

# **Stress Distribution of Prestressed Concrete Structures as Influenced by Time and Temperature**

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Synopsis: It is well known that creep and shrinkage of concrete cause long term deflections of continuous prestressed concrete bridges. A large amount of research has been conducted in this area. While the bridges continue to deform with time, the moment distributions of the bridges remain essentially unchanged under the influence of creep if the bridge temperature is uniform. The change of stresses is caused by prestress losses due to the creep, shrinkage of concrete and the relaxation of steel. However, rate of creep of concrete increases with temperature. The seasonal and daily temperature variations of a prestressed concrete deck not only induce thermal stresses but also significant stress redistribution due to non-homogeneous creep rates in the concrete.

The paper presents some recent findings from Steady State analyses. The provision in the latest European code EN1992 (EC2) for thermal creep is discussed. The amount of moment redistribution will be quantified against a range of thermal creep rates. The results show that stresses could be significantly underestimated if temperature and thermal creep are not considered in design. The advancement of EC2 as well as its limitation in modelling the long term behaviour of prestressed concrete bridges are also discussed. The paper also addresses the problems that can be created when the structural continuity is created for the first time due to the remedial works after the bridge has been in service many years. Under these circumstances creep can have a major effect on both serviceability and safety.

Keywords: bridge; relaxation; steady state; thermal creep; thermal stress

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### INTRODUCTION

The primary consequence of creep in concrete structures is the development of time-dependent deformations. There are however occasions when significant stress changes in time also occur. It is generally believed that creep is beneficial to the behaviour of statically indeterminate structures in that stresses and bending moments will be kept close to design values derived from equilibrium and compatibility considerations. This may be seen readily to be true for the two-span prestressed beam of Fig. 1a where the bending moments are changed from the normal design values by imposing a displacement at one support. The instantaneous effect is as shown in Fig. 1b. Thereafter creep returns the bending moments and stresses asymptotically towards the pre-support displacement imposition values, i.e. the original elastic solution. This response is not general and is a consequence of several factors, viz. the prestressing force is taken to be a follow-up load (assumed here not to change in time, i.e. there is no restraint to longitudinal deformations), the creep rate of concrete is homogeneous throughout the beam and the temperature everywhere is uniform. This behaviour is not general and may be seen to be a special case of a more general steady state theory (see following section for more details)<sup>1-3</sup> for non-uniformly heated structures in which creep becomes a non-homogeneous property that is governed by the local temperature. The non-homogeneity is caused by an increasing specific creep (creep strain per unit stress) with temperature. For such cases the long-term, or steady state solution, is no longer the same as the initial elastic solution but is unique to the temperature distribution within the structure.

In the following sections, the prediction of thermal creep is discussed, the influence of creep on structural performance is further described and examples are given to justify the belief that creep is not always beneficial to structural performance. In the final section some cases of significant importance are described in relation to the imposition of restraints, flexural and axial, late in the life of a structure. Previously safe and serviceable structures are shown to be able to adopt a new character and display intrinsically unsafe behaviour thereafter.

### RESEARCH SIGNIFICANCE

While thermal stress caused by temperature fluctuations is considered in bridge structures, temperature effect on creep induced stress redistribution has not been received the same attention. The research examines the material creep properties as reported by researchers and in the latest European code EC2. Two analytical methods to evaluate the maximum moment and stress induced by quasi-permanent load and temperature fluctuations at steady state are proposed. The effect of different thermal creep properties of concrete is also studied. The results show significant increase in support moment of continuous prestressed concrete bridges. The paper also addresses the problems that can be created when the structural continuity is created for the first time due to the remedial works after the bridge has been in service many years. Under these circumstances creep can have a major effect on both serviceability and safety.

### THERMAL CREEP

It is known that the rate of creep is dependent on the temperature. Hannant<sup>4</sup>, Ross and England<sup>5</sup>, Browne<sup>6</sup> and Hansen<sup>7</sup> carried out experimental tests on concrete under sustained load and temperature up to 80°C. They concluded that the rate of creep increases with temperature. This extra creep occurs at temperature higher than 20°C is usually referred to as “thermal creep”. Illston<sup>8</sup> identified transitional thermal creep which he described as an

irrecoverable component of concrete creep brought about by heating concrete to a higher temperature for the first time while under load (“virgin heating”). The latest version of European Code EN1992:Part 1:2004 (EC2 for Buildings)<sup>9</sup> permits a constant creep coefficient for ambient temperatures between -40°C and +40°C. However, EN1992:Part 3:2006 (EC2 for Liquid retaining and containment structures)<sup>10</sup> recognises the effect of temperature on creep by providing a creep coefficient multiplier to factor the creep coefficient obtained at 20°C. If load is applied before heating, the code also provides a formula to calculate the Transitional Thermal Creep strain. These provisions are not included in the current British codes in the UK and represent an advancement of providing a reference for design engineers.

The influence of temperature on creep coefficient as reported by various researchers and by EN1992:Part3:2006 is summarised in Fig. 2 in the form of a creep coefficient multiplier. It is clearly shown in this figure that there is a wide range of thermal creep strain relating to different aggregates and cement types used. England<sup>5</sup> and Hannant<sup>4</sup> reported gravel aggregates in their tests whilst limestone was used by Browne<sup>6</sup>.

The effect of the Transitional Thermal Creep is to increase deformation of the structure and to cause moment redistribution during the heating phase whenever the temperatures are not uniform. However, this strain component will not influence the long term steady state of the structure and hence will not be further discussed in the paper.

### INFLUENCE OF TEMPERATURE CHANGE TO CONCRETE BRIDGE DECK

The temperature of a bridge deck changes daily and seasonally and is between two extreme limits which is usually non-linearly distributed throughout the depth of the deck. Thermal strains created by non-linear temperature changes are not compatible with plane sections beam theory. They therefore give rise to self-equilibrating stresses throughout the depth of the deck. Stresses of this type exist in addition to any other stresses which result from external constraints if the bridge deck is statically indeterminate. The total of these stresses are usually referred to as thermal stresses.

If concrete stress is within the linear creep range ( $\sigma_c < 0.45f_{ck}(t)$  according to EC2), specific creep is independent to concrete stress but dependent on temperature, i.e. thermal creep. The non-uniform temperature distribution in a bridge deck therefore generates non-homogeneous creep response. If the new temperature state is sustained, stresses in concrete will be redistributed because (i) the creep of concrete will progressively eliminate the thermal stresses and (ii) a new stress state will develop eventually to re-establish a compatible set of creep rates i.e. a Steady State of stress is reached.

### STEADY STATE AND MOMENT REDISTRIBUTION

The Steady State is defined as the state when creep is continuing with no change in stress. The existence of a steady state of stress is supported by theoretical arguments and experimental measurements on non-uniformly heated statically indeterminate prestressed concrete beams<sup>11,12</sup>. Fig. 3 shows details of one test in which the bending moments and stresses changed in time while the beam was subjected to sustained loads and temperature crossfall. The recorded time variation of the support loads provided the bending moment information, while internal stress changes were monitored by recording strains on a set of elastic elements mounted within the constant bending moment section of the beam.

The experiment was conducted to determine the magnitude of the steady state support reactions while the supports were maintained co-linear. At a time (125 days) when the steady state was considered to be being approached fairly closely (from below in Fig. 3) a check was performed by imposing support displacements to increase the support reaction values to values estimated to be above the true steady state values. This produced the expected result, viz. the reaction values now reduced with time, thus approaching the steady state from above. A final check was performed (at 185 days) by returning the support displacements to the original co-linear form. This resulted in the steady state again being approached from below, thus confirming that the structure did in fact have a preferred long-term state for reaction values and bending moments. These in combination have been defined as defining the steady state solution to the problem for the given mechanical loads and specified temperature distribution.

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Other important features of this experiment were (a) the steady state bending moment and stresses were considerably different from the initial elastic solution, both before and after heating. (b) the internal states of stress showed linear variation over the beam depth initially but became significantly non-linear as the steady state was approached. The 'recorded' stresses from the experiment showed a very close correlation with predictions from the steady state stress theory<sup>1</sup>, Fig. 3.

When concrete creep properties are not certain during the design stage, the steady state of a concrete structure which is sensitive to creep should be considered as one of the limiting states for design checking.

### STEADY STATE THERMAL CREEP ANALYSIS AT SUSTAINED TEMPERATURE

In the case that a new temperature is sustained following the initial temperature under which the structure is completed, a thermal creep analysis at sustained temperature can be performed to examine the moment and stress distribution at steady state. If the structure is subjected to a cyclic temperature fluctuation throughout its design life such as bridges, a detailed analysis accounting for the temperature cycles as described in following section can be performed.

The thermal creep analysis adopted here is based on the Rate of Creep method. The total strain rate is expressed as

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \sigma f(T) \quad [1]$$

where the dot notation indicates differentiation with respect to the normalised creep,  $c^*$ , which is termed as pseudo-time,  $t^*$ . The normalised creep is

$$c^* = \epsilon_{\text{creep}} / T / f(T) \quad [2]$$

where,  $T$  is the temperature of concrete in  $^{\circ}\text{C}$  when the creep strain,  $\epsilon_{\text{creep}}$ , occurs.  $f(T)$  is a temperature function to collapse all specific creep curves at various temperatures into one curve.  $f(T)$  is usually in the form of

$$f(T) = T + a \quad [3]$$

$a$  is a constant dependent on the concrete material. Refer to the thermal creep strains results in Fig. 2.  $a = 70^{\circ}\text{C}$  for Browne,  $a = 62^{\circ}\text{C}$  for EN1992:3,  $a = 10^{\circ}\text{C}$  for Ross & England,  $a = 0^{\circ}\text{C}$  for Hansen and  $a = -5^{\circ}\text{C}$  for Hannant.

For beam plane sections remaining plane, at steady state, for an element at  $y$  from the sectional centroidal axis, the strain rate is

$$\dot{\epsilon}_{\text{ss}} = \dot{\bar{\epsilon}} + \dot{\psi}y = \sigma_{\text{ss}} f(T(y)) \quad [4]$$

where  $\dot{\psi}$  is curvature rate

$\dot{\bar{\epsilon}}$  is axial strain rate

the subscript "ss" represents value at steady state

Apply equilibrium,

$$P = \int_{y_b}^{y_t} \sigma_{\text{ss}} b \, dy \quad [5]$$

$$M_{\text{ss}} = \int_{y_b}^{y_t} \sigma_{\text{ss}} b y \, dy \quad [6]$$

where  $P$  is prestress force and  $M_{\text{ss}}$  is bending moment at steady state.

By combining the equations,

$$P = \dot{\bar{\epsilon}} l_1 + \dot{\psi} l_2 \quad [7]$$

$$M_{ss} = \dot{\bar{\epsilon}} l_2 + \dot{\psi} l_3 \quad [8]$$

and  $\dot{\bar{\epsilon}} = p_1 M_{ss} + q_1 P \quad [9]$

$$\dot{\psi} = p_2 M_{ss} + q_2 P \quad [10]$$

where  $l_1 = \int \frac{b}{f(T(y))} dy$ ;  $l_2 = \int \frac{by}{f(T(y))} dy$ ;  $l_3 = \int \frac{by^2}{f(T(y))} dy$

and  $p_1 = -l_2/S$ ;  $p_2 = l_1/S$ ;  $q_1 = l_3/S$ ;  $q_2 = -l_2/S$   
 $S = l_1 l_3 - l_2^2$

After establishing the rate of curvature,  $\dot{\psi}$ , a flexibility analysis can be employed using Virtual Power principle<sup>13</sup>. The general form is

$$\int \dot{\psi} [m_i] ds = 0 \quad [11]$$

Where  $m_i$  is the moment distribution of the released structure with a unit moment applied at the support  $i$ , the location of the  $i^{\text{th}}$  redundancy.

The outcome of this analysis yields the continuity moment over the intermediate support in the example of Fig. 4. With this information, the steady state stress distribution throughout the beam can be evaluated after back substitutions of  $\dot{\bar{\epsilon}}$  and  $\dot{\psi}$ .

## STEADY STATE THERMAL CREEP ANALYSIS FOR CYCLIC TEMPERATURE FLUCTUATIONS

At steady state, the relationship of stress distribution at temperature profile T1 (lower temperature) and that at temperature profile T2 (after heating) is defined as

$$\sigma_2 = \sigma_1 + \sigma_\alpha \quad [12]$$

where  $\sigma_\alpha$  is thermal stress, developed due to temperature change from T1 to T2.

The steady state strain rates at T1 and T2 states are expressed as

$$\dot{\epsilon}_1 = \sigma_1 f(T1) \quad \text{and} \quad \dot{\epsilon}_2 = \sigma_2 f(T2) \quad [13]$$

The average compatible strain rates in a complete temperature cycle are

$$\begin{aligned} \dot{\epsilon}_{AV} &= \frac{\dot{\epsilon}_1 k + \dot{\epsilon}_2}{1+k} \\ &= \sigma_2 T_{AV} - \sigma_\alpha \frac{k f(T1)}{1+k} \end{aligned} \quad [14]$$

where,  $k$  is the ratio of the pseudo-time at T1 to that at T2 and  $T_{AV}$  is the average value of the temperature function  $f(T)$  in a temperature cycle expressed as

$$T_{AV} = \frac{kT1 + T2}{1+k} \quad [15]$$

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Hence,  $\sigma_2$  can be expressed as

$$\sigma_2 = \frac{1}{f(T_{AV})} \left[ \dot{\varepsilon} + \dot{\psi} y + \sigma_a \frac{k f(T1)}{1+k} \right] \quad [16]$$

Apply equilibrium,

$$P = \int_{y_b}^y \sigma_2 b \, dy \quad [17]$$

$$M = \int_{y_b}^y \sigma_2 b y \, dy \quad [18]$$

By combining the equations,

$$P = \dot{\varepsilon} I_1 + \dot{\psi} I_2 + I_4 \quad [19]$$

$$M = \dot{\varepsilon} I_2 + \dot{\psi} I_3 + I_5 \quad [20]$$

$$\text{and} \quad \dot{\varepsilon} = p_1 M + q_1 P + r_1 \quad [21]$$

$$\dot{\psi} = p_2 M + q_2 P + r_2 \quad [22]$$

$$\text{where} \quad I_1 = \int \frac{b}{f(T_{AV}(y))} dy; \quad I_2 = \int \frac{by}{f(T_{AV}(y))} dy; \quad I_3 = \int \frac{by^2}{f(T_{AV}(y))} dy$$

$$\text{and} \quad p_1 = -I_2/S; \quad p_2 = I_1/S; \quad q_1 = I_3/S; \quad q_2 = -I_2/S;$$

$$r_1 = (I_2 I_5 - I_2 I_4)/S; \quad r_2 = (I_2 I_4 - I_1 I_5)/S; \quad S = I_1 I_3 - I_2^2$$

By using the Virtual Power concept and the flexibility method, the steady state continuity moment and section stresses can be found.

### AN EXAMPLE OF A TWO-SPAN CONTINUOUS BEAM

An axially prestressed continuous beam of two spans subjected to two temperature crossfall profiles, one linear and one non-linear, has been studied. The details of the beam and the temperature profiles are shown on Fig. 4. Analysis was carried out based on the detailed list in Table 1.

Two types of analysis have been conducted. The simpler one is to assume the temperature of the beam being sustained at an average temperature until the steady state. The other one is to account explicitly for the real nature of temperature fluctuations between two temperature states until the steady state is reached. Case 4 is the simplified approach of the Case 6 for linear temperature crossfall. Case 5 is the simplified approach of the Case 7 for non-linear temperature crossfall. In all cases, the initial temperature crossfall is T1 (uniform at 20°C) and the temperature function for Rate of Creep analysis is  $f(T) = T + 10^\circ\text{C}$ .

### RESULTS AND DISCUSSION

#### Increase in hogging moment at support

An increasing support moment at Steady State is generally observed for non-uniform temperature crossfall. Figs. 5 and 6 show the moment distributions for the linear and non-linear temperature crossfall cases respectively. They show similar behaviour. The initial support moment caused by the quasi-permanent load is 500kNm. The application of temperature crossfall T3 reduces this initial support moment to 125kNm. At steady state, the maximum support moment occurs when the beam temperature reverts to T1. The two figures also show that both methods of (i) cyclic temperature analysis and (ii) sustained average temperature analysis give extremely similar

maximum support moment at T1 which are found as 803 kNm and 795 kNm respectively, 60% larger than that of initial elastic solution under mechanical load only. Therefore, (i) the thermal creep effect on the change of support moment cannot be ignored; and (ii) a sustained average temperature analysis is recommended for practical use because it is a simpler calculation.

### Influence of Self equilibrating stresses

Figs. 7 and 8 show the stress distributions of the support section for the two temperature crossfalls. The non-homogeneous creep rates causes non-linear sectional stresses leading to a set of self-equilibrating stresses which is additional to the one caused by non-linear temperature crossfall. The self-equilibrating stresses of the non-linear temperature increase the tensile stress at the top of the section from that calculated simply from the bending moment. In this case, the bending moment alone is not adequate to define the design limit for continuous bridges. For the span section, Fig. 9 shows that the creep effect is beneficial to reduce the maximum compressive stress at the top but is not desirable in increasing the tensile stress at the bottom. The change of stresses due to creep is considerably smaller in the span section than that in the support section.

### Influence of thermal creep property of concrete

Here, the thermal creep of concrete is defined by the temperature function  $f(T) = T + a$ , where  $a$  is a material parameter. The larger the value of " $a$ ", the less increase in creep rate due to temperature. Fig. 10 shows that for  $a = 120^{\circ}\text{C}$  (rate of creep appears not to increase with temperature), the maximum support moment still increases to a value 22% and 40% larger than its initial (loading only) elastic value for the non-linear and linear temperature crossfall cases respectively. The current EC2 Part 3 implies a value of  $62^{\circ}\text{C}$  for " $a$ ". Based on this assumption, EC2 Part 1 which assumes creep rate to be temperature independent up to  $40^{\circ}\text{C}$  is supported by the results in Fig. 10. However, when  $a < 40^{\circ}\text{C}$ , the support moment increases drastically and an accurate knowledge of the parameter " $a$ " becomes important.

