s' . Compression reinforcement is very effective in reducing shrinkage curvature. By ACI 435R-95, providing 0.25 % additional compression reinforcement in a section where $A_{s,req} = 0.75$ % reduces the curvature by 29 % while adding it to the tension reinforcement increases it by 10 %.

The 'locked-in' tensile stress at the extreme fibre in the uncracked zone from restrained shrinkage is

$$f_{ct} = \kappa_{un} E_{eff} (h - c) \quad (6)$$

using the same E_{eff} as used to calculate κ . This stress can be significant after some time and appears to be overlooked in codes of practice when ascertaining whether the section is cracked under load.

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Estimating shrinkage curvatures at intermediate ages is not easy. It is possible to set up a spreadsheet that divides the shrinkage into increments which are summed to give the total curvature at the point in time required. However, shrinkage curvature is only a small contribution to the total curvature and it will often be acceptable to make an educated guess.

Deflection is obtained from curvature by the relationship

$$\delta = K \kappa L^2 \quad (7)$$

The coefficient K for shrinkage is 0.125 for a span, 0.5 for a cantilever. If the curvature reverses over one or both supports, the coefficient is reduced by the multiplier $1 - \beta/10$ where β is the ratio $(\kappa_A + \kappa_B) / \kappa_C$ in which κ_A , κ_B and κ_C are the curvatures at left support, right support and mid-span respectively.

Confusion arises as both BS 8110-2 and EN 1992-1-1 (but not ACI) state that the shrinkage curvature should be added to the curvature from load. This means that the coefficient K for a simply supported span is taken as 0.104, not 0.125. For a cantilever the difference is even more marked, correctly 0.5 not 0.25. In this paper it is assumed that the various methods are all aiming to give the actual shrinkage curvature; incorporating this curvature into the load calculation will therefore reduce the shrinkage deflection by around 17 % in a span and by 50 % in a cantilever.

Shrinkage deflection calculated by the ACI method is typically $L/1250$ or less, supporting the approach in ACI-318 (8) of including it in the additional deflection from creep.

CONTRACTION OF INSITU CONCRETE OVERLAYS

Casting an insitu concrete overlay onto a precast concrete substrate or base unit is a common way of forming a homogeneous slab. However, the differential between the contraction of the overlay and the lesser or even zero contraction of the substrate produces internal stresses which lead to shortening and deflection of the composite unit, and sometimes to cracking in the overlay. The same theory can be applied when the base unit is prestressed or when it is a steel section (see below).

Consider an overlay cast onto a base section, and firstly allow the contraction of the overlay to take place freely, as if the interface is greased. Then apply a tensile force to the overlay with an equal and opposite compressive force on the base so that the relative movement at the interface is eliminated (Figure 5). The force acts at an eccentricity e to the centroid of the overlay to equalize the curvature in the overlay with that in the base.

By defining the strain in the overlay as ε_r , the force $F = \varepsilon_r A_a E_a$. Applying the equal and opposite force to the base produces a strain at the level of the centroid of the overlay

$$\varepsilon_n - \varepsilon_r = F [1/E_b A_b + z(z-e)/E_b I_b] \quad (8)$$

Eliminating ε_i gives

$$F = \varepsilon_n / [1/E_a A_a + 1/E_b A_b + z(z-e)/E_b I_b] = \varepsilon_n E_b I_b / [E_b I_b/E_a A_a + I_b/A_b + z(z-e)] \quad (9)$$

In this, suffixes a and b refer to the overlay and the base respectively. A and I are derived from the concrete section including the reinforcement multiplied by the modular ratio m . Reinforced concrete is generally assumed cracked below the neutral axis, prestressed sections uncracked throughout. z is the lever arm between the centroid of the overlay and the centroid of the base. The duration is accounted for by using creep-modified values of E .

The eccentricity e can be found by equating the curvatures ($= M/EI$) of the overlay and base. This gives

$$F e / E_a I_a = F (z-e) / E_b I_b \quad (10)$$

whence

$$e = z / (1 + E_b I_b / E_a I_a) \quad (11)$$

$$z-e = z / (1 + E_a I_a / E_b I_b) \quad (12)$$

ε_n is the net contraction $= \varepsilon_a - \varepsilon_b$, ie the difference in contraction between the overlay and the base in the time period in question (see below). In many cases ε_b can be taken as zero, including all concrete (including prestressed) over about one year old. Conversely, if the base is relatively young and the overlay thin (so that early thermal contraction is minimal), ε_a and ε_b will rapidly converge so that ε_n can be taken as zero, ie the differential contraction is not significant.

The expression for F can be clarified by writing $E_b I_b / E_a A_a = m_c I_b / A_a = q^2$ and $I_b / A_b = r^2$; q is a measure of the influence of the overlay on the base and r is the radius of gyration of the base. Then

$$F = \varepsilon_n E_b I_b / [q^2 + r^2 + z (z-e)] \quad (13)$$

The force F is applied as an external force to the base section (not to the composite section), enabling any property to be calculated; most important will usually be the curvature from which the deflection is then calculated. The tensile stresses induced may also be significant.

The method can be used over any period of time. For instance, the effects of early age contraction are virtually immediate, so that short-term values would be used to give the immediate deflection. Long-term values can be used to give the long-term effect,

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although splitting the behaviour into steps with a spreadsheet will be more accurate as F will decline with time. The same spreadsheet approach may be necessary to find the point at which ϵ_b overtakes ϵ_a if the base is prestressed.

For thin overlays in which cracking would be visible and undesirable, it could be controlled with reinforcement. Where the substrate is less than about 4-6 months old and is expected to be above 15 C for the first month, it will probably be enough to provide ρ_{imm} . In other cases, it will be advisable to provide ρ_{mat} (ρ_{imm} and ρ_{mat} are defined below).

CONTRACTION OF INSITU SLABS IN COMPOSITE CONSTRUCTION

In composite construction, contraction of the insitu concrete slab can be significant. The theory for insitu overlays in the previous section applies, with the simplification that as the slab is usually thin relative to the steel beam the eccentricity e can be ignored. The average slab thickness should be used, not the minimum.

Some assumptions need to be clarified. The first is that because the metal decking is bonded continuously to the concrete, it can be treated in the same way as embedded reinforcement. And because the slab is attached to the steel beam which forces it to contract linearly it does not matter if the decking - or the reinforcement - is not concentric in the section. For typical slab profiles with steel decking 0.9-1.2 mm [0.035-0.047 in] thick and light fabric reinforcement, ρ_n ranges from 1.0 to 1.4 %, ie higher than is normal in conventional reinforced concrete slabs. Using the typical value of the modular ratio m of 17.5 shows that the effect of the restraint is a factor of 0.80-0.85.

The force F is applied as an external force to the steel beam (not to the composite section) enabling the curvature and thence the deflection to be calculated. The curvature κ is given by M/EI , so here

$$\kappa = F z / E_b I_b = \epsilon_{\text{cr}} z / (q^2 + r^2 + z^2) \quad (14)$$

The principal current UK guide is the code of practice for design of composite beams BS 5950-3-1 (9). However, shrinkage of the concrete slab is not mentioned as a contribution to deflection. BS 5950-3-1 will be supplanted by EN 1994-1-1 (10), currently available in draft. This states that "calculation of stresses and deformations at the serviceability limit state shall take into account the effects of [*inter alia*] creep and shrinkage of concrete", and further that "the effect of curvature due to shrinkage of concrete should be included when the ratio of span to overall depth of the beam exceeds 20 and the predicted free shrinkage strain of the concrete exceeds 400×10^{-6} " (presumably not when below these limits).

The author has carried out calculations for typical arrangements which show that the shrinkage deflection is significant but that there is relatively little variation between widely different steel beams; all approach span/750. A simple rule would therefore be to

assume shrinkage deflection is equal to span/750 unless it is estimated by a more accurate calculation – in spite of silence in BS 5950-3-1 and the let-off in EN 1994-1-1.

Cracking in composite concrete slabs

Tension is induced in the slab in three ways. First is the internal restraint of the reinforcement and the metal deck. Taking typical values of $\epsilon_{cs} = 400 \mu\epsilon$, $E_b = 200 \text{ GPa}$ (29 000 ksi) and $m = 17.5$ gives $f_{ct1} = 0.7 \text{ MPa}$ (100 psi) for $\rho = 1.0 \%$ and 0.9 MPa (130 psi) for $\rho = 1.4 \%$.

Second is the restraint of the steel beam itself. This produces a tensile stress

$$f_{ct2} = \epsilon_{cr} E_a / [1 + (r^2 + z^2) / q^2] \quad (15)$$

The term $(r^2 + z^2) / q^2$ varies from about 3 for a heavy beam to as much as 15 for a light one. Taking typical values of $\epsilon_{cr} = 325 \mu\epsilon$ and $E_b = 11 \text{ GPa}$ (1670 ksi) gives $f_{ct2} = 0.25\text{-}0.9 \text{ MPa}$ (35-130 psi).

Third is the restraint of the surrounding structure. This will usually be at least 0.25 MPa (35 psi).

This suggests that the resulting tensile stress will be at least 1.2 MPa (175 psi) and could be over 2.0 MPa (290 psi). The tensile strength of concrete under sustained loading is probably in the range $1.5\text{-}2.1 \text{ MPa}$ (220-300 psi) for the grades of concrete normally used in composite construction. It is clear that the possibility of the concrete cracking is very real and should be considered, particularly where composite slabs are to be left exposed to view. This could be controlled by providing a reinforcement content of ρ_{mat} . It might be thought that the metal deck would perform this function, at least at right angles to the corrugations, but evidence from sites suggests otherwise.

