

Deformation Capacity of RC Members with Brittle Details Under Cyclic Loads

by D.V. Syntzirma and S.J. Pantazopoulou

Synopsis: The sequence of failure in reinforced concrete (RC) prismatic members is used as a tool in estimating dependable deformation capacity. Response mechanisms that may limit the response leading to damage localization are identified (web diagonal cracking, bar buckling, disintegration of compressive struts due to load reversal, and anchorage failure of primary reinforcement). Deformation components are additive only if stable hysteretic response controlled by flexure prevails. In all other cases, the deformation component associated with the controlling mode of failure dominates the overall deformability of the member. Because the sequence of failure depends to a large extent on load history, deformation attained at any particular level of load is also load history dependent. This is why experimental values for deformation capacity reported in international literature are characterized by excessive scatter. The proposed methodology is applied to a number of published column tests. Analytical estimates are evaluated through comparisons with experimental results and by parameter studies conducted in order to examine the sensitivity of the estimated displacement limit at compression bar buckling to important design variables.

Keywords: deformation capacity; displacement-based assessment; nonconforming RC members; seismic design of RC

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INTRODUCTION

A primary objective of displacement-based assessment of existing RC structures is to ensure that dependable deformation capacity Δ_R of each individual component exceeds the anticipated deformation demand Δ_d . The requirement that $\Delta_R > \Delta_d$ refers to an ultimate limit state; reversal of the inequality signifies excessive damage and even collapse of the structural component.

Existing structures, designed up to an array of earlier versions of seismic codes as these evolved through the last half century, are generally classed as “low deformation capacity” systems, i.e. they have limited ability to sustain large inelastic deformation reversals without strength loss. In terms of reinforcement amounts, older structural members typically are lacking in properly detailed transverse reinforcement (confinement and shear reinforcement in beams, columns, joints and lap splices). Reconnaissance studies of damage from past earthquakes show that the above requirement of $\Delta_R > \Delta_d$ may not be always secured by old designs, either because the intrinsically available deformation capacity is negligible due to poor detailing, or because the demand is excessive due to inherent conceptual flaws in the structural system (excessive localized flexibility); the latter is seen frequently even in modern construction.

To estimate the dependable deformation capacity of RC members, it is important to determine simple models that are capable of reproducing the most prevailing parametric sensitivities observed in tests of brittle components. This objective has been pursued in the present paper through analytical modeling and evaluation of published experiments. The analysis is based on a comprehensive concept of sorting through the various alternative failure mechanisms that could control member behavior (often prior to development of full inelastic flexural action), seeking the weakest link of member response for a given cyclic displacement history. This framework is referred to hereon as Capacity-Based Prioritizing, or CBP. Departing from earlier concepts that center or expand on the estimation of drift components using cross-sectional and material properties and aspect ratio, the present approach determines the actual pattern of dependable deformations associated with the prevailing failure mechanism. In identifying the weak link of member behavior the basic premise has been that this

mechanism becomes the fuse of the member response, i.e., upon increase of the imposed end displacement, deformation is expected to localize in that fuse, and hence, beyond that stage, all other forms of nonlinear behavior may be irrelevant.

Particular emphasis is placed on alternative failures that would prevail due to poor confinement. Web diagonal tension failure, buckling of compression reinforcement, disintegration of concrete due to reversal of load, and limited anchorage or lap-splice capacity of primary reinforcement are candidate scenarios. The influence that these failure modes may have upon the deformation capacity of a poorly-detailed member depends upon the sequence in which they develop. Thus, the sequence of failure, if seen as a chain of successive events in the response history of the member, *uniquely* defines deformability, although it is not necessarily *unique* in itself, for it depends on the imposed loading history. As a rule, the various strength mechanisms *do not* degrade proportionately with load reversals. This point is reflected repeatedly in the available experimental evidence and it is why, application of the proposed analysis framework affirms conclusively that the process of assessment of deformation capacity is rather complex and cannot easily be treated by unidirectional closed form expressions.

The proposed methodology is tested on a number of tests published in international literature. Relevant experiments concern *brittle* columns with lap splicing in the critical regions under cyclic load reversals. In the paper, comparisons are made between analytical estimates and experimental results, whereas the sensitivity of the proposed methodology to important design variables is also explored through parameter studies.

RESEARCH SIGNIFICANCE

Quantifying the dependable deformation capacity of RC members, particularly members *non-conforming* to modern standards, is a milestone in the process of assessment of seismic resistance of existing structures. Existing methods of calculating deformation capacity are marked by excessive scatter even when applied to well-detailed members. In this paper a method of capacity-based prioritizing of the alternative modes of failure is used to identify localization of failure and to estimate the associated deformability of reinforced concrete members with substandard detailing representative of former construction practices.

FRAMEWORK FOR ESTIMATING DEFORMATION CAPACITY

Deformation capacity of a RC member as defined for the needs of seismic assessment is the maximum relative translation (or relative drift) the member may sustain without excessive loss of lateral load strength. To evaluate this response parameter, the usual approach is to calculate contributions to drift of the various modes of deformation that occur along the member, namely flexural, shear, and rotation due to reinforcement pullout, which are then superimposed as they are generally considered to act cumulatively [e.g. Lehman¹, Inel², Panagiotakos³, Pujol⁴]:

$$\Delta_y = \Delta_y^{flex} + \Delta_y^{shear} + \Delta_y^{slip} \quad ; \quad \Delta_u = \Delta_y + \Delta_p^{flex} + \Delta_p^{shear} + \Delta_p^{slip} \quad (1)$$

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Subscripts y and p in Eq. (1) refer to the yield and plastic (total minus yield) values of the respective terms. In all relevant published works, Eq. (1) is paired with analytical expressions for the various components [e.g. Lehman¹, Inel²]. These are derived from the basic components of deformation (curvature, shear angle, steel-strain along the anchorage) either using the mechanics of the so-called *stick-model* or are otherwise obtained empirically [e.g. Panagiotakos³]. The *stick-model* is a cantilever having the member geometry and a length equal to the member shear-span L_s , subjected to constant shear that simulates the behavior during lateral sway of a typical frame element segment between midspan and the moment resisting end support. The tip displacement of the stick-model divided by its height is the lateral drift, a response parameter directly comparable with the actual member drift. Previous attempts to correlate the various methods that rely on Eq. (1) for calculating deformation capacity at yield and ultimate with experimental data, are marked by excessive scatter, and great discrepancy between analytical estimates and experimental values of deformation capacity [Inel², Panagiotakos³, Syntzirma⁵]. Scatter is great even in well-detailed elements and deteriorates when members *non-conforming* to modern detailing standards (referred to hereon as *brittle*) are considered.

The systematic inaccuracy of what appears to be a consistent mechanics approach points to a fundamental conceptual flaw. The entire procedure for calculating deformability, either using the stick model or its many variations, is based on the assumption of a robust flexural action. From straightforward moment-rotation calculations it is evident that ideally, flexural drift would account for the largest fraction of the total, i.e. it would represent the basic component. But whereas a moment-curvature analysis could suggest a sufficiently ductile behavior up to a compressive strain level of 0.005 (cover spalling), the actual amount of displacement is often limited by other mechanisms of failure owing to sparsity of stirrups, such as diagonal tension failure of the web, buckling of compression reinforcement, disintegration of the compressive struts due to reversal of load, and limited anchorage or lap-splice capacity of primary reinforcement. Thus, the exact pattern of dependable deformations is not simply a matter of cross-sectional properties. Development of full flexural action may not always be possible in actual circumstances, due to the interaction with shear and the likely influences of all the other mechanisms of response. If such mechanisms prevail prior to development of full inelastic flexural action, then calculated inelastic flexural deformations may even be irrelevant.

The sequence in which the various response (and failure) mechanisms will occur defines a unique pattern of deformation output for the member that clearly depends on load-history since the strength values of the various modes of behavior degrade disproportionately with the extent of cracking and the number of load cycles. A critical step in the direction of determining the deformation response is to identify *the weak link of behavior*, where localization is expected to occur. To settle this issue the dependable strengths of the alternative mechanisms of failure are prioritized as described by the following qualitative statement (CBP method):

$$V_{u,\text{lim}} = \min\{V_{\text{flex}}, V_{\text{shear}}, V_{\text{lap}}, V_{\text{buckl}}\} \quad (2)$$

The static relationship between the various strength indices in Eq. (2) refers to the basic flexural stick-model described. These strength terms need be estimated by considering the cyclic load effects and degradation due to cracking. $V_{u,lim}$ is the shear force sustained by the member at failure, V_{flex} is the shear force required to develop the flexural capacity at the support (i.e. $V_{flex}=M_u/L_s$), V_{shear} is the web shear strength, V_{lap} is the shear force sustained when the capacity of the anchorage or lap splice of reinforcement is attained (i.e., $V_{lap}=M_{lap}/L_s$, with M_{lap} the moment sustained at the support at that instant). V_{buckl} is the shear force sustained when compression reinforcement buckles at the critical section; this magnitude is associated with the displacement limit at which bars are expected to buckle, which is a load-history dependent quantity. Therefore, although it is used in comparing strengths, should V_{buckl} control, it would necessarily be associated with realization of the associated displacement limit in the response curve.

Practical Implementation of the Proposed Method for Old-Type RC Members

Considering that localized deformations of any kind other than flexural may prove fatal for member integrity, it is natural to associate the ultimate drift with the onset of localization. To recognize that the contributions of the various modes of deformation to the total drift at yield or ultimate depends on the mode of localization, the familiar Eq. (1) is *modified conceptually* as shown in Eq. (3) by the introduction of weight factors, w_y and w_u , that operate on the individual contributions. *The weight factors represent the relative strength (for strength controlled mechanisms) or the relative deformation capacity (for deformation-controlled mechanisms) of the weak link controlling localization as compared with the mechanism under consideration:*

$$\Delta_y = w_y^f \cdot \Delta_y^f + w_y^{sh} \cdot \Delta_y^{sh} + w_y^{sl} \cdot \Delta_y^{sl} \quad ; \quad \Delta_u = \Delta_y + w_u^f \cdot \Delta_p^f + w_u^{sh} \cdot \Delta_p^{sh} + w_u^{sl} \cdot \Delta_p^{sl} \quad (3)$$

Apart from shear action which is considered a strength-controlled mode of behavior, all other modes are deformation-controlled. Individual components are obtained from basic mechanics [Pantazopoulou⁶]:

$$\begin{aligned} \Delta_y^f &= \frac{1}{3} \cdot \phi_y \cdot L_s^2 \quad ; \quad \Delta_p^f = (\phi_u - \phi_y) \cdot \ell_p \cdot (L_s - 0.5\ell_p) \quad ; \quad \Delta_y^{sh} = \frac{V_c}{0.4 \cdot E_c \cdot 0.8A_g} \cdot L_s \quad ; \\ \Delta_p^{sh} &= \varepsilon_{st} \cdot L_s \quad \Delta_y^{sl} = \frac{\phi_y \cdot D_b}{8} \cdot \frac{f_y}{f_{b,y}} \cdot L_s \quad ; \quad \Delta_p^{sl} = (\phi_u - \phi_y) \cdot \frac{D_b}{4} \cdot \frac{\beta \cdot f_u}{f_{b,u}} \cdot L_s \end{aligned} \quad (4)$$

Parameters ϕ_y and ϕ_u are the yield and theoretical ultimate curvature of the member; V_c is the concrete contribution to the nominal shear strength; E_c is the concrete modulus of elasticity; A_g is the gross cross sectional area and D_b is the diameter of longitudinal tension reinforcement. Variable f_u in Eq. (4) may be *less* than the actual tension capacity of the reinforcement, as the true stress developed in the primary bars may be limited by premature failure of the member. Ratio $\beta=(f_u-f_y)/f_u$ in Eq. (4) is the normalized strength increase from the point of yield to the maximum stress attained by the tension steel

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during the response. Thus, β is a variable, taking on different values throughout the response as the stress in the tension steel increases. l_p is the associated length of plastic hinge, $f_{b,y}$ is the bond stress at yielding of tension reinforcement, and $f_{b,u}$ is the bond stress corresponding to the ultimate state developed along the depth of yield penetration (here $f_{b,u}$ is taken without loss of generality as a fraction of $f_{b,y}$, i.e., $f_{b,u} = \lambda \cdot f_{b,y}$, where λ ranges between 0.5 and 0.8). Variable ε_{st} is the strain of stirrup steel (at stirrup yielding, it is calculated as $V_w/n_{st}A_{st}E_s$, where V_w is the stirrup contribution to shear strength, n_{st} is the number of stirrup layers intersecting a crack plane at 45° with the member axis, A_{st} is the area of stirrup legs in a single stirrup layer, and E_s is the modulus of elasticity of stirrup reinforcement). Clearly, the maximum value for ε_{st} for which the calculation of Δ_p^{sh} is meaningful is at stirrup yielding, for beyond that point the member is considered to have failed in shear, [Pantazopoulou⁶].

