

COMPARISON OF LIVE LOADS USED IN HIGHWAY BRIDGE DESIGN IN NORTH AMERICA
WITH THOSE USED IN WESTERN EUROPE

By ALFIO SENI

SYNOPSIS

The author first discusses the advantages resulting from having all Live Load Specifications approved and enacted by a State Administration, prior to their publication.

The main provisions for live loads in United States, Canada, Great Britain, and France are then defined.

These specifications are afterwards compared: first a quantitative comparison of moments and reactions for different cases, the results being shown in tables and charts.

There follows a qualitative comparison of these specifications in regard of their simplicity and ease of application. In conclusion, the author suggests that North American specifications for Highway Bridge Loadings should be revised.

Keywords: bending moments; bridge decks; bridges (structures); girder bridges; girders; highway bridges; moving loads; standards; structural design.

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2- Presentation of Loadings used in some Countries of North America and Western Europe

In the following, we shall outline the main provisions of the specifications on live loads used in North America (USA and Canada) and Western Europe (Great Britain and France).

A) United States

In this country, the "Standard Specifications for Highway Bridges 1965" of the AASHO are generally used. It must be mentioned that the loadings, there provided, have not been changed since 1944.

This code provides more classes of loadings, the heaviest being designated by the indication HS20-44. The Code provides also another system of loads: The H loads (truck without trailers) the corresponding heavier loading being H20-44.

For the design of highway bridges, for each class of loading, two types of loadings are considered: the truck loading and the lane loading, and the one will be adopted which leads to maximum stresses (Fig. 1).

For the truck loading, on each traffic lane, one single truck is considered on the whole length of the lane. This is very important for the further comparison.

The lane loading is supposed to be the equivalent of a train of H15 trucks, with one H20 truck, the trucks being spaced at 30 ft. one from the other. It will be

shown thereafter that this loading is much lighter than the European loadings; one of the causes is evidently that the truck loads which served to establish the equivalent are much lighter than HS20-44 and at long distances one from the other.

The AASHO code provides that all the live loads shall be increased by an impact factor. It also provides reduction factors for members affected by loading simultaneously more than two traffic lanes.

B) Canada

The specifications of the Canadian Standards Association (CSA) designated as CSA S6-1966, Design of Highway Bridges, are generally used in Canada. The CSA, like the AASHO, is a non-government organization, and the issued standards are not compulsory for the designers in Canada. The live loads for highway bridges, according to the CSA specifications, are exactly the same as those of the AASHO.

C) Great Britain

The loads are designated by the appellation "Standard Highway Loadings" and are established by the British Standards Specification B.S. 153: Part 3A: 1954 (amended in 1959, 1961 and 1966). This loading is issued by the British Standard Institution and was enacted by the Ministry of Transport of Great Britain (Memorandum No. 771 of 1961) so that it is obligatory

3. Comparison of the Load Specifications

A) Quantitative Comparison

For comparing the specifications, from the quantitative point of view, seeing that these are basically different, moment and reactions (shears) were calculated for different elements of various spans.

In table 2 are indicated the values of bending moments for slabs with simple spans between 7 ft. and 12 ft. The values are represented graphically in the chart Fig. 4. The values include the impact, and for each span three values are given, according to AASHO & CSA loading, British HA normal loading, and French A loading.

In table 3, and in the chart Fig. 5 are given the 3 values for bending moments in simple spans of 50 ft. to 150 ft.

In table 4 and chart Fig. 6 are given the maximum positive bending moments in two equal continuous spans of 40 ft. to 120 ft.

The values of end reactions in simple spans of 50 ft. to 150 ft. are given in table 5 and in chart Fig. 7.

In table 6 and in chart Fig. 8 are given the values of the reactions at middle support of two equal continuous spans of 40 ft. to 120 ft.

All the values include the impact factor, as provided by each of the 3 specifications.

If we examine the values given in the tables and the charts above shown, the following conclusions may be drawn:

- a) The values found by applying the British and the French specifications are generally very close to each other, although the loading systems used are quite different.
- b) These values are, in all cases, much higher than those found by applying the AASHO and CSA specifications. The difference is particularly large for girders, and increases with the span.

Regarding the slabs, the differences vary between 25% and 40%.

For simple span beams, the difference concerning the moments varies from 32% to 110%, and concerning the reactions from 51% to 148%.

For continuous two span girders the differences of positive moments vary from 20% to 93%, and of reactions on the middle support from 84% to 120%.

- c) What conclusion should be drawn from this comparison? Are the vehicles used in Europe so much heavier than those used in North America? This is difficult to believe, because all these countries are highly industrialized, on both sides of the Atlantic Ocean, and the weight of the heavy vehicles

which are using the highways cannot be very different.

In other respects, we have seen, in the first part of this paper, which are the load trains used in the specifications, and it is easy to be seen that the weights assumed are much larger in the West European Countries, even for the normal traffic.

It is evident that a Code cannot be provided for the actual loadings of the vehicles in circulation, which are very variable as well regarding the loads, as the spacing of the axles. It is necessary to adopt a conventional loading which must be on the safe side for any span and for any type of actual load.

It follows therefore that, if we admit that the actual loads cannot be very different, either the conventional european loadings are too large, or the American ones are too small. The fact that the results obtained by applying the British and the French loadings, are very near, though the load systems are quite different, cannot be a mere coincidence. That seems rather to demonstrate that the vehicles which served as base for the establishment of these loadings and their arrangement along the bridge, are very similar.

This leads to the conclusion that the comparison shows rather that the HS20 loading is too weak.

This weakness is due not so much to the weight of the HS20 truck itself, but rather to the density of the loads. For the truck loading a single vehicle is allowed on the whole length of the bridge, while the French code provides two heavy trucks closely spaced and the British Code 3 heavy trucks, plus lighter trucks. As for the lane loading, as mentioned before, it is equivalent to a train of light H trucks, only one of them being a H20 truck, and the spacing between the trucks very large.

For this reason, the lane loading has a very reduced value: only 640 lbs./ft., compared with 1,760 lbs./ft. given by the British Code for spans between 20 ft. and 75 ft. impact excluded. The lane loading being too small, it is the truck loading which governs for simple spans up to 140 ft. which are consequently designed for one single truck on a lane, though there is room enough for 3 consecutive HS20 trucks.

If we consider, on a 140 ft. span, 3 consecutive HS20 trucks spaced at 20 ft. intervals, we find that the bending moment is 4,950 kft. including the impact, instead of 2,677 kft. according to the AASHO Code. If we compare this value with those given by European Codes, we find that it is only

5% below the value given by the British Code. (Fig. 9).

It is therefore clear that the low results found by applying AASHO Code are mainly due to the fact that the lane loading is not equivalent to a train of HS20 trucks, but has a much smaller value and therefore the single HS20 truck governs for spans up to 140 ft.

It appears therefore to be very desirable that AASHO undertake an inquiry for establishing the value of the maximum actual loads of the vehicles presently in use in U.S.A., and derive from that a new type of conventional loading, which cover all these loads.

But, in establishing this new loading, there must be considered, not only the present loads, but also a provision for the future increase of the loads.

It must not be forgotten that the bridges and particularly the concrete bridges, are structures made to last a long time. In the opinion of the author, design should provide for 100 years and make the provisions for probable load increases during that period.

This question is very important, particularly concerning concrete bridges. Steel bridges can easily enough be strengthened if they are no longer suffi-

cient for the increased loads, but this operation is much more difficult if it concerns concrete bridges. The strengthening involves very difficult problems, which means that often it is preferable to build a new bridge, instead of strengthening the old one.

After having established the weight of vehicles to be taken in account, it must be decided by an inquiry on site, at what intervals the consecutive trucks have to be placed on a traffic lane.

In conclusion, it appears from the comparison made on the values of the loadings, that those provided by the North American Codes seem to be insufficient and that one has to verify and revise them so to be sure that they correspond to the present loads with a sufficient margin for their future increase.

2- Qualitative Comparison

If we now compare the qualitative structure of the loading systems used in the countries above mentioned, we find out the following:

In U.S.A. and Canada, for the live loads, two types of loadings are prescribed: the first is composed by concentrated loads of two sorts H loadings and HS loadings; the second consists of an uniform load and a concentrated load which is different for moments and for shears.

In France also, there are 2 types of loadings: one being an uniform loading, the other a train of trucks.

In Great Britain, it is normally one single loading type used, consisting of a uniform load, plus a concentrated load. The abnormal load is used only in special cases, and only for checking the bridge design.

Comparing the 3 loadings systems, it appears that the HA loading used in Great Britain is the easiest to apply.

First, there is only one load system, while in the other specifications there are at least two systems who have to be applied to determine the one producing the higher stresses. Secondly, the British loading is simpler, consisting of a uniform load and a single concentrated load, instead of a train of concentrated loads as provided by the AASHO truck loading and the system B loading of the French specifications. And, as everybody knows, it is much simpler and easier to work with uniform than with concentrated loads.

Therefore, it is desirable that, if a revision should be made in the loadings provided by AASHO and CSA code, this revision should refer not only to the values of the loads but also to the type of loading.

It would be desirable to find a variable uniform load which be equivalent to a conveniently spaced train of heavy trucks. To this should be added a concentrated load to compensate

for the additional weight of an overloaded truck, located at the critical point.

