

THE USE OF MODELS IN STRUCTURAL ENGINEERING

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Scale models are becoming increasingly more important in research on structural concrete and are also being used quite widely in the design problem. Presenting a justification for the use of models in these areas the authors discuss the advantages and disadvantages of structural models in general.

Several classifications of models are presented, including elastic versus ultimate strength models, true versus distorted models, complete versus partial structural models, and direct versus indirect models. Emphasis is on the role of each type, and the potentialities of each type are explored for typical structural problems.

The problem of having a model built is examined by discussing the various types of laboratories available for model construction and testing. European practice is reviewed, and recommendations are formulated on what is necessary in the US in order to further facilitate the use of the physical modeling process.

The type of structures for which model analysis is best suited is examined. Emphasis throughout the paper is on providing the practicing engineer a better understanding of the possibilities of using structural modeling techniques. The entire modeling process will be placed in a realistic perspective with respect to the widely used analytical approach.

Keywords: concretes; costs; microconcrete; models; reinforced concrete; research; scale (ratio); simulation; structural design; structural engineering.

□ In 1930, a small scale (1:240) three dimensional model of Hoover Dam (then called Boulder Dam) was built and tested to aid in solving some unprecedented problems (Fig. 1-1). Several models of smaller dam structures built and studied prior to this work provided a background for the extensive Hoover Dam studies. The methods employed for measuring strains, deflections, and deformations in these early efforts provided the first steps toward the more sophisticated techniques used today. The early model tests on dams may have been related to the fact that hydraulic investigations using modeling techniques had already been well established and accepted. Small scale model studies have played a part in the design of most of the major arch dams constructed in the world over the past 25 years, and a great deal of this work has been done in European laboratories.

Many laboratories abroad, such as Laboratorio Nacional De Engenharia

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Civil, Portugal; Instituto Sperimentale Modelli E Strutture, Italy; and Cement and Concrete Association, England, and others, have also conducted many model studies in connection with specific building projects in the past 20 years.

Acceptance of three dimensional small scale models as design aids has been somewhat slower in the United States, but has gained impetus in recent years. Since 1957, Wiss, Janney, Elstner & Associates have conducted nearly sixty structural and wind model studies, most of which have been addressed to specific building projects (Fig. 1-2, 1-3, 1-4 and 1-5). Several universities have also become considerably involved in three dimensional building model studies.

Models have also served a very significant function in solving problems encountered by engineers designing aircraft and space vehicles. In fact, the refinements employed in instrumentation and model construction in the fields of aircraft and space design surpass by far those which the building industry normally will support.

SCALE

The discussion so far has related to "small scale" models, and question naturally arises as to what scale factor constitutes the division between prototype and model. A great deal of the research and development testing which has been performed on structural components in laboratories throughout the world has involved specimens of rather indefinite scalar relationship with respect to interpretation of results for use by designing engineers. For example, many tests have been conducted on reinforced concrete members (columns or beams), say 6×12 in. in cross section, and 10 or 12 ft in length (Fig. 1-6). The results of such tests are frequently interpreted as applicable to members two, and perhaps four times as large with respect to cross section, span, aggregate sizes, or size of reinforcement. Similarly, the results of tests conducted on structural steel shapes are often interpreted for use with members somewhat larger than the test members. Thus, in a strict sense, most of the laboratory specimens prepared for structural research in the past have been models. However, it is seldom necessary for the researcher to concern himself with the laws of similitude in connection with this work.

HISTORY

The use of small-scale models to assist building designers may be one of the oldest design tools ever employed. One does not have to stretch the imagination much to visualize ancient Egyptian, Greek, Persian, or Roman builders constructing small models to assist themselves in planning some of the marvelous structures built some hundreds, or even thousands of years ago. Michelangelo, Leonardo DaVinci, and Galileo are known to have used models to some extent as an aid in their architectural efforts and perhaps even to clarify structural problems.

These designers of the past did not have available the knowledge of structural behavior, material properties, mathematics, or the measurement methods that are known to us today. Thus, evaluation of structural features through model studies by these early builders was probably confined to very general observations regarding the interaction of assemblages of component parts and regarding methods of assembly.

Small-scale structural models have played a much more scientific role as tools used to assist in the analysis of structural behavior in contemporary buildings. Probably the first truly scientific employment of small-scale models for experimental stress analysis came with the development of the photoelastic technique by such men as Mesnager, Coker, and Filon about the turn of the century. These early efforts emphasized basic stress analysis directed more toward machine design than toward structural design in the building field. Until recently, photoelasticity was limited to two dimensional stress fields, but three dimensional analyses are now possible in many cases. Between 1938-1945 Drucker, Mindlin, Frocht, Hetenyi, and others, published a number of papers on three dimensional methods. Later investigators developed other modeling methods for studying structural behavior directed more to the building field.

Methods based on physical measurements of deformations and deflections in two dimensional small models (often severely distorted) were used primarily to determine distribution of moments and forces, rather than for direct stress analysis. These "indirect" model techniques, developed by Beggs, Gottshalk, Eney, and others have been used principally as study or instruction aids and have been employed only infrequently to develop specific solutions related to particular design or building problems. They are restricted to linear elastic problems because the principle of superposition is invoked in the theoretical basis for the techniques.

The perception of many pioneers must be recognized as the basis on which model analysis, as we know it today, has been built. The development of suitable instrumentation for measurement of strain, force, acceleration, and other physical quantities made "direct" model analysis of small scale structures feasible. The electric resistance strain gage has probably given more impetus to model analysis than any other single factor.

We believe that some attempt should be made to define "small scale models", and suggest that the following definition be considered:

"A structural model is any structural element or assembly of structural elements built to a reduced scale (in comparison with full size structures) which is to be tested, and for which laws of similitude must be employed to interpret tests results."

TYPES OF MODELS

Small scale models may be representative of complete structural frameworks which may or may not include nonstructural building features such as partition walls (Figure 1-7). They may be partial frameworks or portions of building or simulations of structural details (Fig. 1-8 and 1-9).

Small scale models may be constructed to yield information regarding only elastic behavior or they may be built to produce information regarding inelastic behavior including ultimate strength (Fig. 1-10, 1-11, and 1-12).

While the majority of model studies have concerned themselves with the behavior of structures under static loads, small-scale models have occasionally also served to study the response of structures (or members thereof) to temperature changes, dynamic forces, such as seismic action, blast loads, or wind-induced vibrations and forces (Fig. 1-13, 1-14, & 1-15). Currently, efforts are under way to develop techniques of using models to predict time dependent deformations of structures.

Elastic models may be constructed of any material for which the stress-strain relationship is essentially linear to the point of anticipated maximum measured strain. Inelastic models should be constructed of materials which simulate the physical properties of the prototype materials as closely as possible in all respects. This ideal simulation should include complete stress-strain relationship, temperature coefficients, creep, and moisture volume change characteristics. These requirements usually can be met readily when working with metallic materials, and considerable progress has been made recently in the more difficult work with cementitious materials. The third paper in this volume explores model materials requirements in more detail.

SIMULATION

One of the most frequent questions asked by engineers who may be considering the use of small scale models for design purposes (aside from matters related to cost) has to do with the level of confidence that can be expected in interpreting data from the model, with respect to the performance of a prototype. This is a difficult question to answer generally, and perhaps a discussion of the factors which will have an influence on correlation between model and prototype behavior will be helpful.

First, if two structures were built which were identical in all respects, including the soils on which they are founded, they would respond identically to all loading (including dead load) in all respects. If they were founded on different soils, and if this were important to the quantities which are to be measured, the response would be different. If the two structures were identical in all respects, including soil response, with the exception of scale, the response to superimposed load would be identical, taking into account the laws of similitude and certain size effects. In this instance, we see the first deviation from an identical relationship because the prototype dead load stresses and deformations are not reproduced in the model through its own dead load. Dead load on the model can be augmented with reasonable ease of the scale if the model is not too small. It is very difficult to provide sufficient dead load permanently to a very small scale model to simulate the dead load stresses and deformations. However, if the model is designed for elastic data only, this deviation does not constitute a major problem, in that

the elastic response of the structure under the application of test loads often can be interpreted as response to either dead or live load.

It is seldom practical, even if possible, to build a model which simulates a prototype in every respect, including detail. Thus, the model analyst finds himself in a position very much like that of the engineer who employs a mathematical model to design structural members. The engineer must make basic assumptions with respect to boundary conditions, material properties, and structural interaction. Similarly, the model designer must exercise judgment in selecting the parts of the structure that should be simulated as closely as possible, and those where deviation or "distortion" will have minimum effect on the measurements being made. He must select a model material which offers the most promise of providing the answers which he is seeking, and at the same time, will enable the model to be constructed economically. If the study is to be confined to elastic considerations, the model material should, of course, be reasonably elastic. It should be machineable, easy to cut or form, and have a relatively low modulus of elasticity so that readable strains and deformations occur under small loadings. Many plastic materials meet these requirements and methyl methacrylate plastic has been used most frequently in structural model work.

If the study is to include ultimate strength or inelastic behavior considerations, the model materials should either be identical to the prototype materials, or simulate them as closely as possible, especially with respect to the characteristics which will be significant to the results of testing. Furthermore, if detail, such as connections, can be important to the outcome, every effort should be made to simulate this detail.

The matter of simulating details constitutes one of the major problems confronting those designing structural models. If one wishes to observe the behavior of a bolted steel connection within a structural assembly, and it is necessary to simulate this connection in all detail, a practical limitation is placed on the scale factor. For example, if the minimum thickness of prototype material is $1/4$ in., and we recognize that it is not practical to work with plastic or metal material much thinner than 0.050 in., the minimum scale factor is placed at about 5. Similarly, if it is desirable for a study to duplicate the cross section of rolled structural steel or aluminum shapes, the minimum thickness of the prototype metal, as related to the approximate 0.050 in. limitation on thickness of model material, in large measure establishes the minimum scale factor which should be considered. Furthermore, the cost of duplicating intricate detail in very small scale can be prohibitive.

Work done thus far on models representing reinforced concrete, (built of mortar, or so-called "micro concrete", and wire reinforcement) has indicated that the practical limitation on geometric scale factor is about 12 for structures of normal dimensions and reinforcement size. A scale factor of 8 or 10 often proves to be the most satisfactory when cost, instrumentation, loading, and usefulness of results are all properly evaluated (Fig. 1-12).

If the model is intended to yield only information regarding elastic response, it is frequently practical to work at much smaller scale than is the case for models designed to observe inelastic or ultimate strength characteristics. However, it is often necessary to accept certain "distortions" from exact duplication, especially when the smaller scales are employed. These need not result in significant variation between model and prototype response to loading. The variation can be minimized through prudent judg-

ment of the engineer in his evaluation of the possible significance of the distortions. For example, if a number of structural steel members are to be assembled together to form a three dimensional truss or space frame, and in the opinion of the engineers involved in the design, the members are to be subjected principally to direct forces with little bending, the members need only be scaled in terms of cross sectional area, provided the model is not intended to yield answers to stability questions. Thus, the members may be made easily and inexpensively with rectangular or square cross sections. If, on the other hand, secondary stresses or stability were considered to be significant in the prototype, the relation between model and prototype stresses could vary considerably. If significant secondary stresses or stability problems are anticipated, much better agreement between the model and prototype is possible by not only simulating the areas of the members, but by also maintaining the relationship of the moment of inertias from member to member in the model.

COSTS

A very significant factor which enters into the decision to conduct a model study is that pertaining to costs. It is difficult to predict categorically the cost of a small scale model study. The expenditure of time and money will be dependent upon the complexity of the structure, the extent and nature of the information which is desired from the study, and the time allocated for the study. It is not feasible to think of these costs in terms of a fixed percentage of construction cost such as frequently is done in establishing design fees.

The factors which contribute and must be considered in developing costs are as follows:

- (a) Model planning (construction, instrumentation and loading)
- (b) Fabrication and instrumentation of the model in preparation for test
- (c) Building or providing the mechanism for load application
- (d) Acquisition of data
- (e) Reduction of data and preparation of report

Our experience indicates that, in general, the percentage of the total cost for each of the above categories is about as follows:

- (a) 10 - 20 percent
- (b) 50 - 60 percent
- (c) 10 - 20 percent
- (d) 5 - 10 percent
- (e) 10 - 20 percent

It is immediately apparent that the major expense is in the construction and instrumentation of the model and providing the loading mechanism. A relatively minor portion of the cost has been that pertinent to the acquisition and presentation of data.

This unbalance is unfortunate in two respects. First, the model construction, while necessary to the project, is not directly productive. We see

no immediate solution to this problem and believe that only more experience and development of model construction techniques will result in methods of model construction placing this cost in more favorable perspective. The other unfortunate aspect is that so little of the time is spent on reduction and analysis of data. This has been a result of limited budgets and hopefully, any reduction in costs and time spent on model construction may be available for use in the analysis and reporting stage.

The costs for the model studies that have been conducted by the senior author's firm have ranged from \$2,000 to \$60,000. Most of the studies have cost from \$5,000 to \$10,000.

The decision to employ small scale structural model studies in the design of structures is usually made rather late in the design process, after other procedures have either proven unsatisfactory or doubts have developed in the minds of designers. Therefore, these projects are often conducted on a premium basis because of the short time available to perform the work. It would be much more satisfactory to all concerned if the decision to include model studies were made early in the design process to permit a more comfortable time period for the conduct of this work.

Another interesting aspect has to do with the source of payment for model studies, or similar research projects which may be conducted in connection with the design of structures. Unless the project is very large in magnitude, it is seldom feasible for the cost of such studies to be borne by the architect or engineer as part of his design fee. Many architects and engineers are reluctant to admit to the owner, the ultimate beneficiary of such studies, that payment for the model studies should be in addition to the normal design fee. It is difficult to convey to owners the nature of such benefit and convince them that the money is very well spent. Some of our fellow professionals feel that such a request implies lack of competence on their part to perform the task for which they have been retained.