## CHANGE OF STRUCTURAL FORM

There are often good reasons that necessitate changing the structural form of an existing structure. These can relate to sequential construction or the need to satisfy different requirements from the original design, but more frequently they are brought about during the retrofitting of structures after a period of service in order to improve serviceability or extend service life. Examples include the making of simply supported beams continuous over internal supports and the provision of additional prestress late in the life of a structure. Creep of concrete can play a dominating role in defining structural behaviour and safety after such changes have been made even though it may not have been important beforehand.

When the statically determinate, or indeterminate, form of a reinforced concrete or prestressed concrete structure is changed in this manner, time-dependent changes to the support actions and internal stresses and moments can be expected. These changes may be unimportant in the short term but can create seriously changed behaviour in the longer term with the possibility of violating the original design parameters. Special care is therefore necessary to examine in detail the nature and possible extent of the changes, in time, and to determine whether they are likely to impair future serviceability and/or safety.

In analysing problems of this type it is frequently impossible to obtain closed form solutions and one must resort to numerical step by step techniques. It is also important to adopt a representation for creep that is appropriate for allowing significant stress redistribution to occur with time. In this context formulations based on the effective modulus concept are inappropriate since they are most suitable to those cases in which stresses show little or no change in time. Both 'superposition' and 'effective modulus' methods overestimate creep recovery under a falling stress regime. As a consequence they underestimate the magnitude of stress changes in time. 'Rate of flow'<sup>16</sup> which distinguishes explicitly between non-recoverable flow and delayed elastic strain is a highly recommended approach. A simplification of this method is the 'rate of creep' approach. This combines the delayed elastic and flow strains in a strain rate formulation which produces results in which stress changes are overestimated. This method thus constitutes a safe engineering approach. Equation 1 defines the total strain rate, with respect to time, in relation to the elastic and creep properties of concrete. It is used in this section to investigate two cases of change of structural form.

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In the first, a rotational restraint is imposed as a result of making two simply supported prestressed concrete beams continuous over the common support. In the second, a segmentally constructed prestressed unit of varying cross-sectional area is studied following the imposition of overall displacement restraint. This example serves to highlight some of the features introduced into the post-repair behaviour of the Koror-Babeldaob bridge<sup>14</sup> in the republic of Palau in 1996, as described later. This bridge collapsed six weeks after the remedial work was completed.

### (i) Rotational restraint.

Fig. 11 shows two simply supported prestressed concrete beams made continuous over the common support after the two beams were subjected to their design loading; a uniformly distributed load in this case. Here the joining of the two beams creates a rotational restraint over the common support and leads to a time-dependent build up of bending moment there. Fig. 12 gives the details. The long-term steady state support bending moment is of the same magnitude as would have been predicted from an elastic calculation, for the same loads and prestress, and of continuous construction initially. The deformed shape, however, would be different.

### (ii) Axial restraint.

The influence of imposing an overall axial displacement restraint on a prestressed concrete unit of sequential construction and varying cross-sectional area is investigated. This example may be considered as a simplification to some features of the retrofitting carried out in 1996 to the Koror-Babeldaob prestressed concrete box girder cantilever bridge in the Republic of Palau. In this segmentally constructed bridge the piers to the two cantilever arms were founded on piled foundations with raked piles<sup>15</sup>, Fig. 13. The retrofitting was to overcome sagging at the cantilever tips at an age of 19 years and was brought about by several measures as indicated in Fig. 14. Firstly exterior prestressing tendons placed inside the box section, and passing over two deviator beams in each cantilever arm were used to lift the central section after first making it continuous at the crown hinge. At this time the two ends of the cantilevers were also subjected to axial jacking forces to add extra compression to compensate for earlier stress losses due to creep. From this time the bridge acted as continuous between the two piled abutments and was therefore also subjected to axial restraint. The ‘hogging’ moments in the central region would have enhanced the restraining forces by reason of deflecting the bridge profile further from the straight line shortest distance between the two abutments. This bridge collapsed six weeks after the remedial works had been completed.

Although the definitive reasons for the collapse have not been published, this example serves to demonstrate that the remedial work changed the behaviour of the bridge in a substantial way, from that associated with normal prestressed concrete in which the prestress acts as a ‘follow-up’ loading with little change in time, to a dramatically altered behaviour for which the prestressed segments lost stress rapidly due to the overall longitudinal displacement boundary constraint imposed, in addition to the rotational constraint at the crown section. The mixed boundary conditions of prestress forces locally on each segment and overall displacement control on axial deformations thus led to the dramatic behavioural changes, with a rapid destressing of the concrete that is not normally associated with prestressed concrete. The changes described here refer to the influence of creep only. Inclusion of shrinkage will bring about an even more rapid loss of stress than shown in the example of Fig. 11.

The figures and discussion of the next section describe the detailed nature of the behavioural changes caused by the overall displacement control imposed.

The example of Fig. 15 addresses the axial length change restraint in a simplified manner, while retaining the important geometrical features of the bridge, viz. the parabolic variation of cross-sectional area between the crown section and the abutments. Similar to the bridge, each segment of Fig. 15 is prestressed locally to a uniform stress (the same in each element) by a common proportion of prestressing steel. (*The prestressing tendons are assumed to be fully bonded to the surrounding concrete.*) Thereafter a range of different conditions has been studied; all relate to changes of concrete and steel stresses. They are:

- (i) The normal prestressed concrete situation, i.e. no restraint to axial shortening.
- (ii) The imposition of an overall length change restraint, to simulate at least approximately the making of the actual bridge monolithic at the crown hinge.

- (iii) The application of an axial jacking force before applying overall displacement restraint between the two ends. This is to simulate the crown jacking forces applied to the actual bridge before monolithic construction was created.
- (iv) The application of a short term temperature change to determine the distribution of thermal stresses in the axially restrained condition. These stresses could be expected to apply as a short-term variation to the time-varying stresses caused by creep, whenever a temperature change took place. Their magnitude would also relate to the temperature at which the bridge was made continuous at the crown section.
- (v) The imposition of overall length change restraint for elements of the same cross-sectional area. This comparison allows the importance of the influence of the variable geometry to be assessed.

Table 2 sets out the details of the individual cases studied.

## DISCUSSION

Fig. 1 shows the typical response of introducing moment continuity late in the life of a structure. At uniform temperature the steady state solution for the time-dependent moment variations is the same as the normal elastic solution for the same continuity applied initially. When temperatures are sustained but not uniform a different steady state solution is reached that is uniquely related to the form of the temperature distribution and the prestress applied.

Figs. 16 to 20 allow several important observations to be made for the example of Fig. 15 when conflicting boundary conditions (namely restriction to free axial movements) are imposed on a prestressed structure.

Overall displacement restraint is responsible for an immediate departure from normal prestressed behaviour; causing concrete to undergo a rapid loss of prestress. Comparison of Cases (A) and (E) in Fig. 16(a) show the very significant difference in concrete stress as creep occurs. In Case (E) the theoretical limit to the concrete stress is zero. The steel stress however shows no variation over the same period, while steel force is progressively transferred to the boundary constraints.

The variation of cross-sectional area with position creates another important feature. The restraint to overall length change results in an interchange between creep and elastic strains within the segments collectively, but not uniformly. The elastic strain rates, and hence the stress changes, are at all times distributed in the inverse ratio of the segment stiffnesses. This causes additional, and rapidly developing, stress relief (beyond that for relaxation at constant strain) thus causing tension in some of the more slender segments. Fig. 16(b), Case (B), shows this effect for each segment following the application of the length restraint while Fig. 16(c), Case (C), shows a similar effect to exist even after a compressive axial jacking force of  $P = 35 \text{ MN}$  was applied before the length restraint was imposed. The more stocky segments (e.g. segment 10) experience behaviour that is more closely associated with normal prestress behaviour (compare Case (A) of Fig. 16(a)).

Comparison of Fig. 17(a), Case (A), with Fig. 17(b), Case (B), reveal the dramatic changes to concrete stress that result from the combined influence of axial restraint and varying cross-sectional area, on a segment by segment basis, as creep develops. The separate influence of cross-sectional area may be seen by comparing Case (B), Fig. 17(b), with Case (E), Fig. 18.

The benefits of the axial jacking force applied before imposing the length restraint may be seen by comparing Case (C), Fig. 19, with Case (B) of Fig. 17(b). It is of interest to note here that whereas the initial effect of the jacking was to increase the stress in segment 1 by more than  $5 \text{ MN/m}^2$  with the development of creep this difference progressively reduced to around  $1 \text{ MN/m}^2$  and both solutions eventually revealed the development of tensile stresses. Clearly the most important parameter, controlling change of concrete stress, is that of the overall length restraint that was imposed.

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Finally the segment stresses corresponding to a short-term uniform temperature change of  $10^{\circ}\text{C}$  are displayed in Fig. 19, Case (D). Again, the highest stresses are seen to concentrate towards the more slender segments as a result of the non-uniform stiffness throughout the set of segments.

### CONCLUSIONS

Concrete creep can be detrimental to structural performance, not only from the standpoint of serviceability, but from safety considerations also. Non-uniform temperatures cause differential creep and stress redistribution to occur in prestressed concrete structures. The consequences often lead to violations of the original design unless explicit recognition is paid to the long-term changes of stress as part of the design criteria. The self-equilibrating stresses due to thermal creep and thermal stresses of non-linear temperature crossfall will increase the maximum stress over the support section from that calculated from the bending moment. Research presented here shows that if the material constant “a” of the temperature function is less than  $40^{\circ}\text{C}$ , the thermal creep will cause significant stress redistribution (see Fig. 10) and its effect on prestressed concrete structures should not be ignored. The thermal creep multiplier as specified in EC2 Part 3 implies “a =  $62^{\circ}\text{C}$ ” which is less than those from other researchers. If it is valid, the creep effect to the steady state stresses at the low end of the bridge temperature range should still be checked. A support moment of 40% larger than that of a elastic solution is still possible.

Imposed constraints will lead to stress changes in time. Flexural restraints, common to the creation of moment continuity, should be incorporated in the initial design even though they may be imposed late in the service life of a structure. Displacement restraints in the direction of the prestress should be avoided because they generate a conflict of boundary conditions between the follow-up loading required of the prestress from the tendons and the overall displacement constraints required for the structure. The result can be a rapid and serious time-dependent reduction in concrete prestress, leading to failure in tension, or more likely in shear as the result of removing the pre-compression required of a safe design.

The time-dependent stress reductions depicted in the example of Fig. 6, where tensile stresses are shown to develop in some prestressed concrete segments simply as a result of introducing an overall length restraint, suggests that a full and detailed analysis of the Koror-Babeldaob bridge is warranted to determine the most likely true effects of the dual restraints imposed on the structure by the remedial works. The overall length restraint would bring about similar stress reductions with time as displayed in the example of Fig. 6, while the external prestressing tendons introduced to cause uplift over the central region of the span (already made continuous at the crown joint) would exacerbate the rate of stress decay in the concrete by reason of trying to increase the length of the bridge profile. Failure of this bridge was inevitable eventually.

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Table 1 – Temperature history of the analyses

| Cases | Temperature history                                                                  | Temperature profile | Type of analysis                       |
|-------|--------------------------------------------------------------------------------------|---------------------|----------------------------------------|
| 1     | Static at $T_1 = 20^\circ\text{C}$                                                   | uniform             | elastic                                |
| 2     | Instant change from $T_1$ to $T_3$                                                   | linear              | Thermo-elastic                         |
| 3     | Instant change from $T_1$ to $T_5$                                                   | non-linear          | Thermo-elastic                         |
| 4     | Sustain temperature at $T_2$ until steady state, then instant change back to $T_1$   | linear              | Thermal creep at sustained temperature |
| 5     | Sustain temperature at $T_4$ , until steady state, then instant change back to $T_1$ | non-linear          | Thermal creep at sustained temperature |
| 6     | At $T_1$ after cyclic temperature between $T_1$ and $T_3$                            | cyclic linear       | Cyclic thermal creep analysis          |
| 7     | At $T_1$ after cyclic temperature between $T_1$ and $T_5$                            | cyclic non-linear   | Cyclic thermal creep analysis          |

The quasi-permanent load is sustained throughout the whole time history.

See Fig. 4 for details of temperature profiles.

The initial temperature of all cases is  $T_1$  which is uniform at  $20^\circ\text{C}$ .

$T_2$  is a linear temperature crossfall from  $30^\circ\text{C}$  to  $20^\circ\text{C}$  (average of  $T_1$  and  $T_3$ )

$T_3$  is a linear temperature crossfall from  $40^\circ\text{C}$  to  $20^\circ\text{C}$ .

$T_4$  is a non-linear temperature crossfall between  $30^\circ\text{C}$  and  $20^\circ\text{C}$  (average of  $T_1$  and  $T_5$ ).

$T_5$  is a non-linear temperature crossfall between  $40^\circ\text{C}$  and  $20^\circ\text{C}$ .

Table 2 – Combinations of conditions studied.

Results are displayed in Figs. 16 to 20.

| Case                                                      | (A)* | (B)* | (C)* | (D)* | (E)* |
|-----------------------------------------------------------|------|------|------|------|------|
| Uniform initial prestress, $f = 8\text{MN/m}^2$           | x    | x    | x    |      | x    |
| Overall length restraint                                  |      | x    | x    | x    | x    |
| Axial jacking force, $P = 35\text{ MN}$                   |      |      | x    |      |      |
| Uniform temperature change, $\Delta T = 10^\circ\text{C}$ |      |      |      | x    |      |

\* Cross-sectional area of  $i$ th segment is given by:

$$A_i = A_1[1 + 2.5\{(i-1)/(N-1)\}^2], \text{ for } A_1 = 6\text{m}^2,$$

where  $N$  is total number of segments; see Fig. 15.

# All concrete segments have the same cross-sectional area

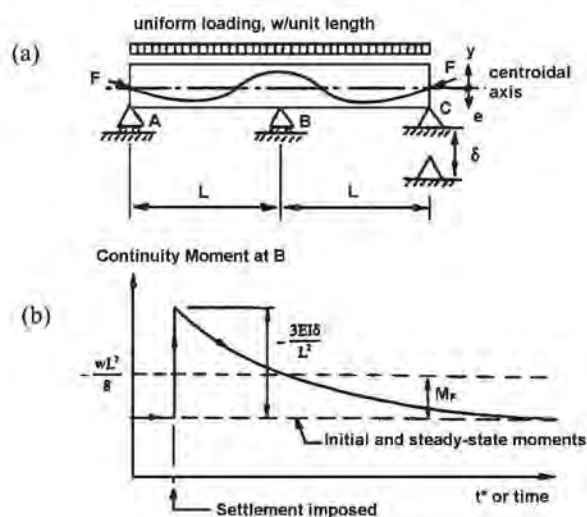


Fig. 1—(a) Change of continuity moment due to support settlement and variation due to creep thereafter.<sup>14</sup>

(b) Two-span prestressed concrete beam with sustained support displacement.<sup>14</sup>

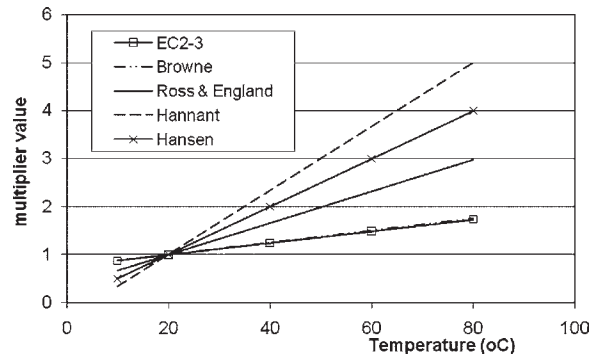


Fig. 2—Creep coefficient multipliers at temperature up to 80 °C according to various researchers.

