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Further Examples for the Design of Structural Concrete with Strut-and-Tie Models

Editors
Karl-Heinz Reineck
Lawrence C. Novak

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PREFACE

The contributions and the examples were prepared by members of ACI Subcommittee 445-A, Strut-and-Tie Models, and were presented in two sessions at the ACI Spring Convention in Chicago, 23 March 2010, co-sponsored by Joint ACI-ASCE Committee 445, Shear and Torsion. The papers of this special publication were reviewed in accordance with American Concrete Institute policies. The editors express their sincere thanks to all reviewers. The cooperation of the authors in the preparation of the manuscripts and their revision considering comments by reviewers is greatly appreciated.

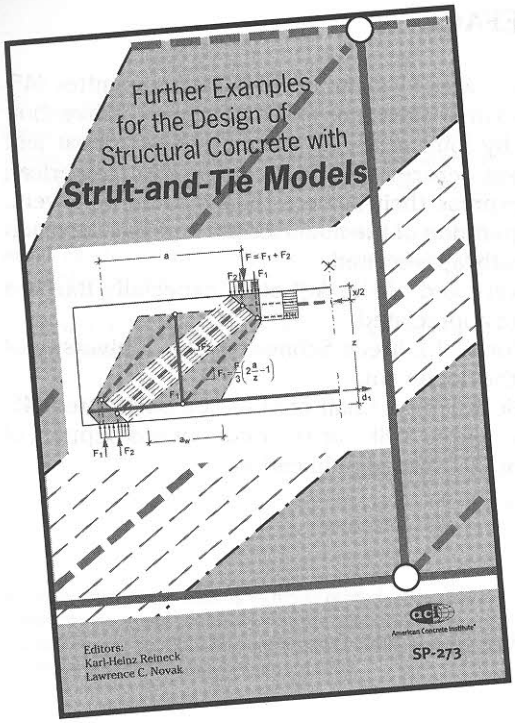
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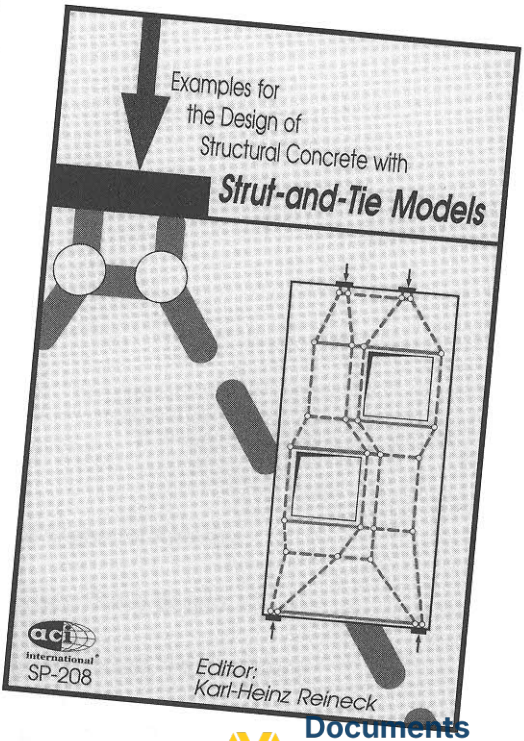
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Introduction

Strut-and-tie models provide an excellent tool for the design of structural concrete where sectional design procedures for flexure and shear do not apply. This is the case in discontinuity regions (D-regions), which may exhibit structural damage or even brittle failure if not properly designed and detailed. This special publication presents further examples for the use of strut-and-tie models following Appendix A of ACI 318-2008 for designing structural concrete members with D-regions. This SP is a follow-up of ACI SP 208, which was published in 2002 when the Appendix A was first introduced into ACI 318. The major part of this Special Publication presents 13 different examples designed with strut-and-tie models using Appendix A of ACI 318-2008. Most of these examples were taken from design practice of buildings and bridges:

- The Examples 1 and 2 dealing with hammerhead bent caps, and Example 3 dealing with a foundation grade beam represent D-regions, which so far have been designed by sectional design for flexure and shear.
- The Example 4 deals with the design of end-regions of prestressed beams, which represent a classical problem in the design of prestressed concrete beams.
- The Examples 5 and 6 deal with the design of beams in buildings which contain critical D-regions.
- The Example 7 covers a dapped end beam with a curved bar node, which is a valuable addition to the nodes presented in Appendix A of ACI 318.
- The Example 8 presents a dapped girder end with a special steel device, the so-called Cazaly hanger.
- In Example 9 two link beams are designed of a high-rise building.
- The Example 10 presents a strut-and-tie design for a storey-high propped cantilever deep beam with a large opening, and additionally comparisons are made with plastic and non-linear truss analyses, finite element analysis, and with the results from a scale model test.
- The Example 11 presents the design with strut-and-tie models for a 3D-structure, which is a pile cap with tension piles.
- The Examples 12 and 13 present the design of diaphragms in two bridges, a segmental concrete girder bridge and an extradosed cable stayed bridge.

All examples were selected to demonstrate the potential of strut-and-tie models to solve uncommon design problems. The different design steps are shown, starting with the important step of finding a model, followed by the dimensioning of the struts and ties, up to the steps with detailing and anchoring the reinforcing bars.

The Papers 14 and 15 deal with general topics rather than presenting numerical examples.

Paper 14 points out the importance of detailing by describing the collapse of the Concorde Overpass that was partly due to improper hanger reinforcement details. The evolution of the hanger reinforcement details in dapped end beams given in different versions of the PCI Design Handbook is presented, and the results of a series of experiments on dapped end beams with different hanger reinforcement details are described.

Paper 15 deals with some controversial topics that were discussed in ACI Subcommittee 445-A "Strut-and-Tie Models". The sectional shear design of beams leads to discrepancies with the design of adjacent D-regions by the strut-and-tie models, but this is avoided by designing the whole member with the truss model. Furthermore, two important D-regions in beams are dealt with, which is a point load applied near the support and the indirect support. Finally, a list of references is given for further guidance in application of strut-and-tie models.

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Example 1:
Bridge Pier – Hammerhead Bent Cap

Robin G. Tuchscherer

Michael D. Brown

Oguzhan Bayrak

Synopsis

The design of a hammerhead bent cap is presented. The design is performed according to the ACI 318 Appendix A Strut-and-Tie Modeling (STM) provisions. The objective of this paper is to present a design example for designers who do not have a large amount of experience with strut-and-tie modeling. Strut-and-tie modeling is relatively new to ACI 318 as provisions were first included in the 2002 edition. The procedure may be used for the design of both reinforced and prestressed concrete structures. The following example is intended to supplement the basic example problems available in textbooks. For additional background information, a review of textbook-level material [ACI SP 208 (2002), Schlaich et al. (1987)] is recommended.

Robin G. Tuchscherer is a Project Engineer at Datum Engineers, Inc.in Austin, Texas. He received his PhD and MSE from the University of Texas at Austin (2008 and 2006 respectively); and his BSCE from the University of Wisconsin-Milwaukee (1999).

Michael D. Brown is a Staff Engineer at Whitlock, Dalrymple, Poston, and Associates, Austin, Texas. He received his PhD from the University of Texas at Austin in 2005. His BSCE (2000) and MSE (2002) are also from the University of Texas at Austin. He is an associate member of ACI-ASCE Committee 445 Shear and Torsion.

Oguzhan Bayrak, FACI, is an Associate Professor in the Department of Civil, Environmental, and Architectural Engineering and holds the Charles Elmer Rowe Fellowship in Engineering at the University of Texas at Austin, where he serves as Director of the Phil M. Ferguson Structural Engineering Laboratory. He is a member of ACI Committees 341, Earthquake-Resistant Concrete Bridges, and E803, Faculty Network Coordinating Committee, and Joint ACI-ASCE Committees 441, Reinforced Concrete Columns, and 445, Shear and Torsion.

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1 Description of design task

1.1 Geometry and Loads

The external geometry of the hammerhead bent cap used for this example is shown in Fig. 1-1. In general, the bent cap dimensions are controlled by the size of the bearing plates. For this example, the bent cap in loaded asymmetrically in order to account for the critical unbalanced load case of live load on one side of the bridge. The loads acting on the cap include the factored dead load, factored lane load, and include an allowance for impact. The factored self-weight of the bent cap was divided equally, and is also included in the applied loads.

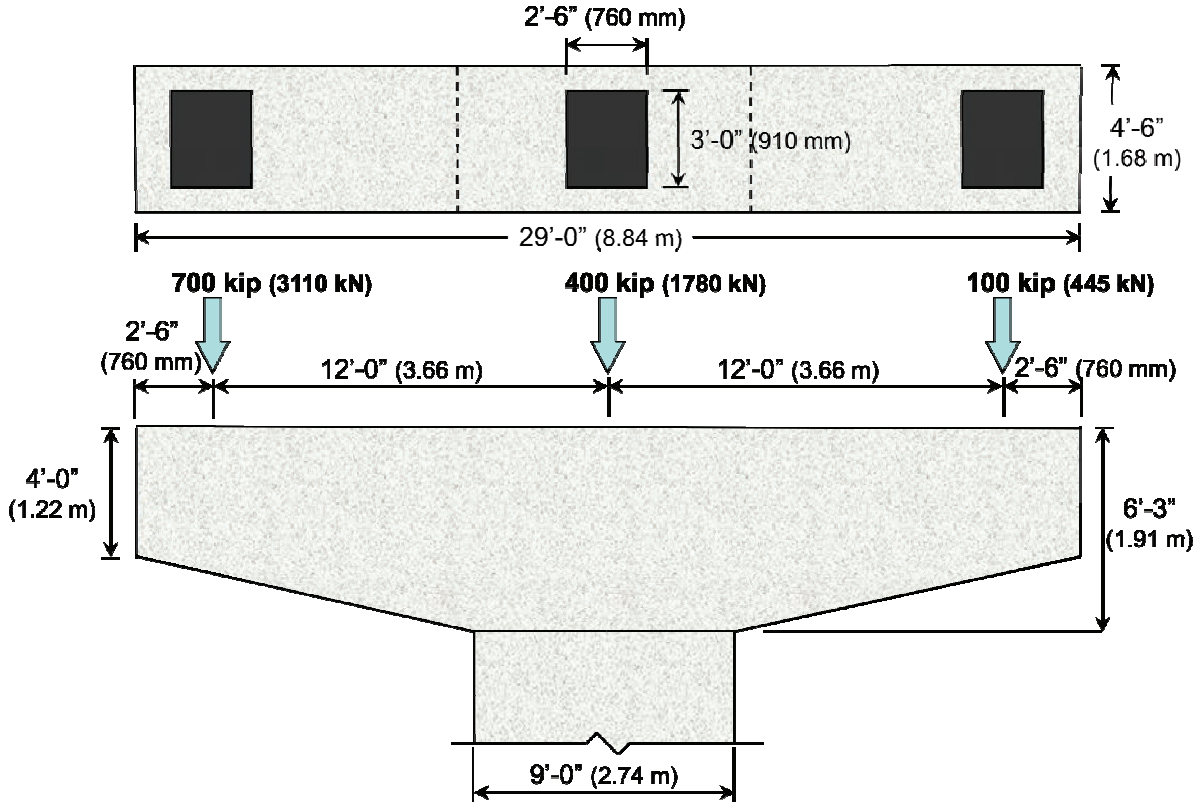


Fig. 1-1: Geometry and loads for hammerhead bent cap

1.2 Materials

Concrete: f_c' = 5,000 psi (34.5 MPa)
Reinforcement: f_y = 60,000 psi (414 MPa)

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1.3 Statement of design problem

A deep beam design must be treated differently than a sectional design (or slender beam design) because the assumptions used to derive the sectional theory are no longer valid. A deep beam is a member whose shear span-to-depth ratio, a/d, is relatively small such that nonlinear shearing strains dominate the behavior. According to MacGregor (1997), a deep beam can be defined as follows:

> *…a beam in which a significant amount of load is carried to the supports by a compression thrust joining the load and the reaction. This occurs if a concentrated load acts closer than about 2d to the support, or for uniformly loaded beams with a span-to-depth ratio, l_n/d, less than about 4 to 5.*

Strut-and-tie modeling (STM) is a relatively simple and effective method that can be used for the design of complex regions of discontinuity. A strut-and-tie model idealizes the complex flow of stresses in a structural member as axial elements in a truss member. Concrete struts resist the compressive stress fields and reinforcing steel ties resist the tensile stress fields. Struts and ties intersect at regions called nodes. Struts, ties, and nodes are the three elements that comprise a STM and they must be proportioned to resist the applied forces. Failure modes of a STM reflect the failure modes of a deep beam region and include the: (i) crushing of concrete in a strut or at the face of a node; (ii) yielding of a tie; (iii) or anchorage failure of a tie.

Further examples illustrating the case of a load near a support are evaluated and presented in ACI Special Publication 208 (2002). The purpose of the example presented herein is to provide further guidance for the design of a cantilever deep beam structure.

2 Design procedure

The shear span-to-depth ratio of the hammerhead bent cap in this example is less than two. As such, the structure may be classified as a deep beam region and may be designed according to the strut-and-tie provisions in Appendix A. For this example, the structure is first designed using a “simple” strut-and-tie model, which is subsequently referred to as Model 1. Next, the analysis is further developed using a “refined“strut-and-tie model, which is subsequently referred to as Model 2.

The following steps are used:

Step 1: Determine appropriate truss model

Step 2: Proportion tie

Step 3: Determine node geometry and check strength of nodes

Step 4: Check strength of struts

Step 5: Proportion crack control reinforcement

Step 6: Check tie anchorage

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3 Design Calculations: Simple Model 1

3.1 Step 1: Determine appropriate truss model

The design of a hammerhead bent cap using a simple truss is presented in Fig. 1-2. The strut-and-tie model is initially approximated by aligning the struts and ties with the centroid of the compression zone and main tensile reinforcement, respectively. The loading pattern shown is asymmetric in order to account for an unbalanced live load. An unbalanced live load can occur at either side of the structure depending which traffic lanes are considered. Thus, struts and ties on both sides of the bent will both be designed for the critical stresses that occur at the side with the higher loading. In addition, the pier must also be designed to resist the bending moment and axial load. This can be done using a conventional beam-column design and is not presented here.

Typically in structures of this type, the column is slightly narrower than the bent cap. This detail ensures that the longitudinal bars that extend from the column will fit inside the reinforcing cage in the bent cap. For the interest of simplicity, this example problem will contain a bent cap and column of the same width.

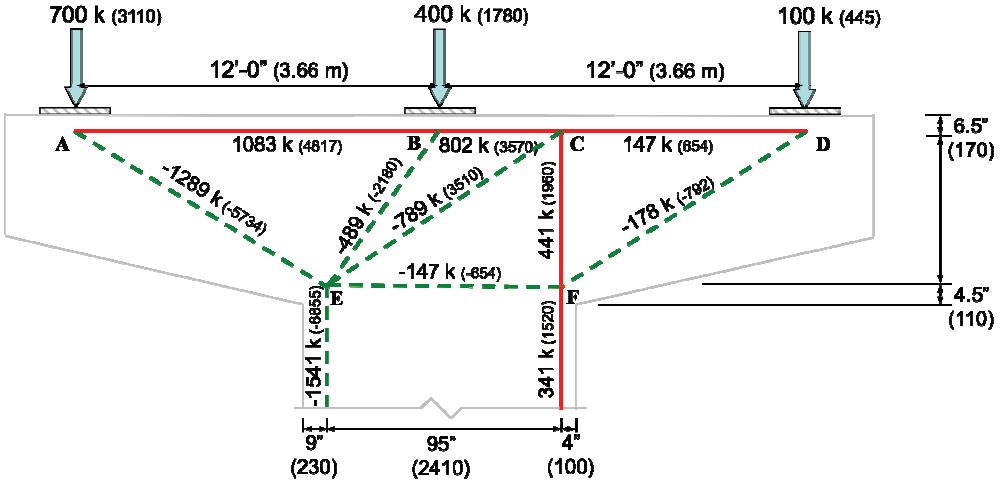


Fig. 1-2: Simple truss model 1 (force in kips and kN; dimensions in in. and mm)

According to the *lower bound theory of plasticity*, the capacity of a STM is always less than the actual capacity of the structure provided the following requirements are met: (i) the model is in equilibrium, (ii) sufficient deformation capacity exists to distribute forces according to the assumed model, and (iii) the stresses applied to the elements do not exceed their yield or plastic flow capacity. The choice of the model is left to the discretion of the designer. However, if the orientation of the model varies significantly from the actual stress field, then the structure must undergo substantial deformation in order to develop the poorly assumed model. Thus, it is good practice for the STM to agree well with the dominant mechanism of force transfer in the structure. Large deviations from 45° of the angle between a strut and tie will demand excessive strains in the reinforcement together with extremely wide crack openings at failure. Accordingly, the ACI 318-08 STM provisions limit the angle between a strut and tie to 25°.

