

Deflection Control for FRP-Reinforced Concrete Members: An Overview

by A. Scanlon

Synopsis: Design for deflection control is a critical part of the design of FRP reinforced members due to the relatively low modulus of elasticity and elastic-brittle nature of FRP reinforcement. This paper provides an overview of design for deflection control including a brief review of the basic mechanics of beam flexure and a review of methods to account for cracking, tension stiffening, shrinkage and creep. The basis for selecting appropriate deflection control criteria is also discussed and a framework is presented to incorporate deflection control into the overall design process.

Keywords: beam; creep; deflection; design criteria; FRP reinforcement; shrinkage; slab; tension stiffening.

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INTRODUCTION

The well-known statement by Hardy Cross (1952) that “strength is essential and otherwise unimportant” applies to FRP-reinforced concrete to at least the same extent that it applies to steel-reinforced concrete. It is essential that adequate strength be provided for FRP reinforced members, as for all structural members, but the usefulness and viability of the structural system will be determined largely by its performance under service loads during its service life.

Deflection control is a crucial design criterion for FRP members and it is essential that this aspect of design is approached from the basis of a sound understanding of mechanics, variability of factors affecting deflection, and the significance of prescribed limits. Deflection of FRP members is influenced by the low modulus of elasticity compared with steel reinforcement, and its effect on immediate and long-term stiffness, as well as the ratio of strength to service load level.

This paper provides an overview of deflection control approaches that have been developed over the years and incorporated in various national design codes, and compares the behavior of members reinforced with steel bars and members reinforced with FRP reinforcement. In some cases general approaches developed for steel-reinforced concrete can be extended to FRP-reinforced concrete. However, in other cases a more fundamental approach is needed. The intense interest over the past few years in FRP research has provided impetus for re-examination of deflection issues.

MECHANICS

Deflection of slender flexural members in which shear deformations are negligible, depends on curvature Φ , which for small slope, ($\theta \ll 1$), is given by

$$\text{Curvature, } \Phi = \frac{d^2 y}{dx^2} \quad (1)$$

and the slope is given by

$$\text{Slope, } \theta = \frac{dy}{dx} \quad (2)$$

where y is the displacement transverse to the member x -axis, and deflection Δ is

$$\text{Deflection, } \Delta = y \quad (3)$$

From elementary engineering mechanics, the curvature at any section for a linear elastic isotropic homogeneous beam material is given by,

$$\Phi = \frac{d^2 y}{dx^2} = \frac{M}{EI} \quad (4)$$

where M is the internal moment at the section, E is the modulus of elasticity, and I is the moment of inertia (second moment of area about the centroid) of the cross-section.

If the bending moment distribution is known, the deflection at any location can be obtained by solving Eq. (4), applying boundary conditions, using methods described in introductory structural analysis textbooks (e.g. direct integration, moment-area method, unit load method (virtual work), direct stiffness, etc.). For the common design case of a uniformly distributed load, w , the midspan deflection is given by

$$\Delta = \frac{K}{384} \frac{w\ell^4}{EI} \quad (5)$$

or

$$\Delta = \frac{5\ell^2}{48EI} (M_m + 0.1(M_\ell + M_r)) \quad (6)$$

or

$$\Delta = \frac{\ell^2}{96} (\Phi_\ell + 10\Phi_m + \Phi_r) \quad (7)$$

where K = boundary condition factor, M_m , M_ℓ , M_r are moments at midspan, left support, and right support respectively, Φ_ℓ , Φ_m , Φ_r are curvatures at left support, midspan, and right support respectively.

For an uncracked section, the moment of inertia I_{uc} can be calculated based on a transformed section replacing the area of reinforcement A_r by a transformed area $(n-1)A_r$ where n = modular ratio = E_r/E_c . A reasonable approximation for the moment of inertia, I_g , can be obtained by considering the gross section ignoring the effect of reinforcement.

If the flexural tensile strength (modulus of rupture f_r) is exceeded at a section a crack will occur and propagate towards the compression zone resulting in a redistribution of stresses as shown in [Fig. 1](#).

For a rectangular section, ignoring the small amount of residual tensile stress close to the neutral axis and assuming perfect bond, the depth of the neutral axis is given by kd where,

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \quad (8)$$

where $\rho = A_r/bd$ is the reinforcement ratio.

The cracked transformed section moment of inertia is given by

$$I_{cr} = \frac{1}{3} b(kd)^3 + nA_r(d - kd)^2 \quad (9)$$

Ignoring for the present the effect of shrinkage restraint stresses, I_g and I_{cr} represent upper and lower bounds on the stiffness of a reinforced concrete section when the linear stress-strain assumption is valid, a reasonable assumption at service load levels. From Eqs. 8 and 9 it can be seen that the cracked stiffness is a function of the product of modular ratio and reinforcement ratio, np .

TENSION STIFFENING

As load increases beyond the initial cracking load, cracking progresses through the depth and along the member length, resulting in a gradual softening of load-deflection response. The term “tension stiffening” refers to the contribution of concrete between cracks to the overall stiffness of the member (not to be confused with “tension softening” which refers to post-peak response of a plain concrete specimen in tension in a deformation controlled test (e.g. Evans and Marathe 1968). The tensile force in the bar at the crack is gradually transferred to the adjacent concrete through bond stress. The resulting variation in reinforcement stress, concrete stress, and bond stress between cracks is shown schematically in [Fig. 2](#) for an axially loaded prism.

Tension stiffening can be represented by an average stress-strain diagram for concrete in tension representing the post-peak residual stiffness provided by concrete between cracks (Scanlon and Murray 1971, 1974). Since the early 1970s, many variations of the tension stiffening stress-strain diagram have been proposed. [Figure 3](#) shows several examples including 1) the stepped (or “sawtooth”) diagram as originally proposed by Scanlon and Murray; 2) nonlinear (e.g., Vecchio and Collins 1978; Fields and Bischoff 2004); 3) linear diagram (Lin and Scordelis 1975); 4) bilinear (Damjanic and Owen 1984); and 5) a constant small residual stress at high strain levels.

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While this approach is often applied in finite element analysis, a moment-curvature diagram including tension stiffening can also be generated using the tension stiffening stress-strain diagram for concrete in tension, strain compatibility and equilibrium. Figure 4 shows an example of a moment-curvature diagram constructed using Branson's equation and diagrams based on three different assumptions for the tensile concrete stress strain diagram (Scanlon and Murray 1971). Note that the initial post-peak response using Branson's equation is much stiffer than that using the other approach. It is interesting to note that the stepped (or sawtooth stress strain diagram) has been applied recently to a variety of mechanics problems by Rots and his co-workers (Rots et al. 2008). There appears to be scope for further development of this approach considering specific bond characteristics of FRP reinforcement for both embedded bars and externally applied sheets.

An alternative approach is to construct a moment-curvature diagram based on an empirical function representing the gradual transition from uncracked to fully cracked state. This approach was initially proposed by Branson (1963) who presented an effective moment of inertia equation given by

$$I_e = I_g \left(\frac{M_{cr}}{M_a} \right)^m + I_{cr} \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \right) \quad (10)$$

where M_{cr} = cracking moment, and M_a is the applied moment at the section under consideration.

Branson verified the equation against test results on beams with steel reinforcement ratios of approximately 1.5%, i. e. relatively heavily reinforced, and recommended $m = 4$ for local moment-curvature ($M-\Phi$) relationships, and $m = 3$ when applied to entire members. ACI Committee 435 (1966) reviewed Branson's proposal and recommended that Branson's I_e or the cracked transformed moment of inertia I_{cr} would be satisfactory for deflection calculations for beams. ACI 318 adopted Branson's equation in the 1971 edition of the ACI Code. Prior to 1971 the ACI Code specified I_g for $\rho < 200/f_y$ and I_{cr} for $\rho > 200/f_y$.

Attempts to apply Branson's equation to FRP met with difficulties due to the low values of ρ typical of FRP reinforced members compared to the beams tested by Branson. A correction factor applied to I_g was used to decrease the stiffness obtained by Branson's equation (Ospina and Gross 2006). The correction factor was calibrated against test results.

Recently, Bischoff (2005) presented an alternative version of the effective moment of inertia given by

$$I_e = \frac{I_{cr}}{1 - \left(\frac{M_{cr}}{M_a} \right)^2 \left[1 - \frac{I_{cr}}{I_g} \right]} \leq I_g \quad (11)$$

Comparison with test data shows that this form of I_e produces good correlation when applied to tests on steel reinforced members over a wide range of reinforcement ratios as well as FRP members without the need for additional correction factors (Bischoff and Scanlon 2007; Bischoff et al. 2009).

DRYING SHRINKAGE

Drying shrinkage affects deflection in two ways. Reinforcement placed asymmetrically with respect to the centroid of the cross section can produce shrinkage warping deflection (ACI 209R-92). More importantly, particularly for lightly reinforced members, shrinkage restraint stress produced by embedded reinforcement and support conditions results in a reduction in the load at which cracking is initiated. The effect of shrinkage restraint is well-known (for example, ACI 224R-01) but because it is not specifically included in the ACI 318 deflection calculation procedure the effect is often neglected in deflection calculations. ACI 318-08 specifies a modulus of rupture value of $7.5\sqrt{f'_c}$ for deflection calculations. Scanlon and Murray (1982) recommended use of a reduced effective modulus of rupture equal to $4\sqrt{f'_c}$ to account for restraint stress when using Branson's expression for I_e . More recently, Scanlon and Bischoff (2008) recommended a reduced effective modulus of rupture of $5\sqrt{f'_c}$ along with Bischoff's expression for I_e . They showed that this value gives a reasonable approximation for members with low reinforcement ratios typical of slabs where the effect can be significant.

CREEP

Creep can be defined in terms of deformation due to sustained stress and can be expressed in terms of a time-dependent creep function $\phi(t, t_0)$ which can be expressed as

$$\phi(t, t_o) = k\phi_b(t, t_o)C_u \quad (12)$$

where $\phi(t, t_o)$ is the strain at time t due to a unit sustained stress applied at time t_o , k accounts for a variety of factors including age at loading, concrete composition and relative humidity, $\phi_b(t, t_o)$ is a time function and C_u is the ultimate creep coefficient at time infinity. Creep functions of varying complexity are provided in ACI 209.2R-08 (2008).

Using the creep function, creep strain at time t can be expressed in terms of initial elastic strain due to immediate load applied at time t_o as

$$\varepsilon_c(t, t_o) = \phi(t, t_o)\varepsilon_i(t_o) \quad (13)$$

Creep can also be defined in terms of specific compliance function $C(t, t_o)$ which expresses total strain due to a unit stress as the sum of the initial strain and time-dependent creep strain, i.e.

$$\varepsilon_T(t, t_o) = C(t, t_o)\varepsilon_i(t_o) = \frac{(1 + \phi(t, t_o))}{E(t_o)} \varepsilon_i(t_o) \quad (14)$$

For a stress increment applied at t_o , $\Delta\sigma(t_o)$, the total strain can be expressed as

$$\Delta\varepsilon(t) = \Delta\sigma(t_o)C(t, t_o) \quad (15)$$

For a series of m stress increments, $\Delta\sigma(t_j)$ applied at times t_j , ($j = 1$ to m) the total strain based on the principle of superposition can be expressed as

$$\varepsilon(t) = \sum_{j=1}^m \Delta\sigma(t_j)C(t, t_j) \quad (16)$$

For a continuously varying stress function,

$$\varepsilon(t) = \int_0^t C(t, \tau) \frac{\partial \sigma}{\partial \tau} d\tau \quad (17)$$

For a constant stress history, time dependent strains can be calculated by elastic analysis using an effective modulus of elasticity,

$$E_c(t) = \frac{E_c(t_o)}{(1 + \phi(t, t_o))} \quad (18)$$

In most cases due to the presence of reinforcement, a gradual reduction in concrete stress occurs. In this case the “age-adjusted” effective modulus can be used to calculate time-dependent deformation,

$$E_c(t) = \frac{E_c(t_o)}{(1 + \chi\phi(t, t_o))} \quad (19)$$

where χ is approximately 0.8 for most cases.

In general, a computer-based time-stepping algorithm can be used to determine time dependent response under a specified loading history, however for most design situations a simple multiplier is considered adequate to calculate long-time deflections (ACI 318-08) based on the high level of uncertainty associated with predicting deflections.

Figure 1 demonstrates schematically the influence of tension reinforcement on creep deflection. In **Fig. 1(a)** a constant moment is applied to a gross (unreinforced) cross section. The stress distribution remains constant during the time period. The increase in curvature due to creep is proportional to the creep coefficient due to the

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constant stress condition.

Figure 1 (b) shows the case for a cracked section under constant moment. In this case the tension steel causes a redistribution of stress across the cross-section. In this case the extreme fiber stress in concrete compression decreases and the stress in the tensile reinforcement increases slightly. As a result the neutral axis depth increases and the increase in curvature is somewhat less than directly proportional to the creep function.

ACI 318 provides a basic multiplier of 2 which is about 0.8 times the average ultimate creep function, $C_u = 2.35$ reported by ACI Committee 209 (ACI 209R-92). ACI 440 has proposed a multiplier less than 2 for FRP members based on test data. However since the multiplier is primarily a function of concrete properties this proposal should be investigated more closely before general adoption.

PERMISSIBLE COMPUTED DEFLECTIONS

ACI 318 prescribes limits on computed deflections where the deflections are computed according to the Code provisions. Limits are prescribed for computed deflections under live load. Limits are also prescribed for deflections that occur after installation of non-structural elements. These limits should be considered as minimum requirements for deflection control. Engineers should determine whether more stringent serviceability requirements are required for specific projects, e.g. support of unreinforced masonry, and support of sensitive equipment. It should also be noted that total deflection under sustained load may be an issue related to perceptible sagging, particularly for members loaded at early age under construction loading (Scanlon and Bischoff 2008).

DEFLECTION CONTROL BY MINIMUM THICKNESS REQUIREMENTS

Minimum thickness requirements provide a long-standing design procedure for deflection control. ACI 318 values for one-way and two-way steel reinforced members are based on previous experience and concerns have been raised over the years about the validity of these rules in current practice (e.g. Rangan [1982]; Gilbert [1985], Grossman [1981]; Scanlon et al. [2001]; Scanlon and Choi [1999]; Scanlon and Lee [2006]). Their applicability to FRP members is also questionable and as a result ACI 440 has developed a set of minimum thickness criteria for FRP members in a similar format to ACI 318 (Ospina and Gross 2006).

An alternative approach to developing minimum thickness criteria was originally proposed by Rangan (1982) for one-way members and extended to two-way systems by Gilbert (1985). The advantage of this approach is that it is rationally based and can be linked to specified permissible deflection criteria. The critical deflection for concrete members satisfying ACI Code permissible deflections is usually the time-dependent deflection occurring after installation of nonstructural elements. For this case the deflection equation for a uniformly distributed load can be written as

$$\Delta_{inc} = \frac{\lambda K W_s \ell^4}{384 E_c I_e} + \frac{K W_L (add) \ell^4}{384 E_c I_e} \quad (20)$$

Re-arranging this equation results in an expression for span to depth ratio in terms of deflection to span ratio,

$$\frac{\ell}{h} = \left[\left(\frac{\Delta_{inc}}{\ell} \right)_{allow} \frac{32 \alpha E_c b}{K (\lambda W_s + W_L (add))} \right]^{\frac{1}{3}} \quad (21)$$

where $\alpha = I_e / I_g$, λ is a long-time multiplier, W_s is the sustained load including a portion of live load and $W_L (add)$ is the remaining portion of live load.

An equation of this general form was incorporated in the Australian code based on Rangan and Gilbert's work with an expression for α as a function of reinforcement ratio. Unfortunately, the reinforcement ratio is unknown at the time the thickness is selected

The other term in the equation that is not known at the time the thickness is computed is the dead load which obviously depends on the member thickness. Scanlon and Choi (1999) proposed a constant factor for α for one way members and demonstrated that a reasonable estimate of the member depth quickly converges to the desired member depth.

A modified form of the Scanlon-Choi span-to-depth function was subsequently incorporated in the Australian Design Standard (2001). Scanlon and Lee (2006) extended this approach to cover both one-way and two-way

systems. The advantage of this approach is that it takes into account the significant factors affecting deflection. This approach can accommodate other assumptions such as long time multiplier, loads, and specified deflection to span ratio, and is relatively easy to apply.

VARIABILITY

The difficulty of predicting deflections accurately due to the variability of factors involved is well known. Ramsay et al. (1979), and Choi et al (2004) demonstrated using Monte Carlo simulation that the variability of both immediate and long time deflections is highest when the cracking moment is close to the applied moment. Since this is often the case with FRP members, variability is likely to be an important issue for FRP members. It should also be remembered that variability of deflections in actual structures is likely to be higher than variability of deflections of members loaded in a controlled laboratory setting. Care is therefore required when extrapolating laboratory test results to design recommendations.

COST OPTIMIZATION APPROACH TO DEFLECTION CONTROL

Since deflection control is a serviceability issue, a failure can be considered as a condition in which deflection causes unsatisfactory performance usually incurring some cost to correct. Lee and Scanlon (2007) investigated the application of utility theory which resulted in a cost minimization approach to selecting member depth for deflection control.

Figure 6 shows schematically a plot of cost vs member depth. As the member depth increases, the cost of construction increases due to increased weight and material costs. As member depth increases, the “cost of failure” decreases. The cost of failure can include costs of such corrective procedures as repairing non-structural elements, applying a leveling material, external post-tensioning etc., as well as costs associated with loss of production and damage to the engineer’s reputation. The cost of failure needs consideration of the variability of deflections and the fact that serviceability failure is a gradual occurrence compared to the crisp definition of failure usually associated with strength.

Figure 7 shows a histogram of member deflections obtained using Monte Carlo simulation for a given member. Also shown is a “serviceability loss function” which represents a measure of the cost due to a serviceability failure. Below a certain deflection it is assumed that there is no cost required for repair. As the deflection increases beyond this point, repair costs increase to a maximum value usually associated with complete replacement. The greater the overlap between the deflection histogram and the serviceability loss function, the greater the cost of repair. Total cost plotted in Fig. 6 consists of the sum of the initial construction cost and the repair cost. The optimum member depth is determined where total cost is a minimum.

While this approach is unlikely in the near term to provide a normal design procedure for deflection control, it shows promise as a basis for evaluating deflection control criteria as discussed by Lee and Scanlon.

DEFLECTION CONTROL IN THE DESIGN AND CONSTRUCTION PROCESS

Deflection control should be included as a logical part of the structural design process as illustrated in Fig. 8. After the framing system is selected to suit functional requirements spans are defined and applied loads (superimposed dead load and live load) can be determined. At this stage deflection control requirements should be defined explicitly in terms of a target deflection to span ratio to suit the functional requirements of the structure.

With a first estimate of member thickness, dead load can be determined and used in Eq. (21) to obtain a member thickness that can be expected to satisfy the specified deflection criteria.

The structure can now be analyzed to determine reinforcement requirements for strength, and the reinforcement can be detailed to meet crack control criteria. Having determined member dimensions and reinforcement, deflections can be checked by calculation, most likely by using available software developed to account for the significant factors identified earlier. (Note that software based on linear elastic analysis generally does not account for effects of cracking, creep, and shrinkage.) With the growing availability of software that can be used for deflection calculations, the need for “deemed to satisfy” deflection control criteria is diminishing. In other words, minimum thickness criteria can be used primarily as a tool for initial proportioning of members. The deflection calculation check will then serve as a quality assurance check on the initial selection of member dimensions.

