

Fiber-Reinforced Polymer Reinforcement for Concrete Structures



10th International Symposium

Editors:

Rajan Sen

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Carol Shield

Will Gold

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PREFACE

The American Concrete Institute sponsored the First International Symposium for Fiber Reinforced Polymer (FRP) Reinforcement for Reinforced Concrete Structures at the Spring 1993 ACI Convention in Vancouver, Canada. At this inaugural event the international research community agreed to repeat these symposia every second year, rotating venues in various continents. Since 1993, the symposia have been held in Ghent, Belgium (1995); Sapporo, Japan (1997); Baltimore, USA (1999); Cambridge, England (2001); Singapore (2003); Kansas City, USA (2005); Patras, Greece (2007); and Sydney, Australia (2009). This volume represents the tenth in the symposium series, held in Tampa, Florida, USA, April 2-4, 2011.

This volume contains 72 papers from 16 countries that address wide-ranging topics. The papers have been placed in twelve chapters that correspond to the sessions presented at the symposium. These cover internally reinforced members, strengthening of columns, material characterization, bond, emerging FRP systems, shear strengthening, fatigue and anchorage systems, masonry, extreme events, applications, durability and strengthening.

The technical papers not only emphasize the experimental, analytical and numerical validations of using FRP composites but are aimed at providing insights needed for improving existing guidelines. The increasing maturity and acceptance of FRP is reflected by several papers that provide background information on the recent design codes and guidelines relating to blast and seismic repair. New frontiers of FRP research are explored by several papers that provide information on emerging materials, systems and applications for extreme events such as fire and earthquake that will further consolidate FRP's pre-eminent position. It is our hope that future FRPRCS symposia will feature more papers on sustainability, novel applications, new technologies and long term field data that will not only lead to continuing growth but result in greater acceptance and use of FRP composites technology by practitioners.

A work of this magnitude could not be put together without the help, dedication, and cooperation of numerous people. First we would like to thank the authors for meeting very tight deadlines for submissions, providing an opportunity to showcase the most current work possible at the symposium. Second, the International Scientific Steering Committee consisting of many distinguished international researchers, including chairs of past FRPRCS symposia and key international representatives was instrumental in attracting a large number of high quality submissions to the symposium. The committee signifies the global expansion and advancement of FRP composites research and technology.

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Special thanks to Danny Winters and Chandra Khoe for their help in creating and maintaining the FRPRCS-10 website, <http://frprcs10.eng.usf.edu>, the University of South Florida, Department of Civil and Environmental Engineering for hosting the conference website as well as Barbara Coleman, and other staff at ACI headquarters for their help in putting together the proceedings in time for the ACI 2011 Spring Convention in Tampa.

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ON THE HISTORY AND RELIABILITY OF THE FLEXURAL STRENGTH OF FRP REINFORCED CONCRETE MEMBERS IN ACI 440.1R

Carol K. Shield, Theodore V. Galambos, and Peter Gulbrandsen

Synopsis: The structural reliability of concrete flexural members reinforced with fiber reinforced polymer (FRP) reinforcement is investigated. Reliability indices based on the equations for flexure in ACI 440.1R-03, which uses the load factors from ACI 318-99 are presented. Choice of a resistance factor for flexure for ACI 440.1R-06, which uses the load factors from ACI 318-02 is also presented. Flexural designs using either ACI 440.1R-03 or ACI 440.1R-06 provide sufficient reliability, with reliability indices between 3.5 and 4.8, with the older versions of ACI 440.1R yielding higher reliability. An analysis of curvature of the beams at failure showed that flexural members that fail by FRP reinforcement rupture have ductilities similar to those that fail by concrete crushing, indicating that FRP reinforcement fracture is not necessarily a more brittle failure mode than concrete crushing.

Keywords: Ductility, Flexure, FRP, Reliability

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INTRODUCTION

Historically, steel bars primarily have been used to reinforce concrete. Steel reinforcing bars have performed well in many applications, however, where the members are subjected to corrosive environments, primarily road salt or coastal salts, the steel bars corrode. Bridge decks, wastewater treatment plants, and parking garages are a few example structures that have had severe corrosion problems. The chloride ions from sodium and calcium hydroxide, found in de-icing salts in northern climates and sea water along coastal areas, create an excellent environment for corrosion. As steel corrodes, its volume expands and can produce large enough tensile stresses in the concrete to produce cracking and ultimately spalling of the concrete; in addition, the cross-sectional area of the steel decreases and the safety of the member can be compromised.

Several techniques have been developed to prevent corrosion or at least extend the life of steel. Zinc rebar coatings, cathodic protection, corrosion inhibiting admixtures in concrete, and increased concrete cover are a few methods of preventing or inhibiting corrosion (Matlock and Krauss 1990). Another common method, used frequently in bridge decks and parking garages, is to use epoxy coated steel bars (Race 1995). The epoxy coatings are often times chipped, scratched, and damaged in the field, which can lead to accelerated corrosion in the damaged areas (De Girorgi 1993). Premature corrosion of epoxy-coated bars has been discovered and has led to skepticism about their long-term performance (Ehsani et al. 1996).

Fiber reinforced polymer (FRP) reinforcing bars provide another solution to providing long-lasting reinforcement for concrete. The FRP bars are corrosion resistant throughout the entire depth of the bar. FRP bars are currently being used in bridge decks and other structural elements where corrosion can cause damage. In the United States of America, design of concrete structures reinforced with FRP reinforcing bars is largely based on documents produced by ACI Committee 440. Committee 440 produces a design guideline: ACI440.1R, first published in 2001 with updates in 2003 and 2006 (ACI Committee 440 2001, 2003, 2006), a materials specification: ACI 440.6 (ACI Committee 440 2008a) published in 2008 and a construction specification: ACI 440.5 (ACI Committee 440 2008b) published in 2008.

RESEARCH SIGNIFICANCE

As the field of FRP reinforced concrete matured, Committee 440 has updated the design guideline relying less on committee consensus and more on experimental evidence. When developing ACI440.1R-06, the committee determined that there were a sufficient number of well documented experimental results to perform a reliability analysis and calibrate the resistance factors for flexure in a manner similar to the calibration of the resistance factors for ACI 318. This paper presents the results of a determination of the flexural reliability of structures designed using ACI 440.1R-03 and presents the development of the resistance factors for flexure found in ACI440.1R-06.

ACI 440.1R FLEXURAL STRENGTH EQUATIONS

One of the main differences between steel and FRP is that FRP is linear elastic up until failure, whereas steel yields. This difference means that unlike steel reinforced concrete, concrete crushing is not the only form of flexural failure for FRP reinforced concrete, it is also possible for the FRP reinforcement to fracture. Flexural capacity is determined by one of two different equations in ACI 440.1R depending on the expected governing mode of failure: concrete crushing or reinforcement fracture. The anticipated failure mode can be determined by comparing the FRP reinforcement ratio to the balanced reinforcement ratio. The balanced reinforcement ratio occurs when concrete crushing and FRP reinforcement rupture happen simultaneously and is given by

$$\rho_{fb} = 0.85\beta_1 \frac{f'_c}{f_{fu}} \frac{E_f \epsilon_{cu}}{E_f \epsilon_{cu} + f_{fu}}, \quad (1)$$

where f'_c is the concrete compressive strength, E_f is the modulus of elasticity of the FRP reinforcement, β_1 is the factor relating depth of the equivalent rectangular compressive stress block to the neutral axis depth, and f_{fu} is the design tensile strength of the FRP bar considering reductions for environmental conditions (i.e. $f_{fu} = C_E f_{fu}^*$, where f_{fu}^* is the guaranteed immediate tensile strength of the FRP bar and C_E is the environmental reduction factor).

When the FRP reinforcement ratio is greater than the balanced reinforcement ratio, concrete crushing is the predicted failure mode and the stress distribution in the concrete can be approximated with the ACI rectangular stress block. In this case, the stress in the FRP bar is less than f_{fu} , but can be found from strain compatibility, with the flexural strength given by

$$M_n = A_f f_f \left(d - \frac{1}{2} \frac{A_f f_f}{0.85 f'_c b} \right), \quad (2)$$

where

$$f_f = \sqrt{\frac{(E_f \epsilon_{cu})^2}{4} + \frac{0.85\beta_1 f'_c E_f \epsilon_{cu}}{\rho_f}} - 0.5 E_f \epsilon_{cu} \leq f_{fu},$$

and A_f is the FRP reinforcement area, d is the effective depth of the beam, b is the width of the beam, and ρ_f is the FRP reinforcement ratio.

When the FRP reinforcement ratio is smaller than the balanced reinforcement ratio, FRP reinforcement rupture is the predicted failure mode and the ACI rectangular stress block may not be applicable because the maximum concrete strain (0.003) may not be achieved. In this case, the stress in the FRP reinforcement at failure is f_{fu} and the resulting net tension and hence compression is known ($T = C = A_f f_{fu}$). However, the location of the centroid of the compression is unknown and hence the moment arm is unknown. ACI 440.1R recommends that the moment arm at balance be used as a lower bound estimate yielding a nominal moment capacity of

$$M_n = A_f f_{fu} \left(d - \frac{\beta_1 c_b}{2} \right), \quad (3)$$

where c_b is the location of the neutral axis at the balanced condition given by

$$c_b = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}} \right) d, \quad (4)$$

where ε_{fu} is the strain corresponding to f_{fu} .

In versions of 440.1R prior to 2006, Eqn. (3) was multiplied by 0.8 and appeared as

$$M_n = 0.8 A_f f_{fu} \left(d - \frac{\beta_1 c_b}{2} \right), \quad (5)$$

The Committee explained that the 0.8 was added to the equation to better fit the data. With the reliability calibration of the equation for the 2006 edition, the Committee felt it was better to have this constant reflected in the calibrated resistance factor, so Eqn. (3) is the version of the equation found in 440.1R-06.

ACI 440.1R-03 RESISTANCE FACTORS

The ACI design guideline (ACI 440.1R) is based on load and resistance factor design (LRFD) using factored loads to determine the demand for the ultimate strength limit states and reducing the nominal capacity by a resistance factor so that for flexure,

$$\phi M_n \geq M_u, \quad (6)$$

where M_n is the nominal flexural capacity given by Eqns. (2), (3), or (5) depending on the expected failure mode and version of ACI 440.1R, ϕ is the resistance factor for flexure and M_u is the factored ultimate moment based on the load factors from ACI 318-99 (ACI Committee 318 1999) or ACI318-02 (ACI Committee 318 2002) for ACI 440.1R-03 and ACI 440.1R-06, respectively. As mentioned previously, the resistance factors in ACI 440.1R-03 were not based on a reliability analysis, but instead, were based on committee consensus. Because flexural members reinforced with FRP do not exhibit

traditional ductile behavior, the Committee sought to implement conservative resistance factors to provide higher reserve strength.

ACI 318-99 specifies a resistance factor of 0.7 for steel reinforced concrete members that fail by concrete crushing prior to reinforcement yielding. The committee consensus was that because the reinforcing material is not the weak link in a concrete beam that fails by concrete crushing, it should not matter if the beam is reinforced with steel or FRP. Therefore, Committee 440 set the resistance factor for FRP reinforced concrete beams failing by concrete crushing to 0.70; the same resistance factor as steel reinforced concrete beams failing by concrete crushing in ACI 318-99. When developing ACI 440.1R-03, ACI Committee 440 believed that failure by concrete crushing would be more ductile than failure by FRP reinforcement rupture; therefore, the committee picked a smaller resistance factor, 0.5, for flexural failure by FRP reinforcement rupture.

To transition between the two flexural strength equations (Eqns. (2) and (5)) and associated two different resistance factors, ACI 440.1R-03 calls for an interpolation of the resistance factor when the reinforcement ratio is between 1 and 1.4 times the balanced reinforcement ratio,

$$\phi = \frac{\rho_f}{2\rho_{fb}} \text{ for } \rho_{fb} < \rho_f < 1.4\rho_{fb}. \quad (7a)$$

This definition for the resistance factor in combination with the resistance factors for concrete crushing controlled failures

$$\phi = 0.7 \text{ for } \rho_f \geq 1.4\rho_{fb} \quad (7b)$$

and FRP reinforcement rupture failures

$$\phi = 0.5 \text{ for } \rho_f < \rho_{fb} \quad (7b)$$

define the resistance factors for all cases. The 0.8 factor in Eqn. (5) causes a discontinuity in the nominal flexural capacity (and hence the design flexural capacity) at the balanced reinforcement ratio. If a designer switches from the reinforcement rupture formula, Eqn. (5) to the concrete crushing formula, Eqn. (2) right at the balanced reinforcement ratio. Many designers used the minimum of Eqns. (2) and (5) when the reinforcement ratio was between the balanced ratio and 1.4 times the balanced ratio. ACI 440.1-06 avoids this problem with the removal of the 0.8 factor.