EXTERNAL RESTRAINT

This section uses a rational approach to give guidance on providing sufficient reinforcement to control cracking in direct tension. It is particularly relevant to walls and ground slabs in basements. These are also structures which often need to be water-resisting. It is therefore on the safe side to assume they will be restrained and determine the reinforcement on this basis.

Controlled cracks

If increasing axial tension is applied to a section it will eventually crack across its full depth and the force will be transferred to the reinforcement. If the tensile force (or the strain) continues to increase, provided the reinforcement bridging the crack is stronger than the uncracked section a new crack will form in a different place. And so on. This is controlled cracking. However, if the reinforcement

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is not strong enough it will yield, and the first crack will be uncontrolled and will just get wider.

Design criteria for controlling cracking

Before showing how this understanding can be used to determine the amount of reinforcement to be provided, it is important to consider the criteria to be applied to cracking in these circumstances. A crack which is not controlled has an unlimited width, and will therefore be unsightly at best and will leak if water is present. The target should therefore be that uncontrolled cracks are unlikely to occur. A reasonable interpretation of this would be to limit the probability of an uncontrolled crack occurring to 5 % or 1 in 20. This would require the upper characteristic axial tensile strength $f_{ctm,0.95}$ of concrete to be used.

In order to maintain the 5 % probability, $f_{ctm,0.95}$ should be used with the mean strength of the reinforcement. US deformed high yield steel currently has design strength 414 MPa (60 000 psi) and mean yield strength 490 MPa (71 000 psi). When EN 1992-1-1 comes into use, the design strength is expected to be 435 MPa (63 000 psi) and the mean to be at least 550 MPa (80 000 psi).

Minimum reinforcement in direct tension

The role of reinforcement in controlling cracking can be derived from first principles by making the strength of the reinforcement greater than the strength of the concrete in tension. Applying this principle, the reinforced axial capacity is:

$$F_s = A_s f_s \quad (16)$$

and the uncracked capacity is:

$$F_c = f_{ct} A_c + (m - 1) A_s \quad (17)$$

The second term in (17) is quite small and is neglected at this point, although a correction for it is applied later. Putting $F_s \geq F_c$ gives:

$$\rho \geq 1.0 (f_{ct} / f_s) \quad (18)$$

Two cases need to be considered, early age contractions in immature concrete and total long-term contraction in mature concrete.

Early age contractions

If the contractions are restrained (eg by pouring against an earlier pour), prediction of the contractions is so unreliable that it is safer to assume that cracking will occur and to provide sufficient reinforcement to control it.

BS 8007 derives a similar expression to (18) for the minimum reinforcement:

$$\rho_{\text{imm}} = f_{\text{ct}} / f_y \quad (19)$$

where f_{ct} is the direct tensile strength of the immature concrete, usually taken at the age of three days as 1.6 MPa (230 psi) for grade 28/35 (4000 psi) concrete. Hughes (11) explains that f_{ct} includes a hidden partial safety factor for materials of 1.1, and quotes a more general value $f_{\text{ct}} = 0.12 (f_{\text{cu}})^{0.7}$, where f_{cu} is the 28-day cube strength. With $f_y = 460$ MPa (66 700 psi), $\rho_{\text{imm}} = 1.6 / 460 = 0.35 \%$, the figure which has been familiar to British engineers for some years.

Three factors need to be taken into account at this point. First is the increase in F_c from the second term in (17); with the minimum percentages recommended here, this requires a multiplier of around 1.06. The other two are the variation in tensile strength at a given compressive strength and the variation of compressive strength. EN 1992-1-1 quotes the upper characteristic tensile strength of concrete as 30 % greater than the mean f_{ctm} . The average concrete strength is usually taken as 8/10 MPa higher than the specified minimum, and the variation is typically 20 %. Combining these two variations by the root-sum-of-squares rule gives a multiplier of 1.36. Multiplying this by 1.06 gives 1.44. Combined with the US ratio of design to mean for reinforcement of 414/490 explained above, this results in:

$$\rho_{\text{imm}} = 1.22 (f_{\text{ct,imm}} / f_{\text{sd}}) \quad (20)$$

where $f_{\text{ct,imm}} = 0.12 (f_{\text{cu}} + 10)^{0.7}$. For use with EN 1992-1-1, 1.22 is replaced by 1.14.

Values of $f_{\text{ct,imm}}$ and ρ_{imm} for a range of concrete strengths are shown in Table 1. It can be seen that using either ACI or EN requires more than 0.35 % reinforcement for all likely concrete grades.

Mature concrete

BS 8007 and ACI 350 (12) cannot be relied on for designing basements to be watertight. They are written for reservoirs, ie concrete tanks which in service will be full of water and usually embedded in soil, often with a covering of soil on the roof. In these conditions, shrinkage is minimal and temperature variations small. BS 8007 assumes that if early age contractions are controlled, subsequent movements will be insignificant, or at least less than the reduction from creep (although this is not stated).

In basements, the internal faces are open to atmosphere and therefore temperature movements and shrinkage will occur. The worst case is basements used for car parking, especially if they are naturally ventilated, as the concrete is then exposed to near-ambient temperature and humidity all year round. These circumstances are hinted at in CIRIA Report 139 (13), but a full explanation does not appear to be given in any UK or US industry-standard document.

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Under sustained loading, the tensile strength of concrete is understood to reduce by around 0.7. The dominant cause of full-depth cracking is sustained contraction – long-term shrinkage and seasonal temperature drop added to early age contractions - although more rapid temperature drops frequently contribute. Allocating these 60–40 suggests that a reduction in f_{ctm} for long-term effects of 0.82 can be applied. This gives:

$$\rho_{mat} \geq 1.00 f_{ctm} / f_{sd} \text{ for ACI} \quad (21)$$

$$\rho_{mat} \geq 0.93 f_{ctm} / f_{sd} \text{ for EN} \quad (22)$$

This results in the minimum percentages in the sixth and seventh rows of Table 1, ie 0.67 % rising to 0.80 % for concrete grades of 28/35 and 35/45 MPa (4000 and 5100 psi) respectively. These can be compared to the minimum in ACI 350 of 0.50 % for lengths over 12 m [40 ft] between joints or without joints. However, ACI 224R-01 (14) (clause 3.5.2) states “To control cracks to a more acceptable level, the percentage requirement needs to exceed about 0.60 %”.

The actual formula in EN 1992-1-1 is

$$\rho_{mat} \geq f_{ctm} / f_{yk} \quad (23)$$

in which f_{yk} is the characteristic (ie 95 % probable) strength of the reinforcement, = 500 MPa (72 500 psi). No derivation is given, but it results in values only marginally lower than the author's.

Modern concretes often contain substantial proportions of flyash or slag, which slow down the rate of gain of strength. In immature concrete this means that the assumption of cracking may be overly pessimistic (although still safe). If reinforcement is provided to control cracking, the lower tensile strength may permit the amount to be reduced. In mature concrete, the problem is that the concrete may gain strength considerably after 28 days. Equation (21) should therefore be used with the value of f_{ctm} appropriate to the expected final strength.

In the UK, comparisons can be made with BS 8110-1 (15), which recommends 0.45 % for “sections subjected mainly to pure tension”, although this has previously been thought to apply to structural tension members like hangers, and with the design of continuous reinforced concrete pavements, for which the Highways Agency (16) specifies 0.60 % reinforcement in one layer at the mid-depth.

Both ACI 350 and EN 1992-1-1 make an allowance for ‘non-uniform self-equilibrating stresses’ in thick slabs. Although expressed in different ways, the net effects are very similar. They are illustrated in Figure 7; as the depth of the section increases (x-axis) a central zone can be discounted and the reinforcement provided based on the remaining surface zones alone. The upper lines express the same effect as a reduction

factor. ACI 350 goes further by reducing the reinforcement in the bottom of base slabs supported on soil by 50 % (not shown in the figure).

Crack widths

Two different methods of estimating crack widths are given in UK codes. Both are in BS 8007: bond slipping is assumed for immature concrete, while no slip is assumed for mature concrete. These two methods are combined in a single method in EN 1992-1-1. The important point to note is that all methods first assume that the reinforcement is in the elastic range, ie that the cracks are controlled.

Should we specify a maximum concrete strength?

The data above are based on the assumption that the average compressive strength will be 8/10 MPa (1200 psi) above the specified minimum with a practical scatter of 20 %. This suggests that an upper limit should be written into specifications, as it is clear from Table 1 that the minimum reinforcement content will be underestimated if the compressive strength is too high. This could be enforced by also placing an upper limit on cementitious material content and discouraging too-low a w/c ratio.

Case study

Figures 8 and 9 show the floor slab of a single level basement 215×70 m (700 x 230 ft) used for car parking which has cracked extensively. The slab is 350 mm (14 in) thick, designed as a flat slab on pile caps at two-way centers varying between 5.5 and 6.5 m (18 and 21 ft). The section (Figure 10) shows that full restraint would be a realistic assumption. Total reinforcement is 0.38 % in the middle strips, 0.60 % in the column strips. Concrete grade was specified as 28/35, although no actual cube test results are available.

The structure was built in the mid-1990s, although the cracks were only observed around 2000. They are roughly parallel, at right angles to the long dimension. They start about 27 m (90 ft) from each end and are in groups of two to five spaced at 500 to 1100 mm (20 to 42 in), followed by a gap of 2 m (7 ft) or more before the next group. They are now around 0.5 mm (0.02 in) wide, although filled with a dark grey precipitate, having leaked extensively over the three winters from 2000-1 to 2002-3, having been below the water table for at least part of that time.