MODEL LABORATORIES

As stated previously, there are a number of laboratories in Europe whose principal function pertains to model studies and related work. As of this time, there are no commercial laboratories in the United States whose principal function is directed toward small scale model investigations. A number of models have been built and tested in the laboratories of Wiss, Janney, Elstner and Associates, but this effort has not been the principal activity of this firm. Several universities throughout the United States have performed services employing small-scale models in connection with the design of specific projects. These same universities and others are also making substantial use of small scale models in research. Several of the laboratories of the U.S. Government are also conducting research with the aid of models.

We believe that the problems associated with model design, model construction, instrumentation, data acquisition, and interpretation of laboratory data dictate that this work is best done by persons skilled in laboratory methods and familiar with available laboratory instrumentation and equipment. The writers are aware of some modest effort which has been made on models in architectural or engineering offices. With care and prudence, valuable information can be obtained from such studies performed and eval-

uated by persons who are unskilled in laboratory research. We are convinced, however, that the expenditure of time and money under such conditions cannot be as efficient nor, in general, can the results be as satisfactory as would be the case if the work were done in a properly equipped laboratory by men experienced in laboratory research.

FUTURE RESEARCH NEEDS

Although a great deal of work has been done to develop model materials that simulate foundations for dams (rock), we know very little about simulating foundation conditions by using model soils for building or bridge studies. Consequently, most small scale model studies, except those conducted on dams, do not include foundation studies. The problem has been bypassed by incorporating support conditions which, in the judgement of the engineer, are representative of those he anticipates for his structure. Of course, this is analogous to assuming boundary conditions for a mathematical model. We believe that the support assumptions "built in" to a model are more likely to be representative of those existing in the prototype than the idealized conditions which are usually assumed for the mathematical model.

A great deal is yet to be learned about the similitude relationship between micro concretes and concretes made with normal sized aggregates with respect to creep, drying, shrinkage, and response to temperature change.

A number of problems met in modeling the complete response of concrete structures need further exploration; these include cracking and bond similitude and the basic causes of the often-observed size effect.

Although some effort is being directed toward the use of small scale models for dynamic studies (Fig. 1-13), a great deal of work must still be done to further improve modeling techniques and similitude relationships.

Great interest is being generated currently regarding the performance of buildings under fire exposure. Fire tests on full-scale structures are being proposed by the ASCE Research Council on Performance of Full Scale Structures. Development of reliable modeling techniques for fire studies would provide great impetus to expanded research programs on fire damage.

A number of recent wind studies have been conducted in connection with the design of specific buildings such as the World Trade Center, John Hancock Building, Princeton Athletic Cage (Fig. 1-14), and the First National Bank Building in Chicago (Fig. 1-15). Other studies were conducted in the wake of the 1965 failure of three reinforced concrete cooling towers at Ferrybridge, United Kingdom. Very little data are available for establishing correlation between the wind pressures produced on full scale structures by wind forces and those developed by model studies in a wind tunnel. Several structures have been instrumented recently to obtain data relative to wind forces.

The performance and strength of full scale structures subjected to the destructive action of vertical loading, wind forces, seismic forces, or blast loading can, of course, be best evaluated by tests conducted on the full scale structures. However, such testing is very expensive and all must agree that building and testing small scale models would be a much more satisfactory method of such evaluation in terms of cost, laboratory control, and ease of testing. Considerable effort, time, and expense certainly seem justified in

order to establish firmly modeling techniques and confidence levels so that all of the above may eventually be investigated with the aid of small-scale models. Carefully controlled testing programs involving both full-scale structures and models of different scale would be invaluable in helping to establish the adequacy of correlation and similitude relationships, as well as overall confidence levels to be achieved in the modeling process.



Fig. 1-1—Small scale structural model
of Hoover (Boulder) Dam
(1/300 scale)—1931

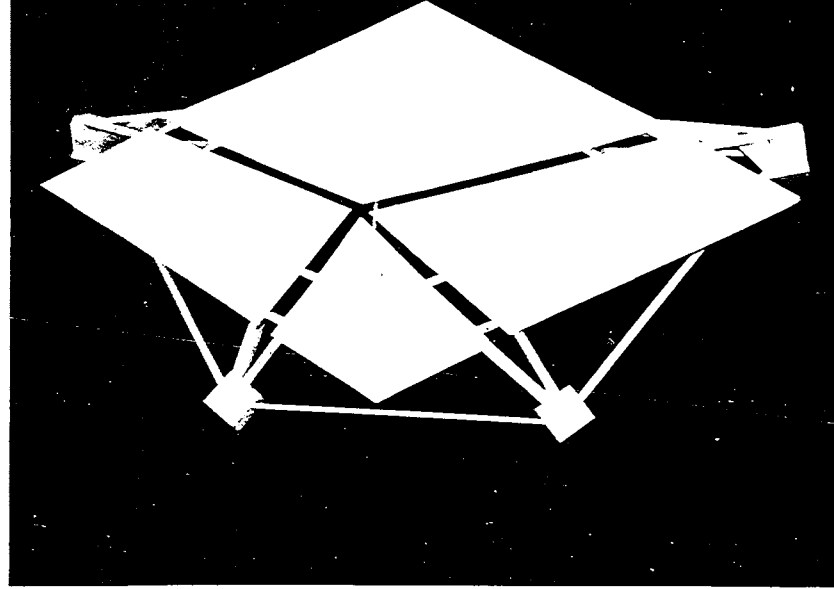


Fig. 1-2—Assembly of hyperbolic paraboloid concrete thin shell elements joined
together to form roof of church (1/30 scale)—Wensler

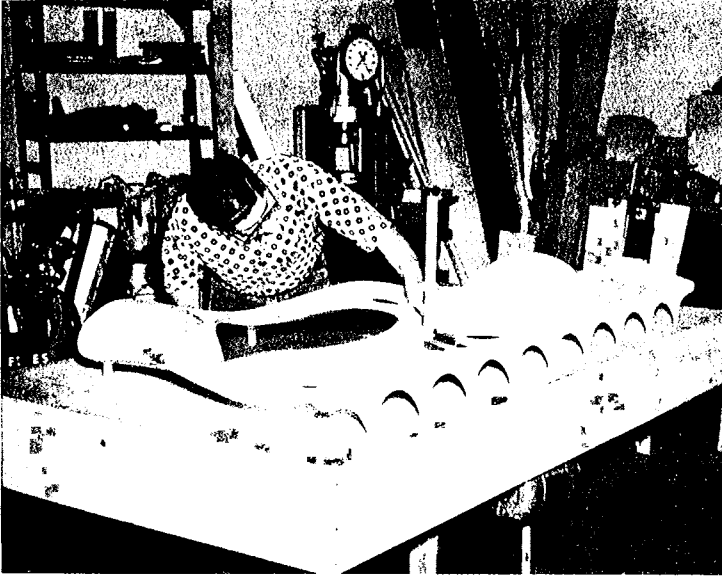


Fig. 1-3—Eastman Kodak Pavilion, New York World's Fair (1/40 scale)—Zetlin

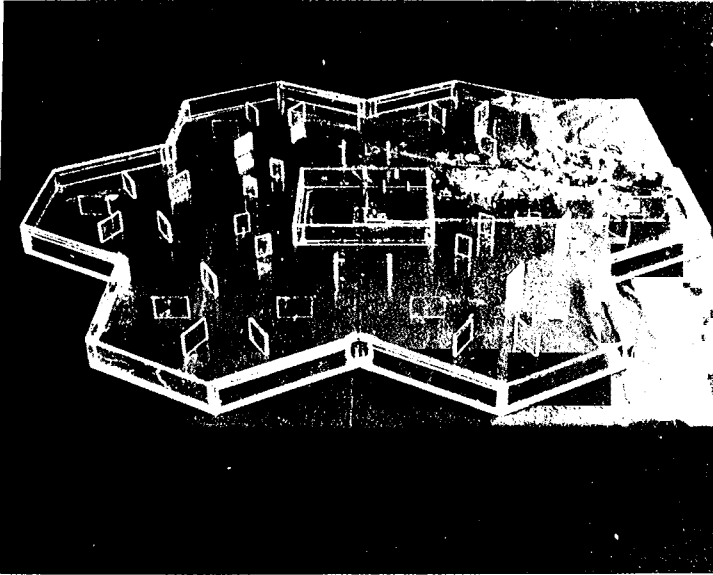


Fig. 1-4—Reinforced concrete flat plate floors of high-rise apartment supported by an array of shear wall columns (1/30 scale)—Fling

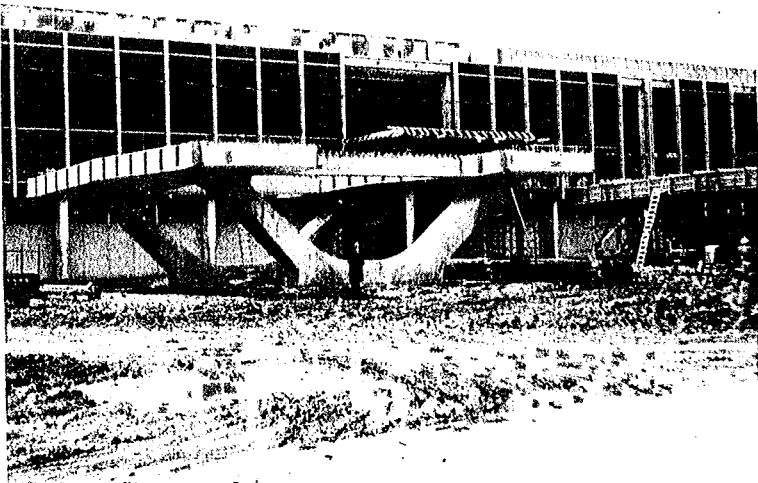
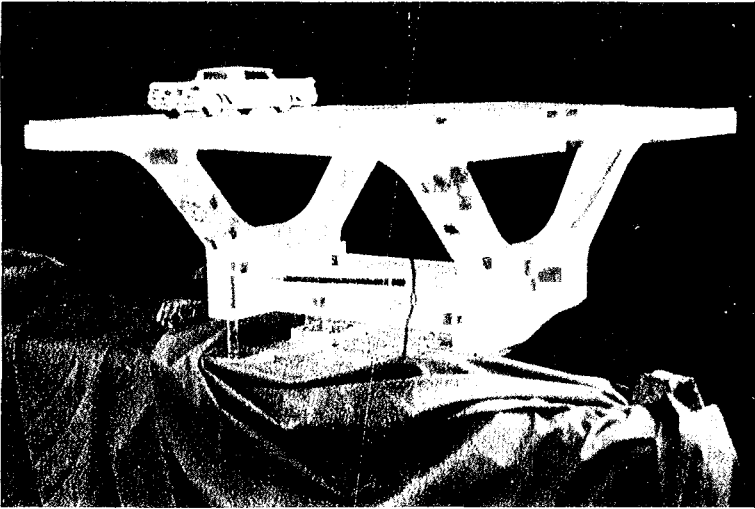


Fig. 1-5—Model and prototype of cast-in-place reinforced concrete support structure for O'Hare Airport access roadway (1/20 scale)—Benesch



Fig. 1-6—Typical reinforced concrete laboratory specimen (eccentrically loaded column)

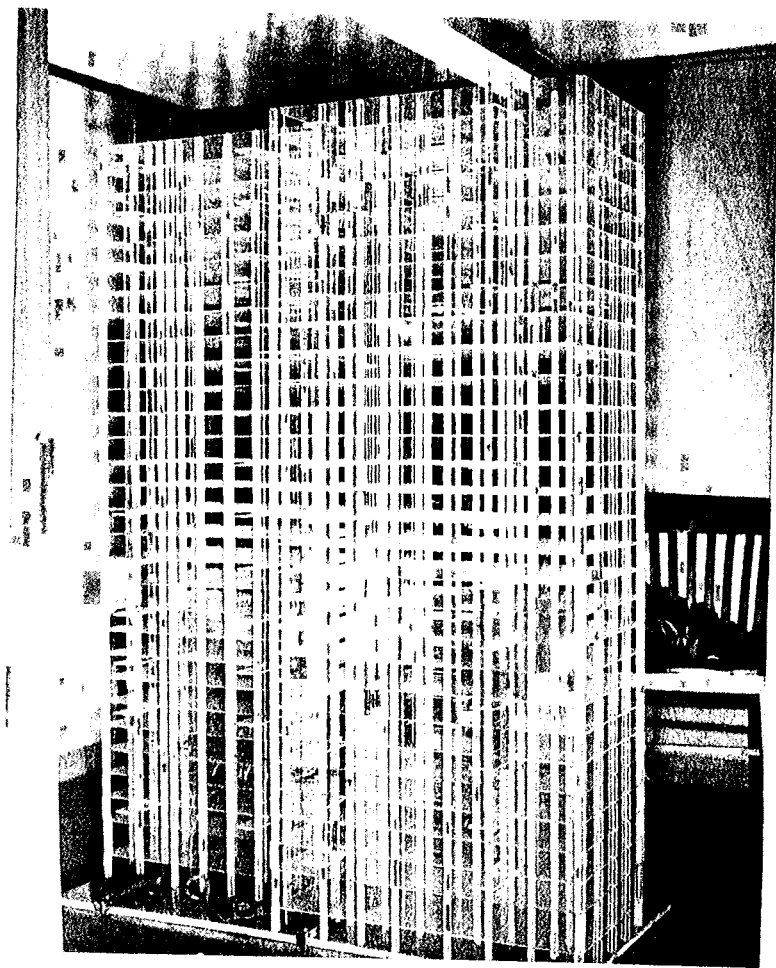


Fig. 1-7—Reinforced concrete flat plate apartment building (1/50 scale)—Benesch

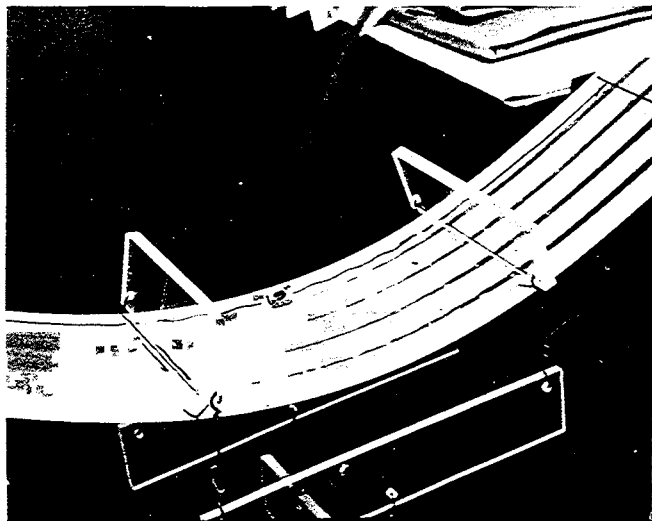


Fig. 1-8—Model of precast unit tested to study buckling characteristics of top flange (1/15 scale)—Material Service Dynacore