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3.2 Step 2: Proportion tie

Tie ABCD is positioned 6.5 in. (165 mm) from the top of the section. The large tensile force in the tie results in two layers of longitudinal bars. The force in tie ABCD is 1083 kip (4817 kN). According to Section A.4 of ACI 318-08, the required area of reinforcement is:

$$F_{nt} = A_{ts} f_y$$

Solving for A_{ts},

$$A_{ts,req} = \frac{F_u}{\phi f_y} = \frac{1083 kip}{0.75 \cdot 60 ksi} = 24.1 in.^2 \left(15{,}500 mm^2\right)$$

Therefore, provide 18 – No. 11 bars $\left(A_{ts} = 28.1 in.^2\right)$ [18 –36M bars ($A_{ts} = 18{,}100 mm^2$)]

It is of interest to note that the strength reduction factor ϕ used to determine the main longitudinal reinforcement is 0.75 compared with 0.9 used for a conventional flexural analysis. This is attributed to the fact that the strength reduction factors in ACI 318-08 coincide with member behavior rather than material.

3.3 Step 3: Determine node geometry and check strength of nodes

3.3.1 Nodes A and D (CCT Node)

Now that the tie reinforcement has been chosen, proportions of the nodal region can be determined. Nodal geometry is based on the cover dimensions, bearing dimensions, and strut angles.

The design of Nodes A and D is based on the concentration of critical stresses at Node A. The node anchors one tie and is therefore considered to be a CCT node. The geometry of Node A is determined as shown in Fig. 1-3. The placement of Tie ABCD and the dimension of the adjacent bearing pad determine the geometry of Node A. The width of the node and strut are assumed to be equal to the width of the bearing pad. The height of a CCT node is typically taken as twice the clear cover distance to the centroid of the reinforcement; or 13 in. (330 mm).

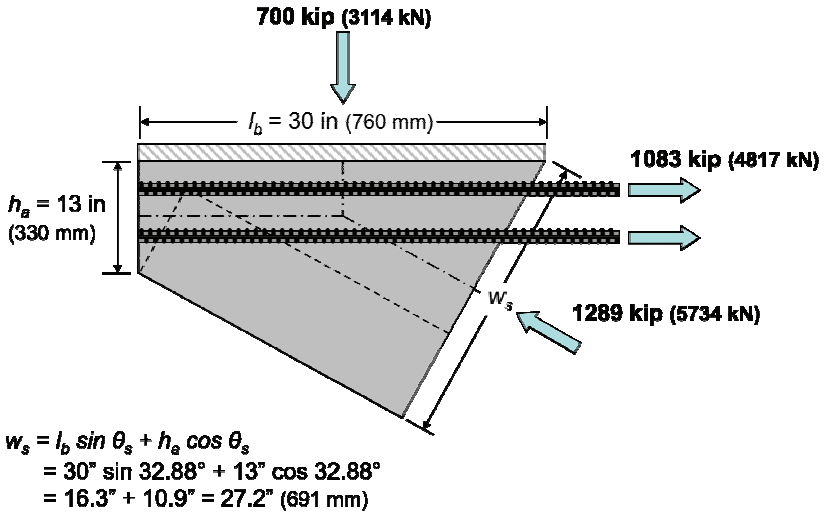


Fig. 1-3: Elevation view of node A

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The angle of the strut is estimated based on the global model. In this case:

$$\theta = \tan^{-1}\frac{64in.}{99in.} = 32.88°$$

According to Section A.5 of ACI 318-08, the allowable stress on any face of a CCT node is:

$$f_{cu} = 0.85 \cdot 0.80 \cdot f_c'$$

The design strength of all the faces of a node must be greater than the strength required from the strut-and-tie model. Based on the allowable stress, the strengths of each nodal face is checked as follows:

For the top face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f_c' A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5ksi \times 30.0in. \times 36.0in. = 2{,}754kip > 700kip$$
$$\left(\phi F_{nn} = 12{,}250kN > 3{,}110kN\right)$$

For the left face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f_c' A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5ksi \times 13.0in. \times 36.0in. = 1{,}193kip > 1{,}083kip$$
$$\left(\phi F_{nn} = 5{,}307kN > 4{,}817kN\right)$$

The strength of the inclined face of the node (strut-to-node interface) is controlled by the allowable stress of a strut and is given in Section 3.4.1.

3.3.2 Node E (CCC Node)

The geometry of node E is determined as shown in Fig. 1-4. The height of the node, h_a, is assumed to be equivalent to the depth of the compressive stress block determined from a conventional flexural analysis. For a rectangular section, the depth of the compressive stress block can be determined as follows:

$$h_a = \frac{A_s f_s}{0.85 f_c' b_w}$$

For this example, the height of node E is estimated to be approximately 9 in. (230 mm).

Similarly, the width of node E can be determined from a sectional analysis of the column considering the interaction between flexure and compression. Aligning the strut with the centroid of the compressive stress block obtained from an ultimate analysis is not necessary for such a large cross-section. Rather, the height of node E can be estimated based on an approximation of the compression zone depth as it varies between the linear-elastic and ultimate stress diagram. For this example, the compression zone is estimated to be 18 in. (910 mm). Thus, the centroid of the strut framing into node F is located 9 in. (460 mm) from the face of the pier.

In order to check node E using strut-and-tie provisions, it is necessary to subdivide the node based on the proportion of compressive force that frames into the region from each direction. In addition, the multiple struts framing into the node from the right side can be resolved into a single force based on summation of vectors. The resulting configuration of node E is illustrated in Fig. 1-4.

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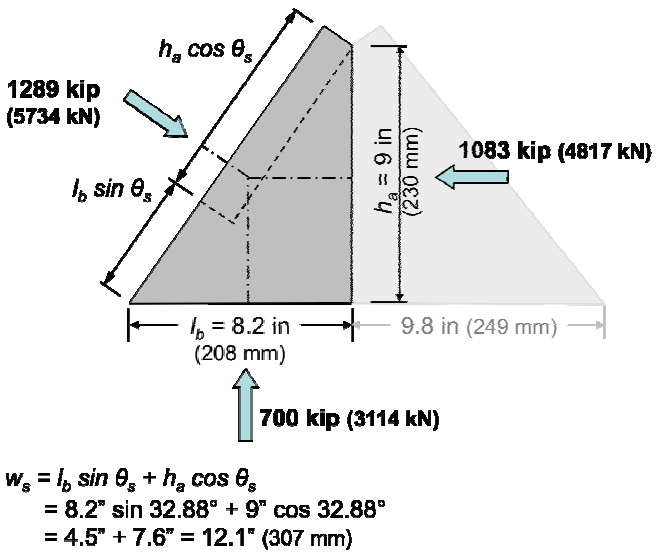


Fig. 1-4: Elevation view of node E

According to Section A.5 of ACI 318-08, the allowable stress on any face of a CCC node is:

$$f_{cu} = 0.85 \cdot 1.0 \cdot f_c'$$

The design strength of all the faces of a node must be greater than the strength required from the strut-and-tie model. Based on the allowable stress, the strengths of the nodal faces are checked as follows:

For the bottom face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f_c' A_{nz} = 0.75 \times 0.85 \times 1.0 \times 5ksi \times 8.2in. \times 54.0in. = 1{,}411kip > 700kip$$
$$\left(\phi F_{nn} = 6{,}276kN > 3{,}114kN\right)$$

For the right face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f_c' A_{nz} = 0.75 \times 0.85 \times 1.0 \times 5ksi \times 9in. \times 54in. = 1{,}549kip > 1{,}083kip$$
$$\left(\phi F_{nn} = 6{,}890kN > 4{,}817kN\right)$$

The strength of the inclined face of the node (strut-to-node interface) is controlled by the allowable stress of a strut and is given in Section 3.4.1.

3.3.3 Node F

By inspection, the critical design of node H would be as a CCC node when the load case in a lane loading on the other side of the bent similar to the analysis presented in Section 3.3.2 for node E. The tensile force at node F is accounted for with the design of the longitudinal column reinforcement.

3.3.4 Node C

Node C is a CTT node framed by tie CF, tie BC, and strut EC. Tie CF is required to maintain equilibrium caused by the unbalanced moment on the pier. A similar condition would exist at the other side of the pier if the loading were unbalanced to the other side of the structure. With a

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conventional design, the unbalanced moment is transferred into the pier by extending the longitudinal column reinforcement vertically into the bent and lapping the hooked ends with the longitudinal reinforcement in the cap. Additionally, column ties and stirrups are provided in order to confine the joint region and develop the flexural strength of each member. A strut-and-tie model may be used to determine the column reinforcement in this region but such analysis is not included in this example.

3.4 Step 4: Check strength of struts

Struts AE and DF connect the exterior applied loads to the support reaction. The design of both of these struts is based on the critically stressed Strut AE. Each interface of strut AE is checked as follows.

According to Section A.3 of ACI 318-08, the effective compressive strength of concrete in a strut shall be taken as:

$$f_{ce} = 0.85 \cdot \beta_s \cdot f_c'$$

The efficiency factor, β_s, is based on the shape of the strut, its location, and the amount of transverse reinforcement crossing its path. The compressive stress field of strut AE will spread laterally such that the width of the midsection will be wider than the width at the node interface (bottle-shaped). Provided that adequate crack control reinforcement is provided, the strength of the strut-to-node interface at Node A is checked as follows:

$$\phi F_{ns} = \phi 0.85 \beta_s f_c' A_{cs} = 0.75 \times 0.85 \times 0.75 \times 5ksi \times 36.0in. \times 27.2in. = 2{,}341kip > 1{,}289kip$$

$$\left(\phi F_{ns} = 10{,}410kN > 5{,}734kN\right)$$

Similarly, the strength of the strut-to-node interface at node F is checked as follows:

$$\phi F_{ns} = \phi 0.85 \beta_s f_c' A_{cs} = 0.75 \times 0.85 \times 0.75 \times 5ksi \times 12.1in. \times 54in. = 1{,}562kip > 1{,}289kip$$

$$\left(\phi F_{ns} = 6{,}948kN > 5{,}734kN\right)$$

3.5 Step 5: Proportion crack control reinforcement

According to Section A.3.3 of ACI 318-08, in order to take advantage of the higher efficiency factor at the strut-to-node interface, the axis of the strut shall be crossed by sufficient crack control reinforcement. Crack control reinforcement is proportioned to resist the transverse tensile force resulting from the spreading of the compressive force in the strut. According to Section A.3.3.1, the requirements of Section A.3.3 can be satisfied as follows:

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i \geq 0.003$$

For a bent cap with such a large cross-sectional area, the shear reinforcement typically consists of four legs of stirrups and longitudinal skin reinforcement. If fourteen – No. 6 (19M) bars are provided, seven at each face, then No. 5 (16M) stirrups with four legs are required at 11 inches center-to-center per the requirements of Section A.3.3:

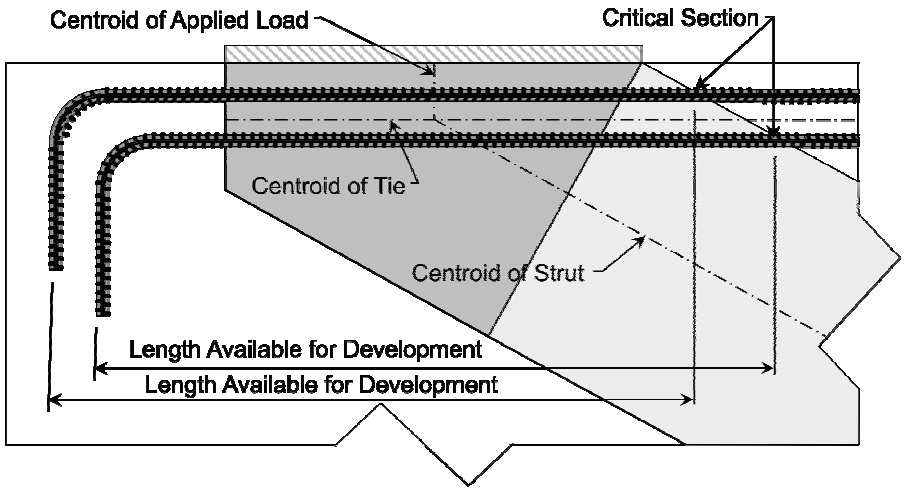
$$\frac{14 \times 0.44in.^2}{54in. \times 55in.} \sin 32.88° + \frac{4 \times 0.31in.^2}{54in. \times 11in.} \sin 57.12° = 0.003$$

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3.6 Anchorage of tie

When using Appendix A of ACI 318, the anchorage requirements of Chapter 12 also apply. Often, ties at CCT nodes must be developed in a short length near the end of a deep beam. For this example, standard 90-degree hooks are considered for the anchorage of tie ABCDE. The critical section for anchoring tie ABCDE is shown in Fig. 1-5. For example, the length available to develop the top bars in tie ABCDE is from the intersection that the bars make with the strut to the exterior face of the bent cap less the required clear cover:



$$L_d = 30in. + \frac{3.3in.}{\tan 31.0^\circ} + 15in. - 2in. = 48.5in.[1240mm]$$

Fig. 1-5: Anchorage of tie ABCDE

According to Section 12.5 of ACI 318-08, the required development length of a standard hooked bar in tension is equal to the following:

$$l_{dh} = \frac{0.02\psi_e f_y}{\lambda\sqrt{f_c'}} d_b$$

$$= \frac{0.02 \times 1 \times 60{,}000\,psi}{1 \times \sqrt{5{,}000\,psi}} 1.41in. = 23.9in. < 48.5in.[607mm < 1240mm]$$

Thus, a hooked bar is adequately developed. If necessary, the required development length may be reduced further provided that the reinforcement is detailed such that the cover and confinement requirements of Section 12.5.3 are satisfied.

According to Section 12.2 of ACI 318-08, the required development length of a straight bar in tension is equal to the following:

$$l_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f_c'}}\right) d_b = \left(\frac{60{,}000\,psi \cdot 1.3 \cdot 1}{20 \cdot 1 \cdot \sqrt{5000\,psi}}\right) 1.41in. = 78in. > 48.5in.[1980mm > 1240mm]$$

Thus, standard hooked bars are required in order to develop the tie.

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4 Design Calculations for alternative Model 2

4.1 Step 1: Determine model

The design of a hammerhead bent cap using a refined strut-and-tie model is presented in Fig. 1-6 and subsequently referred to as Model 2. The layout of Model 2 is similar to Model 1, i.e. the height of the truss, location of the top tie, and location of the bottom strut are identical for both truss models. The main difference between the two trusses is the addition of ties HM and JO.

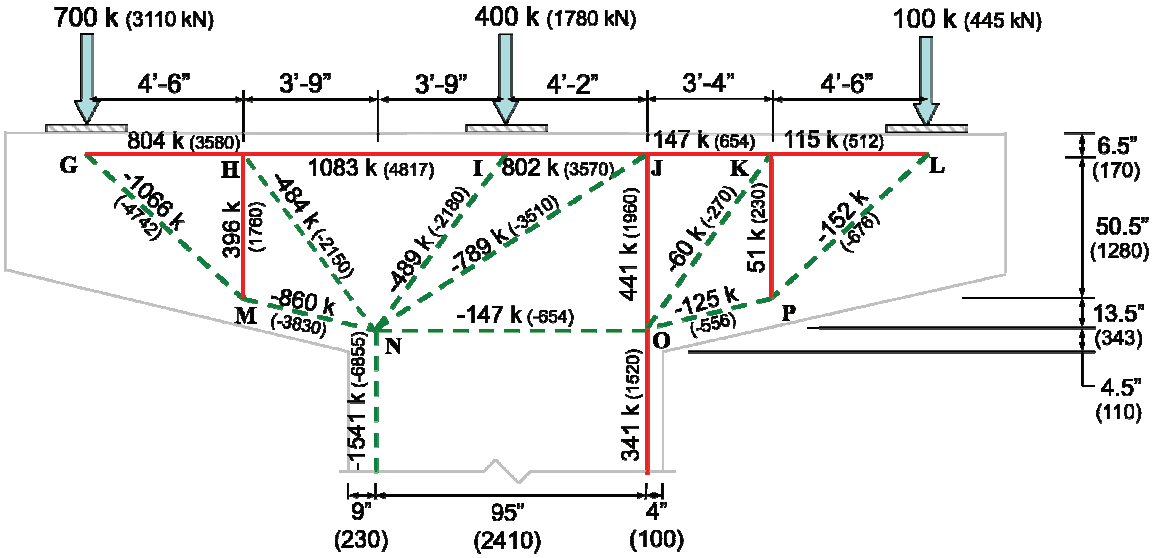


Fig. 1-6: Refined truss model 2 (force in kips and kN; dimensions in in. and mm)

Nodes N and O in Model 2 are at the same location as nodes E and F in Model 1. In order to determine the geometry of node N, the forces in struts MN, HN, IN, JN, and NO would be resolved into two single forces through vector summation. These resultant forces would be identical in magnitude and direction to the forces acting on node E as presented in Section 3.3.2. Since node E has adequate strength, node N also has adequate strength. The same is true for the struts connecting these nodes.