The author believes the explanation is that with $\rho > \rho_{\text{imm}}$ controlled cracks did initially form as predicted for early age contractions but because $\rho < \rho_{\text{mat}}$ they later widened uncontrollably rather than additional new cracks forming. The absence of cracks at each end probably shows the distance needed for full restraint to develop, perhaps aided by the inward pressure of the soil on the perimeter walls. The absence of cracks parallel to the long sides (and of any movement at the construction joints) could be for the same reason – the width is not much more than twice 27 m. The slab was constructed in longitudinal strips generally 5-6 m (16-20 ft) wide, so an alternative explanation is that

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the early age contraction cycle passed without restraint inducing any significant tensile stress and that subsequent contractions have been within the tensile strain capacity.

Managing cracking in basements

Many techniques are available to reduce or control shrinkage effects in concrete, both at early age and when it is mature. These include: limiting pour areas; forming movement joints; incorporating shrinkage gaps; using shrinkage-reducing admixtures; and using shrinkage-compensating cements. All run the risk that if the restraint is underestimated uncontrolled cracking may still occur. The one reliable solution is to provide sufficient reinforcement to control cracking.

Historically, basement-type structures have been sized by first choosing the thickness of concrete, often conservatively on the assumption that the thicker the concrete the more watertight it will be. Reinforcement has then been decided; in the UK prior to BS 8007 even 0.35 % would have been thought excessive.

The analysis above shows that the preferred approach is to select the thickness of concrete as only that necessary to perform structurally; in many cases the minimum of 250 mm (10 in) recommended in BS 8102 (17) will suffice. Reinforcement should then be chosen to satisfy the criteria derived above. It might seem excessive by other current standards, but uncontrolled cracking still occurs far too often, and if it does all the reinforcement that is put in has been wasted.

NOTATION

A_c	area of concrete
A_s	area of reinforcement (total in both faces)
A_s'	area of compression reinforcement
b	width of section
c	depth to neutral axis (consistent with immediate or long-term condition)
d	depth to tension reinforcement
E_c, E_{eff}	modulus of elasticity of concrete; modified for long-term effects
E_s	modulus of elasticity of steel reinforcement
e	eccentricity (defined in Figure 6)
F	force (suffixes c, s refer to concrete, reinforcement)
f_{ct}, f_{ctm}	concrete tensile strength - generally, mean value
f_{cu}	concrete compressive strength, expressed as eg 28/35 MPa [4000 psi] where 28 MPa [4000 psi] is the cylinder strength and 35 MPa is the cube strength
f_{sd}	reinforcement tensile design strength
f_y	reinforcement yield strength
f_{yk}	characteristic reinforcement yield strength (ie 95 % probable)
h	overall depth of section
h_e	effective thickness, = $b h / (2 \times \text{exposed perimeter})$
I_c, I_g	second moment of area of concrete section, gross section
K	coefficient for deflection related to bending moment diagram
k, k_p	coefficients

L	span
M	moment
m	modular ratio between steel and concrete (consistent with immediate or long-term condition, ie $= E_s / E_c$ or E_s / E_{eff})
q, r	parameters (defined in the text)
rh	relative humidity
T_1	temperature drop for early thermal contraction (defined in Figure 1)
S_s	<u>net</u> first moment of area of reinforcement about neutral axis of concrete section (ie deducting S of compression reinforcement)
z	lever arm between centroids of base and overlay
δ	deflection
$\epsilon_{cr}, \epsilon_{cs}$	shrinkage in concrete – restrained, free (unrestrained)
$\epsilon_n, \epsilon_r, \epsilon_s$	strains (defined in the text)
κ	curvature (suffixes A, B, C refer to left support, right support, mid-span)
$\mu\epsilon$	micro-strain, ie strain $\times 10^{-6}$
ν	creep coefficient
ρ	ratio $A_s / b h$
ρ'	ratio $A_s' / b c$
ρ_d	ratio $A_s / b d$
ρ_n	ratio A_s / A_c
σ_{ct}	tensile stress in concrete

suffixes

a	overlay
b	base (or substrate)
cr	cracked
imm	immature, ie in first 3-6 days
mat	mature, ie older than 28 days
prov	provided
req	required
sh	shrinkage
un	uncracked

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Table 1. Minimum reinforcement contents in direct tension for both immature and mature concrete related to strength grade.

Specified strength f_{cu}	MPa (psi)	20/25 (2900)	28/35 (4000)	35/45 (5100)	45/55 (6500)
Immature tensile strength $f_{ct,imm}$	MPa (psi)	1.45 (210)	1.72 (250)	1.98 (285)	2.23 (325)
$\rho_{imm} = 1.22 (f_{ct,imm} / f_{sd})$ [ACI]		0.43 %	0.51 %	0.58 %	0.66 %
$\rho_{imm} = 1.14 (f_{ct,imm} / f_{sd})$ [EN]		0.38 %	0.45 %	0.52 %	0.58 %
Mean mature tensile strength f_{ctm}	MPa (psi)	2.2 (320)	2.8 (405)	3.3 (480)	3.8 (550)
$\rho_{mat} = 1.00 (f_{ct,mat} / f_{sd})$ [ACI]		0.53 %	0.67 %	0.80 %	0.92 %
$\rho_{mat} = 0.93 (f_{ct,mat} / f_{sd})$ [EN]		0.47 %	0.60 %	0.71 %	0.82 %

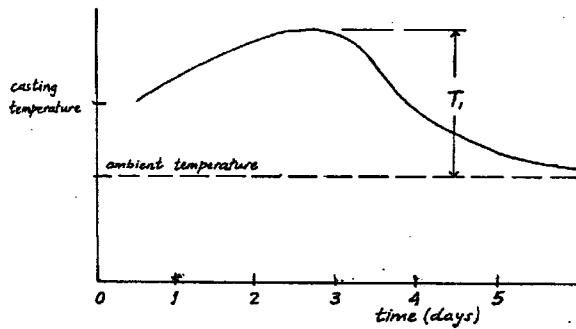


Figure 1. Typical early thermal temperature cycle for a relatively thick section (cycle is more rapid for thin sections)

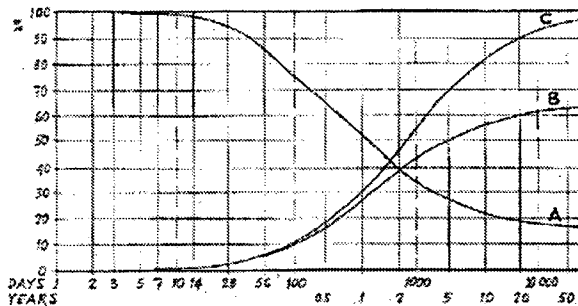


Figure 2. Reduction over time of restrained tensile stress due to creep. Curve A early thermal (from 100%), curve B shrinkage (from free shrinkage C above)

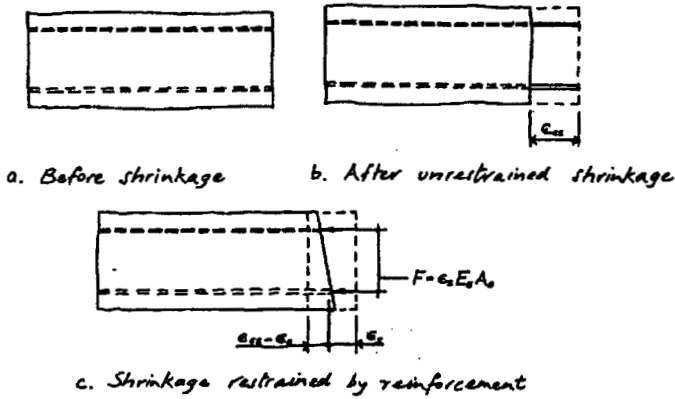


Figure 3. Restrained shrinkage in an asymmetrically reinforced section

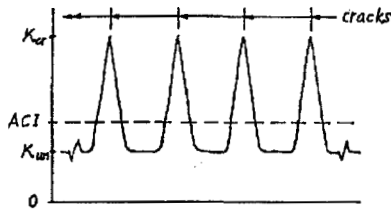


Figure 4. Alternation of shrinkage curvature between cracks and uncracked zones

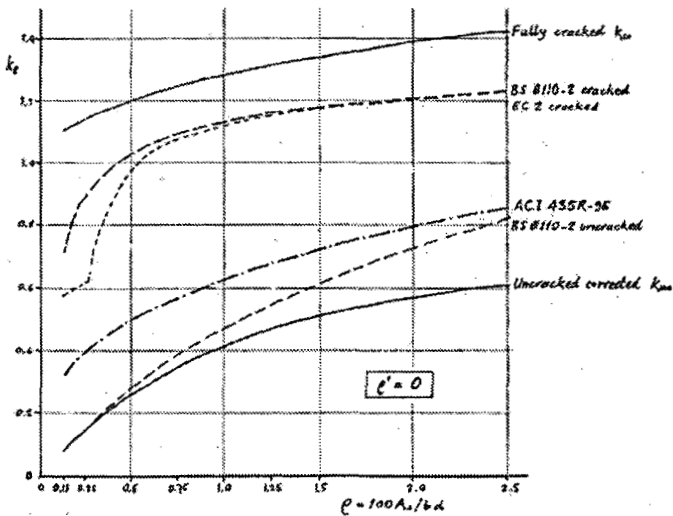


Figure 5. Values of k_r in $k = k_r e_{cs}/d$ against $r_d = 100A_s/bd$ for rectangular sections by various calculation methods ($r' = 0$)

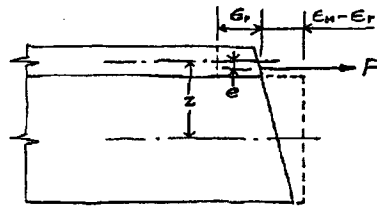


Figure 6. Contraction of in-situ concrete overlay (or topping)

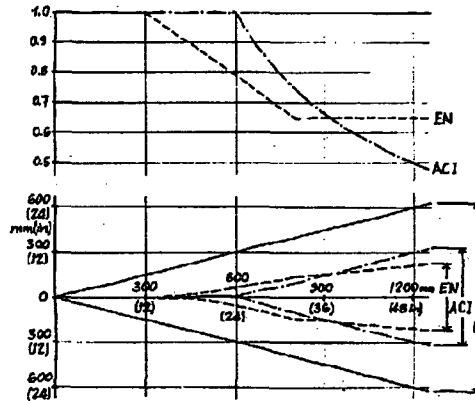


Figure 7. Reduction factor (upper) and illustration of surface and internal zones (lower) against increasing overall depth of section



Figure 8. General view of a typical basement car park.
Note transverse cracks in floor in foreground.