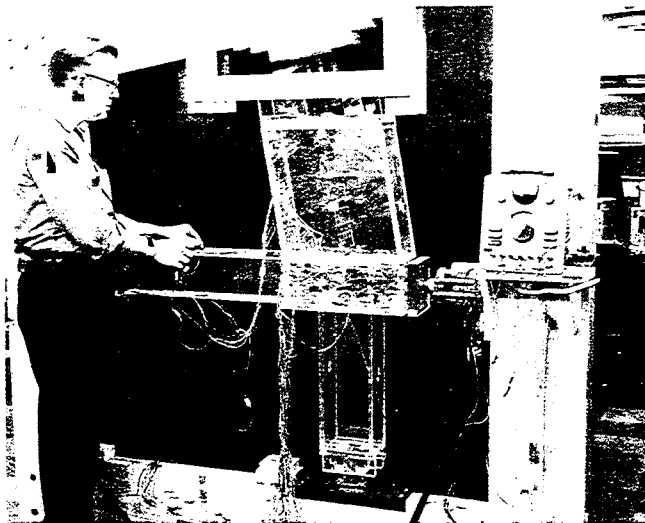


Fig. 1-9—Details of column—caisson connection for high-rise office building (1/8 scale)—Murphy

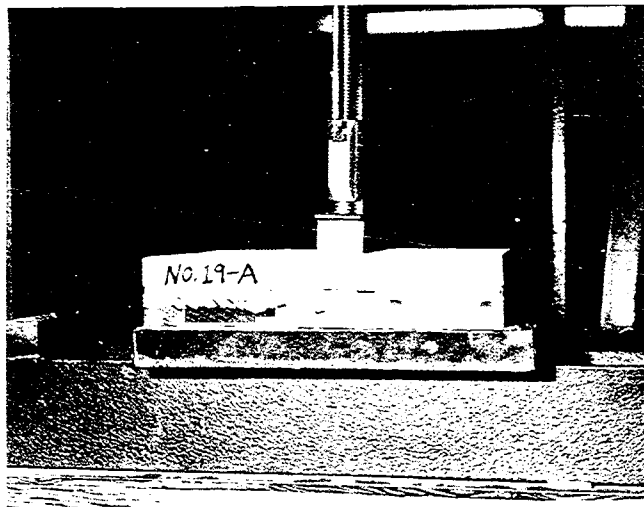


Fig. 1-10—Micro-concrete model (1/8 scale) of reinforced concrete punching shear specimen—Office of Civil Defense

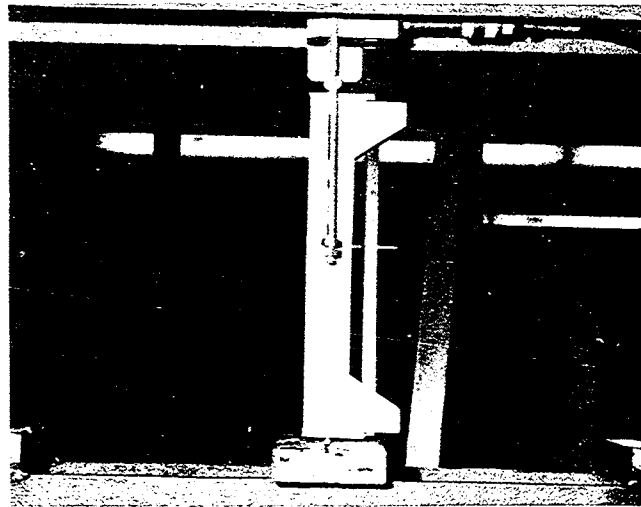


Fig. 1-11—Micro-concrete model (1/8 scale) of reinforced concrete column, test specimen—Office of Civil Defense

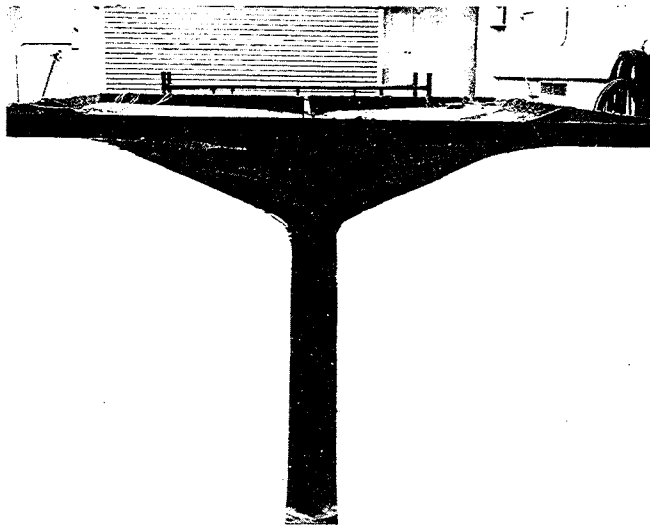


Fig. 1-12—Micro-concrete structure umbrella hyperbolic paraboloid shell roof structure (1/10 scale)—Port of New York Authority

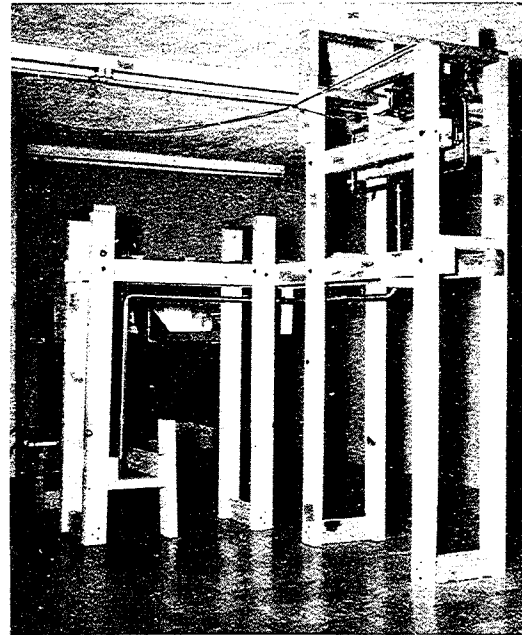


Fig. 1-13—Model of power plant steam pipe system (1/20 scale) built to study dynamic response—Sargent & Lundy

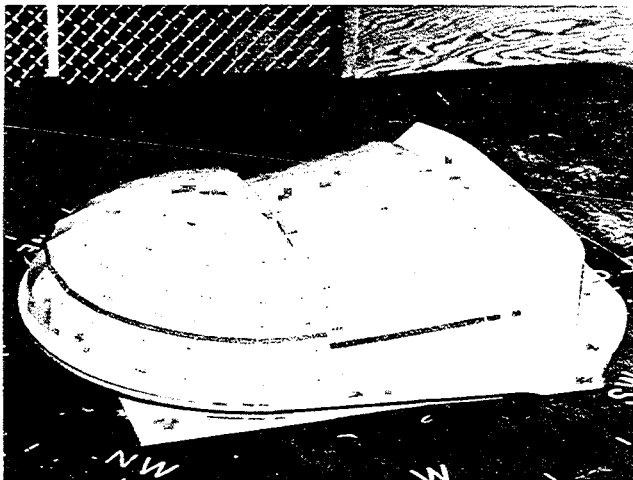


Fig. 1-14—Model (1/100 scale) of structure built for determination of wind forces—Severud, Perrone, Fischer, Sturm, Conlin, Bandel

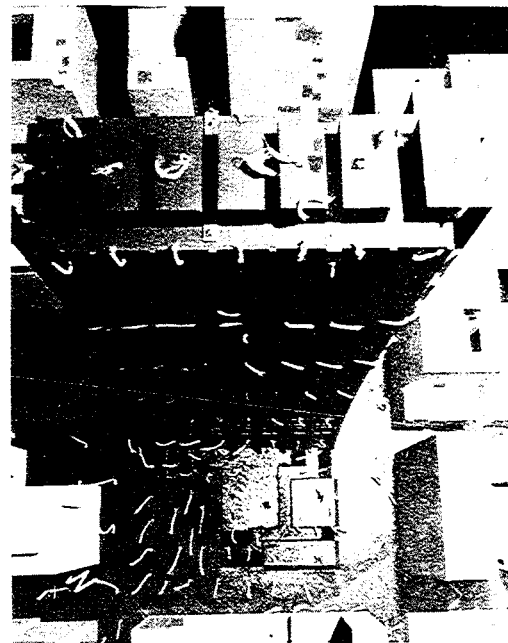


Fig. 1-15—Model (1/200 scale) of high-rise structure in wind tunnel—Murphy

PRINCIPLES OF MODEL ANALYSIS

By PAUL ZIA, RICHARD N. WHITE
AND DAVID A. VANHORN

This paper presents the principles of model analysis with particular reference to concrete structures. Emphasis is placed on the use of direct models for the study of both the elastic and the inelastic behaviors up to and including the ultimate strength.

The similitude relationships are summarized and are followed by a discussion of the requirements (such as stress strain relationship, unit weight, failure criterion, etc.) that the modeling materials must meet. Where these requirements can not be completely satisfied, considerations are given to the conditions and implications of the use of distorted models. Special problems associated with modeling for bond, cracking, time dependent and thermal loadings are discussed.

An example dealing with the modeling of a prestressed concrete reactor vessel is presented to illustrate both the application of similitude requirements and some of the limitations of the scaling process.

Keywords: analogies; bond (concrete to reinforcement); cracking (fracturing); concretes; dynamic loads; earthquakes; geometry; loads (forces); micro-concrete; models; mortars (material); nuclear reactors; pressure vessels; prestressed concrete; reinforced concrete; reinforcing materials; reinforcing steels; research; scale (ratio); simulation; static loads; stress-strain relationships; structural analysis; structural design; thermal properties.

□ The classifications of structural models and the proper role played by each have been discussed in the previous paper. To consider the behavior of concrete structures beyond the elastic range up to and including the ultimate strength, one must deal with "direct" models. The purpose of this paper is to summarize the laws of similitude and to consider their implications and other special problems for direct modeling with particular reference to concrete structures.

SIMILITUDE REQUIREMENTS FOR DIRECT MODELING

The basic similitude requirements for the design of structural models have been treated by many authors^{2,5,11,13}. The scale factors for models under static load are summarized in Table 2-1. The scale factor s_i is defined* as the multiplier required to convert the model quantity " i_m " to the

* See page vi

This report was prepared by Subcommittee 2 of the ACI Ad Hoc Committee on Structural Models. The subcommittee chairman is PAUL ZIA, professor, Department of Civil Engineering, North Carolina State University, Raleigh, North Carolina. The other subcommittee members are: RICHARD N. WHITE, associate professor, Cornell University, Ithaca, New York and DAVID A. VANHORN, professor and chairman, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania.

corresponding prototype quantities " i_p ", or

$$i_p = s_i i_m$$

It should be pointed out that, in a static problem, only two fundamental dimensions are involved: F and L. Therefore, in order to have a true model, one can only select two independent scale factors; e.g., s_σ and s_l . Other scale factors are then obtained according to the principles of dimensional analysis as given in the column 4 of Table 2-1.

If an independent scale factor, s_ϵ , is also chosen for the strain, then a distortion is immediately introduced into the model. However, the use of an independent strain scale could provide more freedom in the modeling process. The various requirements and implications resulting from the use of the distorted models will be considered in terms of materials properties, geometry and loading.

Materials Similitude

To completely model the behavior of concrete structures, the stress-strain curve of the model material must be geometrically similar to that of concrete. This implies that the scale factors for stress and strain, s_σ and s_ϵ , must be constant at all levels of stress and strain, both compressive and tensile. Although in principle, one may arbitrarily select the values of s_σ and s_ϵ , provided that there is no large deflection phenomenon, the choice is generally dictated by the properties of the available model material. Furthermore, for a composite material such as reinforced concrete, the stress-strain relationship of the prototype reinforcement must also be linked to that of the model reinforcement through the same scale factors.

The simplest way of modeling concrete is to use concrete made of small size aggregates or cement mortar¹⁰ as shown in Fig. 2-1a. For such cases, the magnitude of s_σ can be made equal to unity, while s_ϵ can be made nearly equal to one by careful mortar design. On the other hand, high strength gypsum plaster and sand mixes^{2,4,5,14,15} have been used as model concrete with one or both of the scale factors, s_σ and s_ϵ , being different from one as shown in Fig. 2-1b, c, d. It is obvious that the corresponding similitude requirement for reinforcement as shown in Fig. 2-1e and f can easily be satisfied by the use of small size steel wires, whereas those requirements shown in Fig. 2-1g and h are more difficult to satisfy. However in the latter case, some success has been obtained by investigators^{2,4} using carefully annealed small size threaded steel rods.

It should be pointed out that while the annealing process re-establishes the distinct yield plateau it does not reduce the elastic modulus of the steel. What is shown in Fig. 2-1g and h for the model reinforcement is the apparent modulus of the threaded rod. The apparent modulus is influenced by the degree of plastification of the steel near the root area of the thread because of stress concentration. It is also dependent upon how the apparent unit stress is determined. In accordance with the practice commonly accepted for the deformed reinforcing bar, the nominal area for the threaded rod is calculated on the basis of unit weight of the rod. This area is slightly larger than the net area at the root of the thread; thus resulting in a lower value of the apparent modulus.

Fig. 2-2 shows the examples of the similitude requirements for the cases of $E_{rp}/E_{rm} = 1$ and $E_{rp}/E_{rm} \neq s_o/s_\epsilon$. It can be seen from Table 2-1 that, for $E_{rp}/E_{rm} = 1$, the stress scale for steel must be equal to the strain scale for concrete if the model is to properly simulate the yield condition of the prototype. On the other hand, if $E_{rp}/E_{rm} \neq s_o/s_\epsilon$, the steel and concrete stresses must be modeled by different scale factors. The consequence of these restrictions is that the required steel area for the model could be substantially different from what would be required in true modeling, depending upon the values of s_o and s_ϵ .