4.2 Step 2: Proportion tie

Tie GHIJKL in Model 2 is in the same location as tie ABCD in Model 1. These two ties carry the same peak tensile force [1083 kip (4817 kN)]. In Model 2, the force in tie GHIJKL decreases at node H. Based on the decrease in force, the area of the longitudinal reinforcement could be subsequently reduced. However, the region of the cap between nodes G and I carries a constant shear force. Longitudinal bar cut-offs are not advised in zones with large constant shear forces. Reduced shear strength and loss of ductility has been reported when bars are cut off in a tension zone. Furthermore, ACI 318 §12.10.5 does not permit reinforcement to be terminated in a tension zone unless the requirements of §12.10.5.1, 12.10.5.2, or 12.10.5.3 are satisfied. In order to simplify the design and construction of this bent cap, the quantity and placement of steel used in tie GHIJKL is kept constant along the tie length; identical to tie ABCD in the previous solution.

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4.3 Step 3: Determine node geometry and check strength of nodes

4.3.1 Tie HM and KP

Interior nodes, which are not bound by a bearing plate, are often referred to as *smeared* nodes. Forces from compressive struts spread, or smear, and are equilibrated by multiple stirrups, or ties. Because a bearing plate does not abut an interior CTT node, a proportioning technique must be employed to determine the extents of the exterior face, l_a. The method that is employed for this example is that recommended by Wight and Parra-Montesinos (2003): the authors propose that any stirrup that intersects an adjacent strut at an angle greater than 25-degrees be engaged as part of the vertical tie of the CTT node (Fig. 1-7).

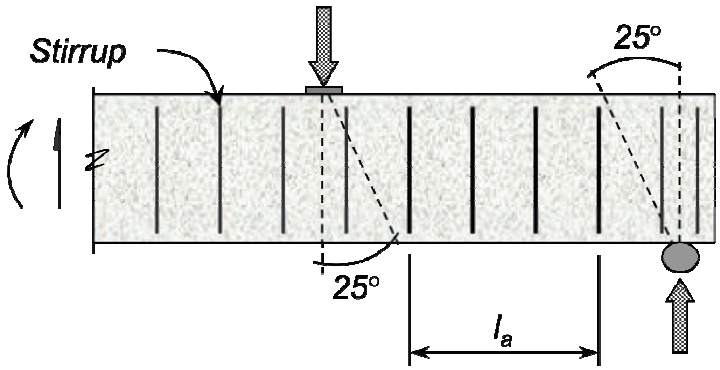


Fig. 1-7: Determination of a CTT vertical tie

Accordingly, the extents of the exterior face of Node H can be determined as follows:

$$l_a = 99in. - (64in.)\cdot \tan 25° - (50.5in.)\cdot \tan 25° = 46in.[1170mm]$$

The design of ties HM and KP is based on the more critically stressed tie HM. Recall that the vertical web reinforcement required for crack control was No. 5 (19M) stirrups at 11 in. (280 mm) – with four legs distributed across the web. Thus, based on the minimum stirrup spacing determined in Section 3.5 four sets of stirrups are included in tie HM. According to Section A.4 of ACI 318, the design strength of tie HM is:

$$\phi F_{nt} = \phi A_{ts} f_y = 0.75\cdot(4\cdot 4\cdot 0.31in^2)\cdot 60ksi = 223kip < 396kip[992kN < 1{,}761kN]$$

Thus, based on the previously determined reinforcement configuration, the capacity of Model 2 is less than the capacity of Model 1. This phenomenon is common; a truss model comprised of a direct strut generally requires less stirrups and more longitudinal reinforcement than a truss model that includes a vertical intermediate tie. Either model is an appropriate solution. However, a solution based on a direct strut typically is simpler and meets the requirements of ACI 318-08. In addition, a direct strut is the predominant mechanism of shear transfer for beams with an a/d ratio less than two (Birrcher et al., 2009). As such, it is recommended that Model 1 be used for determining the capacity and reinforcement required for the current bent cap design.

In order to complete the example, the vertical web reinforcement required for Model 2 is determined. According to Section A.4 of ACI 318-08, if No. 5 (19M) stirrups are provided with four legs distributed across the web, then they must be spaced as follows:

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$$s = \frac{46in.\cdot\left(4\cdot 0.31in^2\right)\left(0.75\cdot 60ksi\right)}{396kip} = 6.5in.[170mm]$$

Provide No. 5 stirrups at 6 in. center-to-center (19M @ 150 mm) with four legs distributed across the web.

4.3.2 Node H (CTT Node)

The height of node H, h_a, is determined by the location of the tie reinforcement in the same manner as described for Model 1. The width of the node is taken to be equivalent to the width of the bent cap. The length of the node, *la*, is based on the technique presented in Section 4.3.1. A drawing of node H is shown in Fig. 1-8.

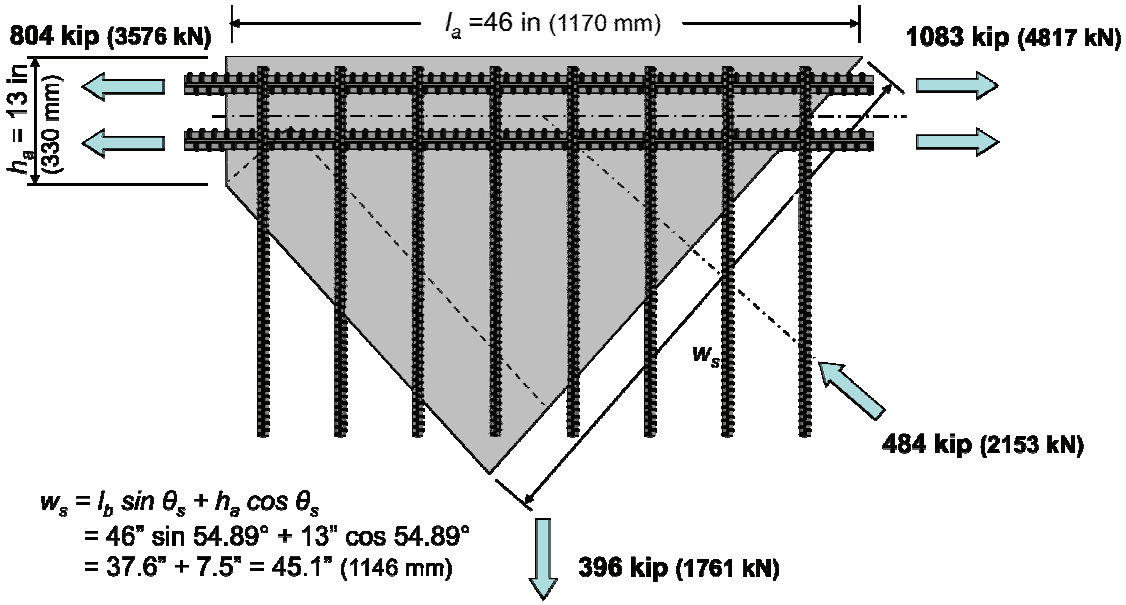


Fig. 1-8 Elevation view of Node H

According to Section A.5 of ACI 318-08, the allowable stress on any face of a CTT node is:

$$f_{cu} = 0.85\cdot 0.6\cdot f_c'$$

The design strength of all the faces of a node must be greater than the strength required from the strut-and-tie model. Based on the allowable stress, the strengths of the nodal faces are checked as follows:

For the top face of the node:

$$\phi F_{nn} = \phi 0.85\beta_n f_c' A_{nz} = 0.75\times 0.85\times 0.6\times 5ksi\times 54in.\times 46in. = 4751kip > 396kip$$
$$\left(\phi F_{nn} = 21{,}130kN > 1{,}761kN\right)$$

For the left face of the node, the strength required is 1083 kip – 804 kip = 279 kip (1241 kN):

$$\phi F_{nn} = \phi 0.85\beta_n f_c' A_{nz} = 0.75\times 0.85\times 0.6\times 5ksi\times 13in.\times 54.0in. = 1343kip > 279kip$$
$$\left(\phi F_{nn} = 5{,}974kN > 1{,}241kN\right)$$

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According to ACI 318-08, the allowable stress at the face of a CTT node is less than the allowable stress in a bottle-shaped strut. Thus, the strength of the inclined face (strut-to-node interface) of node H is limited as follows:

$$\phi F_{nn} = \phi 0.85 \beta_n f_c' A_{nz} = 0.75 \times 0.85 \times 0.6 \times 5 ksi \times 45.1 in. \times 54.0 in. = 4{,}658 kip > 484 kip$$
$$\left(\phi F_{nn} = 20{,}720 kN > 2{,}153 kN\right)$$

4.4 Step 4: Check strength of struts

The strength of strut HN is controlled by the allowable stress at the face of node H as presented in Section 4.3.2.

4.5 Detailing requirements: crack control reinforcement and anchorage

The detailing requirements prescribed for Model 1 also apply to Model 2 and are presented in Sections 3.5 and 3,6. In addition, it is important to realize that the differential in the longitudinal tie forces at Node H must be adequately developed. For this example, this is accomplished by providing continuous longitudinal top bars. A summary of the reinforcement arrangement for the two models is illustrated in Fig. 1-9.

5 Summary

A single hammerhead bent cap was designed according to the provisions of Appendix A using either a simple strut-and-tie model (Model 1) or a more refined strut-and-tie model (Model 2). It was shown that the required amount of web reinforcement was almost doubled when the more complicated Model 2 was used to design the cap. This is attributed to the fact that web reinforcement is not required to maintain equilibrium for a truss containing a single strut between the load and support. A truss model containing a vertical tie in between the load and support requires more web reinforcement; however, it also requires less longitudinal reinforcement. For this example, the amount of longitudinal reinforcement was kept constant between the two models based on detailing and constructability requirements. A typical cross section of the bent cap and elevation is shown in Fig. 1-9. Both designs meet the requirements of Appendix A and are considered to be satisfactory.

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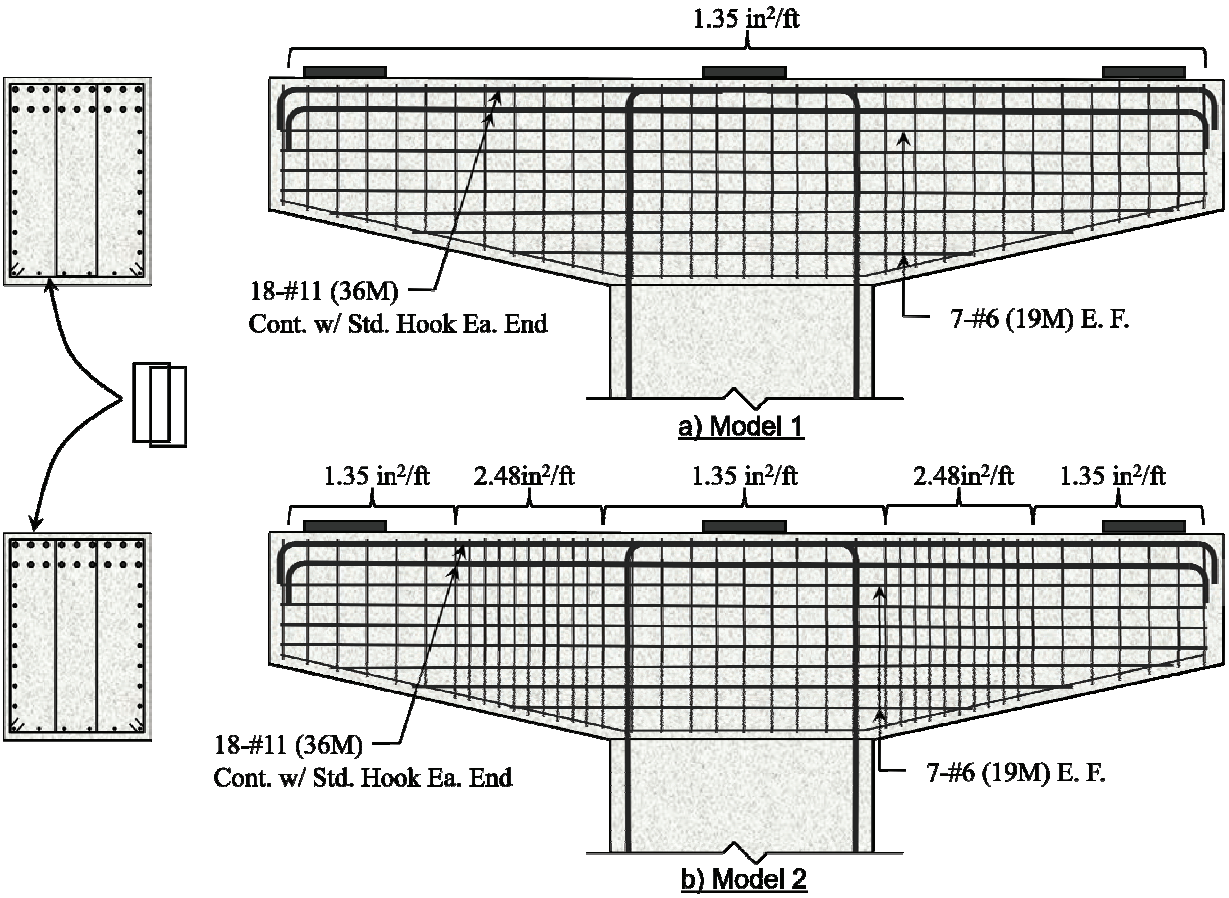


Fig. 1-9: Summary of reinforcement requirements for a) simple model 1 and b) refined Model 2
**Column intermediate and transverse reinforcement not shown for clarity*

Notation

A_{cs} = cross-sectional area at one end of a strut, taken perpendicular to the axis of the strut

A_{nz} = area of a face of a nodal zone or a section through a nodal zone

A_{si} = total area of surface reinforcement at spacing s_i in the i-th layer crossing a strut with reinforcement at an angle α_i to the axis of the strut

A_s = area of tension reinforcement

A_s' = area of compression reinforcement

A_{ts} = area of nonprestressed reinforcement in a tie

b_w = web width

d = depth from the extreme compression fiber to the centroid of the tensile reinforcement

d_b = nominal diameter of bar

f'_c = specified compressive strength of concrete

f_{ce} = effective compressive strength of concrete in a strut or nodal zone

f_s = stress in tension reinforcement

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f_y = specified yield strength of reinforcement
F_{nn} = nominal strength at face of nodal zone
F_{ns} = nominal strength of a strut
F_{nt} = nominal strength of a tie
F_u = factored force acting in a strut, tie, or nodal zone
h_a = thickness of member in which an anchor is located, measured parallel to anchor axis
l_b = length of bearing area
l_d = development length in tension of a deformed straight bar
l_{dh} = development length in tension of a deformed bar with a standard hook
s_i = center-to-center spacing of reinforcement in the i-th layer adjacent to the surface of the member
w_s = width of a strut at the nodal interface
β_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone
β_s = factor to account for the effect of crack control reinforcement on the effective compressive strength of concrete in a strut
θ_s = angle between axis of strut, compression diagonal, or compression field and the tension chord of the member
λ = modification factor related to the unit weight of concrete
φ = strength reduction factor
ψ_e = factor used to modify development length based on reinforcement coating
ψ_t = factor used to modify development length based on reinforcement location

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Example 2: Four-Column Bent Cap

Christopher S. Williams

Michael D. Brown

Oguzhan Bayrak

Synopsis

The design of a statically indeterminate bent cap is presented. The cap resists six concentrated loads and is supported on four columns. Analysis, design, and detailing are all presented. Each of these steps is based on the ACI 318-08 Appendix A Strut-and-Tie Modeling (STM) provisions. Two load cases are considered and the controlling forces in the STMs are used to develop the final design. The objective of the example problem is to help familiarize engineers with the basic STM procedure through a fairly simple beam design.

Christopher S. Williams is a graduate student at the University of Texas at Austin. He received his BS from Southern Illinois University Carbondale. His research interests include sustainability of bridge infrastructure and design of concrete structures.

Michael D. Brown is a Staff Engineer at Whitlock, Dalrymple, Poston, and Associates, Austin, Texas. He received his PhD from the University of Texas at Austin in 2005. His BSCE (2000) and MSE (2002) are also from the University of Texas at Austin. He is an associate member of ACI-ASCE Committee 445, Shear and Torsion.