PROFESSIONAL FACTORS FOR FLEXURE

The accuracy of the nominal flexural capacity equations, Eqns. (2), (3) and (5), was obtained by comparing experimentally determined results found in the literature with the predictions from the appropriate equation. A literature search identified 181 tests of FRP reinforced concrete beams tested to flexural failure by 19 different researchers. A full description of the database can be found in Gulbrandsen (2005). In order to perform the comparison, the literature needed to document the measured material properties (f_{fu}^* , E_f ,

and f'_c) as well as the geometrical parameters (b , L , d , and nominal reinforcement size). Sixty-two flexural tests from 9 different research articles contained all of the necessary information (Nakono et al. 1993, Wang et al. 1998, Toutanji and Saafi 2000, Benmokrane et al. 1996, 1997, Li and Wang 2002, Zhao et al. 1997, Nawy and Neuwerth 1971, Pecce et al. 2000).

The 62 tests obtained from the literature covered three different reinforcing materials: glass, aramid and carbon FRP. The tensile strength of the FRP reinforcement varied not only with fiber material, but also with bar size, with smaller bars having apparently larger strengths. Over the three materials, reinforcement strengths from 500 to 2070 MPa were included in the tests. The moduli of elasticity of the reinforcement ranged from 41 GPa (glass) to 150 GPa (carbon). Nominal bar diameters in the database ranged from 3 to 19 mm. The bars used in the experiments had a variety of surface finishes from spirally wrapped to braided to sand coated. The depth of the beams in the database varied from 145 mm to 510 mm, with the majority of beams in the 150 to 255 mm range. The width of the beams in the database varied from 90 to 500 mm, with the majority of the widths between 100 and 200 mm. The depth to width ratios for the beams in the database varied between 0.29 and 2.5, with the majority of the beams falling between 1 and 1.5. The depth to bar diameter ratios of the beams in the database varied between 9.9 and 54.2, with about 40 percent lying between 20 and 30. Ratios of the FRP reinforcement ratio to the balanced ratio ranged from 0.73 to 2 for beams that failed by FRP reinforcement rupture and between 0.93 to 16.36 for beams that failed by concrete crushing. It is important to note that reinforcement rupture failures occurred at reinforcement ratios up to 2 times the balanced reinforcement ratio. A total of 12 beams in this study with $\rho_f \geq 1.4 \rho_b$ failed by reinforcement rupture. The concrete compressive strengths for the beams in the database varied between 22.8 and 75.8 MPa.

The bar size distribution was biased, with the distribution for tests that failed by reinforcement rupture; approximately eighty five percent of the tests that ended in FRP reinforcement rupture used FRP reinforcement that was smaller than or equal to a No. 3 bar. The bar size distribution for the concrete crushing failures was more evenly distributed; the bar size used most frequently was a No. 5 bar and was used in approximately thirty five percent of the tests.

The professional factors (i.e. the mean value of the test-to-predicted ratio) were determined independently for beams that failed by FRP reinforcement rupture and beams that failed by concrete crushing. The mean value for the ratio of the test-to predicted flexural strength for all of the beams that failed by FRP reinforcement rupture was 1.11 based on Eqn. (5) and 0.89 based on Eqn. (3). The coefficient of variation was 16% for both equations. The mean value for the ratio of the test-to-predicted flexural strength for all the beams that failed by concrete crushing was 1.19 based on Eqn. (2) with a coefficient of variation of 16%.

RELIABILITY ANALYSIS

Load and resistance factor design has its basis in the underlying assumption that loads and resistances are random variables (Nowak and Collins 2000). Statistics on these random variables can be used to predict the reliability of structural members. The reliability of a member is determined by the probability of the load effect exceeding the resistance of the member. As the probability of the load effect exceeding the resistance decreases, the reliability increases. Because there is always some chance of failure, the resistance cannot always exceed the load effect. The basic equation to avoid failure or achieve safety is given by

$$R \geq Q \quad (8)$$

where R is the resistance and Q is the load effect (Nowak and Collins). The resistance depends on the material properties and fabrication procedures. The predicted load effect depends on load models and analysis assumptions. The structure is safe (desired performance) if the resistance exceeds the load effect. Conversely, the limit state is exceeded and failure occurs when the resistance is less than the load effect. The limit state function $g(R, Q) = R - Q$ can be expressed in terms of reduced variables

$$g(Z_R, Z_Q) = \mu_R + Z_R \sigma_R - \mu_Q - Z_Q \sigma_Q, \quad (9)$$

where Z_R and Z_Q are termed the reduced variables, μ_R and μ_Q are the means of the resistance and load effect, respectively, and σ_R and σ_Q are the standard deviations of the resistance and load effect, respectively (i.e. $R = \mu_R + Z_R \sigma_R$ and $Q = \mu_Q + Z_Q \sigma_Q$). For any specific value of $g(Z_R, Z_Q)$, Eqn. (9) represents a straight line in the space of reduced variables Z_R and Z_Q . The line of interest for reliability analysis is the line that corresponds to $g(Z_R, Z_Q) = 0$ because it separates the domains representing safety and failure. The reliability index can be defined as the shortest distance from the origin of reduced variables to the line $g(Z_R, Z_Q) = 0$ (Nowak and Collins 2000). Using geometry illustrated in Fig. 1, the shortest distance between $g(Z_R, Z_Q) = 0$ and the origin is

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}, \quad (10)$$

where β is the inverse of the coefficient of variation of the function $g(Z_R, Z_Q) = 0$ when R and Q are uncorrelated. For normally distributed random variables R and Q , it can be shown that the reliability index is related to the probability of failure by

$$P_f = \phi(-\beta), \quad (11)$$

where ϕ represents the value of the cumulative distribution function (CDF) of a normal random variable (Nowak and Collins 2000).

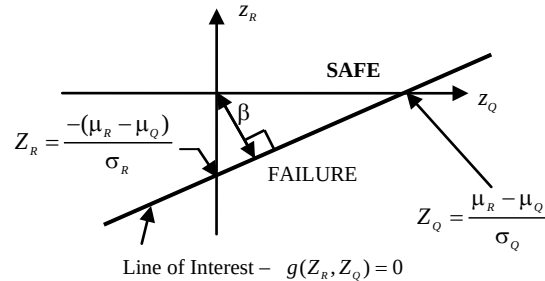


Figure 1 Reliability Index

If both R and Q are continuous random variables, then each has a probability density function (PDF). Moreover, the quantity $R-Q$ is also a random variable with its own PDF. Figure 2 displays the PDF's of all three random variables, safety margin, load effect, and the resistance. The reliability index (β) depends only on the means and standard deviations of the random variables. Therefore, β is called a second moment measure of structural safety because only the first two moments (mean and variance) are required to calculate β . If the random variables are all normally distributed and uncorrelated, and the limit state function is linear, Eqns. (10) and (11) are exact. Otherwise, the equations provide an estimate of the reliability index and the probability of failure (Novak and Collins 2000).

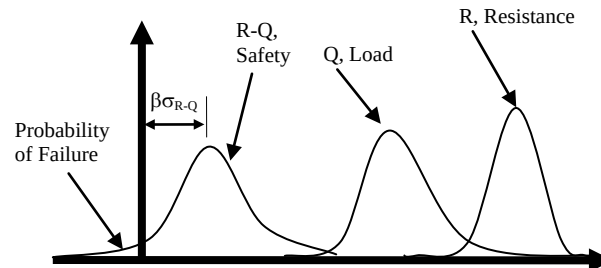


Figure 2 PDF's of the Safety Margin, Load Effect, and Resistance

There are two typical procedures used to determine the reliability index: 1) the Hasofer-Lind method (Hasofer and Lind 1974) to determine the design point (i.e. the point on the line $g(Z_R, Z_Q)$) that minimizes the distance from the line $g(Z_R, Z_Q)$ to the origin or 2) Monte Carlo simulation to determine the probability of failure of a given design. Using Monte Carlo simulation, the results from previous testing and the known distributions for the material and geometrical properties can be used to establish the probability density function characteristics to generate samples of numerical data which can reduce the uncertainty required to obtain the desired reliability estimate, and

correspondingly, the probability of failure (Nowak and Collins 2000). Estimating relatively small probabilities of failure using Monte Carlo simulation while limiting the uncertainty in the estimate, requires a large number of simulations to be conducted. The required sample size depends on the desired coefficient of variation and the relative magnitude of the probability to be estimated (Nowak and Collins 2000). For the types of probabilities typical for structural design a large number of simulations (5,000,000) need to be performed for each trial design to determine an estimate of the reliability of that design. Whether using the Hasofer-Lind method or Monte Carlo simulation, the means and coefficients of variation for all of the random variables (i.e. material properties, geometrical properties, and loads) must be known.

Determination of Statistical Parameters

The means and coefficients of variation for the area of GFRP reinforcement, design GFRP tensile strength, f_{fu}^* (varied according to bar size), and the design GFRP modulus of elasticity were calculated from reports of over 400 bar tests provided by FRP manufacturers. Nominal (design) values for each of these parameters were obtained from the FRP bar manufacturers. The means and coefficients of variation for the width of the beams, depth of the beams, concrete compressive strengths, dead load moments, and live load moments were taken from Nowak and Szerszen (2003a) and are the same values as those used to calibrate the resistance factors in ACI 318-02. Biases for all of these parameters were calculated by taking the mean of the tested quantity divided by the nominal value of that quantity for each parameter. The values of the statistical parameters used in the reliability study are shown in Table 1.

Table 1: Bias and Coefficient of Variation for Parameters used in Reliability Analysis

Parameter	Bias	COV
Eq. 3	0.89	0.16
Eq. 5	1.11	0.16
Eq. 2	1.19	0.16
A_f	1.00	0.03
f_{fu}^* (# 3)	1.18	0.12
f_{fu}^* (# 5)	1.20	0.08
f_{fu}^* (# 6)	1.22	0.07
f_{fu}^* (# 7)	1.12	0.05
f_{fu}^* (# 8)	1.06	0.04
f_{fu}^* (# 9)	1.13	0.05
B	1.01	0.04
d	0.99	0.04
E_f	1.04	0.08
f'_c	1.24	0.10
Dead Load	1.05	0.10
Live Load	1.00	0.18

Three of the limit state equation variables, the environmental service factor, C_E , the material property to define the location of the neutral axis from the depth of the compression block, β_1 , and the ultimate compressive concrete strain, ϵ_{cu} , were considered to be deterministic. The latter two variables were considered to be deterministic also in the calibration of ACI 318-02. The environmental service factor is a non-calibrated coefficient that accounts for long-term exposure based on limited quantifiable data; therefore, the reliability analysis was conducted such that C_E was deterministic.

Design Space for Reliability Indices

Reliability indices were determined using the two methods discussed above (Hasofer-Lind and Monte Carlo simulation) for twenty different beam designs using the design equations of ACI 440.1R-03 to determine the reliability of flexural members designed using that guideline. Additionally, another twenty beam designs were used to determine calibrated resistance factors for the new load factors and design equations appearing in ACI 440.1R-06. In each case, the FRP reinforced concrete design beams reinforced with only tensile reinforcement were designed according to either ACI 440.1R-03 or ACI 440.1R-06. For the designs based on ACI 440.1R-03, when the FRP reinforcement ratio, ρ_f , was between the balanced reinforcement ratio, ρ_{fb} , and $1.4 \rho_{fb}$, the minimum of Eqns. (2) and (5) was used. For the designs based on ACI 440.1R-06, trial resistance factors of 0.5, 0.55 and 0.6 were each evaluated, with the goal of determining resistance factors that would yield a reliability index between 3.5 and 4 for the new load factors and equations in ACI 440.1R-06.