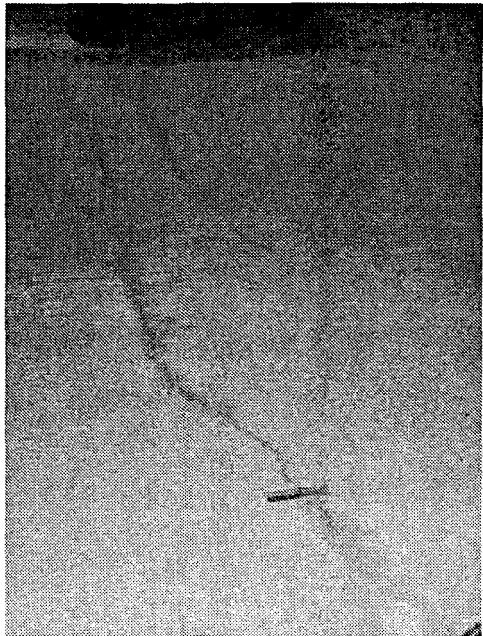


Figure 9. Close-up of cracks in Figure 8

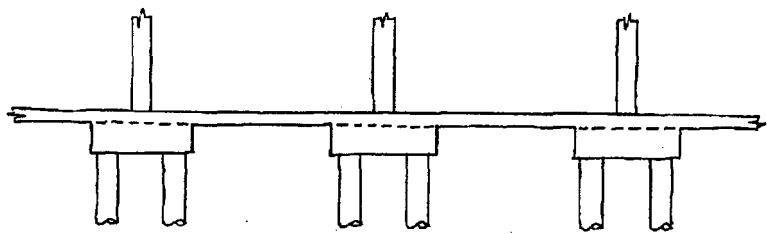


Figure 10. Section through floor construction of car park basement in Figure 8

AS3600 Creep and Shrinkage Models for Normal and High Strength Concrete

by R. I. Gilbert

Synopsis: The Australian Standard for Concrete Structures, AS3600-2001 (1), is currently under review, with the intention to expand its scope to include concrete with characteristic compressive strengths up to 100 MPa. The procedures in the existing Standard for the estimation of the deformation characteristics of concrete, including the tensile strength, the elastic modulus, the creep coefficient and the shrinkage strain, are not applicable for high strength concrete. To provide reasonable agreement with the test data for Australian high strength concretes, new models for predicting the instantaneous and time-dependent material characteristics had to be developed. In this paper, the new models that have been adopted for inclusion in the next edition of AS3600 (due for release in 2006) are presented.

Keywords: creep; creep coefficient; deformational characteristics; elastic modulus; high strength concrete; material properties, shrinkage; tensile strength

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INTRODUCTION

A revised edition of the Australian Standard for Concrete Structures, AS3600 (1) was released in 2001 with amendments introduced to facilitate the use of 500 MPa reinforcing steel in the design of concrete structures. The revision also increased the upper limit of the applicable range of the 28-day characteristic compressive strength of concrete (f'_c) from 50 MPa to 65 MPa. A subsequent revision of AS3600 (2) is currently underway, with the intention to expand the scope of the Standard to include concrete with f'_c up to 100 MPa. The procedures in the Standard for estimating the strength and deformation characteristics of concrete, including the tensile strength, the elastic modulus, the creep coefficient and the shrinkage strain, were developed for normal strength concrete and are not applicable for high strength concrete.

With the move towards higher strength reinforcing steels, the serviceability limit states are more often the critical design consideration and reliable models for estimating the deformation characteristics of concrete are essential. For example, when 500 MPa steel is used in a concrete structure, instead of 400 or 450 MPa steel, a smaller quantity of reinforcement is required in a given situation to satisfy the requirements of adequate strength. Under in-service conditions, this results in less stiffness after cracking, and consequently greater deflections, and higher strains in the tensile reinforcement, and consequently wider cracks. To adequately predict deflections and crack widths, methods of analysis that realistically account for cracking and the time-dependent deformations caused by creep and shrinkage of the concrete are required, as are appropriate material modeling rules.

This paper is primarily concerned with the instantaneous and time-dependent strength and deformation characteristics of concrete with f'_c up to 100 MPa. The factors that have been considered in the current review of the relevant clauses of AS3600 (2) are discussed and the models that have been developed for predicting the tensile strength, the elastic modulus, the creep coefficient and the shrinkage strain for the full range of applicable concrete strengths ($20 \text{ MPa} \leq f'_c \leq 100 \text{ MPa}$) are presented. These models provide a reasonable compromise between being relatively simple to use and providing a reliable estimate of material properties.

COMPRESSIVE AND TENSILE STRENGTH

The strength of concrete is specified in Australia in terms of the lower characteristic compressive cylinder strength at 28 days (f'_c). This is the value of compressive strength exceeded by 95% of all standard cylinders tested at 28 days after casting under standard conditions. The standard compressive strength grades of concrete, proposed for inclusion in AS3600 (2), are 20 MPa, 25 MPa, 32 MPa, 40 MPa, 50 MPa, 65 MPa, 80 MPa and 100 MPa.

The ratio of the mean compressive strength to the lower characteristic compressive strength at age 28 days ($f_{cm,28}/f'_c$) depends on the quality control during batching and the quality and variability of the concrete constituent materials (aggregates and binder, in particular). In the absence of more accurate data, and if quality control is good, AS3600 (2) adopts the assumption that the ratio varies linearly from about 1.25 for 20 MPa concrete to about 1.1 for 100 MPa concrete. The in-situ strength of concrete depends on many factors, including the workmanship during construction, the degree of compaction, the curing and finishing procedures, the environment and degree of exposure, the load history and more. In the absence of more accurate information, AS3600 (2) takes the in-situ compressive strength to be 90% of the cylinder strength.

The tensile strength, f_{ct} , is defined in AS3600 (2) as the maximum stress which concrete can withstand when subjected to uniaxial tension. Direct uniaxial tensile tests are difficult to perform and tensile strength is usually assessed through either flexural tests on prisms or indirect splitting tests on cylinders. In flexure, the apparent tensile stress at the extreme tensile fiber of the critical cross-section under the peak load is calculated assuming linear elastic behavior and taken to be the flexural tensile strength (or modulus of rupture), $f_{ct,f}$. The flexural tensile strength $f_{ct,f}$ is significantly higher than f_{ct} due to the strain gradient and the post-peak unloading portion of the stress-strain curve for concrete in tension. The uniaxial tensile strength, f_{ct} , is usually in the range of 50 to 60% of the measured flexural tensile strength. The indirect tensile strength measured from a split cylinder test $f_{ct,sp}$ is also higher than the uniaxial tensile strength (usually by about 10%) due to the confining effect of the bearing plate in the standard test.

Tensile strength test data shows a relatively large scatter of results and a high degree of variability (3). In the absence of test data, formulas are often given in codes of practice for estimating the lower characteristic flexural tensile strength $f'_{ct,f}$ and the lower characteristic uniaxial tensile stress f'_{ct} . The formulas proposed for inclusion in AS3600 (2) are $f'_{ct,f} = 0.6\sqrt{f'_c}$ and $f'_{ct} = 0.36\sqrt{f'_c}$. The mean and upper characteristic values may be estimated by multiplying $f'_{ct,f}$ or f'_{ct} by 1.4 and 1.8, respectively. In serviceability calculations, mean values of tensile strength should be used in most situations rather than characteristic values (4).

ELASTIC MODULUS

The value of the elastic modulus, E_c , increases with time as concrete gains strength and stiffness. It is common practice to assume that E_c is constant with time and equal to its value calculated at the time of first loading. For stress levels less than about $0.4f'_c$, and for stresses applied over a relatively short period (say up to 5 minutes), the numerical estimate of the elastic modulus specified in AS3600-2001 (1) for concrete strengths up to 65 MPa was originally proposed by Pauw (5) and is given by

$$E_c = \rho^{1.5} 0.043 \sqrt[3]{f_{cm}} \quad (\text{in MPa}) \quad (1)$$

where ρ is the density of concrete (not less than 2400 kg/m^3 for normal weight concrete) and f_{cm} is the mean compressive strength in MPa at the time of first loading. Whilst Equation (1) has been shown to provide a good estimate of the elastic modulus for normal strength concrete, it overestimates E_c when f_{cm} exceeds about 40 MPa and is generally felt to be unsuitable for higher strength concretes (6), (7) and (8). Carrasquillo et al. (6) proposed

$$E_c = [3320 \sqrt[3]{f_{cm}} + 6900](\rho/2320)^{1.5} \quad (\text{in MPa}) \quad (2)$$

which is claimed to be more suitable for high strength concrete (when f_{cm} is in the range 50 - 100 MPa). Plots of both Equations (1) and (2) for normal weight concrete with $\rho = 2400 \text{ kg/m}^3$ are shown in Figure 1.