Oguzhan Bayrak, FACI, is an Associate Professor in the Department of Civil, Environmental, and Architectural Engineering and holds the Charles Elmer Rowe Fellowship in Engineering at the University of Texas at Austin, where he serves as Director of the Phil M. Ferguson Structural Engineering Laboratory. He is a member of ACI Committees 341, Earthquake-Resistant Concrete Bridges, and E803, Faculty Network Coordinating Committee, and Joint ACI-ASCE Committees 441, Reinforced Concrete Columns, and 445, Shear and Torsion.

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1 Description of design task

1.1 Geometry and Loads

The geometry of the bent cap is presented in Fig. 2-1 below in both a plan and an elevation view. Since the bent cap is symmetrical, only half of the member is shown. The bent cap is supported on four columns and, in turn, supports twelve pretensioned girders (six each from adjacent spans). Often, caps such as this are supported on columns with a diameter of 30 in. (762 mm). To simplify the analysis of the nodes located at the column-beam interface, square columns with areas equal to that of the 30 in. (762 mm) circles are used. These equivalent square columns are 26.6 in. by 26.6 in. (676 mm by 676 mm). The column dimension in Fig. 2-1 reflects the size of the equivalent square columns. The bearing areas that support the pretensioned girders are 19 in. by 26 in. (483 mm by 660 mm) with the smaller dimension parallel to the span of the bent cap.

Two factored load cases are presented for the bent cap design. The first load case, shown in Fig. 2-1, is symmetrical with 242 kips (1076 kN) acting on each load plate. The second load case, shown in Fig. 2-2, lacks symmetry: the loads acting on the right side of the cap are smaller than the loads acting on the left side. The total load acting on the cap for each case is equal. The second load case represents the presence of an asymmetric live load on the deck. It is important to note that the live load arrangement could be changed such that a third load case where the "mirror-image" of the loads shown in Fig. 2-2 can be obtained. The presence of this third load case will be accommodated in design.

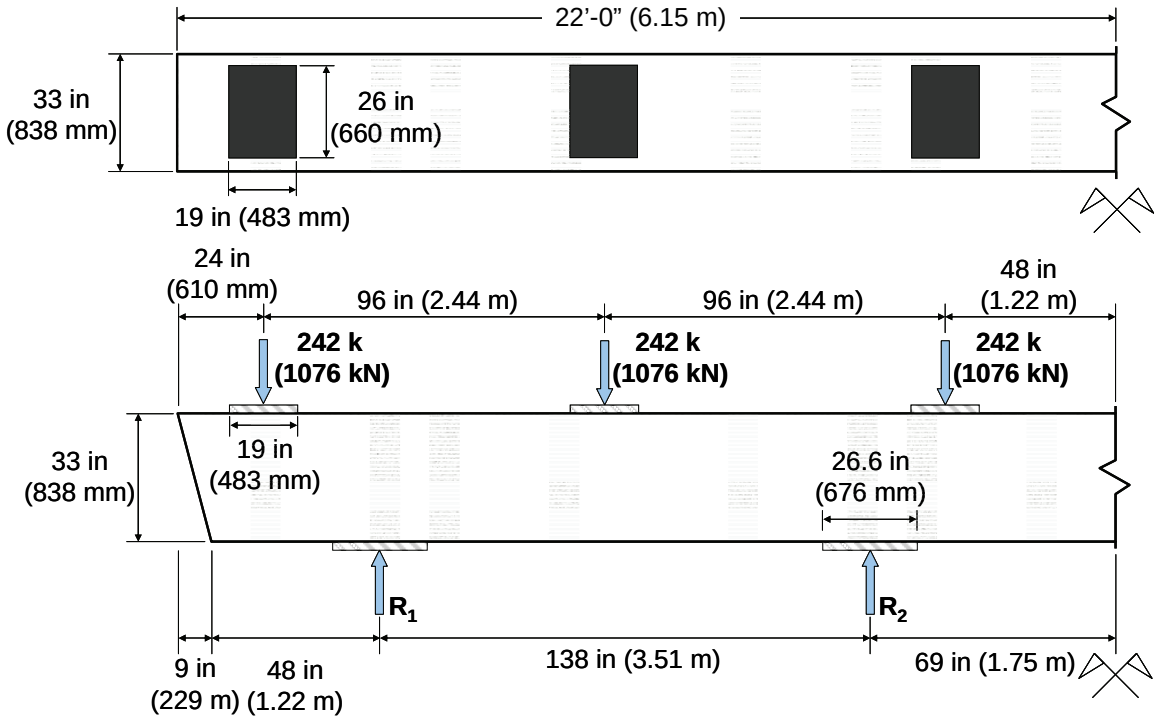


Fig. 2-1: Geometry of bent cap and first load case

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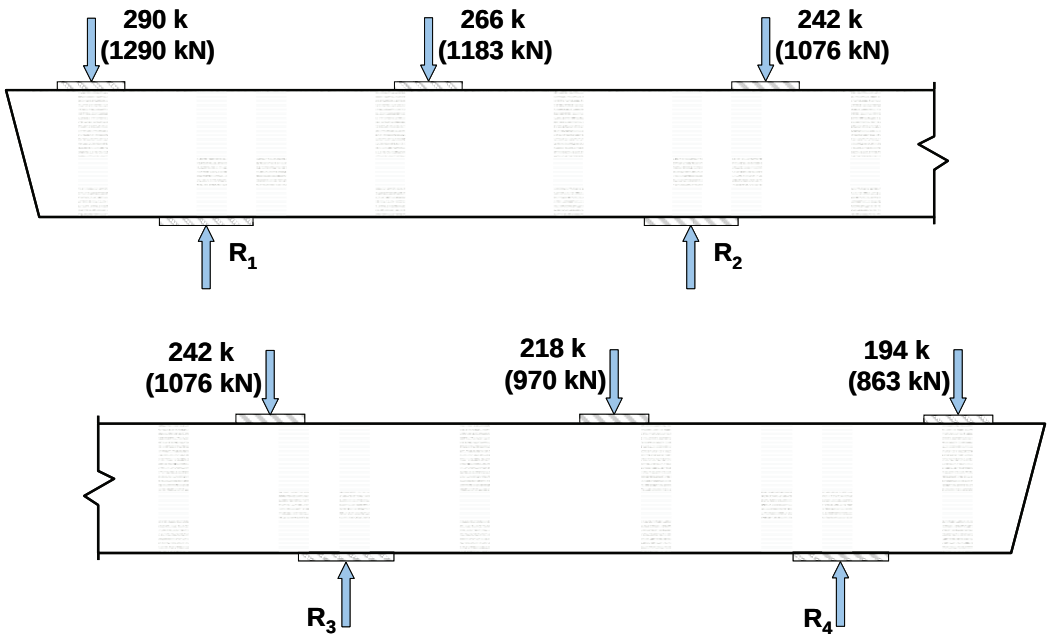


Fig. 2-2: Second load case

1.2 Materials

Concrete: f'_c = 5,000 psi (34.5 MPa)

Reinforcement: f_y = 60,000 (414 MPa)

2 Design procedure

The bent cap contains deep beam regions as defined in the ACI 318-08 provisions (regions with a shear span-to-depth ratio less than two). Only a small portion of the cap is considered a B-region. The shear forces in this region are relatively small and will not control the design. Therefore, the entire member will be designed using the strut-and-tie provisions. This will lead to a conservative result according to Birrcher et al. (2009).

The following steps will be used during the design procedure:

Step 1: Determine external reactions

Step 2: Determine appropriate strut-and-tie model

Step 3: Proportion tie

Step 4: Determine node geometry and check strength of nodes

Step 5: Check strength of struts

Step 6: Proportion crack control reinforcement

Step 7: Provide necessary anchorage for ties

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3 Design calculations

3.1 Step 1: Determine external reactions

In order to develop a strut-and-tie model for the bent cap (Fig. 2-3), the column reactions must be determined. This is accomplished by performing a linear-elastic analysis of the continuous beam. For the first load case, the reactions are symmetrical. The reaction R_1 is 407 kips (1810 kN) and R_2 is 319 kips (1419 kN). For the second load case, R_1 is 480 kips (2135 kN), R_2 is 316 kips (1406 kN), R_3 is 323 kips (1437 kN), and R_4 is 333 kips (1481 kN). These reactions are shown acting on the strut-and-tie models in Fig.s 2-4 and 2-5.

3.2 Step 2: Determine appropriate strut-and-tie model

The strut-and-tie model used for the design of the bent cap is presented in Fig. 2-3. Since some truss members of both the top and bottom chords of the model will be in tension and require longitudinal tension reinforcement, the height of the strut-and-tie model is determined by the locations of this reinforcement. The centroids of the top and bottom longitudinal reinforcement are assumed to be located 3 in. from the top and bottom faces of the bent cap. This should allow for the required clear cover as specified in Section 7.7 of ACI 318-08.

For shear span-to-depth ratios less than about two, the internal forces in the concrete are expected to be transferred via direct struts from the loads to the supports. Struts AF, EI, KP, and OS model this behavior. In order to comply with Section A.2 of ACI 318-08, the angle between the axis of a strut and a tie must not be less than 25°. Two truss panels with two diagonal struts are a better representation of the actual concrete behavior, as compared to a direct strut, in the longer spans of the bent cap.

Fig. 2-3 displays the geometry of the chosen strut-and-tie model. Fig. 2-4 shows the internal member forces for the symmetrical first load case, and Fig. 2-5 shows the member forces for the second load case. Since the second load case represents an asymmetric live load that could act on either side of the beam, the entire bent cap is designed using the larger forces on either side. When the forces of both the first and second load cases are compared, the second load case is seen to always control the design. For this reason, the second load case will be used for the remainder of the problem.

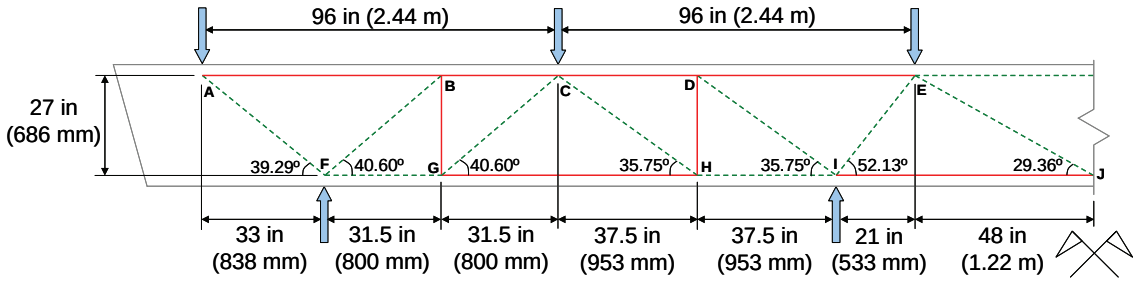


Fig. 2-3: Geometry of strut-and-tie model

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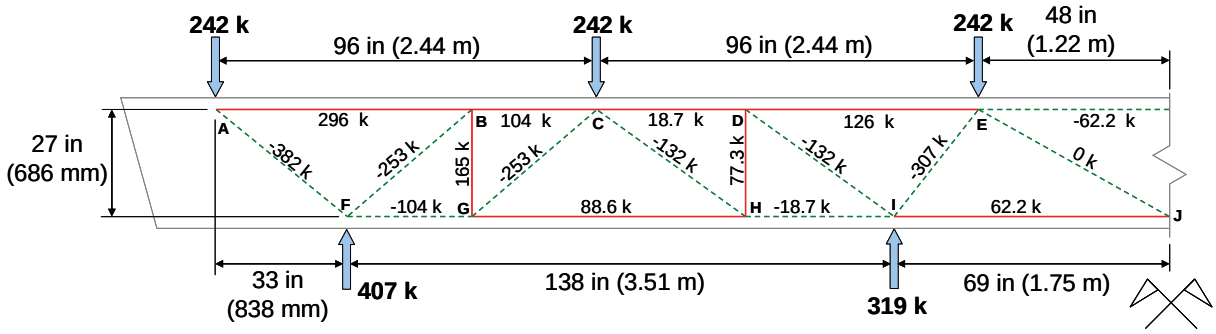


Fig. 2-4: Strut-and-tie model for first load case (1 kip = 4.448 kN)

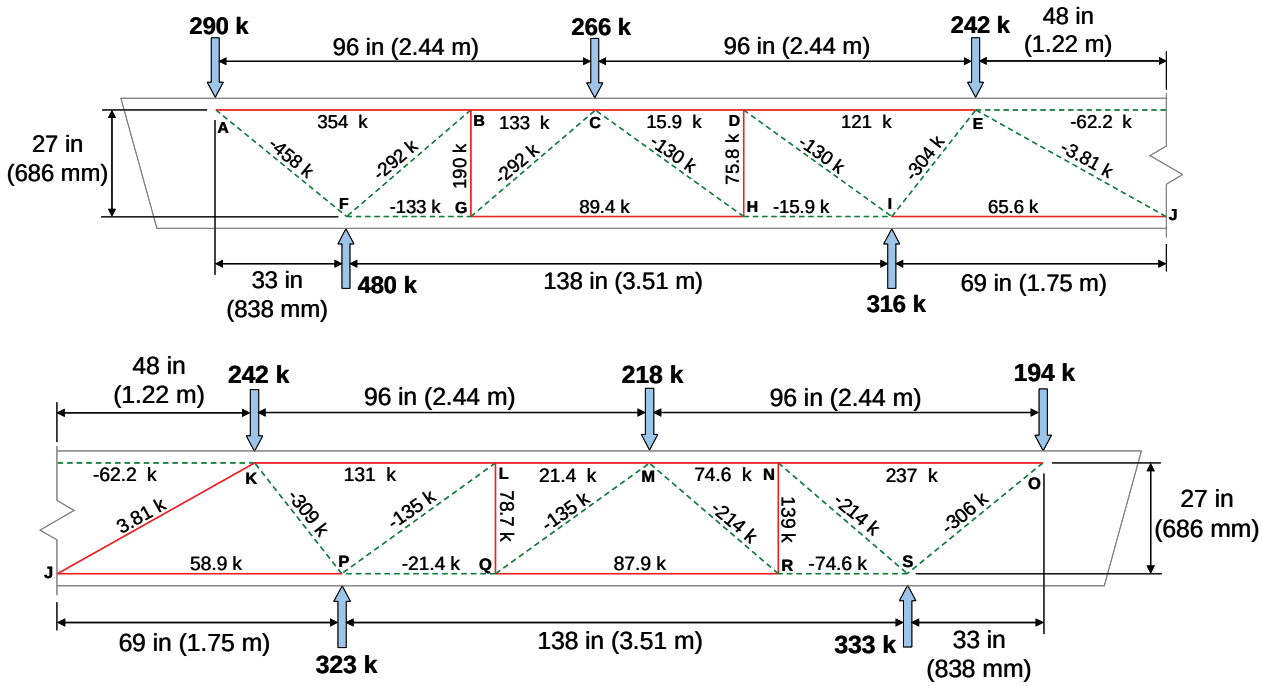


Fig. 2-5: Strut-and-tie model for second load case (1 kip = 4.448 kN)

3.3 Step 3: Proportion Tie

For such a bent cap, maintaining a constant amount of longitudinal steel along the length of the beam is common practice. Therefore, a constant amount of steel will be used along the cap's length in this example problem. As previously stated, the centroid of the top and bottom longitudinal reinforcement is located 3 in. from the top and bottom surfaces of the cap. The amount of steel along the top chord of the strut-and-tie model is determined by the force in Tie AB, 354 kips (1575 kN). One layer of reinforcement is expected to be adequate. According to Section A.4 of ACI 318-08, the nominal tie strength is taken as

$$F_{nt} = A_{ts} f_y$$

Determining the required area of steel gives

$$A_{ts,req} = \frac{F_u}{\phi f_y} = \frac{354\ \text{kip}}{(0.75)(60\ \text{ksi})} = 7.87\ \text{in}^2\ (5075\ \text{mm}^2)$$

Provide 6 – No. 11 bars: $A_{st} = 6(1.56\ \text{in}^2) = 9.36\ \text{in}^2$ [6 – 36M bars (A_{st} = 6036 mm^2)]

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For the lower chord of the strut-and-tie model, the force in tie GH, 89.4 kips (398 kN), controls. The required area of reinforcement is

$$A_{ts,req} = \frac{F_u}{\phi f_y} = \frac{89.4\,\text{kip}}{(0.75)(60\,\text{ksi})} = 1.99\,\text{in}^2\ (1284\,\text{mm}^2)$$

Provide 6 – No. 6 bars in order to satisfy spacing requirements per Section 10.6.4 of ACI 318-08: $A_{st} = 6(0.44\,\text{in}^2) = 2.64\,\text{in}^2$ [Provide 6 – 19M bars (A_{st} = 1704 mm^2)]

3.4 Step 4: Determine node geometry and check strength of nodes

3.4.1 Nodes F and S (CCC)

Now that the tie reinforcement has been chosen, the geometry of the nodes should be determined from the bearing dimensions, location of the longitudinal reinforcement, and angles of the struts.