This variable uniform load should satisfy more requirements:

- a) It has to be conservative for all the possible types of present loads with sufficient provisions for the future.
- b) It may be applied to all the members of the bridge: slabs, main girders, floorbeams, stringers, arches, etc., and for any material used.
- c) It has to be applicable to any span, the variation of the load taking in account all the possible locations of the loads along the bridge.
- d) It must include the impact factor.

Once the new loading established, it should be recommended that it be approved and enacted by the government, so to become an obligatory law in all the respective country.

TABLE 1. HIGHWAY LOADING. TYPE HA

Equivalent U.D.L. to be used in conjunction with the knife edge load.

Loaded length, ft	U.D.L. for beams per linear foot of lane	U.D.L. for longitudinal slabs per linear foot of lane	U.D.L. for transverse slabs and cross girders per linear foot of lane	Loaded length, ft	U.D.L. for beams per linear foot of lane	U.D.L. for longitudinal slabs per linear foot of lane	U.D.L. for transverse slabs and cross girders per linear foot of lane
3	24 200	24 200	22 700	12	4 870	3 250	2 600
4	17 000	17 000	11 800	13	4 540	2 950	2 400
5	12 250	12 250	7 700	14	4 210	2 700	2 300
6	9 660	8 850	5 800	15	3 880	2 500	2 200
7	8 280	6 550	4 600	16	3 550	2 400	2 200
8	7 250	5 200	3 900	17	3 220	2 300	2 200
9	6 440	4 520	3 400	18	2 880	2 250	2 200
10	5 800	4 000	3 100	19	2 540	2 200	2 200
11	5 200	3 600	2 800	20-75	2 200	2 200	2 200

Loaded length, ft	Load, lb/lin. ft	Loaded length, ft	Load, lb/lin. ft	Loaded length, ft	Load, lb/lin. ft	Loaded length, ft	Load, lb/lin. ft	Loaded length, ft	Load, lb/lin. ft
80	2 160	175	1 530	270	1 210	430	1 005	1 200	660
85	2 126	180	1 500	275	1 200	440	1 000	1 300	630
90	2 080	185	1 480	280	1 190	450	990	1 400	600
95	2 040	190	1 460	285	1 175	460	985	1 500	580
100	2 000	195	1 440	290	1 165	470	975	1 600	560
105	1 960	200	1 420	295	1 155	480	970	1 700	540
110	1 920	205	1 400	300	1 145	490	965	1 800	520
115	1 880	210	1 380	310	1 130	500	960	1 900	510
120	1 860	215	1 360	320	1 115	550	935	2 000	490
125	1 830	220	1 350	330	1 105	600	910	2 100	480
130	1 800	225	1 335	340	1 090	650	880	2 200	470
135	1 770	230	1 320	350	1 080	700	860	2 300	460
140	1 740	235	1 305	360	1 070	750	835	2 400	450
145	1 710	240	1 290	370	1 060	800	810	2 500	440
150	1 680	245	1 280	380	1 050	850	790	2 600	430
155	1 650	250	1 265	390	1 040	900	770	2 700	420
160	1 620	255	1 250	400	1 030	950	750	2 800	410
165	1 590	260	1 240	410	1 020	1 000	730	2 900	410
170	1 560	265	1 225	420	1 015	1 100	690	3 000	400

TABLE 2 MOMENTS IN SIMPLE SPAN SLABS (IMPACT INCLUDED)

Span ft.	AASHO & CSA k.ft.	Great Britain k.ft.	France k.ft.
7-0	5.85	7.55	7.44
8-0	6.50	8.52	8.25
9-0	7.15	9.52	9.05
10-0	7.80	10.62	9.75
11-0	8.45	11.70	10.35
12-0	9.10	12.78	10.95

TABLE 3MOMENTS, SIMPLE SPAN, ONE LANE (IMPACT INCLUDED)

<u>Span ft.</u>	<u>AASHO & CSA k.ft.</u>	<u>Great Britain k.ft.</u>	<u>France k.ft.</u>
50	808	1,025	1,070
60	1,024	1,395	1,490
70	1,240	1,821	1,972
80	1,450	2,268	2,480
90	1,660	2,714	3,020
100	1,865	3,175	3,570
110	2,066	3,647	4,110
120	2,270	4,158	4,640
130	2,470	4,680	5,140
140	2,677	5,208	5,620
150	2,826	5,738	6,100

TABLE 4

POSITIVE MOMENTS, ONE LANE, TWO EQUAL CONTINUOUS SPANS
(IMPACT INCLUDED)

<u>Span ft.</u>	<u>AASHO & CSA k.ft.</u>	<u>Great Britain k.ft.</u>	<u>France k.ft.</u>
40	465	561	535
50	645	805	817
60	820	1,092	1,142
70	995	1,421	1,510
80	1,165	1,770	1,900
90	1,340	2,115	2,305
100	1,510	2,473	2,725
110	1,680	2,838	3,150
120	1,845	3,234	3,560

TABLE 5END REACTIONS, SIMPLE SPAN, ONE LANE (IMPACT INCLUDED)

<u>Span ft.</u>	<u>AASHO & CSA k.</u>	<u>Great Britain k.</u>	<u>France k.</u>
50	75.2	82.0	85.5
60	77.2	93.0	99.6
70	78.2	104.0	112.6
80	79.4	113.4	124.4
90	79.7	120.6	134.0
100	80.0	127.0	142.5
110	80.1	132.6	149.5
120	80.2	138.6	154.5
130	81.0	144.0	156.5
140	84.0	148.8	159.5
150	87.8	153.0	163.0

TABLE 6

MAXIMUM REACTION AT MIDDLE SUPPORT, ONE LANE ,
TWO EQUAL CONTINUOUS SPANS (IMPACT INCLUDED)

<u>Span ft.</u>	<u>AASHO & CSA k.</u>	<u>Great Britain k.</u>	<u>France k.</u>
40	84.5	137.0	155.5
50	87.2	164.5	178.0
60	89.0	192.0	193.5
70	97.5	219.5	199.5
80	106.0	243.0	206.0
90	114.5	261.0	209.0
100	122.0	277.0	210.5
110	130.5	291.0	212.0
120	139.0	306.0	213.0

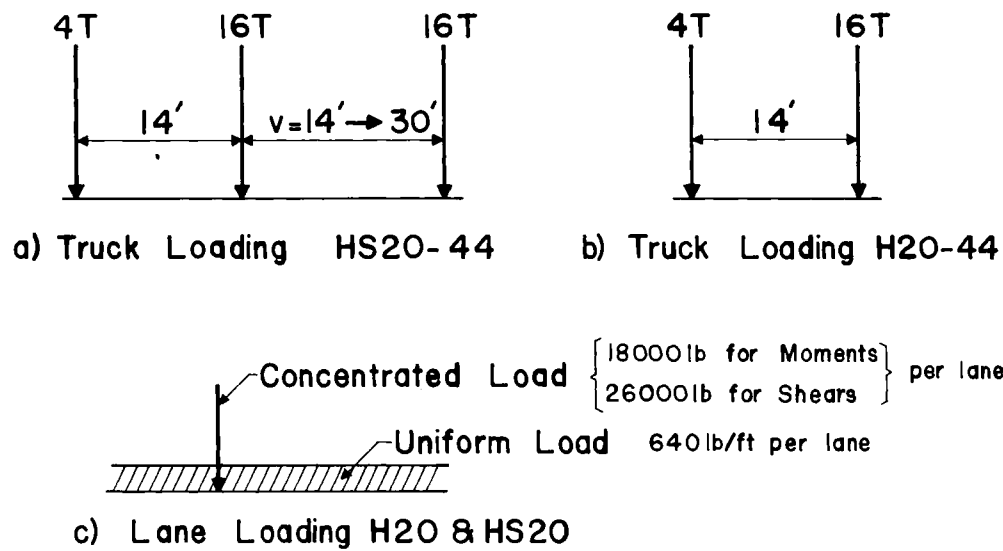


FIG. 1

AASHO & CSA LOADINGS

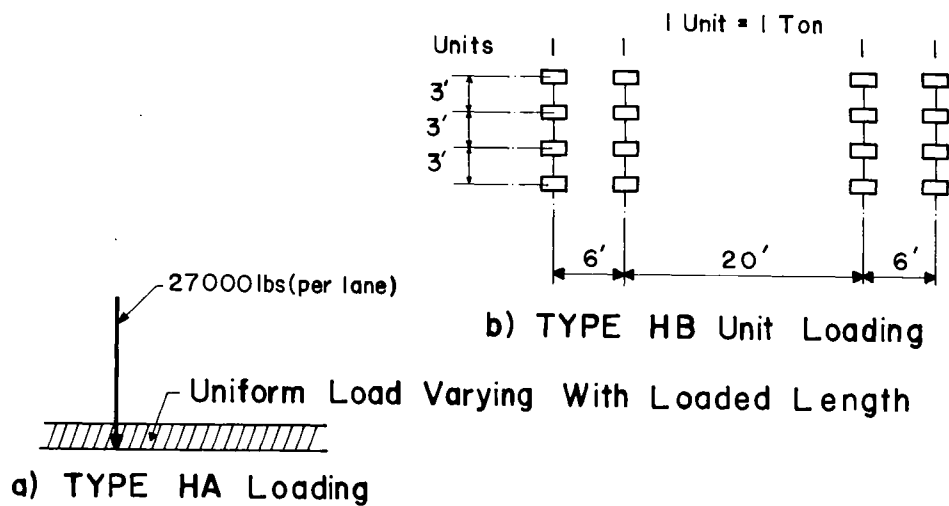


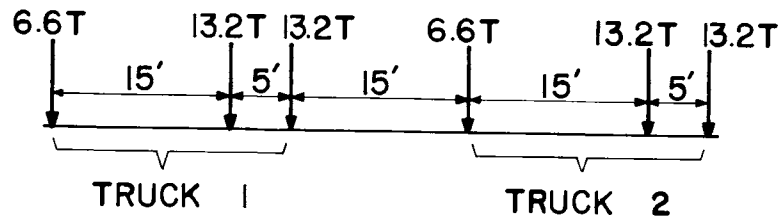
FIG. 2

BRITISH LOADINGS

$$A = 350 + \frac{320,000,000}{L^3 + 60L^2 + 225,000}$$

L (meters), A (kg)