Acknowledging that Equation (1) overestimates E_c for high strength concrete, Mendis et al. (7) proposed the following modification:

$$E_c = \rho^{1.5} 0.043 \eta \sqrt[3]{f_{cm}} \quad (\text{in MPa}) \quad (3)$$

where the modification factor $\eta = 1.1 - 0.002 f'_c \leq 1.0$. Equation (3) is also plotted in Figure 1. However, the modification factor does not adequately cater for the inadequacies of Equation (1) and Equation (3) also overestimates E_c for some aggregate types, particularly for concrete strengths above 65 MPa.

An alternative approach (4) is the use of Equation (1) when $f_{cm} \leq 40 \text{ MPa}$ and Equation (4) when $f_{cm} > 40 \text{ MPa}$.

$$E_c = [0.024 \sqrt[3]{f_{cm}} + 0.12] \rho^{1.5} \quad (\text{in MPa}) \quad (4)$$

Both equations provide similar estimates of E_c at the transition point ($f_{cm} = 40 \text{ MPa}$). Equation (4) is also plotted in Figure 1 for normal weight concrete and the values of E_c

for in-situ concrete at age 28 days for each standard strength grade when $\rho = 2400 \text{ kg/m}^3$ are given in Table 1 (where f_{cm} is taken as 90% of the mean cylinder strength).

The value of E_c given by Equations (1) or (4) is applicable for stress levels up to about $0.4 f_{cm}$ for normal strength concrete and up to about $0.6 f_{cm}$ for high strength concrete and for stresses applied over a relatively short time period (up to about 5 minutes). In general the faster the load is applied, the larger is the value of E_c . For stresses applied over a longer time period (say up to one day's duration), significant increases in deformation occur due to the rapid early development of creep. Yet in a broad sense, loads of one day's duration are usually considered to be short-term and the effects of creep are often ignored. This may lead to significant error. If *short-term* deformation is required after 1 day's loading, it is suggested (9) that E_c given by either Equations (1) or (4) be multiplied by 0.8.

AS3600-2001 (1) suggests that values of E_c given by Equation (1) have a range of ± 20 percent and this is also true for Equation (4). Typical variations in E_c with age for concrete cured under standard conditions are shown in Table 2.

CREEP OF CONCRETE

Discussion

When concrete is subjected to a sustained stress, creep strain develops gradually with time. Creep increases with time at a decreasing rate. In the period immediately after first loading, creep develops rapidly, but the rate of increase slows appreciably with time. Creep is generally thought to approach a limiting value as the time after first loading approaches infinity. About 50% of the final creep develops in the first 2-3 months and about 90% after 2 to 3 years. After several years under load, the rate of change of creep with time is very small. Creep has its origins in the hardened cement paste and is caused by several different mechanisms (9).

Many factors influence the magnitude and rate of development of creep, including the properties of the concrete mix and its constituent materials. In general, as the concrete quality increases, the capacity of concrete to creep decreases. For a particular stress level, creep in higher-strength concrete is less than that in lower-strength concrete. An increase in either the aggregate content or the maximum aggregate size reduces creep, as does the use of a stiffer aggregate type. Creep also decreases as the water-to-cement ratio is reduced.

Creep depends on the environment, and increases as the relative humidity decreases. Creep is also greater in thin members with large surface area-to-volume ratios, such as slabs. However, the dependence of creep on both the relative humidity and the size and shape of the specimen decreases as the concrete strength increases. Near the

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surface of a member, creep takes place in a drying environment and is therefore greater than in regions remote from a drying surface. In addition to the relative humidity, creep is dependent on the ambient temperature. A temperature rise increases the deformability of the cement paste and accelerates drying, and thus increases creep.

In addition to the environment and the characteristics of the concrete mix, creep depends on the loading history, in particular the magnitude of the stress and the age of the concrete when the stress is first applied. When the sustained concrete stress is less than about $0.5f_c$ (and this is usually the case in concrete structures at service loads), creep is proportional to stress and is known as linear creep. The age of the concrete when the stress is first applied, τ , has a marked influence on the magnitude of creep. Concrete loaded at an early age creeps more than concrete loaded at a later age.

The Creep Coefficient

The capacity of concrete to creep is usually measured in terms of the creep coefficient, $\phi(t, \tau)$. In a concrete specimen subjected to a constant sustained compressive stress, σ_o , first applied at age τ , the creep coefficient at time t is the ratio of creep strain to instantaneous strain and is given by $\phi(t, \tau) = \varepsilon_c(t, \tau)/\varepsilon_e$. Therefore, the creep strain is $\varepsilon_c(t, \tau) = \phi(t, \tau) \varepsilon_e = \phi(t, \tau) \sigma_o/E_c$. For stress levels less than about $0.5f_c$, the creep coefficient is a pure time function, independent of the applied stress, and the creep coefficient increases with time at an ever-decreasing rate. As time approaches infinity, the creep coefficient is assumed to approach a final value $\phi^*(\tau) = \phi(\infty, \tau)$, which usually falls within the range 1.5 - 4.0 (9).

The most accurate way of determining the final creep coefficient is by testing or by using results obtained from measurements on similar local concretes. Testing is often not a practical option for the structural designer. In the absence of long-term test results, the final creep coefficient may be determined by extrapolation from relatively short-term test results, where creep is measured over a relatively short period (say 28 days) in specimens subjected to constant stress. Various mathematical expressions for the shape of the creep coefficient versus time curve are available from which long-term values may be predicted from the short-term measurements. The longer the period of measurement, the more accurate are the long-term predictions. Some of the more useful expressions for $\phi(t, \tau)$ are presented by Gilbert (9).

If testing is not an option, analytical methods are available for predicting the creep coefficient. These predictive methods vary in complexity (9). Some are simple and easy to use, and provide a quick and approximate estimate of $\phi(t, \tau)$. Such a method is included in AS3600 (1). Others are much more complicated and attempt to account for the many parameters that affect the magnitude and rate of development of creep. Unfortunately, an increase in complexity does not necessarily result in an increase in accuracy, and predictions made by some of the more well-known methods differ widely (9). The simple approach contained in AS3600 (1) does not account for such factors as aggregate type, cement type, cement replacement materials and more, but it does provide

a ball-park estimate of the creep coefficient for normal strength concrete.

The AS3600-2001 (1) Model for Creep Coefficient

AS3600 (1) defines a basic creep coefficient $\varphi_{cc,b}$ as the ratio of final creep strain to elastic strain for a specimen loaded at 28 days under a constant stress of $0.4 f'_c$. In the absence of tests, the Standard specifies $\varphi_{cc,b} = 5.2, 4.2, 3.4, 2.5$ and 2.0 for characteristic strengths of $f'_c = 20, 25, 32, 40$ and 50 MPa, respectively. According to AS3600 (1), the creep coefficient at any time φ_{cc} may be calculated from

$$\varphi_{cc} = k_2 k_3 \varphi_{cc,b} \quad (5)$$

provided that φ_{cc} is used in conjunction with E_c at 28 days (irrespective of the age at loading); the concrete is not subjected to prolonged periods of temperature in excess of 25°C ; and the sustained stress level does not exceed $0.5 f'_c$. Where the concrete is likely to be subjected to prolonged period of temperature in the range $25\text{--}40^\circ\text{C}$, a 25 % increase in φ_{cc} is here recommended.

The factor k_2 depends on the hypothetical thickness, t_h , the environment and the time after loading and is given in Figure 6.1.8.2(A) in the Standard (1). The hypothetical thickness is defined as $t_h = 2A/u_e$, where A is the cross-sectional area of the member and u_e is that portion of the section perimeter exposed to the atmosphere plus half the total perimeter of any voids contained within the section. The factor k_3 depends on the age at first loading and may be taken from Figure 6.1.8.2 (B) in the Standard (1). The Standard also suggests that the actual creep coefficient has a range of approximately $\pm 30\%$ of the value predicted.

The above procedure was calibrated from creep data for normal strength concretes, but has been shown to underestimate the early development of creep in structural members and to underestimate creep of concrete loaded at very early ages (4). That is, the factors k_2 and k_3 need to be modified. The method also tends to underestimate creep for 40 MPa and 50 MPa concrete.

The Proposed Model for Creep Coefficient - AS3600 (2)

A modified model for predicting the creep coefficient for concrete (with compressive strength in the range 20 - 100 MPa) was proposed by Gilbert (10) and an updated and simplified version of that proposal is outlined here and has been accepted for inclusion in AS3600 (2). The model agrees well with the limited creep data available for Australian high strength concrete.

The creep strain at any time t caused by a constant sustained stress σ_o shall be calculated from $\epsilon_{cc} = \varphi_{cc} \sigma_o / E_c(28)$, where $E_c(28)$ is the elastic modulus of the concrete at age 28 days and φ_{cc} is the creep coefficient at time t .

The creep coefficient at any time may be calculated from

$$\varphi_{cc} = k_2 k_3 k_4 k_5 \varphi_{cc,b} \quad (6)$$

Although similar in form to the existing equation in AS3600 (1), Equation (6) introduces two new k factors, revises the factors k_2 and k_3 , and modifies and extends the basic creep coefficient ($\varphi_{cc,b}$) as given in Table 3.