The adequacy of the strength of nodes F and S will be checked first. The strength checks for the two geometrically similar nodes are controlled by the stresses at node F. Three struts converge at node F, and no tension reinforcement is anchored. Therefore, node F is a CCC node. The geometry of the node is affected by the placement of the lower chord of the strut-and-tie model and the dimensions of the column. The height, h_a, of the node is often taken as twice the distance from the surface of the member to the centroid of the longitudinal reinforcement. The distance from the free surface to the center of the longitudinal reinforcement along the bottom of the bent cap is 3 in. (76.2 mm). Therefore, the height, h_a, of Node F is 6 in. (152 mm). The length of the bottom face of the node, l_b, is taken as the width of the equivalent square column as discussed in Section 1.1, 26.6 in. (676 mm). The width of the node and the width of the strut (into the page) at the node-strut interface are also taken as the dimension of the square column, 26.6 in. (676 mm).

Fig. 2-6 shows node F and the forces acting on it. As is explained in the commentary to Section A.1 of ACI 318-08, calculations can often be made easier by dividing the node into two parts. As shown in Fig. 2-6, the reaction of the column on the bent cap is divided into two forces acting on the bottom faces of the two parts of the node. The 290-kip (1290-kN) force acting on the left equilibrates the vertical component of the 458-kip (2037-kN) force of Strut AF. The 190-kip (845-kN) force acting on the right equilibrates the vertical component of the 292-kip (1299-kN) force. The lengths of the bottom faces of each portion of the node are determined by keeping the pressure acting on these faces uniform.

The division of the node into two parts causes a small change in the angles of struts AF and BF from those shown in Fig. 2-3. The new strut angles are shown in Fig. 2-6 below. As an example, the calculation to determine the angle of strut AF is as follows:

$$\theta = \tan^{-1}\left[\frac{27\,\text{in}}{33\,\text{in} - \left(\frac{26.6\,\text{in}}{2} - \frac{16.1\,\text{in}}{2}\right)}\right] = 44.22°$$

where 27 in. (686 mm) is the height of the strut-and-tie model, 33 in. (838 mm) is the length of the panel of the strut-and-tie model between nodes A and F, and the other values are as shown in Fig. 2-6 below. This change of the strut angles will also affect the magnitude of the forces acting at the node to some extent. This change in the forces can often be neglected, adding conservatism to the checks, as is done here. Alternatively, the forces can also be adjusted to eliminate this added conservatism.

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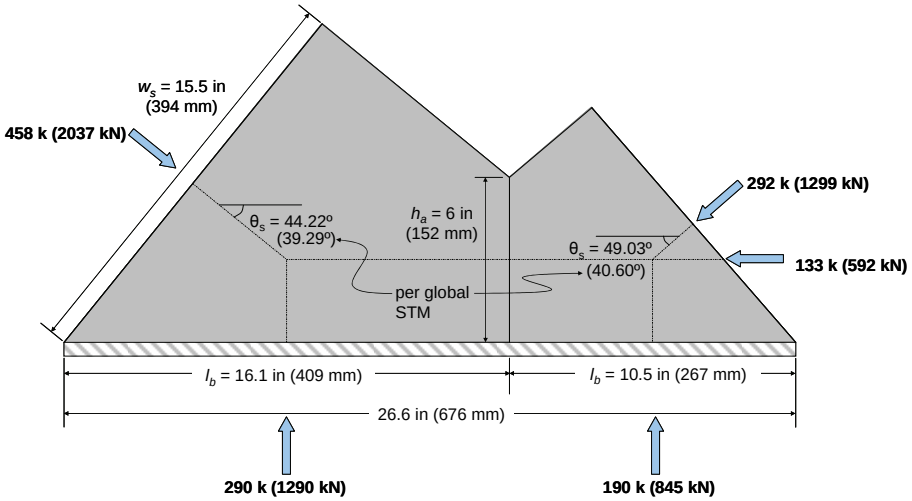


Fig. 2-6: Elevation view of node F

Each side of the node is shown separately in Fig.s 2-7 and 2-8. For the right side of node F, the two compressive forces shown acting on the inclined face in Fig. 2-6 are resolved into one force based on the original strut angles (per global STM) shown in Fig. 2-3. Then, the angle of the resolved strut is adjusted and determined to be 32.52 degrees, as shown in Fig. 2-8 and determined from the following calculation:

$$\tan 28.18° = \frac{27\text{ in}}{x} \qquad x = 50.40\text{ in (1280 mm)}$$

$$\theta = \tan^{-1}\left[\frac{27\text{ in}}{50.40\text{ in} - \left(\frac{26.6\text{ in}}{2} - \frac{10.5\text{ in}}{2}\right)}\right] = 32.52°$$

where 27 in. (686 mm) is the height of the strut-and-tie model and 28.18° is the original angle of the resolved strut.

Equilibrium is satisfied for each side of the node by determining the compressive force acting on the vertical faces using the original strut angles. This force is found to be 354 kips (1575 kN). Both sides of Node F can be treated as separate CCC nodes. The widths of the struts, w_s, are determined using the equations shown in the figures.

$w_s = l_b \sin\theta_s + h_a \cos\theta_s$
= 16.1"sin 44.22° + 6"cos 44.22°
= 11.23" + 4.30" = 15.5" (394 mm)

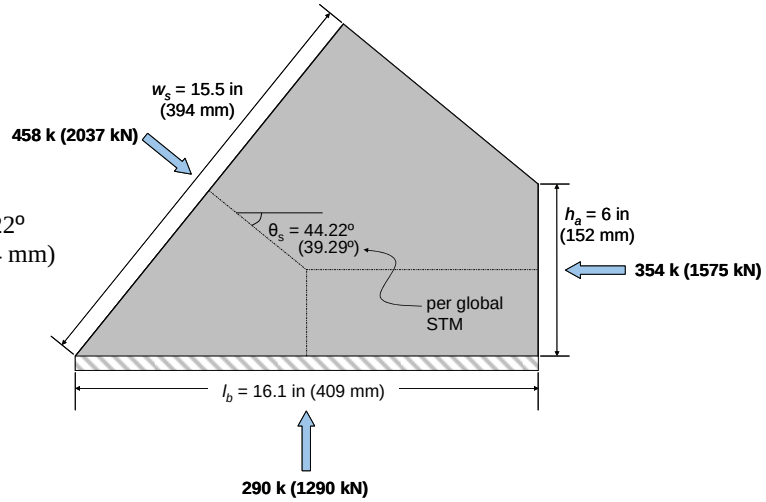


Fig. 2-7: Left side of node F

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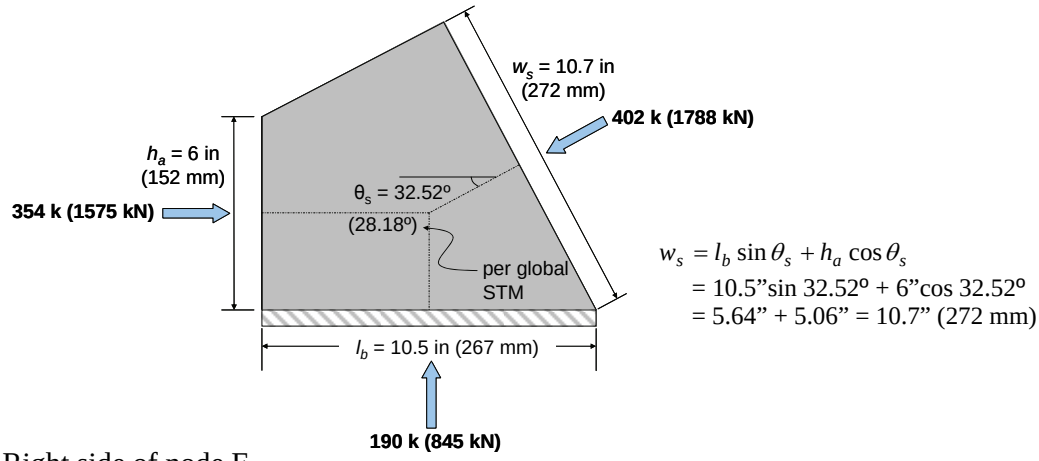


Fig. 2-8: Right side of node F

According to Section A.5 of ACI 318-08, the nominal compression strength of a node is

$$F_{nn} = f_{ce} A_{nz}$$

where the allowable effective compressive stress, f_{ce}, is given by

$$f_{ce} = 0.85\beta_n f'_c$$

For a CCC node, the value of β_n is 1.0 because there is no need in a reduction of strength due to a tension force acting at the node. Following the procedure in Appendix A, the forces acting on each face of the node according to the chosen strut-and-tie model must be greater or equal to the factored strength of the nodal zone. The faces of node F are checked as follows. The width of the node (into the page) is taken as the width of the column, 26.6 in. (676 mm).

For the bottom face of the left side of node F:

$$\phi F_{nn} = \phi 0.85\beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 1.0 \times 5\,\text{ksi} \times 16.1\,\text{in} \times 26.6\,\text{in} = 1365\,\text{kip} > 290\,\text{kip}$$
$$\left(\phi F_{nn} = 6072\,\text{kN} > 1290\,\text{kN}\right)$$

For the vertical face of the left and right sides of node F:

$$\phi F_{nn} = \phi 0.85\beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 1.0 \times 5\,\text{ksi} \times 6\,\text{in} \times 26.6\,\text{in} = 509\,\text{kip} > 354\,\text{kip}$$
$$\left(\phi F_{nn} = 2264\,\text{kN} > 1575\,\text{kN}\right)$$

The bottom face of the right side of node F does not need to be checked since the pressure acting on it is the same as the pressure acting on the bottom face of the left side of node F. The strengths of the inclined faces (node-strut interfaces) of the two portions of the node are controlled by the strut checks performed in Section 3.5.

3.4.2 Nodes A and O (CCT Node)

The adequacy of the strength of nodes A and O will be checked next. The geometry of these two nodes is similar, and the stresses at node A will control the design. Because node A anchors one tie, it is a CCT node. The geometry of the node is affected by the placement of tie AB (or the top chord of the strut-and-tie model) and the bearing dimension supporting the beam above the node. The height, h_a, of the node is taken as twice the distance from the surface of the member to the centroid of the longitudinal reinforcement as with node F. The length of the top face of the node, l_b, is taken is the length of the bearing plate, 19 in. (483 mm). The width of the node and the width of the strut (into the page) at the node-strut interface are taken as the width of the bearing plate, 26 in. (660 mm). The adjusted angle of strut AF was determined in Section 3.4.1 above.

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Fig. 2-9 shows node A and the forces acting on it. The width of the strut, w_s, is determined using the equation shown in the figure.

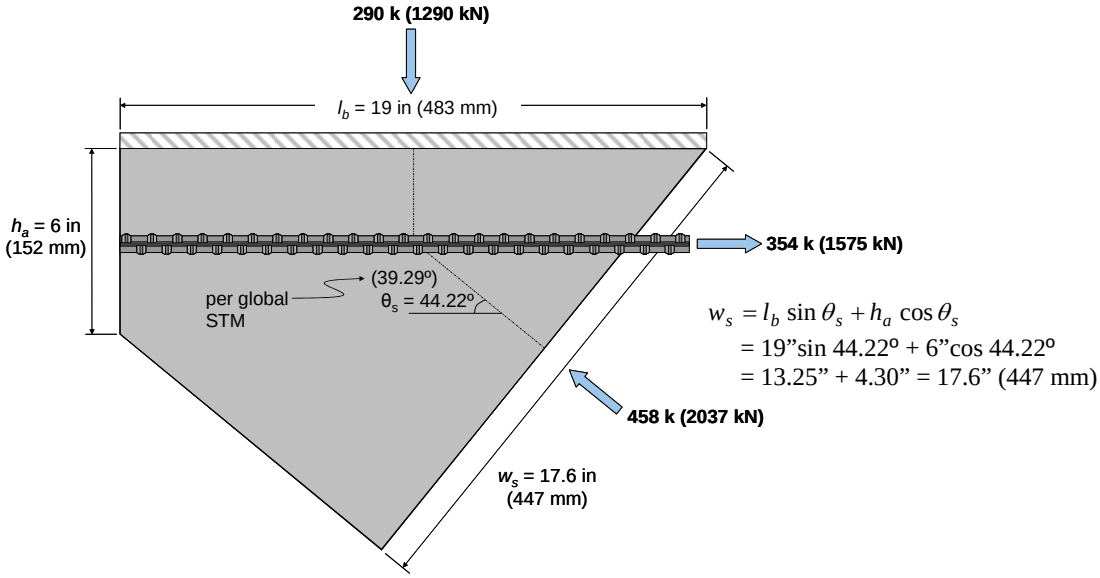


Fig. 2-9: Elevation view of node A

For a CCT node, the value of β_n is 0.80 according to Section A.5 of ACI 318-08. Thus, the allowable effective compressive stress is

$$f_{ce} = 0.85 \times 0.80 \times f'_c$$

The procedure to check the strengths of the faces of node A is the same as with node F. The calculations are as follows. For the top face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5 \text{ ksi} \times 19 \text{ in} \times 26 \text{ in} = 1260 \text{ kip} > 290 \text{ kip}$$
$$\left(\phi F_{nn} = 5605 \text{ kN} > 1290 \text{ kN}\right)$$

For the left face of the node:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5 \text{ ksi} \times 6 \text{ in} \times 26 \text{ in} = 398 \text{ kip} > 354 \text{ kip}$$
$$\left(\phi F_{nn} = 1770 \text{ kN} > 1575 \text{ kN}\right)$$

As with node F, the strength of the inclined face (node-strut interface) is controlled by the strut check that will be performed in Section 3.5.1.

3.4.3 Nodes C and M (CCT)

By inspection, the stresses at node C control compared to those at node M. As will be seen, the calculated strengths of node C are much greater than the required strengths. Thus, node M is expected to have adequate strength as well.

Node C can be divided into two parts using the same procedure as with node F in order to simplify the calculations. Each separate side of the node is shown in Fig.s 2-10 and 2-11. Both can be treated as a separate CCT node. The compressive force acting on the vertical face of each side of the node is determined by equilibrium using the original strut angles shown in Fig. 2-3.

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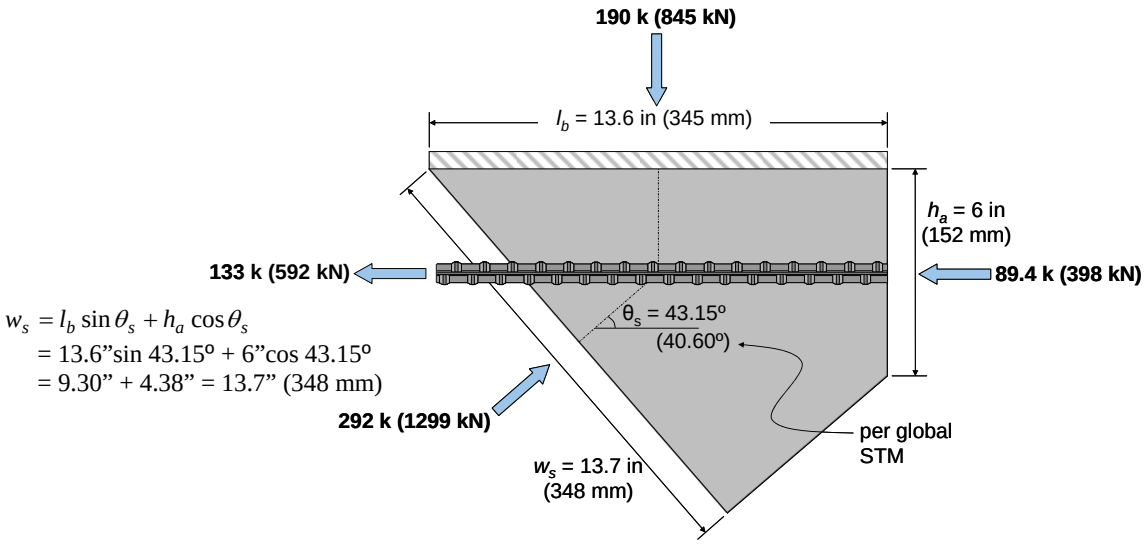


Fig. 2-10: Left side of node C

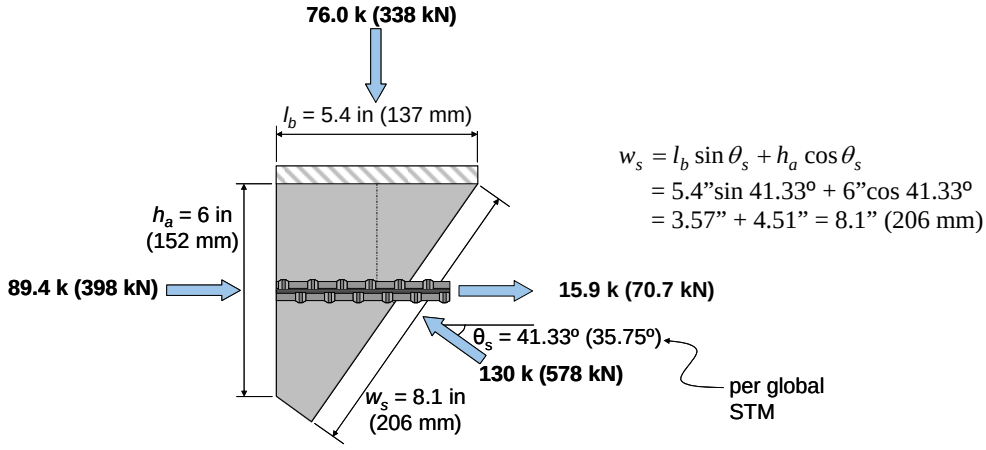


Fig. 2-11: Right side of node C

The allowable effective compressive stress of a CCT node is

$$f_{ce} = 0.85 \times 0.80 \times f'_c$$

The top face of node C does not need to be checked because the strength of the top face of node A, a CCT node, was adequate and a smaller force acts on the top face of node C. The only faces of node C that need to be checked are the vertical faces of the left and right sides of the node. For this check, the left side of the node will control.