During the design process, construction methods that will be used by the contractor are generally not known. It is therefore necessary to consider potential effects of construction procedures (e.g. shoring and re-shoring) on the subsequent deflection performance, before construction begins.

CONCLUSIONS

Deflection control is an important design issue for concrete structures whether reinforced with steel reinforcement or FRP reinforcement. It is particularly important for lightly reinforced members due to the sensitivity of member stiffness to cracking. Extensive research conducted recently on FRP reinforced members has produced renewed interest in the deflection control aspect of design and this trend can be expected to continue and produce new insights.

A framework has been presented for incorporating deflection control as an integral part of the design process and methods are presented to account for effects of cracking, tension stiffening, shrinkage restraint and creep deformation. Additional work is needed to further evaluate the applicability of these approaches to the specific characteristics of FRP materials and their interaction with concrete.

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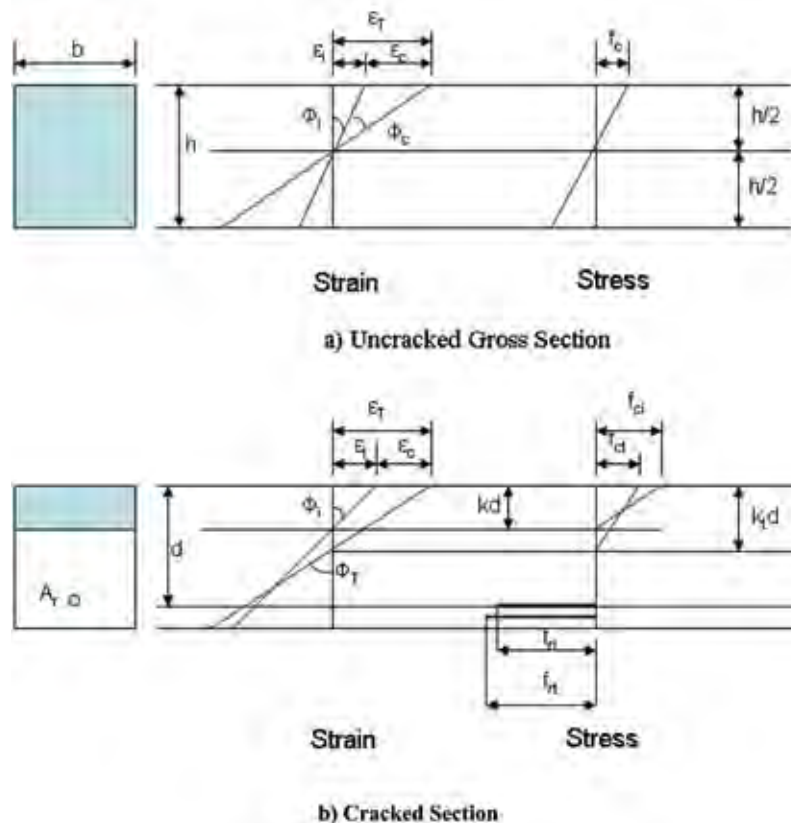


Fig. 1—Stress and strain diagrams on cross section.

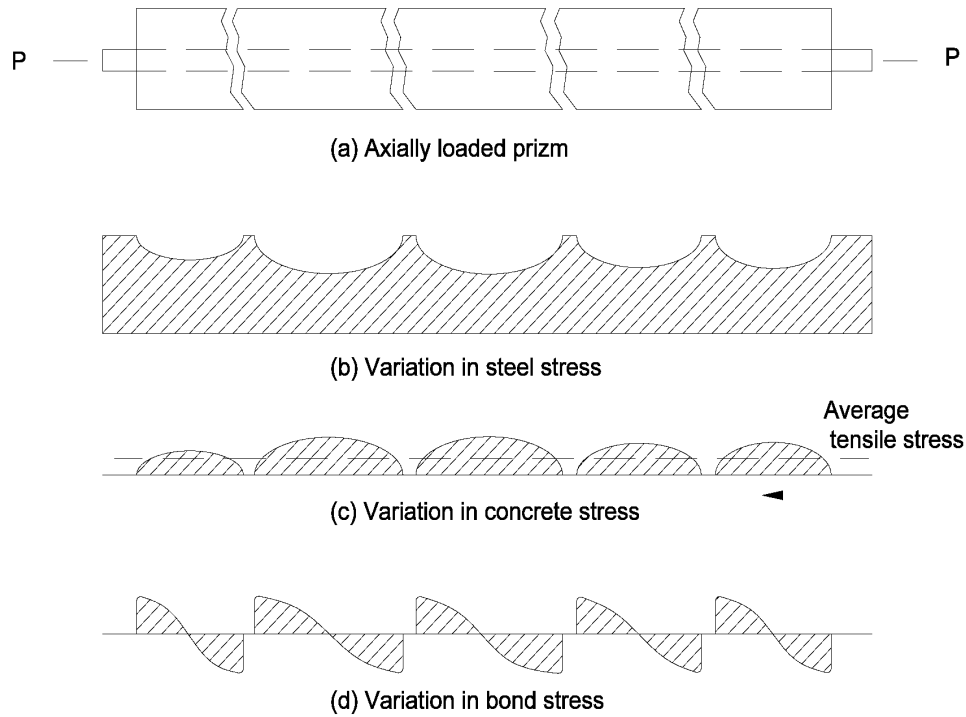


Fig. 2—Stress in steel and concrete—axially loaded cracked prism.

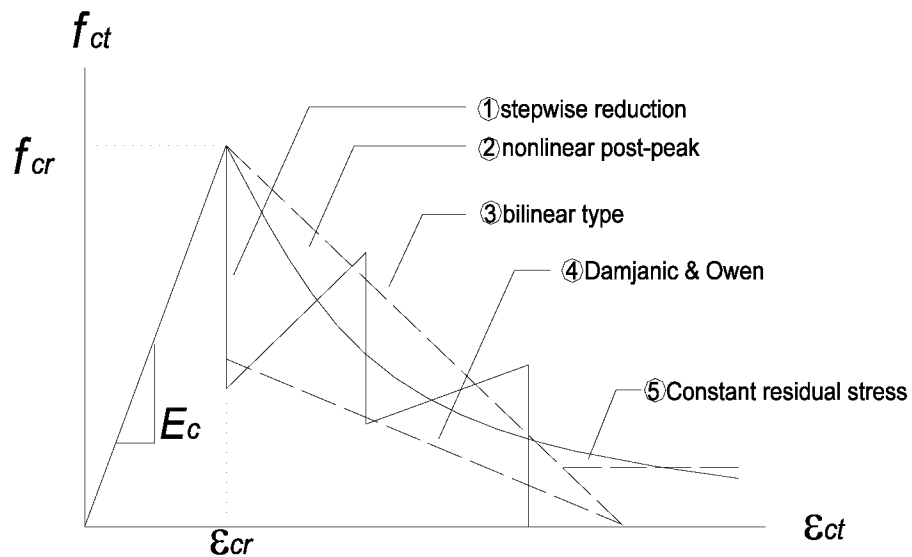


Fig. 3—Tension stiffening models.

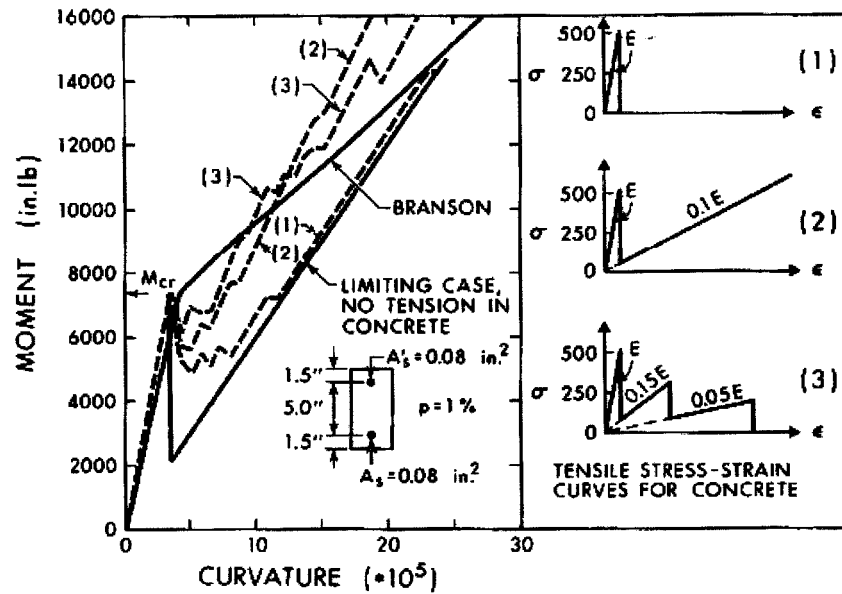


Fig. 4—Moment curvature: Branson and stepped stress strain. (Note: 1 in.² = 645.2 mm²; 1 in.lb = 0.000113 kN.m.)

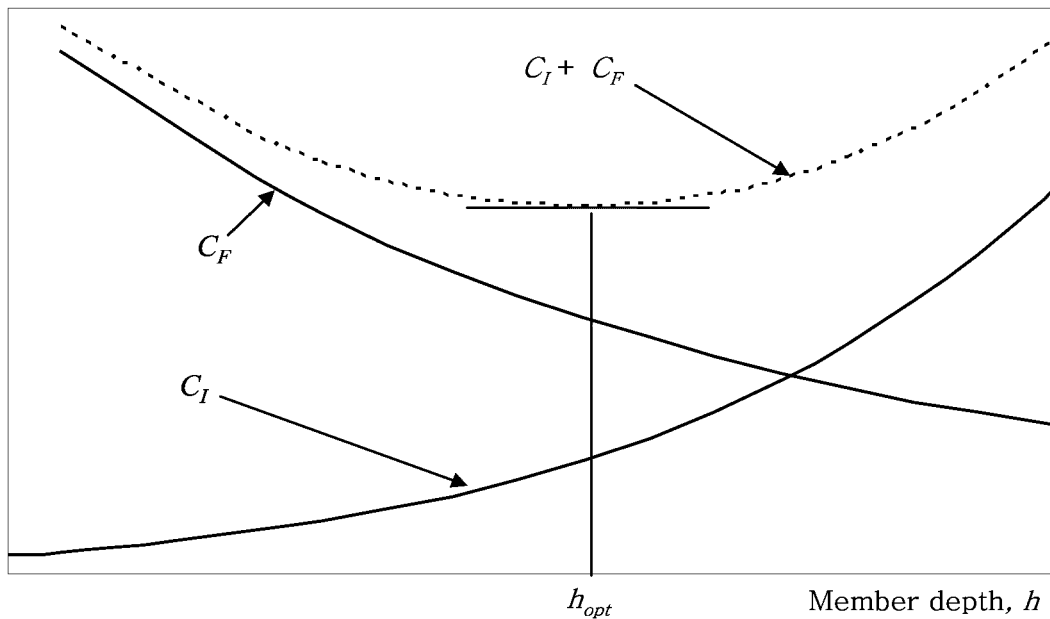


Fig. 5—Cost functions.

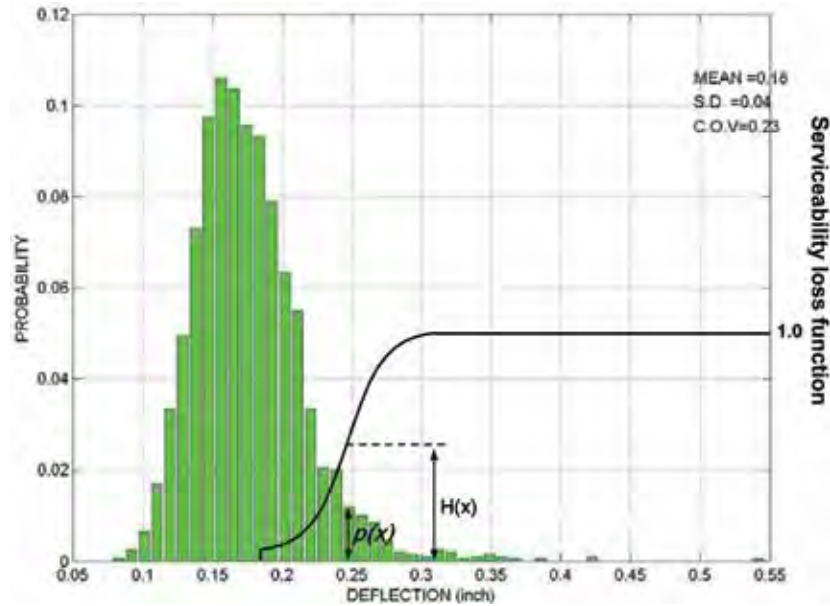
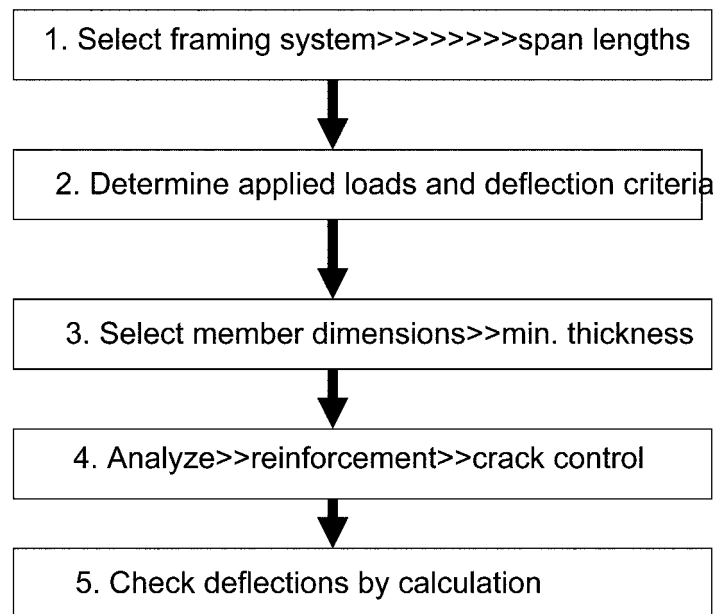


Fig. 6—Deflection histogram and serviceability loss function. (Note: 1 in. = 25.4 mm.)

DESIGN



CONSTRUCTION

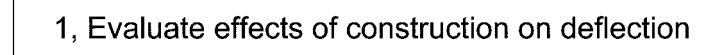


Fig. 7—Design and construction process including deflection control.

Effect of Sustained Loads on Flexural Crack Width in Concrete Beams Reinforced with Internal FRP Reinforcement

by S.P. Gross, J.R. Yost, and D.J. Stefanski

Synopsis: An experimental study was conducted to evaluate the increase in crack width occurring over time in FRP-reinforced concrete as a result of sustained loading. Twelve beams (eight GFRP, two CFRP, and two steel-reinforced) were maintained under a constant sustained service load for nearly three years. Three flexural cracks were monitored on each beam over the duration of the test. The observed increase in flexural crack widths over the study was greater in the FRP-reinforced specimens than in the steel-reinforced specimens. On average, flexural crack widths in FRP-reinforced concrete specimens were observed to double over one year of sustained loading. A simple design approach, based on modification to the existing ACI 440 (2006) crack control procedure, is proposed to account for this observed increase in crack widths with time.

Keywords: crack width; cracking; creep; fiber-reinforced polymer (FRP); serviceability; shrinkage; sustained load; tension stiffening.

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INTRODUCTION

Control of flexural cracking is an important consideration in the design of reinforced concrete members for serviceability. While there continues to be debate over the nature of the relationship between crack width and corrosion of internal reinforcement, there is no doubt that excessive cracking or large crack widths can lead to aesthetic and performance concerns in many types of structures. Control of cracking takes on added importance in the design process when fiber-reinforced polymer (FRP) internal reinforcement is used. Although FRP materials have the advantage of not being susceptible to corrosion, they have a lower elastic modulus than steel. The elastic moduli of commercially available glass FRPs (GFRP) are about 20 to 25% that of steel, while for commercially available carbon FRPs (CFRP) the elastic moduli are about two-thirds that of steel. This lower elastic modulus in turn leads to larger crack widths (and deflections) than would be observed in identical members with an equal amount of steel reinforcement. Thus, serviceability considerations will often control the design of FRP-reinforced concrete whereas steel-reinforced concrete is typically governed by capacity at the strength limit state.

Flexural crack width measurements have been made by numerous researchers on steel-reinforced concrete over the past half-century, and more recently on FRP-reinforced concrete. In most of these studies, instantaneous crack width has been measured after the application of a short-term service level load. Few studies have examined the change in crack widths over time resulting from the application of a sustained load. Furthermore, no mention is made of time-dependent effects in crack control provisions of design codes such as ACI 318 (2008) and design guides such as ACI 440.1R (2006).

Among those studies that have focused on long-term behavior, Lutz et al. (1967) studied the change in crack widths for approximately two dozen flexural cracks on two steel-reinforced beams maintained under a sustained load for a period of five months. One of the beams had compression reinforcement equal to the tensile reinforcement, while the other was singly reinforced. They found that crack widths increased by about 30 to 40 percent over the test duration, with most of the increase coming over the first three to four weeks of loading. On average, the increase in crack width was only 5% greater for the singly-reinforced beam than for the doubly-reinforced beam. The increase in crack widths over time was attributed to several factors, including internal cracking related to changes in the steel and concrete stresses between cracks due to sustained load. Changes in the cracked section curvature due to concrete creep, and shrinkage of concrete between cracks, were thought to play a less significant role.

Illston and Stevens (1972) also monitored the changes in flexural cracking over two years of sustained loading for a series of sixty steel-reinforced concrete specimens. They also found that the crack widths tended to increase with time at a reducing rate, and observed that on average, measured crack widths doubled over a two year period of sustained loading. They attributed the increase to a reduction in average concrete tensile stress between cracks due to a time-dependent change in curvature and the effects of concrete shrinkage, as well as a breakdown in bond with sustained loading.

Bakis and Boothby (2004) investigated the crack width and bond strength of GFRP-reinforced beams under sustained load in a variety of environmental conditions over a period of three years. Loading environments included indoor and outdoor conditions, as well as “aggressive” freeze-thaw and calcium hydroxide saturated environments. Crack widths were found to increase by about 40% on average for the indoor environment and about 60% on average for the outdoor environment over the three year test duration. Crack widths increased more rapidly at early ages for the outdoor environment, while the increase in crack width was more gradual for the indoor environment.

In a previous study by the authors (Gross et al. 2006), crack widths were monitored on six reinforced high-strength concrete beams over a 90 day period of sustained load. Two of the beams utilized GFRP, two used CFRP, and two used steel. Crack widths were found to increase 13%, 15%, and 20% for GFRP, CFRP, and steel, respectively. Almost all of the time-dependent increase in crack width occurred over the first two weeks under sustained load.

RESEARCH SIGNIFICANCE

As a result of the lower elastic modulus of FRP reinforcement as compared to steel and the resulting potential for larger crack widths, consideration of crack control takes on added importance for the design of FRP-reinforced concrete. Current crack control approaches consider only the instantaneous effects of applied load, and do not consider the time-dependent behavior under sustained loads. This study examines time-dependent behavior by measuring the change in crack width over time for FRP-reinforced concrete specimens and companion steel-reinforced concrete specimens. Suggestions are made for how to incorporate time-dependent effects into design estimations of flexural crack width.