The beams were designed with common design criteria such as, keeping the width to height ratio between one-third and one, using a live to dead load ratio of one, two, or three, setting beam lengths from 3 meters to 9 meters, and using a concrete compressive strength of 28 MPa. All beams were designed with one layer of tensile reinforcement in the bottom of the beam, an environmental service factor of 0.80, two or more FRP reinforcement bars, an ultimate concrete compressive strain of 0.003, and a superimposed dead load from approximately 1.5 kN/m up to approximately 4.4 kN/m varying according to the length of the design beam. The three, six, and nine meter long beams were designed with approximately 1.5, 3, and 4.4 kN/m uniform dead loads, respectively. However, these values were adjusted to achieve the desired live to dead load ratio. Beams were designed with several sizes of GFRP reinforcing bars with a modulus of elasticity of 39.4 GPa including No. 3 bars, No. 5 bars, No. 6 bars, No. 7 bars, No. 8 bars and No. 9 bars. The FRP bar design tensile strength f_{fu}^* varied according to bar size. Table 2 shows the FRP design strength (f_{fu}^*) for bar sizes from No. 3 up to No. 9 bar. Table 3 lists the geometrical properties for all 20 beam designs. The geometrical and material properties of the beams were not changed between the ACI440.1R-03 and ACI440.1R-06 analyses, instead, the applied loads were adjusted for the two cases to satisfy exactly M_u equal to ϕM_n . The environmental service factor (C_E) was chosen as 0.8 for GFRP bars not exposed to earth and weather from Table 7.1 in ACI 440.1R-03. The concrete compressive design strength was not varied in the beam designs. FRP reinforced concrete is most economical at lower concrete compressive strengths. By limiting the concrete compressive strength to 28 MPa, it was possible to get a fairly even distribution between FRP reinforcement rupture and concrete compression failures in the beams designed. Serviceability limit

states, such as crack width, often control the design of FRP reinforced concrete beams. For this study the serviceability limit states were ignored in order to determine the reliability of the flexural strength equations.

Table 2: Nominal FRP Tensile Design Strengths

Bar Size	3	5	6	7	8	9
f_{tu}^* (ksi)	110	95	90	85	80	75
f_{tu}^* (MPa)	750	650	620	585	550	520

Table 3: Geometrical Properties of Beam Design Space

Beam	Len ft(m)	b in (mm)	h in (mm)	Bar Size	# of Bars	ρ_f/ρ_{fb}
1	10 (3)	8 (205)	8 (205)	3	3	0.31
2	10 (3)	12 (305)	12 (305)	3	5	0.42
3	10 (3)	14 (350)	14 (355)	3	7	0.29
4	10 (3)	8 (205)	8 (205)	5	2	1.85
5	10 (3)	8 (205)	8 (205)	5	2	1.85
6	10 (3)	8 (205)	10 (255)	5	2	1.38
7	10 (3)	8 (205)	8 (205)	7	2	3.00
8	20 (6)	12 (305)	24 (610)	5	5	0.82
9	20 (6)	20 (510)	30 (760)	5	10	0.4
10	20 (6)	10 (255)	20 (510)	6	3	0.94
11	20 (6)	14 (355)	22 (560)	6	5	1.00
12	20 (6)	16 (405)	26 (660)	6	7	1.02
13	20 (6)	8 (205)	16 (405)	8	2	1.48
14	20 (6)	10 (255)	18 (455)	8	3	1.55
15	20 (6)	12 (305)	20 (510)	8	4	1.52
16	30 (9)	18 (455)	28 (710)	7	7	1.03
17	30 (9)	28 (710)	28 (710)	7	13	1.23
18	30 (9)	10 (255)	28 (710)	9	3	1.07
19	30 (9)	20 (510)	30 (760)	9	7	1.15
20	30 (9)	28 (710)	28 (710)	9	11	1.40

RELIABILITY RESULTS

Reliability indices for each trial design beam were found using both the Hasofer-Lind method and Monte Carlo Simulation. The fundamental idea behind Monte Carlo simulation is to numerically simulate a particular phenomenon and then observe the number of times the event of interest occurs. Simulation techniques are relatively straightforward, but the procedure can become computationally intensive. For each of the beams in the design space, five million simulations were run. The number of FRP

reinforcement rupture failures and the number of concrete compression failures were tallied for each beam design. These were used to determine the total number of failures for each set of 5,000,000 runs. For each beam design, three reliability indices were determined: a reliability index associated with FRP reinforcement rupture, a reliability index associated with concrete crushing, and an overall reliability index, which was associated with beam failure, regardless of the type of failure.

Results for ACI 440.1R-03

Twenty beams were designed using load factors of 1.4 and 1.7 for dead loads and live loads, respectively, and the resistance factors defined in ACI 440.1R-03 (varying from 0.50 to 0.70 depending on the reinforcement ratio). The tabulated results of the reliability index values for each design beam are shown in Table 4. To conserve on space, only the results from the Monte Carlo simulation are shown. The largest difference between the two reliability methods was 0.11; sixteen of the twenty beams had differences smaller than 0.05. In all but four cases, the Monte Carlo results provided smaller reliability indices than the Hasofer-Lind results. The differences between the two methods were relatively small and bolstered the merit of the reliability index values presented.

Table 4: Reliability Indices for Beams Designed using ACI 440.1R-03

Beam	Length ft (m)	Bar Size	Live/ Dead	$\frac{\rho_f}{\rho_{bal}}$	Beta	Beta FRP rupture	Beta Concrete Compression
1	10 (3)	3	1	1.26	3.87	3.90	4.35
2	10 (3)	3	2	0.84	4.38	4.40	4.94
3	10 (3)	3	3	0.84	4.49	4.49	∞
4	10 (3)	5	1	1.85	3.89	4.26	<u>3.94</u>
5	10 (3)	5	2	1.85	3.92	4.29	3.97
6	10 (3)	5	3	1.38	3.87	3.98	4.11
7	10 (3)	7	3	3.00	4.03	4.86	4.03
8	20 (6)	5	1	0.82	4.57	4.59	5.07
9	20 (6)	5	2	0.77	4.59	4.61	4.86
10	20 (6)	6	1	0.94	4.66	4.72	4.94
11	20 (6)	6	2	1.00	4.75	4.75	∞
12	20 (6)	6	3	1.02	4.72	4.72	∞
13	20 (6)	8	1	1.48	<u>3.61</u>	<u>3.68</u>	3.95
14	20 (6)	8	2	1.55	3.73	3.84	3.97
15	20 (6)	8	3	1.52	3.73	3.83	4.01
16	30 (9)	7	1	1.03	4.53	4.54	5.07
17	30 (9)	7	2	1.23	4.11	4.16	4.51
18	30 (9)	9	1	1.07	3.70	3.81	3.97
19	30 (9)	9	2	1.15	3.78	3.89	4.02
20	30 (9)	9	3	1.40	3.79	3.88	4.06

The overall reliability indices varied between 3.61 and 4.75, similar to the reliability of concrete beams with steel reinforcement designed according to ACI 318. The data set can be further subdivided into those beams designed to fail by FRP reinforcement rupture and concrete crushing, recalling that the Committee intended there to be a higher level of reliability associated with FRP reinforcement rupture. For the beams designed to fail by FRP reinforcement rupture ($\rho_f < \rho_{fb}$), the overall reliability indices β were between 4.38 and 4.75. All of these beams had a higher probability of failing by FRP reinforcement rupture than by concrete crushing. All of the beams in the transition region ($\rho_{fb} < \rho_f < 1.4 \rho_{fb}$) had overall reliabilities between 3.7 and 4.72, again with a FRP reinforcement rupture failure more likely than a concrete crushing failure. The beams designed to fail by concrete crushing ($1.4 \rho_{fb} < \rho_f$) had the lowest reliability indices, ranging between 3.61 and 4.03. The 6m beams in this category also had a higher probability of failing by FRP reinforcement rupture than by concrete crushing, whereas the 3m beams in this category were more likely to fail by concrete crushing. In all cases, the reliability indices increased as the live load percentage increased. This is similar to an observation made in Nowak and Szerszen (2003a) that the reliability indices for steel reinforced concrete were low for load combinations with the dead load being approximately eighty to ninety percent of the total load. The smallest reliability index values occurred, in general, for beams with the smallest live-to-dead load ratio and with smaller bars. The correlation with bar size is likely related to the larger coefficient of variation on the bar strength for the smaller bars.

Calibration of ACI 440.1R-06

To be consistent with ACI 318-02, ACI Committee 440 wanted to keep the ACI 318-02 value of the resistance factor of 0.65 for compression controlled failures, so all beam designs based on concrete compression were performed using a resistance factor of 0.65. Designs based on FRP reinforcement rupture were performed using resistance factors between 0.5 and 0.65. Reliabilities of each of the beams were determined using the Hasofer-Lind method, as well as Monte Carlo simulation. Table 5 shows the results from the Monte Carlo Simulation with FRP rupture resistance factor of 0.55.

In versions of 440.1R prior to 2006, the Committee believed that a failure due to concrete crushing would be more ductile than a failure due to rupture of the FRP reinforcement. With ACI 319-02, the determination of ductility of failure was based on the strain in the steel reinforcement, with beams that fail with reinforcement strains of at least 0.005 being given a higher resistance factor. This requirement on development of strain in the steel reinforcement can be thought of as requiring a specific amount of curvature (and hence deflection) prior to failure. The curvature of every design beam at failure was calculated using Response 2000 (Bentz 2000). Response 2000 is a sectional analysis program based on strain compatibility and plane sections remaining plane. A linear elastic stress-strain relationship was used for the FRP reinforcement, the Popovics/Thorenfeldt/Collins material model was used to define the stress-strain relationship of the concrete.

As the curvature of the beam increases, the deflection also increases which yields additional warning to the public. ACI 318-02 states that a tension-controlled section is one where the net tensile strain in the extreme tension steel at nominal strength is greater

than or equal to 0.005. When the concrete and steel strains are at 0.003 and 0.005, respectively, the curvature is equal to 0.008 divided by the depth to the extreme layer of tension reinforcement. Therefore, if the (curvature)×(effective depth) of the FRP reinforced concrete beams is at least 0.008, the curvature would meet the criteria stated in ACI 318-02 for tension-controlled sections. In the commentary, ACI 318-02 also shows an equivalence between steel strains and c/d ratios.

Table 5: Reliability Indices for ACI 440.1R-06

Beam	Length ft (m)	Bar Size	Live/ Dead	$\frac{\rho_f}{\rho_{bal}}$	Beta	Beta FRP rupture	Beta Concrete Compression
1	10 (3)	3	1	1.26	3.45	3.48	3.98
2	10 (3)	3	2	0.84	3.50	3.51	4.33
3	10 (3)	3	3	0.84	3.54	3.55	4.30
4	10 (3)	5	1	1.85	3.83	4.23	3.88
5	10 (3)	5	2	1.85	3.90	4.34	3.94
6	10 (3)	5	3	1.38	3.76	3.86	4.01
7	10 (3)	7	3	3.00	4.01	4.80	4.02
8	20 (6)	5	1	0.82	3.67	3.69	4.33
9	20 (6)	5	2	0.77	3.77	3.78	4.44
10	20 (6)	6	1	0.94	3.75	3.78	4.30
11	20 (6)	6	2	1.00	3.84	3.88	4.33
12	20 (6)	6	3	1.02	3.89	3.92	4.38
13	20 (6)	8	1	1.48	3.58	3.67	3.89
14	20 (6)	8	2	1.55	3.70	3.80	3.98
15	20 (6)	8	3	1.52	3.72	3.83	3.99
16	30 (9)	7	1	1.03	3.58	3.60	4.24
17	30 (9)	7	2	1.23	3.70	3.74	4.12
18	30 (9)	9	1	1.07	3.60	3.62	4.15
19	30 (9)	9	2	1.15	3.72	3.76	4.21
20	30 (9)	9	3	1.40	3.74	3.85	4.00

The values of the ultimate curvatures and curvature× d for each design beam are shown in Table 6. The failure mode predicted by Response 2000 is shown as a “RR” or “CC” for reinforcement rupture or concrete compression, respectively. All beams had a curvature× d of at least 0.008 and were deemed to fail with similar ductility as tension-controlled steel reinforced beams and, in fact, the beams that failed by FRP rupture had larger curvature× d at failure than those that failed by concrete crushing. This led Committee 440 to no longer believe that there should be higher reliability for beams failing by FRP rupture than those failing by concrete crushing. The Committee decided on a target reliability index of 3.5 for flexure, regardless of failure mode. This target reliability is the same value recommended in Nowak and Szerszen (2003b) for ACI 318.

The reliability indices in Table 5 meet the target reliability index using resistance factors of

$$\phi = 0.55 \text{ for } \rho_f < \rho_{fb}$$

$$\phi = 0.3 + 0.25\rho_f/\rho_{fb} \text{ for } \rho_{fb} < \rho_f < 1.4 \rho_{fb}$$

$$\phi = 0.65 \text{ for } \rho_f > 1.4 \rho_{fb}$$

The indices ranged between 3.45 and 4.01. For the beams with reinforcement ratios below $1.55\rho_{fb}$ FRP rupture is more likely than concrete crushing, with the probability of failure of the two different modes becoming less of a difference as the reinforcement ratio approaches $1.55\rho_{fb}$.