a) A Loading



b) B_c Loading

FIG. 3

FRENCH LOADINGS

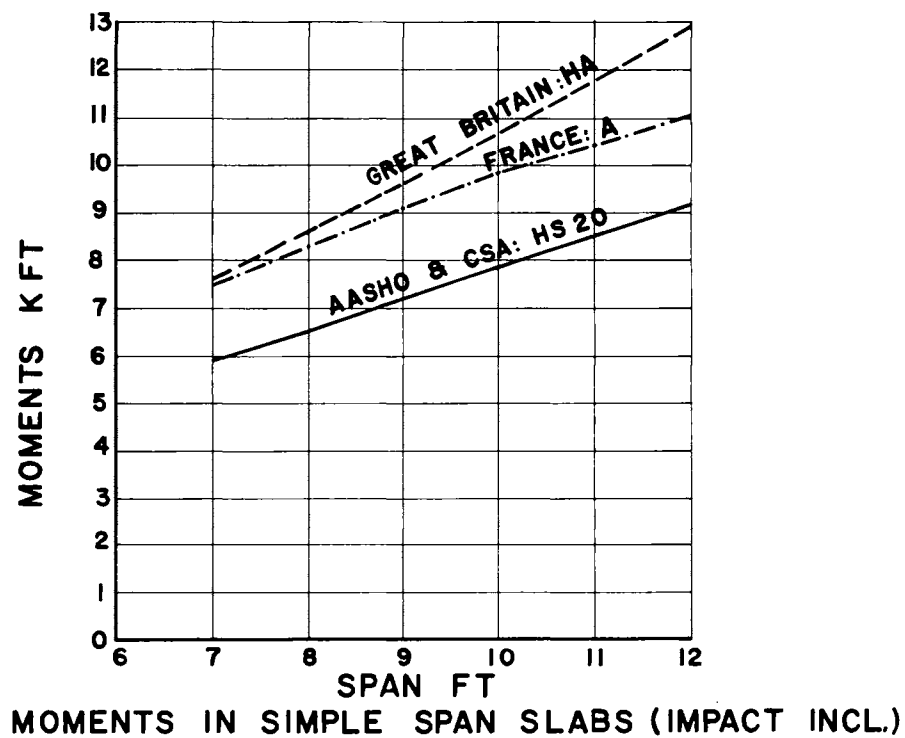
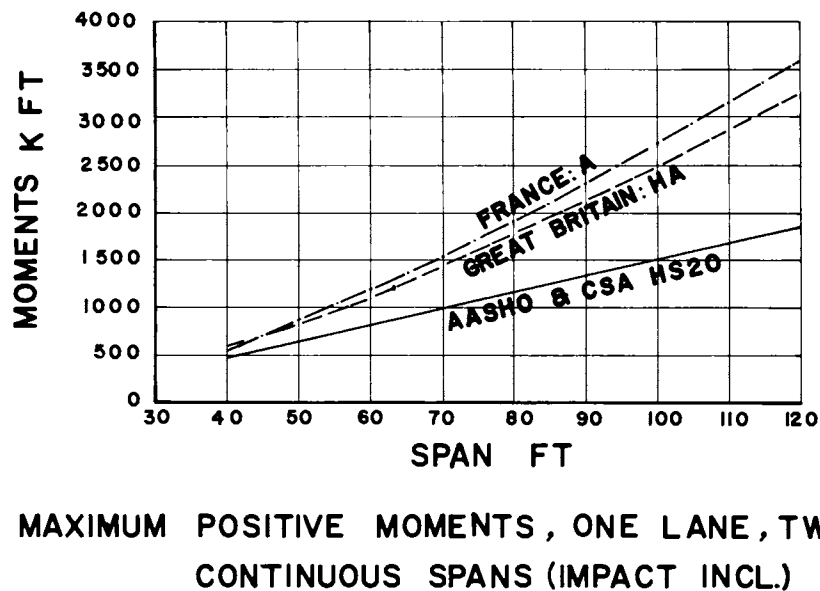
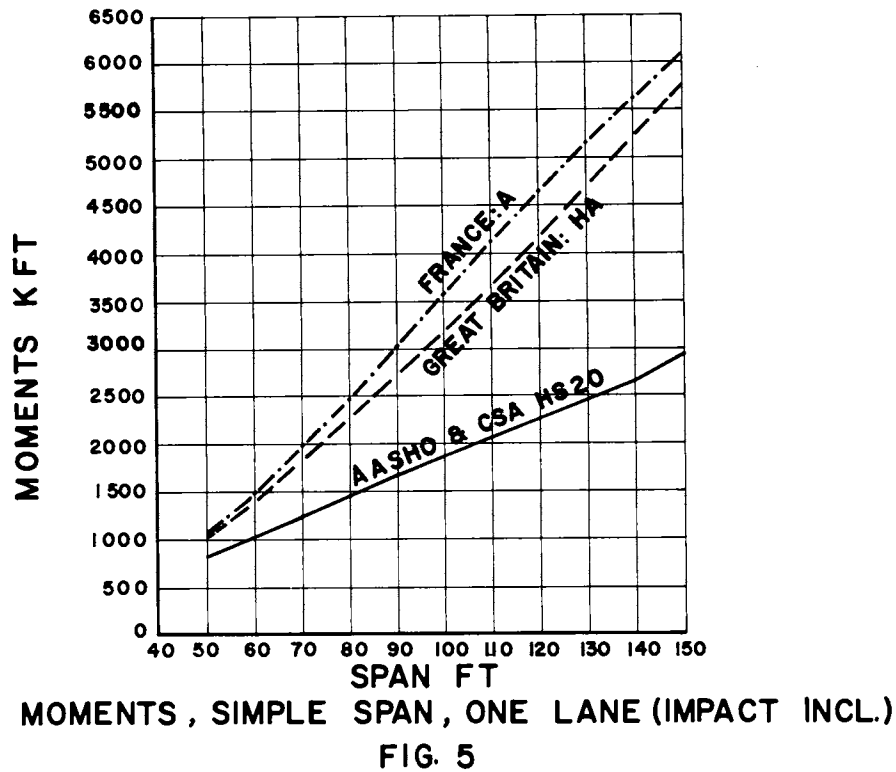
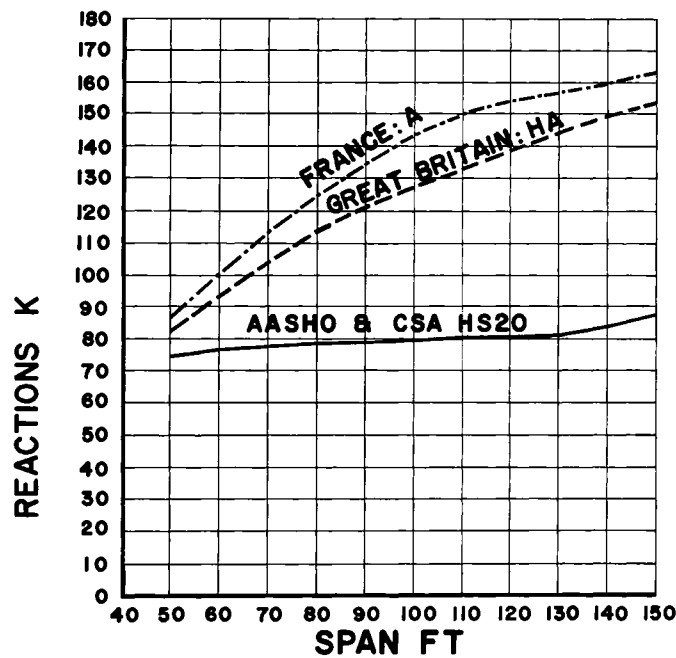


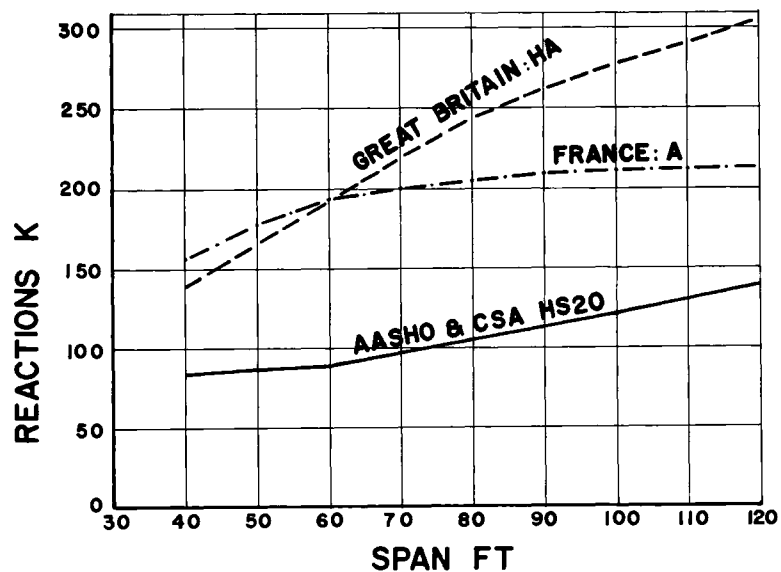
FIG. 4





END REACTIONS - SIMPLE SPAN, ONE LANE (IMPACT INCL.)