ESTIMATION OF FLEXURAL CRACK WIDTH

Several researchers have proposed methods for estimating flexural crack widths resulting from service loads (Kaar and Mattock 1963; Base et al. 1966; Ferry Borges 1966; Gergely and Lutz 1968; Beeby 1971; Frosch 1999). All of the proposed methods are similar in that they predict the flexural crack width as a function of the tensile stress (or strain) in the reinforcement at service load, and geometric parameters such as reinforcement cover, spacing, and the depth of the elastic neutral axis. The methods differ primarily in how they address the effects of these geometric parameters, and in assumptions related to the bond stresses developed between the reinforcement and the surrounding concrete. All of the methods were originally developed for deformed steel reinforcement, but can be modified to consider the use of other reinforcements such as FRP, smooth steel wire, or epoxy-coated steel bars.

Historically speaking, the most commonly used crack width prediction equation is probably the empirical Gergely-Lutz equation (Gergely and Lutz 1968), which was developed using a database of nearly 1000 crack width measurements from several studies:

$$w = (0.076\beta f_s \sqrt[3]{d_c A}) \times 10^{-3} \quad (1)$$

where

- w = probable maximum crack width (at the extreme tension fiber), in.;
- f_s = steel reinforcement stress at service, ksi;
- d_c = concrete cover from extreme tension fiber to the center of the closest reinforcing bar, in.;
- b_w = web width, in.;
- n_b = number of tensile reinforcing bars;
- A = effective tension area of concrete surrounding each reinforcing bar = $2d_c b_w / n_b$ for a single layer of bars, in.²;
- h = beam depth, in.;
- d = effective depth to center of reinforcement, in.;
- c = neutral axis depth under service load, in.;
- β = ratio of distance from neutral axis to extreme tensile fiber and distance from neutral axis to center of tensile reinforcement = $(h - c)/(d - c)$.

Equation (1) was developed for use with deformed steel reinforcing bars but can be generalized for use with other types of reinforcement by replacing 0.076 times the steel reinforcement stress with 2.2 times the reinforcing strain since $(0.076 \times 10^{-3})(29,000) = 2.2$, and by introducing a bond-dependent coefficient (ACI Committee 440 2001)

$$w = 2.2\beta k_b \varepsilon_r \sqrt[3]{d_c A} \quad (2)$$

where

- ε_r = reinforcement strain at service, ksi;
- k_b = bond-dependent coefficient (= 1.0 for deformed steel bar reinforcement, < 1.0 for superior bond relative to deformed steel bars, > 1.0 for inferior bond relative to deformed steel bars).

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Frosch (1999) observed that Eq. (1) was developed on the basis of a limited range of concrete cover distances, and developed an alternative expression that provided better fit to experimental results over a large range of concrete cover:

$$w = 2\beta \frac{f_s}{E_s} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (3)$$

where

E_s = modulus of elasticity of reinforcing steel = 29000 ksi (200 GPa);

s = center-to-center spacing of reinforcing bars within outermost layer, in.

As with Eq. (1), Eq. (3) can be generalized to consider any type of reinforcement (ACI Committee 440 2006):

$$w = 2\beta k_b \epsilon_r \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (4)$$

Equations 2 and 4 are valid in both inch-pound and SI unit systems, since β , k_b , and ϵ_r are all unitless.

Regardless of the expression used, it is well documented that large variations can be expected between measured and calculated crack widths. Thus, calculated crack widths should only be considered as rough estimates of the expected crack width in members in service. ACI Committee 224 (2001) reports that the coefficient of variation of crack width is on the order of 40% and that isolated cracks in beams can exhibit widths double that computed by prediction equations. It is also important to recognize that the generalized forms in Eq. (2) and 4 rely on a service reinforcement strain computed using simple elastic analysis for a reinforced concrete section. Neither the Gergely-Lutz nor Frosch model explicitly considers time-dependent effects.

CRACK CONTROL IN DESIGN CODES

For almost four decades, design codes in the United States have used flexural crack control provisions related to the Gergely-Lutz model. From 1971 through 1995, ACI 318 (1995) relied on a modified form of the Gergely-Lutz expression (Eq. (1)) to compute a “z factor” for use in crack control calculations:

$$z = f_s \sqrt[3]{d_c A} \quad (5)$$

As noted in the ACI 318 commentary, the computed z value is directly related to a maximum crack width w but the form of Eq. (5) essentially allows the designer to perform a crack control calculation without computing a crack width per se. Code-specified limits of 175 kips/in. (31000 kN/m) and 145 kips/in. (25000 kN/m) for z correspond to limiting crack widths of 0.016 in. (0.41 mm) and 0.013 in. (0.33 mm) for interior and exterior exposure conditions, respectively when β is taken as 1.2.

Initial versions of the ACI 440.1R (2001, 2003) design guide for FRP-reinforcement used the modified version of the Gergely-Lutz expression (Eq. (2)). Values of the bond coefficient k_b ranging from 0.71 to 1.83 were reported in the literature (Gao et al. 1998), and the design guide recommended taking k_b as 1.2 for deformed FRP bars in lieu of test results. Though the design guide did not set specific requirements in mandatory language with regard to limiting crack widths, it endorsed the limits of the Canadian Highways Bridge Design Code (Canadian Standards Association 1996) for FRP reinforcement. These limiting values were 0.028 in. (0.71 mm) for interior exposure and 0.020 in. (0.51 mm) for exterior exposure. The wider crack width limits as compared to traditional limits for steel were justified by noting that the non-corrosive properties of FRP can be used to relax crack width requirements when corrosion is the primary reason for such limitations.

Since 2002, ACI 318 has based crack control provisions on the work of Frosch (Eq. (3)). Rather than require computations of crack width, ACI 318 uses the philosophy of specifying a maximum spacing corresponding to an implicit limiting crack width. The maximum spacing can be found by rearranging Eq. (3) as follows:

$$s = 2 \sqrt{\left(\frac{w E_s}{2 f_s \beta} \right)^2 - d_c^2} \quad (6)$$

Note however, that ACI 318-08 (2008) uses a simple bilinear equation as an approximation of Eq. (6)

$$s = \frac{540}{f_s} - 2.5c_c \leq 12 \left(\frac{36}{f_s} \right) \quad (7)$$

where

c_c = clear concrete cover from extreme tension fiber to the edge of the closest reinforcing bar, in.

A comparison of Eqs. 6 and 7 shows that Eq. 7 corresponds to a limiting crack width of about 0.018 in. (0.46 mm) for most common cases when f_s is taken as two-thirds of the yield stress f_y as is suggested in ACI 318-05 (2005) and ACI 318-08 (2008). In ACI 318-02 (2002), prior to an adjustment of load factors for design, f_s was taken as $0.60f_y$, resulting in an effective crack width limit of 0.016 in. (0.41 mm). Note that beginning in 2002, no distinction is made between crack control provisions for interior and exterior exposure conditions, as the premise of crack control shifted to focus largely on aesthetic considerations and not on corrosion of reinforcement.

The latest version of the ACI 440.1R (2006) design guide for FRP reinforcement also transitioned to the Frosch model. Rather than computing a limiting bar spacing, however, Eq. (4) is used directly. A study to recalibrate the default k_b factor on the basis of available test data for FRP-reinforced concrete resulted in a suggested k_b value of 1.4 when the value cannot be established based on specific experimental data (Bakis et al. 2006). The CSA crack width limits of 0.028 in. (0.71 mm) for interior exposure and 0.020 in. (0.51 mm) for exterior exposure were still endorsed.

There continues to be an examination of the crack control provisions by ACI Committee 440 for FRP-reinforced concrete, including debate over the best form in which to present crack control requirements. However, the fact that calculation of crack widths forms the basis for all of the crack control provisions discussed above cannot be ignored. Past and present ACI 318 methods effectively hide the limiting crack width by requiring calculation of other parameters (e.g. z or s), but these methods are still inherently tied to a limiting crack width value.

Furthermore, the importance of long-term effects such as sustained loading should not be understated, yet ACI 318 and ACI 440 methods do not (either explicitly or implicitly) address how long-term effects should be considered. Thus, if crack control provisions that are based only on the short-term loading behavior are narrowly satisfied in a given design, the long-term serviceability performance of the structure might be compromised if the crack widths grow significantly over time.

EXPERIMENTAL PROGRAM

In order to address the above concerns, an experimental program was conducted at the Villanova University Structural Engineering Teaching and Research Laboratory to study the serviceability behavior of FRP-reinforced concrete beams under a period of sustained loading. The study consisted of twelve beam specimens maintained under constant flexural load in a controlled environment for a period of almost three years. Four different reinforcement types were utilized, including two types of GFRP, one type of CFRP, and mild steel. Deflection and flexural crack width were both studied, but only crack width results are reported in this paper.

To facilitate comparison and analysis of the results, all test specimens were designed to have the same dimensions and only the reinforcement was varied. Each beam measured 4 x 8 in. (100 x 200 mm) in cross section and was tested in four-point bending over a simply-supported 6.0 ft. (1.8 m) span, as shown in Fig. 1. All specimens were reinforced with two bars on the tension face of the cross section, using a clear cover of 1 in. (25 mm). Compression reinforcement and transverse (shear) reinforcement were not used in any of the specimens.

Tension reinforcement was either nominal No. 2, No. 3, or No. 4 (No. 6, No. 10, or No. 13) bars, with the properties identified in Table 1. FRP reinforcement properties are reported by the respective manufacturers and represent nominal properties used in design. The four reinforcement types are shown in Fig. 2. The GFRP produced by "Manufacturer X" has an external surface that features a helical wrap and sand coating. The GFRP produced by "Manufacturer Y" features a coarse quartz sand coating. The CFRP produced by "Manufacturer X" features molded ribs along its external surface. The mild steel is ordinary, readily available deformed Grade 60 (415 MPa) reinforcement.

All beams were fabricated in the laboratory using the same set of concrete mixture proportions. Specimens were cast in sets of two or four beams. The mixture proportions were established by trial batching, and tests of the trial batch selected for the study resulted in design properties of 4500 psi (31 MPa) for the concrete compressive strength f'_c and 5.2×10^6 psi (36 GPa) for the concrete modulus of elasticity E_c . Note that this modulus is signifi-

cantly higher than the value computed by the ACI 318 equation $E_c = 57,000\sqrt{f'_c}$ (psi) ($4700\sqrt{f'_c}$ [MPa]).

The test matrix of twelve specimens was split into two groups, as shown in Table 2. Group 1 consists of more lightly reinforced specimens, while Group 2 consists of more heavily reinforced specimens. Each group consisted of four GFRP-reinforced beams (two each using two different manufacturers' bars), one CFRP-reinforced beam, and one steel-reinforced beam. Within Group 1, GFRP- and CFRP-reinforced beams had reinforcement ratios ρ of approximately twice the balanced ratio ρ_{bal} , while Group 2 GFRP- and CFRP-reinforced beams were at approximately three times the balanced ratio. Group 1 and 2 steel-reinforced beams were underreinforced.

Service loads used in testing were established after considering all basic design criteria (flexural strength, shear strength, deflection, flexural crack width, and creep rupture stress limits) in accordance with ACI440.1R-03 (the most current design guidelines at the time) for FRP-reinforced specimens. A superimposed dead load to live load ratio of 3:1 was assumed, and service loads were back-calculated using the loading arrangement shown in Fig. 1. The service moments given in Table 2 include the self-weight of the test specimen and reflect the approximate allowable moments above which serviceability criteria cease to be met. Though the nominal moment capacity M_n of each specimen type within each group varied somewhat as a result of the use of discrete bar sizes in identical cross sections, all six specimens within the same group were loaded to the same service loads so that results can be directly compared.

Two separate service load moments are reported in Table 2 for each group. Approximately 90 days after casting, each beam was temporarily loaded to a moment corresponding to the total service dead plus live load ($M_{s,temp}$). Crack width measurements were recorded and the beam was immediately unloaded. The beam was then reloaded to a lower moment corresponding only to the service dead load ($M_{s,sus}$), and this load was sustained over the duration of the test. This "pre-cracking" to a higher moment than the sustained moment reflects the loading history that a member might actually undergo in the field, and stabilizes the cracking pattern by reducing the potential for additional flexural cracks to form over the duration of the time-dependent study. Temporary service moments correspond to 1.93 and 2.39 times the cracking moment (M_{cr}) for Groups 1 and 2, respectively, while sustained service moments correspond to 1.47 and 1.81 times the cracking moment, respectively.

Beams were loaded within an environmentally controlled room equipped with a strong reaction floor. The temperature and relative humidity were maintained at 72 ± 2 °F (22 ± 1 °C) and $55 \pm 5\%$, respectively throughout the duration of the test. Load was applied to each beam through a loading rack consisting of 0.75 in. (20 mm) diameter threaded rods and 1 in. (25 mm) thick steel plates, anchored to the reaction floor. Hydraulic cylinders and load cells were used for the application of the temporary "pre-cracking" load and for application of the initial sustained load, as shown in Fig. 3(a). Railroad springs were used to maintain the sustained load. Spring elongation was monitored over the duration of the study and the load was maintained by tightening the locking nuts whenever the spring compression (load) changed by 2% from the target value. This ensured a constant ($\pm 2\%$) load throughout the duration of the study.

Flexural cracking in a typical test specimen can be seen in Fig. 3(b). For each test specimen, the three flexural cracks closest to midspan were labeled (A, B, and C) and these three cracks were monitored throughout the duration of the study. Lines were drawn on the concrete surface adjacent to each crack to facilitate locating them for measurement. Crack widths were measured at the extreme tension fiber on the side of the beam (i.e. bottom of the side face), since this is the location on the side face where crack widths should be maximum. Measurements were made using a jeweler's scope with 50x magnification (Fig. 3(c)), which permits measurement resolution of 0.001 in (0.025 mm).

RESULTS

Flexural crack widths are plotted vs. time in Fig. 4 and 5 for Group 1 and Group 2 beams, respectively. For each beam, data is presented for the three monitored flexural cracks (A, B, and C). For each of the steel-reinforced beams, only two flexural cracks occurred so that there is no data for crack A. Measurements were recorded frequently—approximately once a week for 90 days and then once every other week—for a period of approximately 450 days, and that data is presented herein. After that time, crack width measurements were not recorded until just before the beams were unloaded at 1000 days. Because this 1000 day measurement was recorded by a different person than were the original measurements, it is not presented here so as to avoid any potential "reader error". It is worth noting, however, that 1000 day crack widths were generally found to be very consistent with the final measurement points presented in Fig. 4 and 5 (i.e. the crack widths appear to have changed little between 450 and 1000 days).

All measured crack widths increased with time under sustained load. As observed in other studies, the majority

of the increase in crack width occurred very early within the sustained loading period, and smaller incremental increases were observed later in the study. Measured crack widths among the three cracks examined for a single beam showed reasonable agreement in most cases. However, for a few beams such as GX3A and GY3A there was significant scatter between the largest and smallest measured crack widths. Pairs of identical beams (e.g. GX4A and GX4B) generally exhibited reasonable agreement between the measured cracks on the two beams.

Maximum crack widths for each beam are reported in Table 3 at several time stages. The maximum crack width is the largest value recorded on any of the three cracks monitored for each beam and represents the overall maximum crack width for the beam. The first column represents the crack width after the temporary “precracking” load is applied; this data is not shown in the plots of Fig. 4 and 5, which begin with application of the sustained load.

ANALYSIS OF RESULTS

Maximum measured crack widths due to the “precracking” load and associated moment $M_{s,temp}$ in both GFRP and CFRP-reinforced specimens were all less than the ACI 440 (2006) suggested crack width limit of 0.028 in. (0.71 mm) for interior exposure. In fact, only the “precracking” load crack width for beam GX3A exceeded a width of 0.020 in. (0.51 mm) corresponding to the ACI 440 suggested limit for exterior exposure. Time-dependent crack widths due to the sustained load moment $M_{s,sus}$ were also less than these limiting values for all FRP-reinforced specimens except for beam GX3A. Instantaneous and time-dependent crack widths for the steel-reinforced concrete specimens were much lower than for the FRP-reinforced specimens, and were well below the crack width limits assumed in the development of Eq. 7. It should be noted, however, that steel reinforcement stress under the sustained service load are approximately $0.50f_y$ and $0.36f_y$ for specimens ST3 and ST4, respectively, or well less than the $0.67f_y$ typical of service loads in steel-reinforced concrete. These differences result from the decision to use identical service loads for all specimens within a group, regardless of reinforcement type.

Crack widths for Group 1 specimens were generally larger than for comparable Group 2 specimens. This can be attributed to the higher reinforcement strains in the Group 1 specimens at the selected service load values, since these specimens are less overreinforced (or for steel, more underreinforced) than the Group 2 specimens. Figure 6 shows all initial crack width measurements for both “precracking” and sustained load levels plotted against the calculated service strain based on linear-elastic analysis. It can clearly be seen that crack widths increase as reinforcement strains increase, suggesting that the basic forms of Eq. (2) and (4) have merit for initial behavior.

Figure 7 shows the ratio of measured and calculated maximum crack widths for each beam at both of the service load levels. Calculated crack widths for FRP-reinforced concrete are based on Eq. (2) with $k_b = 1.2$ as suggested by ACI 440 (2003), or on Eq. (4) with $k_b = 1.4$ as suggested by ACI 440 (2006). Calculated crack widths for steel assume $k_b = 1.0$ for both Eq. (2) and (4). The average measured/calculated ratio for FRP-reinforced concrete is 0.65 when using Eq. 2 and 0.76 when using Eq. (4), indicating an overprediction of the maximum crack width in both cases. For steel, the average ratios were 1.02 and 1.40 for Eqs. (2) and (4), respectively, indicating an underprediction of crack width.

Alternatively, average k_b values can be computed for each reinforcement type by rearranging Eqs. 2 and 4 and substituting in all measured crack width values. Based on this approach, average k_b values were found to be 0.81, 0.72, 0.85, and 1.02 for GFRP (Manufacturer X), GFRP (Manufacturer Y), CFRP (Manufacturer X), and steel, respectively when Eq. (2) is used. The respective k_b values for Eq. (4) become 1.12, 0.99, 1.13, and 1.40. It is noteworthy that the average k_b values were higher for steel than for any of the FRP specimens implying *better* bond for FRP than steel, however it should be noted that the measured crack widths are so small (≤ 0.005 in. [0.13 mm]) for the steel-reinforced specimens that this is a fairly imprecise calculation for these specimens.

In examining the changes in crack width over time, all measured crack widths are considered rather than just the maximum crack width for a given beam. Crack width growth over one year of sustained load, defined as the difference between the crack width after one year and the crack width after initial application of the sustained service moment $M_{s,sus}$, is shown in Fig. 8 for all monitored cracks. The one year time stage is chosen to facilitate comparison of data from all specimens, since the final data points presented vary within a range from 417 to 453 days of sustained load. As noted previously, there was little observed change in crack widths after one year. Crack width growth is plotted instead as a multiple of the initial sustained moment crack width in Fig. 9. Statistical values related to the growth in crack width are provided in Table 4.

Over this one year period, crack widths in FRP-reinforced beams increased as much as 0.016 in. (0.41 mm), with a median crack width growth of 0.008 in. (0.20 mm). Beams reinforced with GFRP produced by manufacturer X showed more crack width growth than beams reinforced with the other two types of FRP, which is consistent with the larger observed initial cracks with this reinforcement type. Crack widths in steel-reinforced beams grew

substantially less, with a median increase of 0.002 in. (0.05 mm). The median multiple for crack width at one year relative to the instantaneous crack width upon application of the sustained load is 2.00 for all FRP-reinforced specimens, which means that on average the crack widths doubled over one year under sustained load. Multiples were somewhat higher for CFRP-reinforced specimens than for GFRP-reinforced specimens. For steel-reinforced specimens, the median multiple was 1.67.