Table 6: Curvatures

Beam	ρ_f/ρ_{fb}	Failure Mode Response	Failure Mode 440.1R	d (in ¹)	Curvature $\times 10^6$ (1/in)	Curv $\times d$	M_n Response (kip-ft) ²	M_n ACI 440.1R (kip-ft) ²
1	1.26	CC	CC	5.94	3330	0.020	13.9	11.7
2	0.84	RR	RR	9.94	1835	0.018	44.9	37.2
3	0.84	RR	RR	11.94	1518	0.018	75.1	62.6
4	1.85	CC	CC	5.81	2502	0.015	16.7	14.5
5	1.85	CC	CC	5.81	2502	0.015	16.7	14.5
6	1.38	CC	CC	7.81	2201	0.017	27.2	23.3
7	3.00	CC	CC	5.69	2012	0.011	19.4	17.6
8	0.82	RR	RR	21.81	723	0.016	228	197
9	0.77	RR	RR	27.81	558	0.016	581	502
10	0.94	RR	RR	17.75	878	0.016	145	129
11	1.00	CC	CC	19.75	798	0.016	265	238
12	1.02	CC	CC	23.75	668	0.016	443	397
13	1.48	CC	CC	13.62	1034	0.014	92.6	81.9
14	1.55	CC	CC	15.62	919	0.014	154	137
15	1.52	CC	CC	17.62	885	0.016	234	208
16	1.03	RR	CC	25.69	565	0.015	562	547
17	1.23	RR	CC	25.69	596	0.015	1010	912
18	1.07	RR	CC	25.56	517	0.013	346	333
19	1.15	RR	CC	27.56	493	0.014	861	797
20	1.40	CC	CC	25.56	518	0.013	1170	1032

¹ 1in=25.4mm

² 1 kip-ft = 1.356kN-m

Table 6 also shows the nominal moment capacities calculated using Response 2000, and the equations in 440.1R-06. The nominal capacities calculated with response match the nominal capacities in ACI 440.1R-06 fairly well, regardless of failure mode, with the

largest difference being 17%. The failure modes predicted by Response 2000 were slightly different than the failure modes predicted by ACI 440.1R-06, with Response 2000 predicting reinforcement rupture for reinforcement ratios between 0.8 and 1.40 times the balanced reinforcement ratio, and concrete compression failures with reinforcement ratios between 1.48 and 3.00 times the balanced reinforcement ratio. The difference between the predicted failure modes was likely due to the difference in concrete material models assumed by the two methods. The Response 2000 prediction of FRP reinforcement rupture for reinforcement ratios up to 1.4 times the balanced ratio is in good agreement with the experimentally observed results discussed earlier.

The difference between the Response 2000 predicted moment capacities and the ACI 440.1R-06 capacities for beams that failed by reinforcement rupture varied between 3% and 17%, whereas the capacities for beams that failed by concrete crushing varied between 9% and 13%. The larger range of variability in the beams failing by reinforcement fracture is likely due to the use of the moment arm at balance in calculating the nominal moment capacity, rather than using an iterative procedure to find the true moment arm. In all cases, the nominal moments predicted using ACI 440.1R-06 were smaller than those predicted using Response 2000.

SUMMARY AND CONCLUSIONS

Reliability indices of beams designed using versions of ACI440.1R prior to 2006 were shown to be in the range of 3.61 to 4.75. Reliability indices for the flexural strength design equations in ACI 440.1R-06 were more uniform than the reliability for the beams designed according to ACI 440.1R-03, with reliability indices varying between 3.45 and 4.01. An analysis of curvatures of FRP reinforced beams showed that similar levels of ductility, defined as curvature $\times$ effective depth, were observed for both FRP reinforcement rupture failures and for concrete compression failures, leading to the conclusion that neither failure is more brittle than the other. The curvature $\times$ effective depth values were larger than those that would be required by ACI318-02 to be classified as a “tension-controlled” failure, indicating that GFRP reinforced beams will have sufficient deformations at failure to provide adequate forewarning to occupants.

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SHEAR CAPACITY OF CONCRETE BEAMS WITH FRP REINFORCEMENT
J. Hegger, M. Kurth

Synopsis: This paper presents an experimental study on shear performance of concrete I-beams with fiber reinforced polymers (FRP) as internal reinforcement. A total of four beam tests were conducted, including one test without shear reinforcement and three tests with glass fiber reinforced polymers (GFRP) stirrups. In all specimens, GFRP bars were used as flexural reinforcement. The test variable was the ratio of shear reinforcement. In the test without stirrups, diagonal tension failure occurred. Failure due to rupture of the GFRP stirrups rupture was observed in the test with a shear reinforcement ratio of $\rho_w = 0.75\%$ and web crushing failure occurred in the beam tests with $\rho_w = 1.26\%$ and $\rho_w = 2.26\%$ respectively. The experimentally obtained shear strengths were then compared to calculated design values using equations provided in the modified Eurocode 2, ACI 440.1R 06, and CSA S806- 2.

Keywords: FRP reinforced concrete I-beams; shear capacity; web crushing failure.

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INTRODUCTION

Typical shear failure modes that can occur in steel-reinforced concrete (RC) members are diagonal tension failure, shear compression failure, and failure by crushing of the concrete struts. Further failure modes upon testing a concrete beam with fiber-reinforced polymer as internal reinforcement are possible. The FRP shear elements can fail due to either a dowel action rupture or a concentration of stress at a corner in the reinforcement.

RESEARCH SIGNIFICANCE

In recent years, different papers describing shear tests on FRP-reinforced concrete members have been published. Most research has been undertaken to investigate the behavior of FRP-reinforced members without transverse reinforcement, however, also studies were conducted on shear behavior of concrete members with FRP stirrups¹. Due to the lower modulus of elasticity of the FRP-reinforcement compared to steel and the predominantly used rectangular cross section of the test specimens, in the most cases shear compression and diagonal tension failure occurred. Failure by crushing of the concrete struts has only rarely been observed. This Paper evaluates the shear performance of FRP RC I-beams focusing on the maximum shear capacity.

SHEAR DESIGN PROVISIONS

Three shear design provisions for RC members with FRP-reinforcement are summarized in Table 1, containing the modified Eurocode 2 ("Sheffield approach")², ACI 440.1R06³, and CSA-S806-2⁴. These shear design equations are based on the approach, that estimates the shear strength of a concrete member as a sum of the concrete contribution V_c and the contribution from shear reinforcement V_f . The design equations listed in Table 1 were used to calculate the design values of the shear resistance of the tested beams. Subsequently, the design values were compared to the experimental results.

Table 1 – Concrete properties and reinforcement details of test specimens

Reference	Shear design equations
modified Eurocode 2 ²	$V_{Rd,f} = V_{Rd,sf} + V_{Rd,cf}$ (1)
	$V_{Rd,cf} = \left[C_{Rd,c} k \left(100 \rho_{fl} \frac{E_f}{E_s} \phi_s f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] b_w d$ (2)
	$\rho_{fl} = \frac{A_{fl}}{b_w d} \leq 0.02$ (3); $\phi_s = \frac{\varepsilon_{fd,ef}}{\varepsilon_y}$ (4)
	$V_{Rd,sf} = \frac{A_{fw}}{s} \cdot z \cdot \varepsilon_{fd,ef} \cdot E_{fw}$ (5); $V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} / 2$ (6)
ACI 440.1R06 ³	$V = V_c + V_f$ (7)
	$V_c = \frac{2}{5} \sqrt{f'_c} \cdot b_w \cdot c$; where $c = kd$ (8)
	$V_f = \frac{A_{fv} \cdot f_{fv} \cdot d}{s} \leq 0.664 \cdot \sqrt{f'_c} \cdot b_w \cdot d$ (9)
	$f_{fv} = 0.004 \cdot E_f \leq f_{fb}$ (10)
CAN/CSA-S806-02 ⁴	$V_r = V_c + V_{sF} \leq V_c + 0.6 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$ (11)
	$V_c = \frac{130}{1000 + d} \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d \geq 0.08 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$ (12)
	$V_{sF} = \frac{0.4 \cdot \phi_F \cdot A_v \cdot f_{Fu} \cdot d}{s}$ (13)

where $V_{Rd,f}^2$ (V^3 , V_r^4) = shear resistance for members with shear reinforcement, $V_{Rd,cf}^2$ ($V_c^{3,4}$) = shear resistance of concrete, $V_{Rd,cf}^2$ (V_f^3 , V_{sF}^4) = shear resistance of shear reinforcement, $C_{Rd,c}^2$ = empirically derived factor = $0.18/\gamma_c$, γ_c^2 = safety factor for concrete = 1.5, $k^2 = 1 + \sqrt{\frac{200}{d}} \leq 2.0$ with d (effective depth of a cross-section) in mm,

ρ_{fl}^2 = flexural reinforcement ratio, $E_f^{2,3}$ = modulus of elasticity of flexural reinforcement, E_s^2 = modulus of elasticity of steel, ϕ_s^2 = ratio between the maximum strain allowed in the FRP reinforcement $\varepsilon_{fd,ef}$ and the yield strain of steel ε_y , f_{ck}^2 ($f_c^{3,4}$) = characteristic compressive cylinder strength of concrete, $k_1 \sigma_{cp}^2$ = axial load factors, $b_w^{2,3,4}$ = width of the web, A_{fl}^2 = cross-sectional area of flexural reinforcement, A_{fw}^2 (A_{fv}^3 , A_v^4) = cross-sectional area of shear reinforcement, $s^{2,3,4}$ = spacing of shear reinforcement, z^2 = lever arm of internal forces, E_{fw}^2 = modulus of elasticity of shear reinforcement, $V_{Rd,max}^2$ = design value of the maximum shear resistance, α_{cw}^2 = coefficient taking account of the state of the stress in the compression chord, v_1^2 = strength reduction factor for concrete cracked in shear, f_{cd}^2 = design value of concrete compressive strength, $k^3 = \sqrt{2 \rho_{fl} n_f + (\rho_{fl} n_f)^2} - \rho_{fl} n_f$, n_f^3 = modular ratio = E_f / E_c , E_c^3 = modulus of elasticity of concrete, f_{fv}^3 (f_{Fu}^4) = vertical stress in FRP stirrups, f_{fb}^3 = strength of bent portion of FRP bar, λ^4 = factor to account for concrete density, ϕ_c^4 = resistance factor for concrete, ϕ_F^4 = resistance factor for FRP reinforcement.

EXPERIMENTAL STUDY

Test specimens

Four shear tests on two concrete I-beams reinforced with FRP as flexural and shear reinforcement were conducted. These tests were part of an experimental program involving 24 shear tests. The beam specimens were designed to fail in shear and were tested in two successive test phases shown in Figure 1. In the first phase of testing (left diagram), two point loads were applied leading to rupture of the right-hand beam section with low shear reinforcement ratio or without shear reinforcement. In the second phase (right diagram), the left-hand section with a higher shear reinforcement ratio was tested in three-point bending. To prevent failure, the middle beam section was strongly reinforced with steel stirrups. Thus, a total of four tests were conducted (three with and one without stirrups).

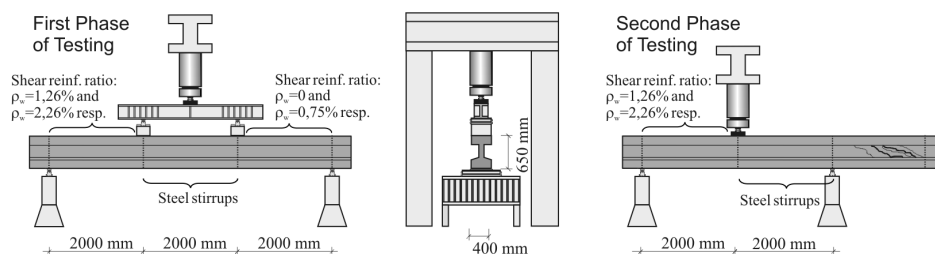


Figure 1 – Specimens and test set-up. (Note: 1 mm=0.0394 in.)