From Eq. (4) it follows that the nominal flexural deformation capacity, $\Delta^fl = \Delta_y^fl + \Delta_p^fl$, may not be realized unless flexure prevails from Eq. (2) as the weakest mode. Clearly there are several alternative combinations, such as premature bar buckling, or inadequate development capacity along the anchorage of a brittle member; this explains the dramatic scatter between experimental values and analytic expressions that are blind to localization. A critical parameter in Eq. (4) is the length of plastic hinge next to the moment-resisting support. The general form is,

$$\ell_p = \beta L_s + \frac{D_b}{4} \cdot \frac{\beta}{(1-\beta)} \cdot \frac{f_y}{f_{b,u}} \quad (5)$$

In most of the available variations of the stick model known in the literature, the second term in Eq. (5), which represents slip/pullout effects on the length of inelastic activity is taken as a constant associated with the ultimate tension capacity of reinforcement [e.g. Priestley⁷]. In reality, the plastic hinge length is a variable that depends on the maximum steel tension stress attained at the critical section (i.e. on the current value of β) and the bond stress conditions along the anchorage, particularly if premature failure terminates the response of the member prior to development of full inelastic flexural capacity.

Strength Estimates for the Various Modes of Failure

The accuracy of the proposed CBP procedure for displacement capacity estimations requires a reliable assessment of the strength terms entering Eq. (2). When correlating with test results these may be calculated from standard theory, using pertinent simplifying assumptions for expediency. One such approach is outlined in the following.

Flexural Strength — The ideal flexural strength is meaningful only if it may be safely assumed that it is supported by all other mechanisms of behavior (i.e., if $V_{flex} < \{V_{shear}, V_{lap}, V_{buckl}\}$). The bending moment sustained when the flexural reinforcement reaches yield for the first time is M_y^m , and in a monotonic loading history the ultimate flexural resistance is M_u^m . Both M_y^m and M_u^m may be calculated from standard theory.

The reduced flexural yield strength that may be sustained after cyclic reversal of load,

need consider that the compression steel has yielded in tension in the preceding cycle [Pantazopoulou⁶]. At this point, cracks will remain open in the compression zone, and hence all the compressive force will be resisted by compression reinforcement. Only if this reinforcement yields in compression may the cracks close so that the concrete may contribute to compression resistance. For equal areas of top and bottom reinforcement, the cyclic yield moment, M_y^c is $M_y^c = A_s f_y (d - d_2)$. For the same case, the ultimate cyclic moment theoretically is $A_s f_u (d - d_2)$, but in the absence of confinement the maximum compressive strain, ϵ_{cu} , is not expected to exceed 0.005 (the cover spalling strain) and it is therefore best to take $M_u^c = M_y^c$. For unequal areas of compression and tension reinforcement, $M_y^c = A_{s2} f_y (d - d_2)$, and $M_u^c = A_{s2} f_u (d - d_2) + (A_{s1} - A_{s2}) f_u j d$ (the last term to be included if confinement is available), where A_{s1} and A_{s2} is area of the tension and compression reinforcement respectively; d and d_2 is the distance from the extreme compression fiber to the centroid of the tension and compression reinforcement and $j d$ is the distance of the tension steel to the centroid of the compression zone on the cross section. The corresponding values of V_{flex} are, $V_{flex,y}^c = M_y^c / L_s$, $V_{flex,u}^c = M_u^c / L_s$.

In the correlation study presented in the following sections, flexural strength is calculated conventionally under the assumption that flexure is the predominant mechanism of failure, so as to point out the dramatic influence on deformation capacity owing to premature occurrence of other failure mechanisms.

Shear Strength — Based on recent tests it has become evident that shear strength of reinforced concrete degrades faster with cyclic load, for higher ratios of shear demand to shear supply. Wood and Sittipunt⁸ had proposed a limit of 60% as a cutoff point in identifying shear failures from other types of failures when processing the experimental literature on walls. Thus, a shear failure is likely to occur when $0.6 V_{shear} < V_{flex}$, even if the nominal check prescribed by the code holds, namely that $V_{shear} < V_{flex}$.

Various models have been proposed to establish shear strength of reinforced concrete as a function of deformation [e.g. Priestley⁹, Moehle¹⁰]. A common working hypothesis is that the shear strength of cracked reinforced concrete comprises a primary contribution, V_w of the web reinforcement (the tension ties of the Ritter–Moërsch truss analogy) and secondary contributions, V_c . Here, these terms are operated on by a strength reduction coefficient, k , which is a function of imposed displacement ductility, μ_Δ [Lynn¹¹]:

$$V_{shear} = k \cdot (V_w + V_c); \quad 0.7 \leq k = (1.15 - 0.075\mu_\Delta) \leq 1.0 \quad (6a)$$

$$V_w = A_{st} \sum_{n_{st}} f_{st,i} \quad (6b)$$

The expression for k was selected from among the various published alternatives due to its simplicity but also because it was found to produce conservative estimates when tested against a large database of experiments on prismatic RC members under reversed cyclic load [Syntzirma⁵]. Note that the reduced shear strength value is 70% of the initial value at a displacement ductility of 6. This is consistent with theoretical investigations based on the diagonal compression field theory, which have shown that at a ductility of shear

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distortions in the order of 2 (yielding of shear reinforcement occurring in a range of shear distortion of 0.004–0.006), the total shear resistance is reduced to 75% of the initial value [Tastani¹²]. (Ductility of shear distortions concerns the ratio of peak to yield shear strain in the plastic hinge region, γ_u/γ_y ; therefore the total displacement ductility is much higher than the reference value of 2). Through k it is recognized that the truss component V_w is also affected by damage accumulation due to the tension–softening of the compressive struts that support the truss function [Lynn¹¹, Vecchio¹³, Martin–Perez¹⁴]. V_w is nonzero only if ties are spaced close enough to secure that any diagonal crack (taken for simplicity at an angle of 45° with respect to the longitudinal axis of the member) is crossed by at least one stirrup layer. In Eq. (6b), n_{st} is the number of stirrups crossing a diagonal web crack, obtained as the integer part of the d/s ratio, where s is the spacing of stirrups. A_{st} is the area of all stirrup legs in a single stirrup layer, crossing the crack plane. $f_{st,i}$ is the maximum stirrup tensile stress developed over the anchorage length of the i -th stirrup. Open stirrups, or inadequately anchored stirrups may not be able to develop their full yield strength, $f_{y,st}$, if the critical section where they cross the crack is very near to their anchorage. Thus, $f_{st,i} = f_{y,st} L_{b,i}/0.7L_b$, where $L_{b,i}$ is the available anchorage length of the i -th tie measured from the point where it is intercepted by the crack to the end hook, and L_b is the standard straight development length for the bar diameter of the ties considered. For open stirrups and usual section heights, the sum in Eq. (6b) corresponds to $0.5n_{st}f_{y,st}$, i.e. only half the stirrups are effective.

The concrete contribution term is attributed to other mechanisms of resistance that get mobilized through diagonal tension of the concrete web, i.e., the dowel action of longitudinal reinforcement spanning across cracks, the frictional interlock between cracked interfaces and reinforcement to concrete bond (tension–stiffening) along the bar between successive cracks. A point of difference between the various models is whether the contribution of axial compression to shear resistance (which is believed to delay opening and affect inclination of cracks) be accounted for under V_c or whether it be added separately to highlight its significance as a distinct mechanism of resistance (referred to as arch action). It can be shown that shear distortion imposes a tensile strain on all reinforcements, the magnitude of which may be estimated from equilibrium and compatibility requirements. If the axial stress level in the member is low or negligible, the net stress in all longitudinal reinforcement may be tensile, particularly for low aspect ratios (high shear demands) [Tastani¹²]. Based on the 45° truss model, the tensile strain increment in the longitudinal reinforcement resulting from shear action alone is, $\epsilon_s = 0.5V_s/E_sA_s$ where V_s is the shear force that is actually resisted by the stirrups (Eq. 6b). Similarly, strain owing to shear develops in transverse reinforcement: ϵ_{st} is given as $V_s/E_s \cdot n_{st} \cdot A_{st}$ (defined in Eq. 4). Considering these strain values as additive to the flexural strains, it is concluded that unless the compression strains caused by flexure or axial load in the compression zone are very high, the compression reinforcement may carry a net tensile strain upon load reversal (i.e., a full reversal of load will not produce symmetric strains in tension and compression). This means that compression reinforcement is principally susceptible to sideways buckling as described in the following section. A simple assumption is that upon load reversal, if the normalized axial load is less than $(\rho_{s1} - \rho_{s2})f_y/f_c$ it should be assumed that the cracks remain open and therefore the V_c term in Eq. (6a) should be taken zero (this is similar to the limit $N/A_g f_c' < 15\%$ prescribed by Codes

for critical zones of beams). Hence:

$$\text{if } \frac{N}{A_g \cdot f_c'} \geq (\rho_{s1} - \rho_{s2}) \frac{f_y}{f_c'} \Rightarrow V_c = 0.5 \sqrt{f_c'} \cdot \left(\frac{d}{L_s} \sqrt{1 + \frac{N}{0.5 \sqrt{f_c'} \cdot A_g}} \right) \cdot A_g \text{ (MPa)} \quad (7a)$$

$$\text{otherwise, } V_c = 0 \quad (7b)$$

Thus, the V_c term in Eq. (7) is defined as the product of the principal tensile stress at diagonal cracking by the gross section area. N is the axial load and ρ_{s1} , ρ_{s2} are the tension and compression reinforcement ratios respectively. The tensile strength of concrete is taken as $0.5 \sqrt{f_c'}$ (MPa) ($6 \sqrt{f_c'}$ in psi).

Anchorage and Lap-Splice Strength — Premature failure of a lap-splice or anchorage effectively limits the force developed in the reinforcing bar to a value lower than its axial strength. In brittle construction, common bond-related problems are owing to (a) the practice of splicing the main column reinforcement just above the base of each floor (i.e., within a critical section), usually providing small lap lengths without the necessary confinement through stirrups, (b) the use of smooth reinforcement, where anchorage capacity depends on frictional mechanisms mobilized along the anchored length.

The force F_s , that a lap-splice or anchorage zone of length L_b may develop, is equal to the total frictional force that develops on the bar lateral surface within the length L_b . The frictional force is proportional to the normal clamping force, through the frictional constant μ_f ; for concrete, μ_f is usually taken between 1 and 1.5 [Priestley⁷]. In the absence of stirrups in the lap region, the clamping pressure is only provided through the tensile resistance of the concrete cover, f_t' , developing over a crack path of length p as defined by Priestley⁷. Therefore, the force that an unconfined lap splice may develop equals to, $F_s = p f_t' \cdot L_b$, and the corresponding bar stress $f_s = p f_t' \cdot L_b / A_b$, where A_b is the cross sectional area of one lapped bar. Even if $f_s \geq f_y$, the actual strength that may be supported by the lap length without stirrups will disintegrate upon load reversal. The lap region being also a critical column segment, will be located alternately in the compression and tension zone of the column cross section as the direction of the seismic force reverses. For axial compressive strains in excess of 0.0015–0.0020, tensile cracks parallel to the direction of the compressive force are expected to occur (with axial compressive strains in that range, the corresponding transverse strains are approximately half that value, i.e. Poisson's ratio near peak stress ≈ 0.5). Upon reversal of the load, the lap region will try to develop tension force in the bar, but with the cover cracked, the clamping force will be diminished.

Therefore, even if the theoretical flexural strength is reached once, upon the first cycle of loading to a full reversal, response is expected to degrade rapidly. Using the familiar k_r concept of ACI 318¹⁵ the development capacity of each lapped longitudinal bar is obtained from the maximum clamping force that stirrups may provide, determined as

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$\mu \cdot n_{st} \cdot A_{tr} f_{yst} / n_b$ where n_b is the total number of bars restrained by a total of n_{st} stirrups in the direction considered along the lap length L_b and A_{tr} is the area of stirrup legs along one direction of restraint (for stirrups to be accounted for in this calculation, they should interrupt the crack path p mentioned in the preceding). A coefficient of friction equal to 1.4 was suggested when transverse reinforcement is used as a restrainer. Thus, the axial force supported by each lapped longitudinal bar is,

$$F_s = 1.4 \cdot A_{tr} \cdot f_{y,st} \cdot n_{st} / n_b + p \cdot f_t' \cdot L_b \quad (8)$$

The second term is to be ignored if the maximum axial compression strain exceeds 0.002.

Dependable Ductility at Buckling of Compression Steel

Upon reversed cyclic loading the strength of compression reinforcement is affected primarily by the load history and the arrangement of stirrups thereby influencing the value of V_{buckl} in Eq. (2). Here $V_{buckl} = M_{buckl} / L_s$ where M_{buckl} the flexural moment of the cross section when compression steel reaches imminent buckling conditions.