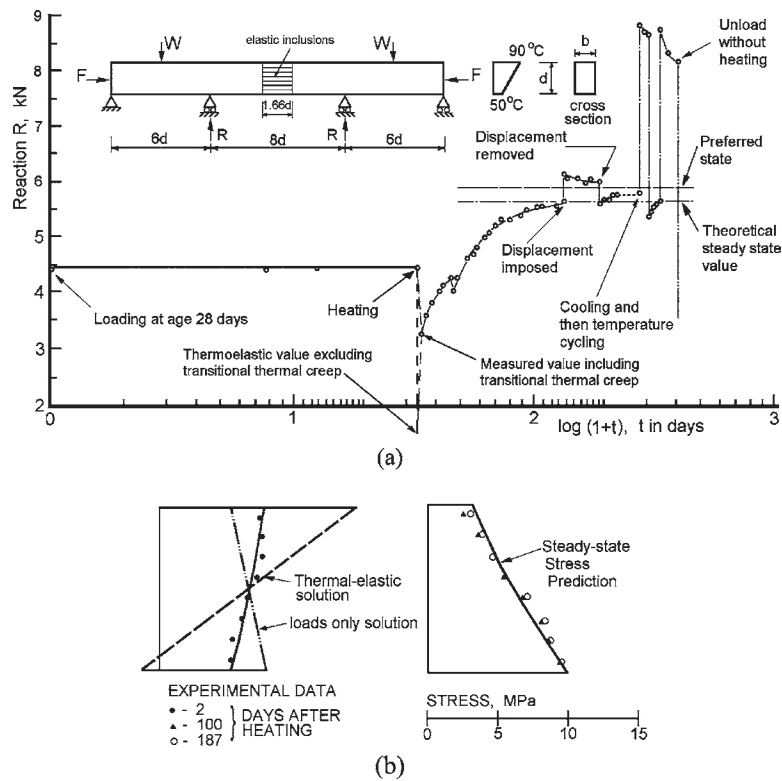


Fig. 3—(a) Time-dependent variation of support reactions for nonuniformly heated prestressed concrete continuous beam, illustrating the existence of a ‘preferred’ state in the long term.<sup>12</sup> Experimental results:  $d = 132$  mm,  $b = 102$  mm,  $W = 8$  kN,  $F = 95$  kN. Imposed displacements at internal supports, 0.24 mm. (b) Time-dependent stress changes in central span of beam in part (a). Comparison of recorded stresses (from strains of elastic inclusions) at 100 and 187 days with a steady-state stress prediction.<sup>12</sup>

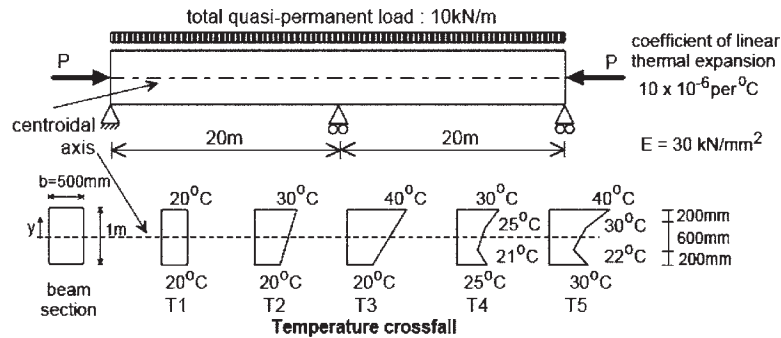
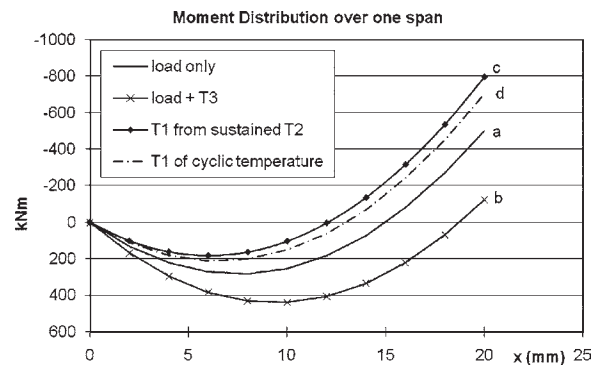
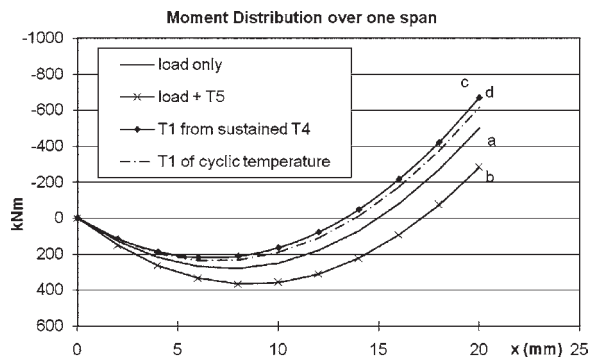


Fig. 4—Configuration of the continuous beam example and temperature crossfall.

Fig. 5—Bending moment diagram of the beam: (a) elastic at temperature  $T_1$ ; (b) thermo-elastic (after application of temperature  $T_3$ ); (c) at  $T_1$  after reaching steady state at sustained temperature  $T_2$ ; and (d) at  $T_1$ , after reaching steady state, for continuous cyclic temperature fluctuations between  $T_1$  and  $T_3$ .Fig. 6—Bending moment diagram of the beam: (a) elastic at temperature  $T_1$ ; (b) thermo-elastic (after application of temperature  $T_5$ ); (c) at  $T_1$  after reaching steady state at sustained temperature  $T_4$ ; and (d) at  $T_1$ , after reaching steady state, for continuous cyclic temperature fluctuations between  $T_1$  and  $T_5$ .

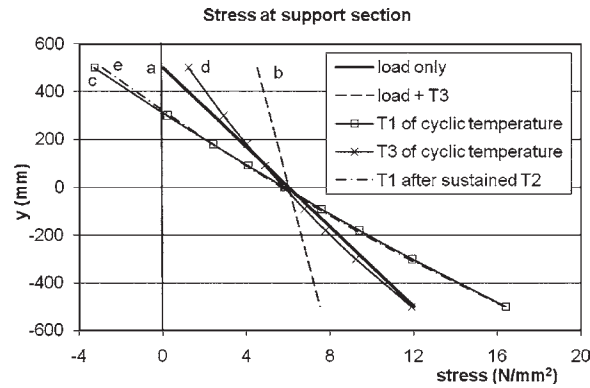


Fig. 7—Stress at support section of the beam: (a) at temperature T<sub>1</sub> initially; (b) thermo-elastic after application of temperature T<sub>3</sub>; (c) and (d) at T<sub>1</sub> and at T<sub>3</sub> after reaching steady state for continuous cyclic temperature fluctuations between T<sub>1</sub> and T<sub>3</sub>; and (e) at T<sub>1</sub> after reaching steady state at sustained temperature T<sub>2</sub>.

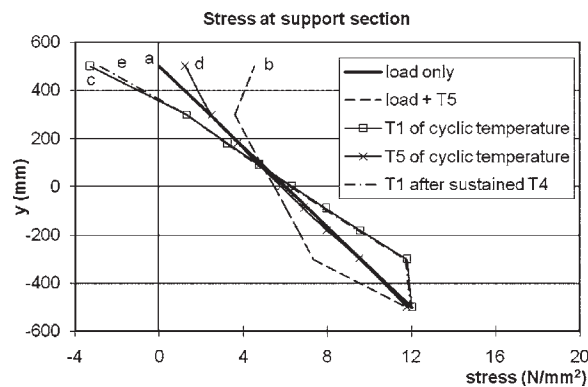


Fig. 8—Stress at support section of the beam: (a) at temperature T<sub>1</sub> initially; (b) thermo-elastic after application of temperature T<sub>5</sub>; (c) and (d) at T<sub>1</sub> and at T<sub>5</sub> after reaching steady state for continuous cyclic temperature fluctuations between T<sub>1</sub> and T<sub>5</sub>; and (e) at T<sub>1</sub> after reaching steady state at sustained temperature T<sub>4</sub>.

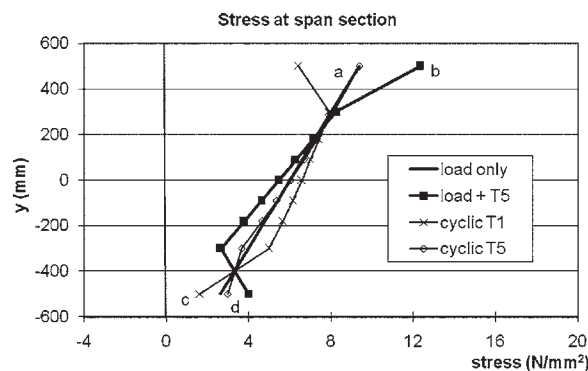


Fig. 9—Stress at span section (0.4 L from free end) of the beam: (a) at temperature T<sub>1</sub> initially; (b) thermo-elastic (after application of temperature T<sub>5</sub>); and (c) and (d) at T<sub>1</sub> and at T<sub>5</sub> after reaching steady state for continuous cyclic temperature fluctuations between T<sub>1</sub> and T<sub>5</sub>.

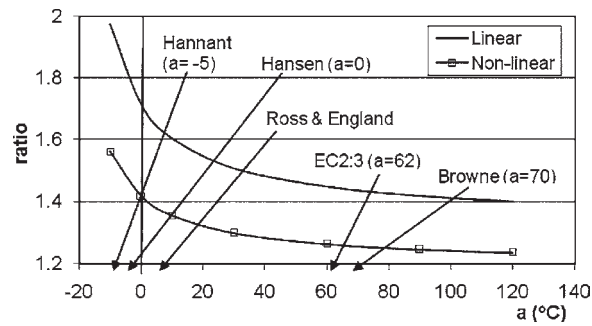


Fig. 10—Ratio of support moment vs temperature function constant “a” (where  $f(T) = T + a$ ) for: (a) linear temperature crossfall of  $T_3$ ; and (b) nonlinear temperature crossfall of  $T_5$ . Values of “a” from results of various sources are indicated.

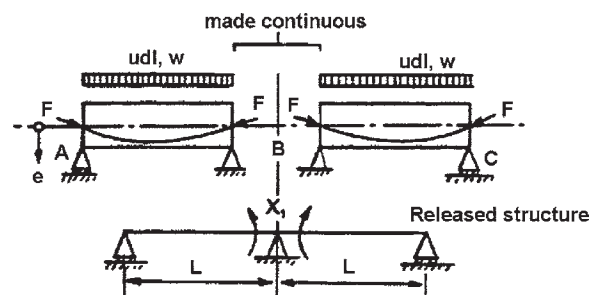


Fig. 11—Two simply supported prestressed concrete beams made continuous.<sup>15</sup>

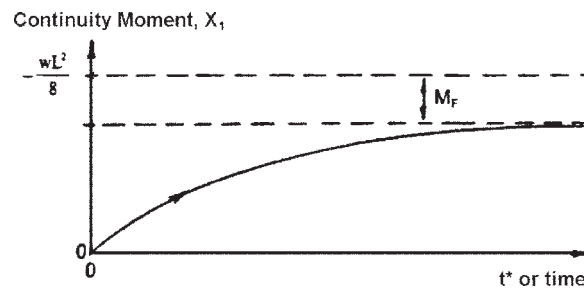


Fig. 12—Variation of continuity moment,  $X_1$ , with creep after creating structural continuity at support B in Fig. 11.<sup>15</sup>

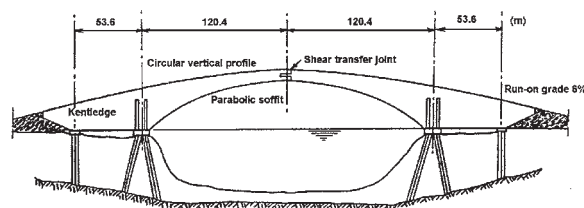


Fig. 13—General layout details for Koror-Babeldaob prestressed concrete balanced cantilever box girder bridge.<sup>15</sup>

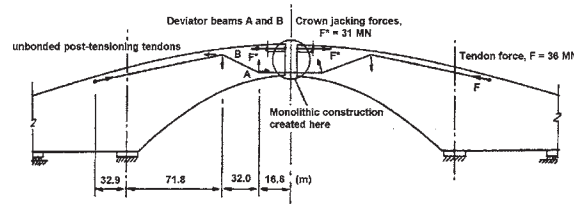


Fig. 14—Nature of remedial works showing unbonded post-tensioning tendons, location of deviator beams, crown jacking forces, and the creation of monolithic connection at the crown joint. Arrows indicate direction of forces applied to box girder.<sup>15</sup>

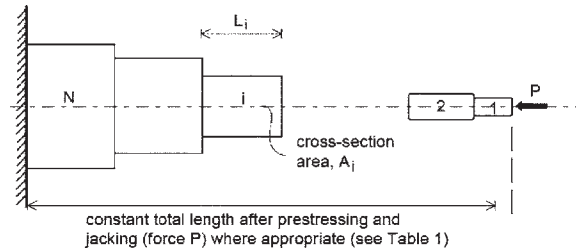


Fig. 15—Segmentally constructed and symmetrically prestressed concrete element, followed by application of an axial jacking force  $P$  and then restrained against overall length change thereafter. Each segment prestressed initially to a uniform concrete stress of  $8 \text{ MN/m}^2$ . Jacking force  $P = 35 \text{ MN}$ . Numerical examples are for  $N = 10$  and  $L = 11.5 \text{ m}$ . For further details, see Table 1.

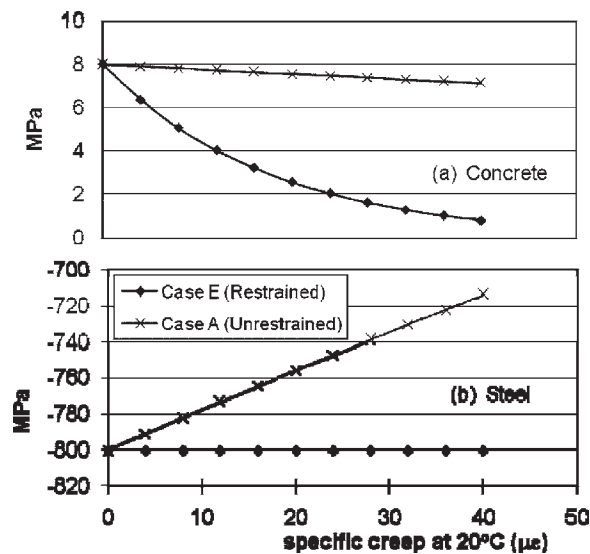


Fig. 16(a)—Changes of concrete and steel stresses with creep; Case (A), normal prestress conditions. Case (E) represents relaxation of stress under full strain restraint in each segment. All segments exhibit identical behaviour.

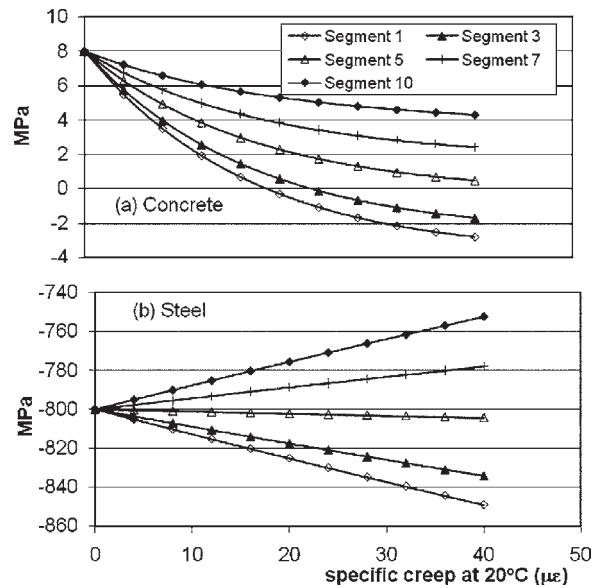


Fig. 16(b)—Changes of concrete and steel stresses with creep for prestress and imposed overall length restraint, Case (B).

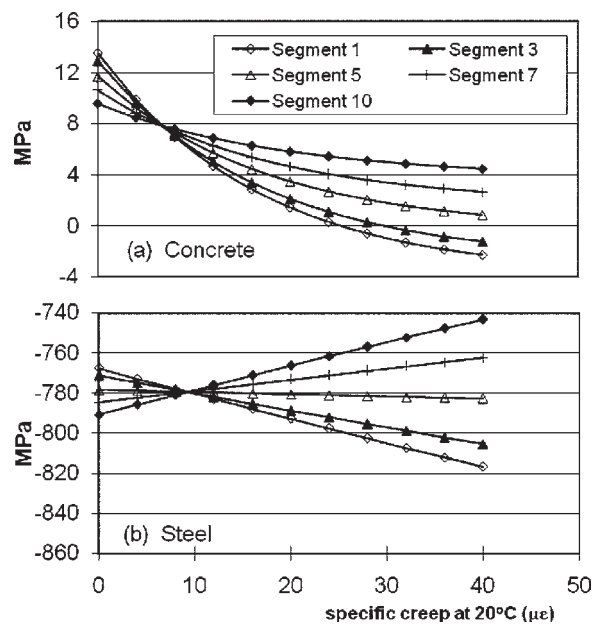


Fig. 16(c)—Changes of concrete and steel stresses with creep for prestress, axial jacking and imposed overall length restraint, Case (C).

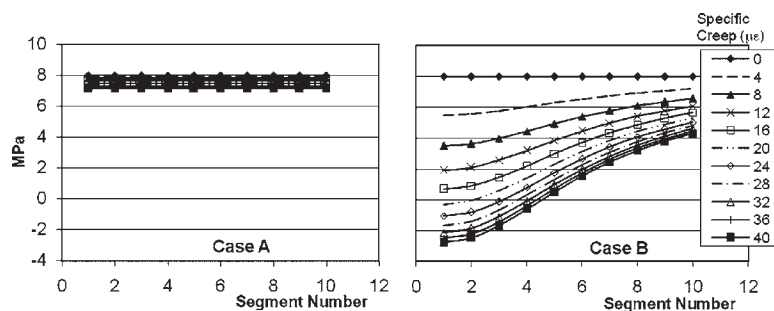


Fig. 17—Variation of concrete stresses with creep within each segment: (a) prestress only, Case (A); and (b) prestress plus overall length restraint, Case (B).

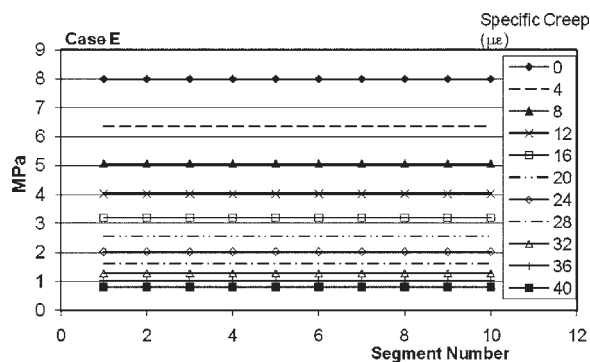


Fig. 18 —Variation of concrete stress within each segment during relaxation at constant strain in each segment, Case (E).

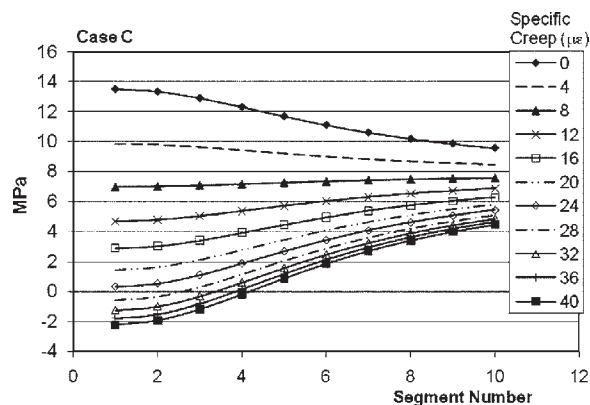


Fig. 19—Variation of concrete stress with creep in each segment for prestress, axial jacking force and overall length restraint, Case (C).