The beams had a total height of 650 mm (25.6 in.) and a web width of 100 mm (3.9 in.). To keep the longitudinal stiffness unchanged, all beams were provided with the same longitudinal reinforcement (six 32 mm [1.3 in.] GFRP bars). To ensure that a shear failure preceded a flexural failure, the flexural reinforcement ratio of 8.5 % was required. Figure 2 shows the geometry and reinforcement details of the test specimens. Since the tests were part of an extensive research project, the specimen designation can be explained as follows. The first letter, S, indicates the test (S for shear test), while the first number represents the beam number (1 for the first beam). The second letter indicates the reinforcement type (A for GFRP reinforcement type A). The third letter stands for the concrete strength (N for normal strength concrete), the second number indicates the shear reinforcement ratio, and the third number represents the serial number.

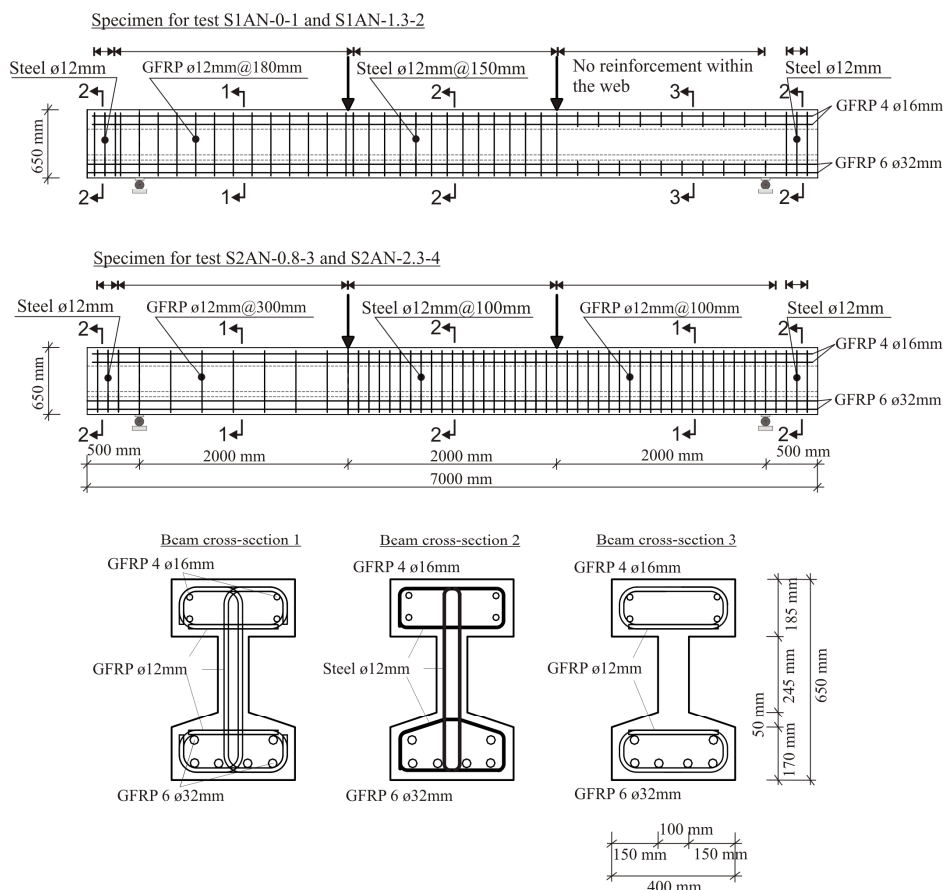


Figure 2 – Geometry and reinforcement details of test specimens. (1 mm=0.0394 in.)

The transverse reinforcement for the tests S1AN-1.3-2, S2AN-0.8-3, and S2AN-2.3-4 consisted of 12 mm (0.5 in.) GFRP stirrups. The part of the beam for the test S1AN-0-1 did not contain stirrups in the web. The stirrups' spacings were 100, 180, and 300 mm (3.9, 7.1, and 11.8 in.).

Material properties

The stirrups have a height of 600 mm (23.6 in.) and 135 mm (5.3 in.) respectively. The mandrel diameter D is 84 mm (3.3 in.) (7ϕ where ϕ is the bar diameter). This shear reinforcement that is composed of four parts can be seen in Figure 3. The GFRP bars and stirrups exhibit surface deformations to enhance bond performance between reinforcement and surrounding concrete.

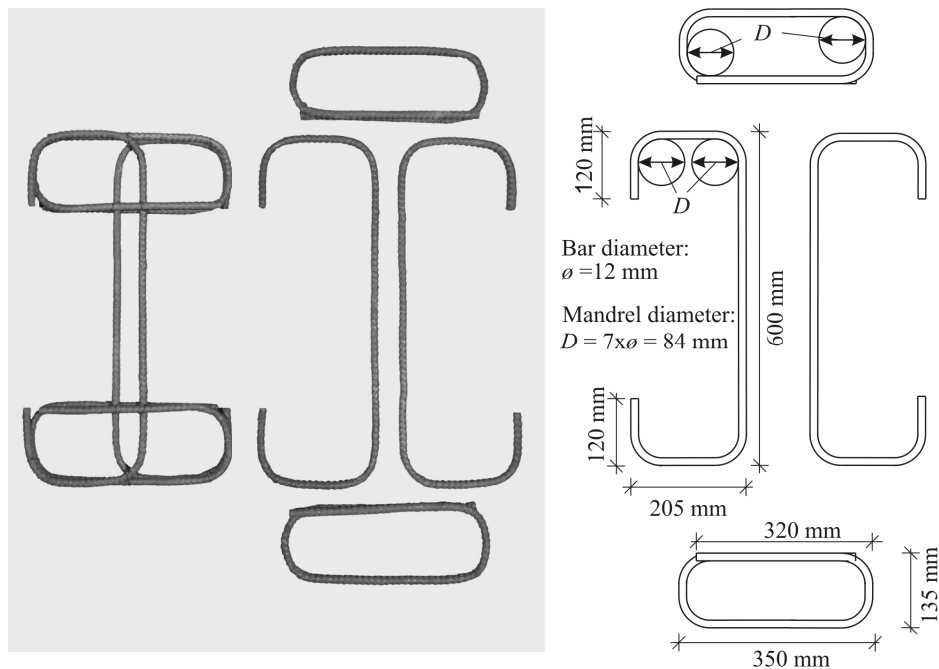


Figure 3 – Details of GFRP stirrups (1 mm=0.0394 in.)

To manufacture the beam specimens, a ready mixed concrete with a maximum aggregate size of 8 mm (0.32 in.) was used. The properties of the test specimens summarized in Table 2 include mean value of compressive strength (f_{cm}), mean value of splitting tensile strength ($f_{ctm,sp}$) and modulus of elasticity of the concrete (E_{cm}), diameter and modulus of elasticity of the flexural and shear reinforcement (ϕ_f , ϕ_s , E_{fm} , E_{fwm}), spacing of shear reinforcement (s) and shear reinforcement ratio (ρ_w).

Table 2 – Concrete properties and reinforcement details of test specimens

Test specimen	Concrete			Flexural reinforcement		Shear reinforcement			
	f_{cm} (Mpa)	$f_{ctm,sp}$ (Mpa)	E_{cm} (Mpa)	ϕ_f (mm)	E_{fm} (Mpa)	ϕ_s (mm)	s (mm)	Ratio $\rho_{fw} = A_{fw}/(b_w \cdot s)$ (%)	E_{fwm} (Mpa)
S1AN-0-1	34.4	2.87	26200	32	59000	-	-	-	-
S1AN-1.3-2	38.4	2.87	27700	32	59000	12	180	1.26	57000
S2AN-0.8-3	34.1	3.00	25600	32	59000	12	300	0.75	57000
S2AN-2.3-4	33.5	2.84	24700	32	59000	12	100	2.26	57000

Note: 1 mm = 0.0394 in.; 1Mpa = 0.145 ksi.

The mean values of the modulus of elasticity of the flexural and shear reinforcing elements provided in Table 2 were determined in the laboratory at the Institute of Structural Concrete (IMB) according to test method B.2 and B.5 respectively (see ACI440.3R-04⁵).

Instrumentation, test set up, and procedure

The strains in the flexural and shear reinforcement were measured using electrical resistance strain gauges. To measure the deflection of the beam, Linear Variable Differential Transducers (LVDTs) were used. The shear crack width was monitored during the test by a non-contact optical 3D deformation measuring system Aramis and by using crack gauges. The results of the crack width measurements are not presented in this paper. The formation and propagation of the cracks and corresponding loads were marked and recorded during the test. As described above, the beams were tested in four-point bending over a clear span of 6000 mm (236 in.) and then in three-point bending over a span of 4000 mm (158 in.). In each test, the point loads were located at a distance of 2000 mm (78.7 in.) from the support, which corresponds to a shear-depth ratio of 3.51. In the four-point bending tests the load was applied through a spreader beam. The load was applied at a rate of 5.0 kN/min (1.12 kips/min) up to about 60% of the failure load. Then, displacement-controlled rates of 0.5 up to 1.0 mm/min (0.019 up to 0.038 in./min) were used to avoid a sudden shear failure, which would complicate the result interpretation. In the four-point bending tests S1AN-0-1 and S2AN-0.8-3 after the major shear crack has developed, the specimens were released and additional LVDTs were applied for measuring the shear crack widths. During the tests, all gauges and LVDT readings were recorded using data acquisition systems.

Particular experimental results

The results obtained for all of the beams tested are summarized in Table 3 which contains the shear force at the onset of diagonal cracking (V_{crack}), the maximum imposed shear force (V_{max}), the inclination angles of the diagonal shear cracks (β_r), average stirrup strains at failure ($\epsilon_{fw,average}$), and mode of failure.

Table 3 – Summary of test results

Test specimen	V_{crack} kN (kips)	V_{max} kN (kips)	β_r degrees	$\epsilon_{fw,average}$ ‰	Failure mode*
S1AN-0-1	91 (20.5)	126 (28.4)	20-28	-	DT
S1AN-1.3-2	93 (20.9)	427 (96.1)	31-41	7,0	WC
S2AN-0.8-3	75 (16.9)	332 (74.7)	28-48	8,3	GR
S2AN-2.3-4	83 (18.7)	486 (109)	32-62	6,2	WC

* DT is diagonal tension failure, WC is web crushing failure, and GR is GFRP stirrup rupture

Shear-carrying mechanism in the test without shear reinforcement (S1AN-0-1) –At low load levels, only small vertical and slightly inclined flexural shear cracks appeared, so that shear transfer across cracks by mechanical interlock was ensured. As loading continued, a large diagonal cracks developed and at the same time the deflections increased suddenly. The load was then carried by the concrete compression zone, and by the longitudinal bars in terms of dowel action. When the load was further increased, arch action occurred, and the horizontal component was ensured by the tensile flexural reinforcement. Finally, diagonal tension failure occurred, when the shear crack extended through the compressive zone towards the load points at a shear force of 126 kN (28.4 kips) (Figure 4a).

Shear-carrying mechanism in test with lower shear reinforcement ratio (S2AN-0.8-3, $\rho_w = 0.75\%$) – The crack pattern of the tested FRP reinforced beam at the imposed maximum load corresponded to that of a steel-reinforced beam, so that the truss analogy can be assumed. Because the FRP stirrups do not yield, the shear reinforcement contributed to the shear resistance up to failure. In contrast to beams with rectangular cross-sections, the bend portions of the stirrups were sufficiently anchored in the flanges of the tested I-beams. Consequently, the stirrups did not fail due to a concentration of stress at a corner in the reinforcement. In fact, the specimen failed due to rupture of the GFRP stirrups at the straight portion at a shear force equal to 332 kN (74.7 kips), however, the failure did not occur suddenly (Figure 4c).