FIG. 7



MAXIMUM REACTIONS AT MIDDLE SUPPORT, ONE LANE,
TWO EQUAL CONTINUOUS SPANS (IMPACT INCLUDED)

FIG. 8

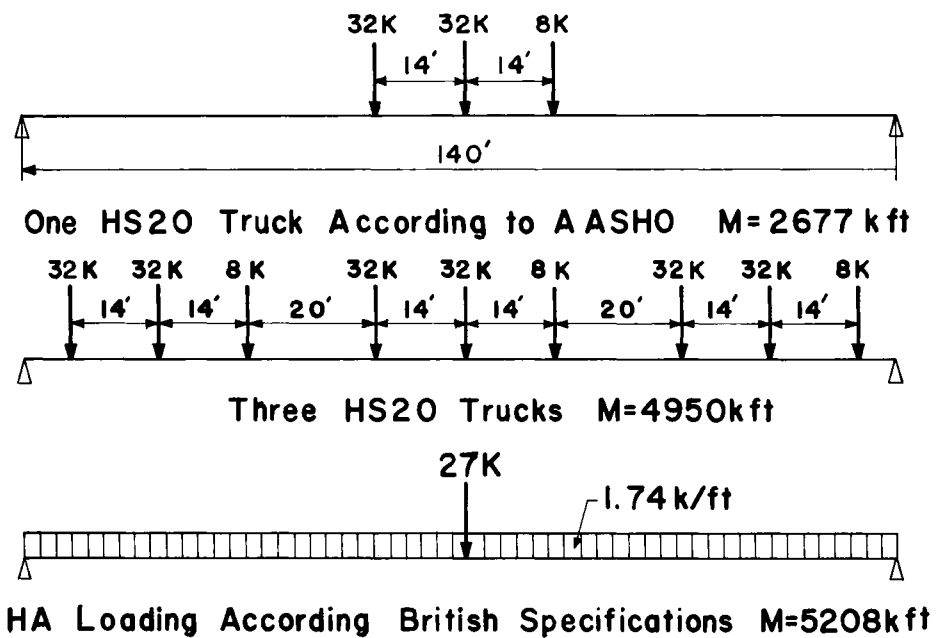


FIG. 9

DISCUSSION

By F. SAWKO

Professor, Department of Civil Engineering, University of Liverpool

I agree with Mr. Seni's comment that the abnormal (HB) vehicle complicates design of bridges in Great Britain. In my opinion however, this complication is really essential since in practice many such vehicles at some time or other during the life of every bridge pass over them and occasionally the weight of the vehicle is very much greater than the 180 tons. Recently I have had the task of assessing the carrying capacity of two structures which have been completed in 1967 under the passage of a 400 ton vehicle train. The two bridges proved incapable of carrying this heavy loading and I would like to ask Mr. Seni whether in his opinion the maximum weight of vehicles crossing the bridge should be restricted? Industry finds it convenient and economic to transfer very heavy loads and there would probably be repercussions if any restriction were to be placed on the maximum load.

Referring again to the passage of very heavy abnormal vehicles over bridges, would it not be desirable in Mr. Seni's opinion to provide a set of influence surfaces which would form a permanent record of the structural action of the bridge during the design stage? Such a record would enable a rapid assessment of the carrying capacity of bridges without the necessity of a complete reanalysis of the structure.

By KARL G. TAMBERG

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In his paper Professor Seni discusses the important question of the heavy vehicle loading of highway bridges. The writer would like to make the following supplemental comments:

1) It would appear advantageous to have select governmental organizations specify live loads which should be used in the design of bridge structures, since roads and bridges together constitute a public highway facility, and the licensing of heavy vehicles which are permitted to travel on the facility should not be considered separately from the problem of specifying corresponding design loads. In other words, there is a need to synthesize practices followed in the licensing of heavy vehicles and bridge design. It seems logical that governmental organizations should be directly responsible for policies pursued in regulating the operation of heavy vehicles by specifying corresponding design loads. The need for the overall planning of the highway transportation complex has become evident (1,2). On the other hand, it is believed that a substantial amount of research into such areas as heavy vehicle operations, the design and performance of pavements and bridges, political jurisdictions, and local economic conditions awaits completion before it will become feasible, in such countries as the United States and Canada, to consider the foundation of a central agency which is solely in charge of controlling the operation of heavy vehicles and the specification of corresponding live loads. A concrete step towards intelligently controlling heavy vehicle operations, which also provides some information for fixing design loads, has been taken recently in the United States by the specification of maximum desirable dimensions and weights of vehicles operated on the Federal-aid systems (1). In the Province of Ontario the Department of Highways provides the

co-ordinating force in bridge design, and virtually no bridges are being designed for less than an HS20-44 loading in an attempt to render them capable of supporting modern highway traffic.

2) Through a comparison of design loads employed in the United States, Canada, Great Britain and France, Professor Seni points out that design loads in Europe are substantially heavier than those generally used in North America, and states that it is difficult to believe that such large differences in loading are warranted. The writer would agree that bridge design loadings used and associated bridge design practices followed in North America are in need of thorough study, but would at the same time also like to point out that failures of bridges which have occurred can virtually never be attributed to their overloading. Could Professor Seni quote an example where a soundly designed and constructed HS20-44 bridge has collapsed or has been excessively damaged due to overloading? The writer is not aware of such a case in Canada. Furthermore, the operation of heavy vehicles can be controlled through legislation (3), and it is questionable whether the unrestricted operation of extremely heavy vehicles is economically justifiable. Preliminary estimates carried out in the Province of Ontario indicate that the costs of damage to the public may outweigh by a considerable margin the cost of benefits to be reaped from the unrestricted operation of heavy vehicles which would require the employment of a design loading much in excess of an HS20-44 loading. Pending further study results, the

writer cannot therefore agree at this time, on a universal basis, with the author that "the HS20 loading is too weak".

3) Nevertheless, it must be emphasized that the validity of employing an HS20-44 design load is in need of further verification. Research studies relating to the design and performance evaluation of pavements and bridges, characteristics and conditions of heavy vehicle operation, associated cost streams, and the economy of all modes of transport must be undertaken in order to accumulate information on the basis of which types and magnitudes of bridge design loadings should be determined. It is essential that a complete logical reference framework be developed, utilizing principles of systems engineering, which provides guidance for the rational derivation of design loads. But, the fact that some European countries use design loads which are heavier than those generally used in North America does not provide a convincing argument that North American design loads should be revised.

4) The writer agrees that if a revision should be made in the loadings specified by the AASHTO and CSA codes, this revision should not only refer to the values of the loads but also to the type of loading - provided that more realistic and/or simpler loading systems can be derived. A loading system similar to the British HA loading, if applicable, would indeed be the simplest to apply. Preliminary studies which have been carried out with respect to the derivation of transformed highway loads are reported upon in references (4) to (7), but further work must be undertaken

before it can be finally determined under what conditions and whether heavy vehicle loads are best synthesized in terms of representative concentrated loads (design trucks) and/or equivalent uniformly distributed loads to be used without or together with one or more concentrated loads. A point worth noting is that the representation of truck loads in terms of uniformly distributed loads (with or without a concentrated load) is difficult if used for the design of skewed slab-type structures. It is therefore questionable whether a design loading similar to the British HA loading is adequate for the design of such structures. Furthermore, it may be observed that the HA loading is used both for the calculation of moments and shears and has a constant U.D.L.-value for loaded lengths 20 to 75 ft., which all seems to result in an overly crude approximation of the load effects generated by heavy vehicles in different structures. It should also be kept in mind that the automated design of bridge structures, utilizing analytical and/or experimental means of analysis (8,9), has become a reality, and that in view of this, a spectrum of critical axle configurations (5,6) should perhaps be derived to constitute live loadings in preference or supplementary to other equivalent load systems.

Although the space here is limited, it is hoped that the comments made draw to the reader's attention some of the complexities involved in establishing live loads. Professor Seni's paper makes the reader aware that the designers must be bewildered by the differences encountered between loadings used in various countries. It is thus hoped that the need for further research to clarify this problem is universally recognized. It would not

only be desirable that studies be made of bridge design loadings in North America, but there is undoubtedly also a need to carry out studies in Europe.

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AUTHOR'S CLOSURE

The author wishes to thank Professor Sawko and Mr. Tamberg for their discussion.

Professor Sawko asks if the maximum weight of vehicles crossing the bridges should be restricted. The author's opinion, expressed in the paper, is that the system of live loads should cover the loads of the vehicles presently in use, with some provision for a future increase.

It is obvious that the design cannot provide for occasionally extremely heavy vehicles, like the 400 ton vehicle train quoted by the discussor. For such cases, the bridge has to be checked assuming that no other live loads should be placed on the structure and allowing an overstress of the members. The use of ultimate strength design should be very suitable for such cases.