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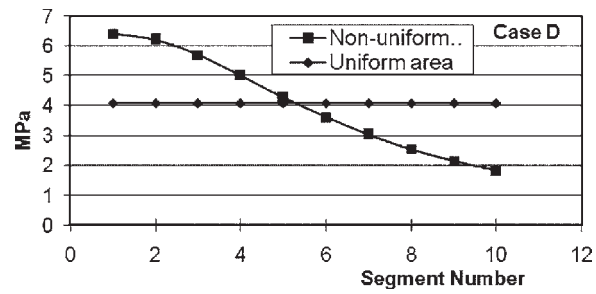


Fig. 20—Variation of concrete thermal stresses in segments for a 10°C temperature rise, under restraint to overall length change, Case (D).

# Numerical Approach to Viscoelastic Analysis of Concrete Structures Using Equilibrium and FEM

by M. Sassone, D. Bigaran, and C. Casalegno

Synopsis: The formulation of creep problems in concrete structures, using linear viscoelasticity leads to integral equations that, generally, can not be solved in closed form. Methods of analysis developed to overcome this problem include the AAEM algebraic method, the method based on the theorems of linear viscoelasticity, or methods that assume simplified creep models. But when the structural problem is complex, involving non-homogeneities, construction steps, changes in static scheme, complex geometry, high prestressing, different materials, a computational approach based on the numerical solution of the systems of equations is required.

Because of the form of the viscoelastic law, in which the present strain is a functional of the whole stress history, the stress history of the structure needs to be stored during calculation. Furthermore the numerical methods necessary to solve the integral equations are not immediately compatible with linear and non-linear finite element analysis and the substitution of integral equations with rate-type approximate laws, seemed to be necessary to allow the use of FEM solvers.

In this paper a set of structural problems is described from the point of view of mathematical and computational formulation, and the general method based on coupling integral equations with the equilibrium method, and then with finite element method, is shown. The proposed formulation allows to perform the step-by-step analysis of any kind of viscoelastic structure, without regard to its complexity, and the computational load, in terms of memory requirements, appears to be no longer prohibitive.

Keywords: computational approach; creep; creep analysis; creep-induced structural effects; design tool; integral equations; structural analysis; viscoelasticity

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**Davide Bigaran** holds a PhD in structural mechanics and is researcher at the Politecnico di Torino. His doctoral thesis concerned the reliability assessment of concrete structures at serviceability limit state, with particular reference to the effects of changes in static scheme on the long term stress distributions. He is part of the Creep Analysis Research Group at the Politecnico di Torino and he works on the analysis of creep effects on complex structures.

**Carlo Casalegno** has obtained a degree in architecture with a thesis on numerical solutions of viscoelastic problems. He presently works on the development of software implementation of numerical algorithms.

### INTRODUCTION

The solution of structural problems involving creep and shrinkage phenomena in concrete structures has been an important task for engineers since the first formulation of the mathematical model of linear viscoelasticity. If on one hand the definition of a suitable formulation of creep laws involved scientists and researchers in past decades and many prediction models have been developed, starting from experimental data and from the direct observation of the long term behavior of concrete structures (Muller 1993, Bazant & Baweja 2000, Gardner & Lockman 2001), the development of structural analysis procedures, based on the creep models, is, on the other hand, of great interest for engineers who need to investigate the effects of creep and shrinkage on the structures they design.

From an historical point of view the evolution of the research on this topic, as in many other research field, has been dramatically influenced by the diffusion of computer technology, starting from the early seventies of past century. Before this event the research was mainly oriented on finding closed form solutions of simple analytic formulations of the creep models. The theory of heritage and the theory of aging, the use of exponential formulations of creep have been developed and largely adopted because of their capability to generate closed form solutions of structural problems. This approach has been particularly profitable from both the theoretical and the application points of view, because a set of fundamental and general theorems have been proven, independently from the specific creep model adopted, and the methods based on this theorems are very important in conception and design of concrete structures (CEB MC 1990, Chiorino & al. 1984, Chiorino 2005).

When more refined creep models needed to be developed, in order to better fit the experimental data, the use of closed form solutions become more difficult and different numerical approaches have been proposed. Of course numerical methods are useful only if automatic computation tools are available, so that only in the early seventies, thanks to Bazant work, the approach called general method could be developed.

The structural problems involving creep phenomena can be described in terms of integral equations systems and then solved by applying a suitable numerical algorithm. In this kind of problems the state of the system at time  $t$  can not be evaluated on the basis of the state at time  $t-\Delta t$  (or on the basis of a small set of previous states) but is expressed as a functional of the whole stress history of the structure. In the past the necessity of storing the stress history has been considered a problem due to the request of a large amount of memory. Nowadays the computer technology allows to perform such analyses relatively quickly, at least for the usual structural problems; hence the use of the general method can be considered as a powerful and complete tool of analysis.

### RESEARCH SIGNIFICANCE

This research is part of the activity of the Creep Analysis Research Group, at the Politecnico di Torino, which main task is to diffuse in researcher and engineers community the knowledge of the theory and of the applications of viscoelasticity. In 2005 the group founded his own web site with the aim to give the engineers the possibility to download a set of design aids related to creep analysis. The first design aid consisted in the Creep.exe program, a freeware computer program capable to generate the compliance, the relaxation, the aging coefficient and the redistribution functions for the B3, GL2000 and CEB90 creep models. The program has a suitable graphic interface and all the parameters of creep models can be edited by the user. A version the finite element viscoelastic solver

presented in this paper has been also implemented into a computer program and has been upload on the web site of the group, giving the possibility to download it and to use it for free. The importance of this kind of tools and design aids has already been underlined by other authors and some computer application has been proposed in the past as a design aid for researchers and practitioners (Kristek & al. 2001, Sassone & Chiorino 2005). The project of the group is to develop in the future a set of different computational tools and to make them available by scientists and practitioners, together with the theoretical descriptions of the underlying concepts.

## COMPUTATIONAL APPROACHES BASED ON THE GENERAL METHOD

When dealing with structural analysis of elastic or inelastic structures, the general formulation of the problems is based on continuum mechanics. In the most general case, the set of equations that gives the general formulation of the elastic (or inelastic) structural problem, in terms of a set of partial differential equations, is obtained by grouping the strains-displacements differential relation, in the linear form, or in the non-linear large-displacements and large-strains form, with the description of the stresses tensor and with a constitutive linear (elastic) or non linear (elastic or inelastic) relation. A suitable set of boundary conditions expressed in terms of restrained or imposed displacements (kinematic BC) and imposed loads (static BC) is necessary to make the mathematical problem well formulated.

When the structural problem involves only one dimensional bodies, under certain restrictions, i.e. when we deal with De Saint Venant bodies, the corresponding formulation can be reduced to a set of ordinary differential equations. This formulation can be extended to other one dimensional bodies, as the beam on elastic soil or as concrete-concrete or steel-concrete composite beams. In the first case an elastic restraint condition is assumed on the bottom of the beam, meanwhile in the second case there is a slip between the two parts and the cross section does not remain plane.

If a structure can be described as made of one-dimensional members (as it happens in continuous girders, frames, bridges, etc.), and it is statically indeterminate, two main approaches are possible in order to obtain the displacement and the stresses: the compatibility (forces) method and the equilibrium (displacements) method, which are dual to each other. In the first case the  $n$  redundant reactions  $X_i$  are the unknowns and a system of  $n$  linear algebraic equations of compatibility has to be written; the influence coefficient  $a_{ij}$  represents the displacement at point  $i$  due to a unit force at point  $j$ . In the second case the unknown are the  $m$  nodal displacements  $\eta_i$ , and a set of  $m$  linear algebraic equations of equilibrium are required; a stiffness coefficient  $k_{ij}$  represents the nodal force at node  $i$  due to a unit displacement at node  $j$ .

The equilibrium method is the conceptual frame of the finite element method: the structure needs to be divided in elements connected to each other in a set of nodal points, and the unknowns are represented by the nodal displacements. Depending on the number of DOF a system of equilibrium equations is written for each node and the relation between nodal displacements and nodal forces is expressed by the stiffness matrices of the elements. The behavior of each finite element is then described with the aid of a set of exact or approximated shape functions, that define the displacement field of the element. Starting from the displacement field, the strain field can then be obtained by derivation, the stress field by applying a stress-strain relation, and finally the nodal forces can be obtained by integration of the stresses.

All the approaches explained above can be formulated for elastic structures as well as for inelastic structures and viscoelastic structures. In case of inelastic structures the constitutive law usually defines the stresses or internal forces as non-linear functions of the strains or relative displacements. The law is then expressed in terms of a variable tangent stiffness. The solution of a inelastic problem can be obtained by means of an iterative algorithm, as the Newton-Raphson (NR) or the Modified Newton-Raphson (MNR) by updating iteratively the structural stiffness and by reducing the so called "out of balance" forces.

If a specific formulation of the structural problem, in terms of partial differential equations, ordinary differential equations or algebraic equations is coupled with a viscoelastic integral type creep law, the mathematical formulation of the problem is expressed in terms of partial integral-differential equations, ordinary integral-differential equations or just integral equations. After the viscoelastic problem is formulated in the correct mathematical form it is then possible to choose a suitable computational approach. In the case of viscoelastic structures, as it will be explained in the sequel, the constitutive law, given by the creep model, defines the strain as a functional of the whole stress history, and can not be inverted in closed form, except in some particular case (when the creep model is particularly

simple). That's the reason for the most frequently adopted formulation of creep problems is based on the compatibility equations. But that's not the only possibility, and in next paragraphs some example of the application of the general method to concrete structures will be given.

#### GENERAL METHOD OF SOLUTION FOR CONTINUOUS BEAMS

As an example of application of the general method, consider a plane beam with general restraint conditions. The differential relation between displacements, internal forces and external loading is expressed by a four order ordinary differential equation, that can be written as a system of four first order equations (left). The first equation expresses the internal compatibility, third and fourth equations express the indefinite equilibrium, meanwhile second equation introduces the constitutive elastic law. Staring from this formulation the elastic law can be easily replaced by the integral viscoelastic law, obtaining the equations on the right. The second equation becomes of integral-differential type and the symbol of partial derivative means that all the variables are function of time  $t$  and of the space coordinate  $x$ .

$$\left\{ \begin{array}{l} \frac{dv}{dx} = \varphi \\ \frac{d\varphi}{dx} = \frac{M}{EI} \\ \frac{dM}{dx} = -T \\ \frac{dT}{dx} = -q_y \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \frac{\partial v}{\partial x} = \varphi \\ \frac{\partial \varphi}{\partial x} = \frac{1}{I} \int_{t_0}^t J(t, \tau) \frac{\partial M}{\partial \tau} d\tau \\ \frac{\partial M}{\partial x} = -T \\ \frac{\partial T}{\partial x} = -q_y \end{array} \right.$$

In order to be well formulated, the system of equations needs to be associated with the correct number of boundary conditions, that can be static or kinematic.

The generality of this kind of formulation becomes obvious if we consider that all the physical parameters, as the compliance function  $J_{(t, \tau)}$ , the external loading  $q$ , can be variable in time, as well as along the  $z$  longitudinal axis of the beam. Even the boundary conditions can change in time, as an effect of the modification in restraints, meanwhile the area moment of inertia  $I$  can be slightly variable along  $z$ . This formulation can then take in account the non homogeneity due to different environmental conditions, in different parts of the beam or in different times, or due to different concrete age (as in segmental bridge construction).

From the computational point of view the problem can be numerically approached with the aid of a finite differences scheme, for what concerns the integration in space, and of the trapezoidal rule for the integration in time. By substituting the derivatives with finite number of difference quotients the system of four differential equations is transformed into a system of  $4N$  algebraic equations, where  $N$  is the number of discretization points, and the solution of the equations system gives the time increments of the four variable of state at each time step  $t$ .

#### GENERAL METHOD OF SOLUTION FOR COMPOSITE BEAMS

A second and more interesting application of the general method concerns the behavior of steel-concrete, or concrete-concrete composite beams. These structures are usually analyzed assuming that the two (or more) parts of the beam are longitudinally connected at the interface surface, by means of a mechanical connection as studs or other kind of device. The connection can be designed in order to be perfectly rigid, i.e. to avoid any longitudinal slip between the connected parts. That is specifically the situation of concrete-concrete composite beams, in which the mechanical interlock of concrete surfaces plays an important role. If the connection is assumed flexible, the slip between the parts appearing when the beam is loaded can not be neglected and the cross section is no longer globally plane. In this case the Newmark formulation of the behavior of composite beams can be adopted, taking in account explicitly the phenomenon. The mechanical model is based on the assumption that only longitudinal slip occurs and that the curvature of the steel and concrete parts are the same in each cross section. The behavior of the connection is described by a constitutive law, usually expressed in terms of relation between longitudinal shear stress and slip per unit length, and it can be assumed elastic.

From the theoretical point of view, the assumption that the curvature is the same in both the steel and concrete parts of the cross sections implies some restrictions in the boundary conditions: the moments applied to the ends of the beam, as the ones deriving from the continuity of the girder, must be applied on the cross section in order to respect the kinematic assumption, i.e. in parts proportional to the stiffness of the steel beam and the concrete slab. When this is not realistic, for instance if the continuity involves only the slab meanwhile the steel beams are simply supported, the assumption can be removed and a more general mechanical model need to be adopted. In order to allow different curvatures in the parts of the cross sections the connection must be assumed elastic even in the transverse direction and a separation from the parts of the beam must be possible. Of course the transverse behavior of the connection is unilateral, because the surface at the interface can separate but can not compenetrare to each other. The mechanical model involving transverse deformability is then intrinsically non linear, even when the materials and the connection itself are assumed to behave elastically.

Starting from the geometric and kinematic configuration represented in Figure 1, in which the displacements  $u$  and  $v$  in  $x$  and  $y$  directions and the rotation  $\varphi$  are defined for both the parts of the beam, and from the static configuration represented in Figure 2, where the internal forces  $N$ ,  $T$ , and  $M$  are defined for both the parts, a system of differential, linear and non linear algebraic equations can be written.

It involves twelve equations: eqs. 1 to 4 express the constitutive relation between axial force and bending moment in the two parts of the beam; eqs. 5 and 6 are only internal compatibility equations; eqs. 9 and 10 are indefinite equilibrium equations; eqs. 7, 8, 11 and 12 represent the constitutive laws of the connection. Equations 11 and 12 are intrinsically non linear.

$$\left\{ \begin{array}{l} 1) \frac{du_1}{dx} - \alpha_1 \frac{1}{E_1} N_1 - \beta_1 \frac{1}{E_1} M_1 = 0 \\ 2) \frac{du_2}{dx} - \alpha_2 \frac{1}{E_2} N_2 - \beta_2 \frac{1}{E_2} M_2 = 0 \\ 3) \frac{d\varphi_1}{dx} - \beta_1 \frac{1}{E_1} N_1 - \gamma_1 \frac{1}{E_1} M_1 = 0 \\ 4) \frac{d\varphi_2}{dx} - \beta_2 \frac{1}{E_2} N_2 - \gamma_2 \frac{1}{E_2} M_2 = 0 \\ 5) \frac{dv_1}{dx} - \varphi_1 = 0 \\ 6) \frac{dv_2}{dx} - \varphi_2 = 0 \\ 7) \frac{dN_1}{dx} - K_x \cdot (u_1 - u_2) = -p_{x1} \\ 8) \frac{dN_2}{dx} + K_x \cdot (u_1 - u_2) = -p_{x2} \\ 9) \frac{dM_1}{dx} + T_1 = 0 \\ 10) \frac{dM_2}{dx} + T_2 = 0 \\ 11) \frac{dT_1}{dx} - f(v_1 - v_2) = -p_{y1} \\ 12) \frac{dT_2}{dx} + f(v_1 - v_2) = -p_{y2} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \frac{\partial u_1}{\partial x} - \alpha_1 \int_{t_0}^t J(t, \tau) \frac{\partial N_1}{\partial \tau} d\tau - \beta_1 \int_{t_0}^t J(t, \tau) \frac{\partial M_1}{\partial \tau} d\tau = 0 \\ \frac{\partial u_2}{\partial x} - \alpha_2 \frac{1}{E_2} N_2 - \beta_2 \frac{1}{E_2} M_2 = 0 \\ \frac{\partial \varphi_1}{\partial x} - \beta_1 \int_{t_0}^t J(t, \tau) \frac{dN_1}{d\tau} d\tau - \gamma_1 \int_{t_0}^t J(t, \tau) \frac{dM_1}{d\tau} d\tau = 0 \\ \frac{\partial \varphi_2}{\partial x} - \beta_2 \frac{1}{E_2} N_2 - \gamma_2 \frac{1}{E_2} M_2 = 0 \\ \frac{\partial v_1}{\partial x} - \varphi_1 = 0 \\ \frac{\partial v_2}{\partial x} - \varphi_2 = 0 \\ \frac{\partial N_1}{\partial x} - K_x \cdot (u_1 - u_2) = -p_{x1} \\ \frac{\partial N_2}{\partial x} + K_x \cdot (u_1 - u_2) = -p_{x2} \\ \frac{\partial M_1}{\partial x} + T_1 = 0 \\ \frac{\partial M_2}{\partial x} + T_2 = 0 \\ \frac{\partial T_1}{\partial x} - f(v_1 - v_2) = -p_{y1} \\ \frac{\partial T_2}{\partial x} + f(v_1 - v_2) = -p_{y2} \end{array} \right.$$

Even in such complex, non linear, twelve equations mechanical model, the elastic behavior of one (or both, in case of concrete-concrete composite structures) part of the beam can be easily replaced by a viscoelastic integral creep law. The formulation of the viscoelastic structural problem is obtained by assuming the variables of state as depending of time and by substituting the elastic law with the viscoelastic integral relation in the equations that describe the concrete parts of the beam. Of course, as well as in the case of the concrete continuous beams, the substitution can be easily made because the kinematic variables, rotations and displacements, are expressed as

functions of the static variables, moments and forces, and the creep integral law is directly compatible with the model.