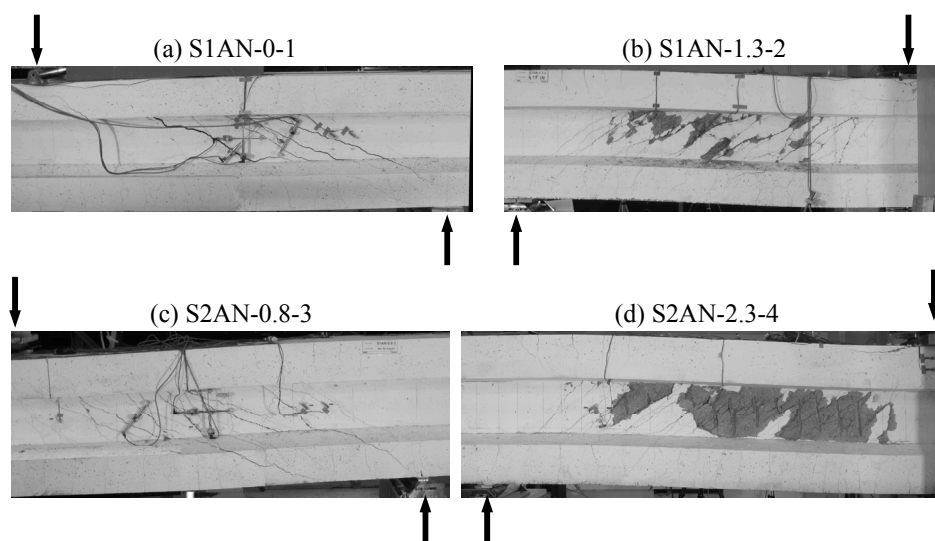


Figure 4 – Failure of test specimens:
(a) S1AN-0-1, (b) S1AN-1.3-2, (c) S2AN-0.8-3, (d) S2AN-2.3-4

Shear-carrying mechanism in test with higher shear reinforcement ratios ($\rho_w = 1.26\%$ and $\rho_w = 2.26\%$) – In the tests S1AN-1.3-2 and S2AN-2.3-4 the crack pattern immediately before failure was comparable to that observed in test S1AN-0.8-3. However, the specimens with higher shear reinforcement ratios exhibited a more distributed crack pattern. The failure occurred suddenly and was due to crushing of the concrete struts within the web at a applied shear force of 427 kN (96.1 kips) and 486 kN (109 kips) respectively.

Cracking pattern at failure – The final crack patterns can be seen in Figure 5. Because of the distributed crack pattern, the major shear crack was not identified and a range of the inclination angles of the shear cracks are highlighted. The smaller the stirrup spacing, the greater the number of induced cracks.

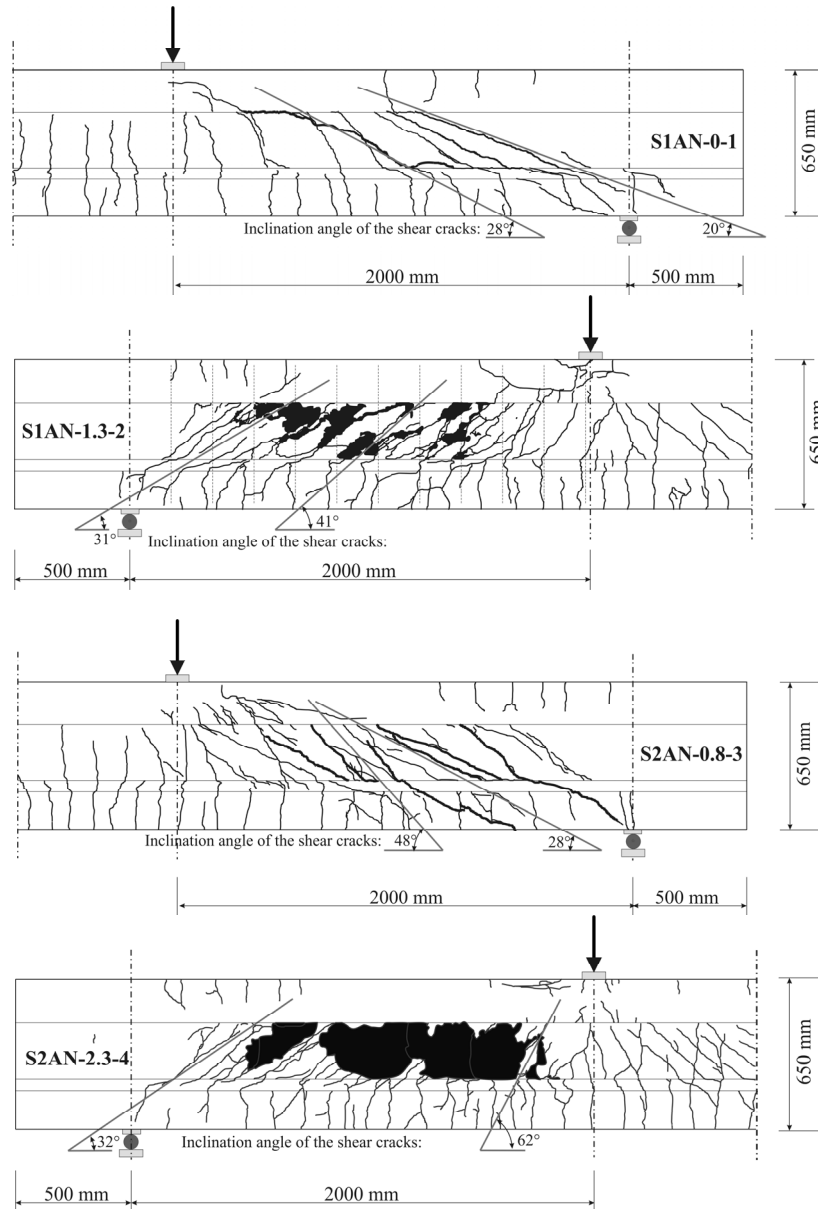


Figure 5 – Cracking pattern at failure for test specimen (1 mm = 0.0394 in.)

Deflection – Figure 6 shows the applied shear force-deflection relationship for the tested beams at mid-span. Due to the different test setups, the graphs were separated. The left-hand diagram depicts the curves measured in the four-point bending tests, whereas the results of the three-point bending tests are shown in the right-hand diagram.

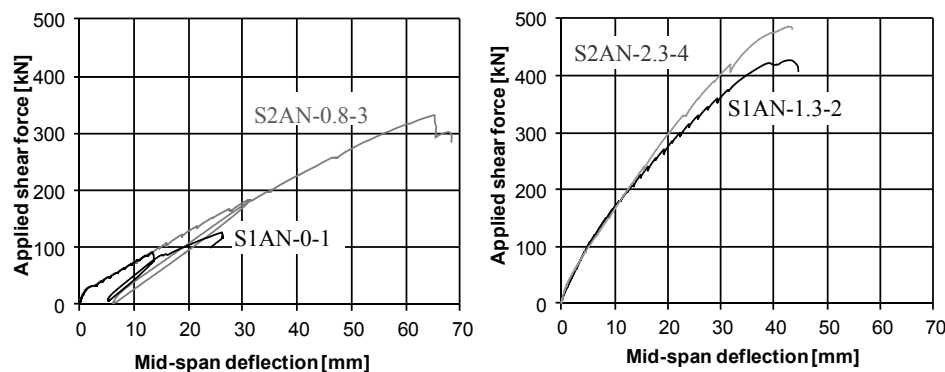


Figure 6 – Applied shear force-deflection relationship for test specimens (1 mm = 0.0394 in.; 1 kN = 0.225 kips)

In the four-point bending tests S1AN-0-1 and S2AN-0.8-3 the bending stiffness was similar up to the load level at which shear cracks appeared. Thereafter, in the test S1AN-0-1, the web failed in shear, and arch action occurred. At the same time the flexural stiffness decreased. However, in test S2AN-0.8-3 the included shear reinforcement resulted in a shear force-deflection curve, that was almost linear up to failure. The reinforcement ratio of 0.75% increased the shear resistance by about 160%. In the three-point bending tests S1AN-1.3-2 and S2AN-2.3-4, both specimens exhibited similar flexural behavior up to about 50% of the shear resistance. Then, the beam with higher shear reinforcement ratio ($\rho_w = 2.26\%$) showed a higher stiffness. Both beams failed due to crushing of the concrete struts, however, the shear resistance of S2AN-2.3-4 was about 15% higher.

Flexural strains – Figure 7 depicts the relationship between the applied shear force and the flexural strains (flexural reinforcement and concrete) at mid span corresponding to Figure 6 in two diagrams. In test S1AN-1.3-2 the concrete strain gauge failed. In the other tests the maximum compressive strains were less than 0.0035, so that failure of concrete in compression did not occur. Furthermore, the longitudinal reinforcement did not rupture, even though strains about 0.007 were measured. Consequently, the specimens failed in shear prior to reaching their flexural capacity.

Strains in GFRP stirrups – The measured maximum and the average stirrup strains at failure are listed in Table 3. For calculating the average strains, all stirrup gauges were considered. In the tests S2AN-0.8-3 and S2AN-2.3-4 a total of three stirrup strain gauges failed shortly before the shear capacity was reached (Figures 8 and 9). The maximum stirrup strain in the straight portion measured at failure was 14.8, 12.9, and 11.0 ‰ for S1AN-1.3-2, S2AN-0.8-3, and S2AN-2.3-4 respectively. Strains at the bend portion of

the stirrups were not measured, because this part of the shear reinforcement was embedded in 230 mm high flanges. Thus, low strains were expected at the bend portion.

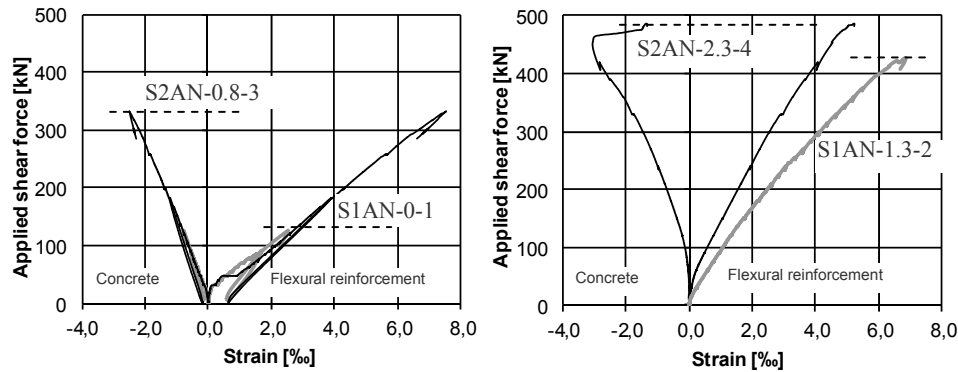


Figure 7 – Applied shear force-flexural strains relationship (1kN = 0.225 kips)

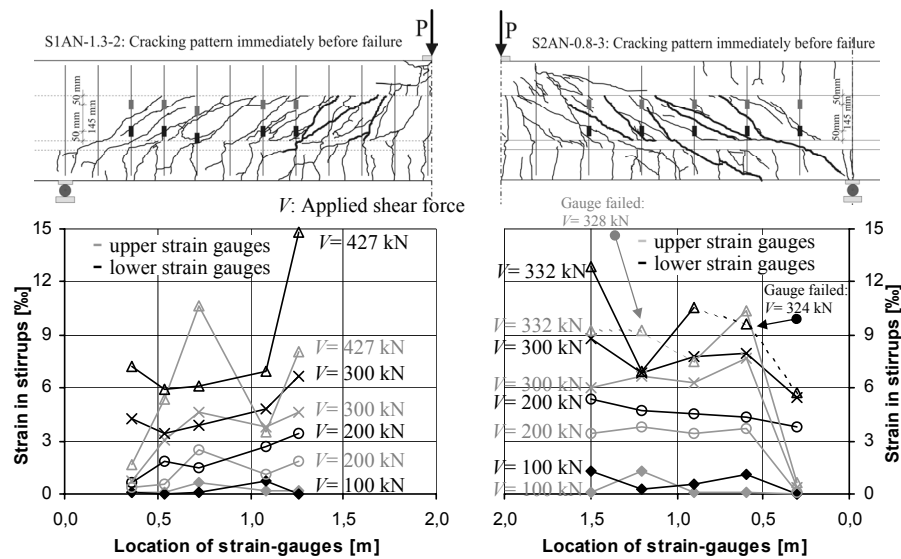


Figure 8 – Stirrup strain distribution along shear span:
S1AN-1.3-2 (left) and S2AN-0.8-3 (right) (1kN = 0.225 kips; 1 m = 39.4 in.)

The average strain measured at failure was 7.0, 8.3, and 6.2 ‰ for S1AN-1.3-2, S2AN-0.8-3, and S2AN-2.3-4 respectively. The higher the shear reinforcement ratio, the lower the average strain. However, influence of the shear reinforcement ratio on the failure mode, in particular in the tests with higher ratios from about 1.3%, has not been identified. Although, in the tests S1AN-1.3-2 and S2AN-2.3-4 different average stirrup strains were observed, in both cases failure due to crushing of the concrete struts occurred.