The required strength is calculated as follows:

89.4 kip + 133 kip = 222.4 kip (989 kN)

For the vertical face of the left side of node C:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5\,\text{ksi} \times 6\,\text{in} \times 26\,\text{in} = 398\,\text{kip} > 222.4\,\text{kip}$$

$$(\phi F_{nn} = 1770\,\text{kN} > 989\,\text{kN})$$

The strengths of the inclined faces (node-strut interfaces) are controlled by the strut checks.

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3.4.4 Nodes I and P (CCT/CCC)

By inspection, there are higher stress values at node P than at node I. As with nodes C and F, node P can be divided to simplify the node checks. The two parts of the node are shown in Fig.s 2-12 and 2-13. The two struts at the right side of node P are resolved into one force as with node F to determine the stress and angle of the inclined face of the right side of the node. Looking at each Fig., the left side is classified as a CCT node while the right side is a CCC node. In addition, the required strength for the vertical face for the left side of the node is greater than that for the right side. For these reasons, the left side of node P will control the strength checks.

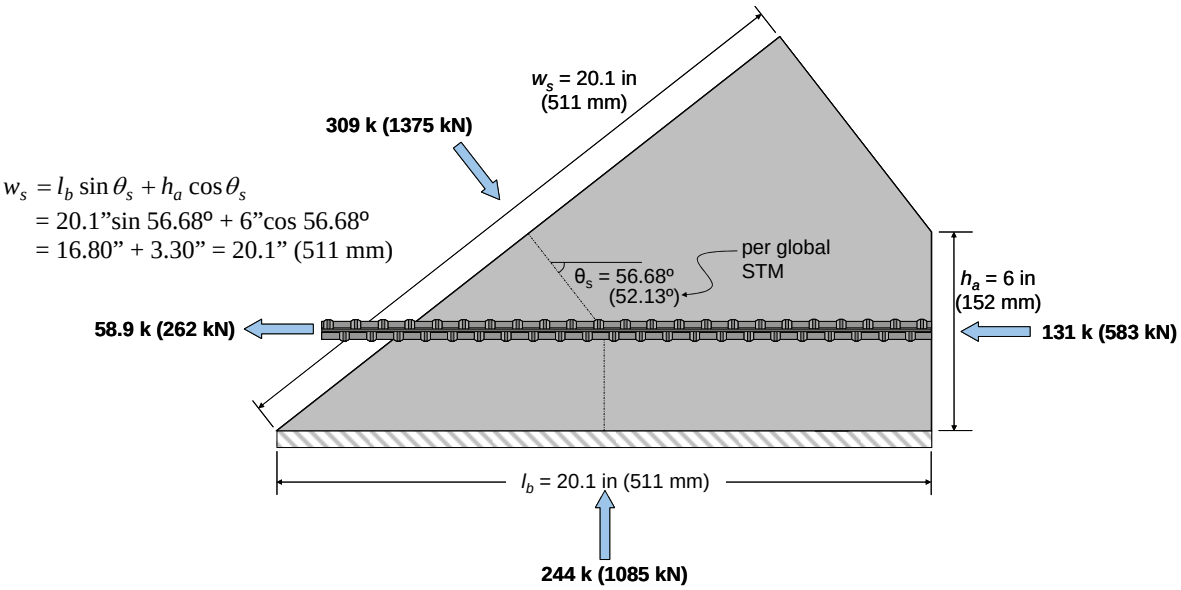


Fig. 2-12: Left side of node P

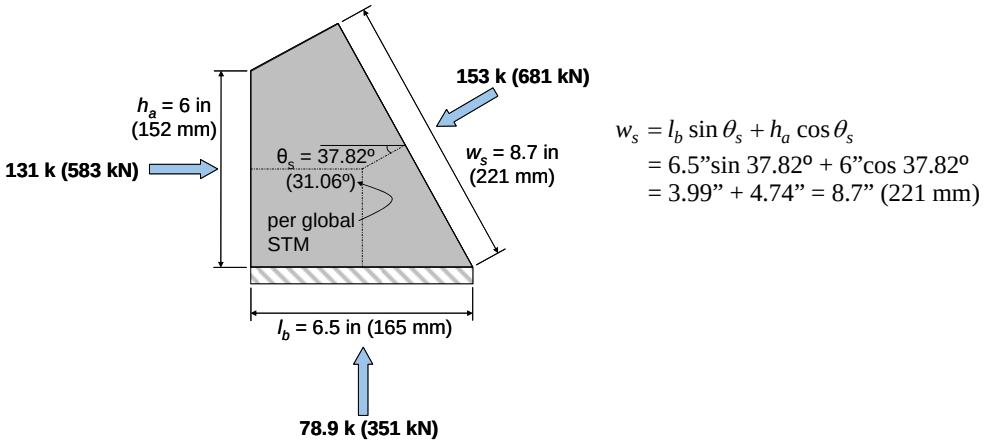


Fig. 2-13: Right side of Node P

The faces of the left side of node P are checked as follows:

For the bottom face of the left side of node P:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5\,\text{ksi} \times 20.1\,\text{in} \times 26.6\,\text{in} = 1363\,\text{kip} > 244\,\text{kip}$$

$$(\phi F_{nn} = 6063\,\text{kN} > 1085\,\text{kN})$$

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For the vertical face, the required strength is calculated as follows:

131 kip + 58.9 kip = 189.9 kip (845 kN)

For the vertical face of the left side of node P:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.80 \times 5 \text{ ksi} \times 6 \text{ in} \times 26.6 \text{ in} = 407 \text{ kip} > 189.9 \text{ kip}$$
$$\left(\phi F_{nn} = 1810 \text{ kN} > 845 \text{ kN}\right)$$

The strengths of the inclined faces (node-strut interfaces) of node P are controlled by the strut checks.

3.4.5 Ties BG and NR

Comparing nodes B and N, node B will control the node strength checks. Similarly, by inspection, the stress of tie BG will control the design of vertical stirrups at this location.

Node B is an interior node with no bearing plate that can be used to define its geometry. For this reason, another method must be used to determine the length of the top face of the node, l_a. According to Wight and Parra-Montesinos (2003), assuming that struts BF and CG form fan shapes so that the ends of the struts at Nodes B and G are spread over a large area, the stirrups that intersect the struts at an angle greater than or equal to 25 degrees can be considered as a part of the vertical tie entering the CTT nodes. Using this method, the maximum length of the top face of node B can be calculated as follows:

$$l_a = 63 \text{ in} - 2 \times 27 \text{ in} \times \tan 25° = 37.8 \text{ in (960 mm)}$$

where 63 in. (1.60 m) is the total length of the two panels of the strut-and-tie model between nodes F and C and 27 in. (686 mm) is the height of the strut-and-tie model.

Next, this distance should be used to determine the necessary shear reinforcement comprising tie BG. No. 4 (13M) stirrups with four legs across the web are used. Using the equation for the strength of a tie from Section A.4 of ACI 318-08, the required spacing of the stirrups comprising tie BG is calculated as follows:

$$\phi F_{nt} = \phi A_{ts} f_y = 0.75 \times \frac{4 \times 0.20 \text{ in}^2 \times 37.8 \text{ in}}{s} \times 60 \text{ ksi} = 190 \text{ kip (845 kN)}$$

Solving for s gives:

$$s = \frac{0.75 \times 4 \times 0.20 \text{ in}^2 \times 37.8 \text{ in} \times 60 \text{ ksi}}{190 \text{ kip}} = 7.16 \text{ in (182 mm)}$$

Therefore, provide No. 4 stirrups at 7 in. center-to-center (13M at 180 mm) with four legs distributed across the web of the bent cap.

3.4.6 Nodes B and N (CTT)

An elevation view of node B is shown in Fig. 2-14. The length of the top face, l_a, is taken as 35 in. (889 mm) based on the 7 in. (178 mm) spacing between the stirrups. Any more stirrups comprising tie BG would be greater than the limit for l_a determined in Section 3.4.5 above. The angle of strut BF is taken from Fig. 2-6.

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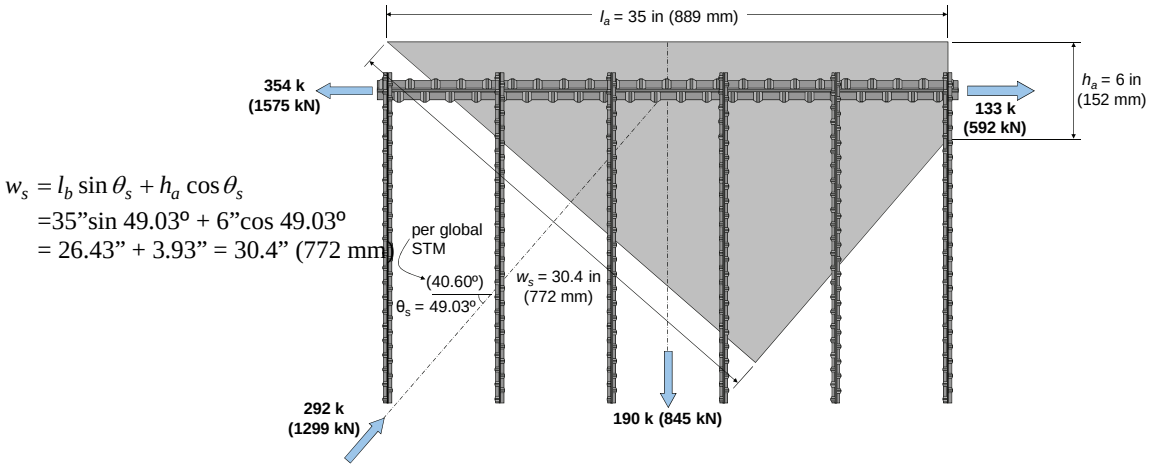


Fig. 2-14: Elevation view of node B

For a CTT node, the value of β_n is 0.60 according to Section A.5 of ACI 318-08. Thus, the allowable effective compressive stress is

$$f_{ce} = 0.85 \times 0.60 \times f'_c$$

The node check calculations are as follows. The width of the node (into the page) is taken as the width of the bent cap, 33 in. (838 mm). For the top face of node B:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.60 \times 5 \text{ ksi} \times 35 \text{ in} \times 33 \text{ in} = 2209 \text{ kip} > 190 \text{ kip}$$

$$(\phi F_{nn} = 9826 \text{ kN} > 845 \text{ kN})$$

For the right face of node B, the required strength is calculated by taking the difference between the tie forces on the left and right as follows:

354 kip – 133 kip = 221 kip (983 kN)

For the right face of node B:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.60 \times 5 \text{ ksi} \times 6 \text{ in} \times 33 \text{ in} = 379 \text{ kip} > 221 \text{ kip}$$

$$(\phi F_{nn} = 1686 \text{ kN} > 983 \text{ kN})$$

Because node B is a CTT node with a β_n factor of 0.60, the strength of the inclined face (node-strut interface) is controlled by the node check. For the inclined face of node B:

$$\phi F_{nn} = \phi 0.85 \beta_n f'_c A_{nz} = 0.75 \times 0.85 \times 0.60 \times 5 \text{ ksi} \times 30.4 \text{ in} \times 33 \text{ in} = 1919 \text{ kip} > 292 \text{ kip}$$

$$(\phi F_{nn} = 8536 \text{ kN} > 1299 \text{ kN})$$

3.4.7 Other nodes

The other nodes of the strut-and-tie model can be checked using the same procedure. Some nodes for which explicit calculations are not provided herein can be deemed to have adequate strength by inspection. All nodes have adequate strength for the applied factored loads on the bent cap.

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3.5 Step 5: Check strength of struts

3.5.1 Struts AF and OS

Due to symmetry, struts AF and OS have similar geometry. The large stress in strut AF will control the strut check. According to Section A.3 of ACI 318-08, the nominal compressive strength of a strut is

$$F_{ns} = f_{ce} A_{cs}$$

where the allowable effective compressive strength, f_{ce}, of the strut is given by

$$f_{ce} = 0.85\beta_s f'_c$$

The struts crossing the strut-and-tie model diagonally are expected to have a larger width at the middle of the strut than at their ends. These struts are known as bottle-shaped struts and have a β_s value of 0.75 if the web reinforcement provided satisfies Sect. A.3.3, ACI 318-08.

Strut AF is checked as follows. Both ends of the struts are ensured to have an adequate strength capacity. The areas at the ends of the struts are the same as the areas of the inclined faces (node-strut interfaces) of the nodes. For the node-strut interface at node A:

$$\phi F_{ns} = \phi 0.85\beta_s f'_c A_{cs} = 0.75 \times 0.85 \times 0.75 \times 5\,\text{ksi} \times 17.6\,\text{in} \times 26\,\text{in} = 1094\,\text{kip} > 458\,\text{kip}$$

$$(\phi F_{ns} = 4866\,\text{kN} > 2037\,\text{kN})$$

For the node-strut interface at node F:

$$\phi F_{ns} = \phi 0.85\beta_s f'_c A_{cs} = 0.75 \times 0.85 \times 0.75 \times 5\,\text{ksi} \times 15.5\,\text{in} \times 26.6\,\text{in} = 986\,\text{kip} > 458\,\text{kip}$$

$$(\phi F_{ns} = 4386\,\text{kN} > 2037\,\text{kN})$$

Therefore, strut AF is found to have adequate strength.

3.5.2 Struts BF/FG and NS/RS

As shown in Fig. 2-8, struts BF and FG were resolved into one force. By inspection, struts BF and FG will control the strut check compared to struts NS and RS. The resolved strut is checked as follows:

For the node-strut interface at node F:

$$\phi F_{ns} = \phi 0.85\beta_s f'_c A_{cs} = 0.75 \times 0.85 \times 0.75 \times 5\,\text{ksi} \times 10.7\,\text{in} \times 26.6\,\text{in} = 680\,\text{kip} > 402\,\text{kip}$$

$$(\phi F_{ns} = 3025\,\text{kN} > 1788\,\text{kN})$$

For the node-strut interface at node B, the node check controls because node B is a CTT node. The node check was performed in Section 3.4.6.

3.5.3 Other struts

The other struts of the strut-and-tie model can be checked using the same procedure. Some struts for which explicit calculations are not provided herein can be deemed to have adequate strength by inspection. All struts are found to have sufficient strength for the applied factored loads on the bent cap.

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3.6 Step 6: Proportion crack control reinforcement

According to Section A.3 of ACI 318-08, a minimum requirement for web crack control reinforcement must be satisfied in order to use a β_s factor of 0.75 for bottle-shaped struts. For concrete with a specified compressive strength not exceeding 6000 psi (41.4 MPa), this requirement can be met by satisfying the following equation:

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i \geq 0.003$$

where α_s is the angle between the crack control reinforcement and the strut it crosses. The angle between strut EJ and the vertical and horizontal web reinforcement will control the amount of required steel because it is furthest from 45°.