When the structural problem is correctly formulated from the mathematical point of view, the solution can be searched by means of a numerical approach. The formulation of the problems in terms of a set of first order differential or integral-differential equations is the standard form of the problem and it makes easier to find the suitable numerical solution. If we consider that the system of equations contains differential equations, integral-differential equations and non linear equations, the solution can be build by assembling three different algorithms. In order to integrate the differential equations the Crank-Nicholson second order finite differences scheme has been applied; the trapezoidal rule is the algorithm adopted for the solution of integral equations; finally the well known Newton-Raphson or Modified Newton-Raphson iterative methods have been applied in order to take in account the non linearity of eqs. 11 and 12. All these numerical algorithms are well known and relatively easy to be implemented from the computational point of view. Only the NR procedure can show problems of numerical convergence when the equations are strongly non linear, as it effectively happens in the case of the connection because they act as a unilateral restraints.

#### EQUILIBRIUM (DISPLACEMENTS) METHOD

In order to analyse more complexes structures, always made of one-dimensional members, the equilibrium method is the most suitable approach. The computational procedure is obtained by coupling the equilibrium method with the integral formulation of the creep law. As it is well known, in the frame of the displacements method the unknowns are the displacements and the constitutive laws are formulated by expressing the static parameters (stresses and nodal forces) as functions of the kinematic parameters (strains and nodal displacements). This is in contrast with the creep law formulation, in which the strain at time  $t$  is expressed as a functional of the stress history from the time  $t_0$  to the time  $t$ . Due to this fact, it is not possible to write a "viscoelastic stiffness matrix" by substituting the elastic law with the viscoelastic law. As it will be shown in the sequel, the solution can be obtained by writing explicitly the numerical algorithm and by transforming the integral formulation in the corresponding discrete representation. As in previous examples the numerical approximation is only related with the integration scheme and to the time discretization. The solution of the viscoelastic problem is then obtained incrementally as a sequence of elastic analyses, the so called step-by-step procedure, in which the elastic modulus is updated at the actual value and the effects of the stress history are taken into account at each step as external nodal actions, calculated on the basis of solutions of previous steps.

Let eqs. (1) be the local matrix relation between nodal forces and nodal displacement in a plane beam element, referred to the local coordinate system:

$$\{Q^*\}_{6 \times 1} = [K^*]_{6 \times 6} \{\delta^*\}_{6 \times 1} \quad (1)$$

In order to introduce a constitutive law expressed by an integral viscoelastic relation, it is necessary to rewrite eq. (1) in the inverse form. As it is well known, eq. (1) says that, given a set of six independent components of the nodal displacements, and assuming the deformation to be congruent, six components of the nodal forces will appear. The inverse form can not be written in a perfectly dual way, because we can not assume six independent components of the nodal forces from which calculate the corresponding displacements: the beam element must be equilibrated, so three equilibrium equations link to each other the nodal forces components. Furthermore the set of nodal displacements we can obtain from a given stress field is defined excluding the three components of a rigid body movement. From this considerations the inverse form of the constitutive law of the element can be written as follow:

$$\begin{cases} u_1 = u_0 \\ v_1 = v_0 \\ \varphi_1 = \frac{\tilde{\eta}_{33}}{E} M_1 + \frac{\tilde{\eta}_{36}}{E} M_2 + \varphi_0 \\ u_2 = \frac{\tilde{\eta}_{44}}{E} N_2 + u_0 \\ v_2 = v_0 + \varphi_0 \ell \\ \varphi_2 = \frac{\tilde{\eta}_{63}}{E} M_1 + \frac{\tilde{\eta}_{66}}{E} M_2 + \varphi_0 \end{cases} \quad (2)$$

in which  $u_0, v_0, \varphi_0$  are the component of the rigid body movement, that will be assumed equal to zero, and  $\eta_{ij}$  are the following coefficients of influence of a plane beam with a hinge at node 1 and a support at node 2,:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\ell}{3I} & 0 & 0 & -\frac{\ell}{6I} \\ 0 & 0 & 0 & \frac{\ell}{A} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\ell}{6I} & 0 & 0 & \frac{\ell}{3I} \end{bmatrix} \quad (3)$$

The three implicit equilibrium equations are the following:

$$\begin{cases} N_1 = -N_2 \\ T_1 = \frac{M_1 + M_2}{\ell} \\ T_2 = -T_1 \end{cases} \quad (4)$$

The third, fourth and sixth equations of (2) are now capable to incorporate an integral expression of the material constitutive law, instead of the elastic one, represented by the constant modulus  $E$ :

$$\begin{cases} \varphi_{1,t} = \tilde{\eta}_{33} \int_{t_0}^t J(t, \tau) dM_1 + \tilde{\eta}_{36} \int_{t_0}^t J(t, \tau) dM_2 \\ u_2 = \tilde{\eta}_{44} \int_{t_0}^t J(t, \tau) dN_2 \\ \varphi_{2,t} = \tilde{\eta}_{63} \int_{t_0}^t J(t, \tau) dM_1 + \tilde{\eta}_{66} \int_{t_0}^t J(t, \tau) dM_2 \end{cases} \quad (5)$$

For general aging viscoelastic models, as those usually employed for concrete, these equations can not be inverted in a closed form and the corresponding viscoelastic stiffness matrix can not be written.

In order to overcome this problem, in the frame of a general computational method, i.e. without introducing any simplified formulation of creep models, one possibility consists in assuming a viscoelastic law written in terms of relaxation functions, instead of compliance functions. In this case there is no need to write eqs. (1) in the inverse

form, in terms of coefficients of influence, rather than of coefficients of stiffness. No real advantages would be obtained from this formulation: first, from the point of view of computational load, the "viscoelastic stiffness matrix" is again an integral linear operator and it has to be applied to the whole displacement history; secondly the relaxation functions need to be numerically calculated before the analysis, for all the couples  $(t, t_0)$  of the time discretization; finally we must consider that relaxation functions by themselves give the exact solutions of viscoelastic problems only when a strain field is impressed and the second theorem of viscoelasticity applies, as well as the compliance functions give the exact solution in for cases in which first theorem applies. In all the other cases the problem is expressed in terms of integral equations that need to be solved numerically. Hence, from the computational point of view, there is no difference between using compliance or relaxation functions in dealing with general problems. Only by the formal point of view the use of relaxation functions in the frame of the displacements methods could be advantageous. In fact the use of relaxation functions allows the formulation to be totally written in closed form, in terms of a system of integral equations, and the choice of a suitable numerical algorithm can be postponed at the end of the procedure.

If the compliance functions are used, and the beam element properties are expressed in the form of eq. (5), we find an obstacle and we are obliged to introduce at this step the numerical algorithm, together with a suitable time discretization. The algorithms generally adopted for the solution of integral equations are based on the substitution of the integral with the sum of a finite set of finite increments. As it will be explained in the sequel, the solution is obtained by considering separately, at each step of the analysis, the effect of the current increment and the effect of all previous increments.

The numeric algorithm used in this paper is the so called trapezoidal rule, first described in Bazant (1972a), which is a second order scheme based on the mean value of the compliance function between loading time  $t_{i-1}$  and loading time  $t_i$ :

$$\varepsilon_{t_k} - \varepsilon_{0,t_k} = \frac{1}{2} \sum_{i=1}^k [J(t_k, t_i) + J(t_k, t_{i-1})] \Delta \sigma_i \quad (6)$$

The current strain increment can be obtained by subtracting the strain value at time  $t_{k-1}$  from the strain value at time  $t_k$ :

$$\begin{aligned} \Delta \varepsilon_{t_k} - \Delta \varepsilon_{0,t_k} &= \frac{1}{2} [J(t_k, t_k) + J(t_k, t_{k-1}) - J(t_{k-1}, t_k) - J(t_{k-1}, t_{k-1})] \Delta \sigma_{t_k} + \\ &+ \frac{1}{2} \sum_{i=1}^{k-1} [J(t_k, t_i) + J(t_k, t_{i-1}) - J(t_{k-1}, t_i) - J(t_{k-1}, t_{i-1})] \Delta \sigma_i \end{aligned} \quad (7)$$

By substituting the discrete sums into eqs. (5) one obtains:

$$\begin{cases} \Delta \varphi_{1,t_k} = \tilde{\eta}_{33} JJ_{kk} \Delta M_{1,k} + \tilde{\eta}_{36} JJ_{kk} \Delta M_{2,k} + \tilde{\eta}_{33} \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{1,i} + \tilde{\eta}_{36} \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{2,i} \\ \Delta u_{2,t_k} = \tilde{\eta}_{44} JJ_{kk} \Delta N_{2,k} + \tilde{\eta}_{44} \sum_{i=0}^{k-1} JJ_{ki} \Delta N_{2,i} \\ \Delta \varphi_{2,t_k} = \tilde{\eta}_{63} JJ_{kk} \Delta M_{1,k} + \tilde{\eta}_{66} JJ_{kk} \Delta M_{2,k} + \tilde{\eta}_{63} \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{1,i} + \tilde{\eta}_{66} \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{2,i} \end{cases} \quad (8)$$

in which:

$$\begin{aligned} JJ_{ki} &= J(t_k, t_i) + J(t_k, t_{i-1}) - J(t_{k-1}, t_i) - J(t_{k-1}, t_{i-1}) \\ JJ_{kk} &= J(t_k, t_k) + J(t_k, t_{k-1}) - J(t_{k-1}, t_k) - J(t_{k-1}, t_{k-1}) \end{aligned}$$

These equations can then be inverted with respect to  $\Delta M_1$  and  $\Delta M_2$ , by reintroducing the stiffness matrix coefficients:

$$\begin{cases} \Delta M_{1,t_k} = \frac{1}{JJ_{kk}} \left( \tilde{K}_{33} \Delta \varphi_{1,t_k} + \tilde{K}_{36} \Delta \varphi_{2,t_k} + \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{1,t_i} + \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{2,t_i} \right) \\ \Delta N_{2,t_k} = \frac{1}{JJ_{kk}} \left( \tilde{K}_{44} \Delta u_{2,t_k} + \tilde{K}_{44} \sum_{i=0}^{k-1} JJ_{ki} \Delta N_{2,t_i} \right) \\ \Delta M_{2,t_k} = \frac{1}{JJ_{kk}} \left( \tilde{K}_{63} \Delta \varphi_{1,t_k} + \tilde{K}_{66} \Delta \varphi_{2,t_k} + \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{1,t_i} + \sum_{i=0}^{k-1} JJ_{ki} \Delta M_{2,t_i} \right) \end{cases} \quad (9)$$

and in compact form:

$$\{\Delta Q\}_{t_k} = \frac{1}{JJ_{kk}} [\tilde{K}] \{\Delta u\}_{t_k} + \frac{1}{JJ_{kk}} \{\Psi\}_{t_k} \quad (10)$$

This matrix relation is then the expression of the viscoelastic behaviour of a plane beam element, in discrete form. The first term is the instantaneous elastic relation between the increment of displacement and the increment of force; the second is the effect at time  $t_k$  of all previous increments of stress, acting as the effect of an impressed strain field.

The local stiffness matrices, as well as the nodal forces vectors and the vectors representing the effect of the stress history, can then be assembled and the global stiffness matrix and global nodal forces vectors are obtained. The global matrix and the global vectors are then partitioned as usual in order to take into account the external restraints:

$$\{\Delta F_F\}_{t_k} = [K_{FF}]_{t_k} \{\Delta u_F\}_{t_k} - \{\Psi_F\}_{t_k} + [K_{FR}]_{t_k} \{\Delta u_R\}_{t_k} \quad (11)$$

$$\{\Delta F_R\}_{t_k} = [K_{RF}]_{t_k} \{\Delta u_F\}_{t_k} + [K_{RR}]_{t_k} \{\Delta u_R\}_{t_k} - \{\Psi_R\}_{t_k} \quad (12)$$

where the subscript **F** means *free* and the subscript **R** means *restrained*. When the incremental displacements are calculated, the stresses in the elements can be obtained by applying eq. (8) again.

#### SOFTWARE IMPLEMENTATION OF THE ALGORITHM

The algorithm described above has been implemented in the Matlab programming environment, which is one of the most widely used mathematical tools. The whole application is composed of a set of different modules. The first module is the Compliance Functions Generator, i.e. the subroutine that generates the compliance values  $J(t, t_0)$  for a given time scale starting from the creep models parameters. The creep models available in the software are the B3 model, the GL2000 model and the CEBMC90 model. According to the models the subroutine also generates the shrinkage functions. The second module is the Time Scale Generator, that defines the sequence of time intervals. The time scale is based on a logarithmic subdivision, but when the problem involves changes in the static scheme, segmental construction or loads applied at different ages, the whole time scale is divided into a set of sub-scales: each sub-scale is based on a logarithmic time subdivision and starts with an initial very short time step. Every time an instantaneous event, as a new load or a change in the scheme, happens, the time scale restarts with a short initial step. This module gives information to the third module: the Finite Element Model Generator, that handle the FEM model on the basis of the geometric and mechanical input data and on the changes occurring during the analysis. This module changes and updates the loads, the restraint conditions, the number of elements and eventually the shape itself of the model, at specified times, if it is required by the analysis. If elements are added to or removed from the model, due for instance to a step in construction, or to a change in the propping system, the global stiffness matrix and the boundary conditions have to be updated. The fourth and most important module is the Incremental Creep Solver: when all the input data are ready and the sequence of steps has been defined, the incremental step by step procedure starts and the finite element solver is called at each step to perform the instantaneous pseudo-elastic analysis. The solution, in terms of displacements, reactions and internal forces is finally given by the sums of the increments calculated at each time step.

With an equilibrium (or finite elements) formulation and with its software implementation, the solution of complex viscoelastic problems becomes possible. The procedure has been applied to the analysis of a box girder bridge, taking into account the effects of different creep and shrinkage in the top flange and the bottom flange, due to their different thickness. The importance of the effects of sectional non homogeneity has already been underlined by other authors [Kristek & al. 2006, Chiorino & al. 2007]. In many cases the long term deflection trend resulted quite different from the early age trend, as an effect of the interaction between creep, shrinkage, prestressing and structural non-homogeneities.

The first model analysed is the cantilever box beam shown in Figure 3, subjected to the dead load. The cantilever spans 20.00 meters and is made of ten segments with variable bottom flange thickness. The geometrical properties of the structure analysed are shown in Figure 3. The top slab is 8.00 m wide and 0.20 m thick for all the length of the beam, meanwhile the bottom slab is 4.00 m wide and has a thickness variable from 0.20 m at the free end to 1.10 at the restrained end. In order to perform the analysis of the box girder taking into account the wall thickness variability in the cross section and along the beam, the structure has been modelled as an equivalent truss, as shown in Figure 4. The chords represent the top and bottom slab, meanwhile the truss web elements, depicted in dashed line, are just the rigid links necessary to guarantee the internal congruence of the beam. The upper flange is prestressed by a set of ten tendons with variable length in order to eliminate in each segment the tensile stresses due to dead load. Tendons are modelled as elastic elements, coaxial to the concrete elements, and the prestressing force is obtained by applying an imposed strain field to the element. The prestress losses due to creep and shrinkage are then explicitly taken in account. The construction, the prestressing of tendons and the application of dead load, are supposed to be simultaneous. The segments have then the same age and the same time of loading. The creep and shrinkage model adopted in the analysis is the CEB MC 90.

The effects of creep and shrinkage on the structure are shown in Figure 5. In order to evaluate the effect of the environmental conditions, three different values of the  $2A_c/u$  ratio are assumed, corresponding to the theoretical value and to values increased of 20% and 40%. In fact the presence of an asphalt layer on the top slab, the solar heating on exterior surfaces, the relative humidity inside the box, all these phenomena can modify the creep and shrinkage in the different parts of the section, introducing further structural non-homogeneities. The blue curves represent the vertical delayed displacement of the cantilever end, considering only shrinkage, loading and prestressing. Due to the difference in the top and bottom flanges thickness, the top flange shrinks faster than the lower one; in Figure 6 the evolution of shrinkage is expressed as a function of the  $2A_c/u$  ratio, showing that after 1000 days (three years) the shrinkage in a slab 20 cm thick can be four times the shrinkage into a slab 1.10 meters thick. Hence, shrinkage produces an initial uplift of the cantilever end, followed by a downward movement. The displacement inversion happens after about ten years (3000 days). If an increased value of  $2A_c/u$  is assumed for the bottom flange the effect of differential shrinkage increases almost proportionally. In the red curves the effect of creep is taken in account: as it is expected the presence of creep reduces the effect of shrinkage and an initial downward deflection appears: in the early age of the structure the creep prevails and the movement is downward, due to the residual hogging moment after prestressing; after that the shrinkage prevails and the beam raises up before moving downward again. This trend is largely influenced by the value of the shrinkage functions in the flanges: if different values of the creep coefficients in the bottom flange are assumed, the overall behaviour can change significantly.

The second example of application of the computational procedure is represented by the structure shown in Figure 7. Two cantilever box beams are restrained at the end sections at time  $t_I = 100$  days. The introduction of a new restraint, as well as every other possible change in the static scheme, is obtained by changing the stiffness matrix at the time step corresponding to time  $t_I$ . In this example the restraint is introduced by changing the stiffness of the elements that connects the cantilever ends from a very low value to a very high value. The dimension of the stiffness matrix is then preserved, because no new elements are added to the model. In a more general case, as when the segmental construction has to be modelled explicitly, considering every new segment being build and loaded at a specific time, the rank of the stiffness matrix can change many times in the analysis. At the left end of the girder the translation along the axis of the beam is freed, in order to consider the presence of an expansion joint.

The results of the analysis are shown in Figure 8. As in the previous analysis the blue curves represent the evolution neglecting creep and taking into account only shrinkage, meanwhile in the red curves the creep is considered. Before

the change of static scheme the behaviour is of course the same as in the cantilever beam. In both the cases of differential shrinkage, (blue curves) and differential creep (red curves), when the end sections are connected and the reciprocal rotation is restrained, the uplift stops, and a slow downward deflection begins. As an effect of the new restraint, the tensile stresses in the top slab increase significantly because of shrinkage, but this phenomenon is mitigated by creep, as it can be seen comparing the red and the blue curves. As it can be expected from theoretical results concerning homogeneous structures, this results confirm that the introduction of a new internal restraint transforms the initial 'creep' problem (evolution of strains under almost constant stresses) to a 'relaxation' problem (evolution of stresses with almost constant strains).