Since the use of steel wire is by far the most popular choice for model reinforcement, the similitude relationships as shown in Fig. 2-1e, 1f, Fig. 2-2a and 2b are encountered more often than others. If large deformation effects are important considerations in prototype, then the choice of modeling scheme is immediately limited to true modeling, Fig. 2-1a and 2-1e.

The difficulties encountered in modeling the reinforcement with respect to bond and cracking behaviors are even greater. This aspect of modeling will be discussed subsequently.

If the modeling of the ultimate flexure strength is the primary concern, some of the stringent conditions mentioned above can be somewhat relaxed, depending upon the mode of failure. For underreinforced beams, the reinforcement properties, i.e., the yield point and its corresponding strain, must be properly modeled whereas an exact modeling of the concrete properties is not necessary. On the other hand, for overreinforced beams, the modeling of the concrete properties including its maximum strength and its ultimate strain must be exact while the similitude requirement for reinforcement may be relaxed. It is obvious that in order to model "balanced" beams, the modeling of both the steel and the concrete properties must be carefully controlled not only with respect to the shape of the stress strain curves but also the magnitudes of the critical stresses and strains. The same is true for modeling moment-rotation relationships.

The required scale factor of one for Poisson's ratio is a rather restrictive condition. If it is faithfully adhered to, it is almost necessary that the prototype and the model materials be identical. Fortunately, for one-dimensional structural elements in which the effects of flexural and axial stiffnesses predominates, failure to satisfy this requirement is unimportant. But for two and three-dimensional structural elements in which the effects of shear and twisting are significant, serious error may result if the required condition for Poisson's ratio is relaxed.

Whenever the dead load stress is of importance, the scaling of mass density must be carefully considered. The scale factor for mass density depends upon the scale factors for stress and linear dimension. If no scaling is involved in stress (i.e. $s_\sigma = 1$) then the material for small scale models must have a density s_l times that of the prototype material. This condition is usually impossible to satisfy in practice. If the condition is important, it is often met artificially by using additional distributed load. On the other hand, if the stress scale and the dimensional scale are chosen to be equal, the model material can be of the same density as that of the prototype material.

Geometry Similitude

For geometrical similarity between the prototype and the model, it is necessary that all linear dimensions of the model be scaled down from the corresponding dimensions of the prototype by a constant ratio. Practical limitations prevent the scaling of many structural details, particularly in reinforced concrete structures. The scaling of fine aggregates, cement particles and steel deformation is impossible. Fortunately, deviations from those minute details are not serious problems so long as the similitude requirements for materials properties are satisfied.

What may become an important consideration in designing the model is the size effect on the strength characteristics and the failure criterion of the model material. It is well known that the strength and the mode of failure vary as the size of the test specimen changes.^{8,12} Thus, a prototype and its model may behave differently because of the sheer difference in their sizes. Presently available data^{4,8,14} seem to indicate that model beams with the least dimension in the order of one inch are not affected significantly by the reduction in size. Further reduction in the model size may cause serious discrepancy in the model results.⁸

The scale factor for displacement is the product of the scale factors for the strain and the linear dimension. It can be seen readily that if the strain scale, s_ϵ , is one (true model), the model displacement will be geometrically similar to the prototype displacement but in a reduced amount in the same proportion as all other linear dimensions. If the linear scaling factor is large, then the model deformation may be so much reduced as to require much more sensitive instrumentation for its measurement. This difficulty may be partly overcome if the model material is such that the strain scale s_ϵ is less than one. In such a case, the deformation of the model will be geometrically distorted. While the increased deformation will facilitate the instrumentation requirement, the model designer must assure himself that the structural behavior of the model will not be adversely affected by the change in geometry as in the case of column stability or shell buckling.

Loading Similitude

To completely simulate the applied force, not only the magnitude but also the distribution of the load must be properly scaled. In general, the condition for concentrated loads is not difficult to comply with but the distributed load would require elaborate setup to achieve, such as using vacuum or hydrostatic pressure.

It should be noted that the loading similitude is independent of the strain scaling factor s_ϵ . Furthermore if $s_l \gg 1$, then the applied force on the model will be only a small fraction of that expected on the prototype; values of $s_\sigma > 1$ will further decrease the model load. This points clearly to the

advantage of small scale models where the loading system can be much simplified.

SPECIAL CONSIDERATIONS

Similitude Requirements for Bond

There are several important aspects of concrete structure behavior which are either wholly or strongly dependent upon proper bond between reinforcement and concrete. These include:

- (a) Complete bond failure under single overloads, or as caused by repeated or alternating loads
- (b) Combined failure modes in which bond plays a relatively minor, but not insignificant role
- (c) Progressive destruction of bonding action as induced by effects of the loading
- (d) Transfer of prestress in pretensioned concrete
- (e) Cracking behavior including the size, distribution and growth of cracks

Accurate modeling of all aspects of prototype bond behavior may not be required in all applications, but the flexibility and applicability of a modeling technique is greatly restricted if bond similitude is questionable.

The basic similitude requirement for bond between concrete and its reinforcement, for true models, is that bond stresses developed by the model reinforcing be identical to those of the prototype reinforcing. It also requires that ultimate bond strength of model and prototype reinforcement be identical. This would be possible only when $s_{\sigma} = 1$.

The scale of the model is an important factor in the bond similitude problem. For large models, #2 and #3 deformed bars which satisfy ASTM requirements may be within the desired range of size as model steel. For smaller models, one is forced to utilize either wire (smooth, rusted or deformed), threaded rod, or stranded cables (for prestressed models). It should be noted, however, that even the use of standard deformed bars does not necessarily insure true bond modeling because of the difference in bond strength of small and large size bars.

Modeling of bond is seriously complicated by our limited knowledge of the bond mechanism in prototype concrete. According to the current notion⁹ of the bond behavior of prototype reinforcing, one must attribute bond strength mainly to the mechanical wedging action afforded by the protruding deformations of the bars. Based upon the current ACI Code, the ultimate bond strength per inch of bar length is

$$U_u = 30 \sqrt{f'_c} < 2500D$$

It would be grossly misleading if one attempts to model this relationship with the types of small reinforcement mentioned previously. In the first place, the above expression is dimensionally inhomogeneous. Secondly, it was not intended to be applied to the small size reinforcement as it was derived experimentally from tests on ordinary bars. If the above expression were valid for all size bars, the ultimate bond strength of small size reinforcement (0.25 in. diam. or below) would always be dictated by the 2500D limitation. However, from the work of Lutz⁹ and others, it can be stated that the

ultimate bond strength is not independent of bar size for any range of diameters. Thirdly, the geometric dissimilarity between the wire, threaded rod, stranded cable and the prototype deformed bars is so great that their bonding action is undoubtedly different. For wires and stranded cables, the lack of protruding lugs greatly lessens the tendency to produce a splitting failure in concrete. On the other hand, pull out tests on threaded rods often produce a complete shearing of the surrounding concrete at the top of the threads, also with little or no splitting accompanying the failure. The effect of concrete cover upon bond strength, and the increased bond strength afforded by stirrups and other steel which tends to prevent splitting from developing adjacent to bars under high bond stress, are also difficult to predict and to model.

In spite of these difficulties, it should be noted that it may be possible to provide model steel which will duplicate local bond behavior of the prototype steel even though its ultimate bond strength is not identical to that of the latter. Cracks normal to the direction of the reinforcement can open through the mechanism of localized slipping of reinforcing without necessarily having longitudinal splitting cracks. Therefore, partial satisfaction of similitude would be provided by this type of model steel.

Another method to improve bond modeling is strictly dependent upon a detailed knowledge of the bond behavior of model reinforcements. If this information is available, then the bond strength of a certain combination of prototype bars may be modeled by using the proper number and size of model bars. The philosophy of this type of modeling is to provide the correct amount of total bond strength instead of being concerned with scaling unit bond stress. It is felt that this approach has considerable merit and will provide the best answer once a detailed knowledge of both prototype and model reinforcing bond behavior is at hand. True bond stress distribution bears little resemblance to the computed average unit bond stress; it will always be difficult to attempt to model such a poorly defined quantity.

Results of studies on bond behavior of various model reinforcements will be presented in the paper "Reinforcement for Small Scale Direct Models of Concrete Structures", in this volume. These results show that pull out bond strengths of prototype bars can be duplicated at a small scale with certain types of deformed model reinforcements, and that the main bond characteristics of prototype bars can be represented with this type of steel.

Cracking Similitude

The inelastic load deflection response of a reinforced concrete structure is often strongly dependent upon the degree and manner of cracking. Cracking modes can also influence behavior under reversed or repeated loading, moment redistribution in continuous structures and even the final mode of failure. The modeling of cracking behavior is just as difficult as the modeling of bond; the two are intimately related phenomena.

To insure complete similitude of cracking between model and prototype, it is necessary that both the crack width and the crack spacing are scaled down by the linear dimensional scale factor s_l . Such seemingly simple requirements are exceedingly difficult to achieve for a number of reasons. First of all, it necessitates that $s_\epsilon = 1$. Secondly, higher tensile strength of the model concrete often leads to initiation of model cracking at a higher scaled load level than is desired which, in turn, affects the load deflection response. If the bond properties of the model steel are inadequate, there will be a reduced number of cracks with relatively wider crack widths. Furthermore,

crack spacings in prototype have been shown by Broms and Lutz³ as being almost directly proportional to the effective side concrete cover over the reinforcement. It has not been demonstrated whether their observation is also valid for the very small cover encountered in models.

Very few tests have been conducted to critically compare cracking small scale models with full or medium size prototypes. With rather large size models, successful modeling of overall cracking pattern seems to have been achieved by Borges and Lima¹ as well as Kaar.⁷ The fourth paper in this volume presents results from Ref.¹ However, there is evidence that the total number of cracks appear to decrease with decreasing model size, although the scaled load deflection curves for the prototype and the models were practically identical. Similar tests on much smaller scale beam models conducted at Cornell University and elsewhere have shown the same type of behavior.

Thermal Modeling

An investigation of the behavior of a concrete structure may involve determination of the effects produced by temperature differentials. As a result, not only the temperature but also the thermal properties of the material must be given consideration. Some of the more common thermal properties are the Coefficient of Thermal Expansion α , the Specific Heat c , the Thermal Conductivity k , and the Coefficient of Heat Transfer h .

It may be ideal from a theoretical standpoint to treat these thermal properties separately in a modeling problem. However, it is not uncommon to judiciously treat two or more of the properties together in such a way as to reduce the number of required model design conditions. For example, one may deal with the Thermal Diffusivity $D = \frac{k}{c\rho}$ which combines the Specific Heat and the Thermal Conductivity with mass density. The Thermal Diffusivity is commonly used in studies of thermal stresses in massive structures such as dams. Another example is the use of the Nusselt number in cases involving heat transfer or when surface effects must be considered. The Nusselt number $\frac{hl}{k}$ combines the Coefficient of Heat Transfer and the Thermal Conductivity. This parameter should be equal in model and prototype in order to have properly scaled temperature gradients. A strong restriction arises here if the model and the prototype are made of the same material. If so, the linear scale factor must be one; that is to say, the model must be of the same size as the prototype.

In Table 2-2, the significant quantities for modeling thermal effects are listed, along with related fundamental dimensions. At this point, it should be noted that in a thermal problem, four fundamental dimensions are involved: F , L , T , and θ . Therefore, for a true model, four independent scale factors may be selected. The remaining scale factors are then based on their individual relationships to the four selected. For a problem in which more than four must be established, distortion is introduced.

In the last three columns of Table 2-2, lists of scale factors are presented for cases involving three different types of model material. For the general case, with Poisson's ratio unequal in model and prototype materials, Hovanesian and Kowalski⁶ have derived the similitude relationships for a general stress state or a state of plane strain; and for a state of plane stress. It should be noted that in problems involving only thermal effects, the strain

scale is not an independently selected scale factor, but is dependent upon the scale factors for the temperature and coefficient of thermal expansion. Furthermore, the time scale factor is proportional to the square of the dimensional scale factor, which means that it is possible to model long time thermal effects in a greatly reduced time.

In choosing a suitable material for thermal modeling, one very important consideration is that the material should have thermal and elastic properties which do not change with temperature. Certain properties of metal and plastics are highly sensitive to temperature changes; but mortars, after having reached a certain age, have properties which are influenced by temperature very little. In general, it is usually impossible to satisfy all design conditions in the model and, therefore, a compromise must be made in line with the factors which are of primary importance in the particular problem being investigated.

Dynamic Modeling

In the modeling of concrete structures, it is often necessary to consider the effects of time. Some of the more common time dependent effects are associated with creep and shrinkage in the concrete, relaxation of prestressing steel, soil structure interaction, and impulse or vibratory loadings. For lack of a clear understanding of the mechanisms of creep, shrinkage, relaxation and their interaction, little has been reported on the modeling of these phenomena. On the other hand, there has been a great deal of interest in modeling the dynamic behavior of buildings, bridges, and dams subjected to earthquake or dynamic loadings.

The general similitude requirements for a problem involving earthquake response of structures are summarized in Table 2-3. The significant quantities, along with the fundamental dimensions, are listed in the first two columns. For a typical problem involving time-dependent loadings, since there are three fundamental dimensions (F, L, and T), three independent scale factors may be selected for true modeling (Case III). The other scale factors can then be derived according to the principles of dimensional analysis. For cases requiring the establishment of more than three factors, the problem of distortion is introduced (Cases I and II). Obviously, there are other possibilities which might be more advantageous for a particular case.

In earthquake simulation, one of the primary difficulties lies in the modeling of the random displacements. Some investigators have attempted to approximate the effects with sine-wave vibrations or impacts. However, it has been found that the random character of the actual seismic excitation is of critical importance, and to add to the difficulty, there is only limited data on some of the strong earthquakes.