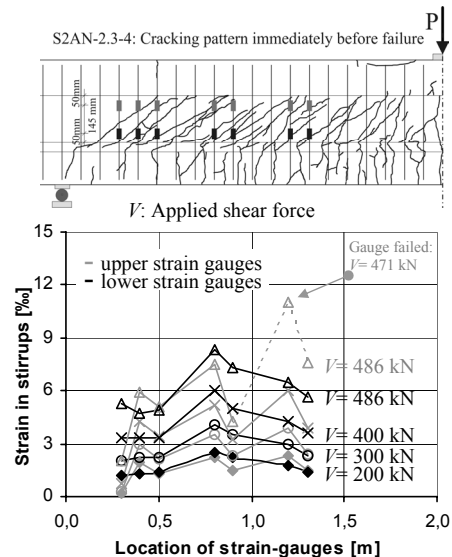


Figure 9 – Stirrup strain distribution along shear span in test S2AN-2.3-4 (1 m = 39.4 in.)

COMPARISON OF DESIGN SHEAR RESISTANCE AND EXPERIMENTAL RESULTS

The shear design equations listed in Table 1 were used to calculate the design shear resistance of the tested beams. Both design values were thereby determined: the shear resistance including the contributions of concrete and stirrups respectively ($V_{pred,1}$) and the maximum shear resistance characterized by the concrete strength ($V_{pred,2}$). Table 4 presents the ratios of the measured shear capacity to the calculated value (V_{exp}/V_{pred}). The relevant values are indicated in bold print. For S1AN-0-1 the shear resistance at onset of diagonal cracking was assumed as the shear capacity, because at this load level the web failed and the additional applied loads were carried by the flanges of the I-beam.

Table 4 – Predicted design shear strength of test specimen

Test	V_{exp} kN (kips)	modified EC 2 ² $V_{exp}/V_{pred,1}$	modified EC 2 ² $V_{exp}/V_{pred,2}$	ACI ³ $V_{exp}/V_{pred,1}$	ACI ³ $V_{exp}/V_{pred,2}$	CSA ⁴ $V_{exp}/V_{pred,1}$	CSA ⁴ $V_{exp}/V_{pred,2}$
S1AN-0-1	91 (20.5)	2.23	-	1.49	-	5.48	-
S1AN-1.3-2	427 (96.1)	2.04	1.60	1.87	1.77	1.84	2.88
S2AN-0.8-3	332 (74.7)	2.37	1.44	2.09	1.50	2.28	2.43
S2AN-2.3-4	486 (109)	1.44	2.14	1.37	2.22	1.21	3.60

Regarding the test without shear reinforcement, the modified Eurocode 2² shear provision slightly underestimates the shear resistance. On contrary ACI440.1R06³ provides an insufficient level of safety. The partial safety factor for material properties should be about 1.8. CAN/CSA-S806-02⁴ greatly underestimates the shear strength of this test specimen. Considering the tests with shear reinforcement all considered guidelines underestimate differently the shear resistance. The corresponding ratio varies between 1.87 up to 3.6. It is particularly noticeable that neither the modified Eurocode 2² nor ACI440.1R06³ classifies properly the failure mode observed in the test with the reinforcement ratio of $\rho_w = 1.26\%$ (S1AN-1.3-2). Although failure due to crushing of the concrete strut occurred in this test, in both guidelines, the shear resistance is governed by the stirrups's strength. However, the third considered guideline CAN/CSA-S806-02⁴ assumes that in all tests with shear reinforcement web crushing failure occurs.

SUMMARY

Four shear tests on two concrete I-beams reinforced with FRP were carried out. In the test with FRP shear reinforcement, diagonal tension failure occurred, when the shear crack extended through the compressive zone towards the load points at a shear force of 126 kN (28.4 kips). In test with a lower shear reinforcement ratio ($\rho_w = 0.75\%$), the specimen failed due to rupture of the GFRP stirrups at the straight portion at a shear force equal to 332 kN (74.7 kips), however, the failure did not occur suddenly. The specimens with higher shear reinforcement ratios ($\rho_w = 1.26\%$, and $\rho_w = 2.26\%$) failed suddenly due to crushing of the concrete struts within the web at a applied shear force of 427 kN (96.1 kips) and 486 kN (109 kips) respectively. Although in both tests web crushing failure occurred, the shear resistance of the specimen with higher reinforcement ratio ($\rho_w = 2.26\%$) was increased by about 15% compared to the beam with the reinforcement ratio of $\rho_w = 1.26\%$. Concerning the comparison between the experimentally obtained shear strength and the predicted design strength, it is particularly noticeable that neither the modified Eurocode 2² nor ACI440.1R06³ classifies properly the failure mode observed in the test with the reinforcement ratio of $\rho_w = 1.26\%$ (S1AN-1.3-2). In both guidelines, the shear resistance is governed by the stirrups's strength.

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DESIGNING FRP REINFORCED CONCRETE FOR DEFLECTION CONTROL

Stuart Veysey and Peter H. Bischoff

Synopsis: This paper investigates the design of fiber reinforced polymer (FRP) reinforced concrete based on ACI 440.1R serviceability requirements related to deflection control of one-way slabs and rectangular beams, and uses this information as the basis for evaluating the minimum member thickness requirements needed to satisfy ACI 318 deflection limits. Serviceability is shown to govern design in most cases, as flexural members designed for deflection control are usually stronger than required. Slabs satisfying deflection requirements have a service load that ranges from 20 to 30% of the nominal member capacity, while service loads for beams range from 35 to 45% of the member capacity. Recommended minimum member thickness values for slabs are too conservative and require revision, while those for beams appear reasonable. A practical approach for design of FRP reinforced concrete members is proposed based on selection of member thicknesses needed to satisfy deflection and strength criteria.

Keywords: beams; deflection; effective moment of inertia; fiber reinforced polymers (FRP); reinforced concrete; serviceability; service loads; strength

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INTRODUCTION

The low stiffness of a fiber reinforced polymer (FRP) bar relative to its ultimate strength can lead to a reinforced concrete member where either the deflection limit or allowable crack width is exceeded before the flexural capacity of the member is reached. Hence, serviceability should often control design when concrete is reinforced with FRP bars (particularly for an over-reinforced member). Contrary to this expectation, the ACI 440.1R-06 Guide for the Design and Construction of Structural Concrete Reinforced with FRP Bars¹ states that serviceability criteria for deflection and crack width are generally satisfied when an FRP-reinforced concrete member is designed for a concrete crushing failure. Note however, that a recent revision to the 440.1R document (successfully balloted at the ACI Spring 2010 Convention in Chicago) now indicates that design of an FRP reinforced cross-section for flexural strength may not satisfy serviceability criteria.

There is also little consensus about the level of service load (relative to the nominal flexural capacity) that can be expected for an FRP reinforced concrete member. Past researchers have assumed that the service load moment M_a can vary anywhere from 70% of the nominal moment capacity M_n (which is fairly typical for a steel reinforced concrete member when strength governs design) down to a low of about 20% of M_n ^{2,3}. At low values of M_a ($0.2M_n$) the service load is usually close to the cracking moment M_{cr} , where tension stiffening from the concrete plays an important role in contributing to the member stiffness, while at high levels of service load ($M_a = 0.7M_n$) tension stiffening is practically nonexistent and the member response follows a more flexible cracked response controlled only by the amount and stiffness of reinforcement used. Hence, the assumed level of service load can significantly impact the calibration and validation of proposed procedures for computing deflection⁴.

This paper investigates the level of service load appropriate for an FRP reinforced concrete member when deflection governs design, and proposes a practical approach that can be used as the basis for designing flexural members that satisfy deflection requirements and have sufficient capacity to satisfy strength requirements. Analysis is limited to deflection of one-way slabs and beams, where the design requirements for strength and deflection are carried out in accordance with ACI 440.1R-06¹ (with the exception of recent revisions to deflection provisions that are outlined further on in the

paper). Creep rupture stress limits under sustained load are also taken into account when assessing serviceability. Requirements for crack control are not considered in this paper.

RESEARCH SIGNIFICANCE

Design of FRP reinforced concrete flexural members is often controlled by serviceability requirements related to deflection. This paper identifies the level of service load (in relation to the cracking load and nominal flexural capacity of the member) corresponding to both deflection and strength criteria being satisfied, and presents a practical approach for design of FRP reinforced concrete flexural members when deflection governs design. Results from this paper may influence future revisions to the ACI 440.1R¹ document that include provisions for minimum member thicknesses recommended for design.

FLEXURAL STRENGTH REQUIREMENTS

ACI 440¹ allows flexural members reinforced with FRP bars to be either under-reinforced and fail by sudden rupture of the reinforcement or over-reinforced such that failure is governed by crushing of the concrete. Over-reinforced members are generally preferred and can provide the flexure member with a marginal amount of ductility before failure¹. Design for flexural strength is carried out such that the design or factored flexural strength ϕM_n equals or exceeds the factored moment M_u , where

$$\phi M_n \geq M_u = \alpha_D M_D + \alpha_L M_L = \alpha_{D+L} M_a \quad [1]$$

The nominal flexural capacity M_n is reduced by a strength reduction factor ϕ , while the dead and live load service moments (M_D and M_L) are multiplied by their respective dead and live load factors ($\alpha_D = 1.2$ and $\alpha_L = 1.6$). The full (dead plus live) service load moment M_a at the critical section is thus increased by using an averaged load factor $\alpha_{D+L} = [1.2(D : L) + 1.6] / [(D : L) + 1]$ which depends on the ratio ($D:L$) of dead to live load.

The strength reduction factor ϕ is assumed to account for the lack of ductility in an FRP reinforced concrete section, and depends on whether the member is under- or over-reinforced.

$$\phi = 0.55 \quad \text{for } \rho_f \leq \rho_{fb} \quad [2a]$$

$$\phi = 0.3 + 0.25 \rho_f / \rho_{fb} \quad \text{for } \rho_{fb} < \rho_f < 1.4 \rho_{fb} \quad [2b]$$

$$\phi = 0.65 \quad \text{for } \rho_f \geq 1.4 \rho_{fb} \quad [2c]$$

The FRP reinforcement ratio is given by $\rho_f = A_f / bd$ and the balanced reinforcing ratio $\rho_{fb} = 0.85 \beta_1 (f'_c / f_{fu}) (E_f \epsilon_{cu}) / (E_f \epsilon_{cu} + f_{fu})$ with an ultimate concrete strain $\epsilon_{cu} = 3000 \mu\epsilon$.

The factor $\beta_1 = 0.85 - 0.05(f'_c - 27.6)/6.9 \geq 0.65$ for a specified compressive strength $f'_c \geq 27.6$ MPa and $\beta_1 = 0.85 - 0.05(f'_c - 4000)/1000 \geq 0.65$ for $f'_c \geq 4000$ psi.

The nominal flexural capacity $M_n = R_n b d^2$. For an over-reinforced member ($\rho_f > \rho_{fb}$)

$$R_n = \rho_f f_f [1 - \rho_f f_f / (1.7 f'_c)] \quad [3a]$$

$$f_f = 0.5 E_f \varepsilon_{cu} [\sqrt{1 + 4(0.85)\beta_1 f'_c / (\rho_f E_f \varepsilon_{cu})} - 1] \leq f_{fu} \quad [3b]$$

The ACI rectangular stress block factors ($\alpha_1 = 0.85$ and β_1) do not apply for an under-reinforced member ($\rho_f < \rho_{fb}$). While $f_f = f_{fu}$ when $\rho_f < \rho_{fb}$, Eq. (3a) needs to be modified to account for the change to the stress block factors α_1 and β_1 . In this case, equivalent stress block factors based on an assumed response for the concrete in compression can be used to determine the nominal flexural strength^{5,6}. These stress block factors depend on the compressive strain in the concrete at failure ($\varepsilon_c < \varepsilon_{cu} = 3000 \mu\varepsilon$).

Substituting the nominal moment capacity $M_n = R_n b d^2$ into Eq. (1) to give $M_a \leq (\phi / \alpha_{D+L}) R_n b d^2$ leads to a general expression for the span-to-depth L/h ratio that satisfies strength requirements⁷. Equation (4a) applies to a flexural member (beam or slab) of width b under a uniformly distributed load w , but is most suitable for a slab strip of unit width (where w/b is the distributed load per unit area acting on the slab).

$$\frac{L}{h} \leq \sqrt{\frac{8(\phi / \alpha_{D+L}) R_n (d/h)^2}{(w/b)(M_a / M_o)}} \quad [4a]$$

This expression is easily rearranged to give the L/h ratio for beams with a given d/b aspect ratio and equivalent w/L loading (defined as the distributed load per unit length of beam divided by the span length).

$$\frac{L}{h} \leq \sqrt[3]{\frac{8(\phi / \alpha_{D+L}) R_n (d/h)^3}{(w/L)(d/b)(M_a / M_o)}} \quad [4b]$$

$M_o = wL^2 / 8$ is the total static moment for a uniformly distributed load, while M_a is the service load moment at the critical section (taken at the support for a cantilever and at midspan for simply supported and continuous members). The M_a / M_o ratio obviously depends on the support conditions and type of loading (Table 1). The averaged load factor α_{D+L} depends on the dead to live load ratio (D:L) as explained earlier. Equations (4a) and (4b) indicate that the L/h ratio of slabs depends on the magnitude of load, while L/h for beams depends on the load and member span as well as the beam aspect ratio. End support conditions (through M_a / M_o) and the d/h ratio also have an effect on L/h .