Table 4 also presents the crack widths after one year under sustained load as a multiple of the initial crack width due to the temporary precracking moment $M_{s,temp}$. The median multiple for all FRP-reinforced concrete specimens was 1.52, and the mean multiple for steel specimens was 1.25. These values indicate that for the majority of monitored cracks, the crack width is higher after one year under a sustained load level equal to about three-quarters of the precracking load than due to the initial effects under the precracking load itself. A crack width prediction considering only elastic effects associated with the highest service load, in this case the precracking load, would not predict the maximum crack width occurring over time.

DISCUSSION OF CRACK WIDTH GROWTH

Figure 10 presents a conceptual basis for the discussion of why crack width increases over time under a sustained load. Figure 10a shows a stabilized crack pattern at service load in a typical beam under a simple third-point loading condition, such that there is a region of constant moment. It is widely recognized that the width of a flexural crack at point C is a function of the differences in reinforcement strain (ϵ_r) and concrete strain at the level of the reinforcement (ϵ_c) between points A and B, each halfway to the adjacent flexural crack (MacGregor and Wight 2005). In particular

$$w = \int_A^B (\epsilon_r - \epsilon_c) dx \quad (8)$$

Since the moment is constant, the crack width may be conveniently examined by considering the differences in strain between two adjacent cracks C and D, instead of between points A and B. First, consider the stresses in the reinforcement and the concrete at the level of the reinforcement, as sketched in Figs. 10b and 10c. The elastic reinforcement stresses at the crack will be relatively high, and the concrete stresses zero, since the section is cracked and the reinforcement resists the entire internal tension force. Midway between the cracks, the concrete will carry some tension and the reinforcement stress will be reduced. The amount of reduction depends on the magnitude of the bond stresses developed between the reinforcement and concrete; high bond stresses may result in a substantial reduction in reinforcement stress relative to the cracked section. At this elastic stage, strains are directly proportional to stresses and as such the strain plots in Fig. 10(d) are directly proportional.

As a limiting value, the reinforcement and concrete stresses that would occur in a fully-bonded uncracked section are also noted in Figs. 10(b) and (c). Although the bond stresses likely cannot develop the concrete stress corresponding to this condition, it is important to note that the difference in magnitude of the reinforcement stresses between the cracked and uncracked limiting conditions would be on the order of 30 to 40 times for GFRP (as assumed in Figs. 10(b) and (c), and only about 8 to 9 for steel, given the material properties used in this study. Thus, there is a substantially more significant gradient in the plot of reinforcement stresses for GFRP as compared to steel, a direct result of the difference in elastic moduli and resulting modular ratios for these materials.

A simple cracked-section analysis using the age-adjusted effective modulus approach (MacGregor and Wight 2005) indicates that at a crack, the reinforcement stresses and strains increase minimally as a result of concrete creep. Assuming a creep coefficient of 2 (that is, a tripling of strains over time under a constant compressive stress) with the material properties in this study yields an increase in reinforcement stress (and strain) at the cracked section of only 3% for GFRP and 6% for steel. These minimal increases can be seen in Fig. 10(b), (c), and (d).

Analysis of changes in stress and strain between the cracks is much more complex, since the concrete resists some of the tensile stresses. However, an age-adjusted effective modulus approach at an uncracked fully-bonded section using a creep coefficient of 2 for both concrete compression and concrete tension indicates that for both the GFRP and steel material properties considered in this study, the reinforcement and concrete strains might be expected to almost triple. The reinforcement stress would also triple whereas the concrete stress would change negligibly because creep is effectively redistributing stress from the concrete to the reinforcement.

Locations between cracks range from a fully-cracked section to a section approaching the fully-bonded uncracked condition (dependent on the extent of bond stresses developed), therefore the change in stresses

and strains at any location between the cracks will vary significantly. However, the net effect will clearly be a more significant change in reinforcement strains (and stresses) than is implied by only considering the behavior at the crack location. Because the reinforcement strains are so much larger than the concrete strains, the change in crack width is approximately equal to the change in the area under the reinforcement strain curve in Fig. 10(d).

Studies focusing on tension stiffening have confirmed that the reinforcement stress between cracks does increase significantly over time, and that this change happens fairly quickly (Scott and Beeby 2005; Gilbert 2007). Though the effects of concrete creep are undoubtedly responsible for this as described above, earlier studies of time-dependent changes in crack width on steel-reinforced concrete (Lutz et al. 1967; Illston and Stevens 1972) suggest that there may be a breakdown in the bond between the reinforcement and concrete over time. Any breakdown in bond would reduce the concrete stress between cracks and increase the reinforcement stress and strain, thus increasing further the area under the reinforcement strain curve in Fig. 10(d) and the associated crack width.

Crack widths will also increase over time as a result of concrete shrinkage, though this effect is time-dependent rather than load-related. The effect of shrinkage on crack widths will depend on the amount of shrinkage, the spacing of adjacent flexural cracks, and the restraint provided by the reinforcement. For example, since GFRP reinforcement has a significantly lower elastic modulus than CFRP or steel reinforcement, it will provide far less restraint and the concrete will shrink more. While shrinkage undoubtedly plays a role in the increase in crack widths over time, it is thought to be less significant in the results of this experimental study since the stabilized cracks were usually only about 3 to 4 in. (75 to 100 mm) apart and since the study began 90 days after casting the concrete, after which much of the shrinkage will have already taken place.

RECOMMENDATION FOR DESIGN

Since the focus of this study is on FRP, modifications to the existing ACI 440 (2006) approach (Eq. (4)) are presented here. The results of this study and past research clearly indicate the crack widths increase over time under sustained load. Thus, two checks should be made in design. The first check should be based on a total service load and should not consider sustained load effects. This check is effectively the same check that is currently recommended by ACI 440.1R. The second check should be based on only the portion of service load which is sustained, but should consider time-dependent effects. The crack width should be computed as the larger of Eq. (9a) and (9b), and compared to established limiting values for design.

$$w = 2\beta k_b \varepsilon_{serv} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (9a)$$

$$w = 2\beta k_t k_b \varepsilon_{sus} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (9b)$$

where

ε_{serv} = reinforcement stress due to total service load, ksi;

ε_{sus} = reinforcement stress due to sustained portion of service load, ksi;

k_t = time-dependent coefficient related to increase in crack growth due to sustained load.

Based on the limited results of this study, it is recommended that k_t be taken as 2.0 for FRP-reinforced concrete. It is recognized that the approach of using a simple multiplier is simplistic in nature, however it is convenient for design. Further work is needed to better understand the different factors that lead to the increase in crack widths with time. As such information becomes available, a refined design methodology may be considered.

CONCLUSIONS

The most important observations made in this research program are noted below:

1. Initial measured crack widths for FRP-reinforced beams tended to be overpredicted by Eq. (2) and (4), while initial measured crack widths for steel-reinforced beams tended to be underpredicted by the same equations when k_b was taken as 1.0 for steel and as values recommended for design by ACI 440 (2003, 2006) (1.2 and 1.4 for Eqs. (2) and (4), respectively).
2. Average k_b values based on initial crack widths were found to be 0.81, 0.72, 0.85, and 1.02 for GFRP

- (Manufacturer X), GFRP (Manufacturer Y), CFRP (Manufacturer X), and steel, respectively when Eq. (2) is used. The respective average k_b values for Eq. 4 become 1.12, 0.99, 1.13, and 1.40 when Eq. (4) is used.
3. Over one year of sustained loading, crack widths in FRP-reinforced beams increased as much as 0.016 in. (0.41 mm), with a median crack width growth of 0.008 in. (0.20 mm). The median multiple for crack width at one year relative to the instantaneous crack width upon application of the sustained load was 2.00 for all FRP-reinforced specimens, indicating that on average the crack widths doubled over one year under sustained load. Crack widths in steel-reinforced beams grew substantially less, with a median increase of 0.002 in. (0.05 mm), equivalent to a 67% increase relative to the initial crack width.
 4. Crack widths grow under sustained load primarily as a result of time-dependent changes in reinforcement and concrete strains between cracks over time. Reinforcement strains increase significantly as a result of concrete creep and possible degradation of bond between the reinforcement and concrete. Shrinkage of concrete also likely plays a role in the increase in crack width over time, though this effect is not load-related.
 5. In order to account for the increase in crack width due to sustained loading, Eq. (9a) and (9b) are recommended for use in the design of FRP-reinforced concrete. The larger computed crack width resulting from consideration of the maximum service load, or the sustained service load with consideration of time-dependent effects, should be used.

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Table 1—Reinforcement material properties

Reinforcement Type	Nominal Cross-Sectional Diameter d_b (in.)	Nominal Cross-Sectional Area A_b (in. ²)	Modulus of Elasticity E_r (ksi)	Guaranteed Tensile Strength f_{tu} (ksi)	Yield Strength f_y (ksi)
#3 GFRP (Manufacturer X)	0.375	0.110	5920	110	N/A
#3 GFRP (Manufacturer Y)	0.375	0.110	6696	111	N/A
#4 GFRP (Manufacturer X)	0.500	0.196	5920	100	N/A
#4 GFRP (Manufacturer Y)	0.500	0.196	6725	103	N/A
#2 CFRP (Manufacturer X)	0.254	0.056	18000	350	N/A
#3 CFRP (Manufacturer X)	0.362	0.101	20000	250	N/A
#3 Steel	0.375	0.110	29000	N/A	60
#4 Steel	0.500	0.196	29000	N/A	60

Note: All values shown are nominal values reported by manufacturer.

1 in. = 25.4 mm; 1 ksi = 6.895 Mpa; 1 in.² = 645.2 mm².

Table 2—Test specimen parameters

Beam(s)	Group 1				Group 2			
	GX3A GX3B	GY3A GY3B	CX2	ST3	GX4A GX4B	GY4A GY4B	CX3	ST4
Reinforcement (Manufacturer)	#3 GFRP (X)	#3 GFRP (Y)	#2 CFRP (X)	#3 Steel	#4 GFRP (X)	#4 GFRP (Y)	#3 CFRP (X)	#4 Steel
ρ	0.0081	0.0081	0.0034	0.0081	0.0145	0.0145	0.0074	0.0145
ρ/ρ_{bal}	2.02	1.85	2.10	0.26	3.05	2.89	3.03	0.47
M_{cr} (kip-in)	21.5				21.5			
M_n (kip-in)	103.4	108.7	115.9	84.2	127.9	134.2	159.2	140.6
$M_{s,temp}$ (kip-in)	41.4				51.3			
$M_{s,sus}$ (kip-in)	31.5				38.9			
$M_{s,temp}/M_{cr}$	1.93				2.39			
$M_{s,sus}/M_{cr}$	1.47				1.81			
$M_{s,temp}/M_n$	0.40	0.38	0.36	0.49	0.40	0.38	0.32	0.36
$M_{s,sus}/M_n$	0.30	0.29	0.27	0.37	0.30	0.29	0.24	0.28
Casting Date	10/14/2005	10/21/2005	11/18/2005	11/11/2005	10/14/2005	10/21/2005	11/18/2005	11/11/2005
Loading Date	1/12/2006	1/19/2006 *	2/17/2006	2/7/2006	1/13/2006	1/19/2006 *	2/17/2006	2/8/2006

* GY3A and GY4B loaded on 1/20/2006

1 kip-in = 113.0 N-m

Table 3—Maximum measured flexural crack widths

	Beam	Maximum Measured Flexural Crack Width (in.)						
		End of Temporary Loading	End of Reloading to Sustained Load	After Time Under Sustained Load				
				30 Days	90 Days	180 Days	1 Year	Final Reading (days)
Group 1	GX3A	0.027	0.021	0.031	0.031	0.033	0.033	0.034 (453)
	GX3B	0.012	0.009	0.017	0.021	0.021	0.023	0.025 (453)
	GY3A	0.019	0.011	0.019	0.021	0.023	0.024	0.024 (445)
	GY3B	0.017	0.013	0.015	0.017	0.018	0.020	0.020 (446)
	CX2	0.015	0.011	0.011	0.014	0.015	0.017	0.017 (417)
	ST3	0.005	0.004	0.004	0.005	0.005	0.006	0.006 (427)
Group 2	GX4A	0.019	0.011	0.017	0.020	0.020	0.022	0.022 (452)
	GX4B	0.010	0.007	0.010	0.012	0.012	0.014	0.015 (452)
	GY4A	0.013	0.008	0.010	0.013	0.014	0.015	0.016 (446)
	GY4B	0.007	0.005	0.008	0.012	0.012	0.014	0.014 (445)
	CX3	0.006	0.005	0.007	0.007	0.008	0.011	0.011 (417)
	ST4	0.004	0.003	0.004	0.005	0.005	0.005	0.005 (426)

1 in. = 25.4 mm

Table 4—Statistics for crack width growth over 1 year under sustained load

	Reinf. Type	n	Minimum	Maximum	Mean	Median	St Dev
$w_{1yr} - w_{Msus}$ (in.)	All FRP	30	0.000	0.016	0.009	0.008	0.004
	GFRP (X)	12	0.006	0.016	0.011	0.011	0.003
	GFRP (Y)	12	0.004	0.013	0.008	0.007	0.003
	CFRP (X)	6	0.000	0.010	0.006	0.007	0.004
	Steel	4	0.002	0.003	0.002	0.002	0.001
w_{1yr} / w_{Msus}	All FRP	30	1.00	8.00	2.48	2.00	1.29
	GFRP (X)	12	1.57	3.29	2.33	2.00	0.62
	GFRP (Y)	12	1.42	3.00	2.14	2.03	0.57
	CFRP (X)	6	1.00	8.00	3.42	2.73	2.58
	Steel	4	1.50	2.50	1.83	1.67	0.45
w_{1yr} / w_{Mtemp}	All FRP	30	0.78	3.25	1.71	1.52	0.60
	GFRP (X)	12	0.78	2.56	1.59	1.52	0.47
	GFRP (Y)	12	1.00	2.40	1.67	1.57	0.49
	CFRP (X)	6	1.00	3.25	2.05	2.08	0.95
	Steel	4	1.20	1.25	1.24	1.25	0.03

1 in. = 25.4 mm

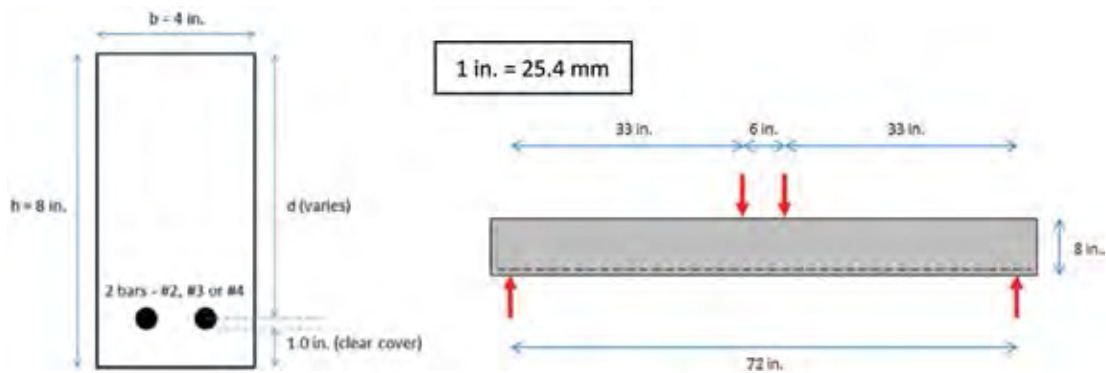


Fig. 1—Test specimen cross section and loading arrangement.



Fig. 2—Reinforcement types used in the study.

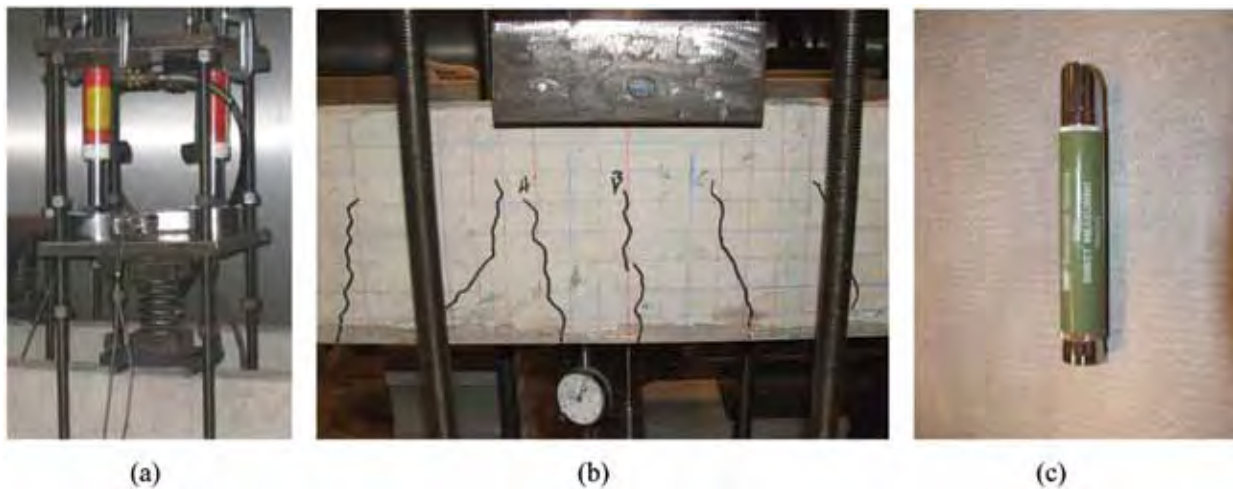


Fig. 3—(a) Loading system; (b) typical flexural cracking in a test specimen; and (c) crack width microscope.