Another important factor which must be considered is the variation of material properties under dynamic loading. It is well known that strain-rate has a very significant effect on quantities such as ultimate strength, yield point, modulus of elasticity, and others. Structural damping is also a factor affecting structural behavior which should be considered. However, although there have been several investigations in this area, the effects of this phenomenon have not been clearly established.

EXAMPLE

The modeling of a prestressed concrete reactor vessel (PCRv) will be discussed to illustrate both the application of similitude requirements and

some of the limitations of the scaling process. The PCRV chosen is the primary containment type vessel as used in gas cooled reactor generating plants. The structural model has played an important role in research on the basic behavior of these very complex structures and as a design tool for specific vessels being built in England, France and the U.S. Since the geometry of primary PCRV's varies considerably from one design to the next, the typical PCRV shown in Fig. 2-3 is merely a representative structure.

The typical PCRV is prestressed with several thousand or more hoop, head, and longitudinal post tensioning elements to resist an internal design pressure of about 500 psi. A considerable amount of mild steel deformed bars are also used in the vessel. The vessel cavity is lined with a thin, leak tight steel plate liner. Interior wall temperature may be in the vicinity of 200 F. under normal operating conditions. A large number of penetrations must be made through the heavy walls and heads of the PCRV. Typical re-fueling penetrations in the upper head are shown in Fig. 2-3. The combination of prestressing, reinforcement, and penetration hardware makes many areas of the vessel highly congested and results in the use of concrete with maximum aggregate size of about 1 in. or less.

Loading conditions that must be considered in design consist of the following:

- (a) Construction induced strains and stresses, including dead load
- (b) Prestress
- (c) Isothermal pressurization and cycling
- (d) Combined thermal and pressurization cycling
- (e) Isothermal creep and shrinkage under various mechanical loadings
- (f) Thermal dependent creep and shrinkage
- (g) Overpressure leading to extensive vessel cracking
- (h) Overpressure leading to ultimate loading
- (i) Earthquake
- (j) Combinations of the above

Model Design

Because the determination of the non linear behavior of a PCRV is normally the motivating factor in using a structural model, a direct model utilizing materials similar to those in the prototype must be used.

The model design for a major structure of this complexity must begin with a decision on the geometrical scale factor s_1 . Current (1968) terminology with regard to scale can be defined as:

Type	Scale factor s_1	Vessel height
Large scale	3-5	25 ft
Small scale	15-20	6 ft
Very small scale	40-50	2.5 ft

If a true model (complete reproduction of all construction details) is contemplated, the very small scale model is immediately eliminated because of the extensive custom miniaturized details required and the need for a very fine aggregate concrete. On the other hand, the very small scale model can

be considered if a representative model (simplified in detail, such as reducing the number of prestressing and reinforcing elements, but maintaining correct steel areas at all sections) is deemed sufficient for answering the questions at hand. The use of a small scale true model may or may not be possible, depending upon the complexity of the prototype, but a representative model may be built at this scale without encountering any overwhelming difficulties. As would be expected, the large scale model presents no significant problems from the standpoint of model design. The disadvantage of the latter is the high cost of building and testing a large structure. Further discussion on the possibilities of meeting similitude requirements will be presented subsequently.

The model design will now be established and the capabilities of the model in predicting response to vessel loadings will be assessed.

A small scale representative model with $s_1 = 20$ will be chosen for this example. In accordance with Table 2-1, all vessel dimensions are reduced by a factor of 20; the model dimensions are given in Fig. 2-4.

The cracking behavior of the vessel under increasing pressure as well as the general non-linear response are both highly dependent upon material properties. Accordingly, every effort will be made to obtain a model concrete with $s_\sigma = s_\epsilon = 1$ (Fig. 2-1a) and model prestressing and reinforcement with $E_{rp}/E_{rm} = 1$. The additional similitude requirements, as given in Table 2-1, include identical stress strain curves for each type of model and prototype materials.

Prototype steel areas are reduced by the factor $s_1^2 = 400$. In order to simplify model construction and reduce the need for a very fine aggregate model concrete, the total number of prestressing elements and reinforcing bars will be reduced by a factor of three or more. Thus each individual prototype steel element will not be geometrically scaled by $s_1 = 20$.

Other features of the PCRV such as the liner and penetrations will be scaled geometrically. In addition, the linear material must meet the requirements of $s_\sigma = s_\epsilon = 1$.

Model Loadings

(a) Construction and Dead Loads -- The required mass density of the model materials is 20 times that of the prototype materials since $s_\sigma = 1$. This cannot be achieved because model and prototype materials will have essentially the same density. Thus the model dead load stresses will be only about 5 percent of prototype stresses, and the inherent distortion must be assessed.

As shown in Table 2-1, any externally applied prototype line loads, concentrated loads, or moments will be scaled by the factors, 20, 400, and 8000, respectively.

(b) Prestress -- Prestressing forces are correctly scaled although the precise distribution of elements is altered slightly. Results of very small scale model tests with only a few prestressing elements have shown excellent agreement with computed elastic response.¹⁷

(c) Pressurization and Cycling -- Since $s_\sigma = 1$, the pressure scale factor is unity and the design pressure = 500 psi. The structure will be designed to respond elastically to this pressure and the model should yield excellent

predictions of prototype response. As with the prestress load, model stresses and strains will be identical to expected prototype stresses and strains; prototype linear displacements will be 20 times model displacements.

(d) Steady State Thermal Gradient Plus Pressure -- This loading condition (200 Deg F interior, 70 Deg F exterior) forces requirements on model thermal properties as specified in Table 2-2. If s_D and s_α are essentially unity for the model concrete, then the scale factors shown in the right hand column of Table 2-2 can be used, with temperatures in model and prototype being identical ($s_\theta = 1$). This will produce undistorted thermal stresses, strains, and displacements which can be measured directly in combination with other stresses since s_σ and $s_\epsilon = 1$.

On the other hand, if the prototype concrete thermal properties α and D cannot be duplicated in the model concrete, the similitude requirements as given in the first column of Table 2-2 must be used. With s_E already set at unity, it then becomes possible to adjust model temperatures by s_θ such that the product s_α and s_θ still equals unity; and thermally induced stresses, strains, and displacements remain undistorted. If $s_\alpha s_\theta \neq 1$, thermal actions cannot be modeled simultaneously with internal pressure, but separate results may be superposed.

The problem of modeling heat transfer at the boundary (scaling of Nusselt's number) will not be discussed here, but it should be examined for any thermal modeling problem.

(e) Creep and Shrinkage (without elevated temperature) -- Modeling creep and shrinkage places severe demands on model concrete properties. If the model concrete has aggregate sizes appreciably smaller than those of the prototype concrete, it will be impossible to design a model concrete with creep properties guaranteed to be identical with those of the prototype concrete. A further complication arises from the greatly reduced thicknesses in the model so that prototype moisture conditions cannot be modeled.

Perhaps the most that can be expected from small scale creep modeling is a reasonable indication of how the prototype vessel will behave in creep. By measuring the basic creep behavior of model and prototype concrete on small specimens with due consideration of the size effect, and by paying very careful attention to model curing conditions, it should be possible to approximately extrapolate the model PCRV creep behavior to that of the prototype. Until we come to a better understanding of the basic creep phenomenon of structural concrete, no more can be expected from a small model. The same statements can be applied to shrinkage behavior.

(f) Thermally Dependent Creep and Shrinkage -- Creep in a PCRV under operating conditions will be accelerated by the elevated temperatures existing in the concrete. Because the thermal creep of prototype concrete is not known adequately, accurate modeling of this phenomenon cannot be accomplished at the present time.

An additional similitude problem to be pointed out is that the time scale for creep deformation should be unity while the time scale for transient thermal effects is $s_1^2/s_D = 400$ for $s_D = 1$. Thus a transient thermal loading in the model would be applied in 0.1 days if the same loading cycle took 40 days in the prototype, and the creep strain accompanying the thermal strain would occur at a grossly incompatible time scaling.

The conclusion to be drawn is that creep modeling for a specific structure should be undertaken on the largest possible model. Small model creep results can be very useful, however, as a comparison to analysis predictions. In this application the small scale model is really a small structure with its own basic creep properties, and scaling is no longer a problem.

(g) Overpressure Leading to Extensive Vessel Cracking -- The model material properties of primary interest for this loading are tensile strength of model concrete, and bond strength between concrete and deformed reinforcement. Concrete compressive strength is of less importance because internal pressure tends to relieve the high initial compression due to prestress. Bond of prestressing steel is not a factor because ungrouted tendons are to be used.

Bond similitude has been discussed previously and will not be further amplified here. The modeling condition of $s_\sigma = 1$ states that tensile strengths of model and prototype concrete be identical. If true cracking similitude is to be achieved, this scaling factor ($s_\sigma = 1$) must be met in the most general sense, that is, for all combined stress states as well as for uniaxial stresses. A major difficulty yet to be overcome is the lack of a sound criteria for failure of prototype concrete. There is a dearth of information on some stress states, such as triaxial stress states, especially those involving principal tension and compression.¹⁸

Perhaps the best that can be achieved currently is to match uniaxial tensile strengths in model and prototype. By careful design of the model concrete, this requirement can be at least approximately met, although the ratio of tensile strength to compressive strength tends to be high in many model concrete mixes. Subsequent papers in this volume provide additional information on model concrete mix properties.

In summary, it is felt that modeling cracking behavior at this scale is feasible, and, if properly done, will yield a reasonable measure of prototype response.

(h) Overpressure Leading to Ultimate Load -- At this state all post-elastic similitude requirements, including those given earlier for the steel reinforcement, liner and prestressing elements, become crucial to the response of the model. Bending actions may produce high compressive stress in the concrete; concrete continues to crack, and steel components yield until excessive vessel cracking and deformations lead to loss of pressure.

This loading is the extreme test on the adequacy of meeting material similitude requirements. With careful choice and workmanship, small scale model results for this type of structural action can be highly satisfactory.

(i) Earthquake -- Modeling of dynamic response of massive structures at a small scale is still in its early developmental stages. Following one of the sets of similitude requirements given in Table 2-3, and using the techniques discussed in the paper by Ferry Borges and Pereira in this volume, one could gain valuable insight into the dynamic response characteristics of the PCRV. However, the confidence level of results extrapolated to the prototype may be low because of the many poorly defined quantities involved. Furthermore, the stresses, strains, and displacements are all distorted in accordance with the scaling factors given in Table 2-3, and inelastic dynamic modeling would be seriously limited because undistorted prestress and the

inadequate dead load effects are automatically superposed on the dynamic stresses.

(j) Loading Combinations -- A number of load combinations need to be considered. Similitude requirements will not change, but additional difficulties may be encountered in meeting several loading condition requirements simultaneously and in accounting for combinations of stresses where distortions of one or more stresses arise. Superposition of individual loading cases will be the best approach for elastic response.

CONCLUDING REMARKS

The similitude requirements for structural models have been summarized with particular reference to reinforced concrete structures. The application of these requirements is illustrated by an example dealing with the modeling of a prestressed concrete reactor vessel. It has been shown that the requirements with respect to materials properties can often impose strong restrictions on the modeling process. In certain instances, such as with bond, cracking, creep, and shrinkage where a basic understanding of the physical phenomena is still lacking, highly accurate modeling is not yet possible. Nevertheless, with proper consideration of the laws of similitude, there have been many successful model studies in which overall prototype responses have been correctly predicted even though certain details of the behavior may not have been reproduced.

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TABLE 2-1—SUMMARY OF SCALE FACTORS FOR REINFORCED CONCRETE MODELS

(1)	Quantity (2)	Dimension (3)	True model (4)	Distorted model		
				$E_{rp}/E_{rm} \neq s_\sigma/s_\epsilon$ (5)	$E_{rp}/E_{rm} = s_\sigma/s_\epsilon$ (6)	$E_{rp}/E_{rm} = 1$ (7)
Materials properties	Concrete stress, σ_c	FL^{-2}	s_σ	s_σ	s_σ	s_σ
	Concrete strain, ϵ_c		1	s_ϵ	s_ϵ	s_ϵ
	Modulus of concrete, E_c	FL^{-2}	s_σ	s_σ/s_ϵ	s_σ/s_ϵ	s_σ/s_ϵ
	Poisson's ratio, ν_c		1	1	1	1
	Mass density, ρ_c	FL^{-3}	s_σ/s_l	s_σ/s_l	s_σ/s_l	s_σ/s_l
	Reinf. stress, σ_r	FL^{-2}	s_σ	$s_\epsilon E_{rp}/E_{rm}$	s_σ	s_ϵ
Geometry	Reinf. strain, ϵ_r		1	s_ϵ	s_ϵ	s_ϵ
	Linear dimension, l	L	s_l	s_l	s_l	s_l
	Displacement, δ	L	s_l	$s_\epsilon s_l$	$s_\epsilon s_l$	$s_\epsilon s_l$
	Angular displacement, β		1	s_ϵ	s_ϵ	s_ϵ
Loading	Area of reinforcement, A_r	L^2	s_l^2	$s_\sigma s_l^2 E_{rm}/s_\epsilon E_{rp}$	s_l^2	$s_\sigma s_l^2/s_\epsilon$
	Concentrated load, Q	F	$s_\sigma s_l^2$	$s_\sigma s_l^2$	$s_\sigma s_l^2$	$s_\sigma s_l^2$
	Line Load, w	FL^{-1}	$s_\sigma s_l$	$s_\sigma s_l$	$s_\sigma s_l$	$s_\sigma s_l$
	Pressure, q	FL^{-2}	s_σ	s_σ	s_σ	s_σ
	Moment, M	FL	$s_\sigma s_l^3$	$s_\sigma s_l^3$	$s_\sigma s_l^3$	$s_\sigma s_l^3$