The factor k_2 in Equation (6) describes the development of creep with time. It depends on the hypothetical thickness, t_h (in mm) (as defined in the Standard) and is given by Equation (7) (and illustrated in Figure 2):

$$k_2 = \frac{\alpha_2 t^{0.8}}{t^{0.8} + 0.15t_h} \quad (7)$$

where t is the time (in days) since first loading and α_2 is given in Equation (8).

$$\alpha_2 = 1.0 + 1.12e^{-0.008t_h} \quad (8)$$

The factor k_3 is modified from that given in AS3600 (1) to predict higher creep for concrete loaded at early ages and is given in Figure 3.

The factor k_4 accounts for the environment and is equal to 0.7 for an arid environment, 0.65 for an interior environment, 0.60 for a temperate environment and 0.5 for a tropical/coastal environment. The factor k_5 is given in Equation (9) and accounts for the reduced influence on creep of the relative humidity and the specimen size as the concrete strength increases (or more precisely, as the water-binder ratio decreases).

When $f'_c \leq 50\text{MPa}$: $k_5 = 1.0$ (9a)

When $50\text{ MPa} < f'_c \leq 100\text{MPa}$: $k_5 = (2.0 - \alpha_3) - 0.02 (1.0 - \alpha_3) f'_c$ (9b)

where
$$\alpha_3 = \frac{0.7}{k_4 \alpha_2} \quad (10)$$

The proposed predictive model gives similar results to AS3600 for normal strength concrete and provides a *ball-park* estimate of the creep coefficient for high strength concrete. However, it must be emphasized that creep of concrete is highly variable with significant differences in the measured creep strains in seemingly identical specimens, tested under identical conditions (both in terms of load and environment). The creep coefficient predicted by Equation (6) should be taken as an average value with a range of ± 30 percent.

To illustrate the use of the model, consider the following example. The final design creep coefficient after 30 years under load ($t = 10950$ days) is required for 65 MPa concrete, located in a temperate environment, first loaded at 28 days and with a hypothetical thickness $t_h = 200$ mm.

From Table 3: $\varphi_{cc,b} = 2.0$

Equation (8) gives: $\alpha_2 = 1.0 + 1.12e^{(-0.008 \times 200)} = 1.226$

Equation (7) gives:
$$k_2 = \frac{1.226 \times 10950^{0.8}}{10950^{0.8} + 0.15 \times 200} = 1.205$$

(or obtain directly from Figure 2).

From Figure 3: $k_3 = 1.1$ and for a temperate environment: $k_4 = 0.6$

Equation (10) gives: $\alpha_3 = 0.7 / (0.6 \times 1.226) = 0.952$

Equation (9b) gives: $k_5 = (2.0 - 0.952) - 0.02 \times (1.0 - 0.952) \times 65 = 0.985$

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and from Equation (6): $\phi_{cc} = 1.205 \times 1.1 \times 0.6 \times 0.985 \times 2.0 = 1.57$

The final creep coefficients ϕ_{cc} (after 30 years) predicted by the above method for concrete first loaded at 28 days, for characteristic strengths of 25 MPa to 100 MPa, for three hypothetical thicknesses, $t_h = 100$ mm, 200 mm and 400 mm and located in different environments are illustrated in Table 4. The numbers shown in brackets for concrete strengths of 25 MPa and 32 MPa are the values of ϕ_{cc} determined using the existing procedure in AS3600 (1). The proposed model and the existing model in AS3600 are in very close agreement for low strength concrete.

The above discussion is concerned with compressive creep. In many practical situations, creep of concrete in tension is also of interest. Tensile creep plays an important part in delaying the onset of cracking caused by restrained shrinkage. The mechanisms of tensile creep are different from those of compressive creep, but at the same stress levels the magnitudes are similar. In design, it is usual to assume that the creep coefficients in tension and in compression are identical. Although not correct, this assumption simplifies calculations and does not usually introduce serious inaccuracies.

SHRINKAGE OF CONCRETE

Discussion

Shrinkage of concrete is the time-dependent strain in an unloaded and unrestrained specimen at constant temperature. Shrinkage arises due to a variety of causes and includes: plastic shrinkage which may occur in the wet concrete soon after its placement; chemical shrinkage caused by hydration and other chemical reactions; thermal shrinkage resulting from the cooling of the concrete soon after setting; and drying shrinkage. Some high strength concretes are prone to plastic shrinkage, which occurs in the wet concrete, and may result in significant cracking during the setting process. This cracking occurs due to capillary tension in the pore water and is best prevented by taking measures during construction to avoid the rapid evaporation of bleed water. Since the bond between the plastic concrete and the reinforcement has not yet developed, the steel reinforcement is ineffective in controlling such cracks.

Drying shrinkage is the reduction in volume caused principally by the loss of water during the drying process. Following an initial phase where drying shrinkage occurs very rapidly in the hours and days after drying commences, it increases with time at a gradually decreasing rate and continues to take place in the months and years after casting. The magnitude and rate of development of drying shrinkage depend on all the factors that affect the drying of concrete, including the relative humidity of the surrounding air, the mix characteristics (in particular, the type and quantity of the binder, the water content and water-to-cement ratio, the ratio of fine to coarse aggregate, and the type of aggregate), and the size and shape of the member. The magnitude of shrinkage

that occurs in the initial phase soon after the commencement of drying depends on the age of the concrete when drying commences and the initial period of moist curing. Poorly cured concrete may suffer large shrinkage strains in the 48 hours after drying commences.

Chemical shrinkage (often termed autogenous shrinkage) results from various chemical reactions within the cement paste and includes hydration shrinkage, which is related to the degree of hydration of the binder in a sealed specimen with no moisture exchange. Chemical shrinkage occurs rapidly in the days and weeks after casting and is less dependent on the environment and the size of the specimen than drying shrinkage. Thermal shrinkage is the contraction that results in the first few hours (or days) after setting as the heat of hydration gradually dissipates.

Concrete shrinkage strain, ϵ_{cs} , which is usually considered to be the sum of the drying, chemical and thermal shrinkage components, continues to increase with time at a decreasing rate. Shrinkage is assumed to approach a final value, ϵ_{cs}^* , as time approaches infinity. Drying shrinkage in high strength concrete is smaller than in normal strength concrete due to the smaller quantities of free water after hydration. However, thermal and chemical shrinkage may be significantly higher.

The AS3600-2001 (1) Model for Shrinkage

For normal strength concrete ($f'_c \leq 50$ MPa), AS3600-2001 (1) suggests that the design shrinkage at any time after the commencement of drying may be estimated from

$$\epsilon_{cs} = k_1 \epsilon_{cs,b} \quad (11)$$

where $\epsilon_{cs,b}$ is a basic shrinkage strain and, in the absence of measurements, may be taken to be 850×10^{-6} ; k_1 describes the development of shrinkage with time and depends on the environment and the concrete surface area-to-volume ratio (as measured by the hypothetical thickness, t_h).

AS3600 (1) states that the actual shrinkage strain may be within a range of plus or minus 40% of the value predicted. In the writer's opinion, this range may be optimistically narrow. The approach does not include any of the effects related to the composition and quality of the concrete. The same value of ϵ_{cs} is predicted irrespective of the water-cement ratio, the aggregate type and quantity, the type of admixtures, etc. In addition, the factor k_1 tends to overestimate the effect of member size and underestimate the rate of shrinkage development at early ages. The method should be used only as a guide for concrete with a low water-cement ratio (<0.4) and with a well-graded, good quality aggregate. Where a higher water-cement ratio is expected or when doubts exist concerning the type of aggregate to be used, the value of ϵ_{cs} predicted by AS3600-2001 (1) should be increased by at least 50%.

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The design shrinkage in AS3600-2001 (1) essentially refers to drying shrinkage. The method is overly simplistic and cannot be extended to include high strength concrete where the mechanisms, causes, magnitude and rate of development of shrinkage are significantly different from normal strength concrete.

The Proposed Model for Shrinkage - AS3600 (2)

The revised model for estimating shrinkage in both normal and high strength concrete was proposed by Gilbert (4,10). The model divides the total shrinkage strain, ϵ_{cs} , into two components, chemical plus thermal shrinkage (termed endogenous shrinkage), ϵ_{cse} , and drying shrinkage, ϵ_{csd} , as given in Equation (12). Endogenous shrinkage is taken to be the sum of chemical and thermal shrinkage and is assumed to develop relatively rapidly and to increase with concrete strength. Drying shrinkage develops more slowly and decreases with concrete strength.

$$\epsilon_{cs} = \epsilon_{cse} + \epsilon_{csd} \quad (12)$$

At any time t_o (in days) after placing the concrete, the endogenous shrinkage is given by

$$\epsilon_{cse} = \epsilon_{cse}^* (1.0 - e^{-0.1t_o}) \quad (13)$$

where ϵ_{cse}^* is the final endogenous shrinkage and may be taken as

$$\epsilon_{cse}^* = (0.06f'_c - 1.0) \times 50 \times 10^{-6} \quad (14)$$

where f'_c is in MPa.

The basic drying shrinkage $\epsilon_{csd,b}$ is given by

$$\epsilon_{csd,b} = (1.0 - 0.008f'_c) \times \epsilon_{csd,b}^* \quad (15)$$

where $\epsilon_{csd,b}^*$ depends on the quality of the local concrete, including the type and quantity of aggregates, cement, cement replacement materials and admixtures. In the absence of more reliable information, $\epsilon_{csd,b}^*$ is specified as 800×10^{-6} for Sydney and Brisbane, 900×10^{-6} for Melbourne and 1000×10^{-6} elsewhere in Australia.