Referring to the influence surfaces proposed by Mr. Sawko, it is obvious that, if included in the design, it should facilitate a further check for such loads. But, as such exceptional loads seldom occur, the long calculations involved for this purpose, make this proposal unpractical. This is especially the case in North America where the lateral distribution of the loads, for the design of the girders, is usually based on empirical coefficients provided by the specifications. Thus, for the influence surfaces, the design calculations should not be useful, a special analysis being necessary, by more exact methods, considering the structure as a grid.

Mr. K. Tamberg's discussion gives numerous references to reports of the Department of Highways of Ontario, which unfortunately were not brought to the public knowledge.

The author is pleased to note that, in spite of some objections, Mr. K. Tamberg expresses a general agreement with the main conclusions of the paper by pointing out that "the validity of employing on HS20-44 design load is in need of further verification" and that, "if a revision should be made, this revision should not only refer to the values of the loads, but also to the type of loading".

Answering to the question asked by Mr. Tamberg, about any bridges having collapsed due to overloading, the author wants to point out the following:

The fact that one cannot quote collapses due to overloads, does not mean that the actual loads may not have largely exceeded the design live loads. The reason of this is that, on the one hand, for concrete bridges, the design stresses are 0.40 of the ultimate or yield strength of the materials, and on the other hand, the design stresses due to live loads are at most 50% of the total stresses. That means that, for reaching the collapse, the live load must be increased 4 times, i.e. an overload of 3 times the design live load. Thus, the fact that collapses or severe damages have not occurred does not mean that the actual live loads have not exceeded the design live loads, and no conclusion may be drawn of it, in favour of the present system of live loads.

The author's opinion is that the design loads must not be defined in terms of the behaviour of existing bridges, but by a thorough inquiry and study. This should be undertaken by the qualified organizations, taking in account the present loads with provisions for their future increase.

The increase of loads is an undeniable fact which worries many technical institutions and personalities (see the article "Bigger bridge loads coming - will we be ready?" by Arthur L. Elliot, Bridge Engineer, California Division of Highways, "Civil Engineering, Feb. 1969".)

COMPARISON OF LOADS AROUND THE WORLD FOR DESIGN OF HIGHWAY BRIDGES

By K. S. RAJAGOPALAN

SYNOPSIS

Design live loads for major bridges in eighteen countries are compared by studying their effects on simple spans of 30 to 140 ft. (9 to 42 meters nominal). A very wide diversity in the design load effects such as maximum bending moments and shear forces is noted--the heaviest effect being about two and one-half times that of the lightest. This wide diversity cannot be easily justified by the normal volumes of traffic loads in these countries. It is hoped a more uniform design load, if specified, will result in economy in some countries and perhaps greater safety in others.

Keywords: bending moments; bridges (structures); highway bridges; impact strength; live loads; loads (forces); moving loads; shear properties; spans; standards; structural design.

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INTRODUCTION

A preliminary comparison¹ of the design loads for major highway bridges as per AASHO, German Code DIN 1072 and the Indian Road Congress revealed a very wide diversity in the resulting maximum live load bending moments and shear forces on simple spans of 30 to 140 ft. (9 to 42 meters nominal). It was then considered that it would be worthwhile to compare the effects of standard loads as obtained in other countries also.

INVESTIGATION

In 1965, Indian Road Congress published a digest of bridge design loads specified in various countries.² The loads used in this investigation have been taken out of the above reference and are summarized in Appendix A.

Some countries might have revised these standard loads since 1965; the author is not aware of any revisions and hence the following computations do not reflect revisions since 1965, if any. However, it is hoped that such revisions, if any, would be few and hence would not totally invalidate the observations in this paper.

In many countries there are different categories of specified loads for different types of bridges. Only the design loads for major bridges are compared in this paper. The general observations that follow are quite applicable to other categories of bridges as well.

Design loads as obtained in a total of thirteen countries are compared explicitly. In addition, Ontario (Canada), New South Wales (Australia) and the Phillipines follow generally the AASHO specifications; Malaysian and Rhodesian specifications are comparable to those of Great Britain.

In the comparisons that follow, the following have been assumed:

1. Effects of loads will be restricted to simple spans of 30 to 140 ft. (9 to 42 meters nominal).
2. Only two lanes of roadway with a total width of 24 ft. (7.2 meters nominal) will be considered.

Simple spans are chosen as samples and as indicative of what is likely to happen in other types of constructions. The above range of spans is selected since a majority of simple span bridges is likely to be covered by this range. Also two lane bridges are considered common and typical in most of the countries.

The maximum bending moments due to the standard loads have been computed. For easier plotting these values are divided by the respective spans. Figure 1 shows, for various countries, the variation of the maximum live load bending moment/span against the span.

The maximum shear forces imposed on simple spans due to the standard loads are represented against the spans in Fig. 2.

The effects of impact factors are not reflected in these figures; different impact factors are specified for steel and concrete bridges in many countries; also, in some cases, for the same type of bridge the impact factors differ from one category of load to another.

OBSERVATIONS

It is conceded, at the outset, that, in various countries, the standard loads for design of highway bridges may have been evolved on factors depending on, among others, the available data of vehicles, their loading capacities, the intensity and frequency of traffic, logistic considerations for general mobilization in case of national emergencies. Also the standard loads might depend on the allowable stresses in members. For instance, a smaller design load may be specified in case smaller permissible stresses are allowed. Hence a heavier design load does not per se lead to a heavier bridge superstructure under identical conditions of spans, road width and type of construction. But it is highly unlikely that the specified load factors will differ drastically from one country to another.

Hence the comparisons in Fig. 1 and 2 should be viewed as reflecting the general trend rather than strictly quantitative.

From Fig. 1 and 2 it can be seen that the variation of the intensities of live load effects differ considerably from one country to another, the ratio of the heaviest to the lightest being between 2 and 2-1/2. It is also seen that the order of countries arranged in the sequence of severity of the effect is different in the case of shear from that of moment; yet in both, design load of Great Britain is the heaviest, and that of AASHO is the lightest over a large range of spans (this is surprising especially because of the large volume of road traffic in USA both in sheer numbers and in terms of loads carried).

Even allowing for different load factors, differences in impact factors, traffic and load intensities that are actually likely to occur from country to country, it is not still easily justifiable why the loading severities should differ so drastically.

In concrete bridges, it may be that the effect of live load is not as significant as that of dead load. Even then a reduction or an increase of live load should similarly affect the dead load (except perhaps in cases of prestressed concrete bridges where the dead load is completely balanced by the prestressing force). Hence, if the standard loads are revised to result in less diversity, an overall economy could result in countries where the loading severity is very high. By a similar token, a greater safety may result in countries with the light intensity of loading effects.

SUMMARY

The loading effects due to standard loads for highway bridge design differ considerably between countries. A greater uniformity might result in a greater economy in some countries and a greater safety in others.

As design practices improve, and as better methods of analysis become available, it becomes necessary to critically reexamine the basic design loads. Also road traffic is increasing in volume and intensity day by day. A reevaluation of design loads may lead to greater economy and additional safety.

ACKNOWLEDGMENT

The author wishes to express his sincere thanks to Professor Phil M. Ferguson, Professor of Civil Engineering, The University of Texas at Austin, for his critical review.

REFERENCES

1. Chandrasekhar, C. S., and Rajagopalan, K. S., "Standard Loadings for Design of Highway Bridges--A Review." Eastern Engineer, India, Sept. 1966.
2. "Bridge Loadings Round the World," Transport-Communications Monthly Review, No. 208, published by Indian Road Congress, Dec. 1965.

TABLE A—Equivalent Distributed Load
(From Bridge Loadings—Italy)