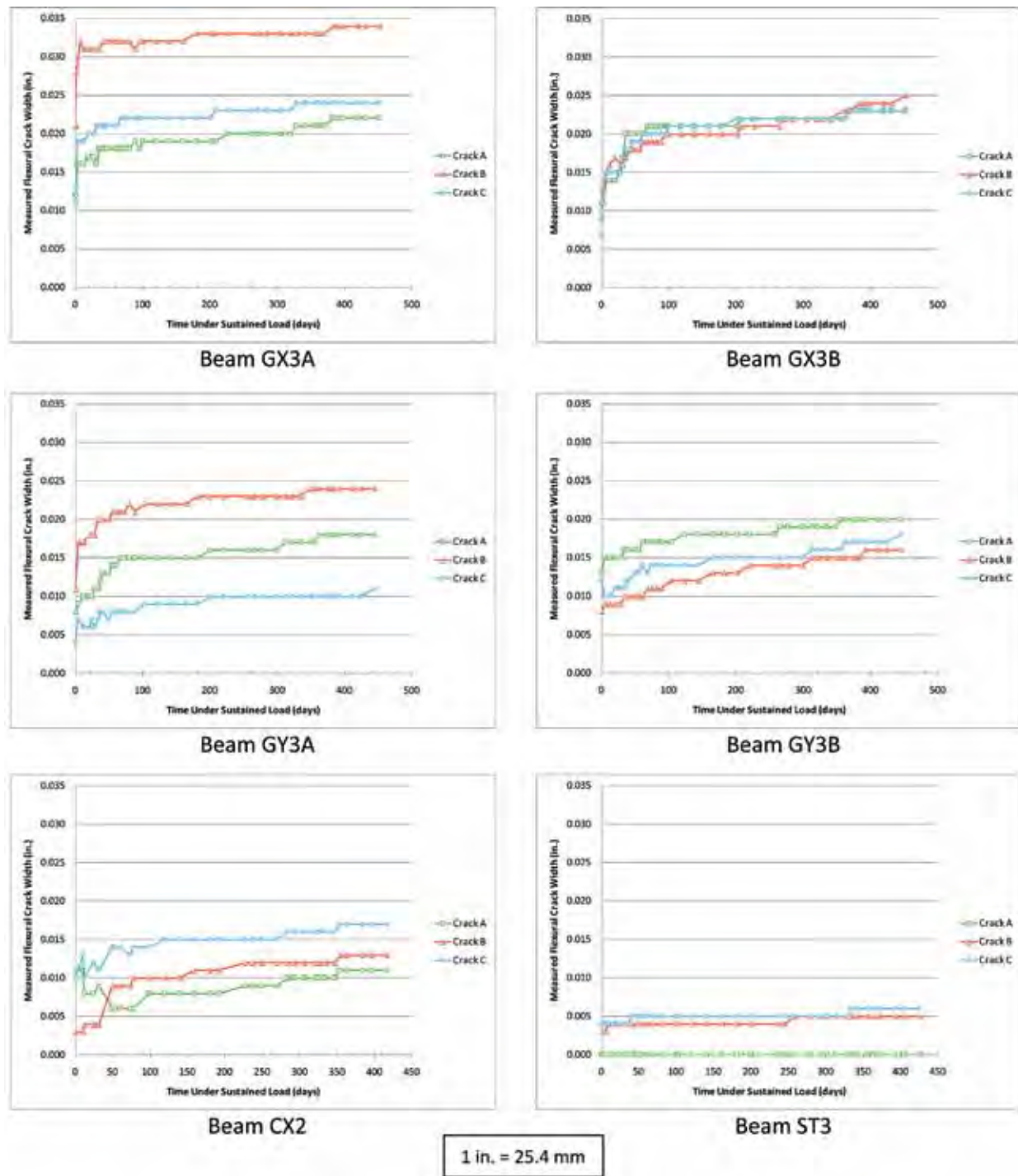


Fig. 4—Crack width measurements for Group 1 beams.

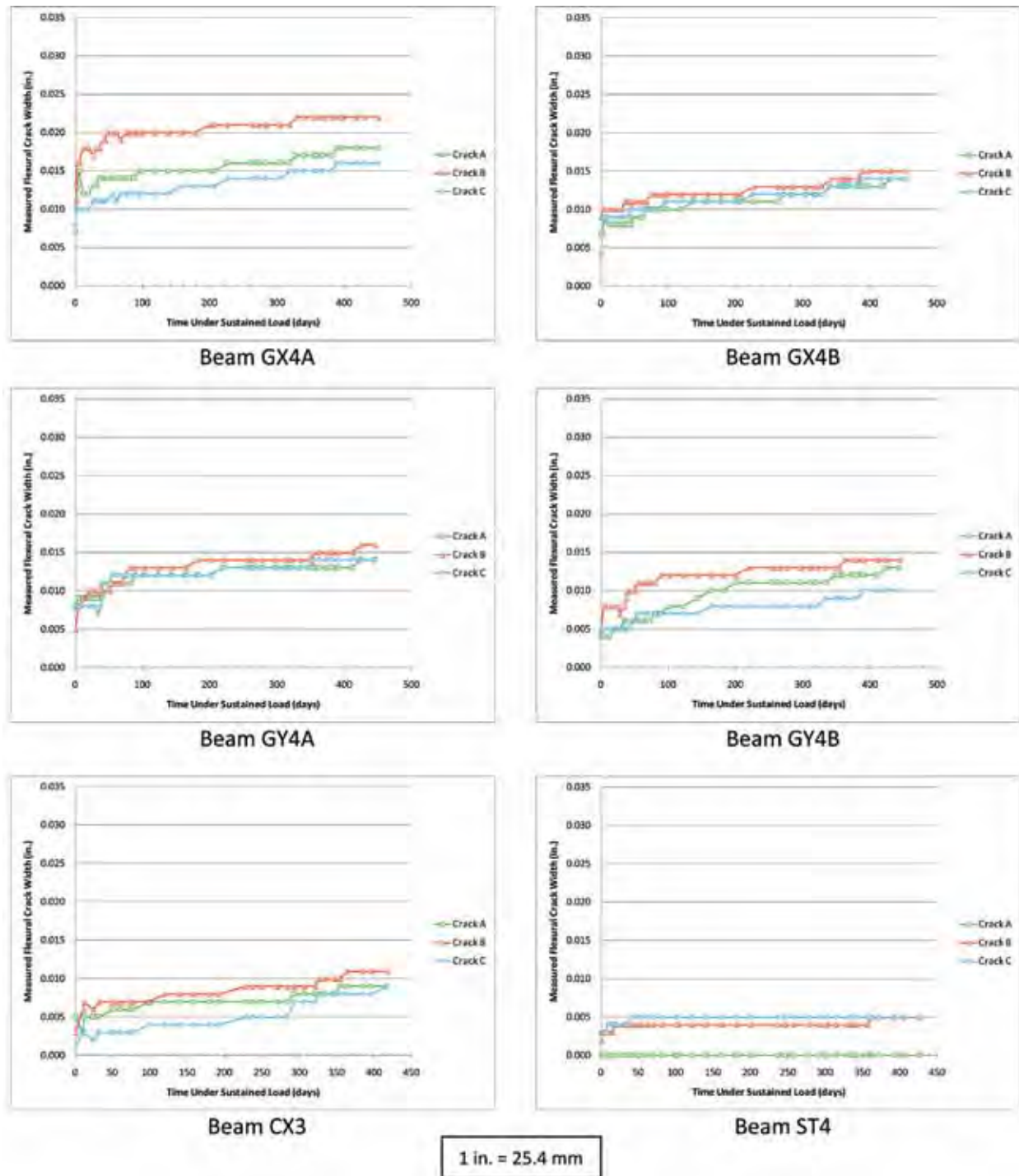


Fig. 5—Crack width measurements for Group 2 beams.

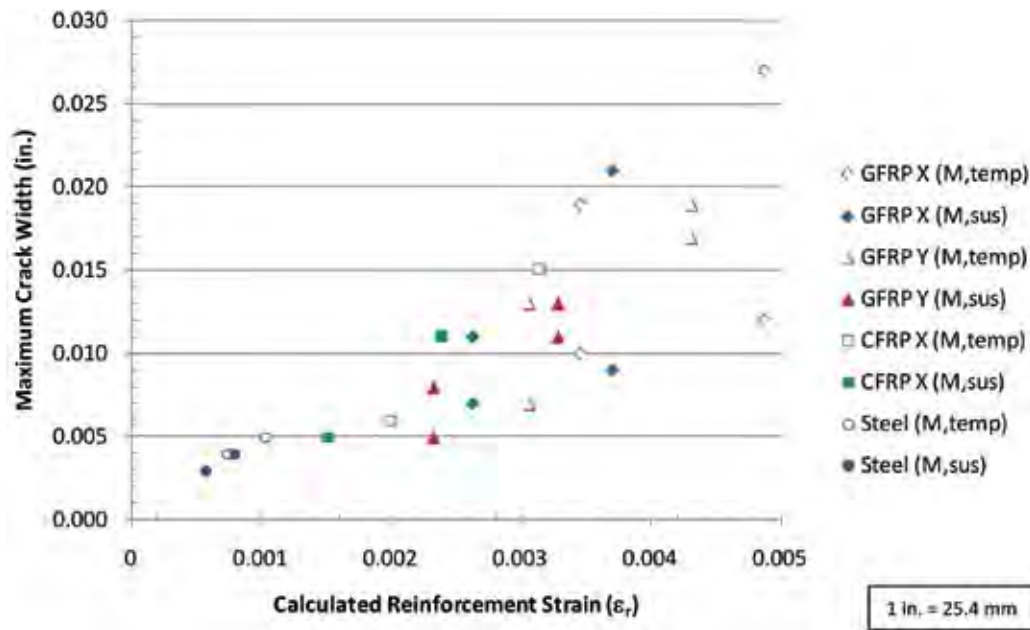


Fig. 6—Measured maximum crack widths versus calculated reinforcement strains.

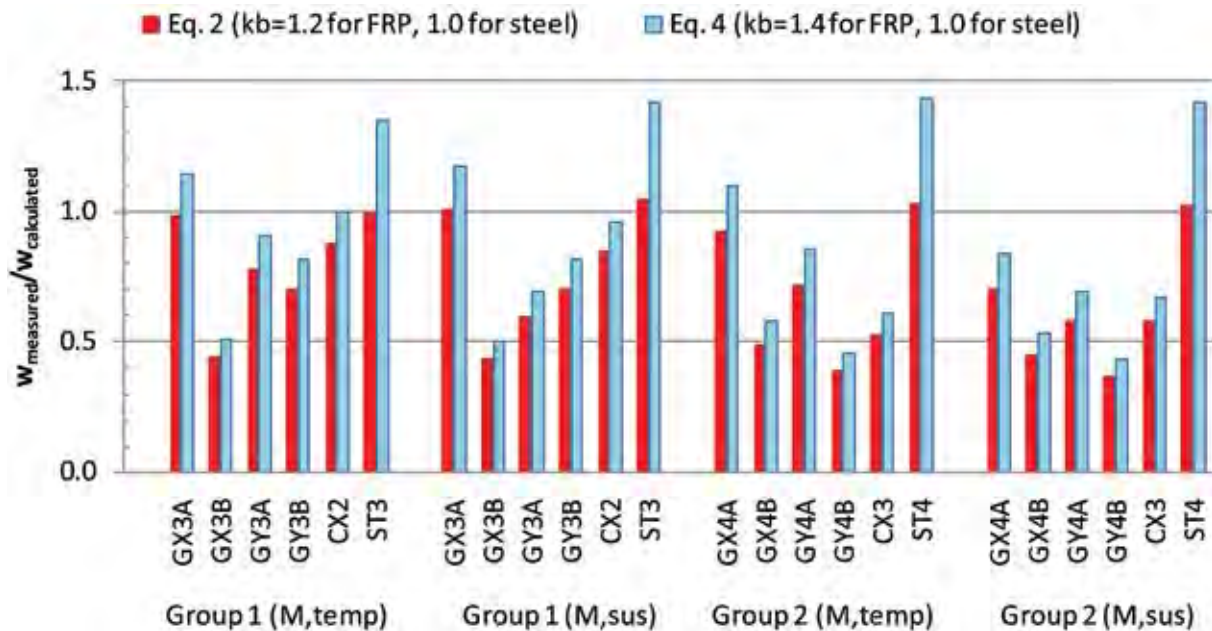


Fig. 7—Measured versus calculated maximum instantaneous crack widths.

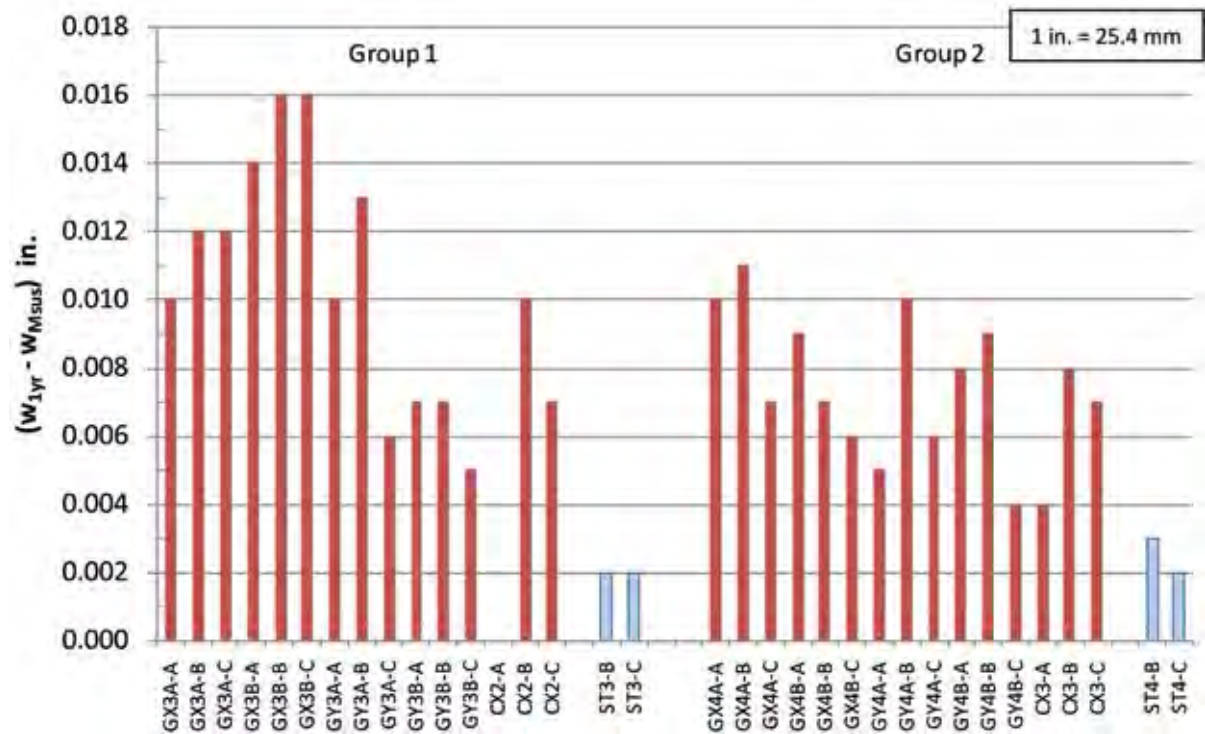


Fig. 8—Crack width growth over 1 year of sustained loading.

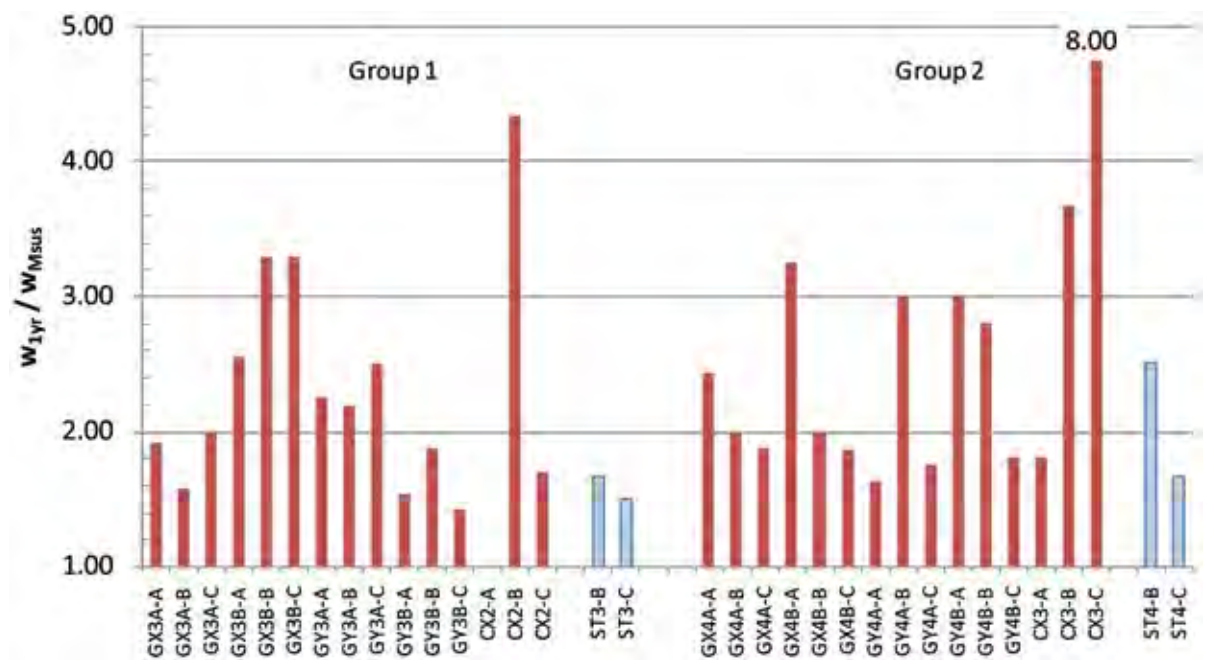


Fig. 9—Crack width growth over 1 year of sustained loading as a multiple of initial crack width.

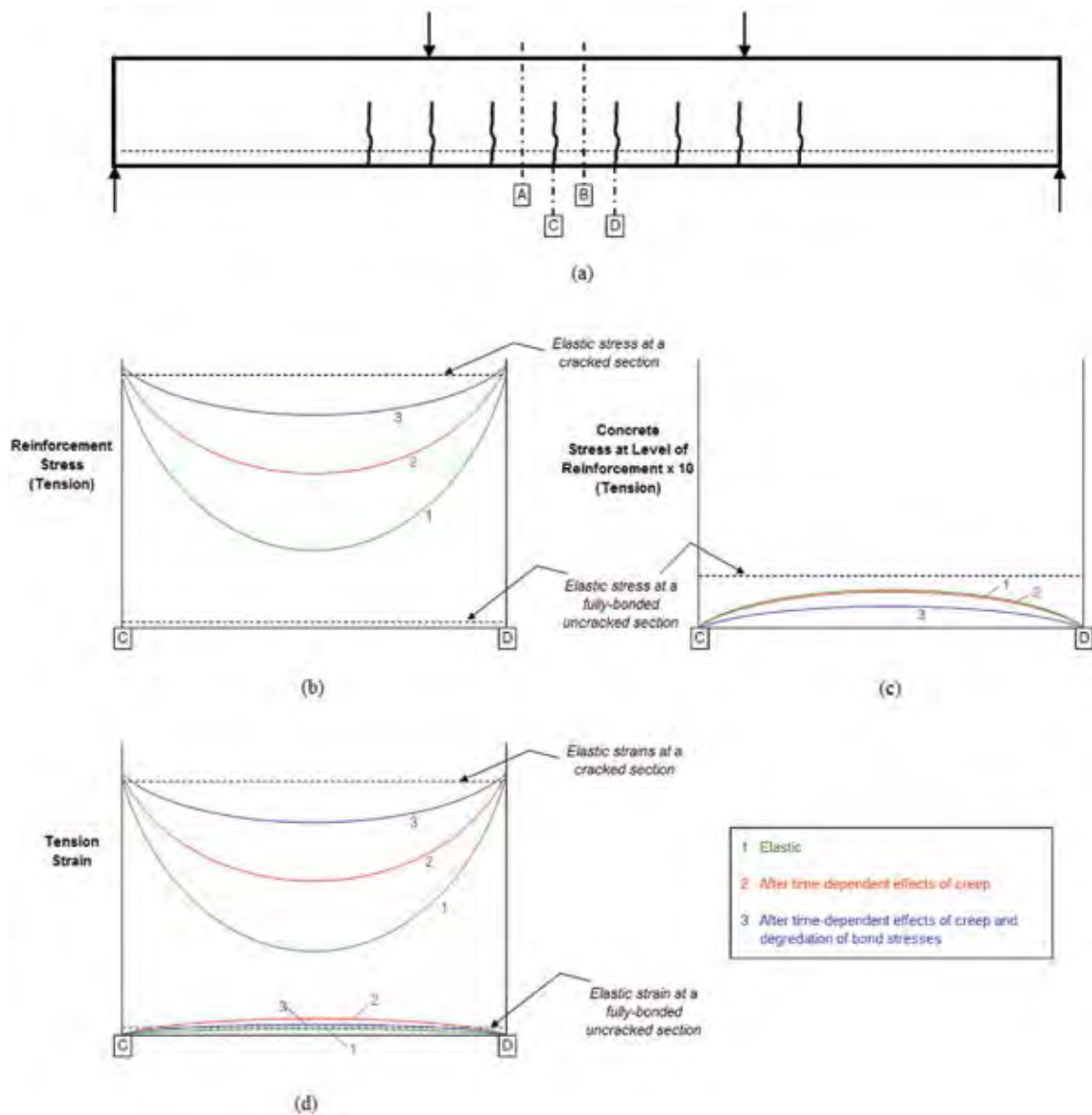


Fig. 10—(a) Stabilized flexural cracking pattern; (b) reinforcement stresses between cracks; (c) concrete stresses between cracks; and (d) reinforcement and concrete strains between cracks.