At any time t_d (in days) after the commencement of drying, the drying shrinkage may be taken as

$$\varepsilon_{csd} = k_1 k_4 \varepsilon_{csd,b} \quad (16)$$

and k_1 is given by Equation (17) and is illustrated in Figure 4.

$$k_1 = \frac{\alpha_1 t_d^{0.8}}{t_d^{0.8} + 0.15 t_h} \quad (17)$$

where

$$\alpha_1 = 0.8 + 1.2 e^{-0.005 t_h} \quad (18)$$

As for creep, k_4 is equal to 0.7 for an arid environment, 0.65 for an interior environment, 0.6 for a temperate inland environment and 0.5 for a tropical or near-coastal environment.

As expressed in Equation (12), the design shrinkage at any time is therefore the sum of the endogenous shrinkage (Equation 13) and the drying shrinkage (Equation 16). The proposed model provides good agreement with available shrinkage measurements on Australian concrete.

To illustrate the model, consider the following example. The design shrinkage strain is required 1000 days after the commencement of drying for 65 MPa concrete, located in Canberra (temperate inland environment) and with hypothetical thickness $t_h = 200$ mm. For this example, t_d is 1000 days and t_o is taken to be 1007 days.

Equation (14): $\varepsilon_{cse}^* = (0.06 \times 65 - 1.0) \times 50 \times 10^{-6} = 145 \times 10^{-6}$

Equation (13): $\varepsilon_{cse} = 145 \times 10^{-6} \times (1.0 - e^{-0.1 \times 1007}) = 145 \times 10^{-6}$

Equation (15): $\varepsilon_{csd,b} = (1.0 - 0.008 \times 65) \times 1000 \times 10^{-6} = 480 \times 10^{-6}$

Equation (18): $\alpha_1 = 0.8 + 1.2 e^{-0.005 \times 200} = 1.24$

Equation (17): $k_1 = \frac{1.24 \times 1000^{0.8}}{1000^{0.8} + 0.15 \times 200} = 1.11$

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Equation (16):
$$\varepsilon_{csd} = 1.11 \times 0.6 \times 480 \times 10^{-6} = 319 \times 10^{-6}$$

Equation (12):
$$\varepsilon_{cs} = 145 \times 10^{-6} + 319 \times 10^{-6} = 464 \times 10^{-6}$$

For specimens located outside Sydney, Brisbane or Melbourne ($\varepsilon_{csd,b*} = 1000 \times 10^{-6}$) with hypothetical thickness $t_h = 200\text{mm}$, the shrinkage strain components predicted by the above model at 28 days after the commencement of drying and after 30 years ($t_d = 10950$ days) are given in Table 5. Typical final design shrinkage strains for specimens with hypothetical thicknesses $t_h = 50, 100, 200$ and 400 mm and located outside Sydney, Brisbane or Melbourne (with $\varepsilon_{csd,b*} = 1000 \times 10^{-6}$) are given in Table 6.

CONCLUDING REMARKS

The Australian Standard for Concrete Structures AS3600-2001 (1) is currently being reviewed, with the intention to expand the applicability of the Standard to cover concrete strengths up to 100 MPa. The revised provisions concerned with the instantaneous and time-dependent deformation characteristics of concrete, as well as the tensile strength, have been discussed. The models that have been recommended for inclusion in the Standard for predicting the direct and flexural tensile strengths, the elastic modulus, the creep coefficient and the shrinkage strain for the full range of applicable concrete strengths ($20 \text{ MPa} \leq f_c \leq 100 \text{ MPa}$) have been presented. These models have been developed to at once provide reasonable estimates of concrete properties and deformation characteristic and at the same time to be relatively simple to use and interpret.

NOTATION

A	cross-sectional area;
E_c	elastic modulus of concrete;
f_c	characteristic compressive strength of concrete in MPa;
f_{cm}	mean compressive strength of concrete in MPa;
$f_{cm,28}$	mean compressive strength of concrete at age 28 days in MPa;
f_{ct}	characteristic uniaxial tensile strength of concrete in MPa;
$f_{ct,f}$	characteristic flexural tensile strength of concrete in MPa;
k_1, k_2, k_3, k_4 and k_5	modification factors for creep and shrinkage;
t	time (in days) since first loading;
t_d	time (in days) since the commencement of drying;

t_h	hypothetical thickness of a concrete element (in mm);
t_o	time (in days) since casting the concrete;
u_e	section perimeter exposed to the atmosphere;
α_1, α_2 and α_3	modification factors for creep and shrinkage;
ϵ_c	creep strain;
ϵ_{cs}	shrinkage strain;
$\epsilon_{cs.b}$	basic shrinkage strain;
ϵ_{csd}	drying shrinkage strain;
$\epsilon_{csd.b}$	basic drying shrinkage strain;
ϵ_{cse}	endogenous shrinkage strain (chemical plus thermal shrinkage);
ϵ_{cse}^*	final endogenous shrinkage strain;
ϵ_e	elastic (or instantaneous) strain;
η	modification factor;
ϕ_{cc}	creep coefficient for concrete (AS3600);
$\phi_{cc.b}$	basic creep coefficient for concrete (AS3600);
$\phi(t, \tau)$	creep coefficient at time t due to a stress applied at age τ ;
$\phi^*(\tau)$	the final creep coefficient due to a stress applied at age τ ;
ρ	density of concrete;
σ_o	constant sustained compressive stress; and
τ	age of concrete at first loading.

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Table 1 Elastic modulus for in-situ concrete E_c when $\rho = 2400 \text{ kg/m}^3$
(Equations 1 and 4).

f'_c (MPa)	20	25	32	40	50	65	80	100
f_{cm} (MPa)	22.5	27.9	35.4	43.7	53.7	68.2	81.9	99.0
E_c (MPa)	24000	26700	30100	32750	34800	37400	39650	42200

Table 2 Increase in elastic modulus with age ($E_c(t)/E_c(28)$) – Gilbert (10).

Cement Type	Age of Concrete in days (t)				
	3	7	28	90	360
Ordinary Portland Cement	0.70	0.84	1.0	1.11	1.15
High Early Strength Cement	0.77	0.87	1.0	1.06	1.10

Table 3 Proposed basic creep coefficient (2).

f'_c (MPa)	20	25	32	40	50	65	80	100
$\phi_{cc,b}$	5.2	4.2	3.4	2.8	2.4	2.0	1.7	1.5

Table 4 Creep coefficients after 30 years for concrete first loaded at 28 days.

f'_c MPa	Final creep coefficient, ϕ_{cc}											
	Arid Environment			Interior Environment			Temperate Inland Environment			Tropical and near-coastal Environment		
	t_h (mm)			t_h (mm)			t_h (mm)			t_h (mm)		
	100	200	400	100	200	400	100	200	400	100	200	400
25	4.82 (4.66)	3.90 (3.88)	3.27 (3.28)	4.48 (4.30)	3.62 (3.56)	3.03 (3.00)	4.13 (3.97)	3.34 (3.28)	2.80 (2.77)	3.44 (3.37)	2.78 (2.73)	2.33 (2.26)
32	3.90 (3.77)	3.15 (3.14)	2.64 (2.66)	3.62 (3.48)	2.93 (2.88)	2.46 (2.43)	3.34 (3.21)	2.70 (2.66)	2.27 (2.24)	2.79 (2.73)	2.25 (2.21)	1.90 (1.83)
40	3.21	2.60	2.18	2.98	2.41	2.02	2.75	2.23	1.87	2.30	1.86	1.56
50	2.75	2.23	1.89	2.56	2.07	1.73	2.36	1.91	1.60	1.97	1.59	1.33
65	2.07	1.75	1.53	1.95	1.66	1.46	1.84	1.57	1.38	1.61	1.38	1.23
80	1.56	1.40	1.29	1.50	1.36	1.25	1.45	1.32	1.22	1.33	1.23	1.14
100	1.15	1.14	1.11	1.15	1.14	1.11	1.15	1.14	1.11	1.15	1.14	1.11

Note: The numbers shown in brackets for concrete strengths of 25 MPa and 32 MPa are the values of ϕ_{cc} determined using the existing procedure in AS3600 (1).

Table 5 Shrinkage strain components ($t_h = 200\text{mm}$, $\epsilon_{csd,b*} = 1000 \times 10^{-6}$).

Environment	f'_c	Shrinkage strain ϵ_{cs} ($\times 10^{-6}$) and shrinkage strain components ϵ_{cse} ($\times 10^{-6}$) and ϵ_{csd} ($\times 10^{-6}$)					
		$t = 28$ days			$t = 10950$ days (30 yrs)		
		ϵ_{cse}	ϵ_{csd}	ϵ_{cs}	ϵ_{cse}	ϵ_{csd}	ϵ_{cs}
Arid	25	25	225	250	25	685	710
	32	45	210	255	45	635	680
	40	65	190	255	70	580	650
	50	95	170	265	100	510	610
	65	135	135	270	145	415	560
	80	180	100	280	190	310	500
	100	235	55	290	250	170	420
Interior	25	25	210	235	25	635	660
	32	45	195	240	45	595	640
	40	65	180	245	70	540	610
	50	95	155	250	100	480	580
	65	135	125	260	145	385	530
	80	180	95	275	190	290	480
	100	235	50	285	250	160	410
Temperate Inland	25	25	195	220	25	585	610
	32	45	180	225	45	545	590
	40	65	165	230	70	500	570
	50	95	145	240	100	440	540
	65	135	115	250	145	355	500
	80	180	85	265	190	260	450
	100	235	50	285	250	150	400
Tropical or near-coastal	25	25	160	185	25	485	510
	32	45	150	195	45	455	500
	40	65	140	205	70	420	490
	50	95	120	215	100	370	470
	65	135	100	235	145	295	440
	80	180	70	250	190	220	410
	100	235	40	275	255	115	370