In the absence of well anchored, stiff and closely spaced ties, elastic symmetric buckling may occur before the bar may reach yielding. An S400 (GR60) compressed bar yields in compression before it buckles if stiff ties are spaced at $s/D_b = 17.5$; the corresponding ratio for S220 (GR33) is 23.7. If ties are very flexible, it is conservative to assume that the length of the buckled bar may extend over the entire length of the plastic hinge, which may encompass several tie spacings. The corresponding elastic axial compressive strain sustained by the bar is $\varepsilon_{s2} = \pi^2 D_b^2 / 16 l_p^2 \leq \varepsilon_y$. If elastic buckling may be safely precluded, the strain ductility of compression reinforcement is calculated considering the following:

(1) Effect of s/D_b ratio on dependable strain — In the plastic hinge region where cracks remain open under cyclic shear reversal, sideways buckling is the expected mode of failure of compression reinforcement. It can be shown that the relationship controlling the critical spacing of ties and the reinforcement stress that may be sustained prior to buckling is given by [Syntzirma¹⁶],

$$s / D_b = \psi \sqrt{E_s / f_s} \quad ; \quad E_s = 200 \text{ GPa}, \text{ if } f_s \leq f_y; \quad E_s = E_r, \quad \text{if } f_s > f_y \quad (9)$$

where ψ is a coefficient taking on the values of 1.5 and 0.785 in the cases of symmetric and sideways buckling, respectively. In the remainder of this study, the value of $\psi = 0.785$ will be used to account for the effect of strain reversal and permanent distortion in the plastic hinge region on the shape of bar buckling. E_r is the double modulus stiffness which is a weighted average between the tangent stiffness of the bar, E_h , and the initial elastic modulus E , to account for the elastic unloading of the tension side of the buckling bar as it bends. For easy reference the ratio of E_r/E is plotted against the ratio E_h/E in Fig. 1a [Papia¹⁷]. Given the tie spacing in a member with poor detailing, the dependable axial compressive strain at which reinforcement is likely to buckle, ε_{buckl} , is calculated

from Eq. (9), and clearly depends upon the strain hardening characteristics of compression steel. Fig. 1b plots the estimated strain ductility at buckling ($\epsilon_{buckl}/\epsilon_{sy}$) versus the corresponding s/D_b ratio for typical examples.

(2) The intrinsic strain capacity of the confined core — The above relationships break down if $E_h=0$. For the bar to survive through the yield plateau region and to enter strain-hardening, the adjacent core must be effectively confined so as to enable load redistribution from the bar to the concrete once the bar stiffness decreases as a consequence of buckling. The axial compressive strain that corresponds to the deformation capacity of the confined core is estimated from the normalized effective confining pressure in the direction of lateral sway, $k_e \rho_{s,st} f_{yst}/f_c$, using pertinent confinement models [Priestley⁷, Pantazopoulou⁶].

Thus, the transverse reinforcement uniquely determines the critical compression strain that the core may sustain without loss of integrity; for a given s/D_b ratio, the critical compression strain in moment-curvature analysis should not exceed the limit $\epsilon_{crit} = \min\{\epsilon_{buckl}, \epsilon_{cu}\}$, where, ϵ_{buckl} is the critical buckling strain obtained from Eq. (9) (or from Fig. 1b) and ϵ_{cu} is the strain in the compression reinforcement when the maximum core strain reaches the limit given by the confinement model (Eq. 11). If M_{buckl}^m is the monotonic moment strength value corresponding to a maximum compressive strain of ϵ_{crit} (either on the extreme fiber or in the centroid of the compression steel depending on the controlling min. limit) then the corresponding shear V_{buckl}^m in Eq. (2) is, $V_{buckl}^m = M_{buckl}^m/L_s$.

(3) Effect of load history on buckling strain — Under reversed cyclic load, buckling of reinforcement may occur in many different ways depending on the overall circumstances of the compression zone, including the load history. A bar in compression may have yielded in tension during the previous cycle. Whether cracks are open or closed depends on the amount of tensile strain, $\epsilon_{s,t}$, sustained by the bar in the previous excursion (Fig. 2).

Full reversal of strains and closure of cracks in the compression zone may occur only in the presence of significant axial load. If ϵ_c is the strain at the extreme fiber of the compression zone, the corresponding strain in the compressed bar is $\epsilon_{s,o2} = (1 - (d_2/x))\epsilon_c$, where x is the depth of compression zone and d_2 is the distance to the compression reinforcement. For symmetric load history (equal imposed drifts in alternating directions), the *strain demand* (and corresponding stress) that must be entered in Fig. 1b or Eq. (9) to evaluate the likelihood of buckling (by comparing with the supply) is:

$$\begin{aligned}\epsilon_{s2} &= \epsilon_{s,t} - \epsilon_y + \epsilon_c \cdot \left(1 - \frac{d_2}{x}\right) = \phi \cdot (d - x) - \epsilon_y + \phi \cdot (x - d_2) \cdot \left(1 - \frac{d_2}{x}\right) \approx \phi \cdot d - \epsilon_y; \\ \mu_{\epsilon,2} &= \epsilon_{s2} / \epsilon_y = \mu_\phi \cdot \frac{1}{1 - x/d} - 1\end{aligned}\tag{10}$$

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where ϕ is the maximum sectional curvature and μ_ϕ is the corresponding curvature ductility experienced during the preceding cycle (this in turn is function of maximum displacement demand, plastic hinge length, axial load ratio, and primary steel area ratios).

EVALUATION OF EXPERIMENTAL EVIDENCE

It was stated before that previous attempts at correlating experimental evidence with closed-form analytical expressions for deformation capacity have generally produced unsatisfactory results characterized by excessive scatter. Apart from the experimental scatter normally expected in concrete, as well as scatter owing to the experimental conditions (support flexibility, etc.), the primary reason for large dispersion of the results is the dependence of cracking and damage on the load history. To illustrate this point, as well as the performance of CBP method in practical calculations of deformation capacity, a correlation study is conducted in this section of the paper. The test data reported by Lynn¹¹ and Aboutaha^{18,19} were used as a point of reference in the investigation. The experimental series comprised seven full-scale column specimens with details typical of pre-1970's construction.

The typical column of the series reported by Lynn¹¹ is illustrated in Fig. 3a having a clear height of approximately 3.00m and a 455mm square cross-section, i.e. an aspect ratio of 3.75 calculated over half the clear span. The four specimens in the reference experimental study had been tested in *double-curvature*, by imposed lateral load reversals and a constant axial load (at low and intermediate levels) as listed in Table 1a, until loss of vertical load carrying capacity.

Longitudinal reinforcement was evenly distributed around the perimeter of the cross-section; bars were $\Phi 31.75\text{mm}$ (#10), with a clear cover of 50mm (2" in.). Two configurations had been used for transverse reinforcement: (a) $\Phi 9.5\text{mm}$ (#3) perimeter stirrups with 90-degree bends at a spacing of 455mm ($\approx d$) and (b) $\Phi 9.5\text{mm}$ (#3) perimeter stirrups and 'diamond' ties with 90-degree bends at a spacing of 300mm, except for the lap-splice region where tie spacing was reduced to 100mm. Two of the specimens under consideration had $20D_b$ longitudinal bar lap-splices just above the top surface of the foundation block. Primary reinforcement had a reported average yield stress of 330MPa; 400MPa for the transverse bars (stress-strain diagrams were provided). Table 1a summarizes the test specimen details. The designation code is as follows: the first numeral is the area ratio of longitudinal reinforcement (3% over the gross area), C or S stands for continuous or spliced reinforcement, L or M for light or moderate axial load, H or D for hoop or diamond shaped stirrup, whereas the last numeral is the spacing of the transverse reinforcement in inches. A fictitious case with identical overall geometry as the specimens was also considered in the present study for completeness: the column is assumed to be axially loaded at intermediate levels, with a $20D_b$ lap-splice at the critical region and hoops at a spacing of 457mm (the point here was to isolate the effect of high axial load on deformation capacity owing to bar buckling).

The geometry and properties of the typical column specimen reported by Aboutaha^{18,19} are given in Fig. 3b and Table 1b. Clear height was approximately 2.70m. Cross section

was rectangular for specimens FC4 and FC5 and square for specimen FC15 leading to an aspect ratio of approximately 6.90 calculated over the clear span. The three specimens in the reference experimental study had been tested in *single-curvature*, by imposed lateral load reversals until loss of vertical load carrying capacity.

Longitudinal bars were $\Phi 25.40\text{mm}$ (#8), having 430MPa reported average yield stress. Three different configurations had been used for transverse reinforcement as shown in Fig. 3b; stirrups were $\Phi 9.5\text{mm}$ (#3) with yield stress equal to 331.20MPa (Grade 40) spaced at 400mm (16in.). The actual stress-strain data was postulated (using the recommendations by Malvar²⁰ for N.A. steels) exploiting all relevant reported information from the reference study. Specimens had 24D_b long lap-splices of primary reinforcement just above the top surface of the foundation block. Table 1b lists peak load, yield and ultimate drift values as obtained from the reported diagrams. Ultimate drift corresponds to a postpeak residual capacity equal to 80% of the peak load.

ANALYSIS RESULTS

As a starting point a typical moment-curvature analysis was conducted assuming a robust flexural response mechanism. Subsequently, these results were re-evaluated considering the effects on moment-curvature of the shear strength model, the lap-splice strength model and the compression reinforcement strength model – as defined in the previous sections of the paper.

Calculated strain ductility at buckling of compression reinforcement is plotted against the s/D_b ratio in Fig. 1b. Interaction diagrams were obtained directly from the experimental stress-strain relationships of longitudinal reinforcement for the tests conducted by Lynn¹¹, and from the assumed stress-strain model for Aboutaha's tests^{18,19}. Survival of reinforcement over the yield plateau (which extended to a strain ductility of 10.9 and 1.6 for the two groups of tests respectively) depends on the effectiveness of confinement.

The ε_{cu} term in Fig. 1b is the strain corresponding to a postpeak residual compressive stress equal to 85% of the strength. Once concrete reaches its ultimate strain capacity the resistance it contributes to buckling of reinforcement ceases, and the phenomenon ensues as controlled by the s/D_b ratio and the stress-strain characteristics of the reinforcement. A conservative approximation to ε_{cu} (Eq. 11) is a lower bound envelope of concentrically loaded confined concrete tests, [Pantazopoulou⁶].

$$\varepsilon_{cu} = \varepsilon_{co} (1 + 24.6 \cdot (k_{eff} \rho_{s,st} f_{y,st} / f'_c)); \quad \varepsilon_{co} = 0.0035 \quad (11)$$

where k_{eff} is the stirrup confinement effectiveness coefficient and $\rho_{s,st}$ is the ratio of shear reinforcement in the direction of sway. Based on the above, the calculated strain ductility of the specimens is negligible, because of poor confinement. Thus, at an axial compressive strain ductility of approximately 2.0 (which corresponds to a curvature ductility of $\mu_\phi \approx 2.60$ according with Eq. (10)) concrete would start to disintegrate; at that stage with the longitudinal steel in the yield plateau ($E_h=0$) bar buckling is imminent.

Figure 4 plots the experimental and calculated load-drift envelopes as a group of curves

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for all cases considered. Each curve represents the solution obtained by allowing for the inclusion of one additional failure mechanism in the original flexural analysis (i.e., flexural, shear, lap-splice, bar buckling) so as to illustrate the variability in deformation capacity and failure mode as predicted by the model. The curve with the lowest deformation capacity is the actual analytical response prediction for the circumstances in consideration. Labels are explained in Fig. 4a for specimen 3SLH18. Calculation of displacements and drifts from deformation measures was done using virtual work principles with the plastic hinge length obtained from Eq. (5).

Note the substantial strength degradation calculated by the model when all failure modes are considered – this confirms the experimental report about shear and/or splice driven failures for all specimens owing to strength disintegration owing to the magnitude of imposed ductility and number of load cycles. Lynn¹¹ reports initiation of yielding of longitudinal reinforcement in the lap splice region for specimens 3SLH18 and 3SMD12. Specimen 3SLH18 had experienced considerable degradation of the splice region before shear failure occurred in the lap zone. Specimen 3CMH18 had failed in shear immediately or shortly after the longitudinal bars reached the yield strain. Similarly, Aboutaha^{18,19} reports splice failure ensuing the formation of vertical splitting cracks along the lap region for all specimens (i.e, FC4, FC5 and FC15).

The proposed analytical procedure is rather conservative in estimating the actual member strengths but reproduces with great accuracy the predominant mode of failure and available deformation capacity. According with the calculated results, not all the specimens tested by Lynn¹¹ would reach their yield strength. Based on the proposed analysis procedure, specimens 3SMH18 and 3SLH18 would experience shear failure assisted by buckling of longitudinal reinforcement and lap-splice disintegration; specimens 3CLH18 and 3CMH18 were estimated to exhibit a shear failure accompanied by bar buckling, whereas specimen 3SMD12 would experience flexural failure supported by the other modes of failure. Similarly, based on the calculated results, the specimens tested by Aboutaha^{18,19} would experience a shear failure. Note that in the absence of axial load in those tests a particularly optimistic typical moment curvature analysis would be obtained should flexure be considered as the predominant response mechanism.

Evidently further investigation of the expressions calculating the strength over the various modes of failure need be carried out in order to improve the accuracy of the analytical estimates, by considering the interaction among different modes – the underside of this is that with many response mechanisms interacting and experiencing simultaneous degradation it is occasionally difficult to distinguish from experimental observation the mode of failure. Yet, the proposed methodology including the expressions used to calculate the terms in Eq. (2) was quite sufficient to identify the deficiencies of current design concept, resulting in very successful estimates of dependable deformation capacity and in properly identifying the likely modes of failure.

INVESTIGATION OF BUCKLING OF COMPRESSION REINFORCEMENT

Strain Ductility at Buckling — In the correlation study, particular emphasis was placed on the calculation of strength of compression reinforcement. The normalized strain at

buckling of longitudinal steel was obtained as a function of s/D_b . The stress-strain relationship for steel in the strain-hardening region was modeled according with Mau²¹. A sensitivity analysis is presented in this section so as to assess the margin of error resulting from variability of the parameters controlling strain hardening. Analysis is done on a member with a 500 mm square cross section; a total of eight $\Phi 20$ bars (3 per side) and a cover of 40mm. The shear span was 1500mm. Concrete strength was taken as $f'_c=25.0\text{MPa}$. Two types of transverse reinforcement were taken into account as shown in Table 2. Four types of steel were considered (Table 3), the three of which represent European construction steels, whereas the fourth type is typical of construction steel used in North America.