Cracking and Deflection Behavior of One-Way Parking Garage Slabs Reinforced with CFRP Bars

by S. El-Gamal, B. Benmokrane, and E. El-Salakawy

Synopsis: This paper describes a laboratory investigation on the flexural behavior of carbon fiber reinforced polymer (CFRP)-reinforced concrete slab specimens extracted from Laurier-Taché Parking Garage (National Capital Region - Canada) after being in service field conditions for about eight years (1997-2005). A total of four specimens, measuring 3 m long × 1 m wide × 0.19 m deep each, are extracted from the parking garage and then tested in flexure under four-point bending set-up. In addition, five FRP bar samples are extracted from the slabs and tested in tension to evaluate the strength or stiffness degradation of the CFRP bars, if any. The test results are presented in terms of deflection, crack widths, strains in concrete and reinforcement, ultimate capacity, and mode of failure. Based on the test results, the current provisions, provided by different FRP codes and design guidelines, are evaluated. The results showed that the CFRP bars as well as the CFRP-reinforced concrete slabs have not been adversely affected after being in service for eight years.

Keywords: composite materials; concrete slabs; cracking; deflection; experimental; fiber-reinforced polymer (FRP); flexure.

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INTRODUCTION

Conventional concrete structures are reinforced with steel, which is initially protected against corrosion by the alkalinity of the concrete. For many structures subjected to aggressive environments, such as marine structures, bridges, and parking garages exposed to de-icing salts, combinations of moisture, temperature, and chlorides result in the corrosion of the reinforcing steel. Since fiber reinforced polymers (FRP) materials are non-corrodible, the problems related to steel corrosion can be avoided. In addition, FRP bars exhibit several mechanical properties such as high tensile strength and modulus that make them suitable for use as structural reinforcement (Benmokrane and El-Salakawy 2002, 2007).

These properties encouraged Public Works and Government Services Canada (PWGSC) to investigate the use of FRP bars as reinforcement of parking garage slabs. In 1997, PWGSC implemented FRP technology in a pilot project by using CFRP bars (Leadline™) as reinforcement for two structural concrete slab panel replacements in the Laurier-Taché Parking garage (Gatineau, Quebec, Canada). The parking garage is a skeletal building made of reinforced concrete columns, post-tensioned girders, and one-way slabs. The 20-m (65 ft.-7 in.) long one-way slabs were continuously-supported over clear spans of 3.6 m (11 ft.-10 in.) or 4.1 m (13 ft.-5 in.) c/c. The CFRP-reinforced demonstration area consists of two adjacent slab panels measuring 8.7 m (28 ft.-6 in.) x 8.2 m (26 ft.-11 in.) as shown in [Figure 1a](#). The slab sections were over reinforced and the design was governed by the serviceability limits (deflection and crack width). The design of the slabs, however, was comparable to the CSA-S806-02 Canadian code on Design and construction of building component with fibre reinforced polymers (CSA 2002) published for the first time in 2002 and which specifies that FRP-reinforced concrete sections should be over-reinforced (Clause 8.2.1). It is worth mentioning that at in 1997 there was no design code on concrete structures reinforced with FRP bars available in Canada.

In early 2003, owing to serious corrosion problems in the garage slabs' reinforcing steel, PWGSC decided to rehabilitate the whole parking garage by replacing all the structural slabs. The demolition and replacement of the slabs, including those reinforced with CFRP bars, started in 2005. In collaboration with the University of Sherbrooke, PWGSC took this opportunity to evaluate the flexural performance of the CFRP-reinforced concrete slabs after being in service for about 8 years. Three slab panels (3.0 m [9 ft.-10 in.] long, 2.0 m [6 ft.-7 in.] wide and 0.19 m [7.5 in.] deep) were extracted from the demonstration area as shown in [Figure 1a](#). The slab panels were supported on hydraulic jacks and a saw cut was made in each slab with the specified dimensions. The slabs were then jacked up and removed from the site by a forklift and placed on a flatbed for transportation to the laboratory. [Figure 1\(b\)](#) shows a photo of the demonstration area after the slab panels had been removed. It should be noted that, prior to cutting, the CFRP-reinforced area was visually examined and there was no signs of deterioration except for some minor transverse hairline shrinkage cracks that appeared within a few weeks of the slab casting back in 1997. It is, also, worth mentioning that the slabs in the entire garage were protected with a thin traffic bearing waterproof membrane layer, while the demonstration slabs with the CFRP bars did not have any protection.

OBJECTIVES

The main objectives of this investigation are (1) to identify and quantify the damage, if any, to the CFRP bars,

concrete, and the interface between CFRP bars and concrete under service conditions; (2) to evaluate the residual mechanical properties of the CFRP bars and compare them to the original properties; and (3) to evaluate the flexural behavior of the slabs, in term of residual strength, stiffness, deflection, cracking, and mode of failure.

These objectives were achieved by conducting a laboratory investigation, which included two phases. The first phase focused on the durability evaluation of the CFRP bars, concrete and the interface between CFRP bars and concrete. The second phase investigated the structural behavior of the slabs under flexural stresses. Each slab panel was cut into two identical segments, which resulted in six slab segments of 3.0 m (9 ft.-10 in.) long, 1.0 m (3 ft.-3 in.) wide and 0.19 m (7.5 in.) deep. Two slab segments were tested in phase I, while the remaining four were tested in phase II. This paper will focus on the second phase of the study, which investigates the flexural behavior of the one way parking garage slabs. More details about the first phase can be found elsewhere (El-Gamal et al. 2007).

LABORATORY INVESTIGATION

Properties of Reinforcement and Concrete

As part of the study, CFRP bar samples were extracted and tested in tension to investigate the degradation in strength or stiffness of the CFRP bars. Three concrete strips measuring 3 m (9 ft.-10 in.) long $\times$ 0.1 m (3.94 in.) wide were cut from one of the slab segments that were used in the durability phase. Each strip contained two 3 m (9 ft.-10 in.) long CFRP bars. Using a small hammer, the two CFRP bars were carefully extracted from each strip producing six 3000 mm (118 in.) long CFRP bars.

The tensile tests were carried out according to Appendix C test method of the CAN/CSA-S806-02 code (CSA 2002) and ACI 440.3R-04 test guide. Five tension CFRP specimens were anchored at each end using steel tubes filled with epoxy. Each specimen was instrumented with a linear variable displacement transducer (LVDT) over a gauge length of 200 mm (8 in.) to capture the specimen elongation during testing. The test was carried out using a universal testing machine and the load was increased monotonically until tensile failure occurred. The results of the tensile tests are given in Table 1. Results show that the mean value of the tensile strength of the CFRP bars extracted from the slabs is similar to that of the virgin bars reported by the manufacturer. Hence, it can be concluded that there was no degradation in the strength or stiffness of the CFRP bars after being in severe service conditions for about eight years. To measure the concrete compressive strength, six 100 mm (4 in.) diameter cores were taken from the slabs and tested in compression. The average concrete strength was 44 MPa (6400 psi) with a standard deviation of 3.6 MPa (520 psi).

Slab Specimens and Test Set-up

Four concrete slab segments (3.0 m [9 ft.-10 in.] long $\times$ 1.0 m [3 ft.-3 in.] wide $\times$ 0.19 m [7.5 in.] deep) were tested in four-point bending over a simply supported clear span of 2.5 m (8 ft.-2 in.) and a shear span of 1.0 m (3 ft.-3 in.) as shown in Figure 2a. All the tested slabs were reinforced with 8-mm (0.32 in.) diameter (area = 49 mm² or 0.08 in.²) CFRP bars as shown in Figure 2b. For each slab, six electrical resistance strain gauges (ESG) were attached to the top surface of the slabs at the middle and the quarter span. Two LVDTs were installed at the mid-span to measure the maximum deflection of the slabs as shown in Figure 2b. In addition, one high-accuracy LVDT (0.001 mm or 4 $\times$ 10⁻⁵ in.) was installed at the position of the first crack to measure crack width. To measure strains in the CFRP bars at mid-span, the bottom concrete cover under one of the main bottom CFRP bars was removed in slabs S1, S3, and S4 and ESGs were attached to the bars.

The load was monotonically applied at a stroke-controlled rate of 1.2 mm/min (0.05 in./min). The loading was stopped when the first flexural cracks appeared. The initial crack widths were measured using a 50X microscope. Afterwards, the high-accuracy LVDT was installed to measure the crack width electronically. A data acquisition system monitored by a computer was used to record the readings of the load, the LVDTs and the ESGs. A similar set-up was used in a previous work (Benmokrane et al. 2004). During loading, the development of cracks on one side of the slabs were marked and recorded. Figure 2c shows a photo of one of the slab segments during testing.

TEST RESULTS

Test results are presented in terms of deflection at mid-span, crack widths, strains in concrete and reinforcement, ultimate capacity, and mode of failure. The test results are analyzed and compared to service ($M_s = 15.64$ kN.m or 11.5 k-ft) and factored ($M_f = 21.91$ kN.m or 16.2 k-ft) mid-span moments calculated according to CSA-S413-94 (CSA 1994). More details about loads and design moments are given in Appendix A.

Deflection

Figure 3 shows applied load and moment versus mid-span deflection for the four tested slabs. It can be noted that the load-deflection curves were bilinear for the four slabs. The first part of the curve up to the cracking moment (15.35 to 18.65 kN.m [11.3 to 13.8 k-ft] - excluding the moment due to the self weight of the slab, 3.49 kN.m (2.6 k-ft) represents the behavior of the uncracked slabs. The second part represents the behavior of the cracked slabs with reduced stiffness. All tested slabs had very similar deflection behavior as expected. The allowable deflection limit by the Canadian Code (CSA-S806-02) for the laboratory-tested slabs is 5.76 mm (0.23 in.) (Benmokrane et al. 2004).

At service moment (15.64 kN.m or 11.5 k-ft), the measured deflections ranged between 1.42 mm (0.056 in.) and 3.82 mm (0.15 in.). For slabs S2 and S3, where their cracking moments were higher than the service moment, the measured deflections at service were small, 2.35 mm (0.093 in.) and 1.45 mm (0.057 in.), respectively. However, for slabs S1 and S4, where their cracking moments were less than the service moment, the measured deflections were higher, 3.82 mm (0.15 in.) and 3.62 mm (0.143 in.), respectively. However, it can be noted that all deflection values were well below the allowable code limits (5.76 mm or 0.23 in.). At failure, the maximum recorded deflections ranged between 55.0 mm (2.2 in.) and 75.34 mm (2.97 in.). It can be noted from Table 2 that the ratio of the measured deflections at failure to those at service ranges between 15 and 38, which indicates that the CFRP-reinforced slabs would give ample warning before failure.

Cracking Propagation

The cracking moments of the four tested slabs are listed in Table 2. These cracking moments include the moment due to the self weight of the slab (3.49 kN.m or 1.32 k-ft). Crack formation was initiated in the flexural span at a moment of 18.84 kN.m (7.12 k-ft) to 22.14 kN.m (8.37 k-ft) which is consistent with the theoretical value, 22.83 kN.m (8.62 k-ft), calculated by the CSA-S806-02 code (CSA 2002).

Figure 4 shows a photo of the tested slabs at failure. Figure 5, shows a schematic drawing of crack propagation at different loading levels. Figure 5a shows the first stage of cracking, from cracking to about $1.6 M_s$, where 2 to 3 vertical cracks appeared around the pure bending zone. By increasing the load to 100 kN (22.5 kips), about 3.2 times the service moment, M_s , more vertical cracks appeared in the shear span and the total number of cracks ranged between 6 and 8 cracks per slab as shown in Figure 5b. By increasing the load to 150 kN (33.75 kips), about $4.8 M_s$, only one to two inclined cracks appeared in the shear span. However, the depth of previous cracks increased and the cracks propagated towards the top surface of the slabs as shown in Figure 5c. The number of cracks in this stage ranged between 8 and 10 cracks. Figure 5d shows the final stage of cracking, from $4.8 M_s$ to failure. In this stage, no new flexural cracks were formed. Only one or two shear cracks were formed before the failure of the slabs.

Crack Width

Figure 6 shows the variation of the maximum measured crack width against the applied load and moment for the tested slabs. For the slab S4, due to a problem in the data acquisition system, the crack width and strains were not recorded after a moment of 20 kN.m (14.8 k-ft). For all slabs, the moment-crack width relationships were almost linear up to failure. Table 2 shows that the maximum measured crack width at failure was 3.22 (0.13), 3.87 (0.15) and 3.21 mm (0.13 in.) for slabs S1, S2 and S3, respectively. Comparison with design provision is given in a separate section below.

Strains in Reinforcement and Concrete

Figure 7 shows the moment-reinforcement strain relationship for slabs S1 and S3. The maximum measured strains in the reinforcement at service moment were less than 4% of the ultimate strains of the CFRP bars. These strains were 506 and 474 micro-strain for slabs S1 and S3, respectively (Table 3). At failure, the measured strains were 9240 and 10200 micro-strain, respectively, which were less than 70% of the ultimate strains of the CFRP bars (15500 micro-strain). For slabs S1 and S3, the ratios of the measured reinforcement strains at service to those at failure were about 5.5% and 4.7%, respectively. This indicates that the slabs would have a very wide range of safety before failure. Furthermore, the measured strains in concrete ranged between -67 to -78 and -2150 to -2680 micro-strain at service moments and at failure, respectively. Table 3 also shows that the ratio of the maximum measure mid-span concrete strains at failure to those at service moments ranged between 28 and 34.

Ultimate Capacity and Mode of Failure

The failure moments of the four tested slabs ranged between 109.0 and 120.5 kN.m. All slabs failed in shear. This can be noticed from the measured concrete and reinforcement strains which were much lower than the ultimate strains of concrete and CFRP bars, respectively. Three slabs (S₁, S₂, and S₄) failed in compression shear failure at moment levels of 112.4 (83.0), 120.5 (88.9) and 109.0 kN.m (80.4 k-ft), respectively. Slab S₃ failed in diagonal tension shear failure at a moment of 109.0 kN.m (80.4 k-ft). It should be mentioned that during cutting this slab to the required dimensions, one of the CFRP bars was cut a few centimeters before the support. This resulted in this mode of failure because the diagonal tension crack started at the cut end of the bar. It can be noted that these failure moments are more than 5 times the factored design moment of the slabs (21.91 kN.m or 16.2 k-ft).

Deformability Factor (J)

Ductility in steel-reinforced concrete elements is quantified by a ratio of values of displacement or curvature at ultimate to those at yielding of steel. Despite the fact that FRP does not yield, concrete flexural members with FRP reinforcement also exhibit substantial deflection prior to failure (ISIS-Canada 2007). Jaeger et al. (1994) introduced the concept of deformability to quantify these deformation characteristics and to provide a means of comparison between the level of safety between ultimate and service states. The deformability factor calculated for the slabs ranged between 20 and 24, which are much greater than (5 times) the minimum deformability factor (4 for rectangular sections) specified by CAN/CSA-S6-00 (CSA 2006). This shows again that the CFRP-reinforced slabs will give ample warning before failure.

DEFLECTION AND CRACKING PROVISIONS

Cracking Provisions

CAN/CSA S806-02 Code—The CAN/CSA S806-02 (CSA 2002) uses the traditional z-factor approach to control crack width in beams and one-way slabs as follows;

$$z = k_b \frac{E_s}{E_f} f_f \sqrt[3]{d_c A} \quad (1)$$

The z factor has to be $\leq 45\,000$ N/mm (260 kips/in.) for interior exposure and $\leq 38\,000$ N/mm (220 kips/in.) for exterior exposure. E_f is the modulus of elasticity of longitudinal FRP reinforcement (MPa); E_s is the modulus of elasticity of steel reinforcement (MPa); f_f is the stress in FRP reinforcement under specified loads (MPa); k_b is a bond-dependant coefficient that accounts for the degree of bond between FRP bar and concrete. For FRP bars having bond behavior inferior to steel, k_b is larger than 1.0, and for FRP bars having bond behavior superior to steel, k_b is smaller than 1.0. According to CSA-S806, it may be taken as 1.2 for deformed rods; d_c is the distance from extreme tension fiber to the centre of the longitudinal bar or wire located closest thereto (mm); A is the effective tension area of the concrete, defined as the area of concrete having the same centroid as that of tensile reinforcement divided by the number of bars.

ACI 440.1R-03—The ACI 440.1R-03 (ACI 2003) calculates crack width in beams and one-way slabs by modifying Gergely-Lutz equation as shown in Eq. 2.

$$w = 2.2 \frac{f_f}{E_f} \beta k_b \sqrt[3]{d_c A} \quad (2)$$

where w is the maximum likely crack width at the tension face of a beam or one-way slab (generally taken as the 90-th percentile crack) (mm), β is the ratio of the distance from the neutral axis to the extreme tension fiber to the distance from the neutral axis to the centroid of the tensile reinforcement, and k_b is a bond-dependant coefficient. Some typical k_b values for deformed GFRP bars cited in ACI 440.1R-03 are between 0.71 and 1.83 based on the work of Gao et al. (1998). ACI 440.1R-03 suggests that designers assume a value of 1.2 for deformed GFRP bars, when k_b is not known. f_f , E_f , d_c , and A are as defined in Eq. 1.

The Canadian Highway Bridge Design Code, CAN/CSA S6-06 (CSA 2006), uses Eq. 3 to calculate crack width values in FRP-reinforced concrete beams and one-way slabs. Equation 3 is a modified version of the crack width equation by Frosch (1999). Clause 16.8.2.3 in CSA S6-06 states the following regarding the crack-control

reinforcement: “When the maximum tensile strain in FRP reinforcement under full service loads exceeds 0.0015, cross-sections of the component in maximum positive and negative moment regions shall be proportioned in such a way that the crack width does not exceed 0.5 mm (0.02 in.) for members subject to aggressive environments and 0.7 mm (0.03 in.) for other members, with the crack width calculated as follows:

$$w_{cr} = 2 \frac{f_f}{E_f} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + (s/2)^2} \quad (3)$$

where w_{cr} is the crack width at the tensile face of the flexural component (mm), f_{FRP} is the stress in the tension FRP reinforcement (MPa), E_{FRP} is the mean modulus of elasticity of FRP reinforcement (MPa), h_2 is the distance from the extreme flexural tension surface to the neutral axis (mm), h_1 is the distance from the centroid of tension reinforcement to the neutral axis (mm), k_b is a coefficient depending on bond between FRP and concrete, d_c is the distance from the centroid of the tension reinforcement to the extreme tension surface of concrete (mm), and s is the spacing of tensile reinforcement (mm).

The CAN/CSA S6-06 states also that the value of k_b in Eq. 3 shall be determined experimentally, but in the absence of test data may be taken as 0.8 for sand-coated and 1.0 for deformed FRP bars. In calculating d_c , the clear cover shall not be taken greater than 50 mm (2 in.).