Bending load in t/ml						Shear load in t/ml					
Span	Civil Loading		Military Loading			Span	Civil Loading		Military Loading		
	Type	Type	Type	Type	Type		Type	Type	Type	Type	Type
m	1	2	4	5	6	m	1	2	4	5	6
1	16.000	24.000	28.000	10.667	38.000	1	16.000	24.000	28.000	12.160	36.000
1.5	10.667	16.000	18.667	9.027	25.333	1.5	10.667	16.000	24.610	10.287	31.680
2	8.000	12.000	15.541	8.188	19.981	2	8.000	12.000	20.860	9.787	26.820
2.5	6.400	9.600	14.193	7.285	18.248	2.5	6.400	9.600	17.830	9.114	22.925
3	5.333	8.000	12.859	7.329	16.534	3	5.333	8.000	15.493	8.770	19.920
3.5	4.571	6.857	11.677	7.561	15.013	3.5	4.898	7.347	13.669	8.620	17.574
4	4.000	6.000	10.658	7.455	13.703	4	4.500	6.750	12.215	8.400	15.705
4.5	3.556	5.333	9.784	7.252	12.579	4.5	4.148	6.222	11.034	8.217	14.187
5	3.200	4.800	9.032	7.143	11.612	5	3.840	5.760	10.058	7.936	12.931
5.5	2.921	4.382	8.381	6.954	10.776	5.5	3.570	5.355	9.497	7.617	12.229
6	2.778	4.167	7.814	6.727	10.047	6	3.333	5.000	9.147	7.289	11.803
6.5	2.651	3.965	7.316	6.486	9.407	6.5	3.314	4.686	8.788	6.968	11.359
7	2.612	3.778	6.877	6.243	8.842	7	3.265	4.408	8.434	6.661	10.917
7.5	2.560	3.605	6.486	6.005	8.839	7.5	3.200	4.160	8.094	6.372	10.487
8	2.500	3.445	6.136	5.776	7.889	8	3.125	3.938	7.770	6.133	10.077
8.5	2.436	3.297	5.990	5.559	7.728	8.5	3.045	3.737	7.464	5.950	9.687
9	2.370	3.160	5.854	5.352	7.561	9	2.963	3.556	7.258	5.813	9.401
9.5	2.305	3.031	5.713	5.157	7.387	9.5	2.925	3.391	7.057	5.717	9.125
10	2.240	2.916	5.599	4.974	7.209	10	2.880	3.240	6.877	5.639	8.873
11	2.128	2.705	5.411	4.639	6.857	11	2.777	2.975	6.609	5.554	8.474
12	2.111	2.521	5.206	4.342	6.518	12	2.667	2.750	6.331	5.511	8.079
13	2.083	2.360	4.997	4.231	6.200	13	2.651	2.556	6.057	5.453	7.700
14	2.122	2.217	4.793	4.137	5.933	14	2.612	2.388	5.809	5.355	7.358
15	2.133	2.091	4.597	4.110	5.710	15	2.560	2.240	5.607	5.234	7.072
16	2.125	1.978	4.420	4.106	5.521	16	2.531	2.109	5.408	5.117	6.798
17	2.104	1.876	4.340	4.120	5.357	17	2.491	1.993	5.216	5.032	6.537
18	2.086	1.784	4.291	4.136	5.195	18	2.444	1.889	5.098	4.982	6.291
19	2.083	1.701	4.230	4.204	5.638	19	2.438	1.795	5.051	4.958	6.059
20	2.080	1.625	4.159	4.253	4.885	20	2.420	1.710	5.006	4.944	5.840
21	2.068	1.555	4.113	4.293	4.739	21	2.395	1.633	4.947	4.920	5.635
22	2.050	1.491	4.060	4.308	4.599	22	2.380	1.562	4.877	4.879	5.443
23	2.028	1.432	4.000	4.304	4.475	23	2.359	1.497	4.829	4.827	5.261
24	2.028	1.378	3.936	4.286	4.363	24	2.333	1.438	4.795	4.778	5.091
25	2.022	1.327	3.870	4.257	4.255	25	2.330	1.382	4.750	4.742	4.930
26	2.036	1.280	3.818	4.220	4.151	26	2.320	1.331	4.702	4.720	4.778
27	2.041	1.236	3.790	4.177	4.051	27	2.305	1.284	4.664	4.709	4.635
28	2.041	1.196	3.766	4.145	3.954	28	2.296	1.240	4.634	4.702	4.500
29	2.036	1.157	3.737	4.118	3.860	29	2.283	1.199	4.600	4.688	4.372
30	2.031	1.121	3.703	4.106	3.771	30	2.267	1.160	4.559	4.665	4.251
31	2.031	1.088	3.666	4.104	3.685	31	2.264	1.124	4.515	4.635	4.137
32	2.031	1.056	3.626	4.105	3.602	32	2.258	1.090	4.477	4.608	4.028
33	2.028	1.026	3.584	4.110	3.522	33	2.248	1.058	4.436	4.591	3.924
34	2.021	0.997	3.567	4.125	3.446	34	2.242	1.028	4.391	4.578	3.825
35	2.012	0.971	3.567	4.137	3.372	35	2.233	0.999	4.357	4.571	3.732
36	2.012	0.945	3.583	4.157	3.302	36	2.222	0.972	4.310	4.568	3.642
37	2.010	0.921	3.612	4.169	3.233	37	2.221	0.947	4.330	4.558	3.557
38	2.017	0.898	3.634	4.174	3.168	38	2.216	0.922	4.314	4.543	3.475
39	2.020	0.876	3.648	4.173	3.105	39	2.209	0.899	4.294	4.523	3.397
40	2.020	0.856	3.657	4.167	3.044	40	2.205	0.878	4.276	4.508	3.323
45	2.015	0.765	3.673	4.107	2.771	45	2.181	0.782	4.212	4.475	2.993
50	2.010	0.691	3.658	4.116	2.541	50	2.163	0.706	4.135	4.429	2.722
55	2.010	0.631	3.646	4.138	2.345	55	2.147	0.643	4.079	4.400	2.496
60	2.004	0.580	3.621	4.107	2.177	60	2.133	0.590	4.041	4.380	2.304
70	2.005	0.500	3.570	4.124	1.903	70	2.116	0.507	3.962	4.337	1.997
80	2.005	0.439	3.602	4.104	1.689	80	2.101	0.444	3.923	4.306	1.762
90	2.003	0.391	3.597	4.107	1.518	90	2.089	0.396	3.878	4.286	1.576
100	2.003	0.353	3.577	4.112	1.379	100	2.079	0.356	3.849	4.268	1.426
120	2.001	0.295	3.583	4.107	1.164	120	2.061	0.298	3.799	4.239	1.197
140	2.002	0.253	3.570	4.103	1.007	140	2.051	0.255	3.765	4.220	1.031
160	2.001	0.222	3.577	4.104	0.887	160	2.043	0.224	3.741	4.205	0.906
180	2.000	0.198	3.572	4.106	0.793	180	2.037	0.199	3.722	4.190	0.808
200	2.001	0.178	3.571	4.086	0.717	200	2.033	0.179	3.707	4.179	0.729

N.B. For any intermediate span, linear interpolation is to be made.