Results of the sensitivity analysis are summarized in Fig. 5. The determining parameter for the shape of the resulting strain ductility vs. stirrup spacing interaction diagrams (steepness of the curves in the range of practical ratios of s/D_b) is the assumed initial slope of the hardening curve E_h of the stress-strain diagram of reinforcement. An increase in the strain at the onset of strain hardening only shifts the curve upwards in the y axis. The critical compression strain for concrete in order to enable load redistribution thereby sustaining the steel over the plateau is plotted in Fig. 5(a–d) using the assumption of a poorly or a well confined cross-section (square hoop, vs. hoop and diamond tie configuration in Table 2). Curves are obtained in accordance with Eq. (11). Evidently, when confinement of concrete is negligible due to the poor stirrup effectiveness and large spacing, bar buckling is controlled by the critical compression strain of concrete. On the contrary, in the case of a well confined section (case 2 in Table 2) substantial strain ductilities are calculated. Figures 5(e–f) plot similar results but with different resulting strain ductility values for the 4th type of steel.

Effect of Load History on Buckling of Longitudinal Reinforcement — From Eq. (10) it follows that the strain capacity of compression reinforcement depends on load history and particularly on the largest previous inelastic excursion in tension prior to load reversal into compression of the bar under consideration. To illustrate the significance of this point, three different displacement histories are considered (Fig. 6). All displacement histories are symmetric cycles of increasing or constant amplitude, all leading to a different displacement ductility at the fourth cycle. Results calculated using Eq. (10) are listed in Table 4 and Fig. 7, where strain ductility is plotted on the vertical axis versus displacement ductility on the horizontal axis.

Apparently, for a specimen that has been subjected to the first loading history (Fig. 6a), even a particularly high ratio of s/D_b would not prevent it from reaching a strain ductility of approximately 2.70 (Fig. 7). On the other hand, a specimen that would have been subjected to the third loading history would need an excessively small ratio ($s/D_b \approx 2.33$) in order to achieve the requested strain ductility of approximately 8.0.

CONCLUSIONS

This paper attempts to interpret the observed scatter in reported deformation capacities of RC elements subjected to lateral load histories, with a particular emphasis on old-type members (with brittle details). It is illustrated that by consistently prioritizing the

potential failure mechanisms of the member it is possible to identify the weakest link of member behavior and the corresponding mode and location of localization of deformations. Through this procedure, referred to in the paper as CBP or Capacity-Based Prioritizing of failure, the dependable deformation capacity associated with the prevailing mode is established with sufficient accuracy. The proposed methodology is used in analyzing the response of brittle column specimens tested under cyclic load as published in the literature. A sensitivity study is carried out to illustrate the effects of load history and concrete core confinement on the axial compression strain ductility of reinforcement. Furthermore, the study illustrates the need for additional investigation to improve the strength estimates of the various failure modes. A particular research need is on the influence of axial load on shear strength, the mechanics of disintegration of lap splices, and the interaction of competing mechanisms of behavior (e.g. shear and lap-splice) when these have similar strengths.

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Table 1a — Reference tests from Lynn et al.¹¹ used in parametric investigation

Spec. ID	f'_c	P	s	l_s	V_u	Δ_y	Δ_u	R.F. Mode ^s	E.F. Mode ^{ss}
3CLH18	25.6	0.09	457	—	268.4	19.1	30.5	S+B	S+B
3SLH18	25.6	0.09	457	635.0	264.0	16.0	30.5	L+S	S+B+L
3CMH18	27.6	0.26	457	—	334.4	23.4	30.5	S	S+B
3SMD12	25.5	0.28	305	635.0	374.0	22.9	45.7	F+L	F+S+B+L
3SMH18 (fictit.)	25.6	0.28	457	635.0	—	—	—	—	S+B+L

f'_c : concrete strength (MPa); P: axial load as % of $f'_c A_g$; s: spacing of stirrups (mm); l_s : lap-splice length (mm); V_u : reported maximum shear force (kN); Δ_y : reported yield displacement (mm) Δ_u : reported maximum displacement (mm); ^sR.F. Mode: Reported Failure Mode; ^{ss}E.F. Mode: Estimated Failure Mode — ^FF for Flexural; S for Shear; L for Anchorage or Lap-Splice; B for Buckling

Table 1b — Reference tests from Aboutaha^{18,19} used in parametric investigation

Spec. ID	f'_c	s	l_s	V_u	R.F. Mode ^s	E.F. Mode ^{ss}
FC4	19.7	406.4	609.6	176.0	L	S
FC5	20.6	406.4	609.6	182.3	L	S
FC15	28.7	406.4	609.6	204.3	L	S

Table 2 — Types of Transverse Reinforcement

Confinement	d_s	s	$f_{y,st}$	Pat.	$\rho_{s,tr}$	$k_{s,tr}$
Poorly confined cross-section	5	280	400.0	h	0.0007	0.2
Well confined cross-section	12	100	400.0	h+d	0.0189	0.6

d_s : diameter of stirrups (mm); $f_{y,st}$: yield strength of stirrups (MPa); Pat.: stirrup pattern (hoop and/or diamond); $\rho_{s,tr}$: volumetric ratio of the transverse reinforcement; $k_{s,tr}$: stirrup confinement effectiveness coefficient

Table 3 — Types of Steel used as Longitudinal Reinforcement

Type of Steel	ϵ_y	ϵ_h [#]	ϵ_u	E_h (MPa)	f_y (MPa)	f_u (MPa)
1 st type	0.0025	0.005	0.040	20,000	500.0	700.0
2 nd type	0.0025	0.005	0.040	40,000	500.0	700.0
3 rd type	0.0025	0.005	0.040	60,000	500.0	700.0
4 th type	0.0025	0.005	0.150	20,000	500.0	700.0

[#] ϵ_h is the strain that signals the onset of the hardening branch

Table 4 — Critical s/D_h Ratios for Estimated Ductilities

Displ. Hist. ID	μ_Δ at the 4 th Cycle	Curvature Duct. μ_ϕ	Strain Duct. μ_ϵ	Critical Ratio s/D_h
No1	2.00	3.33	2.70	10.22
No2	3.00	5.66	5.28	4.78
No3	4.00	7.99	7.87	2.33

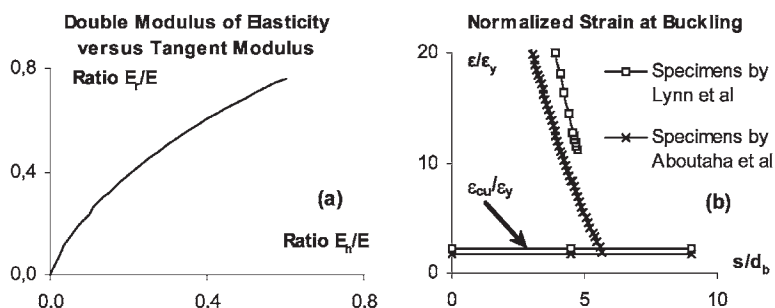


Figure 1 — (a) Calculation of Double Modulus from Tangent Modulus [Papia et al.¹⁷]; and (b) Strain Ductility of Compression Reinforcement versus the s/d_b Ratio; Estimated Dependable Concrete Strain Ductility also plotted.

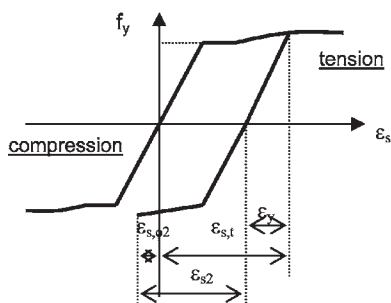


Figure 2 — Critical Compression Strain Under Cyclic Load Reversals.

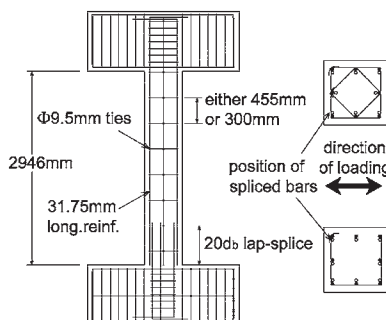


Figure 3a — Typical Column Test Specimen [Lynn et al.¹¹].

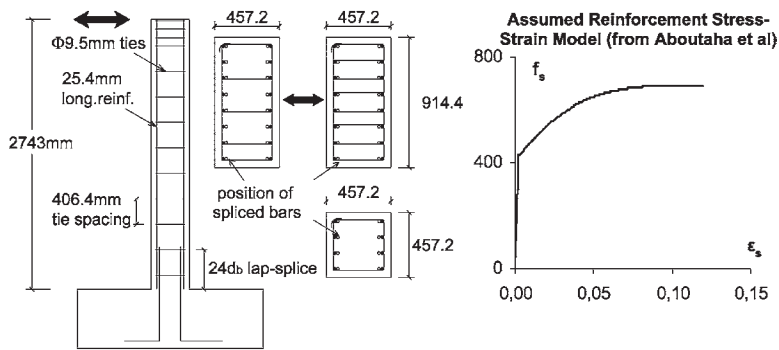


Figure 3b — Typical Column Test Specimen; Assumed Reinforcement Stress–Strain Model [from Aboutaha et al.^{18,19}].

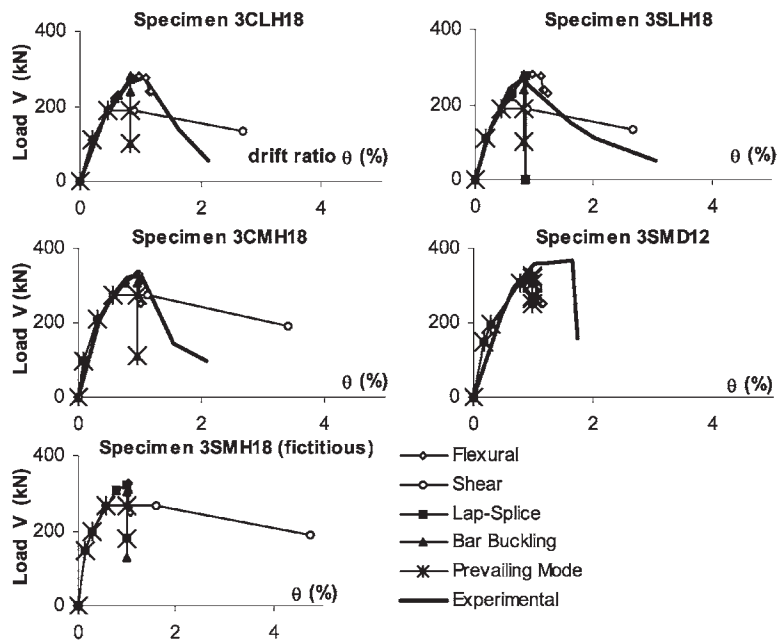


Figure 4a — Correlation of Analytical and Experimental Response Envelopes; Deformation Capacity Limited by Various Modes of Failure.

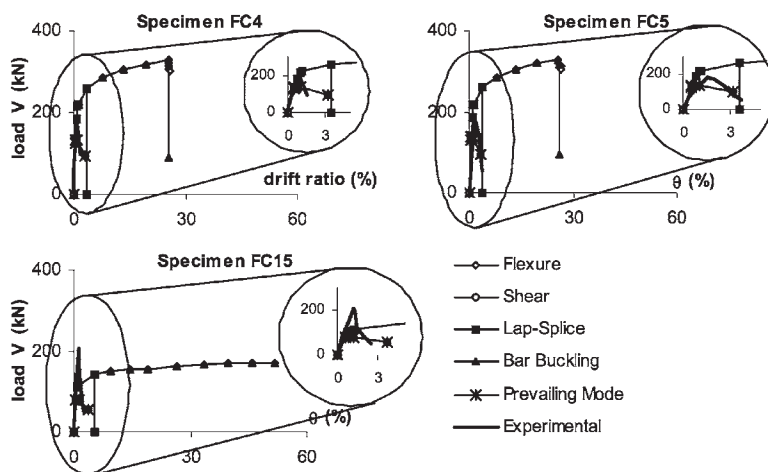


Figure 4b — Correlation of Analytical and Experimental Response Envelopes; Deformation Capacity Limited by Various Modes of Failure.

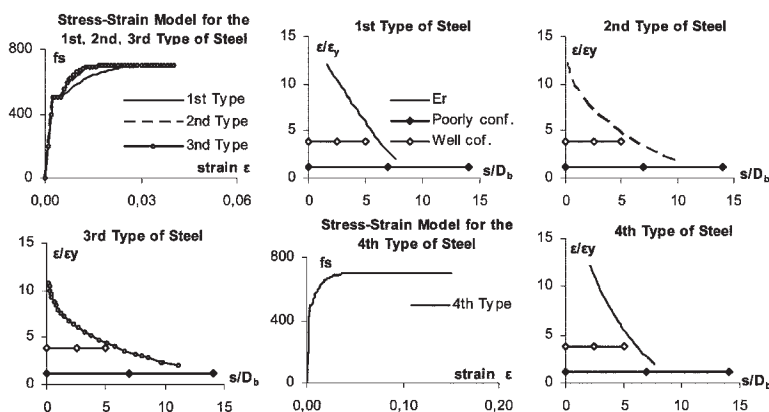


Figure 5 — Stress-Strain Diagram for each Type of Steel; Compression Strain Ductility at Buckling for each Type of Steel.