### CONCLUSIONS

The accuracy of predictions resulting from the analysis of creep and shrinkage effects on concrete structures depends on two main factors:

(1) On the accuracy of creep and shrinkage models and on their capacity to fit the experimental data and to interpret the underlying physical phenomena. From this point of view the discussion is still open, because different creep models have been developed and adopted in standards, with non negligible differences in predictions.

(2) On the sophistication of the structural analysis and on the approximations assumed in the evaluation of creep effects. The structural geometry for instance is usually simplified and reduced to simple schemes, as when large box girders are modelled with one dimensional elements; the interaction between creep and prestressing is frequently neglected, when the prestressing action is considered as a force constant in time; the non homogeneity due to different environmental conditions and to complex building processes, can be very hard to be taken into account.

The choice of a finer or coarser level of analysis depends on the available analysis tools, it means on the formulations and on the algorithms we can apply to the problem. Many simplified or approximate approaches have been introduced in the past in order to overcome the computational load, as the transformation of the integral creep law is in a rate type law, by means of spring dash-pot chains.

In this paper the analysis of creep effects on concrete structures has been approached in the frame of the so called general method, i.e. by starting directly from the linear viscoelastic theory and focusing on the computational methods of solution. Following this approach two main advantages can be obtained:

(1) first, all the approximations and the errors that inherit simplified approaches are avoided, because the analysis procedure starts directly from the creep models. Of course these approximations can be negligible in some case, as when is adopted the algebraic AAEM, that gives very accurate predictions in the most of the situations; anyway, in order to asses the negligibility of the approximations a reference computational procedure is necessary.

(2) secondly, the proposed general method, based on coupling the finite element formulation with a suitable numerical algorithm, allows to numerically solve even very complex viscoelastic problems, almost without any restriction. Complex structures, with complex prestressing sequences, complex construction steps, internal non-homogeneity, differences in material properties or environmental conditions can be numerically analysed, in the frame of the theory of linear viscoelasticity, adopting all the available creep models.

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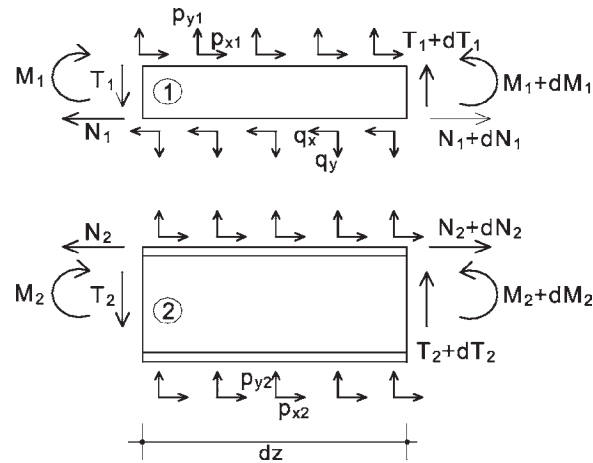


Fig. 1—Static definition of a composite beam.

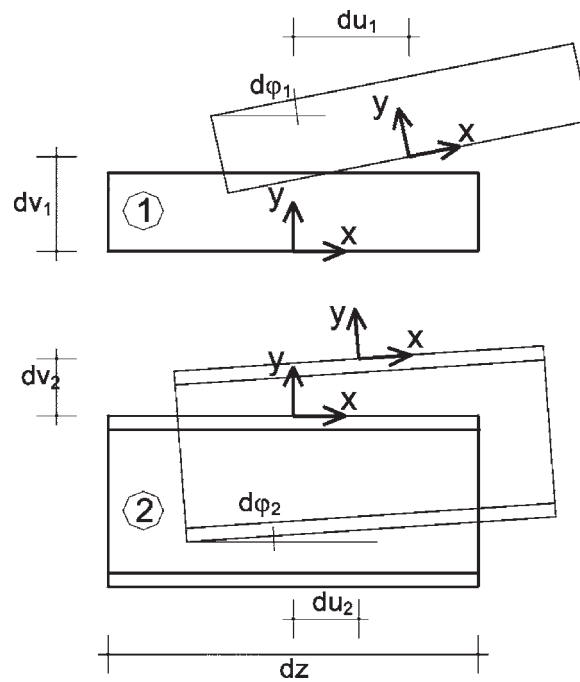


Fig. 2—Kinematic and definition of a composite beam.

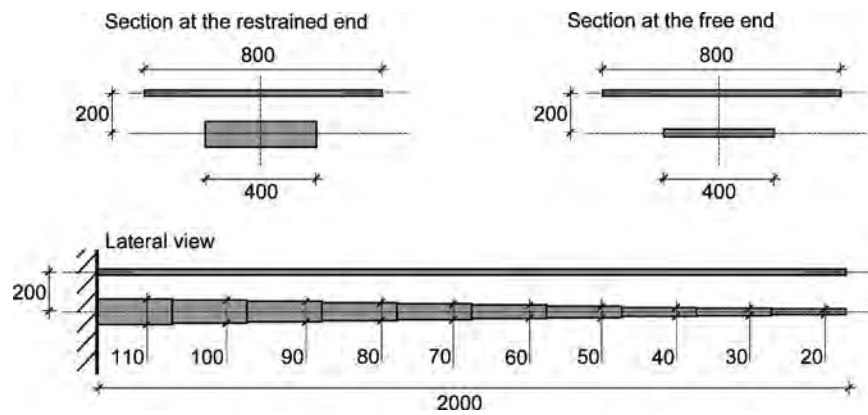


Fig. 3—Geometry of the box girder.

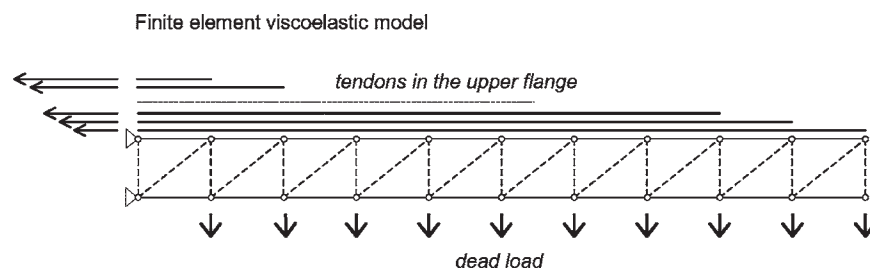


Fig. 4—Viscoelastic finite element truss model with tendons position.

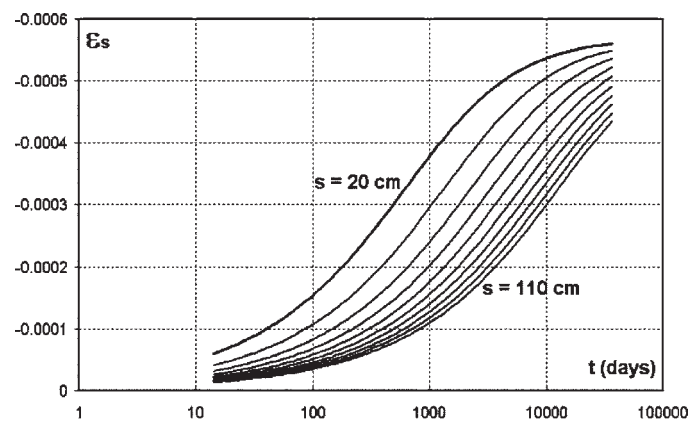


Fig. 5—Shrinkage strains related to the slab thickness.

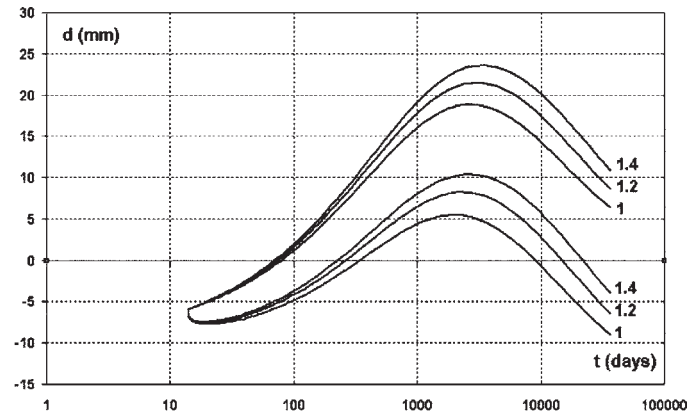


Fig. 6—Vertical delayed displacement of the cantilever.

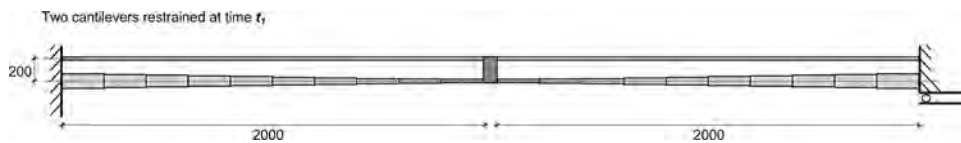


Fig. 7—Geometry of the two cantilever box beams restrained at time  $t_r$ .

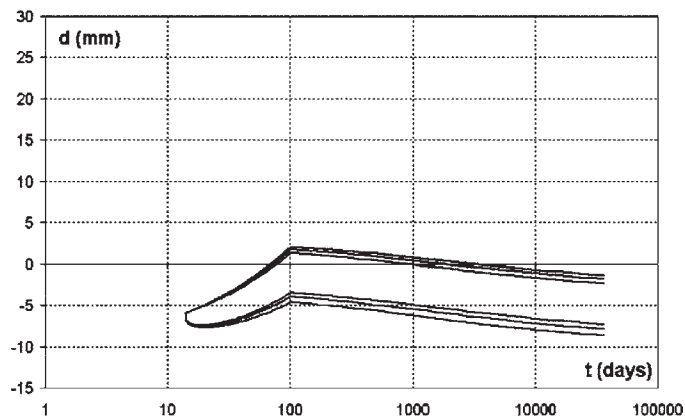


Fig. 8—Vertical displacement of the middle section.

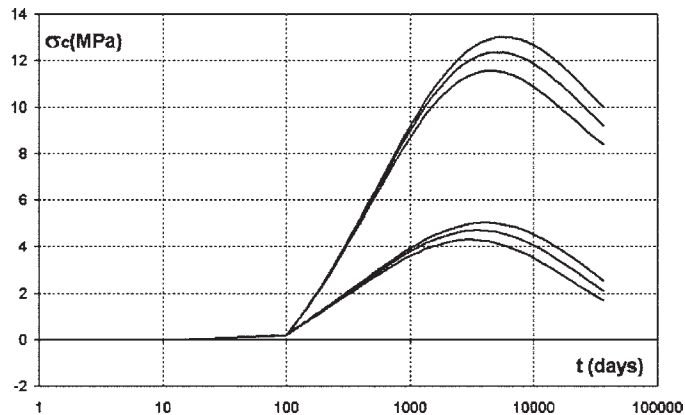


Fig. 9—Concrete tensile stresses in the top slab.



# Removal of Temporary Supports and Creep Effects in Sequential Construction of Bridges

by M. Arici and M.F. Granata

Synopsis: Structural conception of bridges is strictly related to the construction methods involved. A wide application of precasting technologies of reduced dimension elements assembled on site is today the most common choice. During construction, static schemes and strength characteristics of the elements generally change. The connection of different structural elements in subsequent stages, the introduction of delayed restraints and the use of provisional supports modify the structural configuration. Such transient phases can influence significantly the final behaviour of the bridge. In such a way the designer, acting during the constructive stages, can force the structural scheme towards the solution that optimizes elements functionality. In this paper a study about the influence of delayed strains due to creep on the structural conception of prestressed bridges is considered. The effects of additional restraints and support removal on the final solution are evaluated and discussed. The study permits characterizing the phenomena, to define possible simplifications, useful for developing simplified models in the hypothesis of linear viscoelasticity and homogeneous structures. These models allows the use of elastic solutions by means of the redistribution functions.

A case study of an arch-portal bridge is presented. During construction a fundamental role is played by the removal of temporary piers after the introduction of delayed restraints, with a change of the static scheme from the continuous beam behaviour to the arch one.

Keywords: bridge; creep; homogeneous structures; support removal

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### INTRODUCTION

Creep effects modify stress and strain patterns in time, of structures subjected to permanent loads. These effects are very important especially in subsequent construction stages when static schemes vary till the final structural configuration is achieved. The principal effects due to creep in bridge structures construction are:

- a strain increase in time due to permanent loads applied;
- a stress decrease due to permanent geometric actions (distortions) applied;
- a stress redistribution due to a change of the static scheme by the introduction of a new delayed restraint.

In the last case, after loads application at time  $t_0$ , one or more delayed restraints applied at time  $t_1$  prevent viscoelastic deformations that would occur in the section restrained if these restraints were not applied. For  $t > t_1$ , stress pattern evolves approaching the stress distribution corresponding to the application of loads onto the structure in its final static scheme. When more delayed restraints are applied in subsequent times to the structure previously loaded, there is a stress redistribution due to creep that modifies the stress pattern after every single static scheme modification.

If concrete stresses do not exceed about 50% of concrete compressive strength, it is possible to assume a linear ageing viscoelastic model for creep<sup>1</sup> and the validity of superposition principle. This means that creep responses of concrete for steps of stress applied at different times can be superimposed. The development of deformations in the concrete structure is modelled by the compliance function  $J(t, t_0)$  representing the total strain at the generic time  $t$ , due to a unit stress applied at time  $t_0$ . The compliance function  $J(t, t_0)$  is normally the output of creep prediction models adopted by the Codes or by the specialized literature. For every stress history, subdivided into infinitesimal steps  $d\sigma_c$ , the stress-dependent strains due to creep at the generic time  $t$ , can be expressed by the integral relation

$$\varepsilon_c(t, t_0) = \sigma_c(t_0) J(t, t_0) + \int_{t_0}^t d\sigma_c(\tau) J(t, \tau) \quad (1)$$

where  $t_0$  is the time of load application.

Reciprocally, the development of stress is modelled by the relaxation function  $R(t, t_0)$  representing the total stress at the generic time  $t$ , due to a unit strain applied at time  $t_0$ . The relaxation function can be numerically obtained from the compliance function using the Volterra integral equation that states the reciprocal relation between the two functions  $J(t, t_0)$  and  $R(t, t_0)$ . For the superposition principle, considering each increment of permanent geometric actions  $d\varepsilon_c$ , the sum of stress effects due to creep at the generic time  $t$ , can be expressed by the integral relation

$$\sigma_c(t, t_0) = \varepsilon_c(t_0) R(t, t_0) + \int_{t_0}^t d\varepsilon_c(\tau) R(t, \tau) \quad (2)$$

where  $\varepsilon_c(t_0)$  is the concrete initial strain applied at time  $t_0$ .

Changes of static scheme in continuous concrete structures, considered as homogeneous ones, due to the addition of  $n_i$  delayed restraints at time  $t_i$  after load application, can be expressed through a linear combination of stresses, based on the principle of the partial acquisition of the modified system stress distribution.<sup>2</sup> The coefficients of this combination are the redistribution function values  $\xi(t, t_0, t_i)$ : they express the percentage of acquisition of the modified system stress distribution ( $\xi=0$  means no acquisition while  $\xi=1$  means total acquisition, i.e. the final stress

pattern coincides with that of the final elastic scheme considered as it was loaded at time  $t_0$ ). For each single system change at the time  $t_i$ , the redistribution function can be determined through a numerical solution of the Volterra integral equation<sup>3,4,5</sup>

$$J(t, t_0) - J(t_i, t_0) = \int_{t_i}^t J(t, \tau) d\xi(\tau, t_0, t_i) \quad (3)$$

The principle of the partial acquisition of the modified system for ageing materials is strictly valid only when the structure can be considered homogeneous with respect to creep and subjected to rigid restraints. Current structures can be assumed as homogeneous, neglecting local heterogeneities as well as limited variations in concrete properties. This assumption can be done when the analysis concerns the global time-dependent behaviour in terms of sectional forces, external reactions and displacements. On the contrary, a more complex analysis for the description of stress and strain patterns into the section needs to take into account heterogeneities.

Another aspect about creep analysis in homogeneous structures is the modification of static scheme due to the removal of temporary restraints during construction phases. This is a common case when the structural behaviour evolves toward the final service life scheme, through a number of different stages in which temporary supports are used. For restraint removals, creep effects are rather different from those stated by the principle of the partial acquisition of the modified system. It is the case of different typologies of bridges, with static behaviours evolution during construction, when temporary piers are used to support the deck.

In continuous incrementally launched bridges, temporary piers can be placed between the definitive ones, to reduce launch spans and bending moments into the deck, with the advance of the continuous beam above piers.<sup>6</sup>

Temporary supports are used in segmental prestressed portal bridges built by the cantilever method, advancing precast or cast-in-situ segments over slanting piers supported by temporary vertical ones. The temporary piers are removed after the closure in the midspan.<sup>7</sup>

Temporary piers are used also in the construction of arch bridges, together with temporary stay cables, to reduce the length of the cantilever segments before the arch is closed. By removing temporary piers, the arch assumes its definitive configuration. This method can be also used for the construction of a deformable and thin arch, structure often very sensitive to creep effects.<sup>8</sup>

In all these cases, the static behaviour is modified through a history of subsequent addition and removal of restraints. The topic of creep effects due to restraint removals has not been sufficiently discussed in literature, at least for what is in the authors knowledge, because only restraint additions have been considered for static scheme changes. Moreover simplified expressions are not given in literature for the evaluation of the deformation development when static changes occur for either restraints addition or removal.

In this study an evaluation of creep effects produced by supports removal is carried out, taking into account different situations that appear during bridges construction phases: addition and/or removal of restraints after loads application and multiple changes of static scheme in a sequential construction. The study is done by a generalization of the principle of the partial acquisition of the modified system to the case of subsequently or alternate addition and removal of restraints, to evaluate the stress pattern in the final configuration of the bridge. Laws of stress development in time are given together with those concerning deformations. An estimation of these effects allows designers to know the influence on service life condition when an evaluation of serviceability becomes very important both in terms of deformations and stresses.