Providing four legs of No. 4 stirrups spaced at 8 in. (13M at 200 mm) and pairs of No. 4 bars for skin reinforcement spaced at 7 in. (13M at 180 mm) gives:

$$\frac{2 \times 0.20\,\text{in}^2}{33\,\text{in} \times 7\,\text{in}} \sin 29.36° + \frac{4 \times 0.20\,\text{in}^2}{33\,\text{in} \times 8\,\text{in}} \sin 60.64° = 0.0035 > 0.003$$

However, No. 4 stirrups spaced at 7 in. (13M at 180 mm) are required for Ties BG and NR over the length l_a as previously determined in Section 3.4.5. Similar calculations for Ties DH and LQ show that stirrups spaced at 8 in. (203 mm) are adequate for the stresses in these ties.

3.7 Step 7: Provide necessary anchorage for ties

The ties along the top and bottom chords of the strut-and-tie model should be provided necessary anchorage according to Chapter 12 of ACI 318-08. Because the longitudinal reinforcement along the bottom of the bent cap is in compression at its ends, the bars can simply be extended to the ends of the bent cap leaving 2 in. (51 mm) of cover. The longitudinal reinforcement along the top of the beam should be developed properly at Nodes A and O. The available length for development is measured from the point where the centroid of the reinforcement intersects the extended nodal zone as shown in Fig. 2-15. The total available length for development is as follows:

$$L_d = 24\,\text{in} - \frac{3\,\text{in}}{\tan 74.74°} - 2\,\text{in} + \frac{19\,\text{in}}{2} + \frac{3\,\text{in}}{\tan 44.22°} = 33.8\,\text{in}\ (859\,\text{mm})$$

where 24 in. (610 mm) is the distance from the center of the load plate to the end of the bent cap, 19 in. (483 mm) is the length of the load plate, 2 in. (51 mm) is the cover, and the other values are as shown in Fig. 2-15.

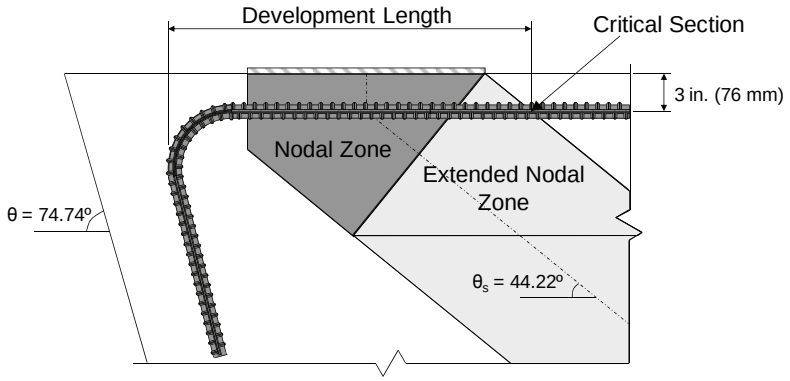


Fig. 2-15: Development length of longitudinal ties

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If straight bars are used to develop the reinforcement, the following development length is required according to Section 12.2 of ACI 318-08:

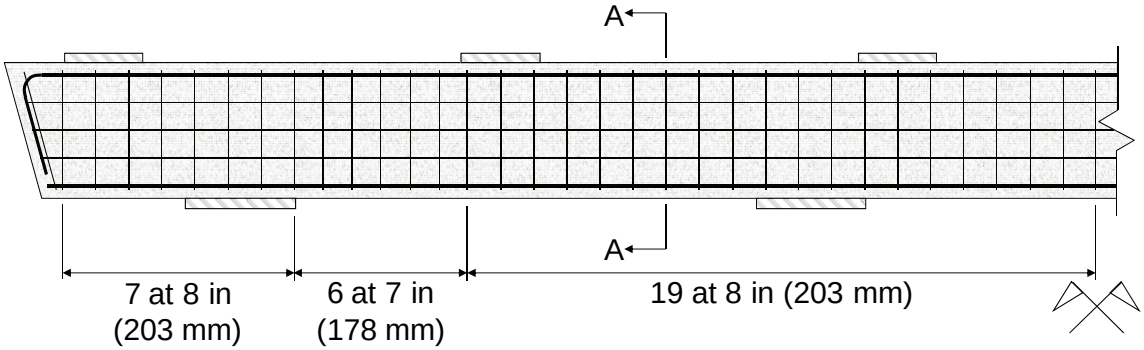
$$l_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f'_c}} \right) d_b = \frac{60{,}000 \text{ psi} \times 1.3 \times 1.0}{20 \times 1.0 \times \sqrt{5000 \text{ psi}}} \times 1.410 \text{ in} = 77.8 \text{ in} > 33.8 \text{ in } (1976 \text{ mm} > 859 \text{ mm})$$

Since this is greater than the available distance, 90-degree hooks are used. Using Section 12.5, the required length of a hook is

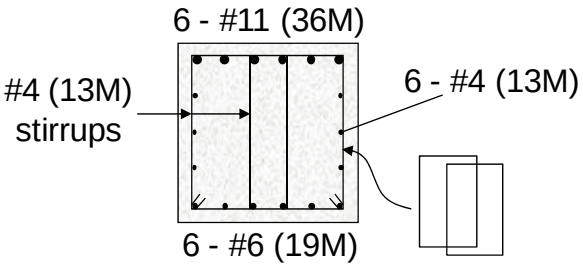
$$l_{dh} = \left(\frac{0.02 \psi_e f_y}{\lambda\sqrt{f'_c}} \right) d_b = \frac{0.02 \times 1.0 \times 60{,}000 \text{ psi}}{1.0 \times \sqrt{5000 \text{ psi}}} \times 1.410 \text{ in} = 23.9 \text{ in} < 33.8 \text{ in } (607 \text{ mm} < 859 \text{ mm})$$

The required length for 90-degree hooks is available. This required length could be multiplied by a factor of 0.7 if the cover requirements are met according to Sect. 12.5.3(a), ACI 318-08.

The final arrangement of reinforcement is shown in Fig. 2-16.



a) Elevation



b) Section A-A

Fig. 2-16: Reinforcement details

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4 Summary

A bent cap resisting six concentrated loads and supported on four columns was designed using the strut-and-tie provisions in Appendix A of ACI 318-08. Two load cases were presented, one accounting for an asymmetric live load acting on the bridge deck supported by this bent cap. A strut-and-tie model was developed and member forces were calculated. In parts of the model, two struts were used between the load and support to prevent the angle between struts and ties entering a single node from being less than 25°. Some nodes were divided into two parts to simplify the node check calculations. In order to provide the necessary transverse reinforcement to carry the load in tie BG, stirrups spaced at 7 in. (178 mm) were required. This spacing was less than that required by the minimum crack control reinforcement provision in Section A.3 of ACI 318-08.

A summary of the reinforcement details is presented in Fig. 2-16.

References

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Example 3:
Strut-and-Tie Design and Detailing of a Foundation Grade Beam

Leonard P. De Rooy
Timothy C. DenHartigh
Robert B. Anderson

Synopsis

This example illustrates strut-and-tie detailing techniques used to produce a practical and constructible design. Information presented herein is based on a 34 story post-tensioned concrete structure in Grand Rapids, Michigan that utilized deep concrete grade beams for its foundation system. The River House condominium project used over 1,900 cubic yards (1,400 cubic meters) of concrete and 380,000 lb (172,000 kg) of reinforcement for the grade beams alone. The grade beam illustrated in this example cantilevers over an existing caisson and a 60 in. (1,500 mm) brick sanitary sewer.

Leonard P. De Rooy is a professor of engineering at Calvin College, Grand Rapids, Michigan. He received his Master's degree from the University of Michigan and Bachelor's degree from Calvin College. Before becoming a professor, he worked as a senior facilities structural engineer for URS Corporation, Grand Rapids, Michigan. He is a member of ACI.

Timothy C. DenHartigh is a senior facilities structural engineer for URS Corporation, Grand Rapids, Michigan. He received his Master's degree from Michigan Technological University in Houghton, Michigan and his Bachelor's degree from Calvin College. His research at Michigan Technological University involved development and bond strength of stainless steel reinforcement. He is a member of West-Michigan ACI chapter.

Robert B. Anderson is a senior structural engineer for URS Corporation, Tampa, Florida. He received his Master's degree in Engineering from the University of Texas at Austin and his Bachelor's degree in Civil Engineering from South Dakota State University. While attending the University of Texas, his research focused on the full scale testing of reinforced concrete models for strut-and-tie nodal regions. His design expertise includes both concrete and steel bridge types ranging from short-span grade separation structures to long-span cable stayed structures. He is a member of ACI Committees 341-Earthquake Resistant Concrete Bridges and 445-Shear and Torsion.

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1 Description of design task

1.1 Description of structure

The River House Condominium is a 34 story post-tensioned concrete structure that is an addition to the Bridgewater complex in Grand Rapids, Michigan. The original Bridgewater complex is an 18 story steel framed office building. During the construction of the original building in 1991, the project scope included two matching 18 story towers. The caissons for the foundations were placed for both towers. However, it was decided to postpone construction of the second tower.

The developer for the River House project hence decided to build a 34 story condominium (left tower in Fig. 3-1 and Fig. 3-2) complex instead of the original planned office space. Post-tensioned concrete was selected after the structural system was value engineered. Construction of this project started in the fall of 2006.

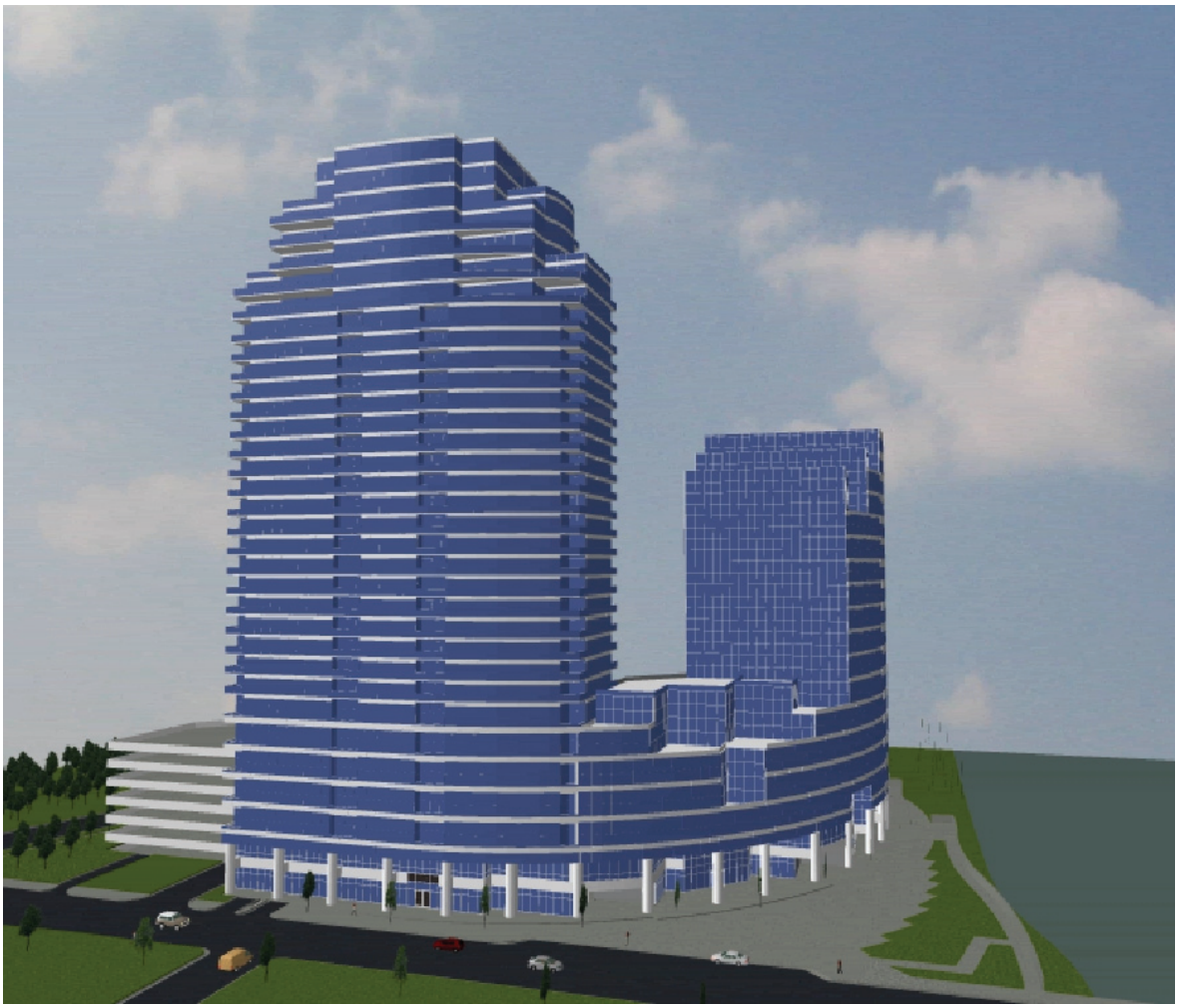


Fig. 3-1: Architectural rendering (with permission by DTS Winkelmann LLC, Grand Rapids, MI)

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Fig. 3-2: Building under construction

1.2 Description of design task

1.2.1 Description of structure

The partial foundation plan for the building showing the grade beam discussed in this example is shown in Fig. 3-3. The foundations for the project were complicated by the fact that some of the existing caissons did not have adequate capacity or were in the wrong location. The existing caissons were originally sized for an 18 story structure as described above in Section 0, thus the lower capacity and incorrect locations. Some of the deficient existing caissons were utilized for their full capacity and supplemented with additional caissons. Other existing caisson capacities could not be verified and thus were not used for resisting loads.

For the grade beam in this example, the existing caisson located near Grid A-10 was not used because its capacity could not be verified. There was also an existing functional sewer line (parallel to Grid 10 - shaded area in Fig. 3-3) located immediately next to this caisson which made it impossible to support the column loads with a simple span grade beam. Therefore, a cantilevered grade beam was designed.

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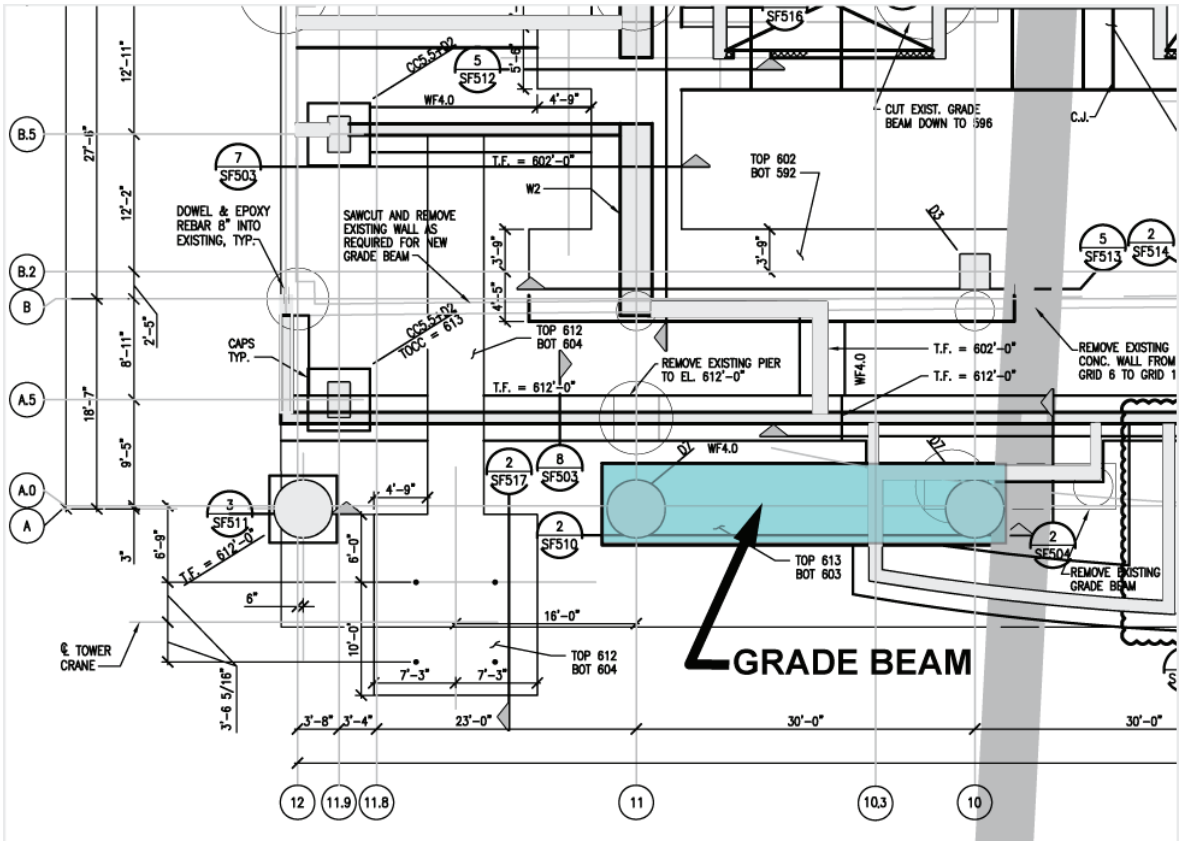


Fig. 3-3: Foundation plan

1.2.2 Geometry and loads

The grade beam is supported by two caissons 21 ft. (6,400 mm) apart and is loaded by two columns: one column is directly over one support (Grid A-11) and the other column is cantilevered 9 ft. (2,700 mm) from the other support (Grid A-10). The final grade beam design has a width of 7 ft 6 in. (2,300 mm) and a depth of 10 ft 0 in. (3050 mm). Both columns that bear on the beam are 42 in. (1,070 mm) in diameter.