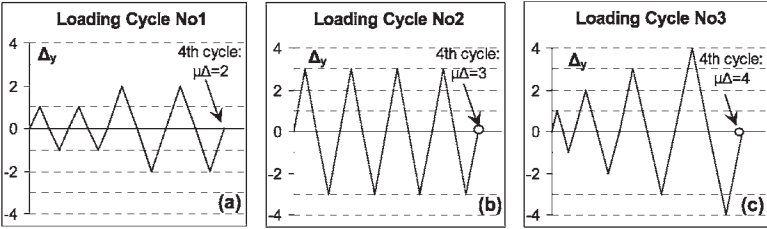


Figure 6 — Assumed loading histories.

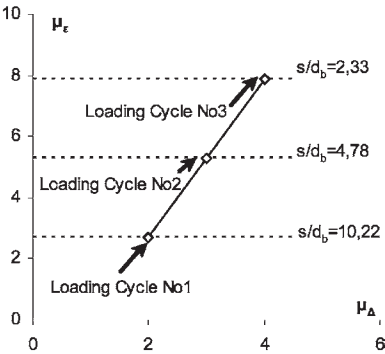


Figure 7 — Strain Ductility at Buckling versus Displacement Ductility.

Effects of Displacement History on Failure of Lightly Confined Bridge Columns

by R.T. Ranf, M.O. Eberhard, and J.F. Stanton

Synopsis: Six nominally identical reinforced concrete columns were subjected to a variety of lateral displacement histories to evaluate the effects of cycling on their failure displacement and failure mechanism. The columns, typical of bridges constructed before the mid-1970s, had circular cross-sections, low axial loads, and little transverse reinforcement. Shear failure caused five of the six columns to lose their axial load carrying ability at drift ratios between 3% and 5%. The sixth column failed in an axial-flexure mode at a drift ratio of 6%. Increasing the number of cycles at each displacement level from one to fifteen decreased the maximum displacement preceding flexure-shear failure by approximately 35%. The effect of cycling on damage accumulation was modeled with the Park-Ang damage model, a Modified Park-Ang damage model, and a Cumulative Plastic Deformation damage model. The Cumulative Plastic Deformation model correlated best with the observed damage, and it was the easiest to implement.

Keywords: bridges; columns; cycling; failure; reinforced concrete; shear

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INTRODUCTION

The San Andreas fault and the Cascadia subduction zone have the potential for generating large-magnitude, long-duration earthquakes that might last several minutes (Heaton and Kanamori 1984, Kramer et al. 1998). Such long-duration earthquakes would impose numerous cycles of displacement on structures (Price 2000). Engineers need tools to evaluate the effects of this repeated cycling on damage accumulation in structures, especially in older reinforced concrete systems.

This paper discusses the effects of cycling on damage in laterally-loaded reinforced concrete columns typical of bridge construction of the 1950s, 1960s and early 1970s. Such columns usually have low axial loads and much less transverse reinforcement than is required in new bridges (Eberhard and Marsh 1997). The effect of cycling was investigated using three damage models: the Park-Ang damage model (Park and Ang 1985); a modified version of the Park-Ang model that does not require an estimate of the yield displacement; and the Cumulative Plastic Deformation damage model.

EXPERIMENTAL PROGRAM

This section describes the methodology followed to proportion the prototype column, the resulting test column properties, the testing setup, the instrumentation, and the testing procedure for the test columns. Ranf et al. (2005) provide details of the experiments.

Column prototype

The prototype was selected to be typical of circular bridge columns built before transverse reinforcement requirements increased following the 1971 San Fernando earthquake. Such older columns have suffered damage in many past earthquakes (Moehle and Eberhard 1999). Table 1 provides specifications and properties of typical

bridge columns built between the late 1940s and the early 1980s in Washington State (Ruth and Zhang 1999).

On the basis of the data provided in Table 1, the yield strength of the longitudinal and transverse reinforcement for the prototype column was chosen as 44 ksi (303 MPa), 10% larger than the specified minimum for columns built before 1977. The concrete strength for the prototype was chosen as 6000 psi (41.4 MPa), 50% larger than the specified minimum for columns presented in Table 1. This increase reflects the effects of long-term strength gain and of the difference between the expected mean and specified strengths (Priestley et al 1992).

The prototype geometry was based on the results of a survey by Ruth and Zhang (1999) of 33 bridges in Washington State that were built between 1957 and 1969. They found that the diameter of 212 of the 216 columns (98%) in these bridges was between 4 and 6 feet (1219 and 1829 mm), with approximately half of these columns having a diameter of 5 feet (1524 mm). The lengths of the majority of columns ranged from 20 to 25 feet (6096 to 7620 mm). Assuming that the distance between the footing and the inflection point is slightly larger than half the column length, these fixed columns have cantilever lengths of about 15 feet (4572 mm) and an aspect ratio of 3. The transverse reinforcement for the surveyed columns consisted nearly always of #4 hoops at 12 in. (305 mm) spacing, independent of the column diameter.

Based on these findings, the prototype column was chosen to be 5 feet in diameter with #4 hoops spaced at 12 in. (305 mm) and an overlap of 24 in. (610 mm). The minimum cover to the outside of the hoop was chosen to be 1.5 in. (38.1 mm), resulting in a volumetric ratio for the transverse reinforcement of 0.12%. This value is far below the current transverse reinforcement requirements of at least 0.56% in non-seismic applications and 1.2% in seismic applications (AASHTO 2004). The longitudinal reinforcement ratio ($\rho_l = A_s/A_g$, where A_s and A_g are the areas of the longitudinal steel and column cross-section) was assumed to be 1.0%, because this value is common in practice. The axial-load ratio ($P/A_g f'_c$, where P and f'_c are the axial load and the concrete compressive strength) was selected as 10%. This value is at the high end of the range for bridge columns in practice.

Test columns

The test columns were designed to be 1/3 the size of the prototype, resulting in a model column height and diameter of 5 feet (1524 mm) and 20 in. (508 mm), respectively. The geometry and reinforcement of the test columns are shown in Fig. 1. Table 2 compares key properties of the prototype and test columns.

The material properties of longitudinal and transverse reinforcement are listed in Table 3. Reinforcing bars with a yield strength of 44 ksi (303 MPa) were not available, so ten D16 grade 420 bars were used as the longitudinal reinforcement. This choice preserved the reinforcement ratio and the scaling for bar size, but resulted in a tension strength that was approximately 50% greater than that of the prototype. Concrete cover and maximum aggregate size were scaled to the nearest available dimension.

The transverse reinforcement for the prototype columns was #4 bars at 12 in. (305 mm) centers with $f_y = 44$ ksi (303 MPa). Scaling all features of the ties was not possible, so instead, the transverse reinforcement was modeled with W2.5 smooth wire hoops at 4 in. (102 mm) centers. With this arrangement, the hoop spacing, transverse bar diameter and transverse reinforcement ratio were scaled to within 15% of their ideal values. However, because this material has a yield strength, f_y , of 80 ksi (552 MPa), the tension strength per bar was twice its ideal value. Furthermore, W2.5 wire was only available in smooth wire, which would be expected to have weaker bond properties than those of a #4 deformed bar.

The six test specimens were cast in four pours, (two for the footings, and two for the columns and hammerheads). Concrete cylinders were stored in the laboratory fog room until they were tested. The compressive strength, f'_c , split cylinder tensile strength, f_{ct} , and elastic modulus, E_c , for each concrete batch are reported in Table 4. Nearly all tests for the material properties were conducted within two days of column testing. However, the elastic modulus for column S1 was determined on the day of testing for column S15. Additionally, the compression tests for column C3R were conducted approximately three weeks after testing.

The average concrete compressive strength of the first four columns tested (S3, C2, C4 and C3R) was 8050 psi (55.5 MPa), 34% larger than the target 56-day strength of 6000 psi (41.4 MPa). This discrepancy can be partly attributed to the average age at testing of the specimens in the group of 192 days. The average strength of the last two columns (S1 and S15) was 5570 psi (38.4 MPa), which was closer to the target value of 6000 psi (41.4 MPa). The strength of the concrete in the footings was nearly 30% higher for S1 and S15 (average of 6360 psi) than for the other four columns (average of 4980 psi).

Test setup

The testing apparatus was designed to maintain a constant axial load while subjecting the column to transverse cyclic displacements (Fig. 2). An axial load of approximately $0.1f'_cA_g$ was applied to each column through a steel cross-head placed on a spherical bearing on the column hammerhead, stressed to the floor by two high-strength rods. The transverse load was applied to the hammerhead using a servo-controlled actuator. The column footing was anchored to the floor using two high-strength rods, each stressed to 125 kips (556 kN).

Instrumentation

The instrumentation scheme is shown in Fig. 3. Axial and lateral forces were measured using load cells. Transverse displacements were measured using five potentiometers at various heights up the column, with two more at the base to monitor footing slip.

Reinforcement strains and relative cross-sectional rotations were measured in columns S3, C2, C4 and C3R, but not in S1 and S15. Eight strain gauges were placed on each of the two longitudinal bars that were nearest to the front and back faces of the column (bars

A and C in Fig. 3). One gauge was also placed on the two longitudinal bars that were nearest to the column sides (bars B and D in Fig. 3). The transverse reinforcement was instrumented with strain gauges at four elevations: 2, 6, 10 and 18 inches (51, 152, 254 and 457 mm) above the top of the footing.

Testing procedure

The naming convention for the columns, provided in Table 5, was based on the characteristics of the imposed displacement histories (Table 6). The histories were chosen to provide a wide range of displacement paths with which to calibrate damage models.

The displacement histories were based on a sequence of nine drift ratios, ranging from 0.27% to 5.72%, each being approximately 1.5 times the previous one. Two series of displacement histories were applied to the columns (“C” and “S”). The main portion of the displacement history for columns C2, C3R and C4 consisted of at least 10 cycles of displacement amplitude at drift ratios of approximately 2%, 3% and 4%, respectively. Column C3R had the additional “R” designation, because after cycles were applied at a drift ratio of 3%, the level of displacement was reduced to a drift ratio of 2%

The S test series consisted of “standard” displacement history, in which the displacement amplitudes increased consistently throughout the test, although at different rates. Columns S1 and S3 were subjected respectively to 1 and 3 cycles at each of the standard drift ratios. The S3 loading therefore resembles closely a loading pattern used by many researchers in the past (e.g., Lehman et al, 2004). Column S15 was subjected to 15 cycles between each standard drift ratio in such a way that the amplitude of each half cycle was a constant multiple of the previous one. This unusual displacement history was developed in order to achieve large values for dissipated energy and accumulated plastic strain while increasing the maximum displacement very gradually. Separating energy from maximum displacement in this way is necessary for evaluating the relative importance of the components of damage models. The damage models are discussed later in this paper.

FORCE-DISPLACEMENT RESPONSE

The effective force-displacement histories for each column are shown in Fig. 4. The effective force is equal to the maximum column moment divided by the length of the cantilever column. It includes the effect of both lateral and vertical loads. The drift ratio is given by the transverse column displacement divided by the cantilever column length.

The variations in the maximum effective forces of the six test columns are primarily attributable to variations in the material properties and axial load. For example, because the concrete strengths for columns S1 and S15 were lower than those for columns S3, C2, C4 and C3R, the axial load was also smaller to maintain the same axial-load ratio. Ranf et al. (2005) showed that the measured flexural strength of the columns can be reproduced accurately with moment-curvature analyses. The ratio of the measured

maximum moment to the calculated maximum moment for the six columns had a mean of 1.1 and a coefficient of variation of 5.1% (Ranf et al. 2005).

Fig. 5 shows the axial load-horizontal displacement history for each column. The target axial-load ratio for each test was 10%. The axial load varied significantly with the horizontal displacement for columns S1, S3, and C4, but the maximum load applied to the columns never exceeded 15% of $A_g f_c'$. The axial load was adjusted continuously for columns S15, C2 and C3R.

The yield displacement of each column was determined experimentally using two methods. For columns with strain gauges (columns S3, C2, C4 and C3R), the initial yield displacement was determined from the point at which the first strain gauge at the column base reached the yield strain of the longitudinal reinforcement. For all six columns, the initial yield displacement was also obtained from the measured force-displacement envelope based on the moment at initial yield, M_y . Table 7 shows that the yield displacements calculated with both methods were similar for the four columns in which both measurements were available.

Table 7 also shows that the yield displacements are clustered into two groups: columns S1 and S15 had an average Δ_y of 0.26 in. (6.48 mm), whereas columns S3, C2, C4, and C3R had an average Δ_y of 0.37 in. (9.27 mm). The difference between these two groups was attributed to additional anchorage slip caused by the presence of strain gauges in the footings of columns S3, C2, C4 and C3R; and to the higher strength and elastic modulus of the footing concrete in S1 and S15.

INFLUENCE OF CYCLING ON DAMAGE ACCUMULATION

All of the columns yielded in flexure, spalled at their bases, and experienced buckling of the longitudinal bars before shear damage was observed. This behavior was consistent with the flexural damage model developed by Berry and Eberhard (2003), which estimated the onset of spalling and bar buckling at drift ratios of 1.87% and 4.00%, respectively. Table 8 shows the maximum column displacement, the hysteretic energy dissipated, and the cumulative plastic deformation for each column at 20% and 50% loss of flexural strength and at loss of axial load.