## INTERACTION BETWEEN ADDITION AND REMOVAL OF RESTRAINTS

The analysis of homogeneous structures with rigid restraints and invariable static scheme can be based on the 1<sup>st</sup> and 2<sup>nd</sup> theorems of linear viscoelasticity for ageing materials.<sup>2,3,9,10</sup> These theorems state the results of viscoelastic analysis, respectively for imposed static actions and for imposed deformations. For the 1<sup>st</sup> theorem, variable imposed static actions produce the corresponding elastic values of internal forces, while deformations vary depending on creep through the compliance function. For constant static actions, the internal force values are constant too and they coincide with the elastic ones, while deformations increase amplifying the elastic values through the product

$E_c J(t, t_0)$ . For the 2<sup>nd</sup> theorem, variable imposed geometric actions (distortions) produce the corresponding elastic values of deformations, while internal forces vary depending on creep through the relaxation function. For constant geometric actions, the deformation values are constant too and they coincide with the elastic ones, while the internal forces are diminishing reducing the elastic values through the ratio  $R(t, t_0)/E_c$ .

The influence of subsequent restraints addition or removal and the interaction between all the different stages are here analysed considering at the time  $t_0$ , imposed external actions: permanent static actions (loads) and geometric permanent actions (imposed distortions). Starting from a generic statically indeterminate structure, let us consider for simplicity one single restraint added at time  $t_1 > t_0$  and one single restraint removed at time  $t_R > t_0$ , varying the order of application and the external action typology. There are no differences if the added and removed restraints are internal or external.

*Case a) Permanent static actions with  $t_R > t_1$ .*

In this case (Fig. 1), after permanent static actions have been applied to the structure at time  $t_0$ , a restraint is added at time  $t_1 > t_0$  and another is removed at time  $t_R > t_1$ . According to the first theorem of linear viscoelasticity, applied loads produce increasing strains due to creep, but the stress field does not change. The delayed restraint applied at  $t_1$  prevents strain increase due to creep and the reaction of the delayed restraint varies in time. By superimposing internal forces constant values due to loads acting on the initial elastic static scheme and variable forces in time due to the new restraint, the stress redistribution is obtained into the structure. So stresses in the modified scheme with the added restraint must be calculated through the redistribution function.

When at time  $t_R > t_1$  a restraint (temporary support) is removed, for equilibrium conditions, the removal is equivalent to the application of a load opposite to the reaction  $X(t)$  of that restraint. Reaction value varies in time due to the redistribution acting on the structure and so, after the support removal, stress state redistribution modifies for  $t > t_R$ .

*Case b) Permanent static actions with  $t_R < t_1$ .*

In this case (Fig. 2) the removal of a restraint takes place before the addition of another one. For the 1<sup>st</sup> theorem of linear viscoelasticity, after load application at time  $t_0$ , there is a sudden stress variation due to the change of structural scheme and then, only deformations vary in time. When the restraint is removed, for equilibrium conditions, the removal is equivalent to the application of a load opposite to the reaction  $X_{el}$  of that restraint. Reaction value varies in time only if the initial static actions vary too, but no stress redistribution occurs in the structure. Instead deformations increase even if static actions are constant. When at time  $t_1 > t_R$  a new delayed restraint is added, the increase of deformations due to creep is forbidden in the restrained point and redistribution starts modifying stress pattern in time.

*Case c) Constant geometric imposed actions with  $t_R > t_1$ .*

In this case (Fig. 3) a geometric action is applied on the structure at time  $t_0$ , a restraint is added at time  $t_1 > t_0$  and another is removed at time  $t_R > t_1$ .

For the 2<sup>nd</sup> theorem of linear viscoelasticity, after load application, stresses decrease in time due to relaxation while deformations do not change. When at a time  $t_1 > t_0$  a delayed restraint is added, for compatibility conditions, initial strains do not change and no stress redistribution acts on the structure due to creep. Only if the delayed added restraint has been forced, the reaction value is nonzero and deformations vary in time together with stresses. When at time  $t_R > t_1$  a restraint is removed, the opposite load to the reaction  $X(t)$  of that restraint is applied on the structure and for the 1<sup>st</sup> theorem there is a further elastic variation in time of stress pattern.

*Case d) Constant geometric imposed actions with  $t_R < t_1$ .*

In this last case (Fig. 4) a geometric action is applied on the structure at time  $t_0$ , a restraint is removed at time  $t_R > t_0$  and another is added at time  $t_1 > t_R$ . For the 2<sup>nd</sup> theorem, after imposed geometric action has been applied, the only creep effect is stresses decrease. The removal is equivalent to the application of loads opposite to reactions, varying in time for the relaxation acting on the structure. As in the previous case c, there is a further elastic variation of stress in time. A successive stress redistribution occurs at time  $t_1 > t_R$ , when the delayed restraint is added.

From the analysis of the interaction between addition and removal of restraints it is possible to know if and when stress redistribution or strain modification occurs. The analytical determination of stress pattern in terms of internal

forces for the case of partial acquisition of the modified system is given in the following section together with an evaluation of the strain pattern in time.

## STRESS AND DEFORMATION ANALYSIS OF STATIC SCHEME CHANGES

Consider a structure with  $k$  redundancies. A constant permanent load  $q$  acts onto the structure at time  $t = t_0$ . According to the 1<sup>st</sup> theorem of linear viscoelasticity, only deformations change in time with the compliance function  $J(t, t_0)$ . If at different times  $t_i > t_0^+$ , the structure is subjected to multiple changes of static scheme by the subsequent addition of  $m$  restraints, the final configuration is a statically indeterminate structure with  $k+m$  redundancies. The stress state of modified system at the generic time  $t$  can be expressed by superimposing the effects of each addition to the previous, through a combination of elastic schemes loaded, whose coefficients are the values of redistribution functions  $\xi(t, t_0, t_i)$  at each time  $t_i$ , for the  $i$ -th restraint addition. The generic internal force  $S(t)$  at time  $t > t_0^+$  can be found by the expression

$$\begin{aligned} S(t) &= S_0 + \sum_{i=1}^m (S_i - S_{i-1}) \xi(t, t_0, t_i) H(t - t_i) \\ &= S_0 + \sum_{i=1}^m \Delta S_i \xi_i(t) H(t - t_i) \end{aligned} \quad (4)$$

where  $S_0$  is the value of the internal force considered in the elastic initial scheme loaded,  $H(t - t_i)$  is the step Heaviside function at time  $t_i$  and  $\Delta S_i = (S_i - S_{i-1})$ . Expression (4) is valid for  $t > t_0^+$  and the introduction of Heaviside functions has here the only aim to underline the times of addition and their chronological sequence, but it is not essential because  $\xi(t, t_0, t_i) = 0$  for  $t \leq t_i$ . The final scheme has  $k+m$  redundancies and at time  $t > t_m^+$  relation (4) gives the internal force value in the final scheme, after the last addition. Every change is independent from the others, previously or successively occurred, and relation (4) states the superposition of stresses. This means that creep redistribution, due to the application at time  $t_i$  of single or multiple restraints, does not depend on the previous history of changes but only on the time  $t_i$ .

It is possible to know the development of deformations by superimposing the responses due to each scheme change. Let  $D(t)$  be the system of deformations (internal strains and external displacements) and  $D_i$  ( $i = 0 \dots m$ ) the elastic solution for the system of deformations in the associated elastic problem, being  $i$  referred to the elastic scheme, i.e.  $D_i$  is not the same that  $D(t_i)$ . If  $E_c$  is the reference elastic modulus, assumed constant in time for simplicity, and keeping in mind relations (1) and (3), the expression that gives the development of deformations can be written:

$$\begin{aligned} D(t) &= D_0 E_c J(t, t_0) + \sum_{i=1}^m (D_i - D_{i-1}) E_c [J(t, t_0) - J(t_i, t_0)] H(t - t_i) \\ &= D_0 E_c J(t, t_0) + \sum_{i=1}^m \Delta D_i E_c [J(t, t_0) - J(t_i, t_0)] H(t - t_i) \end{aligned} \quad (5)$$

where Heaviside functions have the only aim to underline the chronological sequence of deformations development.

Let us assume now that after one single restraint addition at time  $t_1$ , there is a single temporary restraint removal at time  $t_R$ . The effect of restraint removal at time  $t > t_R$  is equivalent to the addition of a static action (load), opposite to the restraint reaction, variable in time if creep stress redistribution occurs. According to the 1<sup>st</sup> theorem, the generic internal force  $S(t)$  at time  $t > t_0^+$  can be found by the expression

$$S(t) = S_0 + (S_1 - S_0) \xi(t, t_0, t_1) H(t - t_1) + (S_2 - S_1) \frac{X(t)}{X_1} H(t - t_R) \quad (6)$$

being  $X(t) = X_0 + (X_1 - X_0) \xi(t, t_0, t_1)$ , while  $S_0, S_1$  are the internal forces and  $X_0, X_1$  the reaction values of the removed restraint, respectively in the initial scheme without the delayed restraint added (scheme 0) and in the scheme with the delayed restraint (scheme 1). In this case, Heaviside function  $H(t - t_R)$  is essential to mark the chronological sequence of scheme changes due to subsequent restraints addition and removal and because  $X(t)$  is nonzero for  $t < t_R$ .

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The last term of relation (6) takes into account the creep effects due to support removal and states that they depend on the reaction values in the elastic schemes and on the redistribution function. Dividing  $(S_2 - S_1)$  by  $X_1$ , in fact, one obtains the internal forces of an auxiliary elastic scheme with the delayed restraint added, the temporary one removed and a unit force applied, opposite to the support reaction  $X_1$ . By applying the principle of superposition in time, the internal force value found in this auxiliary system can be multiplied by the actual value of the reaction  $X(t)$ , that is a function of time  $t$ , due to the redistribution induced by creep effects. In this way the correct values of internal forces can be achieved.

Reaction  $X(t)$  can be seen in this case as a sum of two terms: one constant and another variable in time. The constant term coincides with the value  $X_0$  assumed by the reaction at the time  $t_0$  of load application on the initial elastic scheme. The variable term instead, depends on stress redistribution acting on the structure (Fig. 5). This last term depends on function  $\xi(t, t_0, t_1)$ , if there is stress redistribution. If no redistribution occurs, the stress system suddenly changes and the reaction value of the removed restraint turns immediately to zero. So stress pattern achieves that of the elastic scheme without the removed restraint with the load  $q$  applied.

Through the position

$$\eta(t) = \frac{X_0 + (X_1 - X_0)\xi(t, t_0, t_1)}{X_1} \quad (7)$$

the effect of the removed restraint can be taken into account and expression (6) can be written as:

$$S(t) = S_0 + (S_1 - S_0)\xi(t, t_0, t_1)H(t - t_1) + (S_2 - S_1)\eta(t)H(t - t_R) \quad (8)$$

The expression of  $\eta(t)$  shows that the effect of the removal depends on the previous history of changes.

In the case of a single restraint removal after a single addition, the law that gives the development of deformations can be written as:

$$D(t) = D_0 E_c J(t, t_0) + (D_1 - D_0) E_c [J(t, t_0) - J(t_1, t_0)] H(t - t_1) + (D_2 - D_1) E_c \left[ \eta(t_R) J(t, t_R) + \frac{X_1 - X_0}{X_1} [J(t, t_0) - J(t_R, t_0)] \right] H(t - t_R) \quad (9)$$

The last term of (9) is the deformation response due to the restraint removal and it is related to the stress response given by the last term of equation (8). Relation (9) is found through the integration of the strain responses due to the stress history at the generic time  $t$ , expressed by equation (8).

Now consider the variations of static scheme depending on single or multiple addition of  $m$  delayed restraints and  $n$  successive removals of single temporary ones with  $n \leq k + m$ . Expressions (4) and (8) can be generalized, writing

$$S(t) = S_0 + \sum_{i=1}^{m+n} \Delta S_i \gamma_i(t) H(t - t_i) \quad (10)$$

where

$$\gamma_i(t) = \xi(t, t_0, t_i) \quad \text{or} \quad \gamma_i(t) = \eta_i(t) = \frac{1}{X_{i-1}} \left[ X_0^i + \sum_{j=1}^{i-1} \Delta X_j^i \gamma_j(t) \right] \quad (11a, b)$$

if the  $i$ -th restraint has been respectively added (11a), or removed (11b), at time  $t_i$ .

By the positions (11), expression (10) gives the value of the generic internal force at time  $t$ , for every possible combination of  $m$  restraint additions and  $n$  restraint removals, each term of sum depending on redistribution functions and their combinations  $\eta_i(t)$ . Each scheme modification due to the introduction of delayed restraints at time  $t_i$ , is independent from the previous history, while each modification due to a temporary restraint removal depends on the whole previous history of changes. For this reason a general expression that gives the development of deformations cannot be written without considering the combination and the chronological sequence of subsequent additions and removals.

It is interesting to note that when  $n = k+m$ , subsequent removals lead to a statically determinate final scheme and creep redistribution effects on stress due to the system modifications vanish; after the last removal of temporary restraint, internal forces suddenly become those of the final elastic scheme with permanent loads applied. Moreover, if a delayed restraint that has been introduced is successively removed, the redistribution effect due to the previous changes vanishes also if the structure is still statically indeterminate.

Deformations history depends on the construction phases too, but the final strains pattern can be very different from that of the final elastic scheme, when it is different the sequence of additions and removals. In fact, while stress redistribution stops for a statically determinate scheme and internal forces reach those of the final elastic scheme loaded, deformations vary in time depending on the sequence followed to reach the final configuration.

Let us consider an example of a construction sequence of a beam, for the evaluation of creep effects on stresses and deformations when static scheme changes occur. The beam is composed by three segments, supported by two temporary piers between two definitive ones (Fig. 6). Only dead load  $q$  acts onto the structure. At the beginning of the deck construction, precast segments are simply supported by the definitive and the temporary piers (scheme 0, time  $t_0$ ). In the first phase, two segments are joined in situ (scheme 1, time  $t_1$ ). In a second phase the other two segments are joined (scheme 2, time  $t_2$ ). At this time the deck is continuous over four piers. When concrete joints are hardened enough, the two temporary piers are removed in two subsequent times (scheme 3, time  $t_3$  and scheme 4, time  $t_4$ ). The final scheme is statically determinate with  $m = 2$  and  $n = 2$ .

Expression (10) for this case becomes:

$$S(t) = S_0 + (S_1 - S_0)\xi(t, t_0, t_1)H(t - t_1) + (S_2 - S_1)\xi(t, t_0, t_2)H(t - t_2) \\ + (S_3 - S_2)\eta_3(t)H(t - t_3) + (S_4 - S_3)\eta_4(t)H(t - t_4) \quad (12)$$

where, through position (11b), one obtains

$$\eta_3(t) = \frac{1}{X_2^3} \left[ X_0^3 + (X_1^3 - X_0^3)\xi(t, t_0, t_1) + (X_2^3 - X_1^3)\xi(t, t_0, t_2) \right] \\ \eta_4(t) = \frac{1}{X_3^4} \left[ X_0^4 + (X_1^4 - X_0^4)\xi(t, t_0, t_1) + (X_2^4 - X_1^4)\xi(t, t_0, t_2) + (X_3^4 - X_2^4)\eta_3(t) \right] \quad (13)$$

The development of deformations can be written extending equation (9) to the case of two additions and two removals, obtaining the relation:

$$D(t) = D_0 E_c J(t, t_0) + (D_1 - D_0) E_c [J(t, t_0) - J(t_1, t_0)] H(t - t_1) \\ + (D_2 - D_1) E_c [J(t, t_0) - J(t_2, t_0)] H(t - t_2) + \\ + (D_3 - D_2) E_c \left[ \eta_3(t_3) J(t, t_3) + \frac{X_2^3 - X_0^3}{X_2^3} [J(t, t_0) - J(t_3, t_0)] \right] H(t - t_3) \\ + (D_4 - D_3) E_c \left[ \eta_4(t_4) J(t, t_4) + \left( \frac{X_3^4 - X_0^4}{X_3^4} - \frac{X_3^4 - X_2^4}{X_3^4} \frac{X_0^3}{X_2^3} \right) [J(t, t_0) - J(t_4, t_0)] \right] H(t - t_4) \quad (14)$$

Through relations (12) and (13) it is possible to show that at time  $t > t_4^+$ , stresses become instantaneously those of the final elastic scheme with the load  $q$  applied. In fact, at time  $t > t_4^+$ , expression (12) can be written as

$$S(t) = S_4 - [(S_1 - S_0)(1 - \xi_1) + (S_2 - S_1)(1 - \xi_2) \\ + (S_3 - S_2)(1 - \eta_3) + (S_4 - S_3)(1 - \eta_4)] \quad (15)$$

being  $\xi_i = \xi(t, t_0, t_i)$ . In the same way, relations (13) can be written as

$$1-\eta_3 = \frac{1}{X_2^3} \left[ (X_1^3 - X_0^3)(1-\xi_1) + (X_2^3 - X_1^3)(1-\xi_2) \right] \quad (16)$$

$$1-\eta_4 = \frac{1}{X_3^4} \left[ (X_1^4 - X_0^4)(1-\xi_1) + (X_2^4 - X_1^4)(1-\xi_2) + (X_3^4 - X_2^4)(1-\eta_3) \right]$$

and then, in expression (15) it is possible to separate the terms depending on  $(1-\xi_1)$  and  $(1-\xi_2)$  from the internal forces on the final elastic scheme ( $S_4$ ). From relations (15) and (16) one obtains:

$$S(t) = S_4 - \left[ (S_1 - S_0) + (S_3 - S_2) \frac{X_1^3 - X_0^3}{X_2^3} + (S_4 - S_3) \frac{X_1^4 - X_0^4 + (X_3^4 - X_2^4) \frac{X_1^3 - X_0^3}{X_2^3}}{X_3^4} \right] (1-\xi_1)$$

$$- \left[ (S_2 - S_1) + (S_3 - S_2) \frac{X_2^3 - X_1^3}{X_2^3} + (S_4 - S_3) \frac{X_2^4 - X_1^4 + (X_3^4 - X_2^4) \frac{X_2^3 - X_1^3}{X_2^3}}{X_3^4} \right] (1-\xi_2) \quad (17)$$

$$= S_4 - A(1-\xi_1) - B(1-\xi_2)$$

By inserting the reaction values of the removed restraint in (17), it can be verified that  $A = B = 0$ .