The factored column load directly over the left support is 3,886 k (17,290 kN) and the factored column load at the end of the cantilever is 3,857 k (17,160 kN). The self weight of the grade beam was included in the point loads at the columns.

1.2.3 Materials

Concrete: f'_c = 5,000 psi (34.5 MPa)
Reinforcement: f_y = 60,000 psi (414 MPa)

1.2.4 Statement of design problem

The grade beam was designed to support the column reaction as a cantilever over an existing functional 60 in. (1,500 mm) diameter sewer line and an abandoned caisson, which can be seen in Fig. 3-4. Strut-and-tie reinforcement was detailed to meet The American Concrete Institute (ACI) code requirements as well as to meet constructability considerations, particularly for the node directly below the column at the right cantilevered end.

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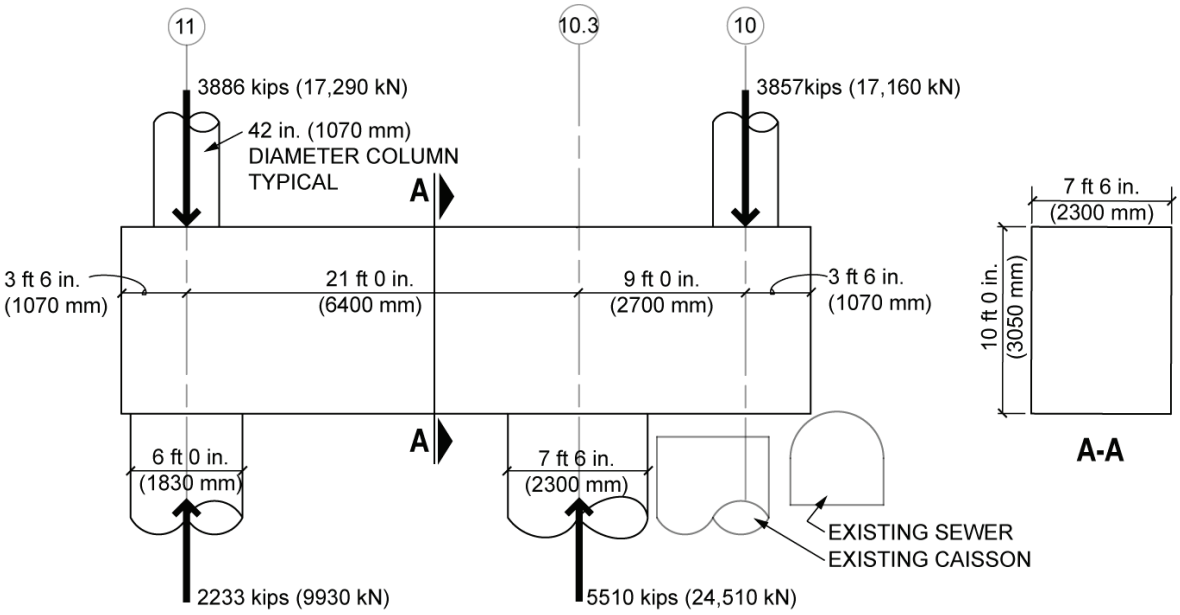


Fig. 3-4: Beam geometry and loads

2 Design Procedure

2.1 Beam Classification

The building was designed and constructed when the 2005 version of the ACI-318 code was the current code adopted by the local building officials. According to the criteria of ACI 318-05 Section 10.7, both the main span and the cantilever portion of the grade beam qualify as deep beams. Various strut-and-tie models were considered during the design process to provide both primary and alternate load paths.

Fig. 3-5 and 3-11 show two different configurations of strut-and-tie models for the transfer of the reaction to the support at the left side of the model. The FIP Recommendations (1999) and ACI SP 208 (ACI, 2002) discuss the “safe” model that may be used to derive the maximum transverse reinforcing (Fig. 3-5) and the direct load path from reaction to support which minimizes the transverse reinforcing requirement (Fig. 3-11). Additional discussion is included in both documents FIP (1999) and ACI (2002) about a shared load path which is a combination of the two configurations presented in Fig. 3-5 and 3-11. Due to the fact that this beam is providing support to 34 floors above and is not able to be inspected after construction, a design decision was made to use the worse reinforcing requirements from the two model configurations. A detailed discussion of the configurations for the two models follows in Sections 2.2 and 2.3.

2.2 Strut-and-tie model

Fig. 3-5 shows the strut-and-tie model for this grade beam. The tension members are shown as solid lines and the compression members are shown as dashed lines. The beam ends were extended 3 ft 6 in. (1,070 mm) past the centre line of the column reaction loads to allow space for the nodes, anchorage of the grade beam reinforcing, and detailing of the connections to the columns and caissons. This example will focus on node N-4 and members E-3 and E-13.

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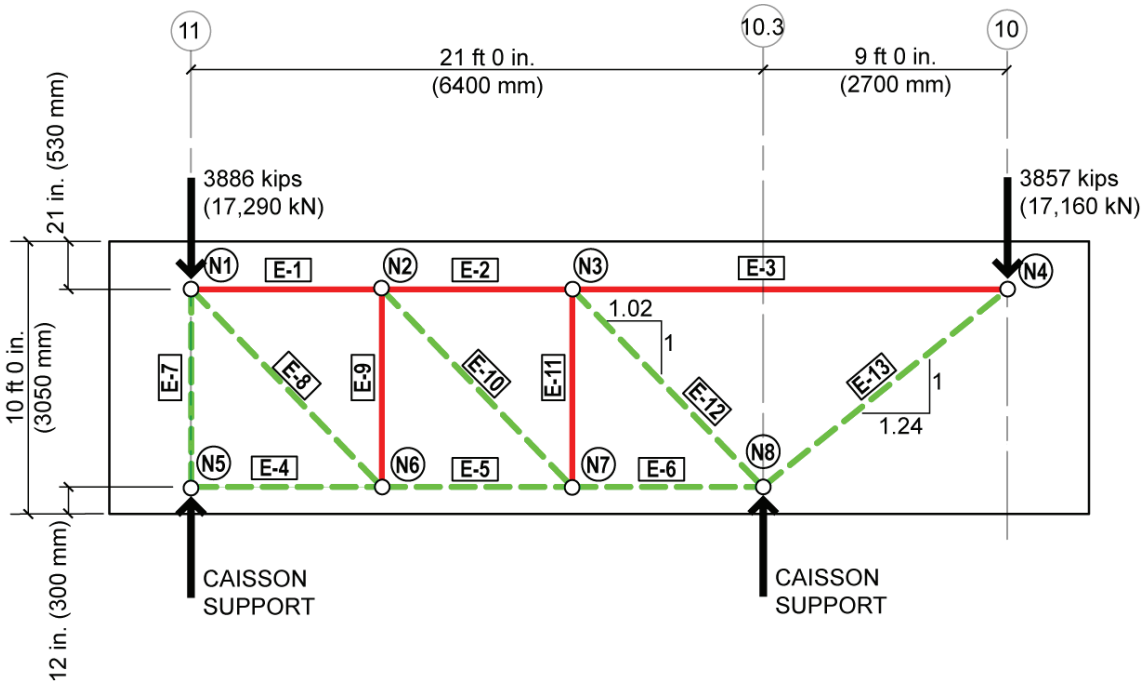


Fig. 3-5: Strut-and-tie model

Fig. 3-5 shows a direct load path from the right side reaction to the caisson support. This maximizes the steel requirement for tie E-3 between nodes N3 and N4. Between the caisson supports at the left side of the Fig. 3-5, the shear demand is much lower versus the right side of the beam. Nonetheless, the design chosen uses a layout of struts with a 45° angle and two intermediate ties (E-9 and E-11) and to the authors seemed best suited to the beam geometry.

The final size of the grade beam was 7 ft 6 in. (2300 mm) wide, 10 ft 0 in. (3,050 mm) deep and 37 ft 0 in. (11,280 mm) long. With a wide beam width relative to the supported column (42 in. [1,070 mm] diameter), there was some concern about utilizing the full width of the beam, which was required for the struts. Fig. 3-6 shows an additional transverse strut-and-tie model created to address the spreading of the concentrated load from the 42 in. diameter column to the 7 ft 6 in. (2300 mm) wide grade beam. The result of this transverse analysis was transverse reinforcing (labelled as “hairpins”) that can be seen in Fig. 3-12 and Fig. 3-13 directly below the column.

Different configurations of the struts and ties were modelled. The diagonal struts were represented as fan shaped (near supports and reaction points) and parallel compression fields with guidance from ACI SP 208 (2002). The horizontal struts at the bottom of the beam were designed as non-reinforced concrete struts. The ties were designed as reinforced members and the nodes are sized to limit the stress levels in the nodes. ACI 318-05, Appendix A, Section A.4.2 requires the axis of the tie reinforcement to coincide with the axis of the tie, therefore, one of the modelling goals was to configure the ties orthogonal and parallel to the beam span which allows for a traditional arrangement of the reinforcing steel in the beam.

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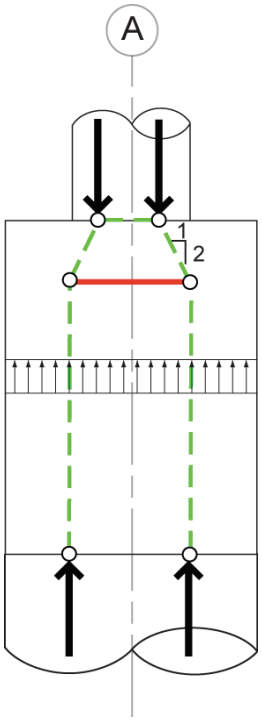


Fig. 3-6: Transverse strut-and-tie model

2.3 General Design Procedure

The strut-and-tie design process is often iterative; one needs to arrange the struts and ties in order to calculate the forces in the members and proportion these elements to handle the forces. If the sizes of the struts and ties change, the over-all dimensions of the beam may change, or the node locations could be altered which, in turn, changes the distribution of the forces.

The beam in this example started out as a 6 ft 0 in. (1,830 mm) wide by 8 ft 0 in. (2,440 mm) deep beam. With a different depth, the struts and ties in the cantilever required a different geometric layout. It soon became clear that regardless of the truss layout, the physical size of the beam had to increase to the current size to accommodate the design of the struts and ties.

3 Design calculations

3.1 Analysis and detailing

3.1.1 Analysis

The program CAST (**C**omputer **A**ided **S**trut-and-**T**ie), (University of Illinois, 2009), was used to layout the strut-and-tie models of all the grade beams in this project. CAST is a program that was developed at the University of Illinois and is available on the strut-and-tie resource web site (http://dankuchma.com/stm/).

The CAST program was used to analyze the strut-and-tie truss members. The resultant tie forces calculated by CAST are shown in Table 3-1. The resultant strut forces are shown in Table 3-2.

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Table 3-1: Tie forces and results

Tie ID (kN)	F_{ut} k (kN)	Required A_{st} in^2 (mm^2)	Bars Provided		Provided A_{st} in^2 (mm^2)	ϕF_{nt} k (kN)	Capacity Demand Ratio
			No.	Size in. (mm)			
E-1	1,596 (7,099)	35.47 (22,884)	48	#14 (43)	108 (69,677)	4860 (21,617)	3.05
E-2	3,192 (11,198)	70.93 (45,761)	48	#14 (43)	108 (69,677)	4860 (21,617)	1.52
E-3	4,788 (21,300)	106.40 (68,645)	48	#14 (43)	108 (69,677)	4860 (21,617)	1.02
E-9	1,653 (7,353)	36.73 (23,697)	48	#8 (25)	37.92 (24.464)	1706 (7,588)	1.03
E-11	1653 (7,353)	36.73 (23,697)	48	#8 (25)	37.92 (24.464)	1706 (7,588)	1.03

Note: $\phi = 0.75$

Table 3-2: Strut forces and results

Strut ID	F_{us} k (kN)	β_s	$\phi F_s = \phi\, 0.85\, \beta_s f'c_s$ k (kN)	Eff. Width in. (mm)	A_{cs} in^2 (mm^2)	ϕF_s k (kN)	Capacity Demand Ratio
E-5	-1,596 (7,099)	0.60	1,913 (8,509)	16 (406)	1,440 (929,000)	2,754 (12,250)	1.72
E-6	3,192 (14,198)	0.75	2,391 (10,635)	18 (203)	1,620 (1,045,000)	3,872 (17,223)	1.21
E-7	2,233 (9,932)	1.00	3,188 (14,180)	42 (1,067)	3,780 (2,439,000)	12,048 (53,590)	5.39
E-8	-2,298 (10,222)	0.75	2,391 (10,635)	22 (559)	1,980 (1,277,000)	4,733 (21,052)	2.06
E-10	-2,298 (10,222)	0.75	2,391 (10,635)	22 (559)	1,980 (1,277,000)	4,733 (21,052)	2.06
E-12	-2,298 (10,222)	0.60	1,913 (8,509)	24 (610)	2,160 (1,394,000)	4,131 (18,375)	1.80
E-13	-6,148 (27,350)	0.60	1,913 (8,509)	48 (1,219)	4,320 (2,787,000)	8,262 (36,749)	1.34

Notes: $\phi = 0.75$; effective thickness = 90.0 in. (2290 mm); strut E-4 is a zero force member

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3.1.2 Sizing the ties

The ties were initially sized to accommodate the forces shown in the tables above. The results for all the members are shown in Table 3-1 and in Table 3-2. The reinforcement required for tie E-3 is shown in the calculations below. The final sizes of the ties were revised when the reinforcing anchorage was considered.

Tie E-3 Factored design force = F_{ut} = 4,788 k (21,300 kN)

ϕ = 0.75 ACI 318-05 Section 9.3.2.6

Select 48 No. 14 reinforcing bars:

A_{st} = 48 bars x 2.25 in^2 = 108 in^2 (69,680 mm^2)

f_y = 60 ksi (413.7 MPa)

ϕF_{nt} = ϕ A_{st} f_y ACI 318-05 Section A.4.1

= 0.75 x 108 in^2 x 60 ksi

= 4,860k > 4,788k => OK (21,620 kN > 21,300 kN)

Capacity vs. demand ratio:

4,860 k / 4,788 k = 1.02 OK (21,620 kN / 21,300 kN = 1.02 OK)

3.1.3 Sizing the concrete struts and nodal zones

The concrete struts and nodal zones are generally designed in an iterative manner. The limiting factor of a strut design is governed by the size of the face of the nodal zone to which the strut is attached. This iterative design process was aided by the use of the CAST program.

3.1.4 Check of node N4 as an example of a nodal zone check

Fig. 3-7 shows the nodal zone N-4, the node below the column at the right cantilevered end. This node contains one tie and therefore is a CCT node by ACI 318-05, Appendix A. The sample calculations below demonstrate how to check the capacity of the node. The thickness of the strut-and-tie members is 90 in. (2,290 mm). For this nodal check, a thickness of 42 in. (1,070 mm) for the column load was used to model the width of the column before the transverse strut-and-tie model of Fig. 3-6. The concrete strength is 5,000 psi (34.5 MPa). The 32 in. (810 mm) dimension shown in Fig. 3-6 for tie E-3 is a geometric function of the placement of the reinforcing steel, whereas the 48 in. (1,220 mm) dimension for strut E-13 is a geometric function of the 42 in. (1,070 mm) column, the 32 in. (810 mm) tie, and the angle of the strut.

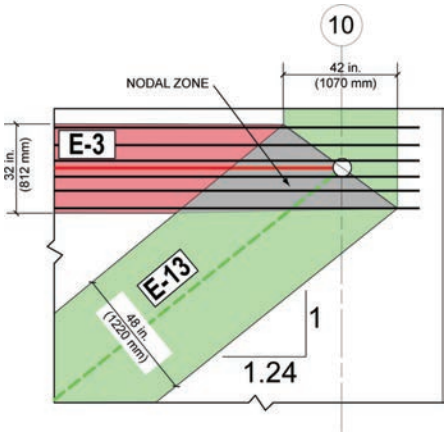


Fig. 3-7: Node N-4

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