Fig. 6 shows the effect of cycling on the maximum displacement at 20% loss of flexural strength. The drift ratio for this damage state decreased as cycling increased, regardless of whether cycling was quantified in terms of dissipated energy or cumulative plastic deformation. In contrast, no trends were apparent when the maximum displacement was quantified in terms of the displacement ductility, because the yield displacement varied among the columns (Table 7). Ranf et al. (2005) show that the trends were similar for 50% strength loss.

All of the columns eventually lost their ability to carry the imposed axial load (Fig. 5). The effect of cycling on loss of axial-load carrying capacity is shown in Fig. 7. As with the other damage states, cycling tended to decrease the displacement at failure. The

exception was column C4 (the only column to have an axial-flexure failure), which failed at a higher drift ratio (6%) than any of the other columns. Five of the six columns had a shear-flexure failure mode, in which a large diagonal crack formed within the hinge region at the base of the columns (Fig. 8a). The sixth column failed in an axial-flexure mode, without the formation of a diagonal crack (Fig. 8b).

CALIBRATION OF DAMAGE MODELS

The effect of cycling on damage accumulation in each column was characterized using three damage models. The Park-Ang damage model (Eq. 1) is the method most commonly used for characterizing the effect of cycling on damage.

$$D = \alpha \frac{\Delta_{\max}}{\Delta_y} + \beta \frac{E_h}{\Delta_y F_y} \quad (1)$$

in which D is the damage index level, $\Delta_{\max}$ is the maximum displacement experienced by the structure, Δ_y is the yield displacement, E_h is the hysteretic energy dissipated during cycling, F_y is the force at yield. The dimensionless coefficients α and β are obtained by calibration against measured test data. To estimate E_h , it is necessary to know (or estimate) a column's cyclic force-displacement history, because this parameter is sensitive to the shape of the hysteresis loops. The Park-Ang model is also sensitive to the accuracy of the estimation of the yield displacement, which appears in the denominator of both terms. As mentioned previously, the yield displacements of columns S1 and S15 were significantly smaller than the yield displacements of the other four columns (Table 7).

To reduce the model's sensitivity to the yield displacement, a Modified Park-Ang model was investigated, which normalizes the data by the column length, L , instead of the yield displacement. This model is given by the equation

$$D = \alpha_m \frac{\Delta_{\max}}{L} + \beta_m \frac{E_h}{L F_y} \quad (2)$$

where α_m and β_m are dimensionless coefficients.

A third model, called the Cumulative Plastic Deformation damage model, was developed to characterize the effects of cycling through the total amount of plastic deformation experienced by the column, and is given by.

$$D = \alpha_p \frac{\Delta_{\max}}{L} + \beta_p \frac{\Sigma \Delta_p}{L} \quad (3)$$

where α_p and β_p are dimensionless coefficients. This model can be implemented on the basis of a column's displacement history, without needing to estimate the force history. Also, the value of $\Sigma \Delta_p$ is insensitive to the shapes of the hysteresis loops.

Each of the damage models was calibrated using a least squares best-fit approximation of the data shown in Table 8. Because column C4 eventually experienced a flexural failure, as opposed to flexure-shear failures of the other five columns, the damage models at the loss of axial load damage state were calibrated both with and without the data from column C4. Figs. 6 and 7 show these best-fit lines, and illustrate that because the data for columns S1, S3, C2, C4 and C3R are clustered together, the least squares fit depends heavily on the data for column S15.

The damage model coefficients for each damage model (α and β in Eq. 1, α_m and β_m in Eq. 2, and α_p and β_p in Eq. 3), and the correlation coefficient (R^2) for the best-fit lines are provided in Table 9 for each damage state. The correlation coefficients are extremely low for the Park-Ang model. This is due to the variation in the values of measured yield displacements of the nominally identical columns. The correlation coefficients for both the Modified Park-Ang damage model and the Cumulative Plastic Deformation damage model are similar for each damage state, particularly at 20% and 50% loss of lateral load. The correlation coefficient was the highest ($R^2 = 0.68$) for the Cumulative Plastic Deformation damage model for the loss-of-axial-load damage state.

SUMMARY AND CONCLUSIONS

Six columns were subjected to a variety of displacement histories to evaluate the effects of cycling on the failure displacement and mechanism of lightly confined reinforced concrete columns. Damage accumulation was evaluated for three levels of column damage: 20% loss of flexural strength, 50% loss of flexural strength, and loss of axial-load carrying capacity. Cycling reduced the displacement at which the flexural strength of the columns decreased (20% or 50%) and when the columns lost their ability to carry the imposed axial load. For example, the maximum displacement preceding flexure-shear failure decreased by 35% when the number of cycles at each displacement level was increased from one to fifteen.

Three damage models were used to characterize the effect of cycling on damage accumulation. The Park-Ang (1985) damage model accounts for cycling in terms of the dissipated hysteretic energy, and it normalizes this data by the yield force and yield displacement. Although the six columns were nominally the same, the measured yield displacements of columns S1 and S15 were significantly smaller than the yield displacements of columns S3, C2, C4 and C3R. Consequently, the Park-Ang damage index, which is sensitive to the estimate of the yield displacement, correlated poorly with the observed damage.

The Modified Park-Ang damage model normalized the displacement demands by the yield force and column length. Although this model reduced the sensitivity of the model to the variations in estimated yield displacement, it was still necessary to estimate the cyclic, force-displacement response to compute dissipated hysteretic energy.

The Cumulative Plastic Deformation damage model quantifies cycling as the summation of the column displacement beyond yield, and normalizes these data by the column

length. Although it is still necessary to estimate the yield displacement, this model is relatively insensitive to inaccuracies in the estimate of the yield displacement. Of the three damage models studied, the Cumulative Plastic Deformation model yielded the best correlation coefficient (highest R^2). This measure has the additional advantage that it can be computed without estimating the force-displacement history for the column. The Cumulative Plastic Deformation damage model is the most effective method for characterizing the effect of cycling on a column's shear strength, because it fit the data best, and it was the easiest to implement.

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NOTATION

A_g – Column cross-sectional area
 A_s – Total area of longitudinal reinforcement
 D – Damage index
 E_c – Elastic modulus of concrete
 E_h – Hysteretic energy dissipated
 F_y – Column yield strength
 f'_c – Concrete strength
 f_{ct} – Concrete tensile strength
 f_y – Yield stress of steel
 L – Column length
 M'_y – Column moment at initial yield
 P – Axial load

α – parameter for Park-Ang damage model
 α_m – parameter for Modified Park-Ang damage model
 α_p – parameter for Cumulative Plastic Deformation damage model
 β – parameter for Park-Ang damage model
 β_m – parameter for Modified Park-Ang damage model
 β_p – parameter for Cumulative Plastic Deformation damage model
 Δ – Column displacement
 Δ_{max} – Maximum column displacement
 Δ_y – Column yield displacement
 ϵ_y – Yield strain of longitudinal steel
 ρ_l – Longitudinal reinforcement ratio
 ρ_t – Transverse reinforcement ratio

Table 1 – WSDOT specifications for bridge columns (Ruth and Zhang 1999)

Specifications Year	Concrete		Reinforcing Steel		
	Class	f'_c (psi)	Longitudinal f_y (ksi)	Transverse f_y (ksi)	Comment
1948	A	3600	40	40	ASTM A15, Intermediate Grade
1951 (1948 Amend. 1)	A	3600	40	40	ASTM A15, Intermediate Grade
1953 (1948 Amend. 2)	A	3600	40	40	ASTM A15, Intermediate Grade
1957	A	3600	40	40	ASTM A15, Intermediate Grade
1963	AX	4000	40	40	ASTM A15, Intermediate Grade
1966 (1963 Amend. 1)	AX	4000	40	40	ASTM A15, Intermediate Grade
1968(1963 Amend. 2)	AX	4000	40	40	ASTM A15, Intermediate Grade
1969	AX	4000	40 ¹	40 ²	¹ ASTM A615, Grade per plans ² ASTM A615, Grade 40
1972	AX	4000	40	40	ASTM A615, Grade 40 except as noted on plans
1974	AX	4000	40	40	ASTM A615, Grade 40 except as noted on plans
1977	AX	4000	60 ¹	60 ²	¹ ASTM A615, Grade 60 except as noted on plans ² Spiral tie. ASTM A615, plain bar Grade per plans
1980	AX	4000	60 ¹	60 ²	¹ ASTM A615, Grade 60 except as noted on plans ² Spiral tie. ASTM A615, plain bar Grade per plans

Table 2 – Details of prototype and test columns

Item	Prototype	Test Column	Remarks
Column Length	15 feet	5 feet	1:3 Scale
Column Diameter	60 in.	20 in.	1:3 Scale
Longitudinal Steel Ratio	$\rho_l = 1.0\%$	$\rho_l = 0.99\%$	Conserved during scaling
Transverse Hoops	No. 4 Grade 40 @ 12-in. spacings	W2.5 wire @ 4-in. spacings	1:3 scale on spacing
Transverse Reinforcement Ratio	$\rho_t = 0.12\%$	$\rho_t = 0.15\%$	Far below current requirements
Cover to Hoop	1.5 in.	0.57 in.	1:2.6 Scale
Concrete Compressive Strength	6000 psi	6000 psi	Target properties
Axial Load	$0.10F_c A_g$	$0.10F_c A_g$	f'_c = actual concrete strength
Maximum Aggregate Size	1 in.	3/8 in.	1:2.7 Scale

Table 3 – Reinforcing steel properties

Property	Longitudinal Steel	Transverse Steel
Average Yield Strength (f_y)	66 ksi (455 MPa)	60 ksi (414 MPa)
Average Ultimate Strength (f_u)	105 ksi (724 MPa)	66 ksi (455 MPa)

Table 4 – Concrete properties near time of testing

Column Name	Column				Footing		
	Age at Test (days)	f'_c (psi)	f_{ct} (psi)	E_c (ksi)	f'_c (psi)	f_{ct} (psi)	E_c (ksi)
S1	16	5270	520	4720	6320	---	5480
S3	159	8150	630	4090	4910	470	3260
S15	26	5870	550	4720	6390	580	5480
C2	199	8260	620	4270	4920	370	3610
C4	188	8170	540	4040	5010	430	3280
C3R	220	7640	600	4930	5080	430	3530

Table 5 – Test column names and description

Column Name	Description
S1	1 standard cycle between each displacement level, with the displacement incremented each half cycle, until failure
S3	3 standard cycles at each displacement level until failure
S15	15 standard cycles between each displacement level, with the displacement incremented each half cycle, until failure
C2	Constant amplitude cycling at a drift ratio of 2%
C4	Constant amplitude cycling at a drift ratio of 4%
C3R	Constant amplitude cycling at a drift ratio of 3%, followed by a decrease in the amplitude of cycling.

Table 6 – Summary of column displacement histories

Drift Ratio (%)	Cycle Numbers at Each Drift Level					
	Column S1	Column S3	Column S15	Column C2	Column C4	Column C3R
0.04		1				
0.07		2-4		1	1	1
0.27	1	5-7	1-15			
0.48	2	8-10	16-30			
0.77	3	11-13	31-45	2	2	2
1.15	4	14-16	46-60			
1.90	5	17-19	61-75	3-14		13-17
3.05	6	20-22		15-17		3-12
3.82				18	3-12	
4.57		23				
5.72					13	

Table 7 – Comparison of strain gauge and force-displacement data

Column	Based on $\varepsilon = \varepsilon_v$		Based on $M_{base} = M'_v$		
	Δ'_v (in.)	Δ_v (in.)	Δ'_v (in.)	Δ_v (in.)	F_v (k)
S1	---	---	0.22	0.26	44.3
S3	0.33	0.39	0.32	0.38	58.8
S15	---	---	0.19	0.23	46.5
C2	0.29	0.35	0.30	0.36	57.2
C4	0.37	0.44	0.29	0.35	57.1
C3R	0.37	0.44	0.31	0.37	56.8

Note: Strain gauges were not installed in columns S1 and S15

Table 8 – Measured displacement capacities

Column	20% loss of flexural strength			50% loss of flexural strength			Loss of axial load		
	Δ_{max} (in.)	E_h (k-in.)	$\Sigma\Delta_p$ (in.)	Δ_{max} (in.)	E_h (k-in.)	$\Sigma\Delta_p$ (in.)	Δ_{max} (in.)	E_h (k-in.)	$\Sigma\Delta_p$ (in.)
S1	2.27	269	15.3	2.34	272	15.4	2.73	381	26.8
S3	2.12	640	33.2	2.12	639	33.2	2.85	709	41.1
S15	1.62	1120	115	1.66	1180	121	1.76	1190	124
C2	2.25	806	57.1	2.28	907	60.6	2.25	929	63.2
C4	2.23	231	5.88	2.29	614	40.5	3.54	852	80.2
C3R	1.78	502	29.1	1.80	745	51.2	1.80	854	69.2
μ	2.05	595	42.6	2.08	726	53.7	2.49	819	67.4
σ	0.28	338	39.5	0.29	305	36.5	0.69	267	33.8
δ (%)	14	57	93	14	42	68	28	33	50

Table 9 – Damage model parameters

Damage Model	Model Parameters	Damage Level			
		20% loss of flexural strength	50% loss of flexural strength	Loss of axial load (including column C4)	Loss of axial load (excluding column C4)
Park-Ang	$1/\alpha$	6.20	6.39	8.19	7.49
	$1/\beta$	-898	-1440	1140	3220
	R^2	0.02	0.03	0.01	0.00
Modified Park-Ang	$1/\alpha_m$	3.91	4.21	5.85	5.42
	$1/\beta_m$	1.48	1.30	0.89	0.87
	R^2	0.43	0.48	0.30	0.55
Cumulative Plastic Deformation	$1/\alpha_p$	3.75	4.00	5.09	5.05
	$1/\beta_p$	775	670	612	438
	R^2	0.47	0.50	0.68	0.66

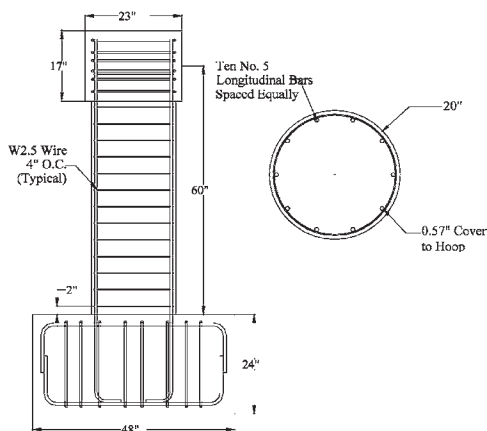


Figure 1 – Test column geometry and reinforcement.