TABLE 2-2—SUMMARY OF SCALE FACTORS FOR THERMAL MODELING

Quantity	Dimension	Scale factor		
		Different materials in model and prototype with $\nu_p = \nu_m$	Different materials in model and prototype with $\nu_p \neq \nu_m$	Same materials in model and prototype and same temperature
Stress, σ	FL^{-2}	$s_\alpha s_\theta s_E$	$c_2 s_\alpha s_\theta s_E$	1
Strain, ϵ		$s_\alpha s_\theta$	$c_1 s_\alpha s_\theta$	1
Elastic modulus, E	FL^{-2}	s_E	s_E	1
Poisson's ratio, ν		1	s_ν	1
Coefficient of linear expansion, α	θ^{-1}	s_α	s_α	1
Thermal diffusivity, D	$L^2 T^{-1}$	s_D	s_D	1
Linear dimension, l	L	s_l	s_l	s_l
Displacement, δ	L	$s_\alpha s_\theta s_l$	$c_1 s_\alpha s_\theta s_l$	s_l
Temperature, θ	θ	s_θ	s_θ	1
Time, t	T	s_l^2 / s_D	s_l^2 / s_D	s_l^2

For general stress state, or plane strain

$$c_1 = \frac{(1 + \nu_p)(1 - \nu_m)}{(1 - \nu_p)(1 + \nu_m)}$$

$$c_2 = \frac{1 - \nu_m}{1 + \nu_p}$$

For plane stress

$$c_1 = \frac{1 + \nu_p}{1 + \nu_m} \quad c_2 = 1$$

TABLE 2-3—SUMMARY OF SCALE FACTORS FOR EARTHQUAKE RESPONSE OF STRUCTURES

Quantity	Dimension	Scale factor		
		Case I	Case II	Case III
Geometry, l	L	s_l	s_l	s_l
Spec. mass, ρ	$FL^{-4}T^2$	s_ρ	s_ρ	s_ρ
Modulus, E	FL^{-2}	s_E	s_E	s_E
Acceleration, a	LT^{-2}	s_a	$s_Q/s_l^3 s_\rho$	$s_E/s_\rho s_l$
Velocity, v	LT^{-1}	$\sqrt{s_E/s_\rho}$	$\sqrt{s_E/s_\rho}$	$\sqrt{s_E/s_\rho}$
Force, Q	F	$s_l^3 s_\rho s_a$	s_Q	$s_E s_l^2$
Time, t	T	$s_l \sqrt{s_\rho/s_E}$	$s_l \sqrt{s_\rho/s_E}$	$s_l \sqrt{s_\rho/s_E}$
Stress, σ ; and pressure, q	FL^{-2}	$s_l s_\rho s_a$	s_Q/s_l^2	s_E
Strain, ϵ		$s_l s_\rho s_a/s_E$	$s_Q/s_l^2 s_E$	1
Displacement, δ	L	$s_l^2 s_\rho s_a/s_E$	$s_Q/s_l s_E$	s_l
Frequency, f	T^{-1}	$(\sqrt{s_E/s_\rho})/s_l$	$(\sqrt{s_E/s_\rho})/s_l$	$(\sqrt{s_E/s_\rho})/s_l$
Poisson's ratio, ν		1	1	1

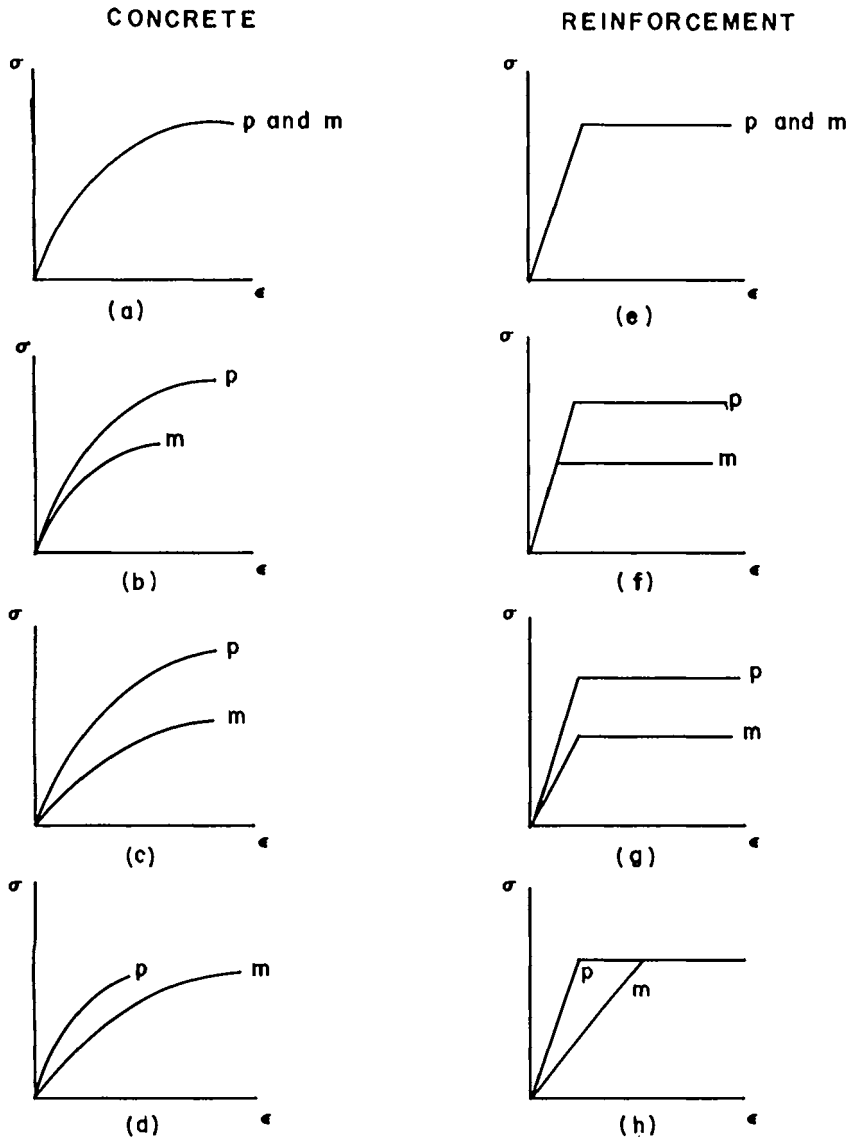


Fig. 2-1—Materials similitude for the case $E_{rp}/E_{rm} = s_\sigma/s_\epsilon$

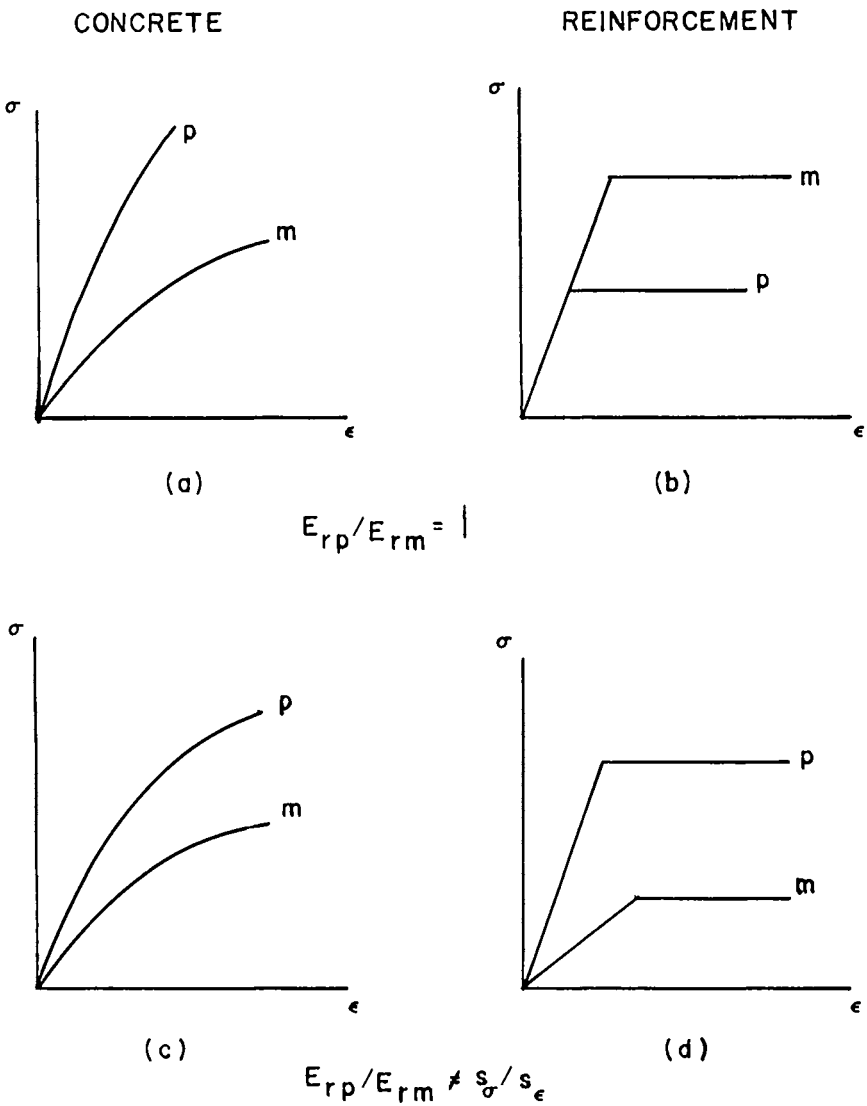


Fig. 2-2—Materials similitude for the case $E_{rp}/E_{rm} = 1$ and $E_{rp}/E_{rm} \neq s_{\sigma}/s_{\epsilon}$

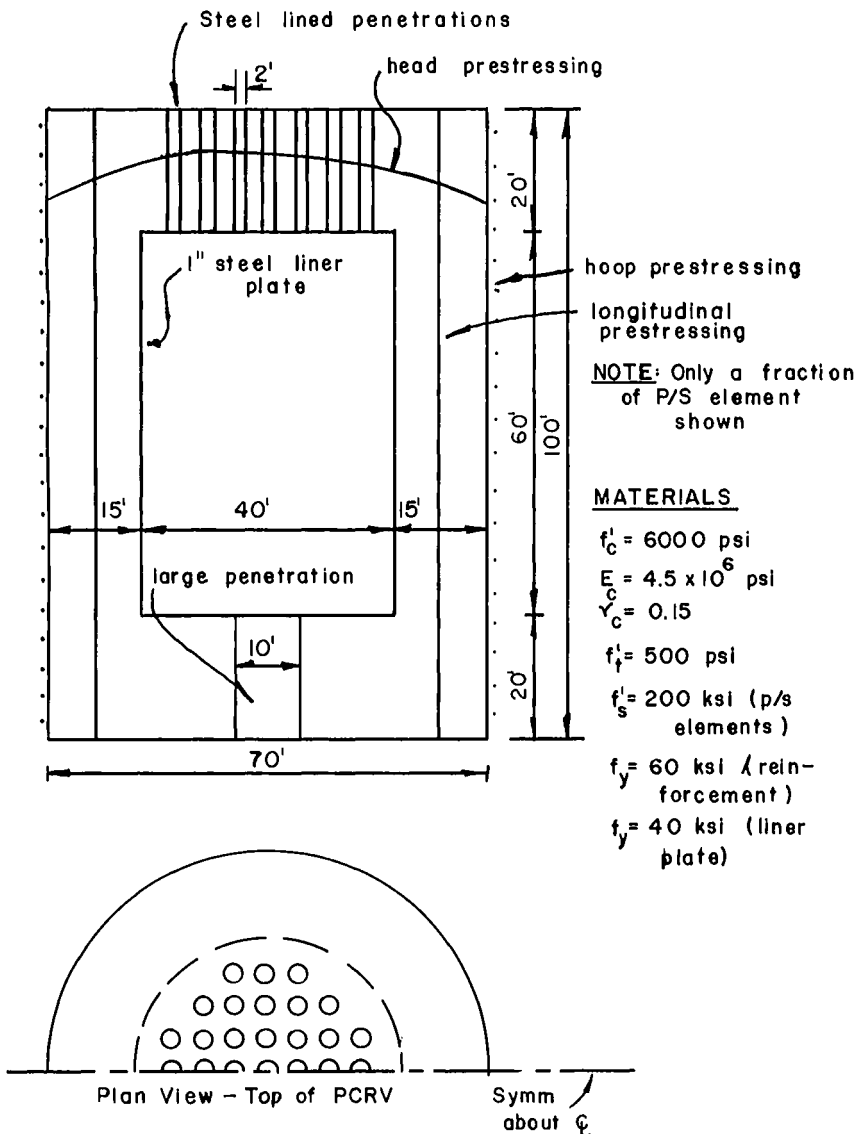


Fig. 2-3—Prototype PCR

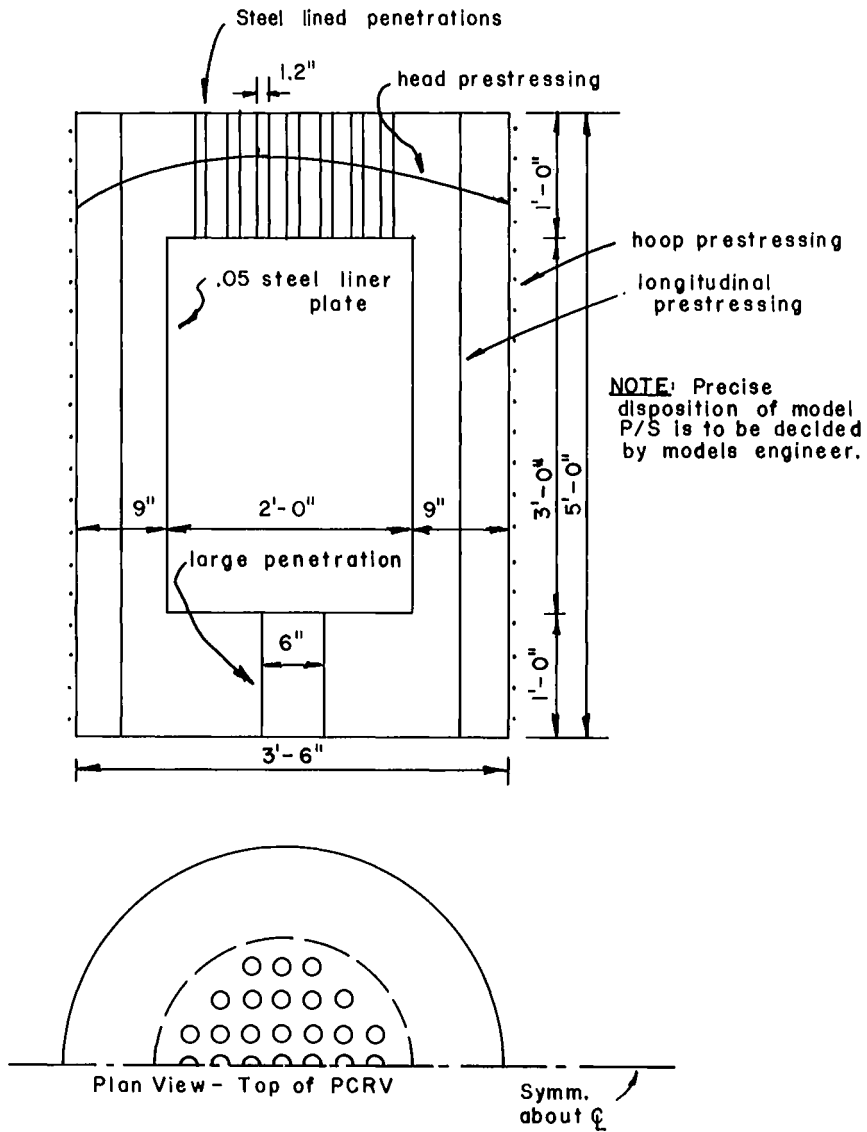


Fig. 2-4—Model PCRV

TECHNIQUES AND MATERIALS FOR STRUCTURAL MODELS

By JAMES E. CARPENTER, FREDERIC ROLL
AND MAIER I. ZELMAN

Techniques and materials for the construction and testing of models of concrete structures are discussed in this paper. Both elastic and ultimate strength models are considered.