So it is possible to see that, being the scheme statically determinate, all the terms depending on redistribution functions  $\xi_i$  vanish and internal forces become instantaneously those of the final elastic scheme for  $t > t_{m+n}^+$ .

Figure 7 shows bending moment in time in the midspan section of the central span. In this way it is possible to follow the values of bending moment given by expression (12) in time, from  $t > t_0^+$ . The chosen times in the sequential construction are:  $t_0 = 28$  days,  $t_1 = 60$  days,  $t_2 = 90$  days,  $t_3 = 120$  days,  $t_4 = 150$  days. Creep curves are those given by CEB Model Code 90,<sup>5,11</sup> for  $RH=80\%$ ,  $h=2A_c/u=400$  mm,  $f_{ck} = 40$  MPa.

If the removal of the two temporary piers is contemporary, times  $t_3$  and  $t_4$  coincide and the effect of removals leads instantaneously to the final elastic scheme.

Figure 8 shows the development of the midspan displacement  $D(t)$ , dimensionless with respect to the value of elastic scheme  $D_4$ , obtained by using the relation (14).

It is so shown how internal forces become instantaneously those of the final elastic scheme, while deformations vary in time and their values depend on the sequence followed and on the values chosen for the times  $t_i$ .

### CASE-STUDY

A case study of the construction of an arch portal bridge is here presented. It is a segmental prestressed bridge built by the cantilever method assembling precast segments. Construction phases are the following (Fig. 9):

- Foundations and abutments are built in the first stage. Two temporary piers are located in their position to support slanting precast piers. Slanting piers are placed between the top of temporary supports and the abutment feet.
- Deck construction begins from the top of the slanting pier and segments are supported by the temporary pier.
- The last segments of lateral spans have to be fixed to the abutments through vertical prestressing cables, to balance the construction of the central span, when other segments are added.
- When the two central cantilevers have been completed, then a closure by a cast-in-situ segment is made and the deck is a continuous beam with four supports (two on the abutments and two over the temporary piers). At the time of closure a delayed restraint in the middle of the central span has been added.
- Finally supports are removed and arch behaviour begins, due to the axial force activated by permanent loads in the structure with slanting piers.

After the temporary piers have been removed, the final scheme can be reached or not, because other restraints can be still added or removed. In this study, the following construction sequence has been examined: closure in the midspan is made with a double internal restraint (relative vertical displacement and relative bending rotation are fixed) and

supports over the abutments are removed. The final scheme is an arch-portal bridge with two hinges and two lateral cantilever spans: it is a statically indeterminate structure with one redundancy. Elastic reference schemes are shown in figure 10.

For a first evaluation of creep effects only the dead load  $q$  is applied and the construction of the two half-bridges is contemporary. Prestressing effects are neglected only with the purpose to underline stress pattern modifications due to creep in the structure subjected to dead load. The load is applied in only one step (in the hypothesis of fast construction of the whole deck with precast segments of the same concrete properties).

The times of construction sequence are:

- $t_0$  : load application – cantilever behaviour;
  - $t_1$  : restraint addition in the midspan section – continuous beam behaviour;
  - $t_2$  : temporary piers removal – arch-portal behaviour with supported lateral spans;
  - $t_3$  : abutments supports removal – arch-portal behaviour with cantilever lateral spans.
- By applying expressions (10) and (11), at the generic time  $t$  one obtains:

$$S(t) = S_0 + (S_1 - S_0)\xi(t, t_0, t_1)H(t - t_1) + (S_2 - S_1)\eta_2(t)H(t - t_2) + (S_3 - S_2)\eta_3(t)H(t - t_3) \quad (18)$$

that is the expression of the generic internal force at time  $t$ .

Results are shown for  $t_0 = 28$  days,  $t_1 = 90$  days,  $t_2 = 120$  days,  $t_3 = 365$  days with the redistribution functions given by CEB Model Code 90,<sup>5,11</sup> for RH=80%,  $h=2A_c/u=400$  mm,  $f_{ck} = 40$  MPa.

In figure 11 the diagrams of internal forces in time are shown for the two hinges arch-portal bridge. Bending moments are shown in the midspan and in the deck section over the slanting pier, while axial force is shown in the midspan section. Internal forces  $S(t)$  are given in terms of dimensionless values with respect to the corresponding values in the final elastic scheme ( $S_3$ ), giving the percentage of partial acquisition of modified system stresses at every time.

It is important to note that the transient phase with stress redistribution, due to the delayed restraint added, continues also after temporary piers and abutment supports have been removed, because the final structure is still statically indeterminate.

In this structure a fundamental role is played by the axial force, because after temporary piers removal, the change of the static scheme leads from the continuous beam behaviour to the arch behaviour. This increase of axial force is the most important effect of static scheme modification, so even if stress redistribution due to creep is important, the main effect is the total change of structural behaviour with the introduction of axial force into the deck in the central span.

Figure 12 shows the development of midspan displacement, obtained through the following relation:

$$D(t) = D_0 E_c J(t, t_0) + (D_1 - D_0) E_c [J(t, t_0) - J(t_1, t_0)] H(t - t_1) + (D_2 - D_1) E_c \left[ \eta_2(t_2) J(t, t_2) + \frac{X_1^2 - X_0^2}{X_1^2} [J(t, t_0) - J(t_2, t_0)] \right] H(t - t_2) + (D_3 - D_2) E_c \left[ \eta_3(t_3) J(t, t_3) + \left( \frac{X_2^3 - X_0^3}{X_2^3} - \frac{X_2^3 - X_1^3}{X_2^3} \frac{X_0^2}{X_1^2} \right) [J(t, t_0) - J(t_3, t_0)] \right] H(t - t_3) \quad (19)$$

The midspan displacement shown in Fig. 12 is due only to the bridge dead load; of course, for a complete evaluation of the final deformed shape due to permanent loads, the effect of prestressing should be also considered.

## CONCLUSIONS

A study about creep effects in structures with temporary supports removal has been presented in the hypothesis of linear viscoelasticity and homogeneous structures. Evaluations about creep effects in structures built through a construction sequence with delayed restraints added and temporary piers removed, have been presented both for the stress and strain systems. While each scheme modification due to the introduction of delayed restraints is independent from the previous history, each modification due to a temporary restraint removal depends on the whole previous history of changes.

When subsequent removals, after restraint additions, lead to a statically determinate final scheme, creep redistribution effects on the stress system, due to the static scheme changes, vanish. After the last removal of temporary restraint, internal forces suddenly become those of the final elastic scheme with permanent loads applied. Moreover, if a delayed restraint that has been introduced is successively removed, the redistribution effect due to the previous changes vanishes also if the structure is still statically indeterminate.

An application to an arch-portal bridge has been shown, taking into account creep redistribution of stress system and temporary support removals. The development of deformations has been also presented, being an useful and important issue for construction works.

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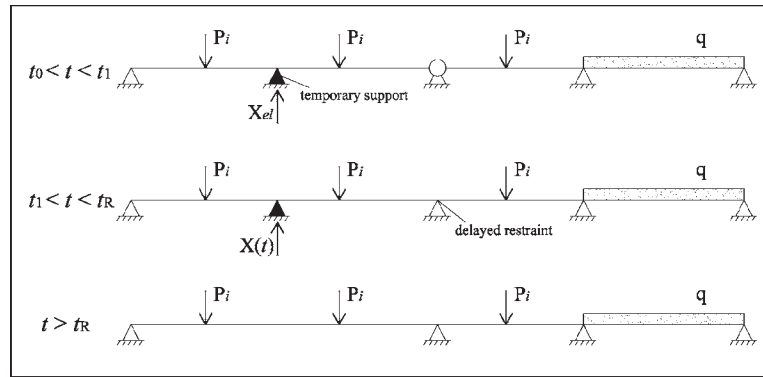


Fig. 1—Case a. Static actions with  $t_R > t_1$ .

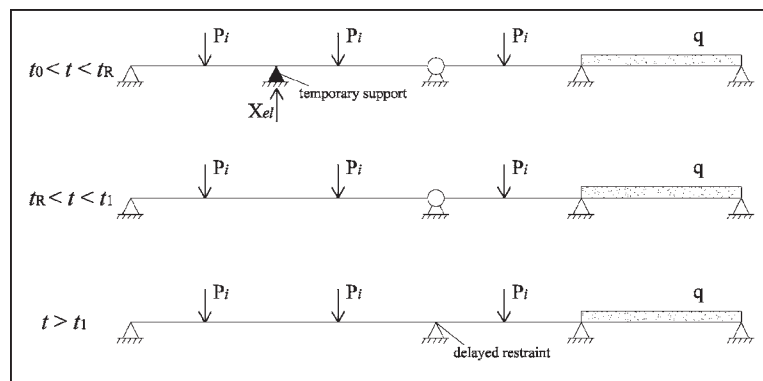


Fig. 2—Case b. Static actions with  $t_R > t_1$ .

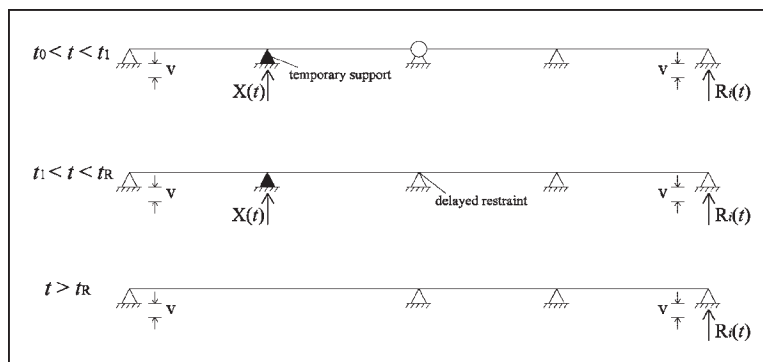


Fig. 3—Case c. Geometric actions with  $t_1 < t_R$ .

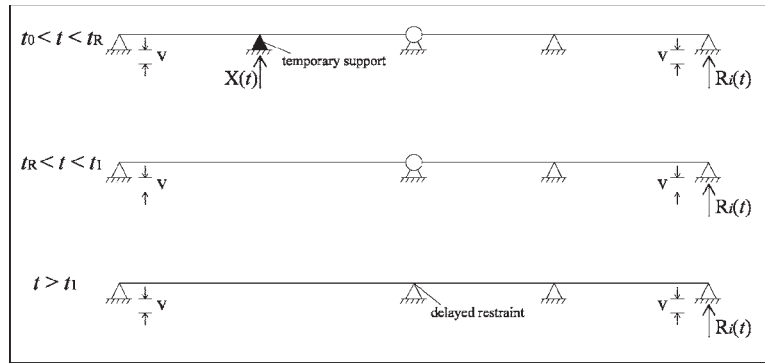


Fig. 4—Case d. Geometric actions with  $t_R > t_1$ .

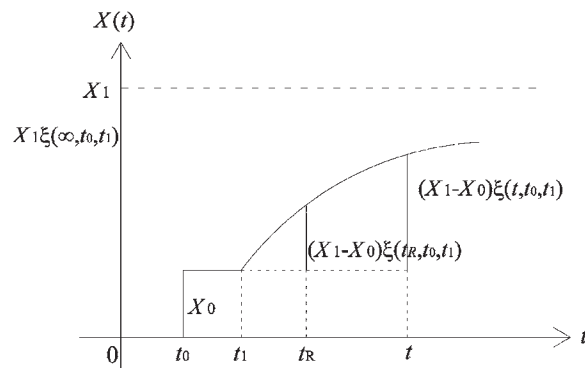


Fig. 5—Reaction  $X(t)$ .

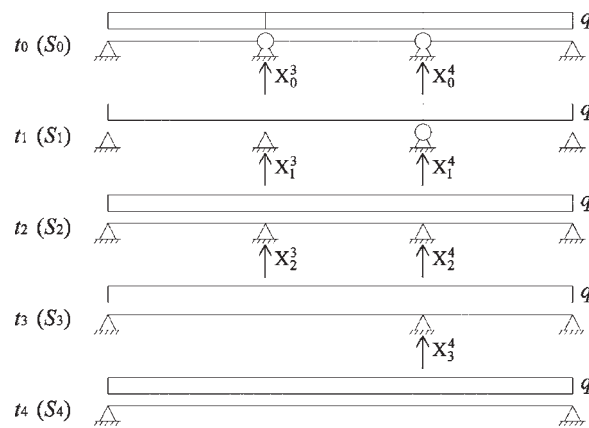


Fig. 6—Example of a beam construction sequence.

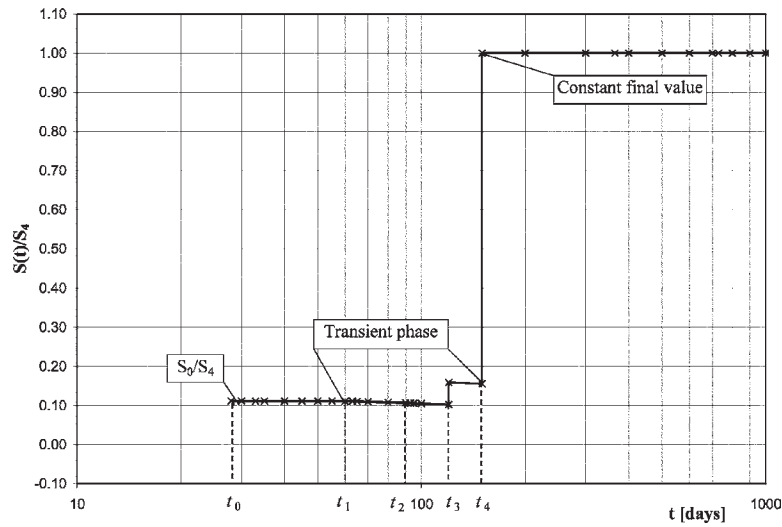


Fig. 7—Bending moment diagram in time (midspan section).

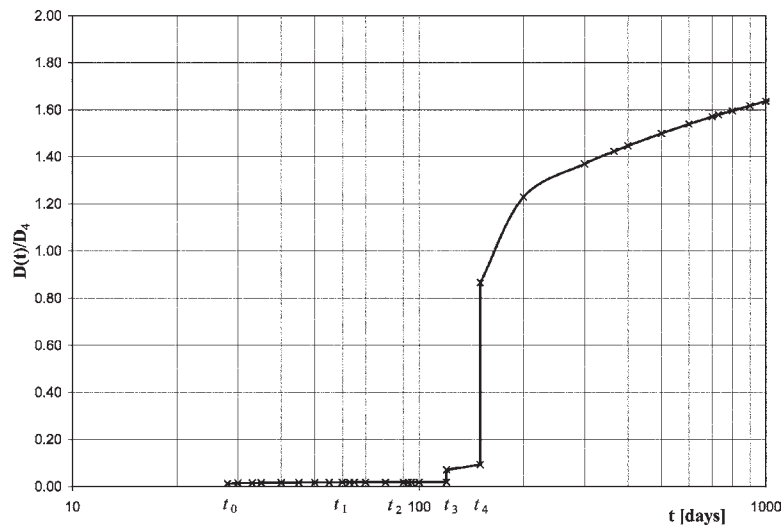


Fig. 8—Midspan displacement diagram in time.

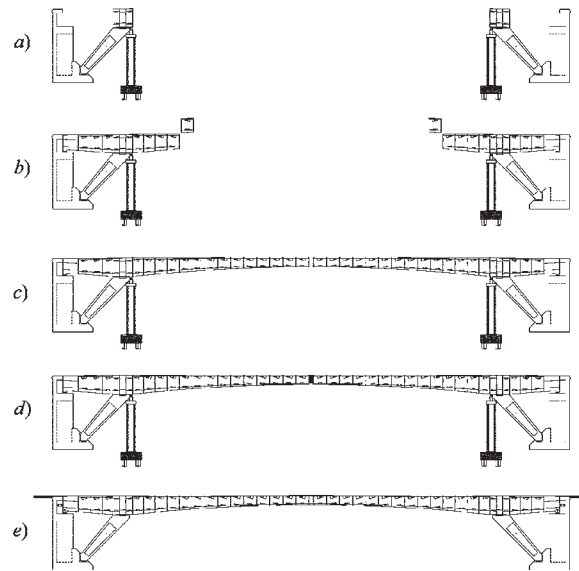


Fig. 9—Construction phases of the arch-portal bridge.

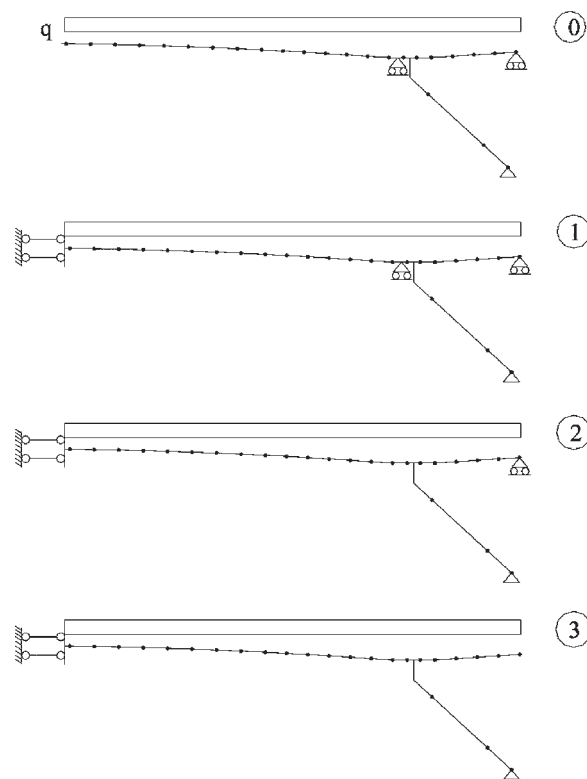


Fig. 10—Static schemes.