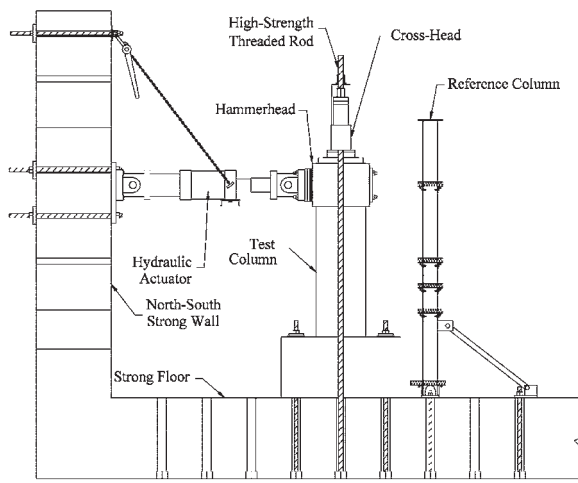


Figure 2 – Testing apparatus.

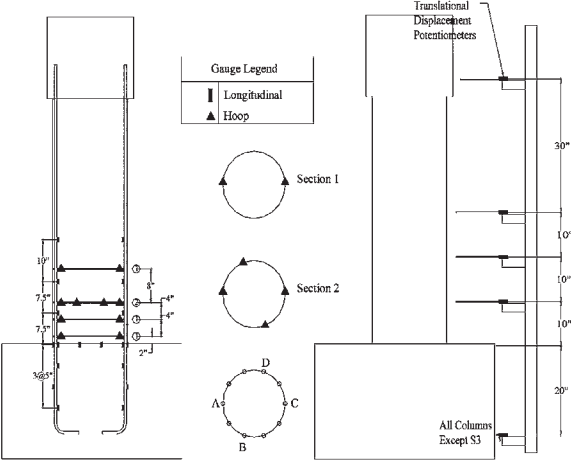


Figure 3 – Column instrumentation.

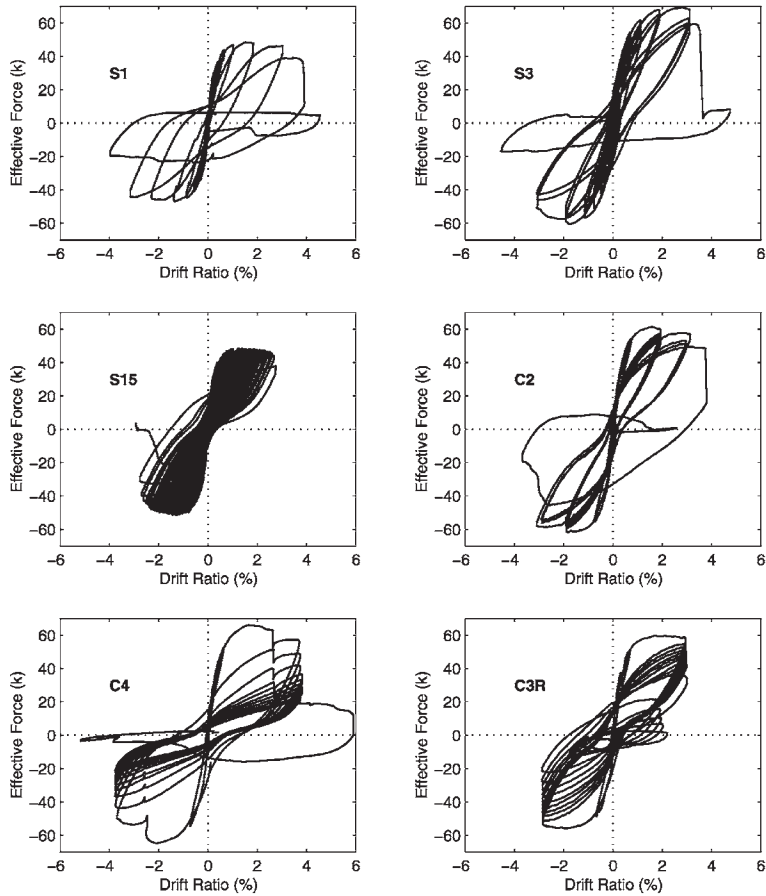


Figure 4 – Effective force – displacement histories.

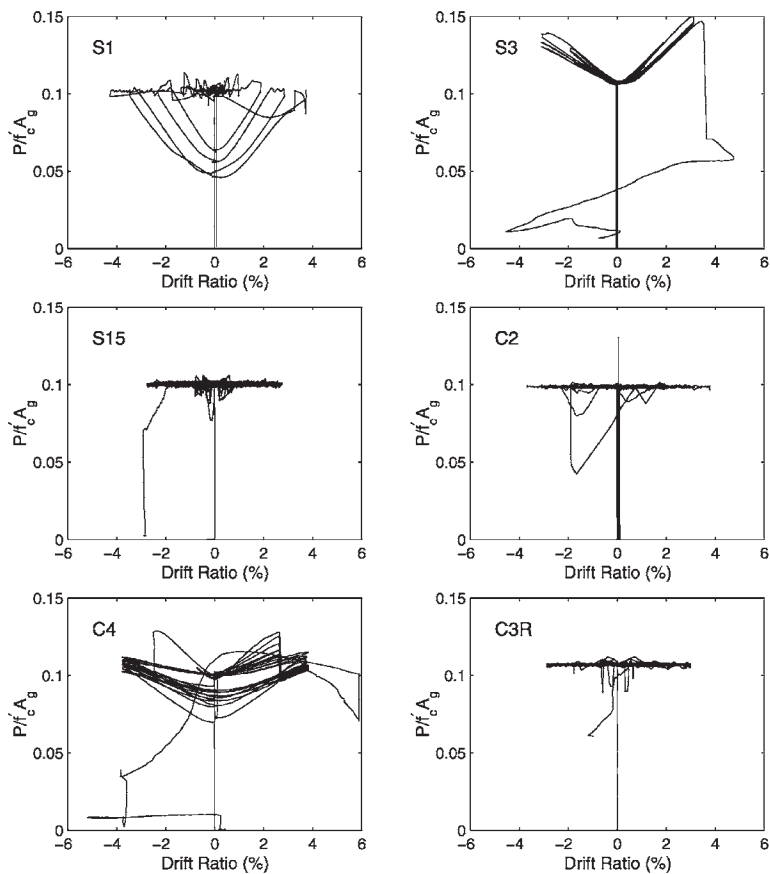


Figure 5 – Axial load versus horizontal displacement histories.

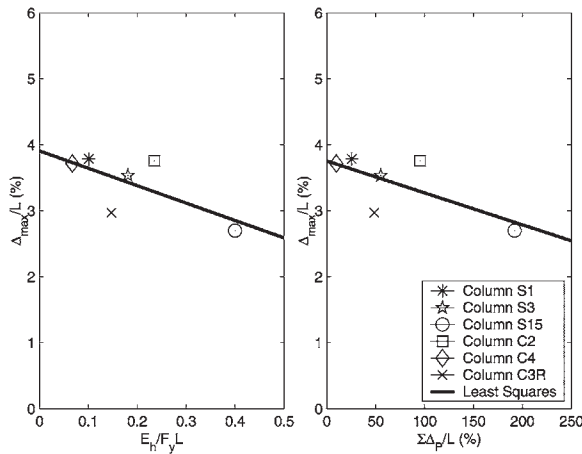


Figure 6 – Drift ratios at 20% loss of flexural strength for the: (a) modified Park-Ang; and (b) cumulative plastic deformation damage models.

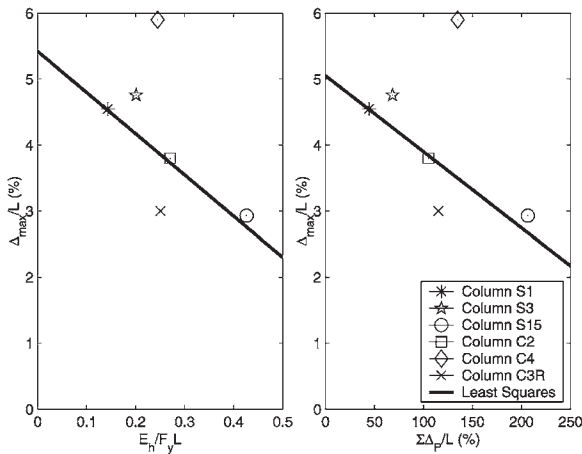


Figure 7 – Drift ratios at loss of axial load for the: (a) modified Park-Ang; and (b) cumulative plastic deformation damage models (calibration excludes column C4).

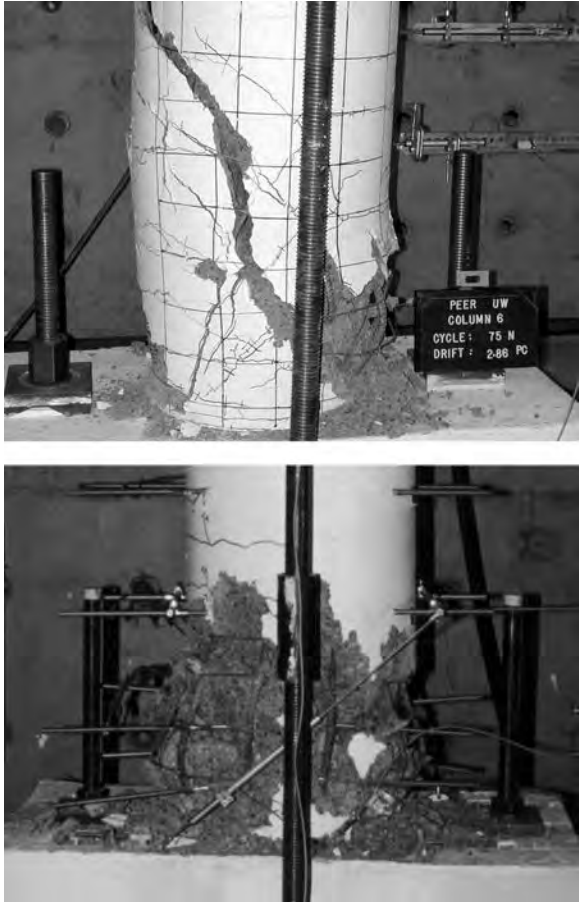


Figure 8 – Examples of: (a) flexure-shear failure; and (b) axial-flexure failure.

Effect of Shear Reversals on Dynamic Demand and Resistance of Reinforced Concrete Elements

by S. Pujol and M.A. Sözen

Synopsis: The effect of shear reversals on the drift capacity of reinforced concrete columns is studied comparing computed limiting drift for monotonically increasing load and limiting drift for cyclic loads. The latter is estimated using models calibrated with data from tests of columns subjected to shear reversals. The comparison indicates that, within the ranges considered, shear reversals cause stiffness decay at displacements that can be as low as one quarter of the displacement capacity for monotonically increasing shear. The effects of stiffness decay on the demand for a single degree of freedom system subjected to strong ground motion are studied. The formulations considered indicate that, if relative reduction in lateral stiffness is associated with an equal relative reduction in damping, shear reversals affect drastically the resistance of the system but not dynamic demand. Ranges are given defining a domain within which the effects of shear reversals may be ignored.

Keywords: capacity spectrum; cyclic load; reinforced concrete

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BACKGROUND AND RESEARCH SIGNIFICANCE

Seismic demand on a reinforced concrete structure depends on the interaction of many variables including characteristics of the ground motion, ratio of mass to stiffness, ratio of strength to weight, and the toughness of the structure, i.e., its ability to sustain strength through displacement reversals in the nonlinear range of response. There is abundant evidence from the laboratory indicating that stiffness and toughness are sensitive to shear and displacement history. During strong ground motion, incremental displacement demand depends on incremental stiffness. Incremental stiffness depends, in the nonlinear range, on displacement. The interaction is circular. This note summarizes an attempt to quantify the effects of shear reversals on both demand and resistance for reinforced concrete systems subjected to earthquake loads.

The interaction between the variables determining the balance between seismic demand and resistance is studied using models of a very simple idealized structure: a column fixed at its base and free to displace, but not to rotate, at its top (Figure 1). The mass of the system is assumed to be concentrated at the top end of the column.