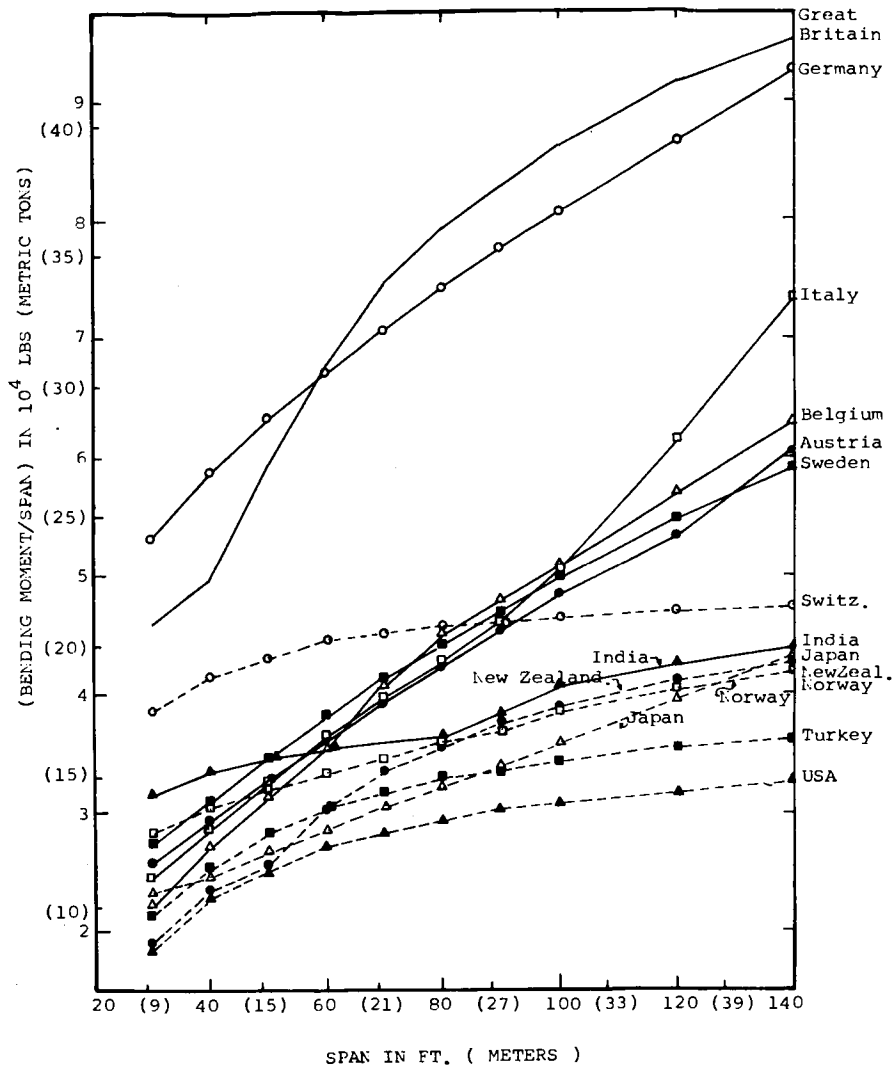


Fig.1. (Bending moment/span) versus span

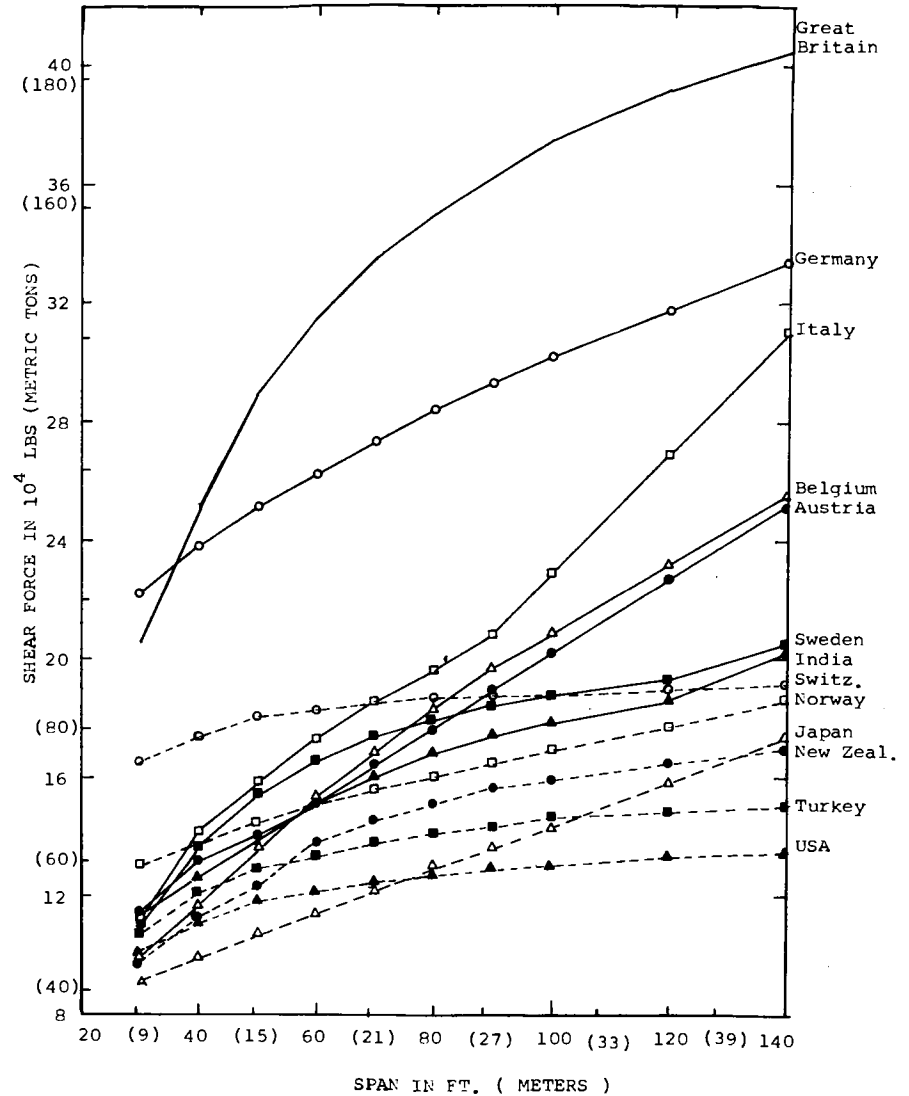
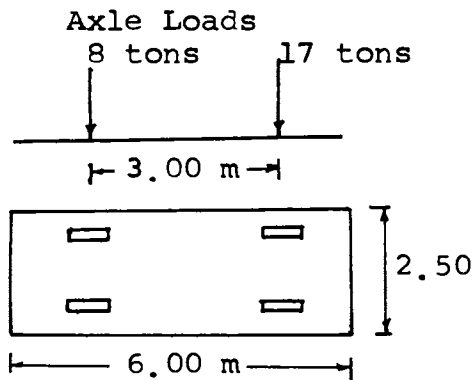


Fig.2. Shear Force versus Span.

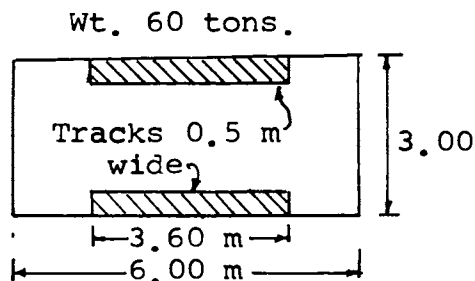
APPENDIX A

Details of loadings for two lane bridges.

AUSTRIA

Bridge Class ITRUCK

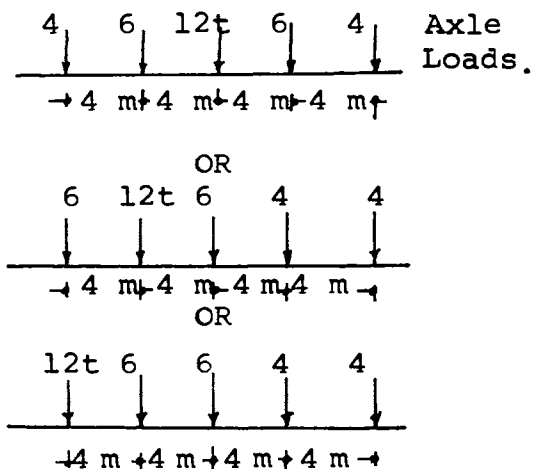
On two adjoining lanes, one truck of 25t each. The rest of the roadway and footpath to be covered with uniformly distributed load of 0.5t per meter².

CATERPILLAR

For a caterpillar, as the only load on the roadway, has a maximum of 0.5m on both sides from the longitudinal center line of the roadway.

All units metric.

BELGIUM

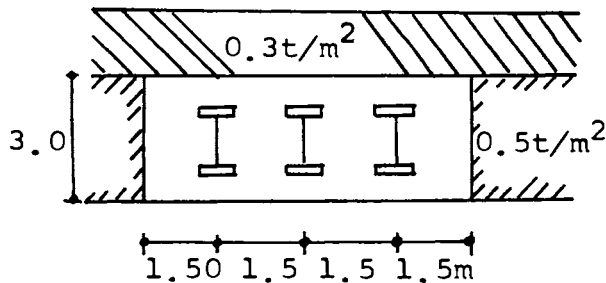


Simultaneously a load of 400 kg. per square meter uniformly distributed on the carriageways and footpath.

Over traffic lane 2.50 m. min. wide to 4.00 m. max. width.

All units metric.

GERMANY (FEDERAL REPUBLIC)

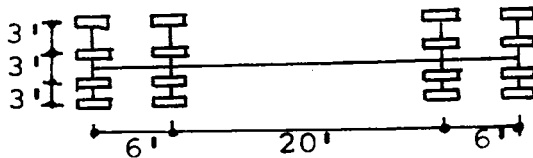
CLASS 60

Wheel load each 10t.
All units metric.

Outside the carriageway, uniformly distributed load of 0.3t/sq. m. and 0.5t/m^2 in the remaining portion of the carriageway.

The load shown in the left is per each lane.

GREAT BRITAIN

HB LOADING

1 unit of HB Loading -
Axle load = 1 ton = 2240 lbs.
All British units.

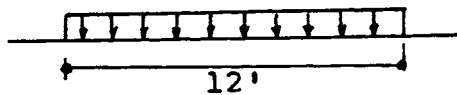
HA Loading consists of a uniformly distributed load plus a single knife edge load. The uniform load varies with the span but between 20 and 75 ft it is constant at 2200 lb per linear ft of 10 ft wide lane. The knife edge load is placed parallel to the supports of the member under consideration and has a value of 27,000 lb for a 10 ft wide lane.

Two lane bridges shall be designed for 2 lanes of HA loading or 1 lane of 45 units of HB Loading together with one lane of $1/3$ HA loading.

INDIA

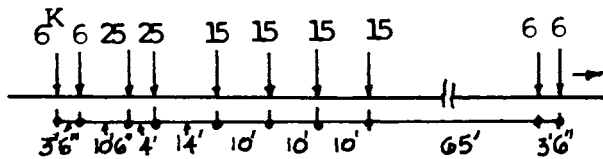
CLASS AA

Total 70 tons.



One train of class AA vehicle or two lanes of class A train of vehicles for two lane major bridges.

1 ton = 2240 lbs.

CLASS A

All axle loads in kips.

ITALY

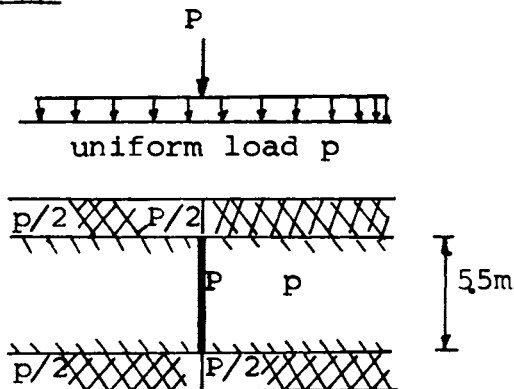
Category I

Equivalent loads for various types of loadings as per table A.

For two lane bridges:

The heaviest of types 4, 5 or 6 flanked by one or more trains of type 1 with crowd loading of 400 kg/m^2 (type 3) on the footpaths.

JAPAN

L-20

Line load P in the main lane of 5.5 m width and $P/2$ in the rest - can be placed anywhere to give worst effects; uniform load in the main lane width is p and on sides $p/2$.

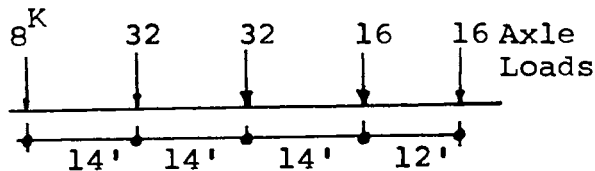
$$P = 5000 \text{ kg/m.}$$

$$p = 350 \text{ kg/m}^2 \text{ for } l \leq 80 \text{ m.}$$

All units metric.

NEW ZEALAND

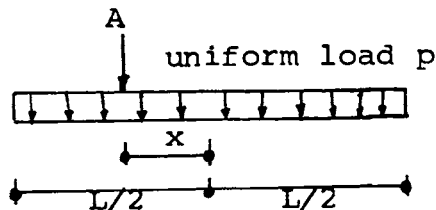
Two trucks in two lanes to produce worst effects.