ACI 440.1R-06—Clause 8.3.1 in the ACI 440.1R-06 states that the maximum probable crack width for FRP reinforced members may be calculated from the following Eq. (4) which is similar to Eq. (3):

$$w = 2 \frac{f_f}{E_f} \beta k_b \sqrt{d_c^2 + (s/2)^2} \quad (4)$$

where w is the maximum probable crack width (mm), E_f is the modulus of elasticity of FRP bar (MPa), f_f is the stress in FRP reinforcement in tension (MPa), k_b is a bond dependent coefficient, β is the ratio of distance from neutral axis to extreme tension fiber to distance from neutral axis to centre of tensile reinforcement (equals h_2/h_1 in Eq. 3), d_c is the thickness of concrete cover measured from extreme tension fiber to centre of bar (mm), and s is the longitudinal FRP bar spacing (mm).

Analysis of crack width data performed by ACI 440 committee (Bakis et al. 2006) on a variety of concrete cross sections and FRP bar manufacturers, fiber types, resin formulations, and surface treatments, average k_b values calculated by Eq. 4 ranged from 0.60 to 1.72, with a mean of 1.10. Data for rough sand-coated FRP bar surface treatments trended toward the lower end of this range. The consensus of the committee, for the case where k_b is not known from experimental data, is that a conservative value of 1.4 should be assumed.

Comparison with Cracking Provisions

Table 3 shows a comparison between k_b values from Eqs. 2, 3 and 4, based on readings from the strain gauges mounted on FRP bars or those came from calculations using a cracked elastic section analysis. The strains in FRP bars used in calculations ranged between 2000 and 9000 micro-strain. It can be noted from Table 3 that the k_b values calculated based on readings of strain gauges are about 1.19 times those calculated based on cracked elastic section analysis. This difference is due to higher strains associate with the cracked elastic section analysis which does not take into consideration the effect of tension stiffening. Figure 8 shows the variation of k_b values versus applied load for three slabs. It can be noted that the k_b varies with load (or strains in FRP bars). Scattered values are obtained at lower load levels where the crack width values are not stable due to the formation of new cracks. With increasing load (strains), the k_b values became more stable. It can be also noted that the difference between the k_b values calculated from the readings of strain gauges and those based on elastic section analysis are big at lower load levels where the tension stiffening effect exists. This difference decreases or almost disappears at higher load levels as shown in Slab 1. It can be noted also from Table 3 and Figure 8 that k_b values from Eqs 3 or 4 are higher than those obtained from Eq. 2. The ratio of k_b values from Eq 3 or 4 to k_b values from Eq. 2 is about 1.11 which is consistent with the observations of Bakis et al. (2006).

The experimental crack width values measured in the tension face of the four tested slabs are compared with the predicted by Eq. 3 (CSA-S6-06) and Eq. 4 (ACI 440.1R-06) as shown in Figure 9. It can be observed that the experimental crack widths of the tested slabs fit well with the predicted curve based on the conservative k_b value of 1.4 recommended by ACI 440.1R-06 when a specific value of k_b is not known for FRP bars.

Deflection Provisions

CAN/CSA S806-02 Code—Clauses 8.3.2 in the CAN/CSA S806-02 (CSA 2002) states that deflections that occur immediately on application of load shall be computed by methods based on the integration of curvature at sections along the span. The moment-curvature relation of FRP reinforced concrete members shall be assumed to be tri-linear with the slope of the three segments being $E_c I_g$, zero, and $E_c I_{cr}$. For the case of two point loading of $P/2$ on a simple span, L , with a shear span, a , the deflection is calculated according to Eq. 5.

$$\delta_{\max} = \frac{PL^3}{48E_c I_{cr}} \left[3 \left(\frac{a}{L} \right) - 4 \left(\frac{a}{L} \right)^3 - 8\eta \left(\frac{L_g}{L} \right)^3 \right] \quad (5a)$$

where

$$\eta = 1 - \left(\frac{I_{cr}}{I_g} \right) \quad (5b)$$

where $\delta_{\max}$ is the mid span deflection (mm), L is the span (mm), a is the shear span (mm), E_c is the modulus of elasticity of concrete (MPa), I_{cr} is the moment of inertia of cracked section transformed to concrete (mm⁴), L_g is the uncracked length (mm).

ACI 440.1R-06—In North American codes, deflection calculation of flexural members are mainly based on equations derived from linear elastic analysis, using the effective moment of inertia, I_e , given by Branson's formula (1965). Many researchers suggested different modifications to Branson's Eq. to improve its performance to be suitable for FRP-reinforced concrete members (Benmokrane et al. 1996; Brown and Bartholomew 1996; Toutanji and Saafi 2000). ACI 440.1R-06 uses an effective moment of inertia formulation, I_e , to calculate the deflection of cracked FRP-reinforced concrete beams and one-way slabs. The procedure entails the calculation of a uniform moment of inertia throughout the beam length, and use of deflection equations derived from linear elastic analysis. The effective moment of inertia, I_e , is based on semiempirical considerations, and despite some doubt about its applicability to conventional reinforced concrete members subjected to complex loading and boundary conditions, it has yielded satisfactory results in most practical applications over the years.

ACI 440.1R-06 uses Eq. 6 to calculate the deflection of FRP-reinforced concrete members:

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 \beta_d I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \quad (6a)$$

where

$$\beta_d = \frac{1}{5} \left(\frac{\rho_f}{\rho_{fb}} \right) \leq 1 \quad (6b)$$

where I_e is the effective moment of inertia (mm⁴), M_{cr} is the cracking moment (N mm), M_a is the applied moment (N mm), I_g is the moment of inertia of gross section (mm⁴), ρ_f and ρ_{fb} are the actual and balanced reinforcement ratio, respectively.

ACI 440 Ballot—ACI 440 Committee,¹ based on the work of Bischoff (2005) and Bischoff and Scanlon (2007), has proposed an alternative expression for the effective moment of I_e that works equally well for both steel and FRP-reinforced concrete members without the need for empirical correction factors as shown in Eq. 7.

$$I_e = \frac{I_{cr}}{1 - \eta \left(\frac{M_{cr}}{M_a} \right)^2} \leq I_g \quad (7a)$$

¹ACI 440 Web Ballot (Feb. 10, 2009 – March 12, 2009).

where

$$\eta = 1 - \left(\frac{I_{cr}}{I_g} \right) \quad (7b)$$

Comparison with Deflection Provisions

Figure 10 and Table 4 show a comparison between the maximum measured deflections of the tested slabs and the estimated from Eqs. 5, 6 and 7. The accuracy of the calculated cracking moment, M_{cr} , is a key aspect to the accuracy of the deflection calculations. The controlling variable for predicting the cracking moment is the modulus of rupture of concrete, f_r . The modulus of rupture used to calculate the M_{cr} in Eqs. 5, 6, and 7 was taken from the corresponding code or guideline as given below in Eq. (8):

$$\text{For CSA S806-02 code (Eq. 5)} \quad f_r = 0.6\lambda\sqrt{f'_c} \quad (8a)$$

$$\text{For ACI 440.1R-06 (Eq. 6)} \quad f_r = 0.62\sqrt{f'_c} \quad (8b)$$

$$\text{For ACI 440 Ballot (Eq. 7)} \quad f_r = 0.8 \times 0.62\sqrt{f'_c} \quad (8c)$$

where f_r is the modulus of rupture (MPa); f'_c is the concrete compressive strength (MPa); and λ is a factor accounting for concrete density. For normal density concrete $\lambda = 1$.

To improve the accuracy of the deflection equations, the moment due to the self-weight of the slabs, 3.49 kN.m (2.6 k-ft), was included in the analysis. This lowered the predicted cracking moment, when compared to an analysis not including the self-weight.

It can be noticed from Figure 10 and Table 4 that, at $1.2 M_{cr}$ (applied moment right after cracking), the CSA/S806 equation gives good predictions of deflection compared to the measured values. Both the ACI 440.1R-06 (ACI 2006) equation and the proposed equation by ACI 440 Ballot underestimate deflection at $1.2 M_{cr}$. At $2.0 M_{cr}$, both the CSA/S806 equation and the proposed equation by ACI 440 Ballot give good predictions. The ACI 440.1R-06 (ACI 2006) equation, however, underestimates deflection at this level. From $3.0 M_{cr}$ up to failure, the deflection predictions from the three Equations fit well the measured ones.

SUMMARY AND CONCLUSIONS

In 1997, PWGSC implemented FRP technology in a pilot project using carbon FRP bars as reinforcement for the replacement of two structural concrete slab panels of the Laurier-Taché Parking Garage. To evaluate the performance of the slabs after being in service field conditions for about eight years (1997-2005), PWGSC extracted three slab segments and sent them to the laboratories of the University of Sherbrooke for evaluation. The investigation included two phases. This paper describes the second phase of this investigation which focused on evaluating the flexural behavior of the CFRP-reinforced concrete parking garage one way slabs.

Based on the results of this study, the following conclusions can be drawn:

1. There was no degradation in strength or stiffness of the extracted CFRP bars compared to values specified by the manufactures.
2. At service moment, the measured deflections were much less than allowable code limits. In addition, the maximum measured strains in the reinforcement at service were less than 4% of the ultimate strains of the CFRP bars. Furthermore, the failure moments of the four tested slabs were more than 5 times the factored design moment.
3. The high ratios of the measured deflections and strains at failure to those at service moment, along with the high values of failure moments show clearly the ample factor of safety of the CFRP-reinforced concrete slabs.
4. The k_b values calculated based on the readings of strain gauges were about 20% higher than those calculated based on cracked elastic section analysis. The k_b values varied with load or strains in FRP bars. Scatter values were obtained at lower load levels where crack widths were not stable due to the formation of new cracks. With increasing load (strains), the k_b values became more consistent.
5. The experimental crack widths of the tested slabs fit well with the predicted one using a k_b value of 1.4

which is recommended by ACI 440.1R-06 when a specific value of k_b is not known.

6. For the slabs tested in this investigation, the results show that at $1.2 M_{cr}$ (applied moment right after cracking), the CSA/S806 equation gives good predictions of deflection compared to the measured values. Both the ACI 440.1R-06 (ACI 2006) equation and the proposed equation by ACI 440 Ballot underestimate deflection at $1.2 M_{cr}$. At $2.0 M_{cr}$, both the CSA/S806 equation and the proposed equation by ACI 440 committee give good predictions. For applied moment greater than three times the cracking moment, the deflection prediction from the three Equations (CSA S806, ACI 440.1R-06, and ACI 440 proposed equation) fit well the measured ones.

These test results give an exceptional opportunity to compare deflection and cracking code provisions to real slabs, which were designed and constructed according to codes and were cut from a real structure after being in service for eight years. In addition, testing four similar slabs gives more credibility to the test results and to the comparison with code provisions. This, in turn, will help to improve the cracking and deflection provisions of FRP-reinforced concrete members.

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APPENDIX A. CALCULATIONS OF SERVICE AND FACTORED DESIGN MOMENTS

Slab thickness = 190 mm (7.5 in.)

Slab self weight = $0.19 \times 23.5 = 4.465 \text{ kN/m}^2$ (93 psf)

Self weight of wearing surface = 0.3 kN/m^2 (6.3 psf)

Span of the slabs in the parking garage = 4.1 m (13 ft-5 in.) (continuous one-way slabs)

Service dead load = $4.465 + 0.3 = 4.765 \text{ kN/m}^2$ (100 psf)

Factored dead load = 4.765×1.25 (NRC 2005, Clause 4.1.3.2) = 5.956 kN/m^2 (124 psf)

Service wheel load = 11.0 kN (2.5 kips) (NBC)

Factored wheel load = 11.0×1.5 (NRC 2005, Clause 4.1.3.2) = 16.5 kN (3.7 kips) (NBC)

Therefore the service and factored moments can be calculated as follows:

$$M_s = 4.765 \times (0.077 \times 4.12) + 11.0 \times (0.21 \times 4.1) = 15.64 \text{ kN.m (11.5 k-ft)}$$

$$M_f = 5.956 \times (0.077 \times 4.12) + 16.5 \times (0.21 \times 4.1) = 21.91 \text{ kN.m (16.2 k-ft)}$$

Table 1—Tensile test results of CFRP bars extracted from the slab

Specimen		Max. load, kN [lb]	Tensile strength, MPa [ksi]	Modulus of elasticity, GPa [ksi]
Reference		—	2800* [406]	155* [22,480]
Extracted bars	1	138.6 [31,158]	2828 [410]	189 [27,412]
	2	139.9 [31,450]	2854 [414]	187 [27,122]
	3	139.1 [31,270]	2838 [411]	196 [28,427]
	4	137.3 [30,866]	2801 [406]	176 [24,221]
	5	129.6 [29,135]	2644† [383]	171 [24,801]
	Mean S.D. Coefficient of variation, %		2830 [410] 22.25 [3.23] 0.79	182.8 [26,513] 8.79 [1274] 4.81

*From the manufacturer

†This value was not taken in the average as it is a low outlier at the 1% significance level.

Table 2—Summary of test results

Slab	Cracking moment, kN.m* [lb.ft]	Failure moment, kN.m [lb.ft]	Deflection, mm [in.]		Crack width at failure, mm [in.]	Maximum strain in reinforcement, microstrain		Maximum strain in concrete, microstrain	
			Service	Failure		Service	Failure	Service	Failure
S1	18.84 [13,895]	109.0 [80,394]	3.82 [0.15]	59.91 [2.36]	3.22 [0.13]	506	9240	-69	-2150
S2	22.14 [16,329]	120.5 [88,876]	2.35 [0.09]	75.34 [2.97]	3.87 [0.15]	—	—	-64	-2220
S3	21.69 [15,998]	109.0 [80,394]	1.42 [0.06]	55.0 [2.17]	3.21 [0.13]	474	10200	-78	-2190
S4	18.98 [13,999]	112.7 [83,123]	3.62 [0.14]	55.34 [2.18]	—	517	—	-78	—

* Including the moment due to the self weight of the slab (3.49 kN.m).

Note: 1 kN.m = 737.5 lb.ft.

Table 3—Comparison between k_b values from Eq. (2), (3), and (4)

Slab	k_b (f_f from strain gauges)		k_b (f_f from calculations)		Ratio of k_b (from strain gauges) to k_b (from calculations)	Ratio of k_b (Eq. 3) to k_b (Eq. 2)
	Eq. (3) and (4)	Eq. (2)	Eq. (3) and (4)	Eq. (2)		
S1	1.77	1.60	1.58	1.42	1.12	1.11
S2	—	—	1.45	1.30	—	
S3	1.55	1.39	1.23	1.11	1.26	
Average	1.66	1.495	1.42	1.28	1.19	1.11

Table 4—Comparison between experimental and predicted deflections

Applied moment	Maximum measured deflection mm [in]					Predicted deflection mm [in]		
	S1	S2	S3	S4	Average value	CSA/S806 (Predic./Exp.)	ACI 2006 (Predic./Exp.)	ACI Ballot (Predic./Exp.)
$1.2M_{cr}$ (25.2 kN.m)*	10.50 [0.41]	11.52 [0.45]	7.19 [0.28]	9.33 [0.37]	9.64 [0.38]	9.10 [0.36] (0.97)	4.28 [0.17] (0.44)	7.45 [0.29] (0.77)
$2.0M_{cr}$ (44.4 kN.m)*	21.40 [0.84]	22.14 [0.87]	18.00 [0.71]	19.68 [0.77]	20.31 [0.80]	21.00 [0.83] (1.03)	16.55 [0.65] (0.81)	19.35 [0.76] (0.95)
$3.0M_{cr}$ (68.3 kN.m)*	34.40 [1.35]	37.14 [1.46]	30.90 [1.22]	31.54 [1.24]	33.50 [1.32]	33.64 [1.32] (1.00)	33.21 [1.31] (0.99)	32.28 [1.27] (0.96)
M_{cr} (92.3 kN.m)*	48.04 [1.89]	51.69 [2.04]	45.90 [1.81]	42.85 [1.69]	47.12 [1.86]	45.80 [1.80] (0.97)	44.38 [1.75] (0.94)	44.70 [1.76] (0.95)

* The moment due to the self weight of the slabs (3.49 kN.m) is subtracted from these values.

Note: 1 kN.m = 737.5 lb.ft.

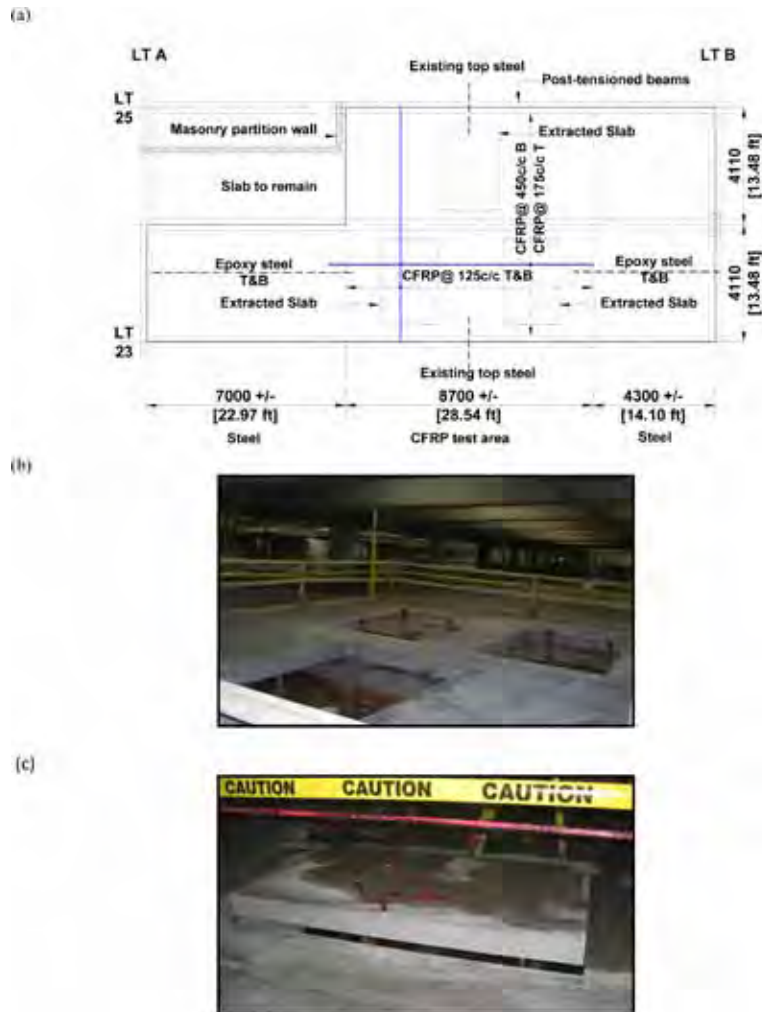


Fig. 1—Test area: (a) test area plane layout; (b) demonstration area after slab removal; and (c) view of an extracted slab. Dimensions are in mm (ft).