Table 6 Typical shrinkage strains after 30 years ($\epsilon_{csd,b^*} = 1000 \times 10^{-6}$)

Environment	f'_c	Final shrinkage strain ϵ_{cs} ($\times 10^{-6}$)			
		$t_h =$ 50 mm	$t_h =$ 100 mm	$t_h =$ 200 mm	$t_h =$ 400 mm
Arid	25	990	870	710	550
	32	950	840	680	530
	40	890	790	650	510
	50	830	740	610	490
	65	730	650	560	460
	80	630	570	500	420
	100	490	460	420	380
Interior	25	920	810	660	510
	32	880	780	640	500
	40	830	740	610	480
	50	770	690	580	460
	65	680	620	530	440
	80	590	540	480	410
	100	480	450	410	370
Temperate inland	25	850	750	610	470
	32	820	720	590	460
	40	780	690	570	450
	50	720	650	540	440
	65	640	580	500	410
	80	560	520	450	390
	100	460	430	400	360
Tropical or near-coastal	25	720	630	510	400
	32	690	610	500	390
	40	660	590	490	390
	50	620	550	470	380
	65	560	510	440	370
	80	500	460	410	360
	100	420	400	370	340

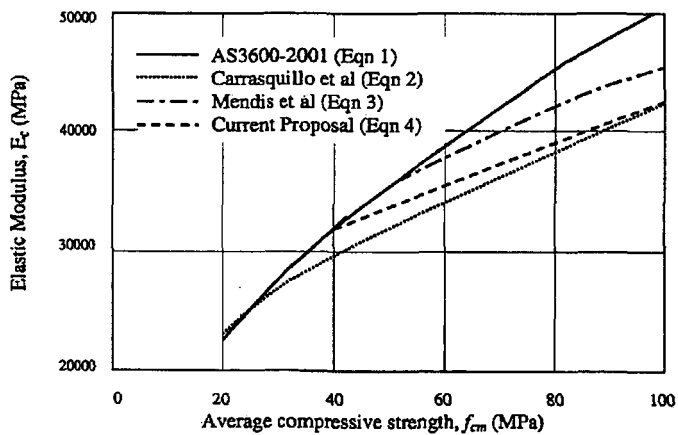


Figure 1 Elastic modulus vs concrete strength (when $r = 2400 \text{ kg/m}^3$).

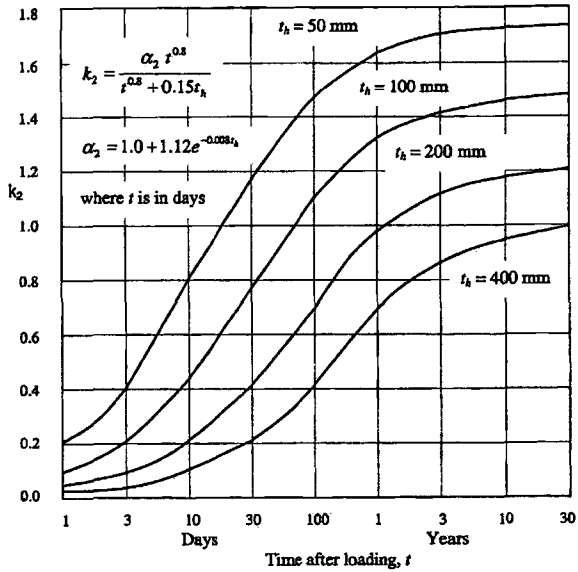


Figure 2 The factor k_2 versus time after loading (2).

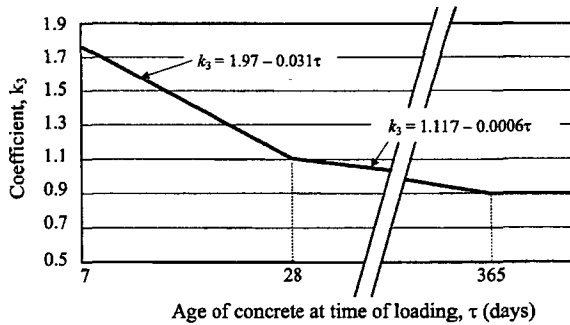


Figure 3 Age-at-first-loading coefficient k_3 (2).

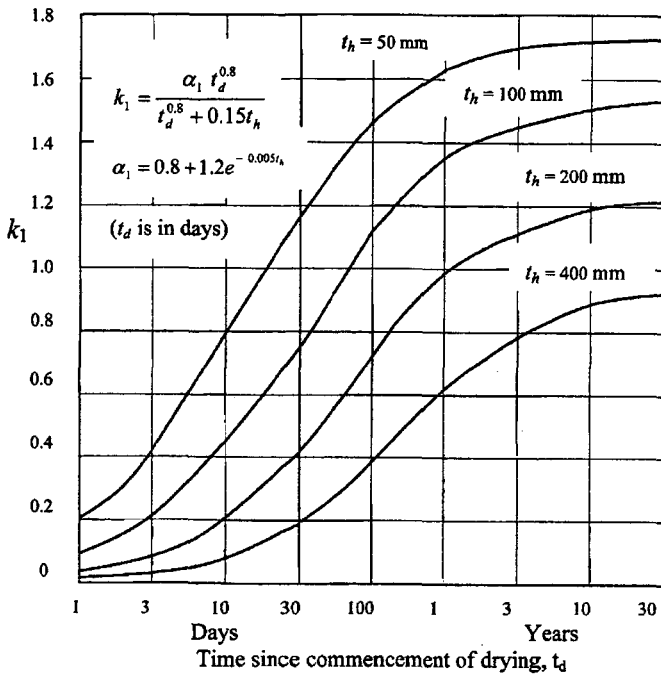


Figure 4 The factor k_1 versus time after loading (2).

Sensitivity of the Models for Predicting Shrinkage of Concrete

by A. Al-Manaseer and S. Ristanovic

Synopsis: The purpose of this study was to conduct a sensitivity study on shrinkage prediction of concrete utilizing the ACI 209 (1), GL 2000 (2), B3 (3), and CEB MC 90-99 (4) models. The sensitivity of a prediction model is function of different parameters utilized in the equations describing the model. The influence of changing input parameters on shrinkage prediction was investigated. The study reveals that the change of relative humidity will result in similar sensitivity for different shrinkage models. The autogenous shrinkage component in the CEB MC 90-99 was found to be most sensitive to strength change at 28 days. The GL 2000 was found most sensitive to cement type while the B3 model was found to be most sensitive to specimen size and type of curing. In general the B3 was the most sensitive model while the ACI was the least sensitive.

Keywords: relative humidity; sensitivity; shrinkage; strength; type of cement; type of curing

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INTRODUCTION

Accurate estimation of concrete shrinkage is significant in the design of concrete structures. Benefits of an accurate prediction model can yield to a safer design, a structure with minimal cracking, extended life and durability. Also, a precise model can reduce the cost incurred from over-design.

The error in determining input parameters can produce significant change in shrinkage prediction values. This change can be quantified by determining the sensitivity of the model used for shrinkage prediction.

SENSITIVITY

Sensitivity is presented as the percentile change in shrinkage prediction value caused by change in input parameter. Equation (1) defines sensitivity as:

$$s(p) = \frac{(\varepsilon(p_1) - \varepsilon(p_2))}{\varepsilon(p_1)} \times 100 \quad [1]$$

$s(p)$ = Shrinkage sensitivity due to change of any input parameter p such as: relative humidity RH, specimen size, 28-day compressive strength f_{cm28} .

p_1 = Initial value of input parameter p

p_2 = Value of parameter p after change

$\varepsilon(p_1)$ = Predicted shrinkage of concrete for the value of input parameter equal p_1

$\varepsilon(p_2)$ = Predicted shrinkage of concrete for after change of input parameter from p_1 to p_2

The input parameters utilized for the different models are summarized in Table 1. The limitations of input parameters, from Ref. 1-4, for each of the four models are given in Table 2.

The study utilizes Equation (1) to derive sensitivity equations for each input parameter listed in Table 1. The sensitivity equations for each model are described next.

The ACI-209 Model Sensitivity

ACI Model-Relative Humidity - Sensitivity of the model due to change of relative humidity (RH) can be determined as follows:

$$s(RH) = \left[1 - \frac{K_{sh(2)}}{K_{sh(1)}} \right] \times 100\% \quad [2]$$

K_{sh} = Relative humidity correction factor for shrinkage as defined in Figure 1.

The results of the sensitivity analysis for different values of RH are shown in Figure 2.

It can be observed from Figure 2 that the sensitivity bi-linearly increases as the change of relative humidity increases and is most sensitive in the range of 80-100%. This rate of increase was found to be higher when the change occurs from a higher initial value of RH.

ACI Model-Specimen Size - The change in shrinkage due to change of specimen size (i.e. volume to surface area ratio, V/S) can be calculated using equation (3):

$$s(V/S) = \left[1 - \frac{1.14 - 0.0035 \left(\frac{V}{S} \right)_{(2)}}{1.14 - 0.0035 \left(\frac{V}{S} \right)_{(1)}} \right] \times 100\% \quad [3]$$

V/S (1) = Initial size of concrete specimen

V/S (2) = Size of specimen after change

The equation is illustrated in Figure 3.

It can be observed from Figure 3 that the specimen size sensitivity linearly increases with change in specimen size. The rate of change in sensitivity is higher for larger initial values of (V/S(1)). It can be observed from analyzing specimen size parameter that if the V/S exceeds 315 the prediction values become very large and do not reflect the actual behavior of concrete.