H20-S16-T16 Truck

NORWAY

Equivalent loading per lane

The equivalent load in each lane.

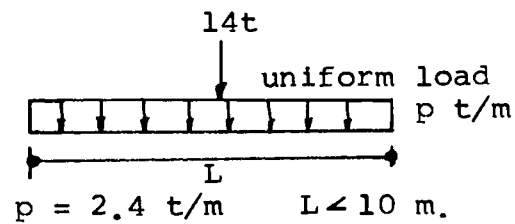


$$A = 12 + 8x/L \text{ tons.}$$

$$p = 0.5 + 35/(L+5) \text{ tons per linear meter of lane.}$$

All units metric.

SWEDEN

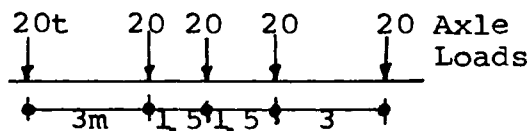
Lane Loading.

$$p = 2.4 \text{ t/m} \quad L < 10 \text{ m.}$$

$$p = 2.4 - 13(L-10)/80$$

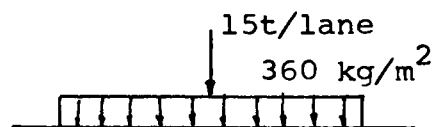
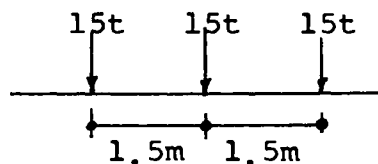
$$\text{as } 10 < L < 90 \text{ m.}$$

$$p = 1.1 \text{ t/m} \quad L > 90 \text{ m.}$$

Single Truck Loading.

All units metric.

SWITZERLAND

Lane Loading.Train Load.

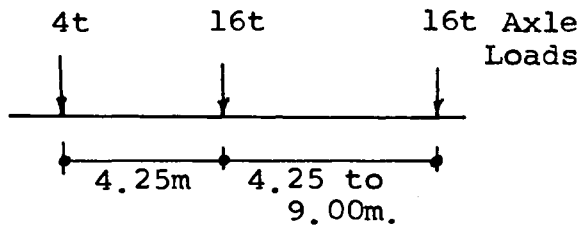
All units metric.

Two lane loading or single truck loading for two lane bridges.

A train in each lane or the lane loading as shown.

TURKEY

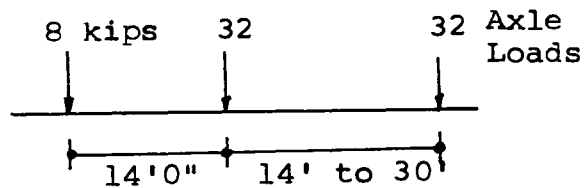
One truck for each lane.

H20-S16 Truck

All units are metric.

U.S.A.

One truck per lane.

H20-S16 Truck

STATISTICAL EVALUATION OF LOAD FACTORS FOR CONCRETE BRIDGE DESIGN

By PRADYOT K. NIYOGI, HARESH C. SHAH, KISHORE D. DOSHI, and WILSON TANG

SYNOPSIS

This paper presents an important aspect of the structural design of concrete bridges under various combinations of loading. 'Load Factors', which are quantities used to define the pattern and magnitude of design loadings, are evaluated from a statistical point of view in such a manner as to assure a desired level of safety for the structure.

An approach is presented in this paper to formulate and develop a procedure for quantitative evaluation of the load factors under dead loads, live loads and earthquake loads, taking into consideration the pertinent factors including the random probabilistic character of loads, within the framework of information available about the loading conditions and the associated 'functional' failure patterns of the bridge structure. Central to the development of such a procedure are the following considerations:

- i. The load factors as defined above, should provide a desired level of safety, i.e., system reliability under various combinations of dead loads, live loads and earthquake loads.
- ii. The procedure should be formulated in such a way that any additional information about the loading environments can be systematically incorporated to obtain a better set of load factors, and the results and confidence should improve with an increase in available data and with increased usage of the method.
- iii. The final results concerning the quantitative evaluation of load factors should be simple and easy to be used by design engineers.

The method developed in this paper for obtaining load factors under different loading configurations to obtain desired reliabilities, is applied to an example of a concrete bridge structure design.

Keywords: bridges (structures); dynamic loads; earthquakes; live loads; load factors; loads (forces); moving loads; probability theory; reinforced concrete; reliability; safety factor; static loads; statistical analysis; structural design.

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ACI Member HARESH C. SHAH is an associate professor of structural engineering at Stanford University.

KISHORE D. DOSHI is the senior research engineer for the Whittaker Corp. in San Diego.

WILSON TANG is a graduate student at Stanford University.

1. Introduction

The present practice of taking statistical nature of loads and resistances into account while designing is to multiply the loads by certain given constant factors, known as "load factors." This method is somewhat indirect, and does not consider the variation of distributions of loads and resistances for specific cases. On the contrary, these factors are arrived at from certain predetermined assumptions about the reliability of the structures and can not be varied even if the designer may wish to do so.

The simplest way of incorporating the statistical nature of the load and resistance parameters of a bridge structure design will be to modify "load factors" in such a way that the designer can select them according to the desired level of reliability and the statistical distribution of structural parameters. An approach is presented in this paper to formulate and develop a procedure for quantitative evaluation of the load factors under dead loads, live loads and earthquake loads. The suggested approach takes into consideration the pertinent factors including the random probabilistic character of loads within the framework of information available about the loading conditions. Central to the development of such a procedure are the following considerations:

- i. The load factors as defined above should provide a desired level of safety, i.e., system reliability under various combinations of dead loads, live loads and earthquake loads.

- ii. The procedure should be formulated in such a way that any additional information about the loading environments can be systematically incorporated to obtain a better set of load factors, and the results and confidence should improve with an increase in available data and with increased usage of the method.
- iii. The final results concerning the quantitative evaluation of load factors should be simple in form and easy to be used by design engineers.

The suggested method is particularly important for structures which will cause extensive life hazard and damage of properties on failure. Bridge structures come under such a category. Examples giving load factors for live loads and dead loads on bridge structures are given. Method is indicated when earthquake loads are present.

2. Statistical Considerations:

Random Loads, Deterministic Resistance

The simplest and idealized case is the one which the applied loads on a structure are random in nature, but the resistance of the structure is deterministic.

Let W_i be the i -th case of loading combination (vis. Dead Load and Live Load, Dead Load, Live Load and Earthquake, and so on). This is actually the combination of the component loads (Dead Load, Live Load, etc.) which have certain mean and variance.

We can write,

$$W_i = \alpha_{i1}\mu_1 + \alpha_{i2}\mu_2 + \dots = \sum_{j=1}^n \alpha_{ij}\mu_j \quad (1)$$

n = number of components of loads

where α_{ij} will be the load factors corresponding to the i -th combination of loading and j -th component of loads, μ_j , are the mean of the component loads.

Let σ_j^2 be the variances of the component loads, and k_i be the factors corresponding to the desired reliabilities and is defined in Fig. 1, assuming the loads have normal distribution.

For the components of loads W_i , we define,

$$l_{ij} = \mu_j + k_i \sigma_j \quad (2)$$

which represents load l_{ij} corresponding to μ_j and σ_j and a reliability corresponding to k_i .

From (2),

$$l_{ij} = \mu_j (1 + k_i v_j) \quad (3)$$

where, $v_j = \frac{\sigma_j}{\mu_j}$ = coefficient of variation.

Considering the combination of all the component loads that constitute W_i , the overall mean and variance may be expressed as

$$\mu_L = \sum_{j=1}^n \mu_j$$

$$\sigma_L^2 = \sum_{j=1}^n (\sigma_j)^2 .$$

Hence, the design load for the i -th loading, W_i can be written as

$$W_i = \sum_{j=1}^n \mu_j + k_i \left[\sum_{j=1}^n (\sigma_j)^2 \right]^{1/2} . \quad (4)$$

This design load W_i has the reliability corresponding to k_i .

To obtain the expression for load factors, W_i should be expressed as in Eq. (1), i.e.,

$$W_i = \sum_{j=1}^n \alpha_{ij} \mu_j$$

so that the coefficients of μ_j are the load factors.

This can be done by some manipulation without much significance of the intermediate steps as follows:

We have

$$W_i = \sum_{j=1}^n \mu_j + k_i \left[\sum_{j=1}^n (\sigma_j)^2 \right]^{1/2}$$

and

$$l_{ij} = \mu_j + k_i \sigma_j.$$

So,

dividing W_i by $\sum_{j=1}^n l_{ij}$, we get,

$$\frac{W_i}{\sum_{j=1}^n l_{ij}} = \frac{\sum_{j=1}^n \mu_j + k_i \left[\sum_{j=1}^n (\sigma_j)^2 \right]^{1/2}}{\sum_{j=1}^n \mu_j + k_i \sum_{j=1}^n \sigma_j} \quad (5)$$

Now,

$$W_i = \sum_{j=1}^n \alpha_{ij} \mu_j = \sum_{j=1}^n \frac{l_{ij}}{\mu_j} \cdot \frac{W_i}{\sum_j l_{ij}} \cdot \mu_j.$$

So,

$$\sum_{j=1}^n \alpha_{ij} \mu_j = \sum_{j=1}^n \left(\frac{l_{ij}}{\mu_j} \right) \frac{W_i}{\sum_j l_{ij}} \cdot \mu_j$$