RESISTANCE OF REINFORCED CONCRETE ELEMENTS

Example

To study the relative relevance of the variables affecting the load-deformation response of a reinforced concrete column, consider an idealized column with the following characteristics:

Axial load (in terms of concrete strength and gross area):	$P = 0.15 * f'_c * A_g$
Ratio of shear span to effective depth:	$a/d = 2.9$
Ratio of reinforcement in tension:	$r_t = 1.5\%$
Ratio of transverse reinforcement:	$r_w = 0.4\%$
Concrete compressive strength:	$f'_c = 5,000 \text{ psi}$
Reinforcement yield stress:	$f_y = f_{yw} = 60,000 \text{ psi}$

Scope

The cases to be considered meet the following criteria:

1. The shear strength for monotonic loading is higher than the shear associated with the development of the full flexural strength of cross sections at the top and bottom of the column.
2. Reinforcement is properly anchored.

3. Bars do not buckle in the range of interest.
4. The axial load does not exceed $0.2 \cdot f'_c \cdot A_g$.

We shall also assume that the effects of shear reversals observed for columns in the laboratory are representative of the effects of reversals from earthquake ground motion. This is a compromise that we make because we have quantified only the effects of displacement histories that have been used in the laboratory (Elwood, 2002; Pujol et al. 1999, 2000).

Strength

The strength of the reference system is quantified as follows:

$$F_y = \frac{A_s \cdot f_y \cdot \left(1 - \frac{2}{5} \cdot \kappa\right) \cdot d + P \cdot \left(\frac{h}{2} - \frac{2}{5} \cdot \kappa \cdot d\right)}{a} \quad (1)$$

Assuming that the reinforcement in tension and compression yield:

$$\kappa = \frac{3}{2} p \quad (2)$$

a is shear span, or in this case, half the column clear height, r_t is the ratio of area of reinforcement in tension to the product $b \cdot d$ and p is the ratio of axial load to the product $b \cdot d \cdot f'_c$. See the section on notation for the definition of other terms.

Assuming $d = (5/6) h$, the expression above is rewritten as

$$v_{max} = r_t \cdot f_y \cdot \left(1 - \frac{2}{5} \cdot \kappa\right) \cdot \frac{1}{\lambda} + \frac{1}{5} \cdot p \cdot f'_c \cdot (3 - 2 \cdot \kappa) \cdot \frac{1}{\lambda} \quad (3)$$

where $\lambda = a / d$.

We assume that the effects of strain hardening and P-Δ effects compensate for one another so that lateral forces remain nearly constant with increasing displacement after yielding of longitudinal reinforcement.

Displacement Capacity

Drift Capacity for Monotonic Load -- Displacement capacity has been observed to be sensitive to number and amplitude of displacement reversals (Pujol, 2002). To evaluate the effect of reversals, we use the displacement capacity for monotonic loads as reference. We express the limiting drift ratio of a column under monotonic lateral load as the sum of three components:

$$\gamma_m = \gamma_f + \gamma_{shear} + \gamma_{slip} \quad (4)$$

The first component is related to bending and can be estimated as (Matamoros, 1999):

$$\gamma_f = \frac{\phi_y \cdot a}{3} + (\phi_u - \phi_y) \cdot d \quad (5)$$

assuming

$$\phi_y = \frac{\varepsilon_y}{(1 - \kappa) \cdot d} \quad (6)$$

and

$$\phi_u = \frac{\varepsilon_u}{\kappa \cdot d} \quad (7)$$

we get

$$\gamma_f = \frac{\frac{\lambda}{3} - 1}{1 - \kappa} \varepsilon_y + \frac{\varepsilon_u}{\kappa} \quad (8)$$

In theory, κ is different at yield and at ultimate. But evidence from the laboratory (Wight, 1973) indicates that our assumption is adequate. We assume that the maximum reinforcement slip is equal to the product of yield strain and half the effective depth. For concrete elements of regular proportions, this assumption is plausible and it is associated with average bond stresses on the order of 12 to 15 $\sqrt{f'_c}$ (f'_c in psi, 145psi = 1MPa) which is close to what has been inferred from measurements (Wight, 1973). We also need to assume the location about which the column pivots due to slip of reinforcement. In order to simplify our calculations and to account for indentation of the concrete in the base (Matamoros 1999), we assume that slip rotation takes place about a point $d/2$ from the point at which we have estimated slip:

$$\gamma_{slip} = \frac{\frac{1}{2} \cdot \varepsilon_y \cdot d}{\frac{1}{2} \cdot d} = \varepsilon_y \quad (9)$$

To estimate the deformation component related to shear we refer to experimental data. Measurements of angular changes in columns subjected to cyclic loads by Matamoros (1999) suggest that shear distortion could reach values well in excess of those one would calculate on the basis of conventional mechanics of materials:

$$\gamma_{shear} = 75 \cdot \frac{v_{max}}{E_c} \quad (10)$$

Assuming $E_c = 57000\sqrt{f'_c}$ (E_c and f'_c in psi, 145psi = 1MPa):

$$\gamma_{shear} = \frac{v_{max}}{760\sqrt{f'_c}} \quad (11)$$

We realize that shear deformations may be lower in elements subjected to monotonically increasing loads but we do not attempt to make adjustments because we are merely trying to establish an approximation for drift capacity for monotonic loading and because we know from observations that our estimates of deflection based exclusively on flexural mechanisms of deformation are smaller than measured deflections (Yamashiro, 1962).

Adding up all the deformation components and rearranging terms:

$$\gamma_m = \frac{\frac{\lambda}{3} - \kappa}{1 - \kappa} \cdot \varepsilon_y + \frac{\varepsilon_u}{\kappa} + \frac{v_{max}}{760\sqrt{f'_c}} \quad (12)$$

Figure 2 shows the variation of γ_m with axial load and aspect ratio for $r_t = 0.015$ and $\varepsilon_u = 0.01$ (Matamoros, 1999). These results indicate that γ_m is not very sensitive to aspect ratio but it decreases with increasing axial load.

Drift Capacity for Cyclic Loads, Approach A -- An early experimental study on this subject conducted by Wight (1973) led to the observation that even if a column may reach a given level of displacement without initial signs of severe disintegration, a severe decrease in lateral stiffness could take place upon application of additional displacement cycles at the same level of deformation. The immediate consequence of this observation

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is very simple: the displacement capacity of a column under displacement reversals is less than that of a column tested monotonically. This concept is almost intuitive but it is often ignored. We study two models for estimation of the drift capacity of columns subjected to displacement reversals with the intention of quantifying the decrease in drift capacity caused by reversals. The models are very different in form. The first model (Pujol et al., 1999) is simpler in approach and is expressed as

$$\gamma_c = \frac{r_w \cdot f_{yw}}{v_{max}} \cdot \lambda \% \leq \lambda \% \leq 4\% \quad (13)$$

This limit represents the displacement at which a column subjected to shear reversals could lose up to 20% of its lateral stiffness.

Drift Capacity for Cyclic Loads, Approach B -- The second approach (Pujol et al., 2000) to estimate drift capacity that we consider was, in fact, not formulated for determination of drift but for estimation of the amount of transverse reinforcement required for a column to sustain cycles at a given level of displacement. We solve for drift assuming the amount of transverse reinforcement is a known parameter. But we must realize the risk of manipulating a design equation. Design equations usually work one-way, from design parameters to section proportions and details. We shall judge on the appropriateness of our use of the formulation by comparison.

This approach is based on the idea that the interaction between normal, shear, and confining stresses in the column core can be represented using a Mohr's circle for planar stresses. The circle is assumed to represent the demand on the core. Core capacity is represented by two terms: "cohesion" and internal friction, to use geotechnical terms. Assuming an angle of internal friction of 37 degrees (Richart, 1929) and equating demand to capacity:

$$c = (\alpha - 3) \cdot \frac{\sigma_a}{4} \quad (14)$$

where c is cohesion, σ_a is an index representing magnitude of normal stresses and α is a function of nominal shear, normal, and confining –transverse– stresses (see the section on notation for definitions).

It is assumed that the effect of shear reversals can be represented as a decrease in cohesion. Despite the fact that such an effect implies that column response depends on number of repetitions, the formulation was developed on the assumption that the reduction in core strength can be expressed as a function of displacement only. Expressing cohesion as a fraction of cylinder compressive strength:

$$c = \frac{f'_c}{7} \cdot \left(1 - \frac{100}{3} \cdot \frac{\gamma}{\lambda} \right) \quad (15)$$

Equating 14 and 15, and expressing σ_a in terms of ratio of reinforcement in tension and axial load:

$$\gamma_c = \frac{3}{100} \cdot \left[1 - \frac{7}{4} \cdot \left(p + r_t \cdot \frac{f_y}{f'_c} \right) \cdot \frac{b \cdot d}{A_c} \cdot (\alpha - 3) \right] \cdot \lambda \quad (16)$$

In the following computations we assume that $b \cdot d / A_c = 7/5$

Comparing drift capacity for monotonic and cyclic loads -- We study the effect of reversals on drift capacity using the formulations above as reference. Figures 3 to 5 show a parametric study of the ratio of calculated displacement capacity for monotonic load to displacement capacity for cyclic load. As expected, the ratio is less than one in many instances regardless which model we use to estimate the effect of shear reversals. Within the ranges considered, our calculations indicate that drift capacity for cyclic load can be as low as 1/4 of the drift capacity for monotonically increasing load. The calculated effect of shear reversals decreases with increasing a/d and r_w , and it increases with increasing r_t . The models described also indicate that, within the ranges considered, drift capacity decreases with increasing axial load for both monotonic and cyclic loads, with the relative effect of shear reversals remaining relatively insensitive to changes in axial load. It is important to note that the trends discussed are common to the two considered models for drift capacity of columns subjected shear reversals.

DEMAND VERSUS CAPACITY

So far we have talked about the capacity of the example system exclusively. Let us consider now the other side of the problem: the demand. Consider characterizing demand in terms of spectral acceleration:

$$S_a = \left(\frac{6}{5} - \frac{2}{7} \cdot \ln(\beta \cdot 100) \right) \cdot 4 \cdot PGA \quad \text{if } T < T_g$$

$$S_a = \left(\frac{6}{5} - \frac{2}{7} \cdot \ln(\beta \cdot 100) \right) \cdot 4 \cdot PGA \cdot \frac{T_g}{T} \quad \text{if } T \geq T_g \quad (17)$$

where β = damping factor, T_g = characteristic period for ground motion.

The function in the parenthesis is intended to account for the effect of changes in damping on linear response spectra, and it follows from work by Newmark and Hall (1982).

We assume $S_d = S_a / \omega^2$ to estimate displacement demand. We use the Capacity Spectrum formulation (Freeman, 1998) to study the relationship between demand and capacity. In this formulation, it is assumed that the nonlinear response of a system can be approximated as the response of a linear system with increased period:

$$T_e = 2 \cdot \pi \sqrt{\frac{M}{F(\Delta)}} \cdot \Delta \quad (18)$$

and increased damping. We use results from work by Gulkan (1974) to estimate the appropriate level of equivalent damping:

$$\beta_\mu(\mu) = \begin{cases} \frac{1}{50} & \text{if } \mu < 1 \\ \left[\frac{1}{50} + \frac{1}{5} \cdot \left(1 - \frac{1}{\sqrt{\mu}} \right) \right] & \text{otherwise} \end{cases} \quad (19)$$

Here $F(\Delta)$ is a function representing the variation of base shear with displacement Δ , and μ is, for elasto-plastic systems, the ratio of total displacement to displacement at yield.

Assuming that the drift ratio at yield is 1% and using equations 17, 18 and 19, we can plot spectral acceleration as a function of displacement ductility or drift ratio in percentage. Figure 6 shows the variation in spectral acceleration with ductility ratio for $PGA = 0.5 \text{ g}$ and $T_g = 0.5 \text{ sec}$. The intersection of the function of displacement thus defined and the force-displacement relationship of our imaginary one-column system, normalized with respect to mass, represents the solution to the problem of estimating system response to the idealized earthquake selected (Figure 6). The use of the monotonic load-displacement curve leads to an estimate of maximum drift demand lower than the monotonic drift capacity, which in this particular case equals 4.7% (Figure 6). The estimate of maximum drift demand obtained would represent a valid solution if we could ignore the effects of shear reversals. But we have observed that reversals may be associated with stiffness loss at displacement levels well below the monotonic displacement capacity. Let us assume that the displacement capacity for cyclic load, defined again as the displacement associated with a stiffness loss of 20%, is 50% of the displacement capacity for monotonic load and that stiffness decay starts at a displacement that is a fraction q of the displacement capacity for cyclic load. Figure 7 shows how the solution to our example problem changes considering stiffness decay due to displacement cycles for $q = 3/4$. The demand and capacity lines do not intersect anymore indicating that the system is not adequate. Notice that stiffness loss has an effect on the demand curve because demand is a function of stiffness (Eq. 17 and 18). As the system softens the period gets longer and spectral acceleration drops (Figure 8). One could also argue that the demand curve should also reflect the fact that damping changes as the area under the load-displacement curve changes. The smaller the hysteretic loops are, the lower damping is. Figure 9 illustrates how different assumptions on the rate of damping decay affect demand. It is clear that large changes in the rate of damping decay do not result in proportional changes in demand. Within the ranges of the variables studied here, and assuming that the relative change in damping is equal to the relative change in stiffness, the effects of increase in period and decrease in damping tend to cancel (Figure 10). The result is a system whose capacity decays with shear reversals while the demand remains