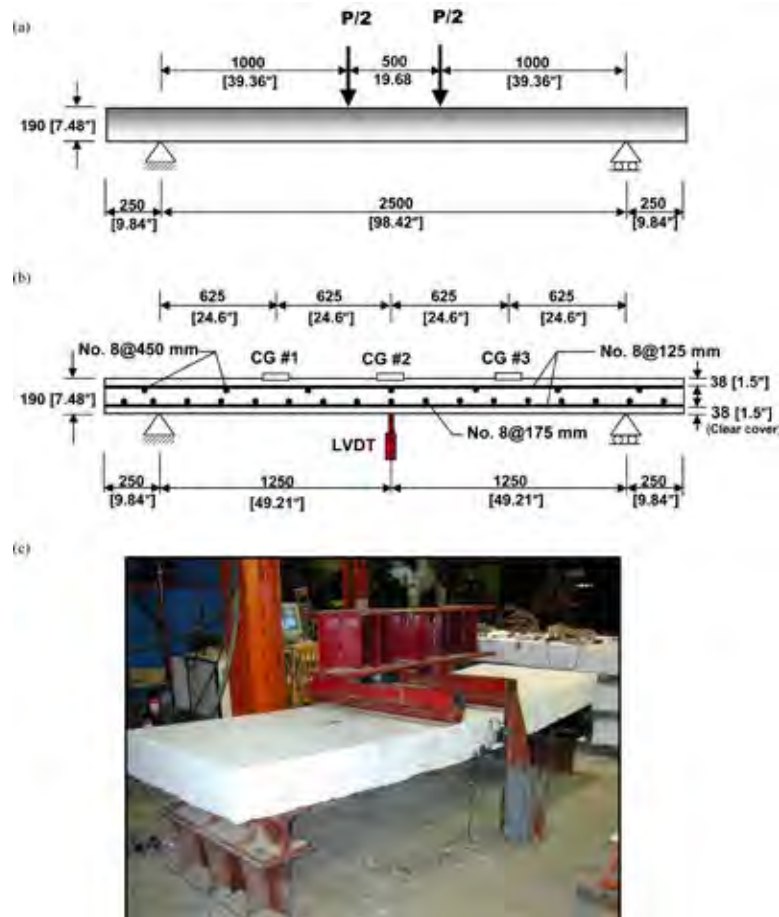


Fig. 2—Test slabs: (a) schematic test set-up; (b) reinforcement and instrumentation; and (c) photo during testing. Dimensions are in mm (ft).

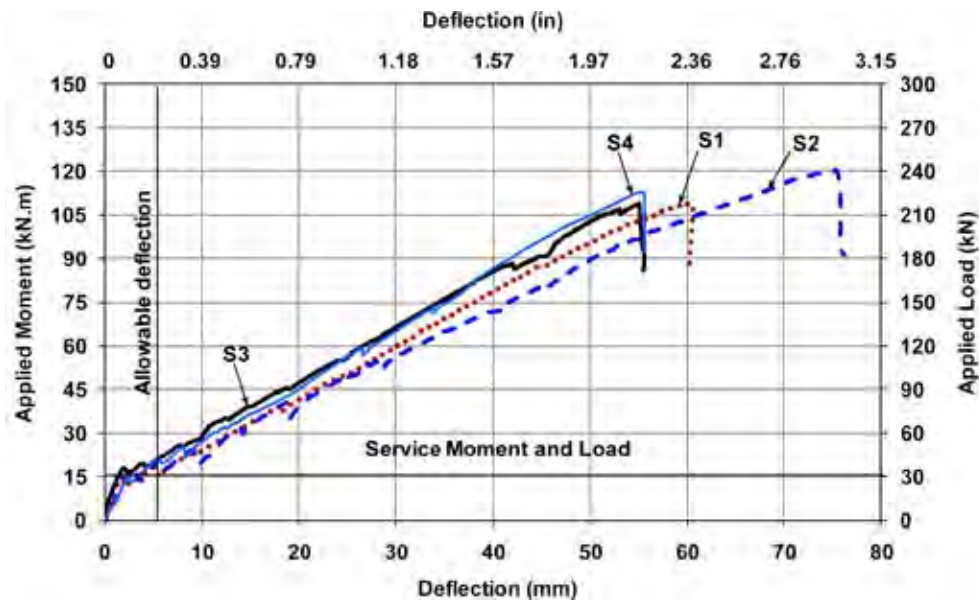


Fig. 3—Moment and load versus mid-span deflection.

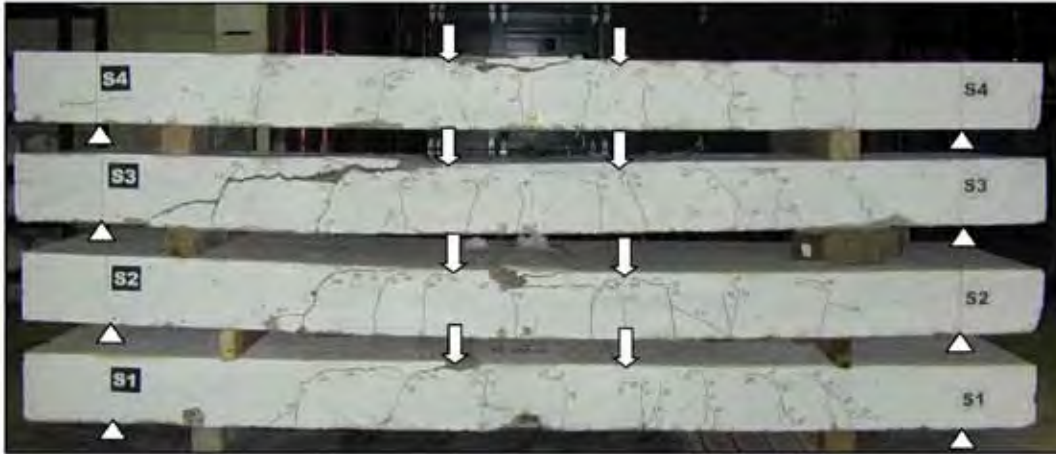


Fig. 4—Tested slabs at failure.

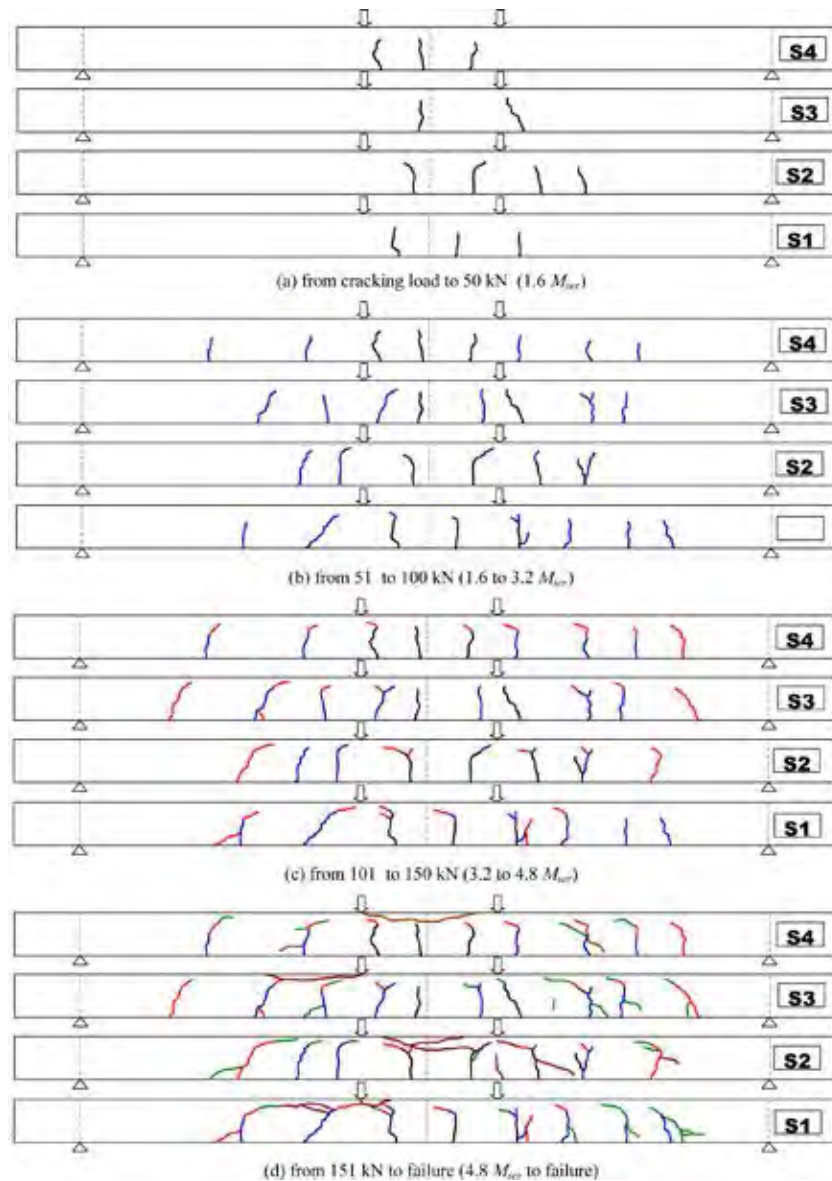


Fig. 5—Propagation of crack at different load levels. (Note: 1 kN = 0.2248 kip.)

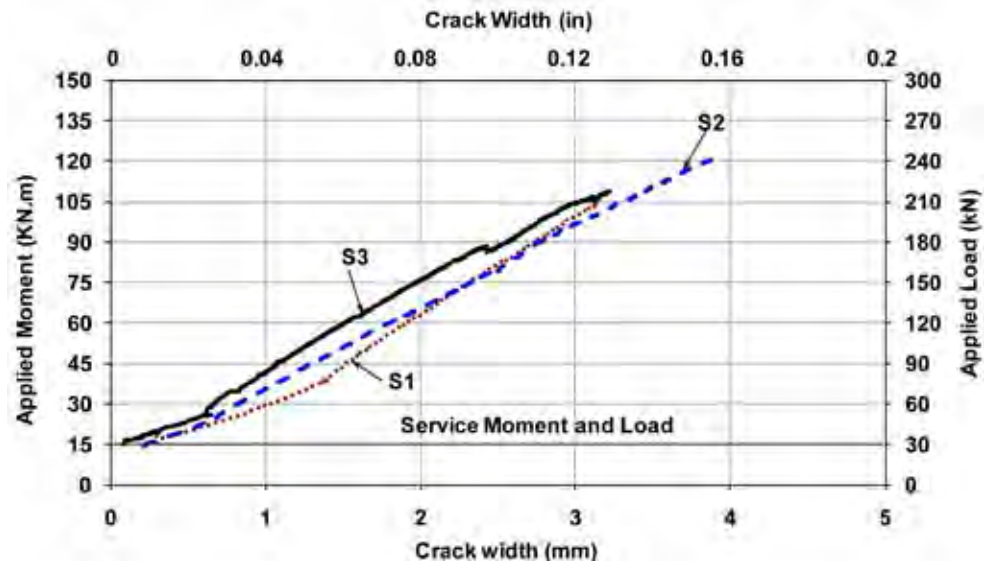


Fig. 6—Applied moment and load versus maximum measured crack width values.

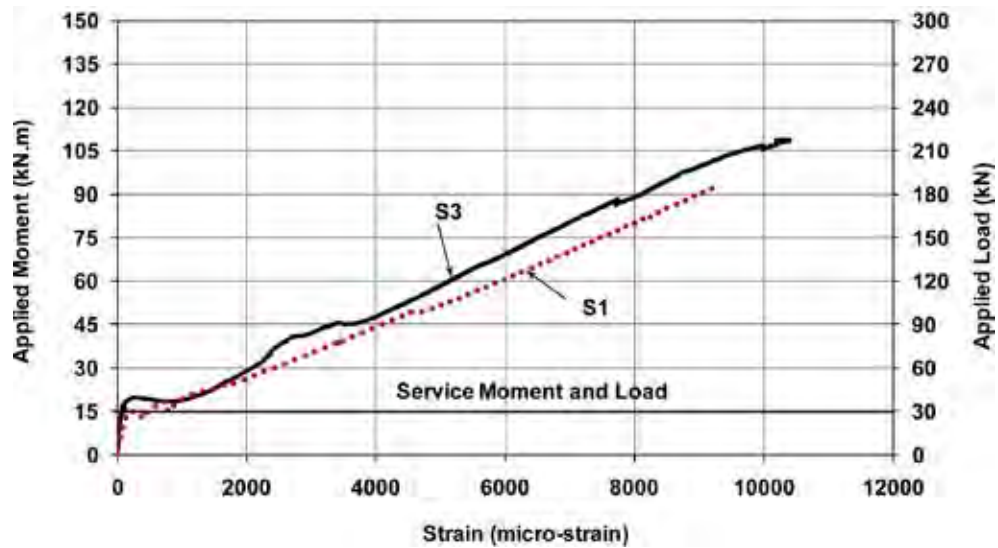


Fig. 7—Applied moment and load versus reinforcement strains.

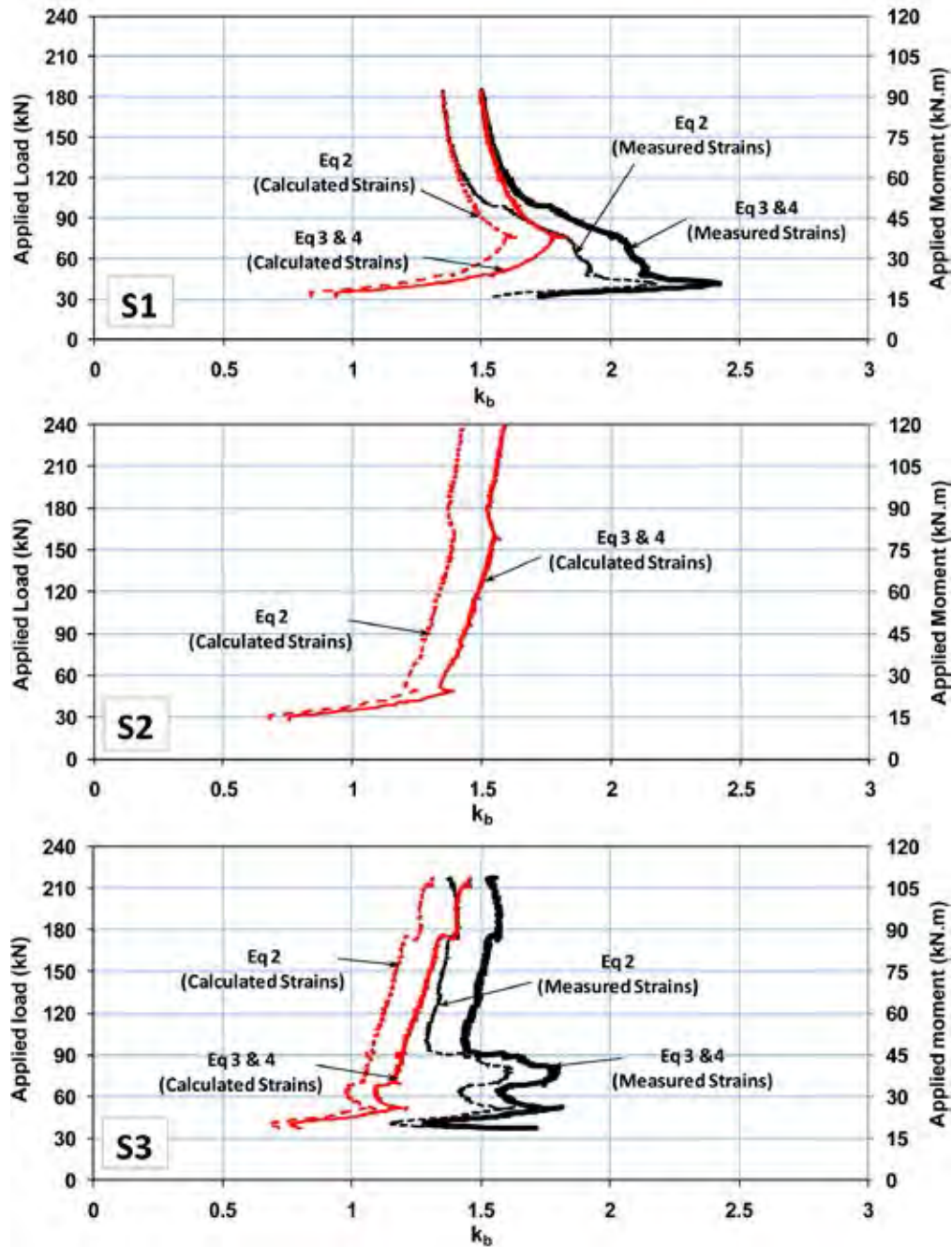


Fig. 8—Load versus k_b values for the test slabs. (Note: 1 kN = 0.2248 kip; 1 kN.m = 738.2 ft.lb.)

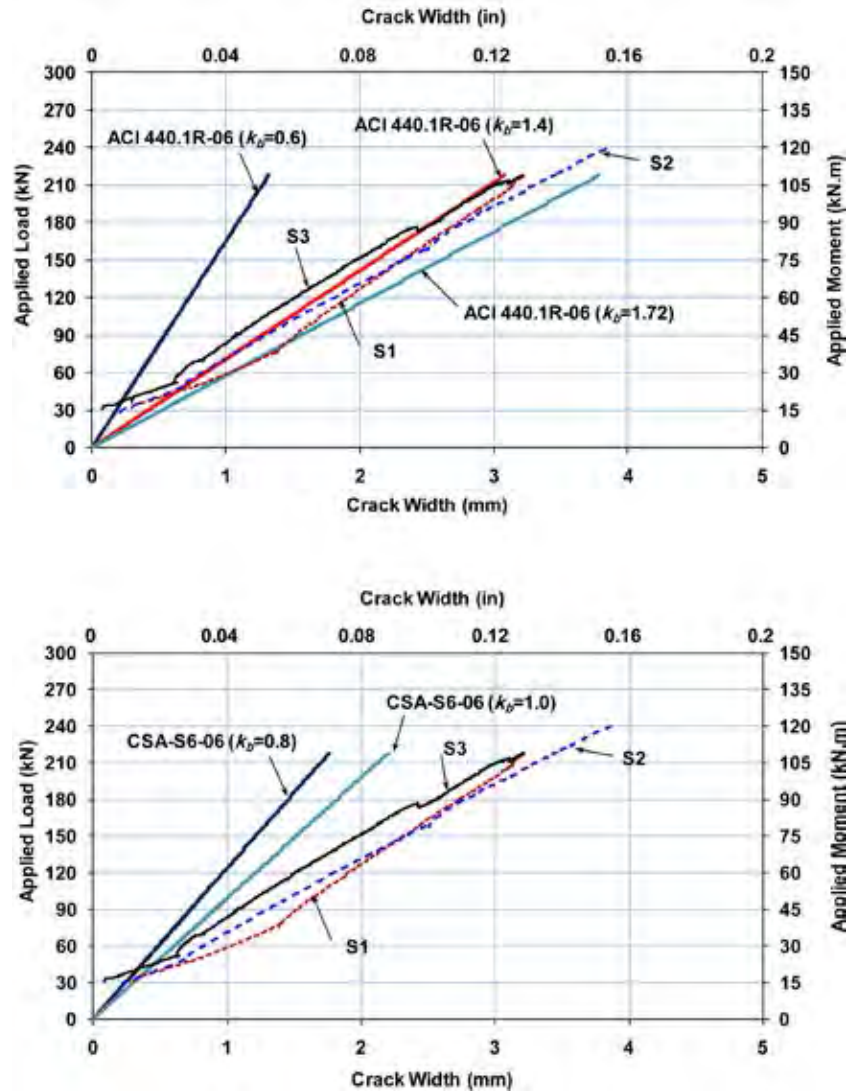


Fig. 9—Load versus crack width estimates. (Note: 1 kN = 0.2248 kip; 1 kN.m = 738.2 ft.lb.)