Elastic models are treated briefly. Time effects in commonly used plastic materials and complications in instrumentation are discussed.

Materials to simulate both the steel and the concrete in ultimate strength models are discussed and references to detailed investigations are given.

Fabrication, instrumentation and testing methods are described.

Keywords: acrylic resins; concretes; fabrication; loads (forces); measuring instruments; micro-cracking; models; mortars (material); polyvinyl chloride; reinforced concrete; reinforcing materials; reinforcing steels; research; structural design; testing; thermoplastic resins; thermosetting resins.

□ When the feasibility and similitude studies have been completed the general outline of the model test program has been established. It then remains for the model test engineer to fill in the outline with the detailed design of the model, the test apparatus, instrumentation and the loading sequence necessary to carry out the aims of the test. These latter detailed operations may not be entirely independent of the earlier studies. For example, it may be found that the available materials will not fulfill some of the specifications, and adjustments must be made in the plans either to relax the specifications, develop new materials or to find another solution.

The interrelationship of all phases of the model test program is illustrated by the next step in the design sequence, that of detailed design of the model. Restricting the discussion to direct models, the aim may be a design study or a research study. For a design study, the model test engineer will have at hand a prototype design which he is being asked to verify or modify. To do so he must insure that the model will predict the behavior of the prototype as closely as possible. In order to design a valid model, he has to keep in mind the reliability of every element going into the model. This reliability may be related to similitude and cost. Balancing these factors becomes the most important part of the design procedure.

For a research study, the model design must cause the desired behavior to predominate. The significance of this problem is discussed in detail in the

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paper "Accuracy of Structural Models." A typical example is the use of mortar models to study the flexural behavior of thin three dimensional structures such as plates and shells. The deflection of these structures under moderate loads is often quite sensitive to the tensile strength of the concrete. Since mortars tend to have tensile strengths higher than those of prototype concretes of the same compressive strength, care must be taken in extrapolating the mortar model deflections to prototype sizes of plates. Other aspects of behavior, such as crack spacing and rotation may be affected also. Viewed another way, this problem emphasizes the need to model all known variables as closely as possible since the need for a model test is based on the fact that the influences of the various factors affecting the behavior are not known in advance.

ELASTIC MODELS

The only similitude requirements for an elastic model material are that it be linearly elastic and that it have a Poisson's ratio identical to the prototype material. The latter requirement is often circumvented or ignored, particularly in beam-type structures where varying Poisson's ratio has little effect on overall behavior. The choice of a material within these limits is then based on such practical considerations as cost, stability, fabrication ease and stiffness.

Materials and Fabrication

Plastics and metals are the most commonly used materials for manufacture of elastic models. Of the two, plastics are more popular because the higher modulus of metals requires relatively large loads to produce measurable strains. Also, plastics are easier to fabricate into complex structural configurations.

The commonly used plastics can be divided into two categories, thermoplastic and thermosetting. Thermoplastics can be formed after being heated to a softening temperature (200 deg F - 300 deg F) but thermosetting plastics cannot be softened by heating once the material is formed. The familiar acrylic plastics and polyvinyl chloride (PVC) are thermoplastics usually purchased in sheet form for model manufacture. Several of these plastics and their properties are listed by Fialho.¹ When purchasing plastics, one must remember that there are usually several types and grades marketed under a single trade name.

The acrylic plastics are probably the most popular model material. They are easily machined and cemented so that accurate models can be quickly assembled. Sheets can be softened by heating and formed into shells of single and double curvature; a vacuum forming machine should be used for most shell forms. Combining the operations, machined edge beams can be cemented onto the formed shell. Fialho¹ and Carpenter, Magura and Hanson² are among those who discuss model fabrication from acrylics in detail.

PVC sheets can be obtained in smaller and more uniform sheet thickness than acrylic, and as a result are particularly suited for shell models. A series of vacuum formed shells of PVC have been tested at Massachusetts Institute of Technology. Little and Hansen³ discuss this operation and other phases of model testing.

The thermoplastics usually have a modulus of elasticity of about 500,000 psi and a Poisson's ratio of about 0.35. The value of the modulus of elasticity is in the right range for convenient use in models, but the Poisson's ratio does not compare well with the 0.15 - 0.20 of concrete. This means that care must be taken in the interpretation of model test results where Poisson's ratio may affect behavior.

Of the thermosetting plastics, casting resins are often used because very intricate models can be cast in prepared molds. Some skill and experience are needed to obtain uniform results. The main advantage over sheet thermoplastics in the manufacture of shell models is that curved surfaces with any desired thickness variation can be cast, whereas the shells made from sheets are the thickness of the original sheet plus whatever changes in thickness result from the forming process. Thickness of the model section is limited by the need to dissipate the heat of polymerization.¹

Fiberglass shells⁴ have also been used, although it is difficult to achieve homogeneous and isotropic properties with this material.

Instrumentation and Test Methods

The following brief summary of instrumentation and test methods will be limited to models of acrylic or PVC plastic, the two most widely used elastic model materials. The use of other plastics would present similar problems to the models engineer.

Two properties of the plastics cause most of the problems in model tests; first the time dependent behavior and second, the low thermal conductivity. The time dependent behavior primarily affects the test procedure, while the low thermal conductivity affects fabrication when heat is used and instrumentation when electrical resistance strain gages are used.

The time dependent behavior manifests itself as creep, or continued deformation with time, when the material is subjected to constant stress. Since most experimental measurements are directly or indirectly related to deformations, the test engineer finds himself in the position of trying to measure values which are changing with time in a model where a constant load is producing a constant stress.

Fortunately, the time dependent change in modulus of elasticity is essentially independent of stress at the stress levels normally encountered in models; thus a modulus of elasticity can be defined for any time after application of load. Once the time dependent modulus is known, the material can be treated as linearly elastic and therefore is suitable for purposes of elastic modeling.

Two testing methods are used to avoid the difficulties due to the time dependent characteristics of the plastic. The first, called the Constant Stress Method, is shown in Fig. 3-1a. A constant load is applied to the model, resulting in a constant distribution of stress (no radical changes in geometry can be allowed in elastic tests.) Strains and other functions of deformation are read at those fixed times after loading for which the modulus of elasticity has been measured. This may be relatively soon after load, with many cycles necessary to read all items of instrumentation; or it may be long enough after loading so that the rate of creep is sufficiently small to allow all the items of instrumentation to be read in one load cycle. The time elapsed before the creep rate becomes negligible is about 6 hr according to Preece and Davies,⁵ but Roll⁶ states that 20 or 30 min is sufficient. The creep behavior of a plastic should be investigated before each model test by running tests on samples of the plastic used in the model.

The second test method, called the Constant Strain Method, is shown in Fig. 3-1b. This method makes use of the fact that if a constant displacement is introduced into the model at one or more points, the resulting strains throughout the model are constant and hence can be read at any time. The load necessary to maintain the imposed displacement diminishes with time, and hence the internal stresses diminish proportionally as shown in Fig. 3-1b.

To determine the relative value of the applied load, an auxiliary device called a strain cell can be used. As described by Carpenter⁷ the strain cell is essentially a load cell with the sensing element made of the same material as the model. Inserted in the loading system, the cell gives a reading constant with time and proportional to the relative load. The cell is calibrated on a statically determinate structure made of the model material.

The behavior of these plastics, like that of other visco-elastic materials, is actually more complex than indicated above. The mechanisms of time dependent effects, and the interactions between them, can become very complicated, particularly for wide variations of imposed stress with time. However, it has been found that where relatively long periods of time elapse between applications of load or deformation, the simple relationships assumed above will be found to predict behavior adequately.

For forming, the low thermal conductivity of the material coupled with a high coefficient of thermal expansion makes it necessary to heat both sides of the sheets and to make provision for dimensional changes when the mold is designed. As an example of the variability of dimensional changes, some grades of plastic will shrink about 2 percent after heating while other grades are preshrunk at the factory. Cooling must be controlled to minimize residual stresses.

The low thermal conductivity, the high coefficient of thermal expansion, sensitivity of the modulus of elasticity to heat, and the low modulus of elasticity itself are among the factors that can cause inaccuracies in the readings of electrical resistance strain gages mounted on plastic. The heat effects become a problem if the gage current is great enough to cause significant heating of the gage. This is more likely to occur when very small wire gages are used. Foil gages are more suited for use of plastics since they have a greater radiating surface and less tendency to overheat. Gage voltages of about 1.5 volts seem to be satisfactory for foil gages at least down to 1/16-in. gage length.

The low modulus of the plastics causes inaccuracies in strain gage readings because the metal alloy gage sensing unit, being much stiffer than the plastic to which it is attached, locally stiffens the plastic and hence is not subjected to the same strain as is the surrounding material. Foil gages are again more satisfactory than the wire gages because their thinner sensing elements exert less stiffening force. Results of actual tests on these several effects are given in the following paper, "Accuracy of Structural Models."

Summary—Elastic Models

The preceding discussion is only a brief outline on materials and fabrication for elastic models for concrete structures. Among the topics that have not been covered are the selection of cements for fabrication, machining methods and tolerances, thickness variations in manufactured sheets, details of heat forming, sensitivity of the material to humidity and strain rate, the elastic "memory" which makes loading response a function of previous loadings and zero drift in instrumentation. More detailed discussion of these points will be found in References.¹⁻⁷

Except for the precautions needed to circumvent the creep problem and the need for extreme care in obtaining and interpreting strain gage readings, tests of elastic models of concrete prototypes can be conducted in much the same way as tests of the prototype would be. Such a test might be summarized as construction of the specimen, construction of the support frame and loading apparatus, instrumentation (probably strain gages, load cells, dial gages, and LVDT's) and application of the loads.

Elastic models, partly because they represent mathematical models of structures more closely than they resemble concrete prototypes, also provide the opportunity to utilize more exotic analysis and measurement techniques. These techniques range in complexity from straightforward measurement of deflection profiles⁸ or curvature⁹ to Moire Analysis,¹⁰ brittle lacquer techniques,¹¹ and photoelasticity.¹² The list is incomplete, and the references cited are only a small portion of the total literature on these subjects. However, for one reason or another these techniques have never become very popular in the United States and Canada for elastic modeling of concrete structures, in spite of the fact that several are standard experimental methods in fields such as machine design.

ULTIMATE STRENGTH MODELS

The development of ultimate strength theory for reinforced concrete has been paralleled by increased use of and research on ultimate strength models. A further increase in use of models can be expected, and to ensure validity of results, research on modeling techniques must be conducted. In addition to having the same areas of incomplete knowledge as the prototype (example: shear), reduced size models also introduce the possibilities of scale effects. A number of these problems are discussed in the paper "Accuracy of Structural Models."

Construction of the ultimate strength model must, by its very nature, follow the construction of the prototype to the largest degree practical. The construction steps will be discussed in the following, with mention of elastic models when common techniques are involved.

Model Concrete

In ultimate strength model studies the engineer must design and produce the various materials, instead of being able to purchase them as is the case for most materials for elastic models. For model concretes, the problem of material design is particularly challenging because the properties of the prototype concrete itself are only partly defined and understood. As a result, the model concrete is usually designed to meet as closely as possible the known requirements as obtained from the similitude study, and by the nature of the model material it is hoped that less clearly defined properties will also be well simulated. For example, compressive strength, tensile strength and shrinkage characteristics might be modeled, and if the constituents of the model concrete are roughly similar in properties to those of the prototype, it would be expected that poorly defined characteristics such as failure criteria would be modeled also. Considerable work is needed to define failure criteria for both prototype and model concrete.

To meet the similitude requirements given in the previous paper, a model concrete, or microconcrete, would be produced by reducing the size of all materials, including cement, by the scale factor. However, this simplified approach has not been successful because the scaled fine aggregate has excessive water requirements and more finely ground cement is not available. As a result of the lack of success of this approach, most small scale concrete for models is now designed as it would be for any structure—a combination of Portland cement, water, aggregate and additives which will display the desired properties at the desired time after casting. In practice, this means Type I or Type III cement, a water-cement ratio of 0.3–0.7, and a maximum aggregate size determined by similitude and by the thickness and reinforcing spacing in the model.

Most investigators try first to produce the design compressive and tensile strengths, keeping in mind that tensile strengths tend to become relatively greater as the scale of the mix is reduced. Conflicting water requirements occur; small amounts of water reduce shrinkage but wetter mixes are easier to place.

Local sands have been used quite successfully as aggregate. The natural gradation can be used below the specified maximum size, or the sands may be sieved into various sizes and recombined gap graded. It has been found advantageous to control the percentage of fines below the 100 mesh.