Table 1–Restraint factor K and ACI 318 design moments for a uniformly distributed load

Support Conditions	K^*	M_a	M_a/M_o^{**}
Simply supported	1.0	$wL^2/8$	1.0
One end continuous	0.85	$wL^2/14$	0.5714
Two ends continuous	0.8	$wL^2/16$	0.5
Cantilever	2.4	$wL^2/2$	4.0

* $K = 1.2 - 0.2M_o/M_a$ except for cantilevers

** $M_o = wL^2/8$

Table 2–Design Properties of Reinforcing Bars and Concrete Material Properties

	GFRP	CFRP	Steel
Design strength f_{fu} : MPa (ksi)	551.6 (80)	2068.5 (300)	413.7 (60)
Elastic modulus E_f : GPa (ksi)	41.4 (6000)	137.9 (20,000)	200 (29,000)

$f'_c = 27.6$ MPa (4000 psi); $E_c = 4730\sqrt{f'_c} = 24.85$ GPa (57,000 $\sqrt{f'_c} = 3605$ ksi);

$f_r = 0.62\sqrt{f'_c} = 3.3$ MPa (7.5 $\sqrt{f'_c} = 475$ psi)

Figures 1 and 2 provide plots of the span-to-depth (L/h) ratios needed to satisfy flexural strength requirements for one-way slabs and beams. Figure 1 shows results for glass FRP (GFRP) reinforced concrete and Fig. 2 shows results for carbon FRP (CFRP) reinforced concrete. The results effectively represent L/h as a function of the reinforcing ratio ρ_f needed to satisfy strength at the critical section, and are plotted in both the under- and over-reinforced regions for a simply supported member with a d/h ratio of 0.85. The concrete has an assumed compressive strength of 27.6 MPa (4000 psi) and properties of the FRP bars are summarized in Table 2. Strength curves in Figs. 1 and 2 are presented for a lightly loaded slab (S1) with a w/b loading of 4.8 kPa (100 lb/ft²), a moderately loaded slab (S2) with a 9.6 kPa (200 lb/ft²) loading, and a heavily loaded slab (S3) with a 24 kPa (500 lb/ft²) loading. Two strength curves are shown for beams; one for a lightly loaded beam (B1) with an aspect ratio d/b of 1.0 and equivalent loading w/L of 3.6 kPa (75 lb/ft²), and the other for a more heavily loaded beam (B2) with $d/b = 2.0$ and $w/L = 14.4$ kPa (300 lb/ft²). The dead to live load ratio is set equal to 2.0 for all cases shown.

Any point along a strength curve will satisfy strength requirements exactly, with more reinforcement being required as the L/h ratio increases (corresponding to a thinner member for a fixed length of span). Observe the difference in behavior between a slab and beam, with a slab having a higher L/h limit because of the lower aspect ratio for a wide and shallow member. Beams with a lower d/b aspect ratio also have a higher L/h limit. Notice also how the minimum member thickness is affected by the level of loading, with L/h decreasing to give a thicker member for a heavier load as expected. The objective now is to determine the point along a strength curve where deflection is being satisfied.

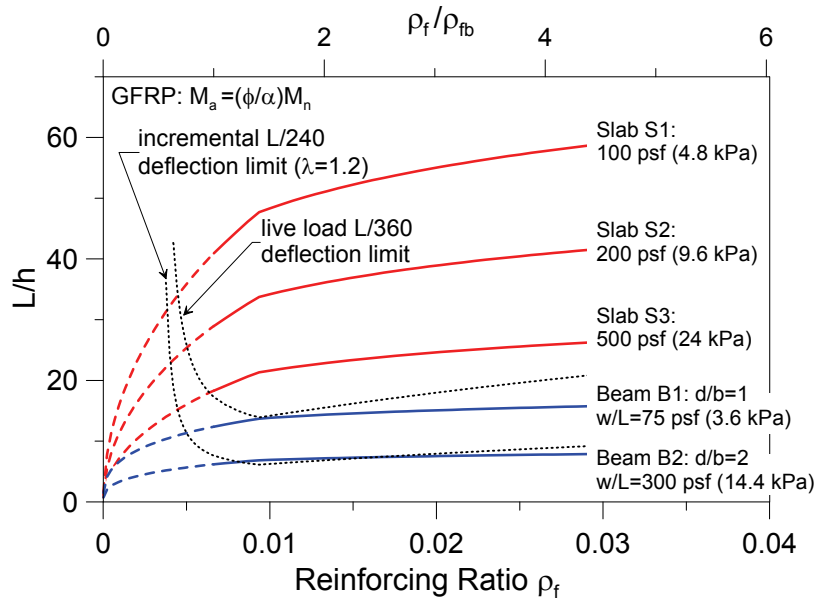


Fig. 1— L/h ratios controlled by strength and deflection for GFRP reinforced concrete members (simply supported case with $D:L = 2$ and $\lambda=1.2$).

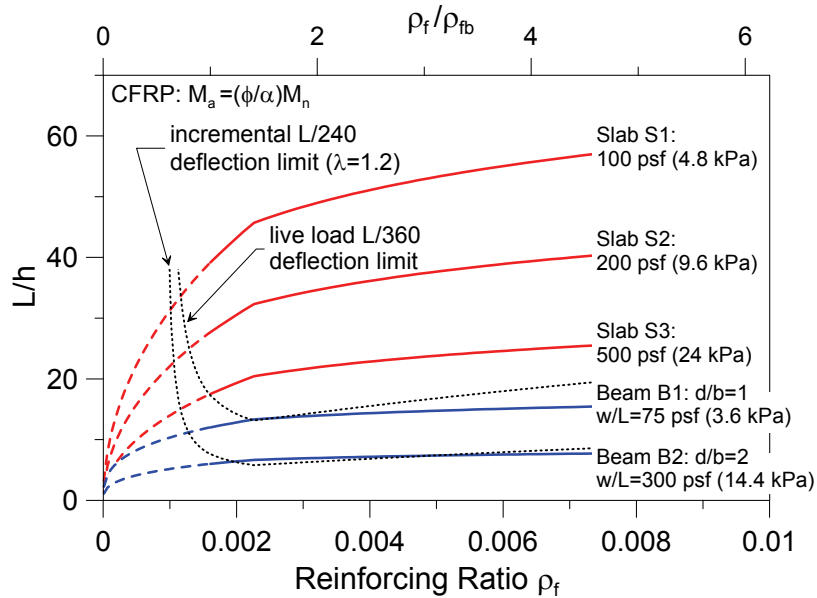


Fig. 2— L/h ratios controlled by strength and deflection for CFRP reinforced concrete members (simply supported case with $D:L = 2$ and $\lambda=1.2$).

DEFLECTION CONTROL REQUIREMENTS

Recommended minimum thicknesses for FRP reinforced concrete members are provided by ACI 440.1R¹ as a guideline only for initial design (Table 3), unlike ACI 318⁸ where the use of minimum thickness requirements can preclude the need for computing deflection. Deflection of FRP reinforced concrete still needs to be computed and compared with allowable limits that depend on the intended use of the structure. Only those criteria prescribed by ACI 318⁸ are considered in this paper, and depend on whether the attached partitions or nonstructural elements are susceptible to damage from excessive deflection (Table 4).

Table 3– ACI 440¹ member thickness recommendations for FRP reinforced concrete and ACI 318⁸ minimum member thickness requirements for steel reinforced concrete.

		Minimum member thickness h			
		Simply supported	One end continuous	Both ends continuous	Cantilever
FRP	One-way slabs	L/13	L/17	L/22	L/5.5
	Beams	L/10	L/12	L/16	L/4
Steel	One-way slabs	L/20	L/24	L/28	L/10
	Beams	L/16	L/18	L/21	L/8

Table 4–ACI 318⁸ deflection limits.

		Deflection Limit
Immediate deflection from live loads	Roofs	L/180
	Floors	L/360
Deflection occurring after attachment of non-structural elements (incremental deflection)	Damage not likely from large deflections	L/240
	Damage likely from large deflections	L/480

ACI 318 deflection limits are based on the immediate deflection due to live load (L/360 and L/180 for floors and roofs, respectively, that are not supporting or attached to nonstructural elements likely to be damaged by large deflections) and the incremental or total deflection occurring after attachment of the nonstructural elements (L/240 when damage is not of concern and L/480 when damage is likely to occur from large deflections). The incremental deflection is comprised of the long-term deflections from sustained loads that occur after installation of the nonstructural elements plus the immediate deflection from any remaining live load. The incremental deflection limit usually governs.

Immediate deflection Δ_i is computed using a generalized elastic deflection equation that depends on the support conditions and type of loading (characterized by a restraint factor K), service load moment M_a at the critical section, member span L , elastic modulus of concrete $E_c = 4730\sqrt{f'_c}$ MPa (57,000 $\sqrt{f'_c}$ psi) and effective moment of inertia I_e .

$$\Delta_i = K \frac{5}{48} \frac{M_a L^2}{E_c I_e} \quad [5]$$

Table 1 provides values of the restraint factor K for different member types subjected to a uniformly distributed load w . Only simply supported members are evaluated in this paper. Nonlinearity of the member stiffness after cracking is taken into account with an effective moment of inertia I_e that models the transition from the gross (uncracked) moment of inertia I_g to the cracked transformed moment of inertia I_{cr} . The expression for I_e used in this paper is based on recent revisions to ACI 440.1R (ACI 440 Web Ballot March 2010) and includes a factor $\gamma = 1.72 - 0.72 M_{cr} / M_a$ to account for the change in stiffness along the length of a member subjected to a uniformly distributed load. The effect of preloading from construction loads is taken into account by using the value of I_e corresponding to the full dead plus live load for all loading conditions considered⁷.

$$I_e = \frac{I_{cr}}{1 - \gamma (M_{cr} / M_a)^2 (1 - I_{cr} / I_g)} \leq I_g \quad [6]$$

For a rectangular cross-section:

$$I_{cr} / bd^3 = k_{cr}^3 / 3 + n_f \rho_f (1 - k_{cr})^2$$

$$\text{with } k_{cr} = \sqrt{(n_f \rho_f)^2 + 2(n_f \rho_f) - n_f \rho_f} \text{ and } n_f = E_f / E_c$$

$$I_g / bh^3 = (1/12) \text{ and } I_{cr} / I_g = 12[k_{cr}^3 / 3 + n_f \rho_f (1 - k_{cr})^2] (d/h)^3$$

$$M_{a,D+L} / M_{cr} = 6(\phi / \alpha_{D+L}) R_n (d/h)^2 / f_r \text{ with } f_r = 0.62 \sqrt{f'_c} \text{ MPa } (7.5 \sqrt{f'_c} \text{ psi})$$

$$I_{e,D+L} / I_g = \frac{I_{cr} / I_g}{1 - \gamma (M_{cr} / M_{a,D+L})^2 (1 - I_{cr} / I_g)} \text{ with } \gamma = 1.72 - 0.72 \left(\frac{M_{cr}}{M_{a,D+L}} \right)$$

Long term or time-dependent deflection Δ_{lt} from the sustained loads is computed by multiplying the immediate deflection value from the sustained loads $\Delta_{i,sus}$ by a long term multiplier λ . This gives $\Delta_{lt} = \lambda \Delta_{i,sus}$. Assuming only the dead load is sustained, the incremental deflection Δ_{incr} is then computed using

$$\Delta_{incr} = \lambda \Delta_{i,D} + \Delta_{i,L} \leq L/240 \text{ or } L/480 \quad [7]$$

$$\text{with } \Delta_{i,D} = K(5/48) M_{a,D} L^2 / (E_c I_{e,D+L}) \text{ and } \Delta_{i,L} = K(5/48) M_{a,L} L^2 / (E_c I_{e,D+L}).$$

Additional long term deflections from sustained live loads can be easily accommodated but are not considered in this paper. The long term deflection multiplier $\lambda \leq 1.2$ as ACI 440.1R-06¹ recommends using only 60% of the time-dependent factors prescribed by ACI 318⁸ for steel reinforced concrete (where $\lambda \leq 2.0$).