Recommendations for mix proportions are given by Aldridge and Breen in a subsequent paper, "Useful Techniques in Direct Modeling of Reinforced Concrete Structures." In another study, Harris, Sabnis and White¹³ conducted a detailed investigation of the properties of Portland cement mortar as applied to structural models. Nomographs are given in the appendix to the report by means of which mortars can be designed to meet specified values of secant modulus E_s , compressive strength f'_c and strain ϵ at 90 percent f'_c . The results, which are valid only for the particular sand used, are expressed in terms of water cement ratio and percent volume of aggregate. Water cement ratios are in the range of 0.4 to 0.7, and aggregate cement ratios by weight are about 4, for f'_c from 4000 psi to 7000 psi. It is reported that these mixes tend to produce ultimate strains somewhat higher than those of the prototype mixes. The mortar mixes also tend to have tensile strengths higher than those of the prototype mixes. Most tests were conducted on 1 in. $\times$ 2 in. cylinders.

Gap graded mortar mixes are used at the Cement and Concrete Association Laboratories in England. A typical mix reported by Somerville, Roll and Caldwell¹⁴ had an aggregate cement ratio of 2.7 and a water cement ratio of 0.5. The aggregate consisted of 50 percent material passing the British Standard No. 7 sieve but retained on the British Standard No. 14 sieve, and 50 percent passing the British Standard No. 52 sieve but retained on the British Standard No. 100 sieve. Using 6×12 -in. cylinders, the compressive strength was found to be about 6300 psi and the split cylinder strength 580 psi, both at 1 yr. A similar mix was used by Long¹⁵ for another project at the Cement and Concrete Association Laboratories.

At the Portland Cement Association Laboratories, Magura¹⁶ used as aggregate a sand with a fineness modulus of 2.76 and 100 percent passing the No. 8 sieve. Using an aggregate cement ratio of 2.75 and a water cement ratio of 0.58 he obtained 28 day strengths of 5300 psi in compression and 740 psi in split cylinder tension. Strengths were based on results for 2×4 in. cylinders.

Test specimens of concrete are intended to give the user a means of determining the strength of the concrete in the structure. In the United States and Canada it is presumed (with few limitations) that the compressive strength of a 6×12 in. cylinder is the same as the compressive strength of the concrete of the same batch when placed in the structure. Similarly, the split cylinder strength for the concrete is assumed to be a measure of the tensile strength of the concrete in the structure. These relationships are generally assumed to be valid for a rather wide variety of prototype sizes.

It has been theoretically predicted and experimentally confirmed that as the size of test cylinders decreases, the indicated strength of the concrete increases. The general trend has been clearly established, but experimental data are not entirely in agreement as to the exact magnitude. In Fig. 3-2 relative strengths according to Harris et al,¹³ Alexander,¹⁷ and Aldridge¹⁸ (Source C, Source M and Source T, respectively) are plotted using the 6 by 12 in. cylinder strength as unit strength. None of the data indicate that cutting cylinder size in half will radically change the indicated strength of the concrete.

Consequently, in spite of the fact that the proper size of cylinder to be used in conjunction with a given size of model has not been determined, it seems that use of a cylinder diameter about the same as the minimum dimension of the section of interest in the model should give satisfactory results. Fortunately, the strengths of many models are not strongly sensitive to concrete strength, and hence exact knowledge of true strength is not always required. True strength here would mean selecting a cylinder size which would indicate concrete strengths which were the same proportion of strength of concrete in the model structure as the 6×12 in. cylinder strengths were to the concrete in a prototype structure.

As an example of this size effect problem, tests at the University of Pennsylvania have shown that compression tests on model cylinders, 2.4 by 4.8 in., resulted in strengths which were 15 to 20 percent higher than the 6 by 12 in. cylinders. The punching shear resistance of perforated reinforced concrete slabs, measured by the term $P_u/bd\sqrt{f'_c}$, showed good correlation between model slabs with a scale factor of 2.5 and corresponding prototype

slabs when the compressive strength of the model cylinders was used. Where the lower compressive strength of the 6 by 12 in. cylinders was used, the term $P_u/bd\sqrt{f'_c}$ for the model was approximately 7 to 10 percent higher than that of the prototype.

Concrete tensile strengths as indicated by small scale tests tend to show more variability than the compressive tests. Harris et al¹³ present results of tensile tests on a number of model concretes. The same increase of indicated strength with reduced test specimen size has also been noted with modulus of rupture specimens.

Gypsum and gypsum sand mixtures are also used as model concretes. The gypsum mixes have the advantage of reaching the desired strength in a matter of hours, thus allowing tests to be conducted much sooner than is the case for Portland cement mortars. The possible disadvantage of conducting the model tests during a period of rapidly changing strength has been overcome by surface sealing with shellac, which prevents drying and nearly arrests strength changes.

The use of a gypsum concrete consisting of a mixture of a commercial gypsum product, sand, and water is described in detail.¹⁹ The gypsum product was a plaster designed to set in about 30 min., and the sand was the same local glacial sand used for the Portland cement mortars. For strengths of 3000 psi to 4000 psi, aggregate plaster ratios were about 0.8 to 1.0, and water gypsum ratios were about 0.3 to 0.35. A retarder was usually added to the gypsum to increase setting time. It was concluded that compressive strengths and stress-strain curves of the prototype concrete could be duplicated. Size effects in gypsum mortars are reported by White and Sabnis²⁰; it is concluded that the sealing process greatly reduces size effects because it leads to higher uniformity of material in the different size specimens.

Ranganatham, Subba Rao and Hendry²¹ used a gypsum sand mix very similar in proportions and in strength. The sand used was a single size passing the British Standard No. 25 sieve but retained on the British Standard No. 36 sieve.

Gypsum plaster alone, without aggregate, has been used as a model material. It has an even earlier set than the gypsum sand mixture, but has the disadvantage of the stress strain curve being nearly linear up to compressive failure, instead of being nonlinear like the stress strain curve of concrete. The brittle nature of the material results in brittle failures of gypsum models in which behavior is governed by concrete properties. Pure gypsum model concrete has excessively high tensile strength compared to the prototype concrete, whereas the gypsum-sand mixes are comparable to cement mortars.

The publications referred to above contain much more detailed discussions of the design criteria and mixing techniques for model concretes than space permits here; these and other available references should be consulted before any new program is started. It will be found that most of the authors use mixes in the range of the proportions mentioned, but recommend preparation of trial batches to determine whether or not local materials will produce a mixture having the desired properties. Among the properties to be considered are stress scale, strain scale, (in particular the strain at maximum stress) tensile strength, Poisson's ratio, shrinkage, creep and workability.

Special purpose mortars of various compositions have been used where unusual properties are required. For example, the foundations in a model dam study may have to be of widely varying modulus to simulate a variety of rocks in the real foundation. A mixture of plaster, diatomite (Fuller's earth) and water is often used²² for this purpose.

Reinforcement

Most deformed bar reinforcement has a well defined yield point and a relatively long yield plateau. The exceptions are the high strength ASTM A431 bars with a nominal yield strength of 75,000 psi, which typically have a rounded stress-strain curve. ASTM A432 bars, the most commonly used high strength steel, can be expected to have a nominal yield strength of 60,000 psi and the sharp yield stress-strain curve.

Model reinforcement must have the same shape of stress-strain curve as the prototype reinforcement even if non-unity stress and strain scales are used. Since some phosphor bronzes which are relatively expensive are the only other materials exhibiting this type of stress-strain curve, the choice of model reinforcement is effectively limited to steel. The steel must meet the following requirements:

1. Stress-strain curve similar to that of prototype, including an appropriate amount of ductility
2. Desired yield strength
3. Proper bond characteristics

Most ultimate strength models will be expected to have stress and strain scale factors equal to unity, and the reinforcing steel will be required to have the same properties as those of the prototype reinforcement. However, as has been shown in the preceding paper, the yield strength and areas of the reinforcement may be adjusted for non-unity stress and strain scaling factors. In either case, though, the problem of providing model reinforcement reduces to satisfying the above conditions.

For reinforcement diameters of 3/8 in. and larger, standard deformed bars can be used for model reinforcement, but unfortunately, no commercially available steel products meet the requirements for the smaller sizes. In general, wire is available in two grades, either a cold drawn product which has a rounded stress-strain curve and a yield strength about 100,000 psi or annealed wire which has a yield strength around 30,000 or 40,000 psi.

The high strength wire can be annealed to produce a relatively sharp yield point at the desired level. If the starting material is the deformed wire manufactured for use in making welded wire fabric, the finished product is a deformed wire with approximately the desired yield point. The inward deformations rolled into these wires do not provide the bond characteristics of the outward deformations of the prototype reinforcement, but they are more satisfactory than plain wires. Fig. 3-3 shows the results of annealing a 0.195 in. deformed wire, at 1000 F for 60 min. The annealing time and temperature were determined by trial. A typical stress-strain curve for an American Society for Testing and Materials (ASTM) A432 bar, marked "prototype", is used for comparison. Here the strain hardening part of the stress-strain curve was not well duplicated.

Another method of producing a deformed wire with the desired yield point is to start with a soft plain wire and cold work it until the yield point is raised to the desired value. If the cold working also produces deformations, both desired results are accomplished at the same time. Harris et al.¹³ describe a knurling device shown in Fig. 3-4 which simultaneously produces deformations on four sides of the wire. Fig. 3-5 shows the stress-strain curves for wires subjected to the knurling operation. Again, a typical stress-strain curve for an ASTM A432 bar is marked "prototype" and used for comparison. This same technique using similar knurling devices has been employed at other laboratories.

Small diameter threaded rods have also been used to simulate deformed bars. The threads, although they could be regarded as outward deformations, apparently provide a more positive bond than do those on standard deformed bars, and as a result still do not exactly simulate deformed bars. These rods, as purchased, have a rounded stress-strain curve with a yield point about 80,000 psi to 120,000 psi.

Examples of the various types of wire are shown in Fig. 3-6, which is taken from Harris et al.¹³ The wires shown are of an intermediate size range. The smallest diameters available are 0.113 in. for the commercially deformed wire, 0.063 in. root diameter for the threaded rods (2-56). Harris et al.¹³ report being able to obtain 0.063 in. diameter commercially deformed wire on special order, although other investigators have had difficulty in obtaining the 0.113 deformed wire listed in catalogs. Wire of about 0.063 in. diameter and smaller may be deformed conveniently by hand pulling through the knurling device of Fig. 3-4; larger wire would require mechanical pulling to achieve the proper amount of deformation.

Very high strength steel to simulate prestressing reinforcement in the prototype is readily available. Piano wire, single wire prototype prestressing material or bicycle spokes can be used for post-tensioning. Stainless steel aircraft control cable has seven wires and in nearly every respect resembles a miniature seven-wire prestressing strand. It is available in convenient sizes and has a stress-strain curve and strengths close to those of prestressing strand.

The general problem of reinforcement bond simulation in reduced scale models has not been solved. This is not surprising, since the bond behavior of the prototype reinforcement is clearly not understood, as was pointed out in the paper "Principles of Model Analysis." Fortunately, the inexact modeling of bond is usually not a serious problem since bond behavior rarely predominates in concrete structures. Depending on the degree of exactness that is required for a particular project, plain wire may be used; rusted wire with slightly better bond characteristics may be needed, or a deformed wire may be needed.

Fabrication

Fabrication of ultimate strength models involves each of the steps of fabrication of the prototype. Ease of fabrication may well determine the scale of the model because of problems of concrete placement and tolerances in accuracy of reinforcement location for very small models.

For most uniform results the mortar should be mixed in a pan-type machine of suitable capacity. Hand mixing is not uncommon, however. The mortar mixes are relatively more difficult to place in the reduced scale

forms than the prototype mixes are in the full-sized forms, and therefore, extreme care must be taken to ensure adequate compaction. Breen²³ recommends the use of workability and set retarding admixtures to aid in placing. Combinations of internal and external vibration, including vibrating tables, appropriate to the size of the model will be needed. The reinforcement cage must be securely fastened to the form to prevent shifting during the placement of the mortar.

Assembly of the reinforcement cage may be done by welding, brazing, or soldering or by tying with fine wire. The heating involved in the first three methods may cause local changes in the properties of the reinforcement, while the fourth method is time-consuming and may not produce cages of the desired rigidity. The assembly method for the reinforcement must be selected considering the size of the cage, the sensitivity of the reinforcement to heating and other factors.

Model prestressing beds are constructed as small scale versions of the full-sized beds. Specially made grips are usually used for stressing and anchorage of tendons. Prestress forces can be measured either by specially constructed low capacity load cells or by observing the extension of the prestressing element. The low forces involved often make it convenient to use a mechanical jacking arrangement, or on occasion a dead weight system. In a nominally unprestressed model a useful technique to maintain accurate location of reinforcement is to lead the reinforcement out of the form through carefully located holes and introduce a slight prestress.

Forms may be constructed of any structurally adequate material, but Plexiglas lends itself particularly to this application. It is easily machined to form leakfree and dimensionally accurate forms. The smooth and non-absorbent surface needs no form oil and produces a dense and similarly smooth surface on the model. Finally, the transparent form material allows the degree of compaction to be observed as casting progresses.

Formwork must meet the usual requirements of rigidity and watertightness. In addition, it may be necessary to include some kind of contraction joints, for example a foam material, to allow the form to shrink with the specimen. If the form restrains the specimen to any great degree the specimen may be damaged or destroyed by the shrinkage forces. It is often expedient to remove model forms at a very early age in order to prevent shrinkage effect damage to the model.

Support Frames

Proper design of the supports for the model under test is an essential part of the model test program. Lack of support frame stiffness may well invalidate the test results by changing reactions or by unintentionally providing a moving reference plane for deformation measurements. For these reasons the importance of adequate support frame design cannot be over-emphasized.

Since support frame design is not usually a part of the model as such, few detailed descriptions of supports will be found in the references and therefore most of the remarks that follow will be general in nature.

With the exception of dams, the supports of the model are rarely considered to be part of the model system. Usually, completely unyielding supports are assumed. Whether or not this assumption is a realistic representation of actual conditions is beside the point. The design of the supports